

A novel notation for quantum cryptography

Applications to some recent quantum cryptographic protocols and their equivalences

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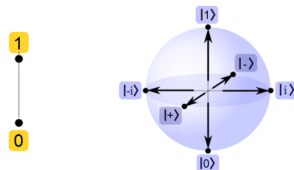
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Introduction

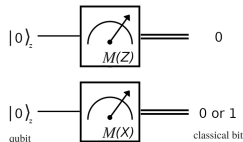
Quantum Information

- The classical bit vs. the qubit



Representation of a classical bit (Left) and a qubit (right) [5].

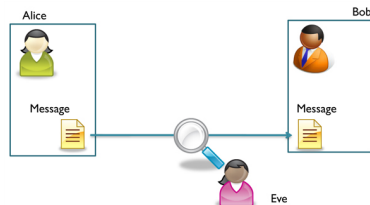
- Mutual unbiasedness



Measuring $|0\rangle_z$ in the Z and X bases [4].

Quantum Cryptography

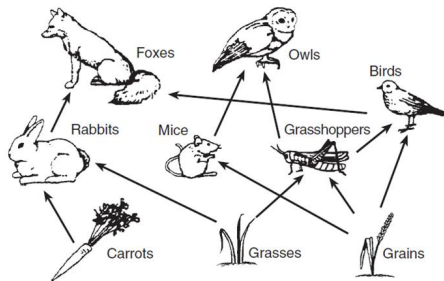
- Quantum cryptographic protocols:
Sending a message securely using
quantum mechanics



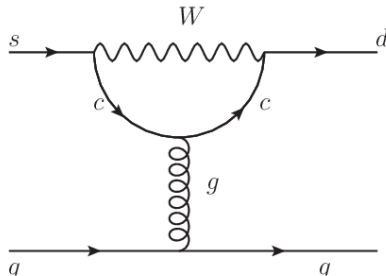
Alice, Bob, and Eve's roles in (quantum) cryptographic protocols [2].

- Dirac notation is not very intuitive

The Diagrammatic Notation



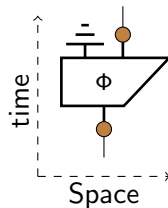
Diagrams in ecology: food webs [3].



Diagrams in particle physics: Feynman diagrams [6].

The Dagrammatic Notation

- Proposed by Coecke and Kissinger in 2017, in *Picturing Quantum Processes* [1].



The Diagrammatic Notation

- Example: mutual unbiasedness
 - Measuring and encoding in the same basis results in the identity

$$\begin{array}{c} \bullet \\ | \\ \bullet \end{array} = \begin{array}{c} \circ \\ | \\ \circ \end{array} = \begin{array}{c} | \end{array} \quad (1)$$

- But measuring and encoding in a different basis results in nothing being sent through

$$\begin{array}{c} \circ \\ | \\ \bullet \end{array} = \frac{1}{2} \begin{array}{c} \circ \\ | \\ \circ \end{array} + \frac{1}{2} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} \quad \begin{array}{c} \bullet \\ | \\ \circ \end{array} = \frac{1}{2} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} + \frac{1}{2} \begin{array}{c} \circ \\ | \\ \circ \end{array} \quad (2)$$

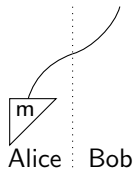
The Aim

- Taking into account the rising popularity of quantum cryptography and the fact that its current notation is insufficient for describing it intuitively we recognize the usefulness of the diagrammatic notation and therefore want to give it a place in the field of quantum cryptography by...
 1. Writing a short handbook-style introduction to this notation for physicists reluctant to read the entire book *Picturing Quantum Processes* [1].
 2. Constructing some recent quantum cryptographic developments and protocols in this new notation.

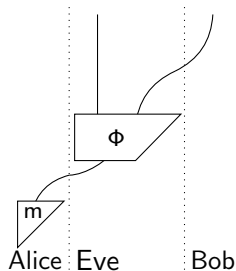
The Classical One Time Pad

The Classical One Time Pad

Ideal situation:



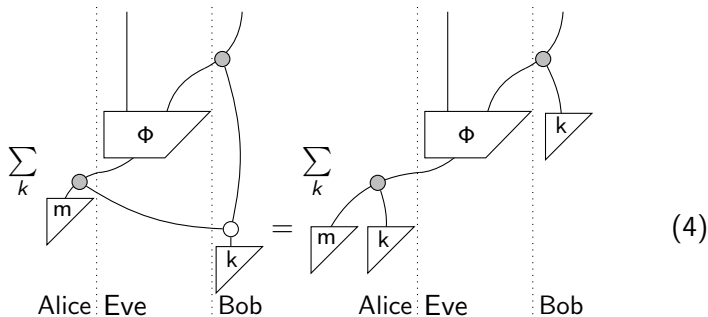
Real situation:



(3)

The Classical One Time Pad

- The One Time Pad solution: xor with secret random variable k



The Classical One Time Pad

- If Eve does not interfere, communication should be provably correct.

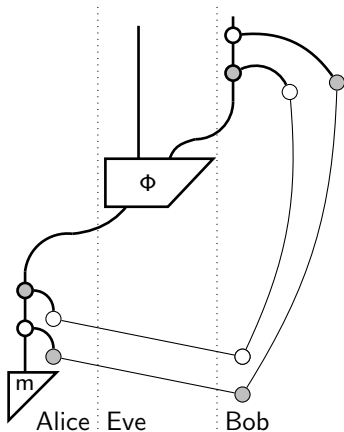
$$\begin{aligned}
 & \sum_k \text{Diagram 1} = \sum_k \text{Diagram 2} = \sum_k \text{Diagram 3} \\
 & = \sum_k \text{Diagram 4} = \sum_k \text{Diagram 5} \approx \text{Diagram 6}
 \end{aligned} \tag{5}$$

The diagrams illustrate the communication process in the Classical One Time Pad protocol, showing the state of Alice, Eve, and Bob's keys and messages at different stages of the protocol. The diagrams are connected by equals and approximation signs, indicating the sequence of operations and the resulting states.

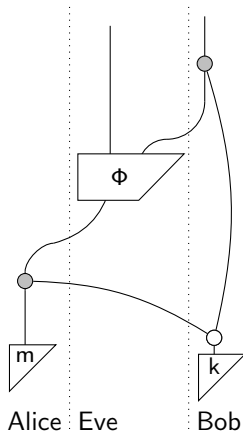
The Quantum One Time Pad

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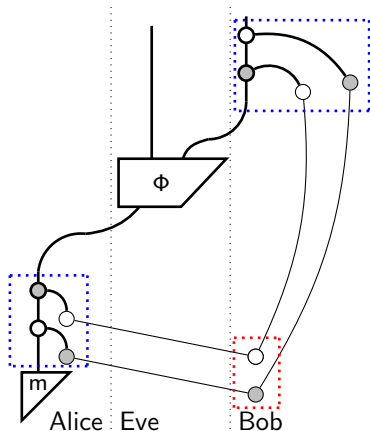
The Classical One Time Pad



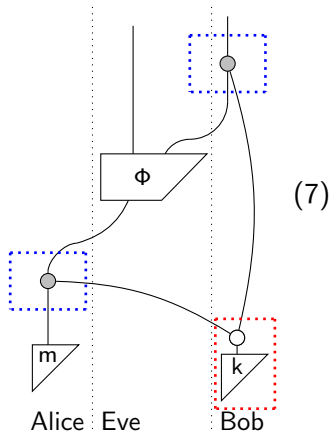
(6)

The Quantum One Time Pad

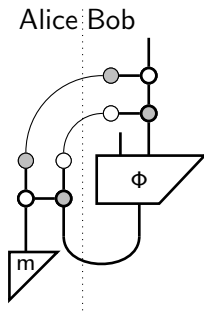
The Quantum One Time Pad



The Classical One Time Pad



Quantum Teleportation is Equivalent to the Quantum One Time Pad

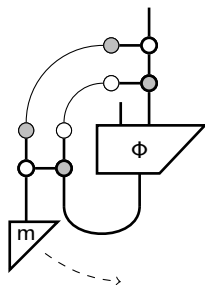


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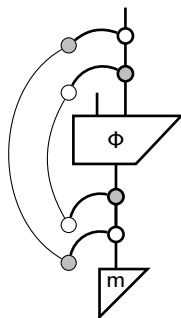
Quantum Teleportation is Equivalent to the Quantum One Time Pad

Quantum Teleportation

The Quantum One Time Pad



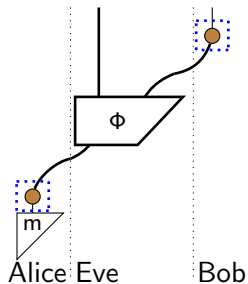
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Quantum Key Recycling

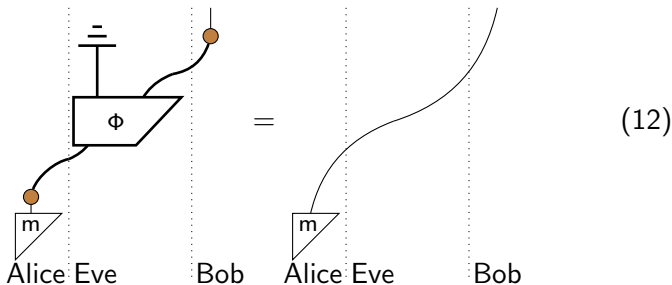
Quantum Key Recycling



(11)

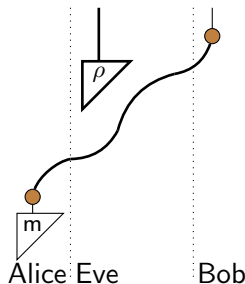
Quantum Key Recycling

- Security proof for quantum key recycling in the noiseless case, the starting point:



Quantum Key Recycling

- With a lot of steps in between, the end result becomes:



(13)

- In words: Eve's part of the diagram separates entirely from Alice and Bob's communication channel!

Discussion and Conclusions

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 - Developed the **classical One Time Pad** diagrammatically and showed that it both works and is secure
 - Developed the **quantum One Time Pad** diagrammatically and showed that it both works and is secure
 - Showed that **Quantum Teleportation** is equivalent to the quantum One Time Pad, and therefore also works and is secure
 - Developed **Quantum Key Recycling** diagrammatically, included a fully fledged security proof and worked out equivalences from a recent paper

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 2. Constructing some recent quantum cryptographic developments and protocols in this new notation.
Yes!

Discussion and Conclusions

- Novelty of this research?
- Role of diagrammatic notation?
- More technical: Classical communication channels in a basis?

$$\left| \blacksquare \right\rangle \quad \left| \square \right\rangle \quad (14)$$

Discussion and Conclusions

- In future research it would be interesting to...
 - Develop a full security proof for Quantum Key Recycling with noise
 - Generally work out more protocols and equivalences in this notation

Questions?

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