

# A novel notation for quantum cryptography

## Applications to some recent quantum cryptographic protocols and their equivalences

Zef Wolffs

External Research Supervisor: Boris Škorić

Internal Thesis Advisor: Jacco de Vries

January 12, 2020

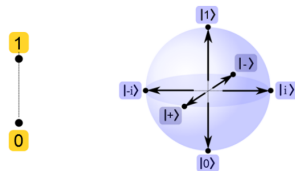




# Introduction

# Quantum Information

- The classical bit vs. the qubit

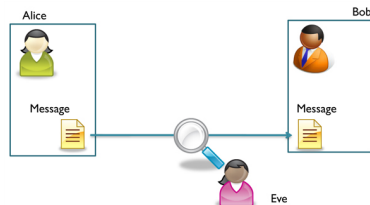


Representation of a classical bit (Left) and a qubit (right) [5].

- Encoding and decoding

# Quantum Cryptography

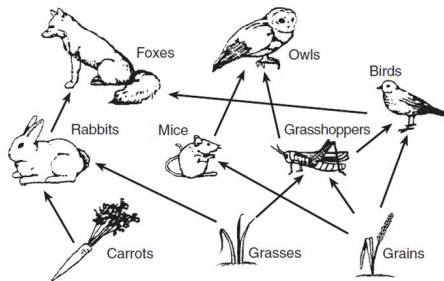
- Quantum cryptographic protocols:  
Sending a message securely using  
quantum mechanics



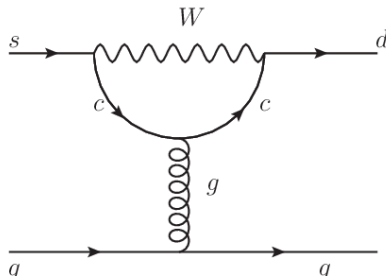
*Alice, Bob, and Eve's roles in (quantum) cryptographic protocols [2].*

- Dirac notation is not very intuitive

## The Diagrammatic Notation



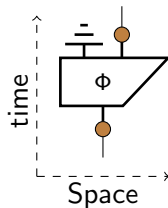
*Diagrams in ecology: food webs [3].*



*Diagrams in particle physics: Feynman diagrams [6].*

## The Dagrammatic Notation

- Proposed by Coecke and Kissinger in 2017, in *Picturing Quantum Processes* [1].





## The Aim

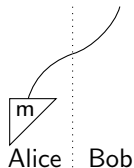
- Taking into account the rising popularity of quantum cryptography and the fact that its current notation is insufficient for describing it intuitively we recognize the usefulness of the diagrammatic notation and therefore want to give it a place in the field of quantum cryptography by...
  1. Writing a short handbook-style introduction to this notation for physicists reluctant to read the entire book *Picturing Quantum Processes* [1].
  2. Constructing some recent quantum cryptographic developments and protocols in this new notation.



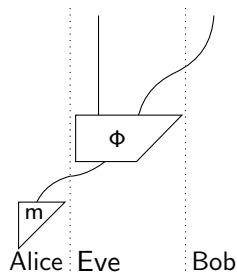
# The Classical One Time Pad

# The Classical One Time Pad

Ideal situation:



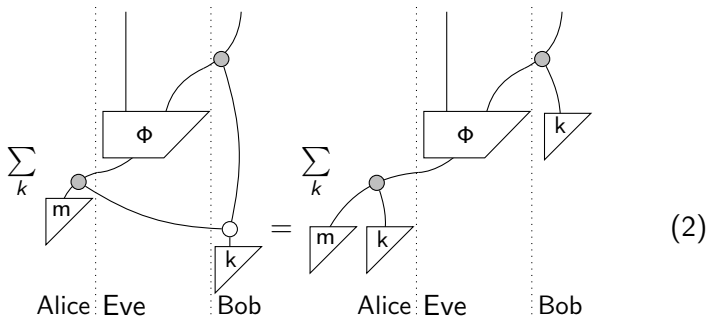
Real situation:



(1)

## The Classical One Time Pad

- The One Time Pad solution: xor with secret random variable  $k$



## The Classical One Time Pad

- If Eve does not interfere, communication should be provably correct.

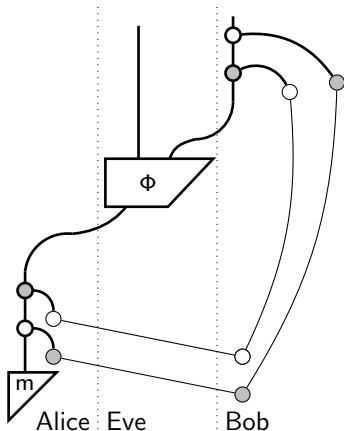
$$\begin{aligned}
 & \sum_k \text{Diagram 1} = \sum_k \text{Diagram 2} = \sum_k \text{Diagram 3} \\
 & = \sum_k \text{Diagram 4} = \sum_k \text{Diagram 5} \approx \text{Diagram 6}
 \end{aligned}
 \tag{3}$$

The diagrams illustrate the simplification of a quantum circuit for the classical one-time pad. The vertical dotted lines represent the stages of the process: Alice's encoding, Eve's potential interference, and Bob's decoding. In the first diagram, Alice's message  $m$  and key  $k$  are combined at a node, which then interacts with Eve's system. The subsequent diagrams show how the summation over the key  $k$  simplifies the circuit, eventually leading to a state where the key  $k$  is effectively decoupled from the message  $m$  and Eve's system, leaving Alice's message and Bob's decoding stage as separate components.

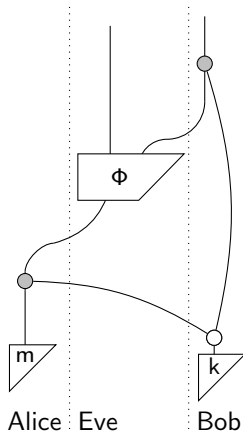
# The Quantum One Time Pad

# The Quantum One Time Pad

The Quantum One Time Pad



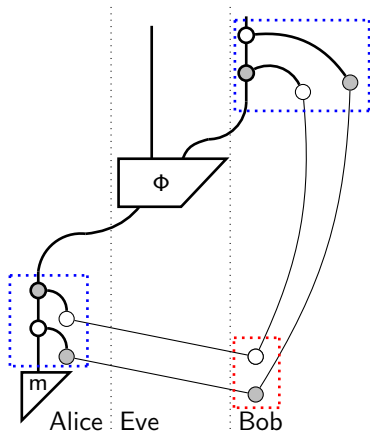
The Classical One Time Pad



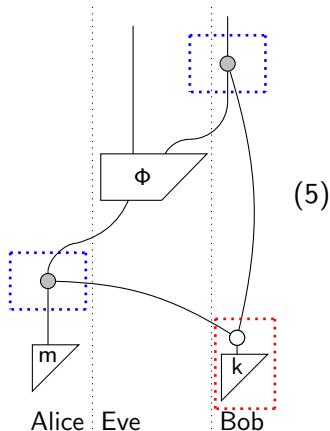
(4)

# The Quantum One Time Pad

The Quantum One Time Pad



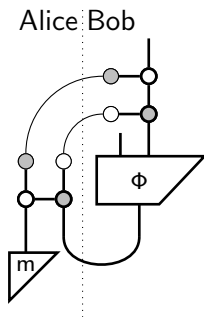
The Classical One Time Pad



(5)

# Quantum Teleportation and Quantum One Time Pad Equivalence

## Quantum Teleportation



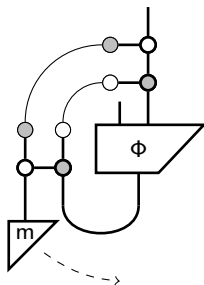
(6)



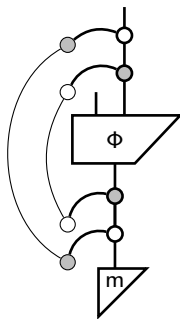
# Quantum Teleportation and Quantum One Time Pad Equivalence

Quantum Teleportation

The Quantum One Time Pad



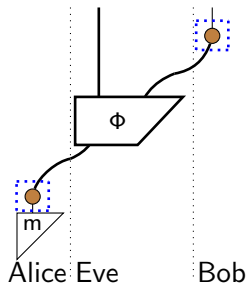
=



(7)

# Quantum Key Recycling

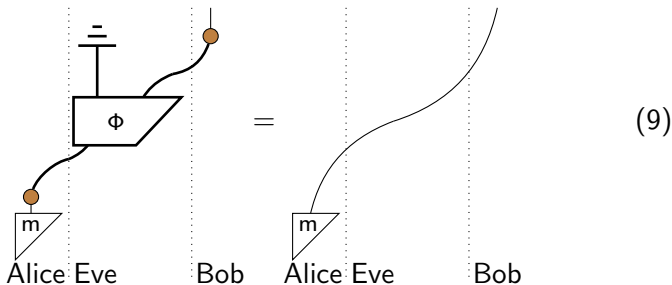
## Quantum Key Recycling



(8)

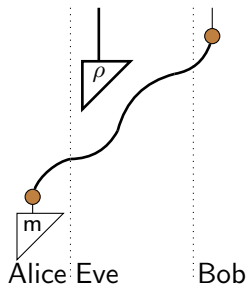
## Quantum Key Recycling

- Security proof for quantum key recycling in the noiseless case, the starting point:



## Quantum Key Recycling

- With a lot of steps in between, the end result becomes:



(10)

- In words: Eve's part of the diagram separates entirely from Alice and Bob's communication channel!



## Discussion and Conclusions

- What novel things did we achieve in this thesis?

## Discussion and Conclusions

- What novel things did we achieve in this thesis?
  - Wrote the first short **handbook-style introduction** to the diagrammatic notation



## Discussion and Conclusions

- What novel things did we achieve in this thesis?
  - Wrote the first short **handbook-style introduction** to the diagrammatic notation
  - Developed the **classical One Time Pad** diagrammatically and showed that it both works and is secure

## Discussion and Conclusions

- What novel things did we achieve in this thesis?
  - Wrote the first short **handbook-style introduction** to the diagrammatic notation
  - Developed the **classical One Time Pad** diagrammatically and showed that it both works and is secure
  - Developed the **quantum One Time Pad** diagrammatically and showed that it both works and is secure



## Discussion and Conclusions

- What novel things did we achieve in this thesis?
  - Wrote the first short **handbook-style introduction** to the diagrammatic notation
  - Developed the **classical One Time Pad** diagrammatically and showed that it both works and is secure
  - Developed the **quantum One Time Pad** diagrammatically and showed that it both works and is secure
  - Showed that **Quantum Teleportation** is equivalent to the quantum One Time Pad, and therefore also works and is secure
  - Developed **Quantum Key Recycling** diagrammatically, included a fully fledged security proof and worked out equivalences from a recent paper [4]

## Discussion and Conclusions

- Did this achieve the aims?

## Discussion and Conclusions

- Did this achieve the aims?
  1. Writing a short handbook-style introduction to this notation for physicists hesitant to read the entire book *Picturing Quantum Processes* [1].

## Discussion and Conclusions

- Did this achieve the aims?
  1. Writing a short handbook-style introduction to this notation for physicists hesitant to read the entire book *Picturing Quantum Processes* [1].  
**Maybe, up to the reader to decide.**

## Discussion and Conclusions

- Did this achieve the aims?
  1. Writing a short handbook-style introduction to this notation for physicists hesitant to read the entire book *Picturing Quantum Processes* [1].  
**Maybe, up to the reader to decide.**
  2. Constructing some recent quantum cryptographic developments and protocols in this new notation.



## Discussion and Conclusions

- Did this achieve the aims?
  1. Writing a short handbook-style introduction to this notation for physicists hesitant to read the entire book *Picturing Quantum Processes* [1].  
**Maybe, up to the reader to decide.**
  2. Constructing some recent quantum cryptographic developments and protocols in this new notation.  
**Yes!**

## Discussion and Conclusions

- Role of diagrammatic notation?

## Discussion and Conclusions

- In future research it would be interesting to...
  - Develop a full security proof for Quantum Key Recycling with noise
  - Generally work out more protocols and equivalences in this notation

# Questions?

- [1] Bob Coecke and Aleks Kissinger.  
*Picturing Quantum Processes*.  
Cambridge University Press, Cambridge, 2017.
- [2] Mathieu Cunche.  
À l'attaque des codes secrets.  
*Interstices.info*, 2011.
- [3] Randi Glaser.  
Food Web Examples.  
*Blendspace.com*, *n.d.*
- [4] Daan Leermakers and Boris Škorić.  
Quantum Alice and Silent Bob.  
<https://eprint.iacr.org/2019/875>, 2019.
- [5] Krzysztof Pomorski, Panagiotis Giounanlis, Elena Blokhina, and Robert Staszewski.  
From Quantum Hardware to Quantum AI.  
*University College Dublin, Dublin*, 2018.
- [6] Kimberley Vos, H. Wilschut, and R. Timmermans.  
Limits on lorentz violation in neutral-kaon decay.  
*Proceedings of the Sixth Meeting on CPT and Lorentz Symmetry*, 2013.