

# **Implementation of Lexical Analysis**

CS143

Lecture 4

# Tips on Building Large Systems

---

- KISS (Keep It Simple, Stupid!)
- Don't optimize prematurely
- Design systems that can be tested
- It is easier to modify a working system than to get a system working

# Outline

---

- Specifying lexical structure using regular expressions
- Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
- Implementation of regular expressions

# Notation

---

- There is variation in regular expression notation
  - At least one A:  $A^+$   $\equiv AA^*$
  - Union:  $A + B$   $\equiv A | B$
  - Option:  $A + \varepsilon$   $\equiv A?$
  - Range:  $'a'+ 'b'+ \dots + 'z'$   $\equiv [a-z]$
  - Excluded range:  
complement of  $[a-z]$   $\equiv [^a-z]$

# Lexical Specification → Regex in five steps

---

## 1. Write a regex for each token

- Number = `digit +`
- Keyword = `'if' + 'else' + ...`
- Identifier = `letter (letter + digit)*`
- OpenPar = `'('`
- ...

# Lexical Specification → Regex in five steps

---

2. Construct  $R$ , matching all lexemes for all tokens

$$\begin{aligned} R &= \text{Keyword} + \text{Identifier} + \text{Number} + \dots \\ &= R_1 + R_2 + \dots \end{aligned}$$

(This step is done automatically by tools like flex)

# Lexical Specification $\rightarrow$ Regex in five steps

---

3. Let input be  $x_1 \dots x_n$

For  $1 \leq i \leq n$  check

$$x_1 \dots x_i \in L(R)$$

4. If success, then we know that

$$x_1 \dots x_i \in L(R_j) \text{ for some } j$$

5. Take  $x_1 \dots x_i$  as token, and go to (3) for next token.

# Ambiguity 1

---

- There are ambiguities in the algorithm
- How much input is used? What if
  - $x_1 \dots x_i \in L(R)$  and also
  - $x_1 \dots x_K \in L(R)$
- Rule: Pick longest possible string in  $L(R)$ 
  - Pick  $k$  if  $k > i$
  - The “maximal munch”



# Ambiguity 2

---

- Which token is used? What if
  - $x_1 \dots x_i \in L(R_j)$  and also
  - $x_1 \dots x_i \in L(R_k)$
- Rule: use rule listed first
  - Pick  $j$  if  $j < k$
  - E.g., treat “if” as a keyword, not an identifier

# Error Handling

---

- What if  
    No rule matches a prefix of input ?
- Problem: Can't just get stuck ...
- Solution:

# Error Handling

---

- What if  
    No rule matches a prefix of input ?
- Problem: Can't just get stuck ...
- Solution:
  - Write a rule or regex for matching all “bad” strings
  - Put it last (lowest priority)

# Summary

---

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve
  - To handle errors
- Good algorithms known
  - Require only single pass over the input
  - Few operations per character (table lookup)

# Summary

---

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities  
**sol: [matches longest possible and highest priority]**
  - To handle errors
- Good algorithms known
  - Require only single pass over the input
  - Few operations per character (table lookup)

# Summary

---

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities  
**sol: [matches longest possible and highest priority]**
  - To handle errors  
**sol: [define a regex for all erroneous or bad string]**

# Finite Automata

---

- Regular expressions = specification
- Finite automata = implementation
- A finite automaton consists of
  - An input alphabet  $\Sigma$
  - A finite set of states  $S$
  - A start state  $n$
  - A set of accepting(final) states  $F \subseteq S$
  - A set of transitions  $\text{state} \xrightarrow{\text{input}} \text{state}$

# Finite Automata

---

- Transition

$$S_1 \xrightarrow{a} S_2$$



# Finite Automata

---

- Transition

$$s_1 \xrightarrow{a} s_2$$

- Is read

In state  $s_1$  on input “a” go to state  $s_2$

# Finite Automata

---

- Transition

$$s_1 \xrightarrow{a} s_2$$

- Is read

In state  $s_1$  on input “a” go to state  $s_2$

- If end of input and in accepting state  $\Rightarrow$  accept

# Finite Automata

---

- Transition

$$s_1 \xrightarrow{a} s_2$$

- Is read

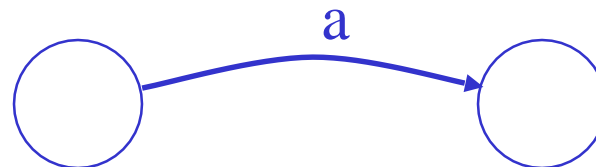
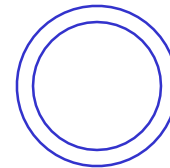
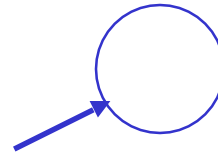
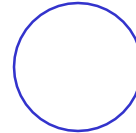
In state  $s_1$  on input “a” go to state  $s_2$

- If end of input and in accepting state  $\Rightarrow$  accept
- Otherwise  $\Rightarrow$  reject

# Finite Automata State Graphs

---

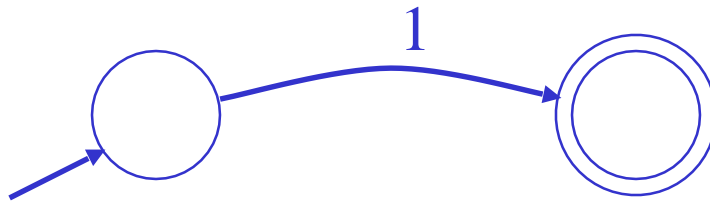
- A state
- The start state
- An accepting state
- A transition



# A Simple Example

---

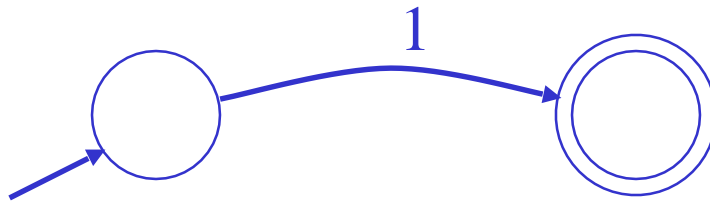
- A finite automaton that accepts only “1”



# A Simple Example

---

- A finite automaton that accepts only “1”



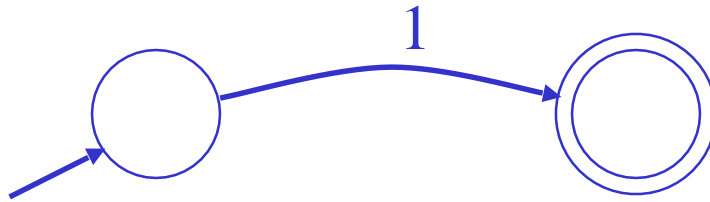
Accept

State	Input
A	$^1$
B	$1^$

# A Simple Example

---

- A finite automaton that accepts only “1”



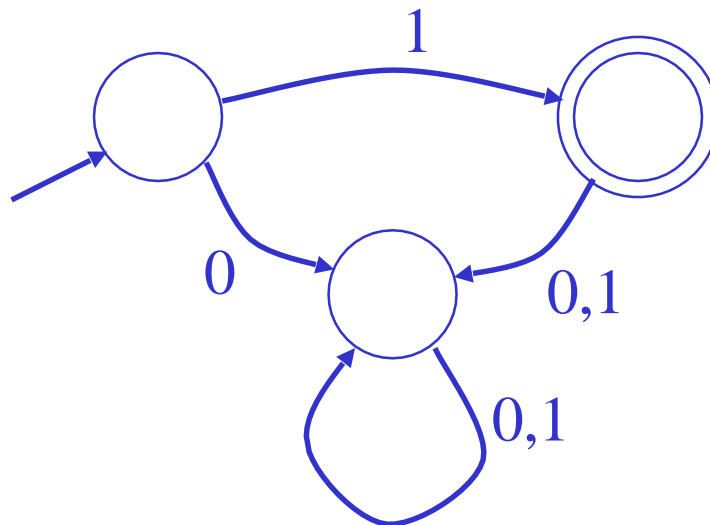
Reject

State	Input
A	$\wedge 0$

# A Simple Example

---

- A finite automaton that accepts only “1”





## Another Simple Example

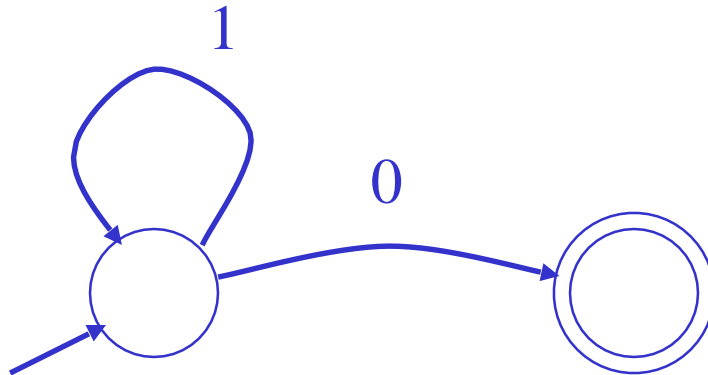
---

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}

# Another Simple Example

---

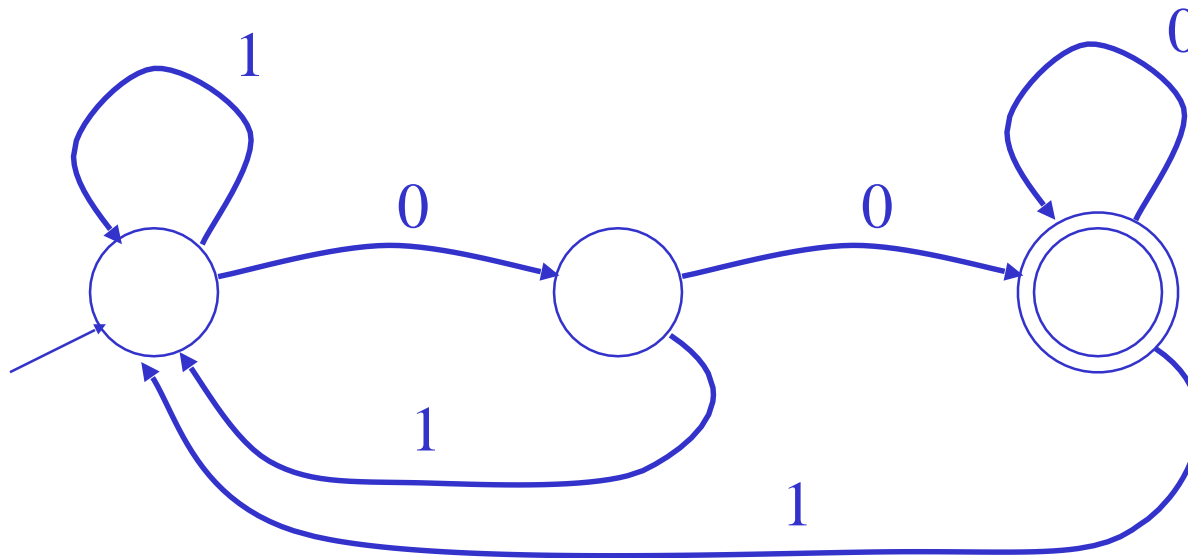
- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}



# And Another Example

---

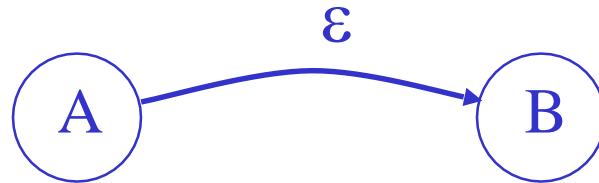
- Alphabet  $\{0,1\}$
- What language does this recognize?



# Epsilon Moves in NFAs

---

- Another kind of transition:  $\epsilon$ -moves



- Machine can move from state A to state B without reading input
- Only exist in NFAs

# Deterministic and Nondeterministic Automata

---

- Deterministic Finite Automata (DFA)
  - Exactly one transition per input per state
  - No  $\epsilon$ -moves
- Nondeterministic Finite Automata (NFA)
  - Can have zero, one, or multiple transitions for one input in a given state
  - Can have  $\epsilon$ -moves

# Execution of Finite Automata

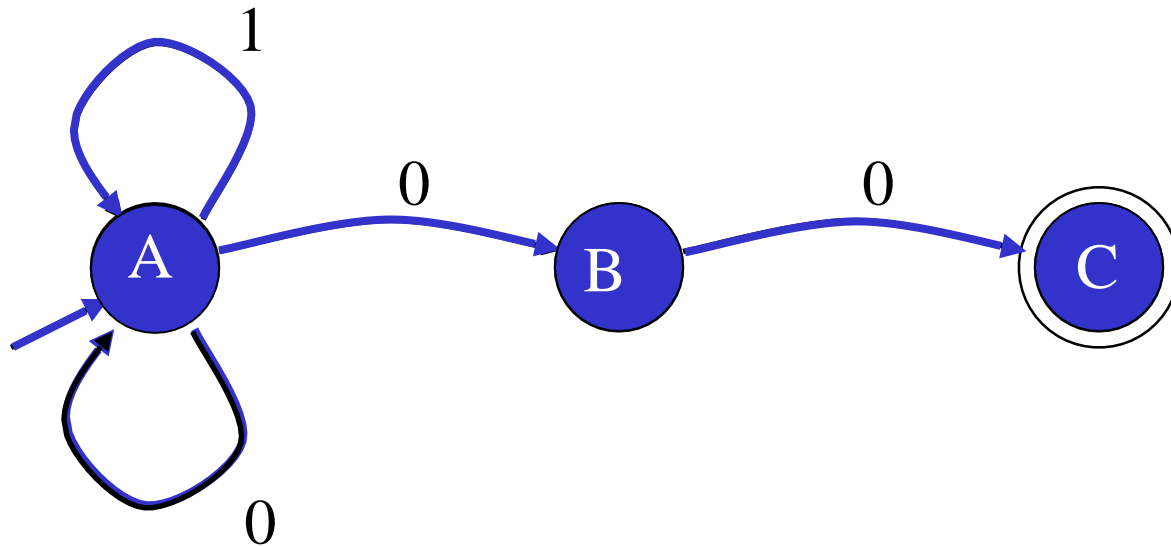
---

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make  $\varepsilon$ -moves
  - Which of multiple transitions for a single input to take

# Acceptance of NFAs

---

- An NFA can get into multiple states



- Input:
- State

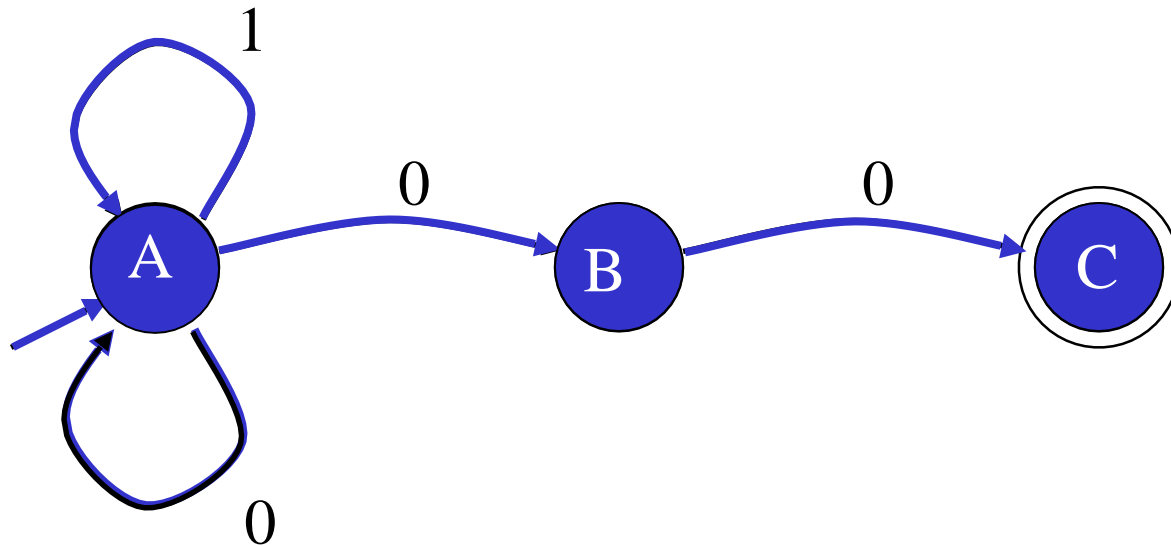
1      0      0

Rule: NFA accepts if it can get to a final state

# Acceptance of NFAs

---

- An NFA can get into multiple states



- Input:                    1                    0                    0
- State:                    {A}                    {A,B}                    {A,B,C}

Rule: NFA accepts if it can get to a final state



# NFA vs. DFA (1)

---

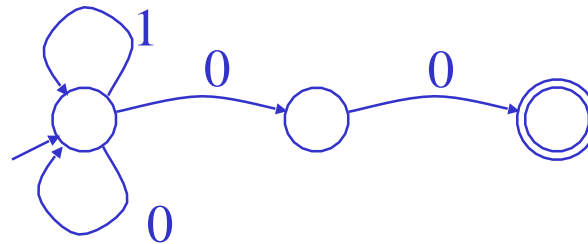
- NFAs and DFAs recognize the same set of languages (regular languages)
- DFAs are faster to execute
  - There are no choices to consider

## NFA vs. DFA (2)

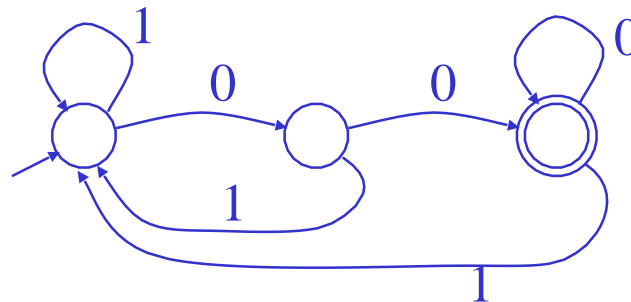
---

- For a given language NFA can be simpler than DFA

NFA



DFA

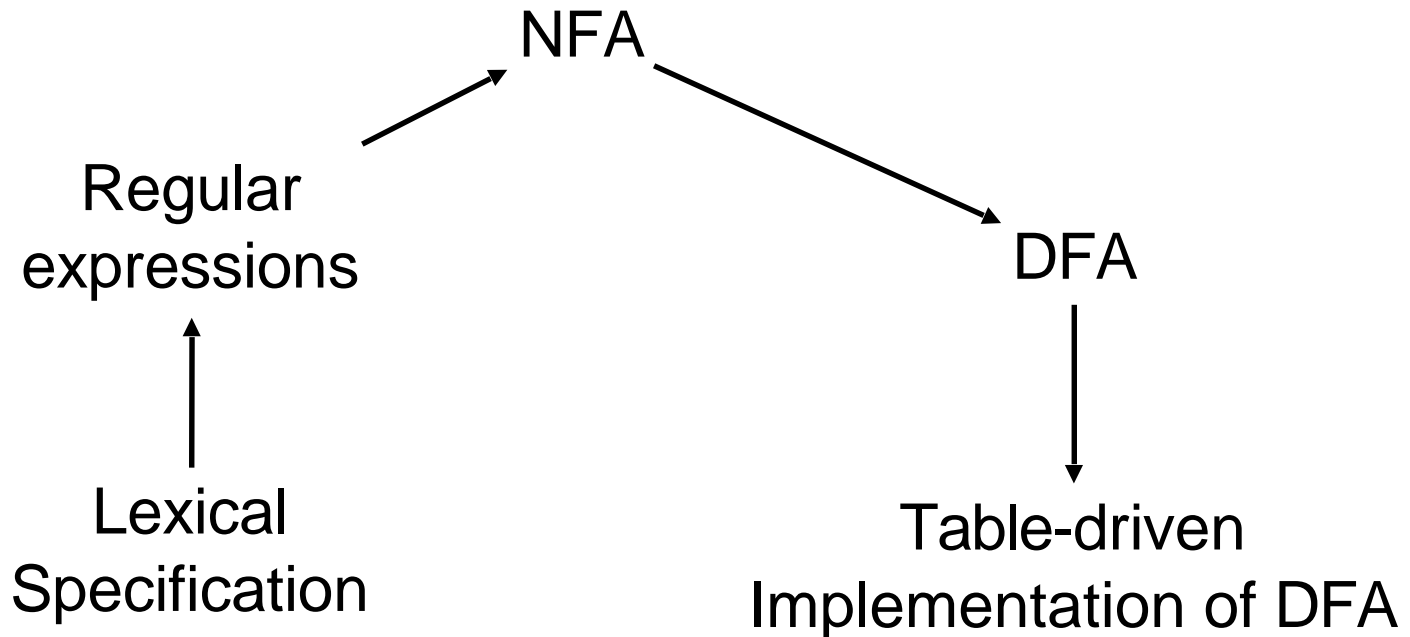


- DFA can be exponentially larger than NFA

# Convert Regular Expressions to Finite Automata

---

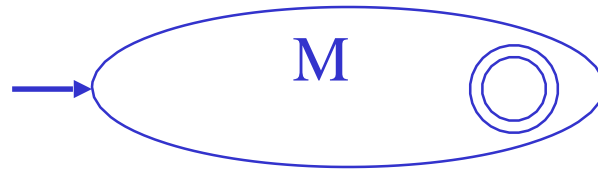
- High-level sketch



# Convert Regular Expressions to NFA (1)

---

- For each kind of regex, define an equivalent NFA
  - Notation: NFA for regex  $M$



- For  $\epsilon$



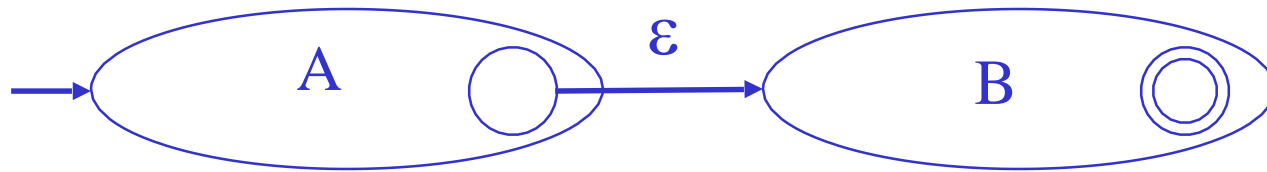
- For input  $a$



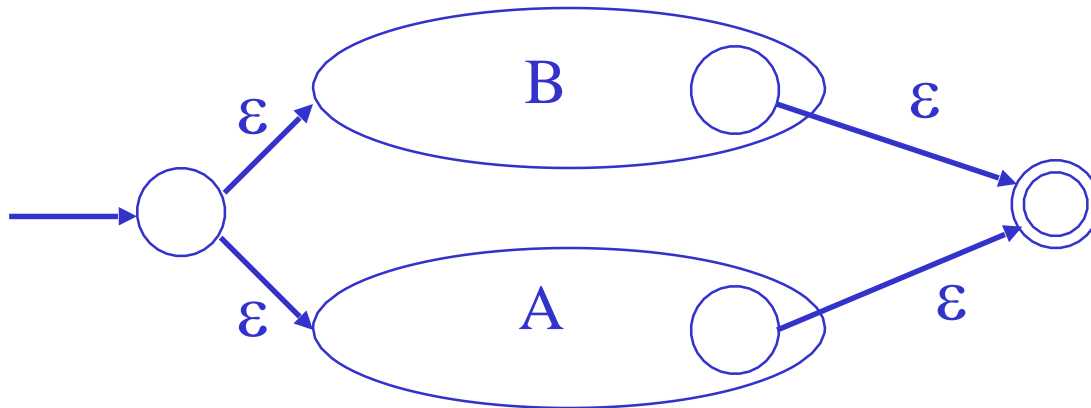
# Convert Regular Expressions to NFA (2)

---

- For  $AB$



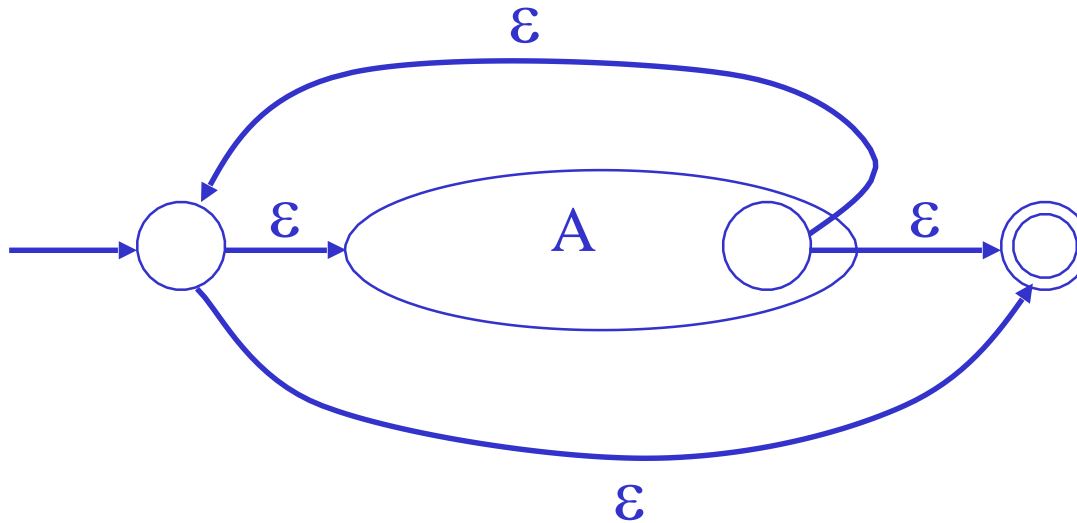
- For  $A + B$



# Convert Regular Expressions to NFA (3)

---

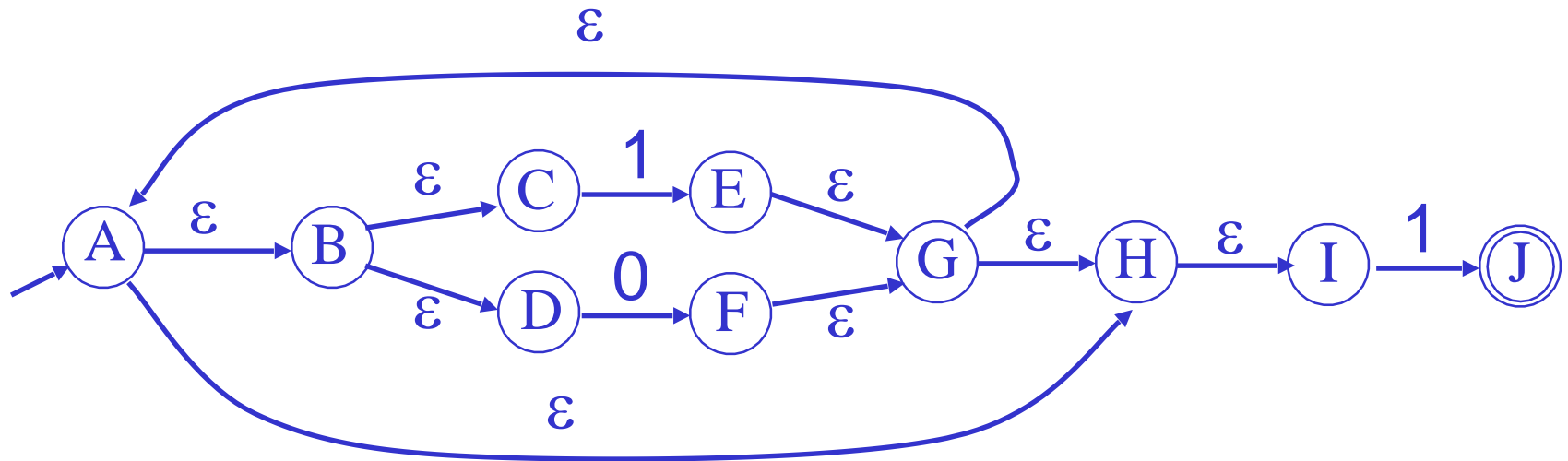
- For  $A^*$



# Example of RegExp to NFA conversion

---

- Consider the regular expression  
 $(1+0)^*1$
- The NFA is



## NFA to DFA. Remark

---

- An NFA may be in many states at any time

How many different states ?

- If there are  $N$  states, the NFA must be in some subset of those  $N$  states

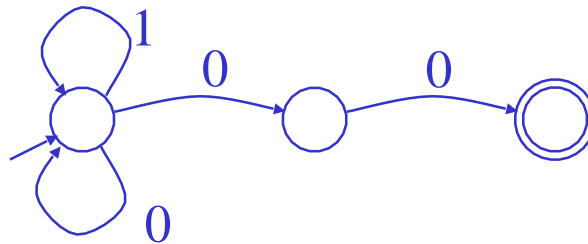


## NFA vs. DFA (2)

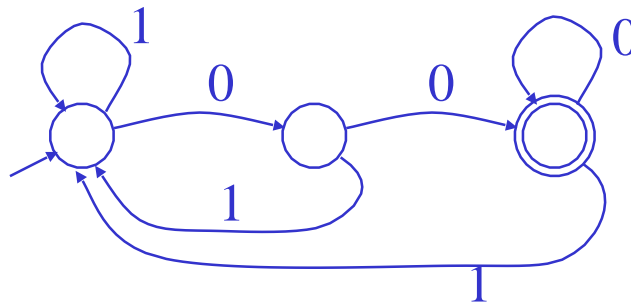
---

- For a given language NFA can be simpler than DFA

NFA



DFA



# Implementation

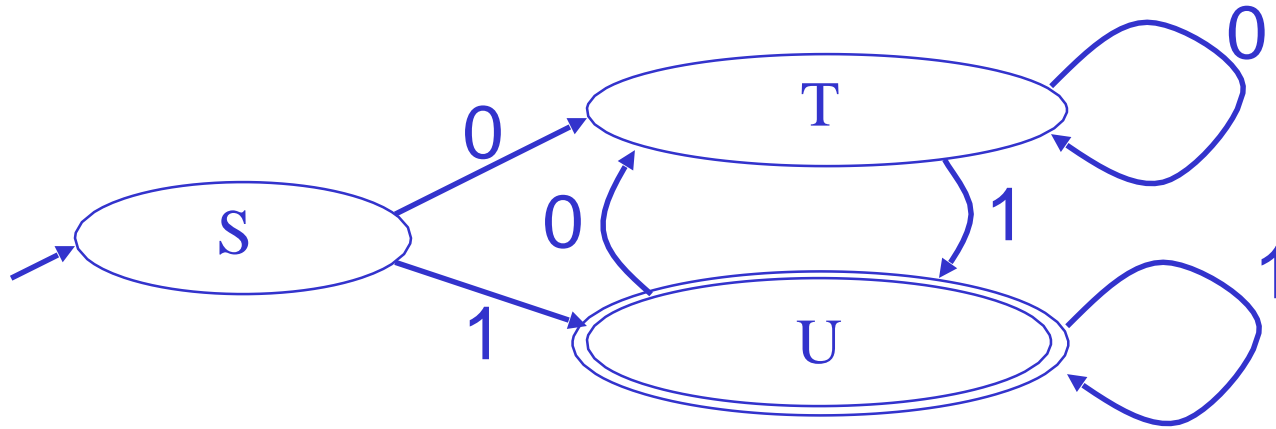
---

- A DFA can be implemented by a 2D table  $T$ 
  - One dimension is “states”
  - Other dimension is “input symbol”
  - For every transition  $S_i \xrightarrow{a} S_k$  define  $T[i,a] = k$

		input symbols	
		0	1
states	a	<b>a</b>	<b>b</b>
	b	a	b
	c	b	b
	d	a	b

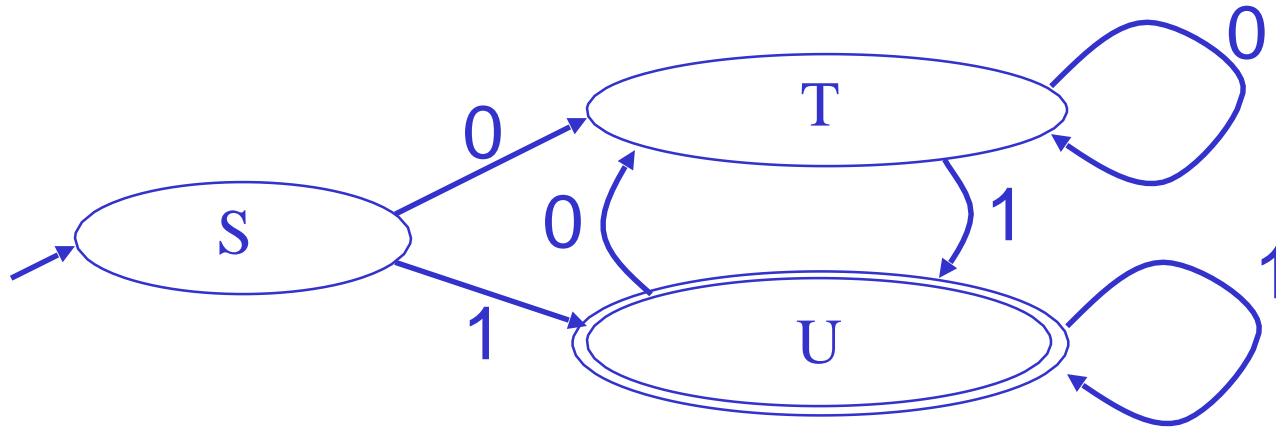
# Table Implementation of a DFA

---



# Table Implementation of a DFA

---



	0	1
S	T	U
T	T	U
U	T	U

## Implementation (Cont.)

---

- NFA  $\rightarrow$  DFA conversion is at the heart of tools such as flex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations