

# A novel notation for quantum cryptography

## Applications to some recent quantum cryptographic protocols and their equivalences

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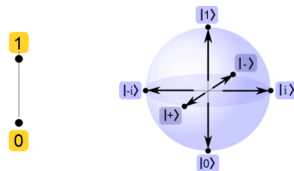




# Introduction

# Quantum Information

- The classical bit vs. the qubit

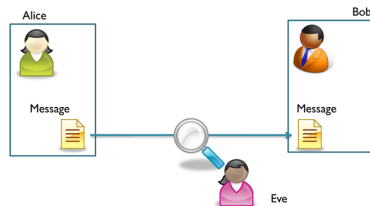


Representation of a classical bit (Left) and a qubit (right) [5].

- Encoding and decoding

# Quantum Cryptography

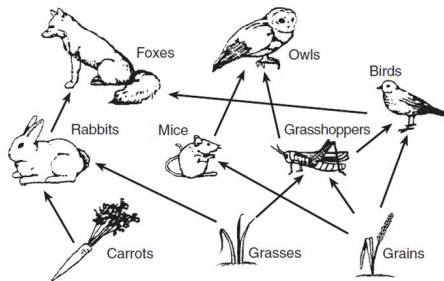
- Quantum cryptographic protocols:  
Sending a message securely using  
quantum mechanics



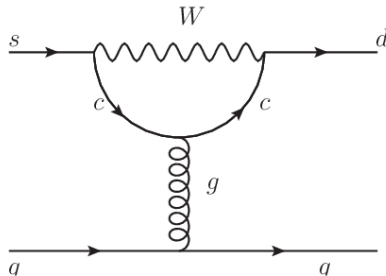
*Alice, Bob, and Eve's roles in (quantum) cryptographic protocols [2].*

- Dirac notation is not very intuitive

## The Diagrammatic Notation



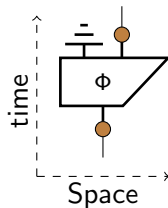
*Diagrams in ecology: food webs [3].*



*Diagrams in particle physics: Feynman diagrams [6].*

## The Dagrammatic Notation

- Proposed by Coecke and Kissinger in 2017, in *Picturing Quantum Processes* [1].





## The Aim

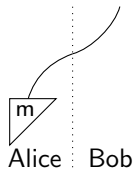
- Taking into account the rising popularity of quantum cryptography and the fact that its current notation is insufficient for describing it intuitively we want to give the diagrammatic method a place in the field of quantum cryptography by...
  1. Writing a short handbook-style introduction to this notation for physicists who do not want to read the entire book *Picturing Quantum Processes [1]*.
  2. Constructing some recent quantum cryptographic developments and protocols in this new notation.



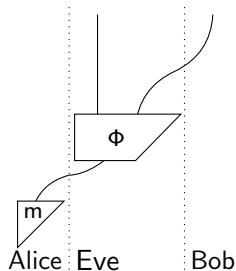
# The Classical One Time Pad

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Ideal situation:



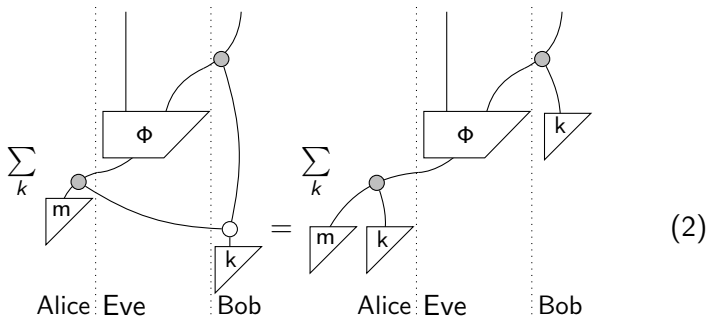
Real situation:



(1)

## The Classical One Time Pad

- The One Time Pad solution: xor with secret random variable  $k$



## The Classical One Time Pad

- If Eve does not interfere, communication should be provably correct.

$$\begin{aligned}
 & \sum_k \text{Diagram 1} = \sum_k \text{Diagram 2} = \sum_k \text{Diagram 3} \\
 & = \sum_k \text{Diagram 4} = \sum_k \text{Diagram 5} \approx \text{Diagram 6}
 \end{aligned} \tag{3}$$

The diagrams illustrate the communication process in the Classical One Time Pad protocol, showing the flow of information (m) and the key (k) through various stages of interaction and summation over the key space  $k$ .

Diagram 1: Alice sends message  $m$  to Eve, who then sends it to Bob. Bob also receives key  $k$ . The diagram shows a path from Alice to Eve to Bob, and a separate path for the key  $k$  to Bob.

Diagram 2: Similar to Diagram 1, but with a different internal structure for the key distribution.

Diagram 3: Similar to Diagram 2, but with a different internal structure for the key distribution.

Diagram 4: Similar to Diagram 3, but with a different internal structure for the key distribution.

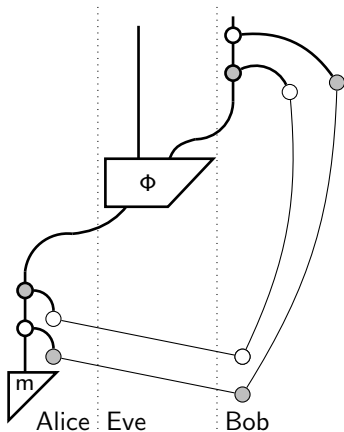
Diagram 5: Similar to Diagram 4, but with a different internal structure for the key distribution.

Diagram 6: Similar to Diagram 5, but with a different internal structure for the key distribution.

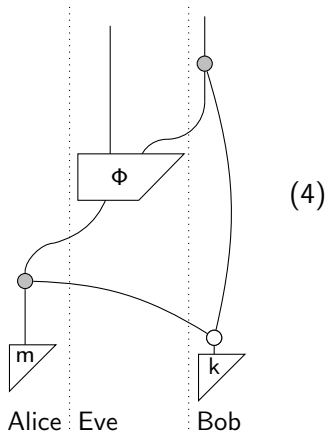
# The Quantum One Time Pad

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The Quantum One Time Pad



The Classical One Time Pad



(4)



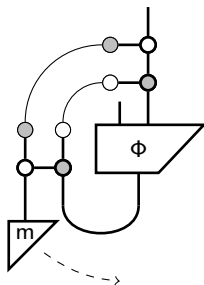




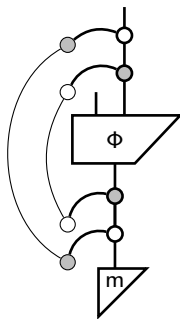
# Quantum Teleportation and Quantum One Time Pad Equivalence

Quantum Teleportation

The Quantum One Time Pad



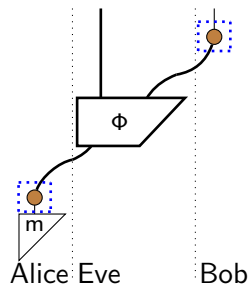
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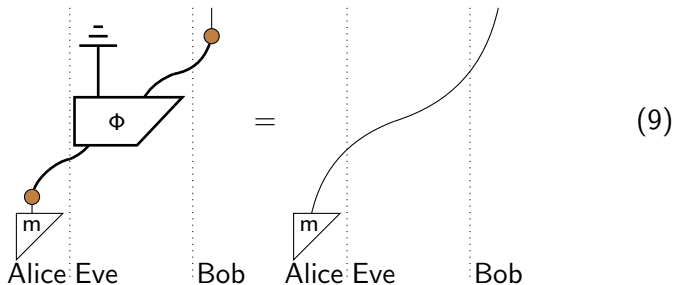
# Quantum Key Recycling



(8)

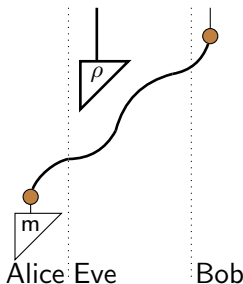
## Quantum Key Recycling

- Security proof for quantum key recycling in the noiseless case, the starting point:



## Quantum Key Recycling

- With a lot of steps in between, the end result becomes:



(10)

- In words: Eve's part of the diagram separates entirely from Alice and Bob's communication channel!



## Discussion and Conclusions

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  - Developed the **classical One Time Pad** diagrammatically and showed that it both works and is secure
  - Developed the **quantum One Time Pad** diagrammatically and showed that it both works and is secure
  - Showed that **Quantum Teleportation** is equivalent to the quantum One Time Pad, and therefore also works and is secure
  - Developed **Quantum Key Recycling** diagrammatically, included a fully fledged security proof and worked out equivalences from a recent paper [4]



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  2. Constructing some recent quantum cryptographic developments and protocols in this new notation.  
**Yes!**

## Discussion and Conclusions

- Role of diagrammatic notation?
- More technical: classical channels have a basis?

$$\left| \blacksquare \right\rangle \quad \left| \square \right\rangle \quad (11)$$

## Discussion and Conclusions

- In future research it would be interesting to...
  - Develop a full security proof for Quantum Key Recycling with noise
  - Generally work out more protocols and equivalences in this notation

# Questions?

## References

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