



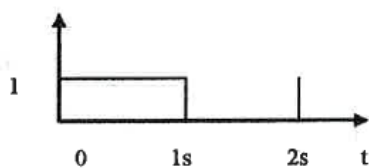
MINISTÉRIO DA EDUCAÇÃO
UNIVERSIDADE TECNOLÓGICA FEDERAL DO PARANÁ
DEPARTAMENTO ACADÊMICO DE ELETRÔNICA

CONTROLE II - TERCEIRO EXERCÍCIO PREPARATÓRIO- 20/06/17
PROF. PAULO BRERO

ALUNO: _____

1ª Questão: Calcule o valor do vetor de estados no tempo, para: a) $t=1s$ (**valor=4,0**) ; b) $t=2s$. (**valor=4,0**)

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \bar{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x}$$



2ª Questão: Calcule o valor da saída para: a) $t=1s$ **valor=1,0** ; b) $t=2s$. (**valor=1,0**)

GABARITO

(I)



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$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \bar{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x}$$

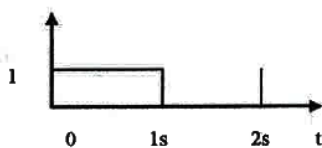
$$A = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s & -1 \\ -1 & s+2 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 1 & s \end{bmatrix}}{s^2 + 2s - 1} = \frac{\begin{bmatrix} s+2 & 1 \\ 1 & s \end{bmatrix}}{(s+2,414)(s-0,414)}$$



2ª Questão: Calcule o valor da saída para: a) $t=1s$ valor=1,0) ; b) $t=2s$. (valor=1,0)

$$\frac{s+2}{(s+2,414)(s-0,414)} = \frac{A}{s+2,414} + \frac{B}{s-0,414}$$

$$A = \frac{(s+2)}{(s-0,414)} \Big|_{s=-2,414}$$

$$A = \frac{-0,414}{-2,828} = 0,1464$$

$$B = \frac{s+2}{s+2,414} \Big|_{s=0,414}$$

$$B = \frac{2,414}{2,828} = 0,8536$$

$$\frac{s}{(s+2,414)(s-0,414)} = \frac{A}{s+2,414} + \frac{B}{s-0,414}$$

$$A = \frac{-2,414}{-2,828} = 0,8536$$

$$B = \frac{0,414}{2,828} = 0,1464$$

$$\frac{1}{(s+2,414)(s-0,414)} = \frac{A}{s+2,414} + \frac{B}{s-0,414}$$

$$A = \frac{1}{s-0,414} \Big|_{s=-2,414}$$

$$A = \frac{1}{-2,828} = -0,354$$

$$B = \frac{1}{s+2,414} \Big|_{s=0,414}$$

$$B = \frac{1}{2,828} = 0,354$$

$$(sI - A)^{-1} = \frac{0,1464}{s+2,414} + \frac{0,8536}{s-0,414}$$

$$\frac{-0,354}{s+2,414} + \frac{0,354}{s-0,414} = \frac{0,854}{s+2,414} + \frac{0,1464}{s-0,414}$$

$$\textcircled{V} (SI - A)^{-1} = \begin{bmatrix} \frac{0,1464}{s+2,414} + \frac{0,854}{s-0,414} & \frac{-0,354}{s+2,414} + \frac{0,354}{s-0,414} \\ \frac{-0,354}{s+2,414} + \frac{0,354}{s-0,414} & \frac{0,854}{s+2,414} + \frac{0,1464}{s-0,414} \end{bmatrix}$$

$$e^{At} = \mathcal{B}[(SI - A)^{-1}]$$

$$\phi(t) = e^{At} = \begin{bmatrix} 0,1464 e^{-2,414t} + 0,854 e^{0,414t} & -0,354 e^{-2,414t} + 0,354 e^{0,414t} \\ -0,354 e^{-2,414t} + 0,354 e^{0,414t} & 0,854 e^{-2,414t} + 0,1464 e^{0,414t} \end{bmatrix}$$

$$\begin{bmatrix} -0,354 e^{-2,414t} + 0,354 e^{0,414t} \\ 0,854 e^{-2,414t} + 0,1464 e^{0,414t} \end{bmatrix}$$

$$\underline{X}(t) = e^{A(t-t_0)} \underline{X}(t_0) + \int_{t_0}^t e^{A(t-\tau)} B u d\tau = \int_{t_0}^t e^{A(t-\tau)} B u d\tau$$

$A = 0$

$$\underline{X}(t) = \int_0^t \begin{bmatrix} 0,1464 e^{-2,414(t-\tau)} + 0,854 e^{0,414(t-\tau)} & -0,354 e^{-2,414(t-\tau)} + 0,354 e^{0,414(t-\tau)} \\ -0,354 e^{-2,414(t-\tau)} + 0,354 e^{0,414(t-\tau)} & 0,854 e^{-2,414(t-\tau)} + 0,1464 e^{0,414(t-\tau)} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} d\tau$$

$$\underline{X}(t) = \int_0^t \begin{bmatrix} -0,2076 e^{-2,414(t-\tau)} + 1,208 e^{0,414(t-\tau)} \\ 0,5 e^{-2,414(t-\tau)} + 0,5 e^{0,414(t-\tau)} \end{bmatrix} d\tau$$

$$\underline{X}(t) = \begin{bmatrix} -0,2076 e^{-2,414t} \left(\frac{e^{2,414t}}{2,414} \right)_0^t + 1,208 e^{0,414t} \left(\frac{e^{-0,414t}}{-0,414} \right)_0^t \\ 0,5 e^{-2,414t} \left(\frac{e^{2,414t}}{2,414} \right)_0^t + 0,5 e^{0,414t} \left(\frac{e^{-0,414t}}{-0,414} \right)_0^t \end{bmatrix} + \begin{bmatrix} 1,208 e^{0,414t} \left(\frac{e^{-0,414t}}{-0,414} \right)_0^t \\ 0,5 e^{0,414t} \left(\frac{e^{-0,414t}}{-0,414} \right)_0^t \end{bmatrix}$$

(11)

$$\underline{X}(t) = \begin{bmatrix} \frac{-0,2076}{2,414} (1 - e^{-2,414t}) + \frac{1,208}{(-0,414)} (1 - e^{0,414t}) \\ \frac{0,5}{2,414} (1 - e^{-2,414t}) + \frac{(-0,5)}{0,414} (1 - e^{0,414t}) \end{bmatrix}$$

(TBL)

$$\underline{X}(1) = \begin{bmatrix} 1,4181 \\ 0,808 \end{bmatrix}$$

$$Y = 1,4281$$

Interval $1 \leq t < 2s$

$$u=0 \quad \underline{X}(1) = \begin{bmatrix} 1,4181 \\ 0,808 \end{bmatrix}$$

$$\underline{X}(t) = e^{A(t-1)} \underline{X}(1)$$

$$\underline{X}(t) = \begin{bmatrix} 0,1464 e^{-2,414(t-1)} + 0,854 e^{0,414(t-1)} \\ -0,354 e^{-2,414(t-1)} + 0,354 e^{0,414(t-1)} \end{bmatrix} \begin{bmatrix} -0,354 e^{-2,414(t-1)} + 0,354 e^{0,414(t-1)} \\ 0,854 e^{-2,414(t-1)} + 0,1464 e^{0,414(t-1)} \end{bmatrix} \begin{bmatrix} 1,4181 \\ 0,808 \end{bmatrix}$$

$$\underline{X}(t) = \begin{bmatrix} -0,0706 e^{-2,414(t-1)} + 1,4962 e^{0,414(t-1)} \\ 0,188 e^{-2,414(t-1)} + 0,6199 e^{0,414(t-1)} \end{bmatrix}$$

p/ $t=2s$

$$\underline{X}(2) = \begin{bmatrix} 2,257 \\ 0,9546 \end{bmatrix}$$

$$Y = 2,257$$