Shengli No.1 High School, Dongying
Shandong, P.R.C.

An Attempt at Optimizing Non-Contact On-Road Charging for Electric Vehicles, Utilizing an Evolutionary Algorithm

Author:
Liu Zebang
Guo Yuheng

Counselor:
Ding Hongjuan

An Attempt at Optimizing Non-Contact On-Road Charging for Electric Vehicles, Utilizing an Evolutionary Algorithm

Abstract

It is believed that the advent of electric vehicles, whose preliminary signs are already being perceived, will bring about great ramifications to technology, the environment, and peoples' lives in general, promising cleaner air, less carbon emissions and less waste of energy and resources. However, such developments are being hampered by the low energy density of the batteries currently used in EVs, which severely constricts EVs' cruising radius, limiting their practical usages. Due to the impasse that the development of battery technologies is facing, we believe that non-contact on-road charging is the ideal solution to this problem. In order to optimize and demonstrate the feasibility of this approach, we have designed an evolutionary algorithm for finding the best design for the coils used in the charging process, which has yielded promising results.

Keywords

wireless charging, evolutionary algorithm, computational physics, electric vehicles, affordable, efficiency

本参赛团队声明所提交的论文是在指导老师下进行的研究工作和取得的研究成果。尽本团队所知，除了文中特别加以标注和致谢中所罗列的内容以外，论文中不包含其他人或本团队已经发表或撰写过的研究成果。若有不实之处，本人愿意承担一切相关责任。

参赛团队签名：

日 期：

An Attempt at Optimizing Non-Contact On-Road Charging for Electric Vehicles, Utilizing an Evolutionary Algorithm

Abstract

It is believed that the advent of electric vehicles, whose preliminary signs are already being perceived, will bring about great ramifications to technology, the environment, and peoples' lives in general, promising cleaner air, less carbon emissions and less waste of energy and resources. However, such developments are being hampered by the low energy density of the batteries currently used in EVs, which severely constricts EVs' cruising radius, limiting their practical usages. Due to the impasse that the development of battery technologies is facing, we believe that non-contact on-road charging is the ideal solution to this problem. In order to optimize and demonstrate the feasibility of this approach, we have designed an evolutionary algorithm for finding the best design for the coils used in the charging process, which has yielded promising results.

I Introduction

In the 21^{st} century, the virulent consequences of the intense industrialization in the last two centuries has surfaced, taking the forms of more drastic changes in temperature, worse droughts and floods, more frequent haze and extreme weather conditions, as well as more serious issues in the supply of fresh water, food and energy. In efforts to ameliorate the situation, people are beginning to perceive electric vehicles, which have virtually no negative environmental impact, as the most probable candidate for replacing the pollutant-emitting petroleum-powered cars. Indeed, corporations like *Mercedes-Benz*, *BMW*, *Ford*, *Tesla* and *General Motors* have already presented their EVs to the general public, harbingering an unprecedented advent of electric vehicles. However, the inefficiency and expensiveness of the energy storage systems installed on EVs severely circumscribes EVs' cruising radius and marketing audience, limiting their practical usages and wider adoption. Even though new types of more efficient batteries are being designed and polished, most of them are still too expensive for ordinary consumers and too difficult to produce in large amounts with a low failure rate.

Therefore, methods other than developing battery technologies are being brought to attention, the most prominent of which is on-road charging. The main idea of this concept is to create priority lanes which have the capability of charging vehicles whilst they drive over them. If properly implemented in a wide range, EV owners would no longer need to worry about running out of power during excursions or long trips, the number of charging stations by the side of highways would be reduced, along with the length of trips, since time would no longer be wasted on refilling the energy storage. Currently, there are two ways of implementing such a charging system: installing conducting surfaces or rails that are connected to the power grid, which are then linked to conductors on the vehicles; and installing into the road recessed coils that transmit power to the vehicles utilizing electromagnetic induction. It is apparent that the latter approach has numerous advantages over the former, as underground coils are more resilient to corrosion and wear, pose no danger of electrocution for passers-by, and are more aesthetically pleasing than bare wires. Considering all these points, we perceive non-contact on-road charging as the ideal solution to the impasse that EV development is currently facing.

Though this approach is promising, it is not without its own problems. The most serious obstacle to its implementation is the relatively low efficiency of inductive charging. In order to make it more efficient, we have studied methods of improving efficiency used in other fields, and found an inspiring case [1]. At the *NASA Ames Research Center*, a team designed an evolutionary algorithm (EA), a stochastic search method inspired by Darwinian evolution that operate on a population of solutions using the principle of natural selection to incrementally produce better solutions, that was used to search the design space and automatically find novel antenna designs that are more effective than would otherwise be developed. The final design produced by the algorithm, which eventually flew on NASA's *Space Technology 5* mission, achieved a 93% efficiency, as opposed to the 38% efficiency achieved by human-designed antennae. Learning from this precedence, we decided to imitate this method and engineer an evolutionary algorithm capable of finding the optimal design for the recessed coils, the most crucial part of the inductive charging system.

After thorough physical analysis of the problem and sound software engineering, we have designed a full-fledged evolutionary algorithm, which takes into account factors such as efficiency, construction cost, stability and power, and from that produces the best possible coil shape. Using the first model me constructed, the efficiency of the system was bounded to less than 17%, but the optimizing power of the evolutionary algorithm was clearly observed. After adjusting the physical model, higher efficiencies were achieved. Results from runs of the perfected algorithm were fruitful, with coil designs that achieved a 60% efficiency and a $6.3kW$ power output, did not include ferrite cores, and were well-rounded in all other respects.

II The Physical Analyses

II.a The Scenario

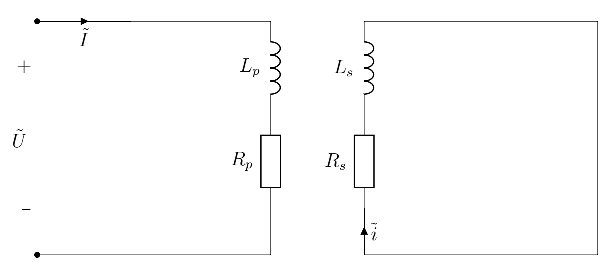
The electric priority lane, or the powered lane, consists of repeating "blocks" as of the recessed underground infrastructure. Each block contains one primary coil used in the inductive charging, and pressure sensors detecting whether a vehicle is above the block, so that the coil is only activated in the presence of a vehicle. Of course, more sensors may be added in order to charge the drivers for using the system. The coils are connected to an AC power source. Each block is $4.5m$ in length and $3m$ in width. The secondary coil installed on the vehicle is also rectangular in shape, $2.25m$ in length and $1.5m$ in width. Note that this is only one possible design for the secondary coil, it may take many shapes. As we have an algorithm that can take the secondary coil's design as an input, this is not a problem.

The vehicle runs at a constant speed of $15m/s$, or $54km/h$. The distance between the front and back tires is $3.5m$.

The distance between the primary coil's plane and the secondary coil's plane is $15cm$. The primary coil has 1 round, while the secondary coil has 10 rounds. The coils are made of copper, with a resistivity of $1.6 \times 10^{-8}(\Omega \cdot m)$ and a diameter of $1mm$.

II.b The Calculations

The major objective of the calculations is to obtain the current flowing through the primary and secondary coils at any given time t as well as the voltage applied to them, so that the work may be calculated using $W = \int U(t)I(t)\,dt$, and therefore obtain the efficiency $\eta = W_s/W_p$.



The main equations follow.

$$U_0 \cos 2\pi f t = L_p \frac{dI}{dt} + \frac{d\,(Mi)}{dt} + IR_p \tag{1}$$

$$\frac{d\,(MI)}{dt} + L_s \frac{di}{dt} + iR_s = 0 \tag{2}$$

$$u = -\frac{d\,(MI)}{dt} \tag{3}$$

I and i denote the current in the primary and secondary coils respectively, u denotes the voltage of the secondary coil, U_0 denotes the maximum voltage applied to the primary coil, M denotes the mutual induction between the primary and secondary coils, L_p, R_p, L_s, R_s denote the self induction and resistance of the primary and secondary coils respectively.

It should be noted that there are intervals during which the primary coils in two consecutive blocks are activated, so the equations need to be altered in that light.

$$U_0 \cos 2\pi f t = L_p \frac{dI_1}{dt} + \frac{d\left(M_1 i\right)}{dt} + I_1 R_p \tag{4}$$
$$U_0 \cos 2\pi f t = L_p \frac{dI_2}{dt} + \frac{d\left(M_2 i\right)}{dt} + I_2 R_p \tag{5}$$
$$\frac{d\left(M_1 I_1\right)}{dt} + \frac{d\left(M_2 I_2\right)}{dt} + L_s \frac{di}{dt} + iR_s = 0 \tag{6}$$
$$u = -\frac{d\left(M_1 I_1\right)}{dt} + \frac{d\left(M_2 I_2\right)}{dt} \tag{7}$$

Also, actually solving the above differential equations in their original forms is an unrealistically complex process, so they must be transformed into a simpler form, using complex numbers.

$$U_0 e^{j\omega t} = \tilde{I}_1 (R_p + j\omega L_p) + \tilde{i}(\dot{M}_1 + j\omega M_1) \tag{8}$$
$$U_0 e^{j\omega t} = \tilde{I}_2 (R_p + j\omega L_p) + \tilde{i}(\dot{M}_2 + j\omega M_2) \tag{9}$$
$$\tilde{I}_1(\dot{M}_1 + j\omega M_1) + \tilde{I}_2(\dot{M}_2 + j\omega M_2) = -\tilde{i}(R_s + j\omega L_s) \tag{10}$$
$$u = -\frac{d\left(M_1 \cdot Re(\tilde{I}_1)\right)}{dt} + \frac{d\left(M_2 \cdot Re(\tilde{I}_2)\right)}{dt} \tag{11}$$

The equations above are linear, therefore are easy to solve even with pen and paper.

Let

$$a = R_p + j\omega L_p \tag{12}$$
$$b = \dot{M}_1 + j\omega M_1 \tag{13}$$
$$c = \dot{M}_2 + j\omega M_2 \tag{14}$$
$$d = R_s + j\omega L_s \tag{15}$$
$$\tilde{U} = U_0 e^{j\omega t} \tag{16}$$

The solution to equations (1)(2) and (3) is

$$\tilde{i} = \tfrac{b}{b^2 - ad} \tilde{U} \tag{17}$$
$$\tilde{I}_1 = \tfrac{d}{ad - b^2} \tilde{U} \tag{18}$$

The solution to equations (8)(9)(10) and (11) is

$$\tilde{i} = \tfrac{b+c}{b^2 + c^2 - ad} \tilde{U} \tag{19}$$
$$\tilde{I}_1 = \tfrac{c(c-b) - ad}{a(b^2 + c^2 - ad)} \tilde{U} \tag{20}$$
$$\tilde{I}_2 = \tfrac{b(b-c) - ad}{a(b^2 + c^2 - ad)} \tilde{U} \tag{21}$$

With these results, it is possible to calculate $I_i(t)$, $u(t)$ and $i(t)$, as long as U_0, f, L_p, L_s, R_i and M_i are given. However, L_p, L_s and M_i are impossible to calculate by hand, due to the arbitrary shape of the primary coil, therefore computer simulations must be conducted to determine those values, especially M_i, which varies over time. According to the equations

$$\psi = MI \tag{22}$$
$$\psi = LI \tag{23}$$

where ψ denotes the flux linkage, if the flux linkages of both the primary and secondary coils are calculated, with a known current flowing through the coils, it is possible to calculate M and L. In order to accomplish that, the magnetic field strength $\overrightarrow{B}$ must be obtained, using the Biot-Savart law, so that the flux linkage may be calculated by integrating $d\psi = N\overrightarrow{B} \cdot \overrightarrow{dS}$ across the coils' areas. The equation that describes the magnetic field generated by a finite straight wire at a given point, derived from the original Biot-Savart law, is as follows:

$$\overrightarrow{B} = \frac{\mu_0 I}{4\pi r}(cos\theta_2 - cos\theta_1)\frac{\overrightarrow{l} \times \overrightarrow{r}}{|\overrightarrow{l} \times \overrightarrow{r}|} \tag{24}$$

where $\overrightarrow{r}$ denotes the distance between the point and the wire, and $\overrightarrow{l}$ denotes the length of the wire. Used under the framework of analytical geometry, it has been transformed into a simple yet powerful algorithm for magnetic field calculations that far exceeded our requirements. The specifics of the said algorithm will now be delineated.

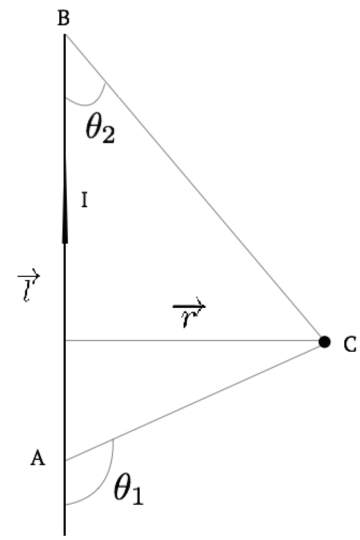


FIG.1 A current carrying wire AB and an arbitrary point C.

The parameters of the algorithm are given on the graph above. $\overrightarrow{l}$ points from A to B, $\overrightarrow{r}$ points from the wire to C.

The linear equation satisfying $(x_A, y_A, 0)$ and $(x_B, y_B, 0)$ is

$$(y_B - y_A)x - (x_B - x_A)y + (x_B - x_A)y_A - (y_B - y_A)x_A = 0 \tag{25}$$

therefore the distance d between AB and C's projection on the xOy plane is:

$$d = \frac{|(y_B - y_A)x_C - (x_B - x_A)y_C + (x_B - x_A)y_A - (y_B - y_A)x_A|}{\sqrt{(y_B - y_A)^2 + (x_B - x_A)^2}} \tag{26}$$

and $|\overrightarrow{r}|$ may be calculated as follows:

$$|\overrightarrow{r}| = \sqrt{d^2 + z_c^2} \tag{27}$$

the cosines may be obtained by taking advantage of the equation

$$cos < \overrightarrow{a}, \overrightarrow{b} >= \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}| \cdot |\overrightarrow{b}|} \tag{28}$$

since

$$\theta_1 =< \overrightarrow{CA}, \overrightarrow{AB} > \tag{29}$$
$$\theta_2 =< \overrightarrow{CB}, \overrightarrow{AB} > \tag{30}$$

therefore

$$cos\theta_1 = \frac{\overrightarrow{CA} \cdot \overrightarrow{AB}}{|\overrightarrow{CA}| \cdot |\overrightarrow{AB}|} \tag{31}$$
$$cos\theta_2 = \frac{\overrightarrow{CB} \cdot \overrightarrow{AB}}{|\overrightarrow{CB}| \cdot |\overrightarrow{AB}|} \tag{32}$$

also

$$\frac{\overrightarrow{l} \times \overrightarrow{r}}{|\overrightarrow{l} \times \overrightarrow{r}|} = \frac{\overrightarrow{CA} \times \overrightarrow{AB}}{|\overrightarrow{CA} \times \overrightarrow{AB}|} \tag{33}$$

and now $\overrightarrow{B}$ is obtained. With this algorithm, it is possible to calculate the entire magnetic field distribution over the primary and secondary coils by approximating the arbitrary shape as series of straight lines put together, and so made possible the calculation of $I_i(t)$, $u(t)$ and $i(t)$, the keys to the final objective.

III The Evolutionary Algorithm

III.a Overview of Evolutionary Algorithms

Evolutionary Algorithms (EA) is a class of search heuristics that imitate the process of Darwinian evolution, and operate on a population of individuals following the principles of natural selection, genetic crossover and mutation. They have been applied in a range of optimization problems, and often produced novel solutions more effective than would otherwise have been devised. The essential principle behind such algorithms, namely the use of statistics from previous attempts in the generation of new solutions, has been wired into the heart of a currently thriving field: machine learning, which is responsible for the famous AlphaGo, the first computer Go program to defeat a professional Go player without handicaps.

The individuals in an EA are dubbed "chromosomes", strings of numbers or characters that contain all the information that describes the attributes of the respective individuals, the same as actual chromosomes. During each generation, chromosomes will mate and produce offsprings, at a possibility based on their fitness scores, a quantitive measure of how well an individual fits the preset requirements, so that the more fit individuals are also more likely to survive and propagate. Chromosomes would also mutate, which means random alterations will be introduced to the chromosomes, in the hope that beneficial mutations might occur and improve their fitnesses.

The algorithm we designed is availiable at *https://github.com/theGreatLzbdd/Nirvana/* .

III.b Original Features in our EA

III.b.1 Radical Mutation for the Unfit

One of the most serious problems an EA may encounter is premature convergence, which means reaching a virtually static state before finding the global optimum of the search space. A large enough population can mitigate this issue, but it would also drastically increase the amount of calculations needed, something undesirable when the evaluation function is expensive. Many different solutions to premature convergences exist, but they are essentially the same in their approaches: to preserve genetic diversity, which makes sense since the goal is to avoid a homogenous population. For example, in the parallel genetic algorithm (pGA) [2] [3], multiple populations develop independently, and communicate with each other via infrequent intra-population matings. In this manner, when a population suffers from premature convergence, intra-population matings, which introduce individuals of a large variety to the said population, may provide a sufficient disturbance able to steer the population away from the point of convergence. However, maintaining multiple populations would still increase the amount of calculations, since there would usually be more individuals, so a better method is required.

Inspired by the pGA, we have developed our own solution to premature convergence: *Radical Mutation for the Unfit (RMU)*. As the name suggests, the intensity and frequency of mutations are dramatically increased for the least fit individuals. Since the very purpose of keeping unfit individuals is to maintain genetic diversity and to provide "building blocks" that are potentially of above average quality [4], the plausibility and soundness of our approach is obvious. Also, because only the worst performing individuals, which are more unlikely to produce offsprings, experience more radical mutations, the best performing individuals usually can feel but faint repercussions of the overly radical mutations, while being disturbed just enough to avoid premature convergence. It would seem as if there were two independent populations, as described in the pGA. However, it should be noted that the process of RMU is only initiated after a certain number of generations has elapsed, due to the more than ample diversity the population exhibits during the first few generations.

III.b.2 Variable Foci of Fitness Evaluation

Our algorithm takes many factors into account when evaluating the fitness of individual coil designs: efficiency, stability, cost and so on, but they are not of equal importance. The first priority of the algorithm is to increase the efficiency of the coils, since an overly inefficient coil is of no value even if all of its other attributes are ideal, therefore efficiency must be raised to a certain level before other factors are considered. So in our algorithm, the effect that efficiency has on a coil's fitness score is greatly magnified until efficiency reaches a certain threshold.

III.c Adapting the problem into the EA framework

Expressing our problem under the EA framework can be broken down into four tasks:

- Expressing an individual coil design as a single string, or chromosome.

- Defining the crossover operation, which is the production of offsprings from parent coil designs.

- Defining the mutation operation.

- Defining the evaluation function.

The solution to these tasks will be described as follows.

Each coil design will be approximated as a combination of a series of straight edges, in order to make the physical calculations possible. Each edge may be represented by its two vertices, which can be transformed into a plain text string under a coordinate system. For instance,

$$(1,0) \rightarrow (4,8) \rightarrow (5,96) \Leftrightarrow 010004080596$$

Note that it can only handle integer coordinates. Also, this representation implicitly contains a direction, even though coil designs are undirected, but this extra information is perfectly harmless once a starting and a finishing node are specified for each coil.

In each generation, a certain number of chromosome pairs will be selected from the pool to undergo the crossover process and produce offsprings, based on the principle of tournament selection [5]. During the process, the two parent chromosomes will first be divided into the same number of segments of same lengths, after which the algorithm would enumerate through the segments and roll a dice to decide whether each particular segment would be exchanged between the parents. The selection process will continue until the number of children is equal to that of parents, when the children would form the next generation.

During the mutation operation, the algorithm would enumerate through all vertices in a single coil, and roll a dice to determine whether each vertex would mutate. If so, the vertex's coordinates would randomly deviate within a certain boundary.

To evaluate each coil design, a "test drive" will be conducted on a "test lane", which consists of two "block"s. During the test drive, the work done by the primary coil and the energy received by the secondary coil will be incrementally calculated as the simulated vehicle moves through the test lane, whereby obtaining the efficiency. Since the whole drive is temporal symmetric, only the former half will be simulated. The procedures for calculating efficiency have already been described in **II.b**.

However, as no algorithm can enumerate through an infinite amount of points, approximations must be made by taking a set of sample points, whose magnetic field strengths will represent that of their adjoining areas. Simple though it may be in principle, the actual process is complicated by the task of finding the set of points that are sure to be within the boundaries of the coil, as there is no obvious method for determining whether a point fits such a requirement. After exhaustive research, we have found the said method, which will be delineated as follows.

Starting from each point that share the same plane with a polygon, an arbitrary ray can be drawn, which may or may not intersect with the polygon. It can be proved, using the Jordan Curve Theorem [6], that if the said point in within the polygon, the number of intersections the ray has with the polygon would always be odd (unless vertices are involved), and vice versa. In fact, if one draws an analogy between our conclusion and the Divergence Theorem (or Gauss's Theorem) [7] in vector calculus

$$\int_U \nabla \cdot \mathbf{F}\, dV_n = \oint_{\partial U} \mathbf{F} \cdot \mathbf{n}\, dS_{n-1} \tag{34}$$

applied under a two dimensional space, the validity of our conclusion instantly becomes obvious, for which a proof will not be given here.

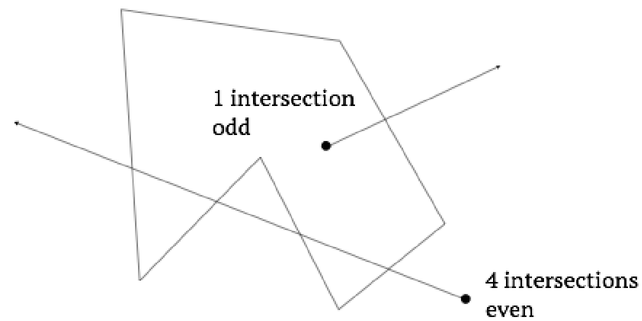


FIG.2 A demonstration of the given theorem.

Therefore, utilizing this conclusion, an algorithm capable of determining a point's geometrical relations with a polygon can easily be devised, which is exactly what has been done.

However, during the preliminary empirical trials of this algorithm, many problems were discovered, For instance, erroneous results were often produced when the polygons took peculiar non-simple shapes, and the efficiency of the algorithm was somewhat unsatisfactory. Hence, we sought for new possibilities, and eventually found a much superior method, one that utilized a concept in analytical geometry called winding number. A point's winding number can determine whether it is inside a polygon by counting how many times the polygon winds around the point; the winding number would be nonzero if it is, zero otherwise. [8] delineates well both the concept of the winding number and an efficient algorithm for its calculation, and it is the basis of our own implementation.

The considered factors in the determination of fitness scores are: efficiency, stability, power and construction cost.

- Stability will be measured in the magnitude of the change in efficiency when the simulated vehicle deviates from the central axis. The reason for taking stability into account is that in real life situations it is difficult for drivers to align their vehicles perfectly with the central axis, and a resulting large decrease in efficiency would be undesirable.

- Construction cost will be measured in the length of the coil. The reason for taking this factor into account is that a low construction cost would accelerate the adoption of non-contact on-road charging, while a high cost would be pernicious to it.

and fitness is calculated according to the equation

$$f = C_1\eta + C_2P - C_3\Delta\eta - C_4t \tag{35}$$

where η is efficiency, P is power, $-\Delta\eta$ is stability, t is construction cost, and C_1, C_2, C_3, C_4 are positive coefficients.

IV Initial Empirical Results

After implementing the evolutionary algorithm and running extensive empirical trials, several important facts had been uncovered, revealing a new direction of research.

Firstly, the evolutionary algorithm had fully demonstrated its capacity for optimization problems. It was able to increase the maximum fitness score several hundred times over the course of hundreds of generations.

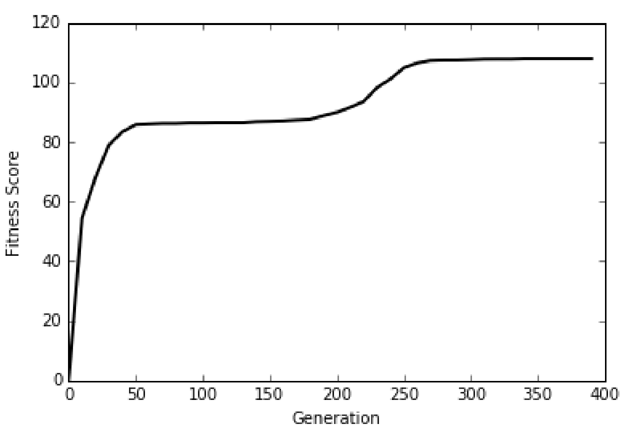


FIG.3 A graph delineating how the fitness score increases over generations.

Visually, it is easy to observe how the edges of the primary coil gravitate towards the secondary coils position, and how the coil takes on increasingly recognizable shapes.

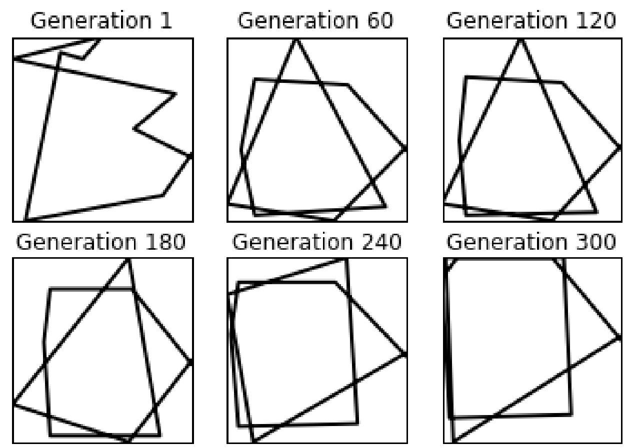


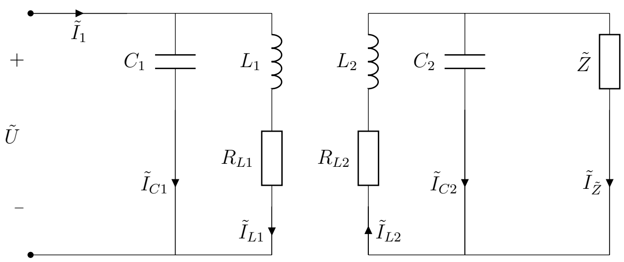
FIG.4 A demonstration of how a coil design evolves over generations.

Secondly, though we began the project with the hope of achieving a high efficiency by nothing more than altering the shape of the primary coil, the best coil the algorithm produced could only operate at a 17% efficiency, marking a theoretical optimum efficiency of the system that cannot be surpassed. Therefore, alterations must be made to the original model, possibly by adding components other than inductive coils.

Having learned these points, we did thorough research into improving the efficiency of wireless charging, and constructed a second model for on-road inductive charging.

V The Second Model

After full consideration, we chose to utilize resonant inductive coupling to increase the efficiency of the system. As seen from other studies [9] [10], the addition of a parallel resonant capacitor can mitigate the deterioration of voltage across wide gaps and significantly increase the system's efficiency and power output. The new system that we designed will be described below.



The resonant frequency of the circuit is

$$\omega = \sqrt{\frac{1}{\sqrt{L_1C_1}} - (\frac{R_{L1}}{L_1})^2} = \sqrt{\frac{1}{\sqrt{L_2C_2}} - (\frac{R_{L2}}{L_2})^2} \tag{36}$$

The Kirchhoff equations of the circuit are as follows

$$\tilde{U} = \tilde{I_{L1}}(R_{L1} + j\omega L_1) + \tilde{I_{L2}}(\dot{M} + j\omega M) \tag{37}$$
$$\tilde{I_{L1}}(\dot{M} + j\omega M) + \tilde{I_{L2}}(R_{L2} + j\omega L_2) + \tilde{I_Z}\tilde{Z} = 0 \tag{38}$$
$$\tilde{I_Z} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2} + \tilde{Z}}\tilde{I_{L2}} \tag{39}$$
$$\tilde{I_{L1}} = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + \tilde{Z_{L1}}}\tilde{I}_1 \tag{40}$$

Let

$$\tilde{Z_{L1}} = R_{L1} + j\omega L_1 \tag{41}$$
$$\tilde{Z_{L2}} = R_{L2} + j\omega L_2 \tag{42}$$
$$\tilde{Z_M} = \dot{M} + j\omega M \tag{43}$$
$$\tilde{Z_{C1}} = \frac{1}{j\omega C_1} \tag{44}$$
$$\tilde{Z_{C2}} = \frac{1}{j\omega C_2} \tag{45}$$

The solution to equations (37)(38)(39) and (40) is

$$\tilde{I_Z} = -\tilde{U}\frac{\tilde{Z_M}\tilde{Z_{C2}}}{(\tilde{Z_{L1}}\tilde{Z_{L2}} - \tilde{Z_M}^2)(\tilde{Z} + \tilde{Z_{C2}}) + \tilde{Z}\tilde{Z_{C2}}\tilde{Z_{L1}}} \tag{46}$$
$$\tilde{I}_1 = \tilde{U}\frac{\tilde{Z_{C1}} + \tilde{Z_{L1}}}{\tilde{Z_{C1}}\tilde{Z_{L1}}}(\frac{\tilde{Z_M}^2(\tilde{Z} + \tilde{Z_{C2}})}{(\tilde{Z_{L1}}\tilde{Z_{L2}} - \tilde{Z_M}^2)(\tilde{Z} + \tilde{Z_{C2}}) + \tilde{Z}\tilde{Z_{C2}}\tilde{Z_{L1}}} + 1) \tag{47}$$

The power of the impedance $\tilde{Z}$ is

$$P_Z = Re(\tilde{I_Z}\tilde{Z}) \cdot Re(\tilde{I_Z}) \tag{48}$$

The total power is

$$P_{total} = Re(\tilde{U}) \cdot Re(\tilde{I}_1) \tag{49}$$

When two primary coils are activated at the same time, the equations are

$$\tilde{U} = \tilde{I_{L1a}}(R_{L1} + j\omega L_1) + \tilde{I_{L2}}(\dot{M}_a + j\omega M_a) \tag{50}$$
$$\tilde{U} = \tilde{I_{L1b}}(R_{L1} + j\omega L_1) + \tilde{I_{L2}}(\dot{M}_b + j\omega M_b) \tag{51}$$
$$\tilde{I_{L1a}}(\dot{M}_a + j\omega M_a) + \tilde{I_{L1b}}(\dot{M}_b + j\omega M_b) + \tilde{I_{L2}}(R_{L2} + j\omega L_2) + \tilde{I_Z}\tilde{Z} = 0 \tag{52}$$
$$\tilde{I_Z} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2} + \tilde{Z}}\tilde{I_{L2}} \tag{53}$$
$$\tilde{I_{L1}} = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + \tilde{Z_{L1}}}\tilde{I}_1 \tag{54}$$

The solution to the equations above is

$$\tilde{I_Z} = -\tilde{U}\frac{(\tilde{Z_{Ma}} + \tilde{Z_{Mb}})\tilde{Z_{C2}}}{(\tilde{Z_{L1}}\tilde{Z_{L2}} - \tilde{Z_{Ma}}^2 - \tilde{Z_{Mb}}^2)(\tilde{Z} + \tilde{Z_{C2}}) + \tilde{Z}\tilde{Z_{C2}}\tilde{Z_{L1}}} \tag{55}$$
$$\tilde{I_{1a}} = \tilde{U}\frac{\tilde{Z_{C1}} + \tilde{Z_{L1}}}{\tilde{Z_{C1}}\tilde{Z_{L1}}}(\frac{\tilde{Z_{Ma}}(\tilde{Z_{Ma}} + \tilde{Z_{Mb}})(\tilde{Z} + \tilde{Z_{C2}})}{(\tilde{Z_{L1}}\tilde{Z_{L2}} - \tilde{Z_{Ma}}^2 - \tilde{Z_{Mb}}^2)(\tilde{Z} + \tilde{Z_{C2}}) + \tilde{Z}\tilde{Z_{C2}}\tilde{Z_{L1}}} + 1) \tag{56}$$
$$\tilde{I_{1b}} = \tilde{U}\frac{\tilde{Z_{C1}} + \tilde{Z_{L1}}}{\tilde{Z_{C1}}\tilde{Z_{L1}}}(\frac{\tilde{Z_{Mb}}(\tilde{Z_{Ma}} + \tilde{Z_{Mb}})(\tilde{Z} + \tilde{Z_{C2}})}{(\tilde{Z_{L1}}\tilde{Z_{L2}} - \tilde{Z_{Ma}}^2 - \tilde{Z_{Mb}}^2)(\tilde{Z} + \tilde{Z_{C2}}) + \tilde{Z}\tilde{Z_{C2}}\tilde{Z_{L1}}} + 1) \tag{57}$$

The power of the impedance $\tilde{Z}$ is

$$P_Z = Re(\tilde{I_Z}\tilde{Z}) \cdot Re(\tilde{I_Z}) \tag{58}$$

The total power is

$$P_{total} = Re(\tilde{U}) \cdot (Re(\tilde{I_{1a}}) + Re(\tilde{I_{1b}})) \tag{59}$$

With the results given above, it is easy to calculate the efficiency of the system.

VI New Empirical Results and Analysis

Since the addition of the resonant capacitors has no effect on the distribution of the magnetic field, the coil designs generated under the previous model are still applicable. After running a number of trials using the new model, many important results were made.

Firstly, we explored the relationship between the efficiency of the coil and the frequency of the AC power source. The exact relationship is somewhat difficult to discern, but it resembles $Axsin^2(\omega x)$ under low frequencies. Under frequencies lower than 20kHz, the peak values increase in a roughly linear fashion, and appear with 3kHz intervals. Under frequencies between 20kHz and 50kHz, the peak values increase at lower speeds, and achieve a maximum value of 60 percent. It should be noted that due to limitations in the amount of computational resources we were able to utilize, we do not have any data on how the curve evolves at higher frequencies, so its entirely possible that higher efficiencies may be attained.

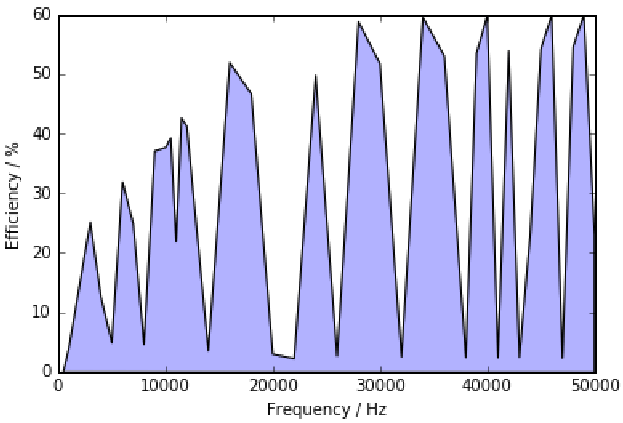


FIG.5 System efficiency under different frequencies.

Frequency / Hz	Efficiency / %
500	0.1004
1000	3.921
3000	25.14
5000	4.839
7000	24.82
9000	37.01
11500	42.58
14000	3.500
16000	51.89
18000	46.64
20000	2.928
22000	2.190
24000	49.77
26000	2.633
28000	58.75
30000	51.70
32000	2.470
34000	59.49
36000	52.98

TABLE.1 System efficiency under different frequencies.

Consequently, we explored the relationship between power output and frequency. It appeared that power generally decreases as frequency increases.

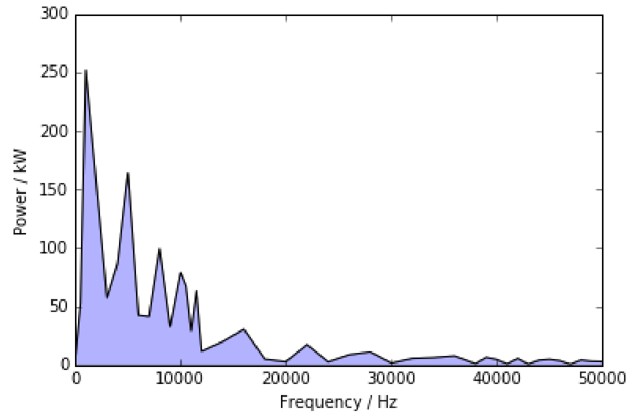


FIG.6 Power output under different frequencies.

Frequency / Hz	Power / kW
500	4.982
1000	53.21
3000	251.8
5000	86.55
7000	42.53
9000	99.51
11500	29.09
14000	11.82
16000	20.02
18000	30.63
20000	4.974
22000	2.805
24000	17.47
26000	2.672
28000	8.730
30000	11.15
32000	1.661
34000	6.287
36000	7.695

TABLE.2 Power output under different frequencies.

With these relations, we were able to predict the frequencies under which the efficiencies are at peak values. With the best coil we were able to produce, a 51.89% efficiency and a $20kW$ power output were achieved under 16000 Hz, with a 1000V peak voltage and a 1Ω load. A 59.49% efficiency and a $6.3kW$ power output was achieved under 34000 Hz, with a 1000V peak voltage and a 1 Ω load. These results are simply remarkable. The highest efficiency is comparable to that of wirelessly charging medical implant devices with a power output of mere milliwatts. Also, the length of the primary coils are 16 meters each, and the change of efficiency when the coil deviates from the central axis is 3 percent.

Just as importantly, the coils are more than affordable. If the coils are made of copper, whose price is calculated at a rate of 70 CNY per kilogram, the raw materials for the whole setup is less than 3 CNY per meter of road. The low price of the system means that wide adoption will be easy.

The absence of ferrite cores is another advantage. Though ferrite cores are able to concentrate the magnetic field flux and increase inductive coupling, they are generally expensive, and the eddy current would seriously heat up the system under high frequencies. In the following part, we will roughly discuss the effect that eddy current would have under alternating magnetic fields.

Consider a solenoid coil of diameter D and height h, with a cylindrical ferrite core installed, connected to an AC power source of frequency f.

Let

$$\tilde{B} = B_0 cos 2\pi f t \tag{60}$$

If we take an infinitesimal cylindrical slice of the core with radius r and thickness dr, then the magnetic flux within the slice would be

$$\Phi = \tilde{B}\pi r^2 \tag{61}$$

The induced electromotive force would be

$$\varepsilon = -\frac{d\Phi}{dt} = 2B_0\pi^2 r^2 f sin 2\pi f t \tag{62}$$

The resistance of the slice, given the material's electrical conductance σ,is

$$R = \frac{2\pi r}{h\sigma dr} \tag{63}$$

Therefore, the ohmic power generated by the slice is

$$dP = 2\pi^3 r^3 h\sigma f^2 B_0^2 sin^2 2\pi f t \cdot dr \tag{64}$$

The ohmic power generated by the entire ferrite core is

$$P = \int dP = \frac{1}{32}\pi^3 h\sigma f^2 D^4 B_0^2 sin^2 2\pi f t \tag{65}$$

The average ohmic power is

$$\overline{P} = \frac{1}{T}\int_0^T P \cdot dt \tag{66}$$

If we denote the root mean square of the magnetic field as B, then

$$B^2 = \frac{1}{2}B_0^2 \tag{67}$$

Therefore,

$$\overline{P} = \frac{1}{64}\pi^3\sigma f^2 B_0^2 D^4 h \tag{68}$$

As can be observed, the average power generated by the eddy current increases quadratically with the power source's frequency, which would result in detrimental overheating and power loss under the higher frequencies. Although our model is overly simplistic, without considering the hysteresis losses and anomalous losses, it demonstrates our point. Since our system usually operates at tens of kilohertz and dozens of kilowatts, the addition of ferrite cores is clearly not a feasible option for our case. Also, calculated at a price of 30000 CNY per cubic meter, the ferrite core alone would cost more than 80 CNY per meter of road, magnitudes more expensive than before.

VII Conclusion

From the empirical results, it can be observed that a simple pair of transmitting and receiving coils, without ferrite cores, is capable of achieving a 60% efficiency, as well as a high stability and a low cost, simply by having the novel shapes derived from our evolutionary algorithm. Therefore, the idea of using non-contact on-road charging as the solution to the short battery lives of electric vehicles is perfectly sound, and with enough resources spent in its name, it is entirely possible that this technology will have an unprecedented impact to our daily lives and to the environment. In fact, firms such as *Ingenieurgesellschaft Auto und Verkehr (IAV)*, *Google* and *General Motors* are already trying to propagate the concept of setting up on-road inductive charging lanes, while firms like *Toyota*, *Ford* and *Daimler* claim an interest, so together with the results made in this paper, it can be said that intense development will unquestionably be seen in this area.

The other noticeable merit of this essay is its new way of using computers in an engineering problem. Traditionally, engineers utilize computers only for immense physical calculations and simulations, while in this essay, computers have actually been active participants in the optimization process, rather than just inert calculators waiting for command inputs. This is representative of an important trend which requires close attention from people of all professions. As the thriving field of artificial intelligence rapidly develops, more and more of such instances will come into being, to an extent where nearly all of the analytical work would be completed by artificial neural networks encapsulating deep-thinking algorithms, with a stream of reasoning that we can't even begin to understand yet produces solutions far superior to human-designed ones. Just as how engineers and theoretical physicists had to become computer operators and programmers a few decades ago, physicists of the future must also double as experts in machine learning, neural networks, data mining and so forth, otherwise this profession would doubtlessly become obsolete in a matter of 15 years. *The Higgs Boson Machine-Learning Challenge* proposed by CERN [11] is one example of the role that machine learning plays in today's physical research, and the role that machine learning will play tomorrow. Therefore, scientists of today should boldly experiment with freshly new methods of research, the same as how Albert Einstein adopted non-euclidean geometry, how Werner Heisenberg adopted matrices, and how Paul Dirac adopted Hamilton's quaternions. It is highly probable that the next major breakthrough in physics would be thanks to the use of machine learning.

Acknowledgements

We give our most sincere gratitude to Shengli Geophysical Research Institute of Sinopec for kindly lending us the workstations on which we ran our evolutionary algorithm.

References

[1] Hornby, G. S., Globus, A., Linden, D. S., & Lohn, J. D. (2006). Automated Antenna Design with Evolutionary Algorithms. *Space*,.

[2] Cohoon, J.P., Hedge, S.U., Martin, W.N., Richards, D.(1988) Distributed Genetic Algorithms for the Floor Plan Design Problem. *Technical Report TR-88-12*. School of Engineering and Applied Science, Computer Science Department, University of Virginia.

[3] Whitley, D. & Starkweather, T. (1990) GENITOR II: A Distributed Genetic Algorithm. *Journal of Experimental and Theoretical Artificial Intelligence 2*: 189-214.

[4] Goldberg, D. (1989). Genetic Algorithms in Search, Optimization and Machine Learning. Reading, MA: Addison-Wesley Professional. ISBN 978-0201157673.

[5] Blickle, T., Thiele, L. (1995) A Mathematical Analysis of Tournament Selection. *In Proceedings of the Sixth International Conference on Genetic Algorithms (1995), pp. 9-16* .

[6] Jordan, C. (1887) Cours d'analyse, pp. 587-594

[7] Gauss, C. F. (1813) Theoria attractionis corporum sphaeroidicorum ellipticorum homogeneorum methodo nova tractata, *Commentationes societatis regiae scientiarium Gottingensis recentiores*, 2: 355-378

[8] Sunday, D. (2001) Inclusion of a Point in a Polygon *http://geomalgorithms.com/a03-_inclusion .html*

[9] Kurs, A., Karalis, A., Moffatt, R., Joannopoulos, J. D., Fisher, P., Soljačić, M. (2007) Wireless Power Transfer via Strongly Coupled Magnetic Resonances. *Science, vol.317, pp.83-86*

[10] Matsuda, Y., Sakamoto, H., Shibuya, H., & Murata, S. (2006). A non-contact energy transferring system for an electric vehicle-charging system based on recycled products. *Journal of Applied Physics*, 99(8).

[11]*http://home.cern/about/updates/2014/05/higgs-boson-machine-learning-challenge*