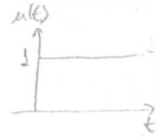
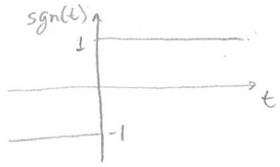
FORMULÁRIO

Funções

a) Degrau unitário: $u(t) = \begin{cases} 0, & t<0 \\ 1, & t \geq 1 \end{cases}$

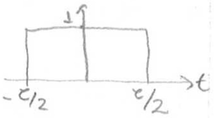


b) Sinal: $\mathrm{sgn}(t) = \begin{cases} -1, & t<0 \\ 1, & t>0 \\ 0, & t=0 \end{cases}$



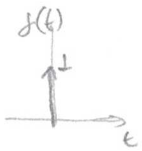
c) Pulso retangular:

$$\mathrm{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| \leq \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$$



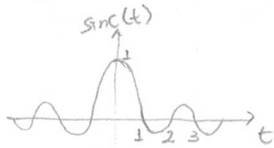
d) Impulso unitário:

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & t \neq 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



e) Função Sinc

$$\mathrm{sinc}(t) = \frac{\mathrm{sen}(\pi t)}{\pi t}$$



Série Trigonométrica de Fourier

$$g_p(t) = a_0 + 2\sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi n t}{T_0}\right) + b_n\, \mathrm{sen}\left(\frac{2\pi n t}{T_0}\right)\right]$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\, dt$$

$$a_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t) \cos\left(\frac{2\pi n t}{T_0}\right) dt \qquad n = 1, 2, 3 \ldots$$

$$b_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\, \mathrm{sen}\left(\frac{2\pi n t}{T_0}\right) dt$$

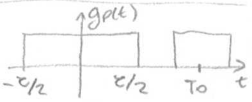
Série exponencial complexa de Fourier

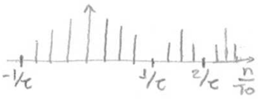
$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n \exp\left(\frac{-j2\pi n t}{T_0}\right)$$

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t) \exp\left(\frac{-j2\pi n t}{T_0}\right) dt \qquad n = 0, \pm 1, \pm 2 \ldots$$

$|C_n| \times \frac{n}{T_0} \rightarrow$ espectro discreto de amplitude

$\arg(C_n) \times \frac{n}{T_0} \rightarrow$ espectro discreto de fase





Transformada de Fourier

(TF é uma qntd complexa)

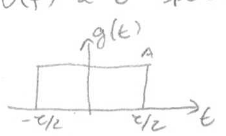
$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot \exp(-j2\pi f t) \cdot dt$$

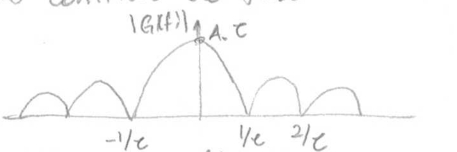
$$G(f) = \underbrace{A(f)}_{\text{real}} + j\underbrace{B(f)}_{\text{imag}} = \underbrace{|G(f)|}_{\text{mód}} \cdot \exp[j \cdot \underbrace{\theta(f)}_{\text{fase}}]$$

$$|G(f)| = \sqrt{A^2(f) + B^2(f)} \qquad \theta(f) = -\mathrm{arc\,tg}\left(\frac{B(f)}{A(f)}\right)$$

$|G(f)|$ é o espectro contínuo de amplitude

$\theta(f)$ é o espectro contínuo de fase





pulso retg ?s possui mais altas frequencias

Propriedades da Transformada de Fourier

a) $g(at) \Leftrightarrow \frac{1}{|a|} \cdot G\left(\frac{f}{a}\right)$ → mudança de escala

b) $g(t-t_0) \Leftrightarrow G(f) \cdot \exp(-j2\pi f t_0)$ → desloc. no tempo

c) $g(t)\exp(2\pi f_c t) \Leftrightarrow G(f-f_0)$ → desloc. na frequência

d) $\frac{d^n}{dt^n} g(t) \Leftrightarrow (j2\pi f)^n \cdot G(f)$ → diferenciação no dom. do tempo

e) $\int_{-\infty}^{t} g(\tau)\, d\tau \Leftrightarrow \frac{G(f)}{j2\pi f} + \frac{G(0)}{2} \cdot \delta(f)$ → integração no dom. do tempo

f) $g_1(t) * g_2(t) \Leftrightarrow G_1(f) \cdot G_2(t)$ → convolução no tempo

g) $G(t) \Leftrightarrow g(-f)$ → dualidade

h) $g_p(t) \Leftrightarrow \frac{1}{T_0}\sum_{n=-\infty}^{\infty} G\left(\frac{n}{T_0}\right) \cdot \delta\left(f - \frac{n}{T_0}\right)$ → Transf. de funções periódicas

Pares de Transformadas

$A.\mathrm{rect}\left(\frac{t}{\tau}\right) \Leftrightarrow A.\tau.\mathrm{sinc}(f.\tau)$

$\exp(-t).u(t) \Leftrightarrow \frac{1}{1+j2\pi f}$

$A.\mathrm{sinc}(2Wt) \Leftrightarrow \frac{A}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$

$\delta(t) \Leftrightarrow 1$

$\delta(t-t_0) \Leftrightarrow \exp(-j2\pi f t_0)$

$\exp(j2\pi f_c t) \Leftrightarrow \delta(f-f_c)$

$\cos(2\pi . f_c t) \Leftrightarrow \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)]$ ①

$\mathrm{sen}(2\pi f_c t) \Leftrightarrow \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$ ②

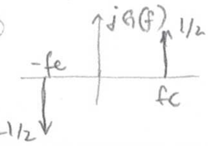
$\exp(-a|t|) \Leftrightarrow \frac{2a}{a^2+(2\pi f)^2}$

$\exp(-\pi t^2) \Leftrightarrow \exp(-\pi f^2)$

$\mathrm{sgn}(t) \Leftrightarrow 1/j\pi f$

$\frac{1}{\pi f} \Leftrightarrow -j\,\mathrm{sgn}(f)$

①


②


Cálculo de TF por diferenc. sucessivas

$$\frac{d^n g(t)}{dt^n} \Leftrightarrow (j2\pi f)^n \cdot G(f) \qquad F\left[\frac{d^n g(t)}{dt^n}\right] = (j2\pi f)^n \cdot G(f)$$

$$G(f) = \frac{F\left[\frac{d^n g(t)}{dt^n}\right]}{(j2\pi f)^n}$$

Densidade Espectral de Energia

$$\Psi_g(f) = |G(f)|^2 \rightarrow \text{DEE}$$

$$E = \int_{-\infty}^{\infty} \Psi_g(f)\, df \quad \therefore \quad \int_{-\infty}^{\infty} |G(f)|^2 df = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

Área total de DEE igual Energ. Total

(Energia de Rayleigh)

Densidade Espectral de Potência

$S_{pg}(f) = |G_P(f)|^2 \rightarrow$ DEP

$P = \int_{-\infty}^{\infty} |G_P(t)|^2 df$ → área total de DEP é igual a potencia media do sinal periódico

$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |g_p(t)|^2 dt$ → Pot media no tempo!

$$C_n = \frac{\tau . A}{T_0} \mathrm{sinc}\left(\frac{n\tau}{T_0}\right)$$

$P = |C_0|^2 + 2 . \sum_{n=1}^{\infty} |C_n|^2$ → Teorema da Potência de Parseval

Função Autocorrelação (sinais aperiódicos)

$$R_g(\lambda) = \int_{-\infty}^{\infty} g(t) \cdot g^*(t-\lambda)\, dt$$

$$R_g(\lambda) = \int_{-\infty}^{\infty} g(t+\lambda) \cdot g^*(t)\, dt$$

Propriedade:

a) $R_g(\lambda) = R_g^*(-\lambda)$ → simetria conjugada

b) $E = R_g(0)$ → Energia total do sinal

c) $|R_g(\lambda)| \leq R_g(0)$ → valor máximo

d) $R_g(\lambda) \Leftrightarrow \Psi_g(f) = |G(f)|^2$ → Transf. Four

Função Autocorrelação (sinais periódicos)

$$R_{gp}(\lambda) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_p(t) \cdot g_p^*(t-\lambda)\, dt$$

$P = R_{gp}(0)$ → Potência media do sinal

$R_{gp}(\lambda) \Longleftrightarrow S_{gp}$ → Relações de Wiener-Khinchin

$R_{gp}(\lambda) = R_{gp}(\lambda \pm n.T_0)$ → Periodicidade ($n = 1, 2, 3$)

Função Cross-Correlação

$$R_{12} = \int_{-\infty}^{\infty} g_1(t)\, g_2^*(t-\lambda)\, dt \quad \text{(sinais aperiodicos)}$$

$R_{12}(\lambda) = R_{21}^*(-\lambda)$ → simetria conjugada

$R_{12}(0) = 0$ → sinais ortogonais → Ex: sen e cos

$R_{12}(\lambda) \Leftrightarrow G_1(f)\, G_2^*(f)$ → Teorema da Correlação

$$R_{12}(\lambda) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_{p1}(t)\, g_{p2}^*(t-\lambda)\, dt \quad \text{(periodicos)}$$

Transformada de Hilbert

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\lambda)}{t-\lambda}\, d\lambda$$

$$g(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{g}(\lambda)}{t-\lambda}\, d\lambda \quad \rightarrow \text{TH, sua inversa}$$

$$\hat{g}(t) = g(t) * \frac{1}{\pi t} \quad \rightarrow \text{TH pode ser uma convolução}$$

$$\hat{G}(f) = -j\,\mathrm{sgn}(f)\, G(f) \quad \rightarrow \text{TH de } \hat{g}(t)$$

Atraso de fase e de Grupo

$$\tau_p = \frac{-\beta(f_c)}{2\pi f_c} \qquad \tau_g = \frac{-1}{2\pi} \left.\frac{\partial \beta(f)}{\partial f}\right|_{f=f_0}$$

Interferência Intersimbolica

$$ISI = \frac{|\tau_{ps} - \tau_{p0}|}{T_b} \cdot 100\% = 10\% \text{ (aceitavel)}$$

Transf. Hilbert

$$m(t)\cos(2\pi f_c t) \overset{TH}{\Longleftrightarrow} m(t)\,\mathrm{sen}(2\pi f_c t)$$

$$m(t)\,\mathrm{sen}(2\pi f_c t) \Longleftrightarrow -m(t)\cos(2\pi f_c t)$$

$$\frac{\mathrm{sen}\, t}{t} \Longleftrightarrow \frac{1 - \cos t}{t}$$

$$\mathrm{rect}(t) \Longleftrightarrow \frac{-1}{\pi} \ln\left|\frac{t - 1/2}{t + 1/2}\right|$$

$$\delta(t) \Longleftrightarrow \frac{1}{\pi t}$$

$$\frac{1}{t} \Longleftrightarrow -\pi\,\delta(t)$$

$$\frac{1}{1+t^2} \Longleftrightarrow \frac{t}{1+t^2}$$