Problems

Problem 2.1 A signal that is sometimes used in communication systems is a *raised cosine pulse*. Figure P2.1 shows a signal $g_p(t)$ that is a periodic sequence of these pulses with equal spacing between them. Show that the first three terms in the Fourier series expansion of $g_p(t)$ are as follows:

$$g_p(t)=\tfrac{1}{2}+\frac{8}{3\pi}\cos(\pi t)+\tfrac{1}{2}\cos(2\pi t)+\cdots$$

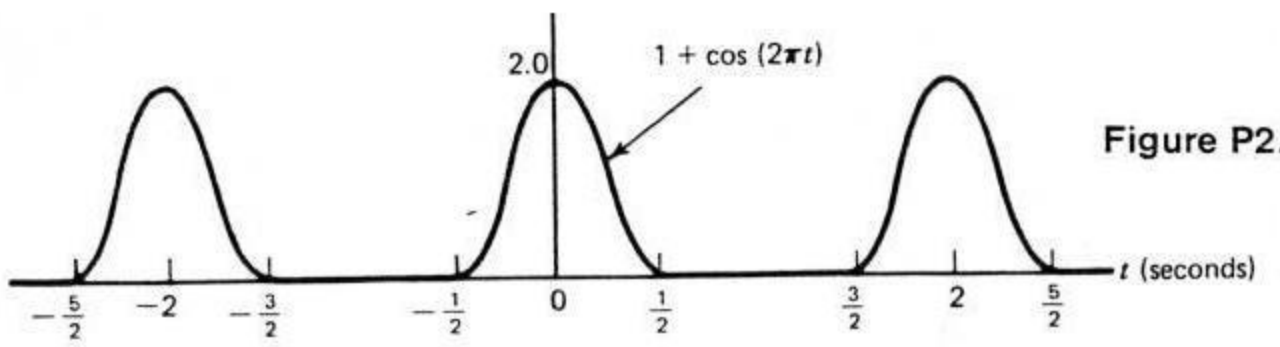

Figure P2.1

Problem 2.5 Determine the Fourier transform of the signal $g(t)$ consisting of three rectangular pulses, as shown in Fig. P2.3. Sketch the amplitude spectrum of this signal for the case when $T \ll T_0$.

Hint: Consider a rectangular pulse of amplitude A and duration T, and use the linearity and time-shifting properties of the Fourier transform.

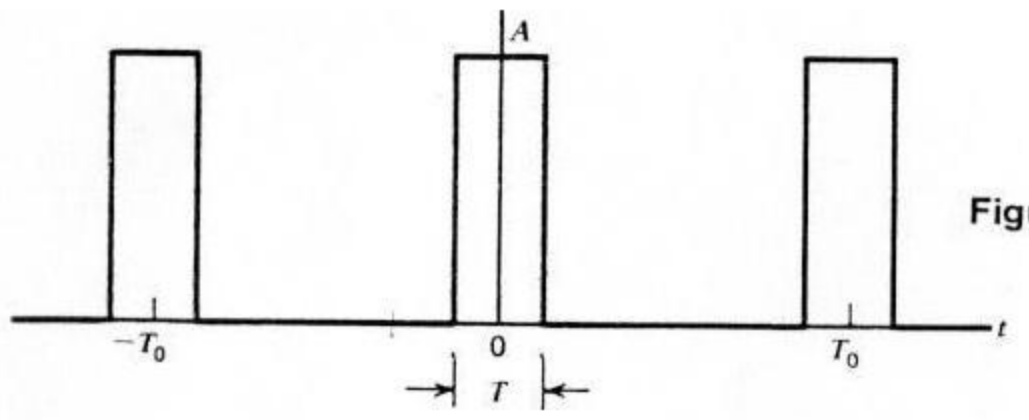

Figure P2.3

Problem 2.15 A signal $g_1(t)$ is defined by

$$g_1(t)=\exp(-\alpha t)u(t)$$

where $\alpha>0$.

(a) Find the function $g_2(t)$ obtained by convolving $g_1(t)$ with itself.
(b) Find the Fourier transform of $g_2(t)$.

Problem 2.16 A signal $x(t)$ of finite energy is applied to a square-law device whose output $y(t)$ is defined by

$$y(t)=x^2(t)$$

The spectrum of $x(t)$ is limited to the frequency interval $-W \leqslant f \leqslant W$. Hence, show that the spectrum of $y(t)$ is limited to $-2W \leqslant f \leqslant 2W$.

Hint: Express $y(t)$ as the product of $x(t)$ by $x(t)$.

Problem 4.8 A carrier wave is frequency-modulated using a sinusoidal signal of frequency f_m and amplitude A_m.

(a) Determine the values of the modulation index β for which the carrier component of the FM wave is reduced to zero. For this calculation you may use the values of $J_0(\beta)$ given in Table A4.1 of Appendix 4.
(b) In a certain experiment conducted with $f_m=1$ kHz and increasing A_m (starting from 0 volts), it is found that the carrier component of the FM wave is reduced to zero for the first time when $A_m=2$ volts. What is the frequency sensitivity of the modulator? What is the value of A_m for which the carrier component is reduced to zero for the second time?

Problem 4.13 A carrier wave of frequency 100 MHz is frequency-modulated by a sine-wave of amplitude 20 volts and frequency 100 kHz. The frequency sensitivity of the modulator is 25 kHz per volt.

(a) Determine the approximate bandwidth of the FM wave, using Carson's rule.
(b) Determine the bandwidth by transmitting only those side-frequencies whose amplitudes exceed 1 percent of the unmodulated carrier amplitude. Use the universal curve of Fig. 4.9 for this calculation.
(c) Repeat your calculations, assuming that the amplitude of the modulating wave is doubled.
(d) Repeat your calculations, assuming that the modulation frequency is doubled.