TRUCK SCHEDULING FOR FOODSTUFFS

ENGSCI 263: OR Project 2019

SYNOPSIS

An analysis of a vehicle routing problem for Foodstuffs NZ's supermarket chains in the Auckland region to determine an optimal truck schedule.

Jaime Wu

Team 14

EXECUTIVE SUMMARY

We have been approached by Foodstuffs NZ to determine the best way to route their trucking fleet to minimise total transportation costs. We approached Foodstuffs trucking schedule as a vehicle routing problem (VRP). First, we used a two-way ANOVA analysis to predict weekday and weekend demands for each supermarket type. I used the weekday with the most demands (Wednesday) to create a weekday routing schedule, and I used the Saturday demands to create a weekend routing schedule. Next, I split the supermarkets into different regions via k means clustering algorithm. For each region, I generated feasible routes by using two knapsack LPs and the cheapest insertion algorithm. I iterated and banned previous routes so I could obtain multiple feasible routes per region. After generating a set of routes, I formulated a mixed-integer program to find routes that were most optimal in minimising the total transportation cost. The optimal weekday routing schedule is about $9223.41, and the optimal weekend routing schedule is about $2945.89. Using bootstrap sampling of previous demand patterns and a time multiplier with various uniform distributions for travel times, we evaluated the effectiveness of our optimal solution to obtain a distribution of the transportation cost for Foodstuffs. We obtained a mean, 95% confidence interval, and prediction interval for both routing schedules. They are $10677.39, $10385.60-$10969.18, and $5473.9-$19184.31 respectively for the weekday schedule, and $8735.68, $8459.70-$9011.66, and $3324.60-$15614.19 for the weekend schedule.

We recommend that Foodstuffs increase the number of trucks to reduce the expense of using Mainfreight wet-leased trucks. If Foodstuffs decides to use our routing schedules, we suggest reducing the uncertainty in the data, i.e. demand and travel times. Foodstuffs can theoretically combine our ANOVA model with their supply chain data to reduce the uncertainty in demand predictions. As far as traffic forecasting is concerned, Foodstuffs could run a pilot trial period to holistically produce a more realistic distribution of the traffic multiplier. My final recommendation for Foodstuffs is that if they want to use our models, they should experiment with setting up a separate routing schedule for each day of the week and perform simulation on those routes.

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1 INTRODUCTION

Foodstuffs NZ operates the New World, Pak 'n Save and Four Square supermarket chains. Each supermarket requires the delivery of a specific number of pallets of goods each day. Pallets of goods are distributed from the central warehouse in Mt Roskill to each of their supermarkets around Auckland. The company's current trucking division contains a fleet of 10 trucks, operating across two four-hour shifts each day, starting at 8 am and 2 pm respectively. Each truck costs $150 per hour to operate, and $200 per hour to operate for every hour exceeding four-hours. On days where there are insufficient trucks to satisfy all demands, additional trucks can be "wet-leased" from Mainfreight for a cost of $1200 for every four-hours (charged in four-hour blocks). Both Foodstuffs and wet-leased Mainfreight trucks can carry up to 12 pallets of goods. The trucks operate on a trip schedule where each truck deliver goods to a selection of stores and then returning to the warehouse. Furthermore, due to Foodstuffs policy, supermarkets can only be visited once per day, so all the demand required by a store must be delivered by one truck. Henceforth, Foodstuffs NZ would like to determine the most optimal trucking schedule to route each truck while minimising the total transportation costs.

2 METHODS

We modelled Foodstuffs trucking schedule problem as a modified version of the vehicle routing problem (VRP), an extension of the well-known travelling salesman problem (VSP). We seek to heuristically generate a series of trucking routes, obtained and optimised through numerous assumptions and simplifications. Then we will use a mixed-integer program to optimise the trucking schedule by selecting routes that minimise the total cost of operation. We would then employ a Monte Carlo simulation of our routing schedule to determine the effectiveness as a solution for day to day scenarios.

2.1 DATA ANALYSIS

We approached data analysis by initially undertaking an exploratory analysis to examine the historical demands of each weekday for each supermarket type. From our exploratory analysis, we inferred that demands are dependent on the type of store and the day of the week. A series of boxplots and violin plots are shown in Appendix 1 to confirm our conclusions. We then formerly undertook a two-way ANOVA using two categorical variables: supermarket type and weekday to model the demands. The primary assumption for a two-way ANOVA is that the categorical variables are independent of each other, that is, we assumed the

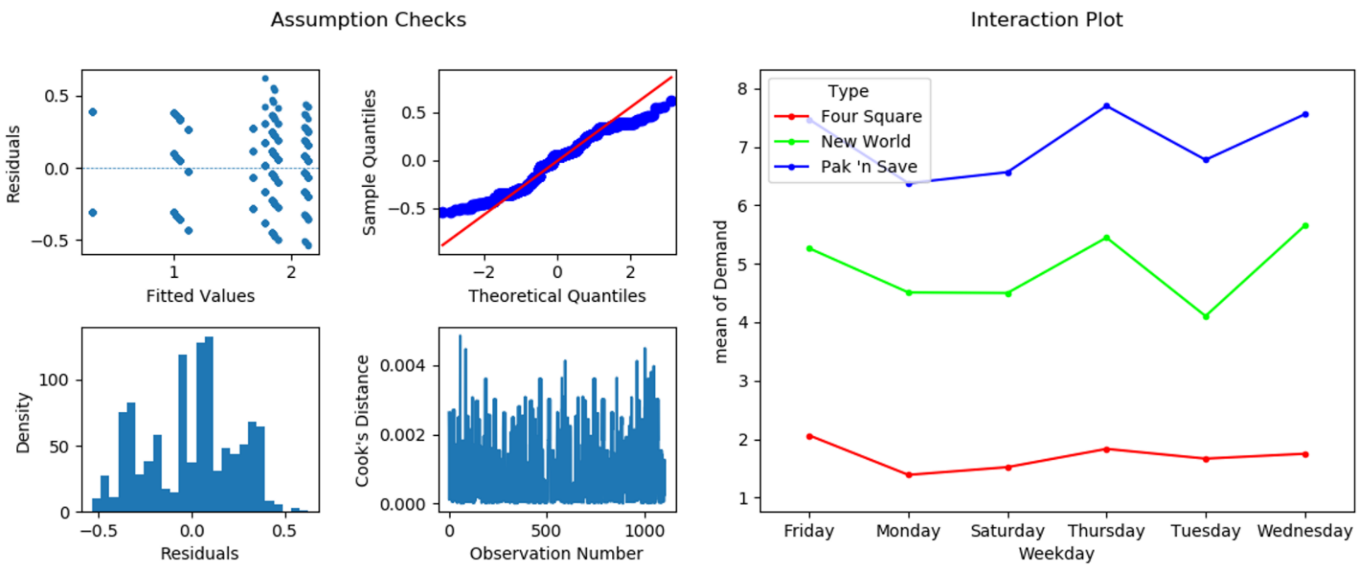


Figure 1: Assumption checks of two-way ANOVA (left). Interaction plot for demand and weekday (right).

supermarket type is not impacted by the day of the week (Appendix 1). We can safely make this assumption as we lack information regarding the supply chain operation of Foodstuffs.

The interaction plot (Figure 1) and our exploratory analysis confirm that if we break up demand by supermarket chains, we see three distinct demand models: Sunday, which is zero for every store; Saturday, which has reduced demand; and the demand over Monday-Friday, which has regular demand.

The normality of the two-way ANOVA (Figure 1) appears to be mostly satisfied, and the variance of residuals seems to be mostly equal. Therefore, we used our ANOVA model for predicting the demand of the different supermarket types based on the day of the week (Figure 2).

In our analysis, we simplified the number of supermarket types by including Fresh Collective Alberton as a Four Square because it is currently still owned and operated under Four Square, despite the rebranding. Therefore, our cleaned demand data set only contains three supermarket types: New World, Pak 'n Save and Four Square.

Supermarket Type	Weekday	Demand
New World	Monday	5
Pak 'n Save	Monday	7
Four Square	Monday	2
New World	Tuesday	5
Pak 'n Save	Tuesday	8
Four Square	Tuesday	2
New World	Wednesday	6
Pak 'n Save	Wednesday	7
Four Square	Wednesday	2
New World	Thursday	5
Pak 'n Save	Thursday	8
Four Square	Thursday	2
New World	Friday	5
Pak 'n Save	Friday	7
Four Square	Friday	2
New World	Saturday	2
Pak 'n Save	Saturday	4
Four Square	Saturday	0

Figure 2: Supermarket demands from ANOVA model

2.2 ROUTE GENERATION

I used our predicted demands from our ANOVA model to generate feasible trucking routes. I approached the problem by first separating the supermarkets into geographical regions. Although I could have manually done this, I decided to use the k means clustering algorithm to generate multiple regions easily. For each region, I solved two knapsack LPs to identify a set of supermarkets that are valid to form a route (node sets). One LP find feasible node sets by maximising the number of supermarkets being visited. The second LP minimises the number of supermarkets to include per node set but has an additional minimum demand constraint. I used two knapsack LPs to increase the number of routes as well as the variation of routes.

The objective function of both knapsack LPs is the sum of all binary decision variables representing whether a supermarket is selected for in the node set. For one knapsack LP, we would maximise this objective function, and we would minimise for the other. Both knapsack LPs is subject to the following constraint: $\sum_i d_i n_i \leq 12$, where the total demand for a route is less than 12. The knapsack LP minimising the objective function is subject to the additional minimum demand constraint: $\sum_i d_i n_i \geq min$, where min refers to the minimum demand that we must have in the node set to ensure a feasible solution. The full formulation of both LPs can be found in Appendix 2. Next, I used the node set to formulate and solve a TSP heuristically using the cheapest insertion algorithm. The outcome was the most optimum route to traverse through the supermarkets with the least amount of time. If the most optimal time for a particular route exceeded four hours, I would discard the solution because the extra-time is charged at $200 per hour, so it would be more expensive for Foodstuffs. Finally, I iterated and resolved the two knapsack LPs while banning all previous optimal solutions to obtain more routes. My route generation model can be found in Appendix 3. Using my model, I generated a set of feasible routes for a weekday solution using the highest demands (Wednesday). I also generated a set of feasible routes for a weekend solution using Saturday demands since demands are zeros on Sundays.

2.3 ROUTE SELECTION

The generated sets routes were first duplicated and assigned a cost of $1200 to accommodate for Mainfreight. Then I used those routes as the binary decision variables of a mixed-integer program to determine the least-cost routing schedule. The binary decision variables represent whether the route is chosen or not, where 1 is chosen, and 0 is otherwise. The objective function of the LP is then to minimise the sum of the total cost. Next, I assumed that the travel time of each route is independent of the shift. Therefore, we have an equivalent of 20 trucks, since each truck operates two four-hour shifts. Therefore, the LP is subject to the following constraints:

1. The sum of all routes containing supermarket_i must be equal to one to ensure that each supermarket is visited once.
2. The sum of all routes traversed by Foodstuffs' trucks must be less than or equal 20.

The full formulation of the mixed-integer LP can be found in Appendix 4. We used this route selection LP to generate an optimum weekday solution based on Wednesday demand, and an optimum weekend solution based on Saturday demand.

2.4 Simulation

We evaluated the quality of our routing schedule by creating a Monte Carlo simulation. We ran our simulation model 1000 times to estimate the distribution of the optimal transportation cost. Our simulation model assumed that the sources of uncertainty were only limited to the demand per supermarket, and the travel time of each route due to traffic congestion.

Generation of realistic supermarket demands was achieved by bootstrapping the demand dataset (resampling the empirical distribution with replacement). Traffic congestion was modelled using a multiplier with various uniform distributions as a result of the insufficient traffic data. A uniform distribution would better represent traffic since we would give an equal probability for all traffic situations to occur. The traffic multiplier distributions we used can be summarised as follows: uniform between 1-1.5 times (weekday mornings), uniform between 1-2 times (weekday afternoons), uniform between 1-1.25 times (weekend mornings), and 1-1.4 times (weekends afternoons). The range of the uniform distributions was pragmatically assumed from prior empirical observation of traffic congestion in the Auckland region.

Our simulation model arbitrarily splits the optimal routing schedules into morning and afternoon shifts, assuming there would be negligible amounts of variation in our final result. For each shift, we then apply the traffic multiplier based on the distributions mentioned earlier and then sum up the total travel time to traverse all routes. The optimal cost can then be determined as follows: $150 per hour of operation, $200 per hour of operation for every hour exceeding four hours. A crucial simplification in our simulation model is that we will use the equivalent of 20 trucks, each with a demand of 12, to traverse all routes and lump sum the demand. It means that we do not need to enumerate through each route and take out supermarkets that make the route infeasible. We ensure the maximum use of Foodstuffs' trucking fleet, irrespective of whether the route will exceed the demand of a truck. If the lumped sum demand is greater than 240, then the excess demand will need to be supported by wet-leasing trucks from Mainfreight. This simplification means that we may double up deliveries but was used as we assumed that it would not affect the optimal solution to a significant degree. The output of our simulation is then a distribution of the optimal transportation cost of our trucking schedules with a sample size of 1000.

3 Results

3.1 Summary of Solution

I used Wednesday demands to generate the optimal weekday routing schedule as it has the highest demand. As we noticed that Sunday has zero demand, the optimal weekend routing schedule used Saturday's demand.

These demands predictions are from our ANOVA model in Figure 2. The full solution of both the weekday and weekend model is presented in Appendix 5. To summarise our solution:

- The optimal transportation cost for the weekday schedule is $2945.89
- The optimal transportation cost for the weekday schedule is $9223.41

3.2 Discussion of Solution

The optimal routing schedule for weekday and weekend is shown in Figure 3. A map showing each route as a straight line, with reduced overlapping, can be found in Appendix 6.

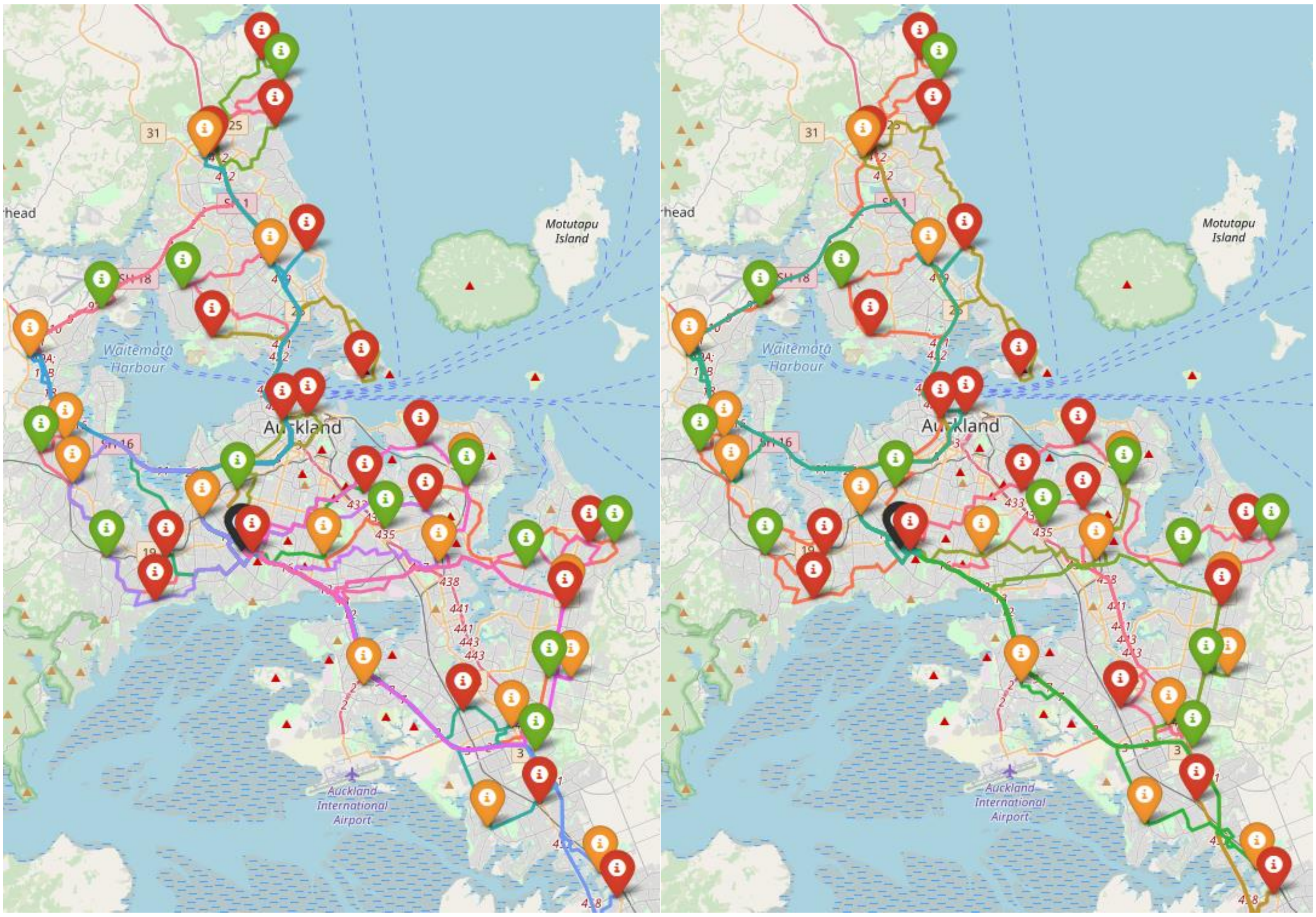


Figure 3: Weekday routing schedule (left). Weekend routing schedule (right).

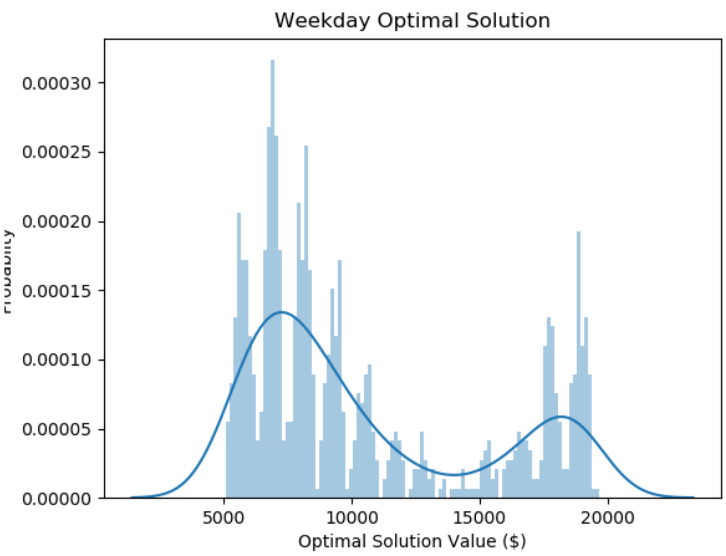


Figure 4: Distribution of the cost of operating the weekday routing schedule

The results of our Monte Carlo simulation of the above trucking schedules are shown in Figure 4 and Figure 5. We noticed that both routing schedules exhibit a general bimodal distribution; however, the intensity of this bimodal distribution is less prominent in the weekend schedule compared to the weekday schedule. We believe that this difference can be potentially linked to the limited variation of the weekend demand data, as shown by our exploratory analysis and our two-way ANOVA (Appendix 1). Additionally, we believe that we may have underestimated demand for some days using ANOVA, which is why we have a bimodal distribution.

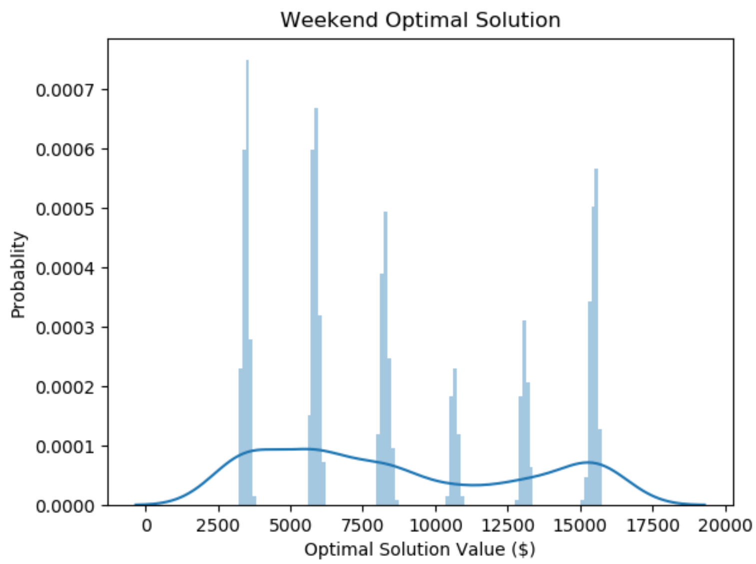


Figure 5: Distribution of the cost of operating the weekend routing schedule

The most important aspect of the two distributions is that they are composed of smaller, normally distributed spikes. We reasoned that these spikes are correlated with demands being integer values. Each spike would correlate with the different combinations of demands per store to make up total demand for a particular simulation. Furthermore, each spike displays some noise and uncertainty, which we believe is linked to our model of the distribution of the traffic multiplier affecting travel time. Although we modelled the distributions of the traffic multipliers as being uniform, we are uncertainty as to why each spike looks normally distributed. Perhaps, there are some inherent supply chain factors that we have not accounted for in our model. Those factors may influence two or more categorical variables being dependent on each other, but our demand model assumed independence.

From the two distributions, we calculated the mean, confidence intervals (CI), and prediction intervals (PI) (Figure 6). In Appendix 7, I have overlaid the intervals with Figure 4 and Figure 5. The mean, CI, and PI of the weekday routing schedule are larger than that of the weekend routing schedule. The result confirms that our model works, given that demand is reduced on Saturdays, and thus we would expect it to have a lower cost. However, I would have expected the weekend costs to be much smaller as we are delivering fewer pallets. I believe that this is due to our ANOVA model predicting Four Square chains as having zero demand in the weekend. The routes we produced took advantage of Four Square's lack of demand, resulting in overly long routes. Accompanying our model prediction is the fact that bootstrapping demand would likely result in Four Square actually having demand. As our model does not consider this, the cost of our weekend solution is higher than expected, and we can amend this by generating a new set of routes while using a different demand model.

```
Weekday
Mean: 10677.394749466646
Conf_Int: 10385.604673457718 10969.184825475575
Pred_Int: 5473.903713625866 19184.30619975948
Weekend
Mean: 8735.677455321187
Conf_Int: 8459.695237765496 9011.659672876878
Pred_Int: 3324.5978219199055 15614.189474744664
```

Figure 6: Python output for mean, confidence intervals, and prediction intervals of both routing schedules

The spread of the distribution for the weekday solution is between \$5000 and \$20000 whereas the spread of this distribution for the weekend solution is between \$3500 and \$16000, so our solution still makes sense despite a slightly wrong demand estimate for Four Square. The small CI interval of approximately \$500, where 95% of sample means will likely lie within, is due to our manual route partitioning into the morning and afternoon shift. The other variances in our simulation are not significant enough to result in a different distribution each time.

The routing schedules from our model is specific to the scenarios modelled, and deviations may invalidate the results, as indicated by our simulation results. Therefore, there are potential problems with Foodstuffs applying our routing schedule in a real-life scenario. The prediction interval of our simulation is quite large for both weekday and weekend schedules, so there will be a great deal of variability in using our solution in real life. We would not expect our solution to provide the optimal cost of \$9223.41 and \$2945.89, rather 95% of the time we would find it between the prediction interval of \$5473.9-\$19184.31 and \$3324.60-\$15614.19.

I think that our solution can be improved if we considered a routing schedule for every day of the week instead of modelling all five days using Wednesday's demands. As inferred from our simulation results, the variability in demand causes most of the spread in our distribution (the different spikes). The uncertainty of traffic does not affect our solution with the same degree, merely the noise of each spike. Therefore, it is likely that the optimal routes will change each day, given that the demands vary daily. However, if we were to model the routing schedule for every day of the week, we may overfit our solution to the current demand data set, which may become less feasible for future uses of our solution. If the project had been given sufficient time, I think we would be able to compare and contrast our current approach of generating a routing schedule, a weekday and weekend solution, against a routing schedule for every day of the week. Furthermore, the distribution of travel times in real life would be bimodal, but we had chosen to use a uniform distribution for implication. If our routing schedule is used for a real scenario, our results may under or overshoot the actual cost, and this cannot be captured in our current model as seen via our simulation.

3.3 CONCLUSION & RECOMMENDATION

With our routing schedule, Foodstuffs is expected to pay between \$5000 and \$20000 for weekdays, and between \$3500 and \$15500 for weekends. However, due to the strong bimodal distribution for the weekday solution, Foodstuffs can expect, on some days, the cost of operation to be much lower at around \$5000 to \$10000, and on other days to be much higher around \$15000 to \$20000. The weaker bimodal distribution for the weekend solution means that I cannot make any claims other than the cost being between \$3500 and \$15500. We concluded that for each given weekday or weekend situation, our ideal routing plan may not be the most optimal, so we have suggestions to proceed below.

As our weekday routing schedule requires the use of wet-lease trucks from Mainfreight, one recommendation would be for Foodstuffs to increase the number of trucks in their trucking fleet as this would reduce cost. It only cost \$150 per hour to operate a Foodstuffs truck, whereas the use of a Mainfreight truck cost \$1200 for a four-hour block.

Due to the large variation in the costs of our solution, I would recommend Foodstuffs to further investigate the sources of variation in our model, namely demand and travel time. Given that Foodstuffs would have information regarding their supply chain, I would highly recommend Foodstuffs to integrate that data into our demand model to reduce variability. As far as traffic forecasting is concerned, Foodstuffs could run a pilot trial period to holistically produce a distribution that would represent the traffic multiplier more realistically.

My final recommendation for Foodstuffs is that if they want to use our models, they should experiment with setting up a separate routing schedule for each day of the week and perform simulation on those routes. From our analysis, we inferred that this improvement will significantly reduce the variability of our current solution.

Ultimately, our simplification of the problem along with its key assumptions meant that our results contain a lot of variation. However, our model produces two critical routing schedules, a weekday schedule and a weekend schedule, for Foodstuffs to minimise the transportation costs while meeting the demands of each supermarket.

4 References

Kalvelagen, E. (2011). Integer Cuts. Retrieved from http://yetanothermathprogrammingconsultant.blogspot.com/2011/10/integer-cuts.html

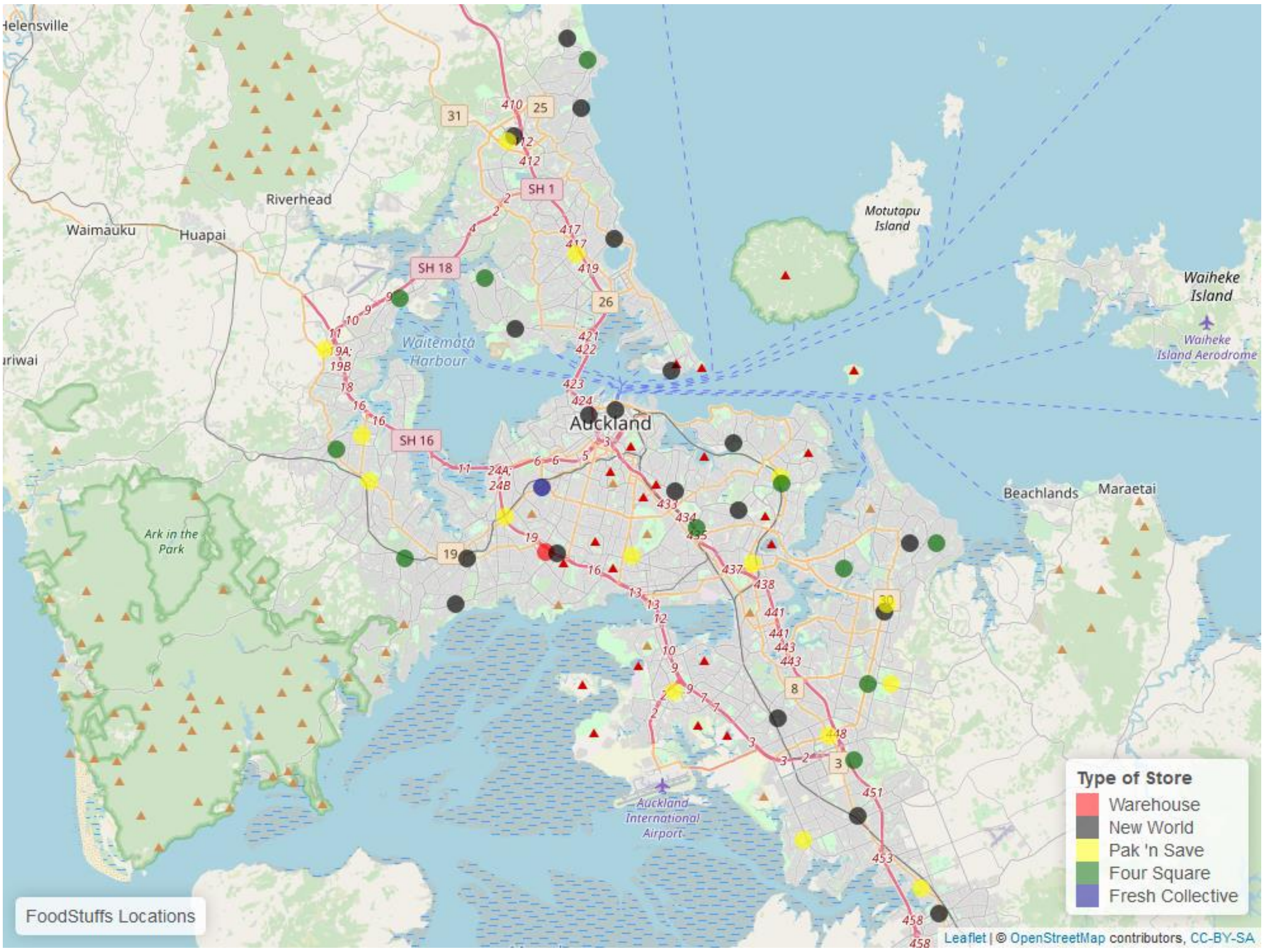
5 Appendices

5.1 Appendix 1

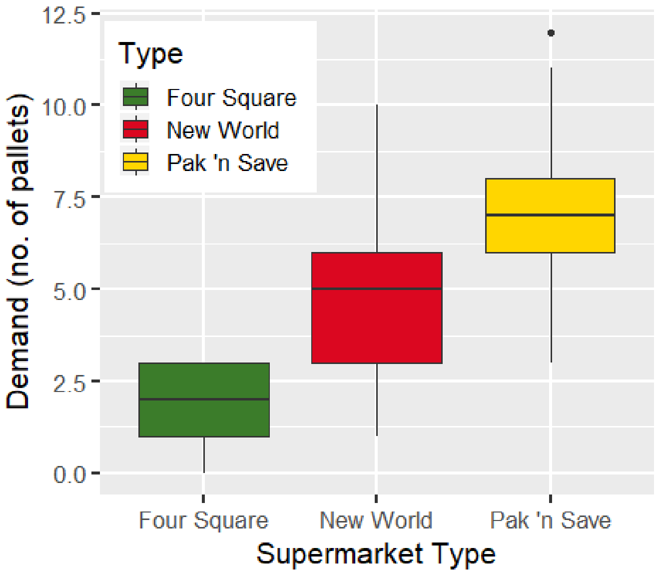
We began data analysis by converting the demand dataset into a tidy format; the dates were changed into the day of the week, and the data was sorted into four categorical variables: Supermarket, type, demand, and weekday.

The distribution of each supermarket chain's demand for all days of the week can be seen in the figures below. We noticed that the demand for all supermarket chains had a similar distribution throughout the weekday, but demand was reduced on Saturdays and zero on Sundays.

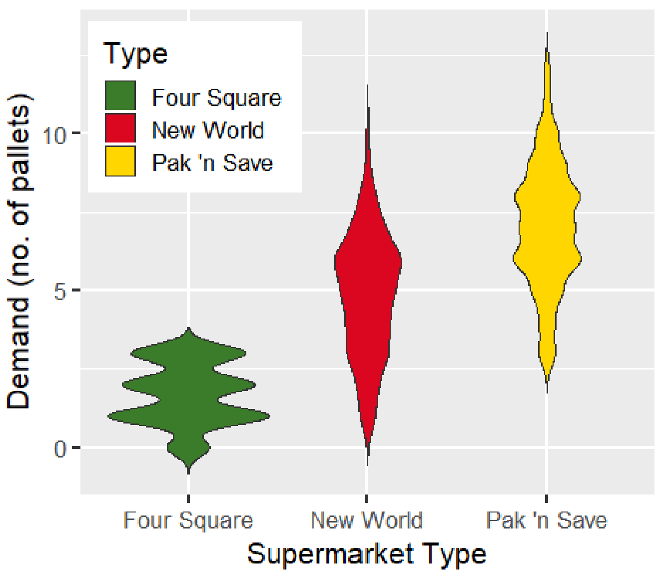
We notice that the demand mostly resembles a normal distribution for Pak 'n Save and New World; however, demand is left-skewed for Four Square. The spread is similar between Pak 'n Save and New World, but much narrower for Four Square. Finally, the median demand from highest to lowest is Pak 'n Save, New World, and Four Square. Intuitively, the demand data is expected as Four Square is more of a convenient store whereas Pak 'n Save, and New World are both what we consider as "supermarkets".



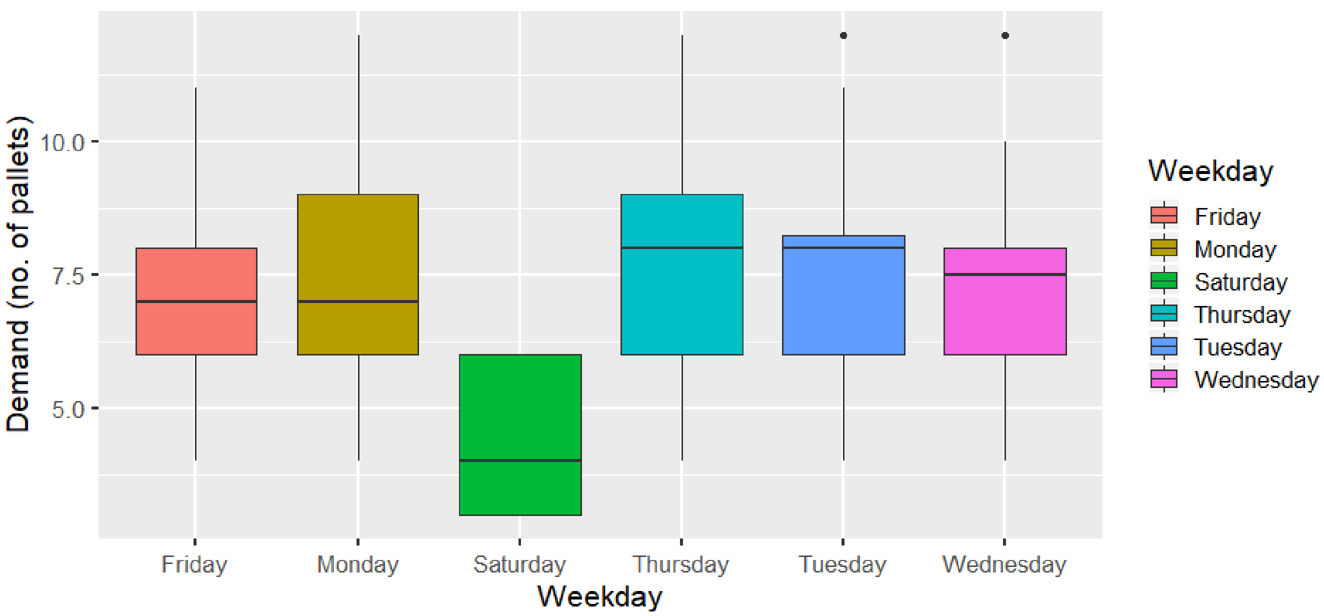
Box Plot Of Demand Across the Supermarket Types



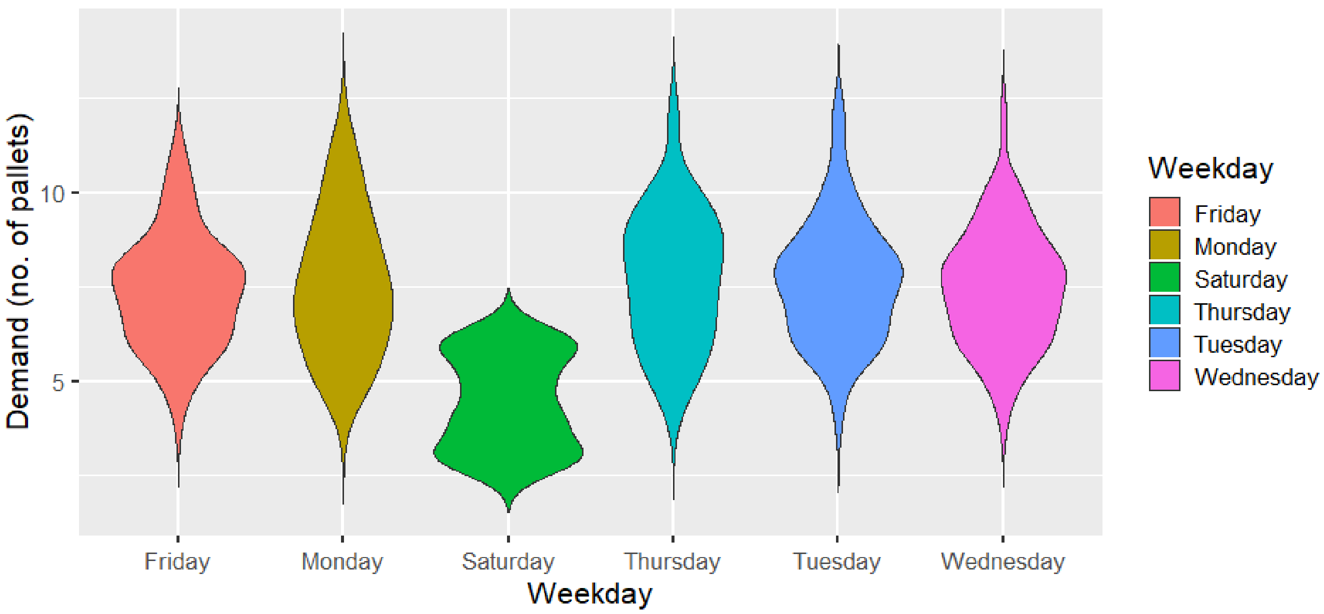
Violin Plot Of Demand Across the Supermarket Types

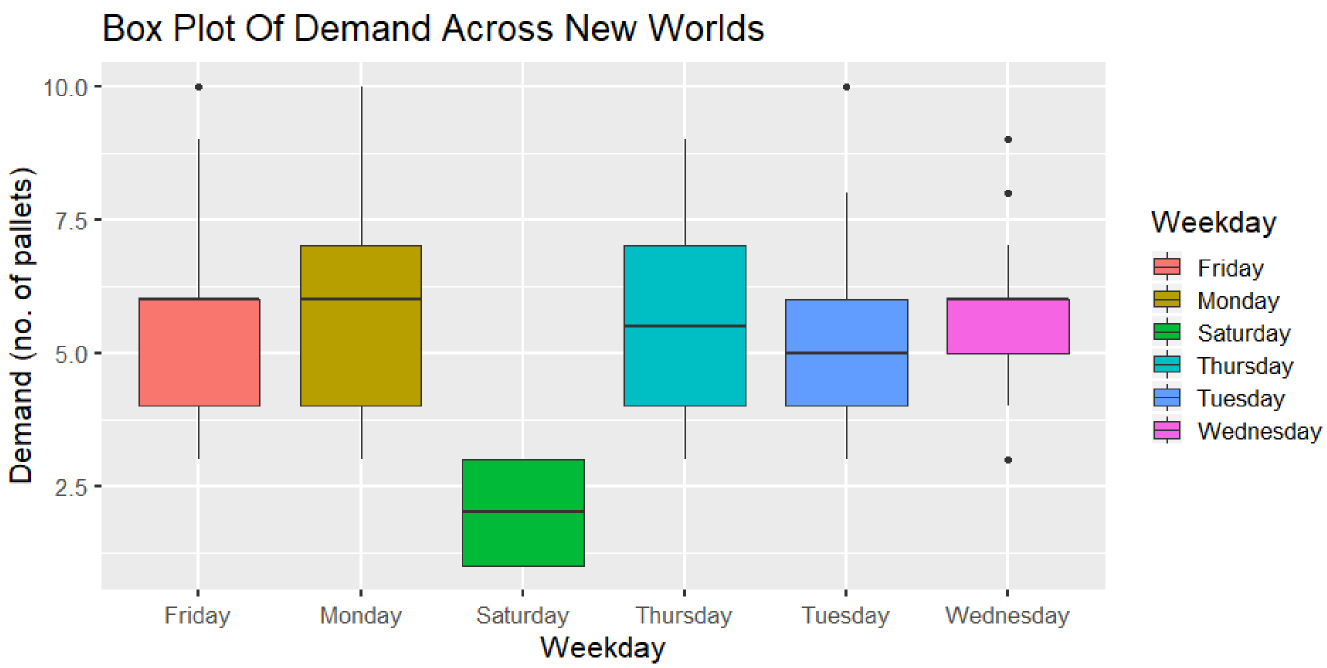


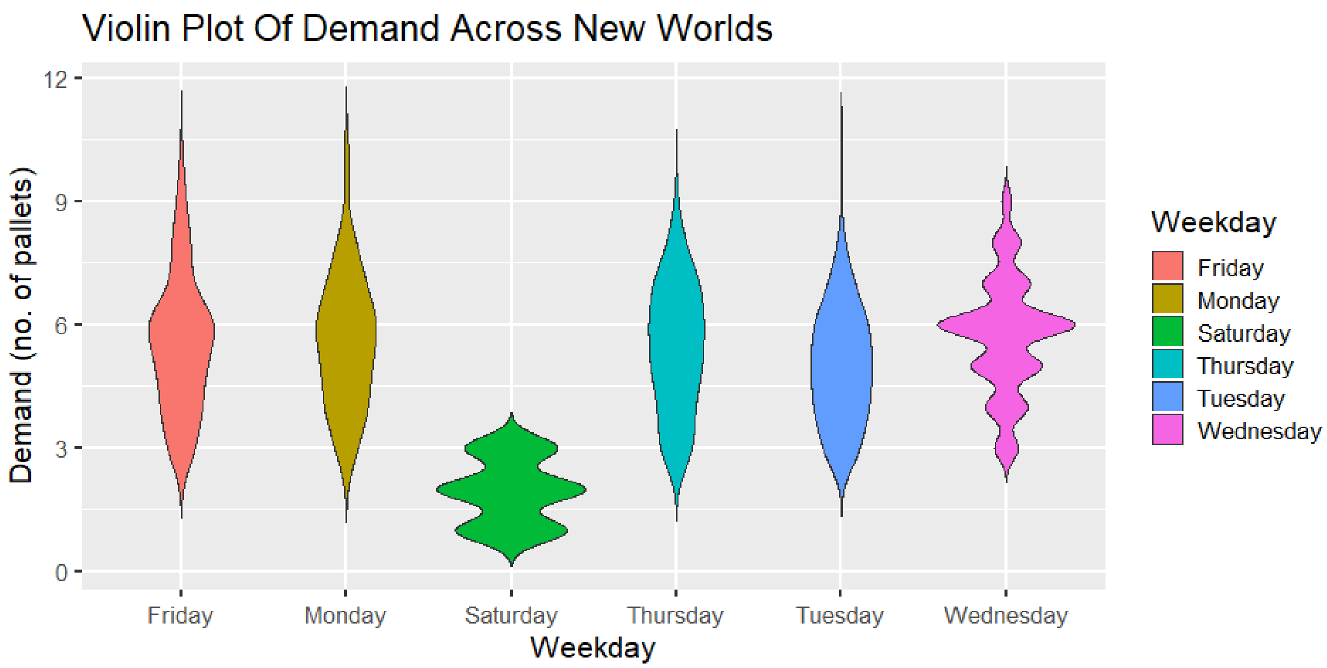
Box Plot Of Demand Across Pak 'n Saves

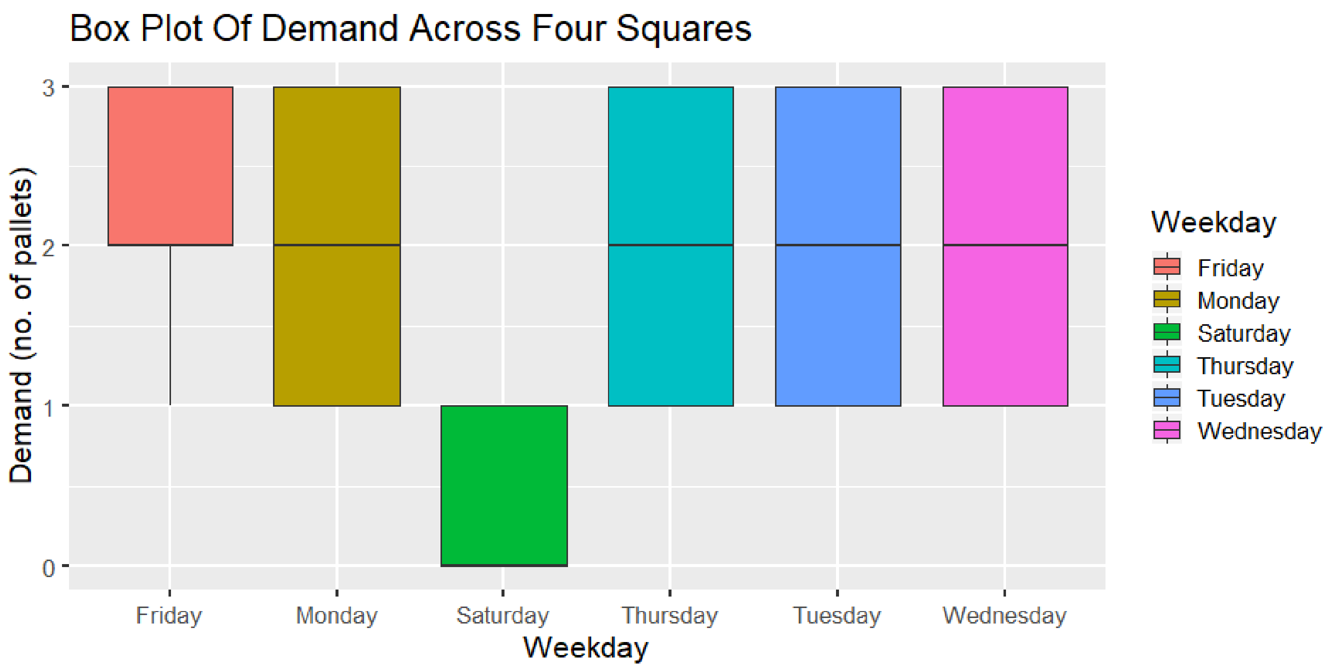


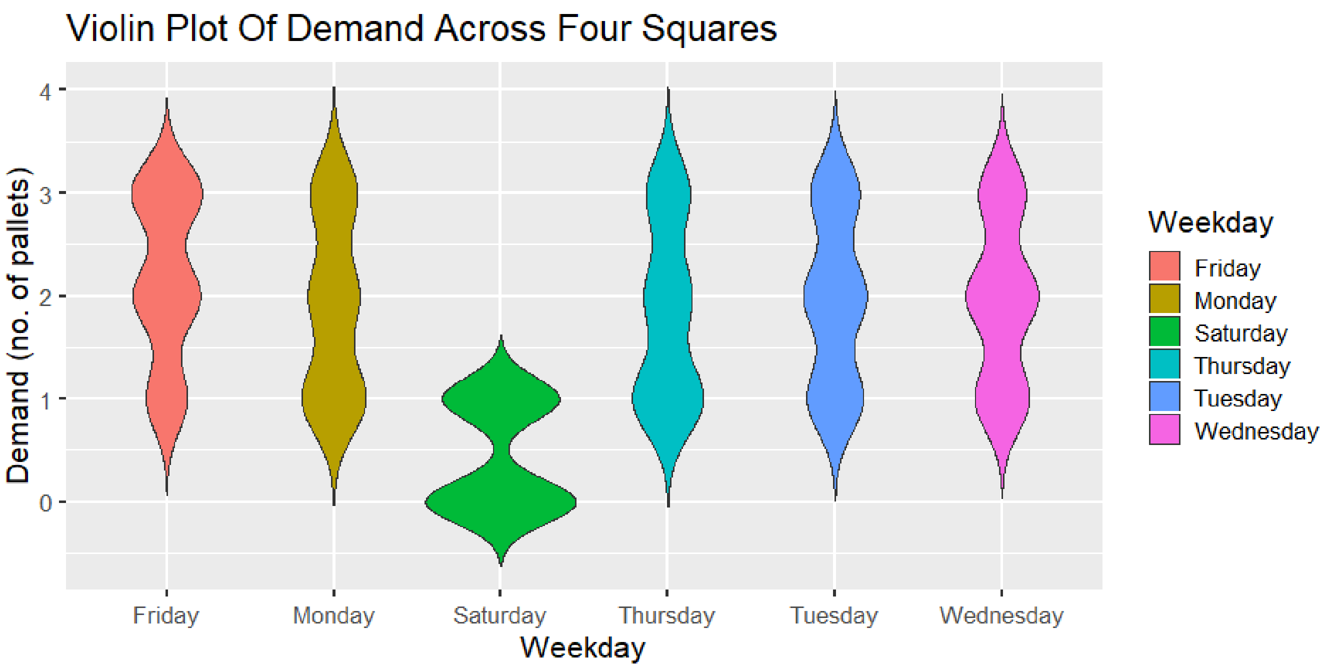
Violin Plot Of Demand Across Pak 'n Saves




Box Plot Of Demand Across New Worlds
Demand (no. of pallets)
10.0
7.5
5.0
2.5
Friday
Monday
Saturday
Thursday
Tuesday
Wednesday
Weekday
Weekday
Friday
Monday
Saturday
Thursday
Tuesday
Wednesday


Violin Plot Of Demand Across New Worlds
Demand (no. of pallets)
12
9
6
3
0
Friday
Monday
Saturday
Thursday
Tuesday
Wednesday
Weekday
Weekday
Friday
Monday
Saturday
Thursday
Tuesday
Wednesday


Box Plot Of Demand Across Four Squares
Demand (no. of pallets)
3
2
1
0
Friday
Monday
Saturday
Thursday
Tuesday
Wednesday
Weekday
Weekday
Friday
Monday
Saturday
Thursday
Tuesday
Wednesday




```
                  sum_sq      df            F        PR(>F)
Weekday        69.084999     5.0   226.802055  8.480841e-166
Type          210.153164     2.0  1724.801704   0.000000e+00
Weekday:Type    4.145087    10.0     6.804040   2.350472e-10
Residual       66.160167  1086.0          NaN            NaN
                            OLS Regression Results
==============================================================================
Dep. Variable:     np.log(Demand + 1)   R-squared:                       0.811
Model:                            OLS   Adj. R-squared:                  0.808
Method:                 Least Squares   F-statistic:                     273.6
Date:                Tue, 22 Oct 2019   Prob (F-statistic):               0.00
Time:                        18:10:41   Log-Likelihood:                -12.840
No. Observations:                1104   AIC:                             61.68
Df Residuals:                    1086   BIC:                             151.8
Df Model:                          17
Covariance Type:            nonrobust
==============================================================================================
                                              coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------------
Intercept                                   1.1196      0.036     31.426      0.000       1.050       1.189
Weekday[T.Monday]                          -0.0975      0.050     -1.936      0.053      -0.196       0.001
Weekday[T.Saturday]                        -0.8163      0.050    -16.202      0.000      -0.915      -0.717
Weekday[T.Thursday]                        -0.1204      0.050     -2.390      0.017      -0.219      -0.022
Weekday[T.Tuesday]                         -0.0722      0.050     -1.433      0.152      -0.171       0.027
Weekday[T.Wednesday]                       -0.0697      0.050     -1.384      0.167      -0.169       0.029
Type[T.New World]                           0.7225      0.046     15.877      0.000       0.633       0.812
Type[T.Pak 'n Save]                         0.9926      0.048     20.767      0.000       0.899       1.086
Weekday[T.Monday]:Type[T.New World]         0.1103      0.064      1.713      0.087      -0.016       0.237
Weekday[T.Saturday]:Type[T.New World]       0.0205      0.064      0.319      0.750      -0.106       0.147
Weekday[T.Thursday]:Type[T.New World]       0.1097      0.064      1.705      0.089      -0.017       0.236
Weekday[T.Tuesday]:Type[T.New World]        0.0005      0.064      0.008      0.994      -0.126       0.127
Weekday[T.Wednesday]:Type[T.New World]      0.1133      0.064      1.761      0.079      -0.013       0.240
Weekday[T.Monday]:Type[T.Pak 'n Save]       0.1074      0.068      1.589      0.112      -0.025       0.240
Weekday[T.Saturday]:Type[T.Pak 'n Save]     0.3742      0.068      5.537      0.000       0.242       0.507
Weekday[T.Thursday]:Type[T.Pak 'n Save]     0.1521      0.068      2.250      0.025       0.019       0.285
Weekday[T.Tuesday]:Type[T.Pak 'n Save]      0.1024      0.068      1.515      0.130      -0.030       0.235
Weekday[T.Wednesday]:Type[T.Pak 'n Save]    0.0791      0.068      1.170      0.242      -0.054       0.212
==============================================================================
Omnibus:                      160.647   Durbin-Watson:                   2.161
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               42.247
Skew:                          -0.138   Prob(JB):                     6.70e-10
Kurtosis:                       2.082   Cond. No.                         28.7
==============================================================================
```

5.2 APPENDIX 2

Knapsack LP: Maximise the number of supermarkets in a route

$$n_i = \begin{cases} 1 & Chosen \\ 0 & Otherwise \end{cases} \quad where \quad i \in \{0, 1, \dots supermarkets\}$$

The binary decision variables n_i represents whether a supermarket (node) is within current node set for a given region. We are maximising the number of supermarkets, so the objective function is:

$$\max \sum_i n_i$$

subject to the following constraints:

$$\sum_i d_i n_i \leq 12$$

Where d_i refers to the demand of that particular supermarket, and the sum of all demands per node set to be less than the capacity of the truck of 12 pallets.

Knapsack LP: Minimise number of supermarkets in route

The formulation for this knapsack LP is equivalent to the formulation above except we minimise the objective function:

$$\min \sum_i n_i$$

Additionally, we add the follow constraint:

$$\sum_i d_i n_i \geq min$$

Where min is the minimum demand, we must have in the node set to ensure that minimising the node set does not actually result in zero supermarkets.

5.3 APPENDIX 3

Forming Multiple Optimal Solutions for Knapsack LPs

```
Algorithm: Banning Previous Optimal Solutions
1: for i to n do
2:     solve current LP
3:     if solution = "optimal" then
4:         use cheapest insertion to find optimal route
5:         determine cardinality of the set of decision variables
6:         add constraint to ban previous optimal solution
7:     else if solution != "optimal" then
8:         break
```

The following constraint to ban previous optimal solution is:

$$\sum_{i|y^*_i=1} y_i - \sum_{i|y^*_i=0} y_i \leq card\{i|y^*_i = 1\} - 1$$

(Kalvelagen, 2011)

Where the first summation term is all the decision variables with a binary value of 1 and the second summation term is all the decision variables with a binary value of 0. The right hand side is one less the cardinality of the set of decision variables with a binary value of 1.

5.4 APPENDIX 4

$$r_i = \begin{cases} 1 & Chosen \\ 0 & Otherwise \end{cases} \quad where \quad i \in \{0, 1, \dots n\}$$

Where r_i refers to a route (including both Foodstuffs' trucks and Mainfreight's wet-leased trucks).

$$n_{ij} = \begin{cases} 1 & Supermarket\ Chosen\ on\ route\ i \\ 0 & Otherwise \end{cases} \quad where \quad i \in \{0, 1, \dots n\}, j \in \{0, 1, \dots 46\}$$

Minimise cost of routing schedule:

$$\min \sum_i c_i r_i$$

Subject to the following constraints:

$$\sum_{ij} r_i n_{ij} = 1$$

$$\sum_i r_i \leq 20$$

Where c_i refers to the cost of taking that route; $150 per hour for Foodstuffs, and $1200 per Mainfreight truck.

5.5 APPENDIX 5

Optimal Trucking Schedule for Weekday

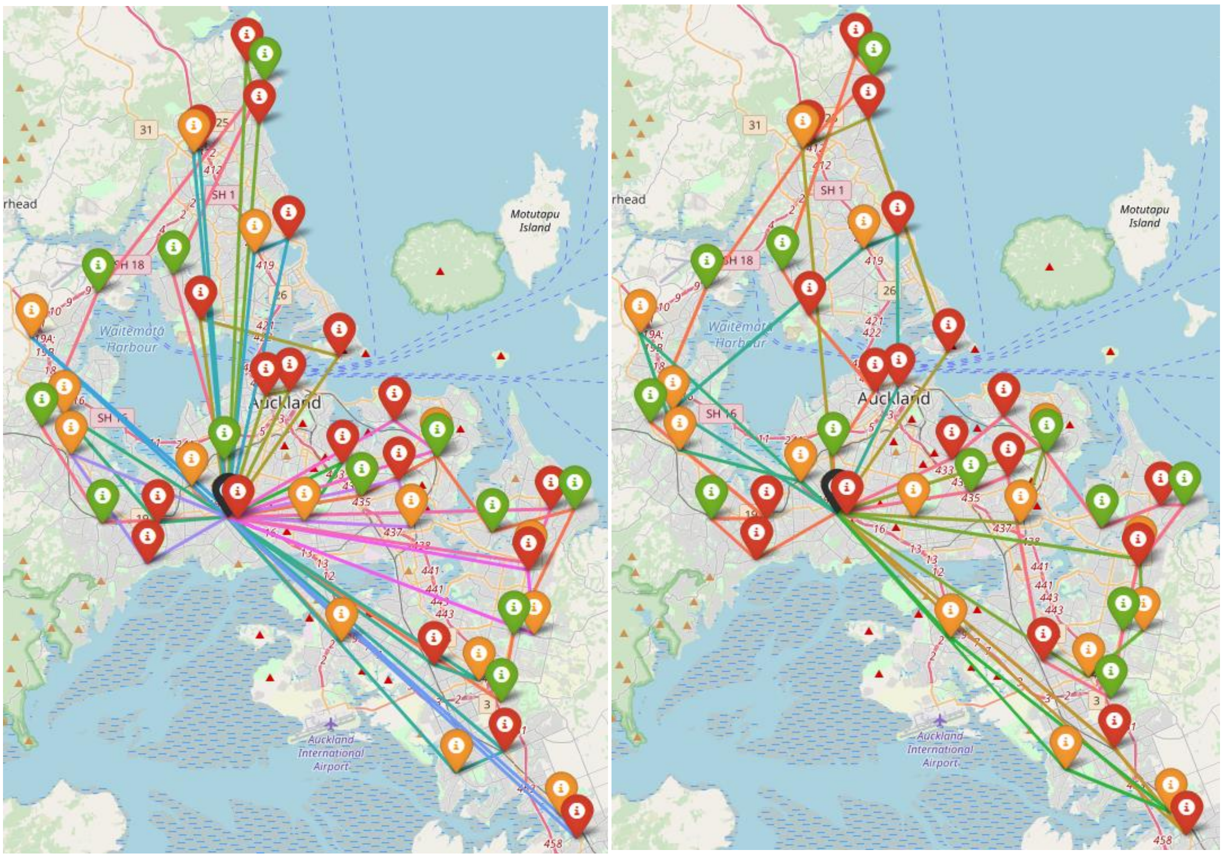
Route	Travel Time	Cost
['Warehouse', "Pak 'n Save Albany", 'Warehouse']	4849.26	202.0525
['Warehouse', "Pak 'n Save Royal Oak", 'Warehouse']	1898.04	79.085
['Warehouse', "Pak 'n Save Westgate", 'Warehouse']	3511.82	146.3258
['Warehouse', "Pak 'n Save Mt Albert", 'Warehouse']	1341.92	55.91333
['Warehouse', "Pak 'n Save Sylvia Park", 'Warehouse']	3273.52	136.3967
['Warehouse', "Pak 'n Save Clendon", 'Warehouse']	3751.94	156.3308
['Warehouse', "Pak 'n Save Manukau", 'Warehouse']	3439.28	143.3033
['Warehouse', "Pak 'n Save Mangere", 'Warehouse']	2483.32	103.4717
['Warehouse', 'New World Birkenhead', 'New World Victoria Park', 'Warehouse']	4619.4	192.475
['Warehouse', 'New World Howick', 'New World Botany', 'Warehouse']	5704.21	237.6754
['Warehouse', 'New World Stonefields', 'New World Papatoetoe', 'Warehouse']	4780.56	199.19
['Warehouse', 'New World Papakura', 'New World Southmall', 'Warehouse']	5536.96	230.7067
['Warehouse', "Pak 'n Save Wairau Road", 'Fresh Collective Alberton', 'Warehouse']	4700.53	195.8554
['Warehouse', 'Four Square Fair Price Henderson', "Pak 'n Save Lincoln Road", 'Warehouse']	3877.44	161.56

['Warehouse', "Pak 'n Save Henderson", 'Four Square Glen Eden', 'Warehouse']	4103.25	170.9688
['Warehouse', "Pak 'n Save Glen Innes", 'Four Square Great Eastern', 'Warehouse']	4519.72	188.3217
['Warehouse', "Pak 'n Save Botany", 'Four Square Pakuranga Heights', 'Warehouse']	5045.14	210.2142
['Warehouse', 'New World Metro Queen St', 'New World Mt Roskill', 'Warehouse']	3186.5	132.7708
['Warehouse', 'New World Green Bay', 'New World New Lynn', 'Warehouse']	2712.54	113.0225
['Warehouse', 'New World Eastridge', 'New World Remuera', 'Warehouse']	4026.57	167.7738
['Warehouse', 'Four Square Lancaster', 'New World Browns Bay', 'Four Square BKs Torbay', 'Four Square Hobsonville', 'Warehouse']	8671.75	1200
['Warehouse', 'New World Devonport', 'New World Albany', 'Warehouse']	7081.89	1200
['Warehouse', "Pak 'n Save Papakura", 'Four Square Cockle Bay', 'Four Square Botany Junction', 'Warehouse']	8095.63	1200
['Warehouse', 'New World Long Bay', 'New World Milford', 'Warehouse']	6894.37	1200
['Warehouse', 'Four Square Everglade', "Pak 'n Save Ormiston", 'Four Square Ellerslie', 'Warehouse']	5822.71	1200

Optimal Trucking Schedule for Weekend

Route	Travel Time	Cost
['Warehouse', 'New World Green Bay', 'Four Square Glen Eden', 'New World New Lynn', 'Four Square Fair Price Henderson', 'Four Square Hobsonville', 'New World Albany', 'Four Square BKs Torbay', 'New World Long Bay', 'Four Square Lancaster', 'New World Birkenhead', 'New World Victoria Park', 'Fresh Collective Alberton', 'Warehouse']	14134.59	588.9412
['Warehouse', 'Four Square Ellerslie', 'New World Stonefields', 'New World Papatoetoe', 'Four Square Everglade', 'Four Square Botany Junction', 'New World Botany', 'Four Square Cockle Bay', 'New World Howick', 'Four Square Pakuranga Heights', 'Four Square Great Eastern', 'New World Eastridge', 'New World Remuera', 'Warehouse']	12377.78	515.7408
['Warehouse', 'New World Mt Roskill', 'Warehouse']	737.44	30.72667
['Warehouse', 'New World Papakura', 'New World Southmall', 'Warehouse']	5536.96	230.7067
['Warehouse', 'New World Devonport', 'New World Browns Bay', "Pak 'n Save Albany", 'Warehouse']	8229.32	342.8883
['Warehouse', "Pak 'n Save Westgate", "Pak 'n Save Henderson", "Pak 'n Save Mt Albert", 'Warehouse']	5315.61	221.4838
['Warehouse', "Pak 'n Save Glen Innes", "Pak 'n Save Sylvia Park", "Pak 'n Save Royal Oak", 'Warehouse']	5150.17	214.5904
['Warehouse', "Pak 'n Save Lincoln Road", "Pak 'n Save Wairau Road", 'New World Milford', 'New World Metro Queen St', 'Warehouse']	6995.65	291.4854
['Warehouse', "Pak 'n Save Botany", "Pak 'n Save Ormiston", "Pak 'n Save Manukau", 'Warehouse']	5936.45	247.3521
['Warehouse', "Pak 'n Save Clendon", "Pak 'n Save Papakura", "Pak 'n Save Mangere", 'Warehouse']	6287.48	261.9783

5.6 Appendix 6



5.7 Appendix 7