

A novel notation for quantum cryptography

Applications to some recent quantum cryptographic protocols and their equivalences

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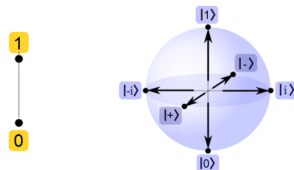
January 12, 2020



Introduction

Quantum Information

- The classical bit vs. the qubit

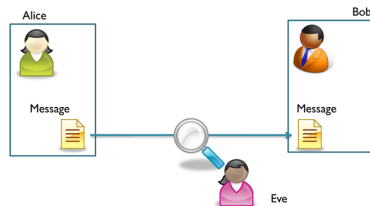


Representation of a classical bit (Left) and a qubit (right) [5].

- Encoding and decoding

Quantum Cryptography

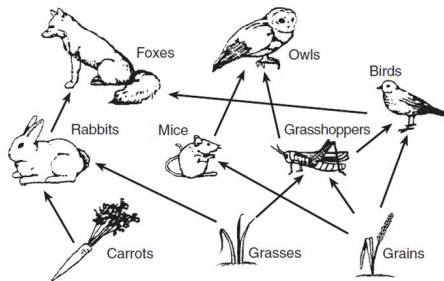
- Quantum cryptographic protocols:
Sending a message securely using
quantum mechanics



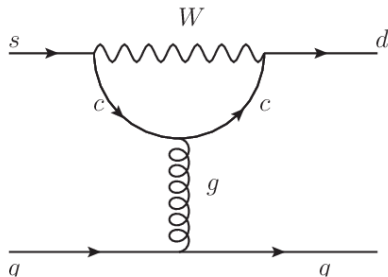
Alice, Bob, and Eve's roles in (quantum) cryptographic protocols [2].

- Dirac notation is not very intuitive

The Diagrammatic Notation



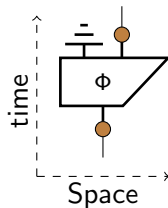
Diagrams in ecology: food webs [3].



Diagrams in particle physics: Feynman diagrams [6].

The Dagrammatic Notation

- Proposed by Coecke and Kissinger in 2017, in *Picturing Quantum Processes* [1].





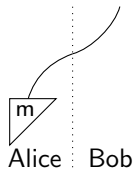
The Aim

- Taking into account the rising popularity of quantum cryptography and the fact that its current notation is insufficient for describing it intuitively we want to give the diagrammatic method a place in the field of quantum cryptography by...
 1. Writing a short handbook-style introduction to this notation for physicists reluctant to read the entire book *Picturing Quantum Processes* [1].
 2. Constructing some recent quantum cryptographic developments and protocols in this new notation.

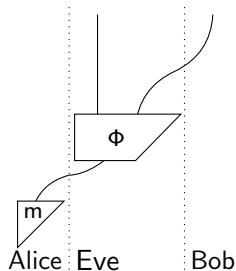
The Classical One Time Pad

The Classical One Time Pad

Ideal situation:



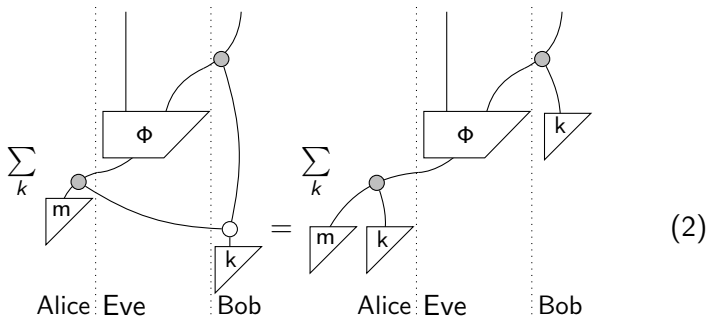
Real situation:



(1)

The Classical One Time Pad

- The One Time Pad solution: xor with secret random variable k



The Classical One Time Pad

- If Eve does not interfere, communication should be provably correct.

$$\begin{aligned}
 & \sum_k \text{Diagram 1} = \sum_k \text{Diagram 2} = \sum_k \text{Diagram 3} \\
 & = \sum_k \text{Diagram 4} = \sum_k \text{Diagram 5} \approx \text{Diagram 6}
 \end{aligned}
 \tag{3}$$

The diagrams illustrate the communication process in the Classical One Time Pad protocol, showing the flow of information (m for message, k for key) and the role of Eve's potential interference. The diagrams are arranged in two rows, with the first row showing the initial state and the second row showing the final state after Eve's potential interference.

Diagram 1: Alice sends message m to Bob via a channel. Eve is present but does not interfere. The diagram shows a solid line from Alice to Bob and a dashed line from Alice to Eve. The sum is over k.

Diagram 2: Alice sends message m to Bob via a channel. Eve is present but does not interfere. The diagram shows a solid line from Alice to Bob and a dashed line from Alice to Eve. The sum is over k.

Diagram 3: Alice sends message m to Bob via a channel. Eve is present but does not interfere. The diagram shows a solid line from Alice to Bob and a dashed line from Alice to Eve. The sum is over k.

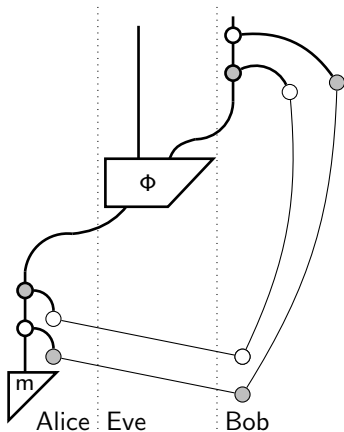
Diagram 4: Alice sends message m to Bob via a channel. Eve is present but does not interfere. The diagram shows a solid line from Alice to Bob and a dashed line from Alice to Eve. The sum is over k.

Diagram 5: Alice sends message m to Bob via a channel. Eve is present but does not interfere. The diagram shows a solid line from Alice to Bob and a dashed line from Alice to Eve. The sum is over k.

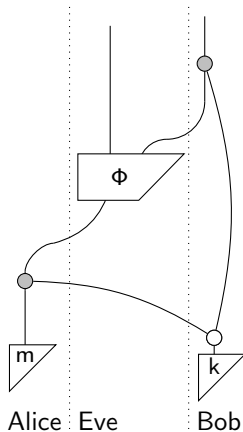
Diagram 6: Alice sends message m to Bob via a channel. Eve is present but does not interfere. The diagram shows a solid line from Alice to Bob and a dashed line from Alice to Eve. The sum is over k.

The Quantum One Time Pad

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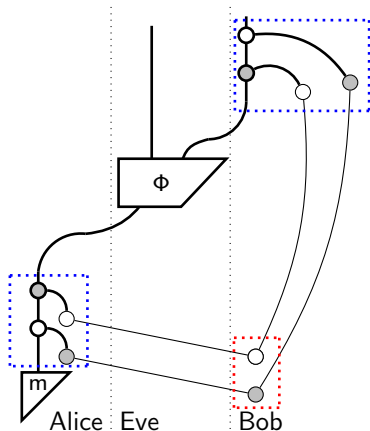
The Classical One Time Pad



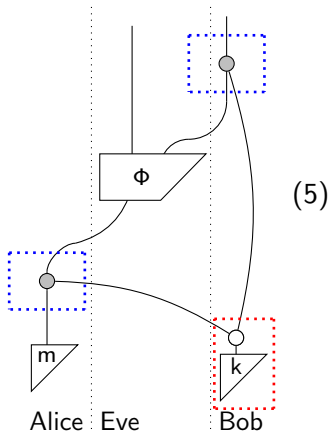
(4)

The Quantum One Time Pad

The Quantum One Time Pad

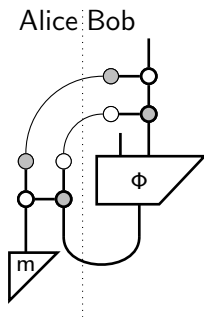


The Classical One Time Pad



(5)

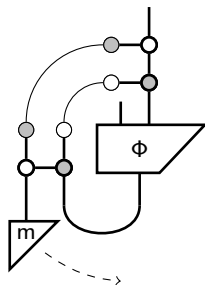
Quantum Teleportation



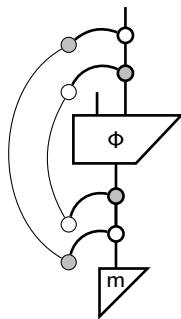
(6)

Quantum Teleportation and Quantum One Time Pad Equivalence

Quantum Teleportation



The Quantum One Time Pad



(7)

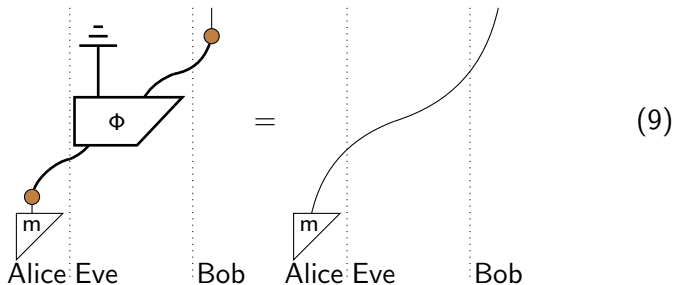
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(8)



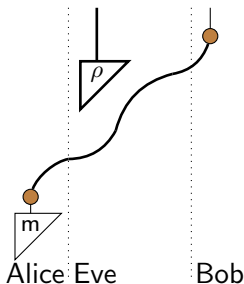
Quantum Key Recycling

- Security proof for quantum key recycling in the noiseless case, the starting point:



Quantum Key Recycling

- With a lot of steps in between, the end result becomes:



(10)

- In words: Eve's part of the diagram separates entirely from Alice and Bob's communication channel!

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 - Developed the **classical One Time Pad** diagrammatically and showed that it both works and is secure
 - Developed the **quantum One Time Pad** diagrammatically and showed that it both works and is secure
 - Showed that **Quantum Teleportation** is equivalent to the quantum One Time Pad, and therefore also works and is secure
 - Developed **Quantum Key Recycling** diagrammatically, included a fully fledged security proof and worked out equivalences from a recent paper [4]

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Yes!

Discussion and Conclusions

- Role of diagrammatic notation?
- More technical: classical channels have a basis?

$$\left| \blacksquare \right\rangle \quad \left| \square \right\rangle \quad (11)$$

Discussion and Conclusions

- In future research it would be interesting to...
 - Develop a full security proof for Quantum Key Recycling with noise
 - Generally work out more protocols and equivalences in this notation

Questions?

