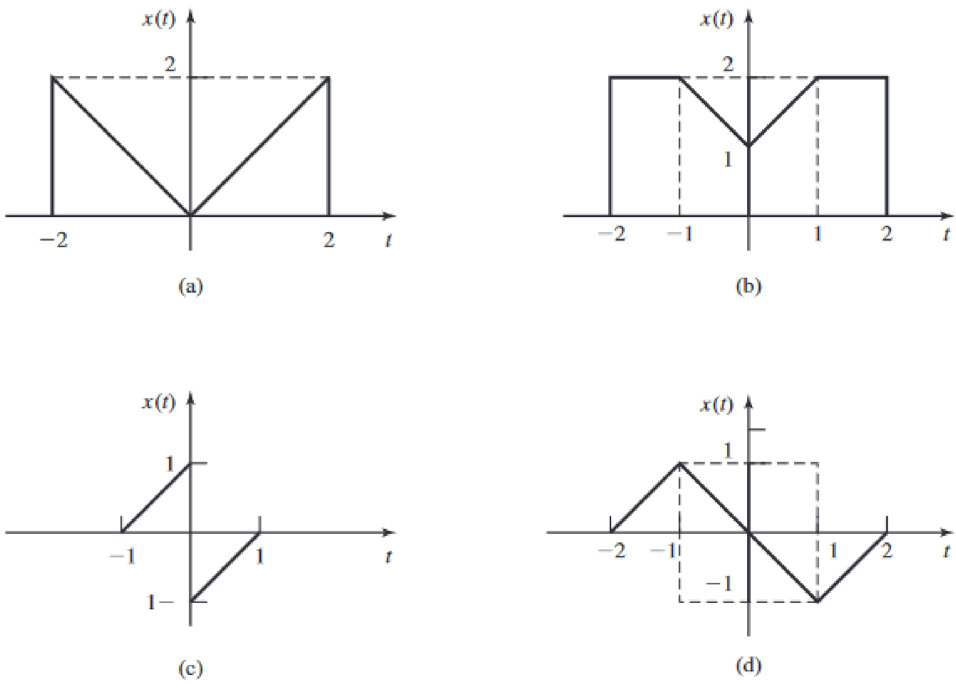
SEMANA 1

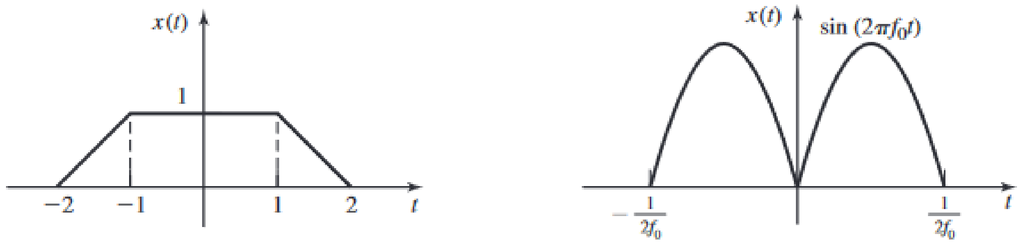
2.2 Determine the Fourier series expansion of the following signals.

1. $x_1(t) = \sum_{n=-\infty}^{\infty} \Lambda(t - 2n)$
2. $x_2(t) = \sum_{n=-\infty}^{\infty} \Lambda(t - n)$
3. $x_3(t) = e^{t-n}$ for $n \le t < n + 1$
4. $x_4(t) = \cos t + \cos 2.5t$
5. $x_5(t) = \sum_{n=-\infty}^{\infty} \Lambda(t - n)u_{-1}(t - n)$
6. $x_6(t) = \sum_{n=-\infty}^{\infty}(-1)^n \delta(t - nT)$
7. $x_7(t) = \sum_{n=-\infty}^{\infty} \delta'(t - nT)$
8. $x_8(t) = |\cos 2\pi f_0 t|$ (Full-wave rectifier output)
9. $x_9(t) = \cos 2\pi f_0 t + |\cos 2\pi f_0 t|$ (Half-wave rectifier output)

2.12 Determine the Fourier transform of the signals shown in Figure P-2.12:



(c) (d)



(e) (f)

Figure P-2.12

Problem 2.12

a) We can write $x(t)$ as $x(t) = 2\Pi(\frac{t}{4}) - 2\Lambda(\frac{t}{2})$. Then

$$\mathcal{F}[x(t)] = \mathcal{F}[2\Pi(\frac{t}{4})] - \mathcal{F}[2\Lambda(\frac{t}{2})] = 8\text{sinc}(4f) - 4\text{sinc}^2(2f)$$

b)

$$x(t) = 2\Pi(\frac{t}{4}) - \Lambda(t) \Longrightarrow \mathcal{F}[x(t)] = 8\text{sinc}(4f) - \text{sinc}^2(f)$$

c)

$$\begin{aligned}
X(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt = \int_{-1}^{0}(t+1)e^{-j2\pi ft}dt + \int_{0}^{1}(t-1)e^{-j2\pi ft}dt \\
&= \left(\frac{j}{2\pi f}t + \frac{1}{4\pi^2 f^2}\right)e^{-j2\pi ft}\Big|_{-1}^{0} + \frac{j}{2\pi f}e^{-j2\pi ft}\Big|_{-1}^{0} \\
&\quad + \left(\frac{j}{2\pi f}t + \frac{1}{4\pi^2 f^2}\right)e^{-j2\pi ft}\Big|_{0}^{1} - \frac{j}{2\pi f}e^{-j2\pi ft}\Big|_{0}^{1} \\
&= \frac{j}{\pi f}(1 - \sin(\pi f))
\end{aligned}$$

d) We can write $x(t)$ as $x(t) = \Lambda(t+1) - \Lambda(t-1)$. Thus

$$X(f) = \text{sinc}^2(f)e^{j2\pi f} - \text{sinc}^2(f)e^{-j2\pi f} = 2j\text{sinc}^2(f)\sin(2\pi f)$$

e) We can write $x(t)$ as $x(t) = \Lambda(t+1) + \Lambda(t) + \Lambda(t-1)$. Hence,

$$X(f) = \text{sinc}^2(f)(1 + e^{j2\pi f} + e^{-j2\pi f}) = \text{sinc}^2(f)(1 + 2\cos(2\pi f)$$

f) We can write $x(t)$ as

$$x(t) = \left[\Pi\left(2f_0(t - \frac{1}{4f_0})\right) - \Pi\left(2f_0(t - \frac{1}{4f_0})\right)\right]\sin(2\pi f_0 t)$$

Then

$$\begin{aligned} X(f) &= \left[\frac{1}{2f_0}\text{sinc}\left(\frac{f}{2f_0}\right)e^{-j2\pi\frac{1}{4f_0}f} - \frac{1}{2f_0}\text{sinc}\left(\frac{f}{2f_0})\right)e^{j2\pi\frac{1}{4f_0}f}\right] \\ &\quad \star\frac{j}{2}(\delta(f+f_0) - \delta(f+f_0)) \\ &= \frac{1}{2f_0}\text{sinc}\left(\frac{f+f_0}{2f_0}\right)\sin\left(\pi\frac{f+f_0}{2f_0}\right) - \frac{1}{2f_0}\text{sinc}\left(\frac{f-f_0}{2f_0}\right)\sin\left(\pi\frac{f-f_0}{2f_0}\right) \end{aligned}$$

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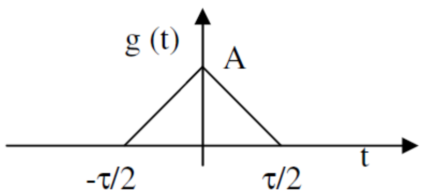
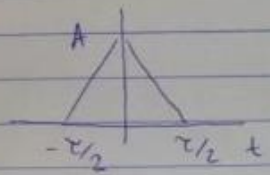
5) Determine a TF do Pulso triangular usando diferenciações sucessivas.

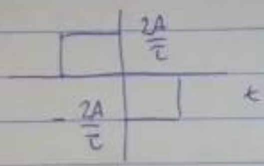


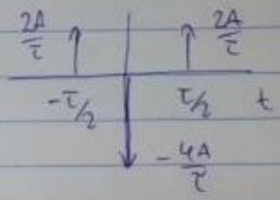
$$g(t) = \begin{cases} A + \dfrac{2At}{\tau}, & -\dfrac{\tau}{2} \le t \le 0 \\ A - \dfrac{2At}{\tau}, & 0 \le t \le +\dfrac{\tau}{2} \end{cases}$$

Ex 5) Pulso Triangular (T.F.)



$$g(t) = \begin{cases} A + \frac{2At}{\tau}, & -\frac{\tau}{2} \le t \le 0 \\ A - \frac{2At}{\tau}, & 0 \le t \le \frac{\tau}{2} \end{cases}$$





$$\frac{d^2 g(t)}{dt^2} = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau}\delta(t) + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

$$\downarrow \text{T.F.}$$

$$(j2\pi f)^2 \cdot G(f) = \frac{2A}{\tau} e^{j2\pi f\frac{\tau}{2}} - \frac{4A}{\tau} + \frac{2A}{\tau} e^{-j2\pi f\frac{\tau}{2}}$$

$$= \frac{2A}{\tau}\left[e^{j\pi f\tau} + e^{-j\pi f\tau} - 2\right]$$

$$\frac{d^2 g(t)}{dt^2} = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau}\delta(t) + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

$$\downarrow \text{T.F.}$$

$$(j2\pi f)^2 \cdot G(f) = \frac{2A}{\tau} e^{j2\pi f\frac{\tau}{2}} - \frac{4A}{\tau} + \frac{2A}{\tau} e^{-j2\pi f\frac{\tau}{2}}$$

$$= \frac{2A}{\tau}\left[e^{j\pi f\tau} + e^{-j\pi f\tau} - 2\right]$$

$$= \frac{2A}{\tau}\left[2\cos(\pi f\tau) - 2\right] = -\frac{8A}{\tau}\,\mathrm{sen}^2\left(\frac{\pi f\tau}{2}\right)$$

$$G(f) = \frac{1}{(j2\pi f)^2} \cdot \frac{-8A}{\tau}\,\mathrm{sen}^2\left(\frac{\pi f\tau}{2}\right) = \frac{A\tau}{2}\,\mathrm{sinc}^2\left(\frac{f\tau}{2}\right)$$

//

SEMANA 2 – faltou o 13 Lista

2.31 Determine whether these signals are energy-type or power-type. In each case, find the energy or power-spectral density and also the energy or power content of the signal.

1. $x(t) = e^{-\alpha t}\cos(\beta t)u_{-1}(t) \quad \alpha, \beta > 0$
2. $x(t) = \mathrm{sinc}(t)$
3. $x(t) = \sum_{n=-\infty}^{\infty} \Lambda(t-2n)$
4. $x(t) = u_{-1}(t)$
5. $x(t) = \frac{1}{t}$

Problem 2.31

1) $x(t) = e^{-\alpha t}u_{-1}(t)$. The spectrum of the signal is $X(f) = \frac{1}{\alpha + j2\pi f}$ and the energy spectral density

$$\mathcal{G}_X(f) = |X(f)|^2 = \frac{1}{\alpha^2 + 4\pi^2 f^2}$$

Thus,

$$R_X(\tau) = \mathcal{F}^{-1}[\mathcal{G}_X(f)] = \frac{1}{2\alpha}e^{-\alpha|\tau|}$$

The energy content of the signal is

$$E_X = R_X(0) = \frac{1}{2\alpha}$$

2) $x(t) = \mathrm{sinc}(t)$. Clearly $X(f) = \Pi(f)$ so that $\mathcal{G}_X(f) = |X(f)|^2 = \Pi^2(f) = \Pi(f)$. The energy content of the signal is

$$E_X = \int_{-\infty}^{\infty} \Pi(f)df = \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pi(f)df = 1$$

3) $x(t) = \sum_{n=-\infty}^{\infty} \Lambda(t-2n)$. The signal is periodic and thus it is not of the energy type. The power content of the signal is

$$\begin{aligned}
P_x &= \frac{1}{2}\int_{-1}^{1}|x(t)|^2 dt = \frac{1}{2}\int_{-1}^{0}(t+1)^2 dt + \int_{0}^{1}(-t+1)^2 dt \\
&= \frac{1}{2}\left(\frac{1}{3}t^3 + t^2 + t\right)\Big|_{-1}^{0} + \frac{1}{2}\left(\frac{1}{3}t^3 - t^2 + t\right)\Big|_{0}^{1} \\
&= \frac{1}{3}
\end{aligned}$$

The same result is obtain if we let

$$\mathcal{S}_X(f) = \sum_{n=-\infty}^{\infty} |x_n|^2 \delta(f - \frac{n}{2})$$

4)

$$E_X = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} |u_{-1}(t)|^2 dt = \lim_{T\to\infty} \int_0^{\frac{T}{2}} dt = \lim_{T\to\infty} \frac{T}{2} = \infty$$

Thus, the signal is not of the energy type.

$$P_X = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |u_{-1}(t)|^2 dt = \lim_{T\to\infty} \frac{1}{T}\frac{T}{2} = \frac{1}{2}$$

Hence, the signal is of the power type and its power content is $\frac{1}{2}$. To find the power spectral density we find first the autocorrelation $R_X(\tau)$.

$$\begin{aligned} R_X(\tau) &= \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u_{-1}(t)u_{-1}(t-\tau)dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{\tau}^{\frac{T}{2}} dt \\ &= \lim_{T\to\infty} \frac{1}{T}(\frac{T}{2} - \tau) = \frac{1}{2} \end{aligned}$$

Thus, $\mathcal{S}_X(f) = \mathcal{F}[R_X(\tau)] = \frac{1}{2}\delta(f)$.

5) Clearly $|X(f)|^2 = \pi^2 \text{sgn}^2(f) = \pi^2$ and $E_X = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} \pi^2 dt = \infty$. The signal is not of the energy type for the energy content is not bounded. Consider now the signal

$$x_T(t) = \frac{1}{t}\Pi(\frac{t}{T})$$

Then,

$$X_T(f) = -j\pi\text{sgn}(f) \star T\text{sinc}(fT)$$

and

$$\mathcal{S}_X(f) = \lim_{T\to\infty} \frac{|X_T(f)|^2}{T} = \lim_{T\to\infty} \pi^2 T \left| \int_{-\infty}^{f} \text{sinc}(vT)dv - \int_{f}^{\infty} \text{sinc}(vT)dv \right|^2$$

However, the squared term on the right side is bounded away from zero so that $\mathcal{S}_X(f)$ is ∞. The signal is not of the power type either.

---

17) O sinal v(t), abaixo, foi obtido no processo de detecção coerente para sinais DSB/SC. Supondo que este sinal é filtrado com o uso de um FPB tipo Butterworth de 1º ordem (onde f_{3dB} é a *freq. de corte* do filtro), determine o valor de f_c para que as componentes indesejadas de v(t) sejam atenuadas em pelo menos 20 dB. Considere que $f_{3dB} = 1{,}5.f_m$.

$v(t) = \cos(2\pi f_m t).\cos(2\pi 2 f_c t) + \cos(2\pi f_m t)$ $\qquad$ $|H(f)| = (1 + (f/f_{3dB})^2)^{-1/2}$

Ex 17) $s(t) \rightarrow \boxed{\times} \xrightarrow{v(t)} \boxed{FPB} \rightarrow m_o(t)$

$\uparrow c'(t)$

$At(dB) = 20\ dB$

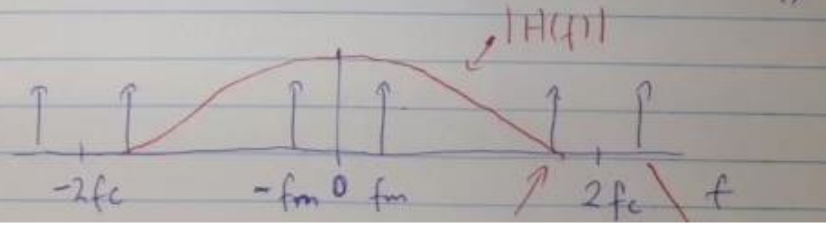
$$v(t) = \underbrace{\cos(2\pi f_m t) \cdot \cos(2\pi 2 f_c t)}_{\text{indesejado}} + \cos(2\pi f_m t)$$

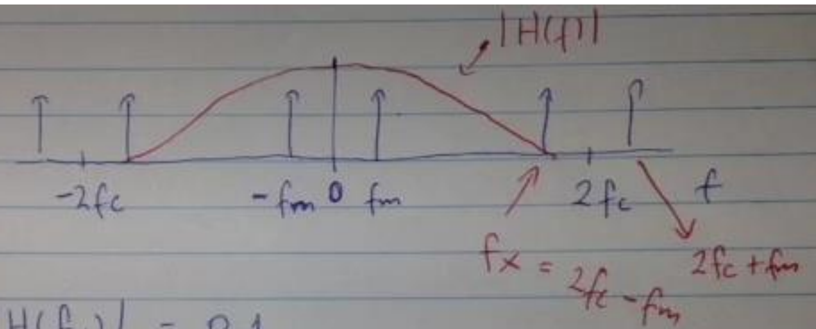
$f_{3dB} = 1{,}5 \cdot f_m$

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^2}}$$

$At(dB) = 20 \log_{10}(At)$

$-20 = 20 \log_{10}(At) \qquad At = 10^{-\frac{20}{20}} = 0{,}1$





$|H(f_x)| = 0{,}1$

$$\frac{1}{\sqrt{1 + \left(\frac{f_x}{f_{3dB}}\right)^2}} = 0{,}1$$

$$1 + \left(\frac{f_x}{f_{3dB}}\right)^2 = 100$$

$f_x \cong 10 \cdot f_{3dB}$

$f_x \cong 15 \cdot f_m$

$f_c = ?$

$2f_c = f_x + f_m$

$f_c = 8 \cdot f_m$

SEMANA 3

2.4 Consider the AM signal

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

produced by a sinusoidal modulating signal of frequency f_m. Assume that the modulation factor is $\mu = 2$, and the carrier frequency f_c is much greater than f_m. The AM signal $s(t)$ is applied to an ideal envelope detector, producing the output $v(t)$.

(a) Determine the Fourier series representation of $v(t)$.

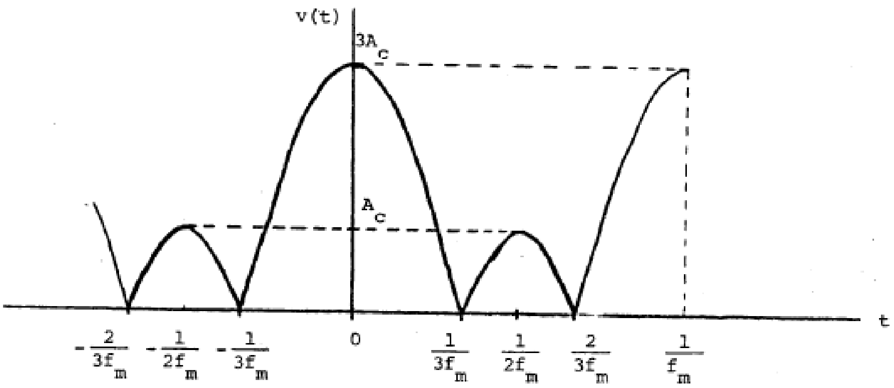
(b) What is the ratio of second-harmonic amplitude to fundamental amplitude in $v(t)$?

Problem 2.4

(a) The envelope detector output is

$$v(t) = A_c|1 + \mu\cos(2\pi f_m t)|$$

which is illustrated below for the case when $\mu=2$.



We see that $v(t)$ is periodic with a period equal to f_m, and an even function of t, and so we may express $v(t)$ in the form:

$$v(t) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos(2n\pi f_m t)$$

where $$a_0 = 2f_m \int_0^{1/2f_m} v(t)dt$$

$$= 2A_c f_m \int_0^{1/3f_m} [1+2\cos(2\pi f_m t)]dt + 2A_c f_m \int_{1/3f_m}^{1/2f_m} [-1-2\cos(2\pi f_m t)]dt$$

$$= \frac{A_c}{3} + \frac{4A_c}{\pi}\sin(\frac{2\pi}{3}) \qquad (1)$$

$$a_n = 2f_m \int_0^{1/2f_m} v(t)\cos(2n\pi f_m t)dt$$

$$= 2A_c f_m \int_0^{1/3f_m} [1+2\cos(2\pi f_m t)]\cos(2n\pi f_m t)dt$$

$$+ 2A_c f_m \int_{1/3f_m}^{1/2f_m} [-1-2\cos(2\pi f_m t)]\cos(2n\pi f_m t)dt$$

$$= \frac{A_c}{n\pi}[2\sin(\frac{2n\pi}{3}) - \sin(n\pi)] + \frac{A_c}{(n+1)\pi}\{2\sin[\frac{2\pi}{3}(n+1)] - \sin[\pi(n+1)]\}$$

$$+ \frac{A_c}{(n-1)\pi}\{2\sin[\frac{2\pi}{3}(n-1)] - \sin[\pi(n-1)]\} \qquad (2)$$

For n=0, Eq. (2) reduces to that shown in Eq. (1).

(b) For n=1, Eq. (2) yields

$$a_1 = A_c(\frac{\sqrt{3}}{2\pi} + \frac{1}{3})$$

For n=2, it yields

$$a_2 = \frac{A_c\sqrt{3}}{2\pi}$$

Therefore, the ratio of second-harmonic amplitude to fundamental amplitude in v(t) is

$$\frac{a_2}{a_1} = \frac{3\sqrt{3}}{2\pi+3\sqrt{3}} = 0.452$$

3.4 Suppose the signal $x(t) = m(t) + \cos 2\pi f_c t$ is applied to a nonlinear system whose output is $y(t) = x(t) + \frac{1}{2}x^2(t)$. Determine and sketch the spectrum of $y(t)$ when $M(f)$ is as shown in Figure P-3.4 and $W \ll f_c$.

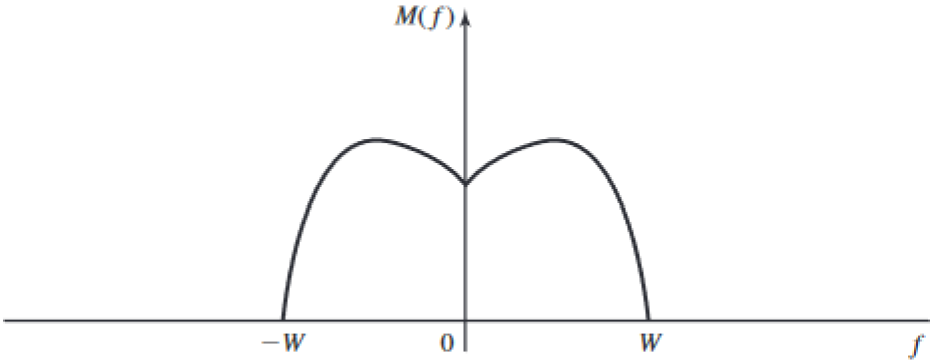


Figure P-3.4

Problem 3.4

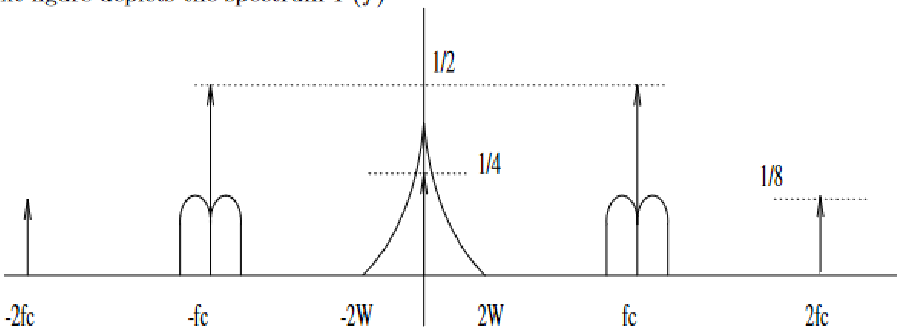
$$y(t) = x(t) + \frac{1}{2}x^2(t)$$

$$= m(t) + \cos(2\pi f_c t) + \frac{1}{2}\left(m^2(t) + \cos^2(2\pi f_c t) + 2m(t)\cos(2\pi f_c t)\right)$$
$$= m(t) + \cos(2\pi f_c t) + \frac{1}{2}m^2(t) + \frac{1}{4} + \frac{1}{4}\cos(2\pi 2 f_c t) + m(t)\cos(2\pi f_c t)$$

Taking the Fourier transform of the previous, we obtain

$$\begin{aligned} Y(f) &= M(f) + \frac{1}{2}M(f) \star M(f) + \frac{1}{2}(M(f-f_c) + M(f+f_c)) \\ &\quad + \frac{1}{4}\delta(f) + \frac{1}{2}(\delta(f-f_c) + \delta(f+f_c)) + \frac{1}{8}(\delta(f-2f_c) + \delta(f+2f_c)) \end{aligned}$$

The next figure depicts the spectrum $Y(f)$



3.13 An AM signal is generated by modulating the carrier $f_c = 800$ kHz by the signal

$$m(t) = \sin 2000\pi t + 5\cos 4000\pi t t$$

The AM signal

$$u(t) = 100[1 + m(t)]\cos 2\pi f_c t$$

is fed to a 50 Ω load.

1. Determine and sketch the spectrum of the AM signal.
2. Determine the average power in the carrier and in the sidebands.

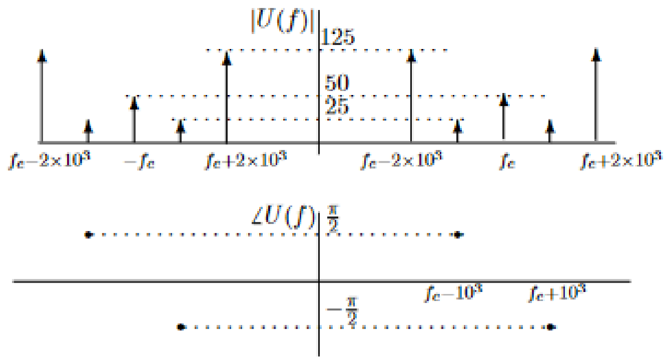
Problem 3.13

1) The modulated signal is

$$\begin{aligned}
u(t) &= 100[1 + m(t)]\cos(2\pi 8\times 10^5 t)\\
&= 100\cos(2\pi 8\times 10^5 t) + 100\sin(2\pi 10^3 t)\cos(2\pi 8\times 10^5 t)\\
&\quad +500\cos(2\pi 2\times 10^3 t)\cos(2\pi 8\times 10^5 t)\\
&= 100\cos(2\pi 8\times 10^5 t) + 50[\sin(2\pi(10^3 + 8\times 10^5)t) - \sin(2\pi(8\times 10^5 - 10^3)t)]\\
&\quad +250[\cos(2\pi(2\times 10^3 + 8\times 10^5)t) + \cos(2\pi(8\times 10^5 - 2\times 10^3)t)]
\end{aligned}$$

Taking the Fourier transform of the previous expression, we obtain

$$\begin{aligned}
U(f) &= 50[\delta(f - 8\times 10^5) + \delta(f + 8\times 10^5)]\\
&\quad +25\left[\frac{1}{j}\delta(f - 8\times 10^5 - 10^3) - \frac{1}{j}\delta(f + 8\times 10^5 + 10^3)\right]\\
&\quad -25\left[\frac{1}{j}\delta(f - 8\times 10^5 + 10^3) - \frac{1}{j}\delta(f + 8\times 10^5 - 10^3)\right]\\
&\quad +125\left[\delta(f - 8\times 10^5 - 2\times 10^3) + \delta(f + 8\times 10^5 + 2\times 10^3)\right]\\
&\quad +125\left[\delta(f - 8\times 10^5 - 2\times 10^3) + \delta(f + 8\times 10^5 + 2\times 10^3)\right]\\
&= 50[\delta(f - 8\times 10^5) + \delta(f + 8\times 10^5)]\\
&\quad +25\left[\delta(f - 8\times 10^5 - 10^3)e^{-j\frac{\pi}{2}} + \delta(f + 8\times 10^5 + 10^3)e^{j\frac{\pi}{2}}\right]\\
&\quad +25\left[\delta(f - 8\times 10^5 + 10^3)e^{j\frac{\pi}{2}} + \delta(f + 8\times 10^5 - 10^3)e^{-j\frac{\pi}{2}}\right]\\
&\quad +125\left[\delta(f - 8\times 10^5 - 2\times 10^3) + \delta(f + 8\times 10^5 + 2\times 10^3)\right]\\
&\quad +125\left[\delta(f - 8\times 10^5 - 2\times 10^3) + \delta(f + 8\times 10^5 + 2\times 10^3)\right]
\end{aligned}$$



2) The average power in the carrier is

$$P_{\text{carrier}} = \frac{A_c^2}{2} = \frac{100^2}{2} = 5000$$

The power in the sidebands is

$$P_{\text{sidebands}} = \frac{50^2}{2} + \frac{50^2}{2} + \frac{250^2}{2} + \frac{250^2}{2} = 65000$$

3) The message signal can be written as

$$\begin{aligned} m(t) &= \sin(2\pi 10^3 t) + 5\cos(2\pi 2 \times 10^3 t) \\ &= -10\sin(2\pi 10^3 t) + \sin(2\pi 10^3 t) + 5 \end{aligned}$$

As it is seen the minimum value of $m(t)$ is -6 and is achieved for $\sin(2\pi 10^3 t) = -1$ or $t = \frac{3}{4\times10^3} + \frac{1}{10^3}k$, with $k \in Z$. Hence, the modulation index is $\alpha = 6$.

4) The power delivered to the load is

$$P_{\text{load}} = \frac{|u(t)|^2}{50} = \frac{100^2(1+m(t))^2\cos^2(2\pi f_c t)}{50}$$

The maximum absolute value of $1 + m(t)$ is 6.025 and is achieved for $\sin(2\pi 10^3 t) = \frac{1}{20}$ or $t = \frac{\arcsin(\frac{1}{20})}{2\pi 10^3} + \frac{k}{10^3}$. Since $2 \times 10^3 \ll f_c$ the peak power delivered to the load is approximately equal to

$$\max(P_{\text{load}}) = \frac{(100 \times 6.025)^2}{50} = 72.6012$$

SEMANA 4--- faltou 6 e 10 lista

3.8 A message signal $m(t) = \cos 2000\pi t + 2\cos 4000\pi t$ modulates the carrier $c(t) = 100\cos 2\pi f_c t$ where $f_c = 1$ MHz to produce the DSB signal $m(t)c(t)$.

1. Determine the expression for the upper sideband (USB) signal.
2. Determine and sketch the spectrum of the USB signal.

$$
\begin{aligned}
u(t) &= m(t)c(t) \\
&= 100(\cos(2\pi 1000t) + 2\cos(2\pi 2000t))\cos(2\pi f_c t) \\
&= 100\cos(2\pi 1000t)\cos(2\pi f_c t) + 200\cos(2\pi 2000t)\cos(2\pi f_c t) \\
&= \frac{100}{2}\left[\cos(2\pi(f_c+1000)t) + \cos(2\pi(f_c-1000)t)\right] \\
&\quad \frac{200}{2}\left[\cos(2\pi(f_c+2000)t) + \cos(2\pi(f_c-2000)t)\right]
\end{aligned}
$$

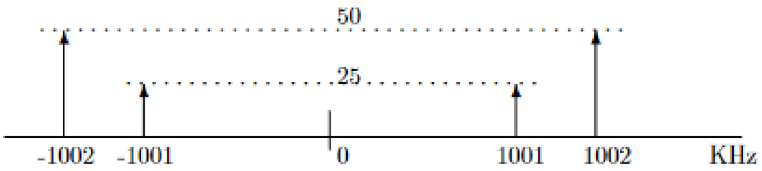
Thus, the upper sideband (USB) signal is

$$u_u(t) = 50\cos(2\pi(f_c+1000)t) + 100\cos(2\pi(f_c+2000)t)$$

2) Taking the Fourier transform of both sides, we obtain

$$
\begin{aligned}
U_u(f) &= 25\left(\delta(f-(f_c+1000)) + \delta(f+(f_c+1000))\right) \\
&\quad +50\left(\delta(f-(f_c+2000)) + \delta(f+(f_c+2000))\right)
\end{aligned}
$$

A plot of $U_u(f)$ is given in the next figure.



3.16 A SSB AM signal is generated by modulating an 800-kHz carrier by the signal $m(t) = \cos 2000\pi t + 2 \sin 2000\pi t$. The amplitude of the carrier is $A_c = 100$.

1. Determine the signal $\hat{m}(t)$.
2. Determine the (time domain) expression for the lower sideband of the SSB AM signal.
3. Determine the magnitude spectrum of the lower sideband SSB signal.

Problem 3.16

1) The Hilbert transform of $\cos(2\pi 1000t)$ is $\sin(2\pi 1000t)$, whereas the Hilbert transform ofsin$(2\pi 1000t)$ is $-\cos(2\pi 1000t)$. Thus

$$\hat{m}(t) = \sin(2\pi 1000t) - 2\cos(2\pi 1000t)$$

2) The expression for the LSSB AM signal is

$$u_l(t) = A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t)$$

Substituting $A_c = 100$, $m(t) = \cos(2\pi 1000t) + 2\sin(2\pi 1000t)$ and $\hat{m}(t) = \sin(2\pi 1000t) - 2\cos(2\pi 1000t)$ in the previous, we obtain

$$\begin{aligned}
u_l(t) &= 100\left[\cos(2\pi 1000t) + 2\sin(2\pi 1000t)\right]\cos(2\pi f_c t) \\
&+ 100\left[\sin(2\pi 1000t) - 2\cos(2\pi 1000t)\right]\sin(2\pi f_c t) \\
&= 100\left[\cos(2\pi 1000t)\cos(2\pi f_c t) + \sin(2\pi 1000t)\sin(2\pi f_c t)\right] \\
&+ 200\left[\cos(2\pi f_c t)\sin(2\pi 1000t) - \sin(2\pi f_c t)\cos(2\pi 1000t)\right] \\
&= 100\cos(2\pi(f_c - 1000)t) - 200\sin(2\pi(f_c - 1000)t)
\end{aligned}$$

3) Taking the Fourier transform of the previous expression we obtain

$$\begin{aligned}
U_l(f) &= 50\left(\delta(f - f_c + 1000) + \delta(f + f_c - 1000)\right) \\
&+ 100j\left(\delta(f - f_c + 1000) - \delta(f + f_c - 1000)\right) \\
&= (50 + 100j)\delta(f - f_c + 1000) + (50 - 100j)\delta(f + f_c - 1000)
\end{aligned}$$

Hence, the magnitude spectrum is given by

$$\begin{aligned}
|U_l(f)| &= \sqrt{50^2 + 100^2}\left(\delta(f - f_c + 1000) + \delta(f + f_c - 1000)\right) \\
&= 10\sqrt{125}\left(\delta(f - f_c + 1000) + \delta(f + f_c - 1000)\right)
\end{aligned}$$

3.23 The normalized signal $m_n(t)$ has a bandwidth of 10,000 Hz and its power content is 0.5 W. The carrier $A \cos 2\pi f_0 t$ has a power content of 200 W.

1. If $m_n(t)$ modulates the carrier using SSB amplitude modulation, what will be the bandwidth and the power content of the modulated signal?
2. If the modulation scheme is DSB-SC, what will be the answer to part 1?
3. If the modulation scheme is AM with modulation index of 0.6, what will be the answer to part 1?
4. If modulation is FM with $k_f = 50{,}000$, what will be the answer to part 1?

1) If SSB is employed, the transmitted signal is

$$u(t) = Am(t)\cos(2\pi f_0 t) \mp A\hat{m}(t)\sin(2\pi f_0 t)$$

Provided that the spectrum of $m(t)$ does not contain any impulses at the origin $P_M = P_{\hat{M}} = \frac{1}{2}$ and

$$P_{\text{SSB}} = \frac{A^2 P_M}{2} + \frac{A^2 P_{\hat{M}}}{2} = A^2 P_M = 400\frac{1}{2} = 200$$

The bandwidth of the modulated signal $u(t)$ is the same with that of the message signal. Hence,

$$W_{\text{SSB}} = 10000 \text{ Hz}$$

2) In the case of DSB-SC modulation $u(t) = Am(t)\cos(2\pi f_0 t)$. The power content of the modulated signal is

$$P_{\text{DSB}} = \frac{A^2 P_M}{2} = 200\frac{1}{2} = 100$$

and the bandwidth $W_{\text{DSB}} = 2W = 20000$ Hz.

3) If conventional AM is employed with modulation index $\alpha = 0.6$, the transmitted signal is

$$u(t) = A[1 + \alpha m(t)]\cos(2\pi f_0 t)$$

The power content is

$$P_{\text{AM}} = \frac{A^2}{2} + \frac{A^2 \alpha^2 P_M}{2} = 200 + 200 \cdot 0.6^2 \cdot 0.5 = 236$$

The bandwidth of the signal is $W_{\text{AM}} = 2W = 20000$ Hz.

SEMANA 5

2.33 A carrier wave of frequency 100 MHz is frequency-modulated by a sinusoidal wave of amplitude 20 volts and frequency 100 kHz. The frequency sensitivity of the modulator is 25 kHz per volt.

(a) Determine the approximate bandwidth of the FM signal, using Carson's rule.

(b) Determine the bandwidth by transmitting only those side frequencies whose amplitudes exceed 1 percent of the unmodulated carrier amplitude. Use the universal curve of Figure 2.26 for this calculation.

(c) Repeat your calculations, assuming that the amplitude of the modulating signal is doubled.

(d) Repeat your calculations, assuming that the modulation frequency is doubled.

Problem 2.33

(a) The frequency deviation is

$$\Delta f = k_f\, A_m = 25 \times 10^3 \times 20 = 5 \times 10^5 \text{ Hz}$$

The corresponding value of the modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{5 \times 10^5}{10^5} = 5$$

The transmission bandwidth of the FM wave, using Carson's rule, is therefore

$$B_T = 2f_m(1+\beta) = 2\text{x}100\ (1+5) = 1200 \text{ kHz} = 1.2 \text{ MHz}.$$

(b) Using the universal curve of Fig. 3-36 we find that for $\beta=5$:

$$\frac{B_T}{\Delta f} = 3$$

Therefore,

$$B_T = 3\text{x}500 = 1500 \text{ kHz} = 1.5 \text{ MHz}$$

(c) If the amplitude of the modulating wave is doubled, we find that

$$\Delta f = 1 \text{ MHz and } \beta = 10$$

Thus, using Carson's rule we obtain

$$B_T = 2\text{x}100\ (1+10) = 2200 \text{ kHz} = 2.2 \text{ MHz}$$

Using the universal curve of Fig. 3-36, we get

$$\frac{B_T}{\Delta f} = 2.75$$

and $B_T = 2.75$ MHz.

(d) If f_m is doubled, $\beta = 2.5$. Then, using Carson's rule, $B_T = 1.4$ MHz. Using the universal curve, $B_T/\Delta f = 4$, and

$$B_T = 4\Delta f = 2 \text{ MHz}.$$

2.38 Figure P2.38 shows the frequency-determining network of a voltage-controlled oscillator. Frequency modulation is produced by applying the modulating signal $A_m \sin(2\pi f_m t)$ plus a bias V_b to a pair of varactor diodes connected across the parallel combination of a 200-μH inductor and 100-pF capacitor. The capacitor of each varactor diode is related to the voltage V (in volts) applied across its electrodes by

$$C = 100V^{-1/2} \text{ pF}$$

The unmodulated frequency of oscillation is 1 MHz. The VCO output is applied to a frequency multiplier to produce an FM signal with a carrier frequency of 64 MHz and a modulation index of 5. Determine (a) the magnitude of the bias voltage V_b and (b) the amplitude A_m of the modulating wave, given that f_m = 10 kHz.

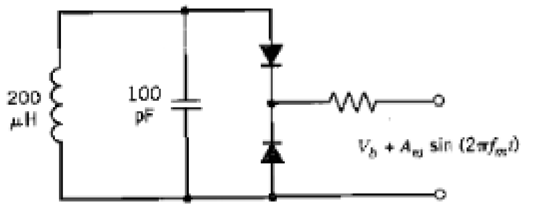


FIGURE P2.38

Problem 2.38

(a) Let L denote the inductive component, C the capacitive component, and C_0 the capacitance of each varactor diode due to the bias voltage V_b acting alone. Then, we have

$$C_0 = 100\ V_b^{-1/2} \text{ pF}$$

and the corresponding frequency of oscillation is

$$f_0 = \frac{1}{2\pi\sqrt{L(C+C_0/2)}}$$

Therefore,

$$10^6 = \frac{1}{2\pi\sqrt{200 \times 10^{-6}\ (100 \times 10^{-12} + 50\ V_b^{-1/2} \times 10^{-12})}}$$

Solving for V_b, we get

$$V_b = 3.52 \text{ volts}$$

(b) The frequency multiplication ratio is 64. Therefore, the modulation index of the FM wave at the frequency multiplier input is

$$\beta = \frac{5}{64} = 0.078$$

This indicates that the FM wave produced by the combination of L, C and the varactor diodes is a narrow-band one, which in turn means that the amplitude A_m of the modulating wave is small compared to V_b. We may thus express the instantaneous frequency of this FM wave as follows:

$$f_i(t) = \frac{1}{2\pi}\,[200 \times 10^{-6}\,\{100 \times 10^{-12} + 50 \times 10^{-12}\,[3.52 + A_m \sin(2\pi f_m t)]^{-1/2}\}]^{-1/2}$$

$$= \frac{10^7}{2\sqrt{2}\pi}\,\{1 + 0.266\,[1 + \frac{A_m}{3.52}\sin(2\pi f_m t)]^{-1/2}\}^{-1/2}$$

$$\simeq \frac{10^7}{2\sqrt{2}\pi}\,\{1 + 0.266\,[1 - \frac{A_m}{7.04}\sin(2\pi f_m t)]\}^{-1/2}$$

$$= 10^6\,[1 - 0.03\,A_m \sin(2\pi f_m t)]^{-1/2}$$

$$\simeq 10^6\,[1 + 0.015\,A_m \sin(2\pi f_m t)]$$

With a modulation index of 0.078, the corresponding value of the frequency deviation is

$$\Delta f = \beta f_m$$

$$= 0.078 \times 10^4 \text{ Hz}$$

Therefore,

$$0.015\,A_m \times 10^6 = 0.078 \times 10^4$$

where A_m is in volts. Solving for A_m, we get

$$A_m = 52 \times 10^{-3} \text{ volts.}$$

3.24 The message signal $m(t) = 10\,\mathrm{sinc}(400t)$ frequency modulates the carrier $c(t) = 100\cos 2\pi f_c t$. The modulation index is 6.

1. Write an expression for the modulated signal $u(t)$?
2. What is the maximum frequency deviation of the modulated signal?
3. What is the power content of the modulated signal?
4. Find the bandwidth of the modulated signal.

Problem 3.24

1) Since $\mathcal{F}[\text{sinc}(400t)] = \frac{1}{400}\Pi(\frac{f}{400})$, the bandwidth of the message signal is $W = 200$ and the resulting modulation index

$$\beta_f = \frac{k_f \max[|m(t)|]}{W} = \frac{k_f 10}{W} = 6 \Longrightarrow k_f = 120$$

Hence, the modulated signal is

$$\begin{aligned} u(t) &= A\cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau) \\ &= 100\cos(2\pi f_c t + +2\pi 1200 \int_{-\infty}^{t} \text{sinc}(400\tau)d\tau) \end{aligned}$$

2) The maximum frequency deviation of the modulated signal is

$$\Delta f_{\max} = \beta_f W = 6 \times 200 = 1200$$

3) Since the modulated signal is essentially a sinusoidal signal with amplitude $A = 100$, we have

$$P = \frac{A^2}{2} = 5000$$

4) Using Carson's rule, the effective bandwidth of the modulated signal can be approximated by

$$B_c = 2(\beta_f + 1)W = 2(6 + 1)200 = 2800 \text{ Hz}$$

3.31 The carrier $c(t) = 100\cos 2\pi f_c t$ is frequency modulated by the signal $m(t) = 5\cos 20000\pi t$, where $f_c = 10^8$ Hz. The peak frequency deviation is 20 kHz.

1. Determine the amplitude and frequency of all signal components that have a power level of at least 10% of the power of the unmodulated carrier component.
2. From Carson's rule, determine the approximate bandwidth of the FM signal.

Problem 3.31

1) The modulation index is

$$\beta = \frac{k_f \max[|m(t)|]}{f_m} = \frac{\Delta f_{\max}}{f_m} = \frac{20 \times 10^3}{10^4} = 2$$

The modulated signal $u(t)$ has the form

$$\begin{aligned} u(t) &= \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t + \phi_n) \\ &= \sum_{n=-\infty}^{\infty} 100 J_n(2)\cos(2\pi(10^8 + n10^4)t + \phi_n) \end{aligned}$$

The power of the unmodulated carrier signal is $P = \frac{100^2}{2} = 5000$. The power in the frequency component $f = f_c + k10^4$ is

$$P_{f_c+kf_m} = \frac{100^2 J_k^2(2)}{2}$$

The next table shows the values of $J_k(2)$, the frequency $f_c + kf_m$, the amplitude $100 J_k(2)$ and the power $P_{f_c+kf_m}$ for various values of k.

Index k	$J_k(2)$	Frequency Hz	Amplitude $100 J_k(2)$	Power $P_{f_c+kf_m}$
0	.2239	10^8	22.39	250.63
1	.5767	$10^8 + 10^4$	57.67	1663.1
2	.3528	$10^8 + 2 \times 10^4$	35.28	622.46
3	.1289	$10^8 + 3 \times 10^4$	12.89	83.13
4	.0340	$10^8 + 4 \times 10^4$	3.40	5.7785

As it is observed from the table the signal components that have a power level greater than 500 (= 10% of the power of the unmodulated signal) are those with frequencies $10^8 + 10^4$ and $10^8 + 2 \times 10^4$. Since $J_n^2(\beta) = J_{-n}^2(\beta)$ it is conceivable that the signal components with frequency $10^8 - 10^4$ and $10^8 - 2 \times 10^4$ will satisfy the condition of minimum power level. Hence, there are four signal components that have a power of at least 10% of the power of the unmodulated signal. The components with frequencies $10^8 + 10^4$, $10^8 - 10^4$ have an amplitude equal to 57.67, whereas the signal components with frequencies $10^8 + 2 \times 10^4$, $10^8 - 2 \times 10^4$ have an amplitude equal to 35.28.

2) Using Carson's rule, the approximate bandwidth of the FM signal is

$$B_c = 2(\beta + 1)f_m = 2(2 + 1)10^4 = 6 \times 10^4 \text{ Hz}$$