

DÜZGÜN DAĞILIM

$$f(x) = \begin{cases} \frac{1}{b-a} & , a \leq x \leq b \\ 0 & , \text{d.d} \end{cases}$$

$$F(x) = P(X \leq x) = \int_a^x \frac{1}{b-a} dt$$

$$= \frac{1}{b-a} \cdot t \Big|_a^x = \frac{x}{b-a} - \frac{a}{b-a} = \frac{x-a}{b-a}$$

$$F(x) = \begin{cases} 0 & , x < a \\ \frac{x-a}{b-a} & , a \leq x \leq b \\ 1 & , x > b \end{cases}$$

DÜZGÜN DAĞILIM

Beklenen Değer

$$E(X) = \int_a^b x \cdot f(x) dx$$

$$= \int_a^b x \frac{1}{b-a} dx$$

$$= \frac{1}{b-a} \frac{x^2}{2} \Big|_a^b$$

$$= \frac{b^2 - a^2}{b-a} \cdot \frac{1}{2}$$

$$= \frac{(b-a)(b+a)}{b-a} \cdot \frac{1}{2} = \boxed{\frac{b+a}{2}} \xrightarrow{E(X)}$$

DÜZGÜN DAĞILIM

Varyans

$$\begin{aligned} E(X)^2 &= \int_a^b x^2 \cdot \frac{1}{b-a} dx = \frac{1}{b-a} \frac{x^3}{3} \Big|_a^b \\ &= \frac{b^3 - a^3}{b-a} \cdot \frac{1}{3} = \frac{(b-a)(b^2 + ab + a^2)}{(b-a) 3} \\ &= \frac{b^2 + ab + a^2}{3} \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 = \frac{b^2 + ab + a^2}{3} - \left(\frac{b+a}{2}\right)^2 \\ &= \frac{4b^2 + 4ab + 4a^2 - 3b^2 - 6ab - 3a^2}{12} \\ &= \frac{b^2 - 2ab + a^2}{12} = \boxed{\frac{(b-a)^2}{12}} \rightarrow \text{Var}(X) \end{aligned}$$

DÜZGÜN DAĞILIM

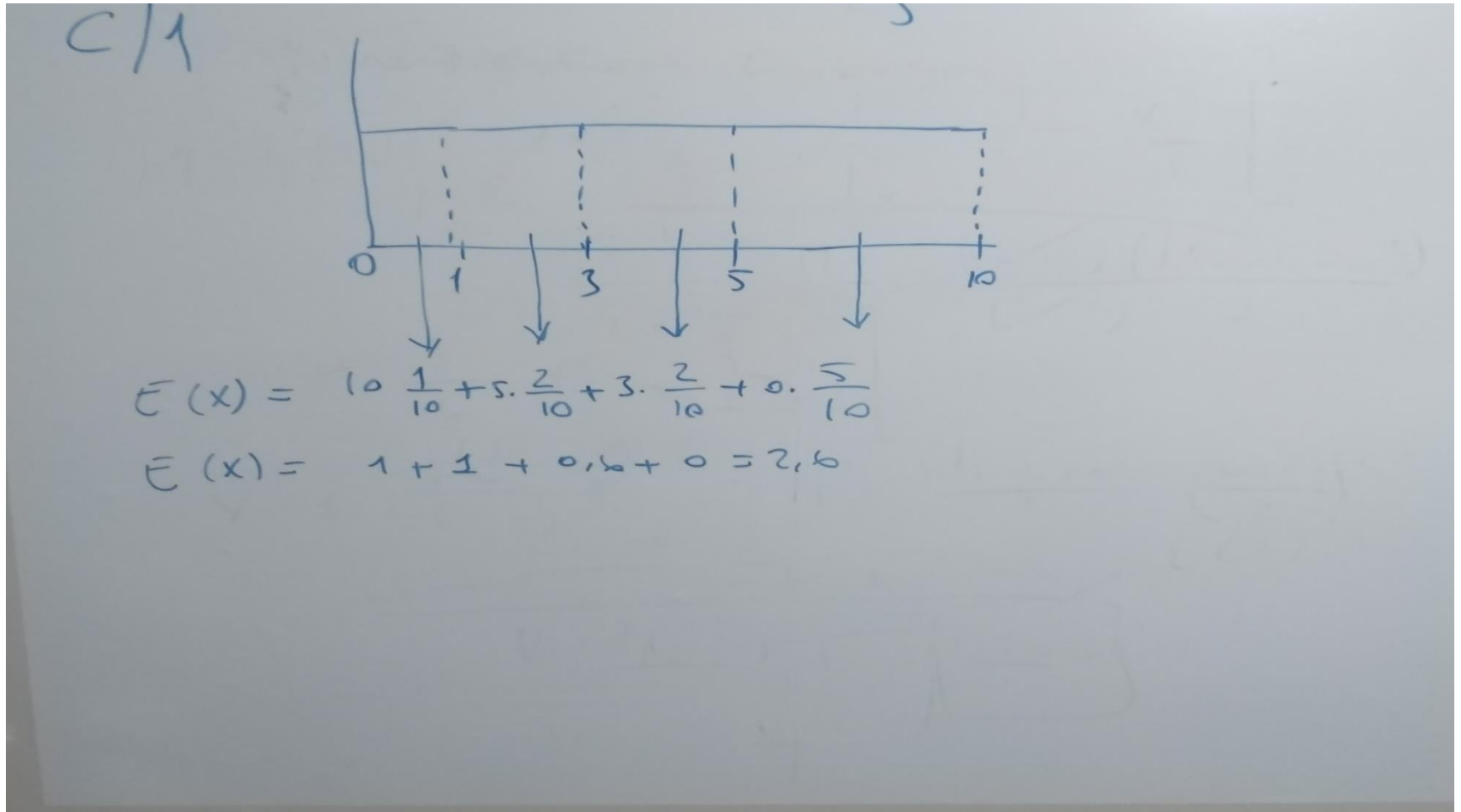
Moment Çıkaran Fonksiyon

$$\begin{aligned} M_x(t) &= \int_a^b e^{tx} \frac{1}{b-a} dx \\ &= \frac{1}{t} \cdot e^{tx} \cdot \frac{1}{b-a} \Big|_a^b \\ &= \frac{1(e^{tb} - e^{ta})}{t(b-a)} \end{aligned}$$

ÖRNEK

- Bir adam bir hedefe atış yapmaktadır. Eğer atış hedefin 1 cm içinde ise 10 puan, 1-3 cm içinde ise 5 puan ve hedefin 3-5 cm içinde ise 3 puan almaktadır. Atışın hedefe olan uzaklığı $(0,10)$ arasında düzgün dağılıma sahipse elde edeceği puanın beklenen değerini hesaplayın.

DÜZGÜN DAĞILIM



ÖRNEK

- Beklenen değer tanımı kullanarak $(0,1)$ aralığında tanımlı X tesadüfi değişkeni için $E(X^n)$ değerini hesaplayınız

DÜZGÜN DAĞILIM

C/2

$$E(X^n) = \int_a^b X^n \cdot \frac{1}{b-a} dx$$

$$= \int_0^1 x^n \cdot \frac{1}{1-0} dx$$

$$= \int_0^1 x^n dx = \frac{x^{n+1}}{n+1} \Big|_0^1$$

$$= \frac{1^{n+1}}{n+1} - \frac{0^{n+1}}{n+1}$$

$$= \frac{1}{n+1} \rightarrow$$

$$n=1 \rightarrow E(X) = \frac{1}{2}$$

$$n=2 \rightarrow E(X^2) = \frac{1}{3}$$

$$\text{Var}(X) = \frac{1}{3} - \left(\frac{1}{2}\right)^2$$

$$\text{Var}(X) = \frac{1}{12}$$

ÜSTEL DAĞILIM

Üstel Dağılım
 $h > 0$

$$F(x) = \begin{cases} h \cdot e^{-hx} & , x > 0 \\ 0 & , x \leq 0 \end{cases}$$

$X \sim \text{Üstel}(h)$

$$\begin{aligned} F(x) &= \int_0^x h \cdot e^{-ht} dt \\ &= h \int_0^x e^{-ht} dt = h \cdot \left[-\frac{1}{h} \cdot e^{-ht} \right]_0^x \\ &= -e^{-ht} \Big|_0^x = -e^{-hx} - (-1) \\ F(x) &= 1 - e^{-hx} \\ F(x) &= \begin{cases} 0 & , x < 0 \\ 1 - e^{-hx} & , x \geq 0 \\ 1 & , x \rightarrow \infty \end{cases} \end{aligned}$$

ÜSTEL DAĞILIM

Üstel Dağılım

Beklenti Değeri

$$E(X) = \int_0^{\infty} x \cdot h e^{-hx} dx$$

$u = x$
 $du = dx$

$$h e^{-hx} dx = dv$$
$$-e^{-hx} = v$$
$$x \cdot e^{-hx} \Big|_0^{\infty} - \int_0^{\infty} -e^{-hx} dx$$
$$\int_0^{\infty} e^{-hx} dx$$
$$-\frac{1}{h} e^{-hx} \Big|_0^{\infty}$$
$$-\frac{1}{h} (0 - 1) = \boxed{\frac{1}{h} = E(X)}$$

ÜSTEL DAĞILIM

Üstel Dağılım

Varyans

$$E(X^2) = \int_0^{\infty} x^2 \cdot h \cdot e^{-hx} dx$$

$$x^2 = u \quad h \cdot e^{-hx} dx = dv$$

$$2x dx = dv \quad -e^{-hx} = v$$

$$x^2 \cdot -e^{-hx} \Big|_0^{\infty} - \int_0^{\infty} -e^{-hx} 2x dx = \int_0^{\infty} 2x e^{-hx} dx$$

$$\int_0^{\infty} 2x e^{-hx} dx \quad \begin{matrix} 2x = u \\ 2dx = dv \end{matrix} \quad e^{-hx} dx = dv \rightarrow -\frac{1}{h} e^{-hx} = v$$

$$2x \cdot e^{-hx} \Big|_0^{\infty} - \int_0^{\infty} -\frac{1}{h} \cdot e^{-hx} 2 \cdot dx$$

$$\downarrow 0 \quad \frac{2}{h} \int_0^{\infty} e^{-hx} dx \quad \frac{2}{h} \left[-\frac{1}{h} e^{-hx} \right]_0^{\infty} = \frac{2}{h^2} (0 - (-1)) = \frac{2}{h^2}$$

$$E(X)^2 = \frac{2}{h^2}$$

$$Var(X) = \frac{2}{h^2} - \left(\frac{1}{h}\right)^2$$

$$Var(X) = \frac{1}{h^2}$$

ÜSTEL DAĞILIM

Üstel Dağılım

Moment Gıkarıcı Fonksiyon

$$\begin{aligned} & \int_0^{\infty} e^{tx} \cdot h \cdot e^{-hx} dx \\ & h \cdot \int_0^{\infty} e^{(t-h)x} dx = h \cdot \int_0^{\infty} e^{-(h-t)x} dx \\ & h \cdot \left. \frac{1}{t-h} \cdot e^{-(h-t)x} \right|_0^{\infty} \\ & \frac{h}{-(h-t)} (e^{-\infty} - e^0) \\ & \frac{h}{-(h-t)} \cdot -1 = \frac{h}{h-t} = M_x(t) \end{aligned}$$

ÜSTEL DAĞILIM

Üstel Dağılım

Belleksizlik Özelliği

$$P(X > s+t | X > s) = P(X > t)$$

$$\begin{aligned} P(X > s+t | X > s) &= \frac{P(X > s+t, X > s)}{P(X > s)} = \frac{P(X > s+t)}{P(X > s)} \\ &= \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda(s+t-s)} = e^{-\lambda t} \\ &\quad \swarrow \\ &\quad P(X > t) \end{aligned}$$

Örnek

- Bir elektronik parçanın ömrü, yıl olarak aşağıdaki olasılık yoğunluk fonksiyonuna sahiptir.
- Bu elektronik parçanın
 - a) En çok 4 yıl dayanması olasılığı
 - b) En az 3 yıl dayanma olasılığı
 - c) Beklenen ömrü
 - d) Ömrünün standart sapması nedir?

$$f(x) = \begin{cases} \frac{1}{3} * e^{-\frac{1}{3} * x}, & x > 0 \\ 0, & d.d \end{cases}$$

ÜSTEL DAĞILIM

Üstel Dağılım

c1/ $f(x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x} & , x > 0 \\ 0 & , \text{d.d} \end{cases}$

$$F(x) = \int_0^x \frac{1}{3} e^{-\frac{1}{3}t} dt = -3 \cdot \frac{1}{3} \cdot e^{-\frac{1}{3}t} \Big|_0^x = -e^{-\frac{1}{3}t} \Big|_0^x = -(e^{-\frac{1}{3}x} - 1)$$
$$F(x) = 1 - e^{-\frac{1}{3}x}$$

a) $P(X \leq 4) = F(4) = 1 - e^{-\frac{1}{3} \cdot 4} = 1 - e^{-\frac{4}{3}} = 0,7364$

b) $P(X \geq 3) = 1 - P(X < 3) = 1 - F(3) = 1 - 1 + e^{-1} = 0,3678$

c) $E(X) = \frac{1}{\frac{1}{3}} = 3$

d) $\text{Var}(X) = \frac{1}{(\frac{1}{3})^2} = 9 \quad S(X) = \sqrt{9} = 3$

Örnek

- Bir radyonun çalışma ömrü (yıl) $\lambda=1/10$ ortalamalı üstel dağılıma sahiptir. Bir radyo satın alındığında
- a) En az on yıl kullanma olasılığı nedir?
- b) On yıl kullandıktan sonra 8 yıl daha radyo kullanma olasılığı nedir?

ÜSTEL DAĞILIM

C2./ Üstel Dağılım
 $\lambda = \frac{1}{10}$

$$a) P(X > 10) = 1 - P(X < 10) = 1 - (1 - e^{(-\frac{1}{10}) \cdot 10}) \\ = 0,3678$$

(Bağımsızlık)

$$b) P(X > 18 \mid X > 10) = P(X > 8) = 1 - P(X < 8) \\ = 1 - (1 - e^{(-\frac{1}{10}) \cdot 8}) \\ = 0,4493$$

GAMMA DAĞILIMI

Gamma Dağılımı

$\forall n > 0$

$$\Gamma(n) = \int_0^{\infty} x^{n-1} \cdot e^{-x} dx$$

$$\Gamma(n) = (n-1) \Gamma(n-1)$$

$$\Gamma(n) = (n-1) \cdot (n-2) \dots \Gamma(1)$$

$$\Gamma(1) = \int_0^{\infty} x^{1-1} \cdot e^{-x} = \int_0^{\infty} e^{-x} dx = -e^{-x} \Big|_0^{\infty} = 1$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(k+1) = k \Gamma(k)$$

$$\Gamma\left(\frac{1}{2}+1\right) = \frac{1}{\sqrt{\pi}} \Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}$$

GAMMA DAĞILIMI

Gamma Dağılımı

$X_1, X_2, \dots, X_n \sim \text{Üstel}(h)$

$X = X_1 + X_2 + \dots + X_n$

$f(x) = \begin{cases} \frac{h(hx)^{n-1} e^{-hx}}{\Gamma(n)}, & x > 0 \\ 0, & x \leq 0 \end{cases}$

$X \sim \text{Gamma}(n, h)$

3. $n = k/2$
 $h = 1/2$
 $X \sim \chi_k^2$

1. $n=1 \rightarrow \frac{h(hx)^{n-1} e^{-hx}}{\Gamma(1)} = h \cdot e^{-hx} \rightarrow \text{Üstel} \sim X$

2. $\forall n \in \mathbb{Z}^+$ $X \sim \text{Erlang}(n, h)$

$f(x) = \begin{cases} \frac{h(hx)^{n-1} e^{-hx}}{(n-1)!}, & x > 0 \\ 0, & x \leq 0 \end{cases}$

$F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-hx} \sum_{i=0}^{n-1} \frac{(hx)^i}{i!}, & x > 0 \end{cases}$

$\Delta, x \rightarrow \infty$

GAMMA DAĞILIMI

Gamma Dağılımı

Beklendi Değer - Varyans

$$E(X) = \int_0^{\infty} x \cdot \frac{h(hx)^{n-1} \cdot e^{-hx}}{\Gamma(n)} dx$$

$$E(X) = \int_0^{\infty} \frac{x \cdot h(hx)^{n-1} \cdot e^{-hx} \cdot (n \cdot h)}{\Gamma(n) \cdot n \cdot h} dx$$

$$E(X) = \frac{n}{h} \int_0^{\infty} \frac{h(hx)^n e^{-hx}}{\Gamma(n+1)} dx = \frac{n}{h}$$

$$E(X^2) = \int_0^{\infty} x^2 \cdot \frac{h(hx)^{n-1} e^{-hx}}{\Gamma(n)} dx \cdot \frac{n(n+1)h^2}{n(n+1)h^2}$$

$$E(X^2) = \frac{n(n+1)}{h^2} \int_0^{\infty} \frac{h(hx)^{n+1} e^{-hx}}{\Gamma(n+2)} dx = \frac{n(n+1)}{h^2}$$

$$Var(X) = \frac{n(n+1)}{h^2} - \frac{n^2}{h^2}$$

$$Var(X) = \frac{n}{h^2}$$

GAMMA DAĞILIMI

Gamma Dağılımı
Moment Gıkarar Fonksiyon

$i=1, 2, \dots, n$ $X_i \sim \text{Üstel}(h)$

$$X = \sum_{i=1}^n X_i \quad X \sim \text{Gamma}(h)$$

$$\begin{aligned} M_X(t) &= \bar{E}(e^{tx}) = \bar{E}(e^{tx_1} \cdot e^{tx_2} \cdot \dots \cdot e^{tx_n}) \\ &= \bar{E}(e^{tx_1}) \bar{E}(e^{tx_2}) \dots \bar{E}(e^{tx_n}) \\ &= M_{X_1}(t) M_{X_2}(t) \dots M_{X_n}(t) \\ &= \frac{h}{h-t} \cdot \frac{h}{h-t} \cdot \dots \cdot \frac{h}{h-t} \\ M_X(t) &= \left(\frac{h}{h-t} \right)^n \end{aligned}$$

Örnek

- Bir süpermarkete gelen müşterilerin gelişler arası süreleri dakikada 0,4 ortalama ile üstel dağılmaktadır. Buna göre
- a)2. müşterinin birinci dakikada gelme
- b)5. müşterinin kaç dakika içinde gelmesi beklenir?

GAMMA DAĞILIMI

Gamma Dağılımı

a) A: "2. müsterinin 1. defa kadar gelmesi"

1. müsterinin geliş süresi $T_1 \sim \text{üstel}(0,4)$

2. müsterinin geliş süresi ile 1. müsterinin geliş süresi arası geçen zaman $T_2 \sim \text{üstel}(0,4)$

2. müsterinin gelişine kadar geçen süre

$$t_2 = T_1 + T_2 \sim \text{Gamma}(2, 0,4)$$

$$F_{t_2}(x) = \begin{cases} \frac{(0,4) (0,4)^{2-1} e^{-0,4x}}{\Gamma(2)}, & x > 0 \\ 0, & \text{öğ} \end{cases}$$

$$\int_0^1 (0,4) \cdot (0,4x) e^{-0,4x} dx = 0,0615$$

b) $T = T_1 + T_2 + \dots + T_5$ $E(T_5) = E(T_1 + T_2 + \dots + T_5)$
 $= E(T_1) + E(T_2) + \dots + E(T_5)$
 $= \frac{1}{0,4} + \frac{1}{0,4} + \dots + \frac{1}{0,4} = 12,5$

BETA DAĞILIMI

Beta Dağılımı

$$\forall \alpha, \beta > 0$$

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx$$

$$B(\alpha, \beta) = B(\beta, \alpha)$$

$$B(\alpha, \beta) = \frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha + \beta)}$$

$$\frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha + \beta)} = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx$$

BETA DAĞILIMI

Beta Dağılımı

$$f(x) = \begin{cases} \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, & 0 < x < 1 \\ 0, & \text{d.d.} \end{cases}$$

$$X \sim \text{Beta}(\alpha, \beta)$$

1. $\alpha = \beta = 1 \rightarrow X \sim U(0,1)$ Düzgün dağılım

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{d.d.} \end{cases}$$

2. $\alpha = 2 \quad \beta = 1 \quad f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{d.d.} \end{cases}$ Üsgenli dağılım

$\alpha = 1 \quad \beta = 2 \quad f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & \text{d.d.} \end{cases}$

3. $\alpha = 2 \quad \beta = 2 \quad f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{d.d.} \end{cases}$

Parabolik dağılım

BETA DAĞILIMI

Beta Dağılımı

Beklener Değer

$$E(X) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} \int_0^1 x \cdot x^{\alpha-1} (1-x)^{\beta-1} dx$$

$$\int_0^1 x \cdot x^{\alpha-1} (1-x)^{\beta-1} dx = \int_0^1 x^{\alpha} (1-x)^{\beta-1} dx$$

$$\Gamma(\alpha+1) = \alpha \Gamma(\alpha)$$

$$\Gamma(\alpha+\beta+1) = (\alpha+\beta) \Gamma(\alpha+\beta)$$

$$= \int_0^1 \frac{\Gamma(\alpha+1) \Gamma(\beta)}{\Gamma(\alpha+\beta+1)} dx$$

$$= \int_0^1 \frac{\alpha \cdot \Gamma(\alpha) \cdot \Gamma(\beta)}{(\alpha+\beta) \Gamma(\alpha+\beta)} dx$$

$$E(X) = \frac{\alpha}{\alpha+\beta}$$

$$E(X) = \frac{\alpha}{\alpha+\beta}$$

$$\int_0^1 \frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha+\beta)} dx = 1$$

BETA DAĞILIMI

Beta Dağılımı

Varyans

$$E(x^2) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 x^2 x^{\alpha-1} (1-x)^{\beta-1} dx$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+\beta+2)}$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\alpha+1 \cdot \Gamma(\alpha+1) \cdot \Gamma(\beta)}{\Gamma(\alpha+\beta+1) \cdot (\alpha+\beta+1)}$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\alpha+1 \cdot \Gamma(\alpha) \cdot \alpha \cdot \Gamma(\beta)}{\Gamma(\alpha+\beta) \cdot (\alpha+\beta) \cdot (\alpha+\beta+1)}$$

$$E(x^2) = \frac{(\alpha+1)\alpha}{(\alpha+\beta)(\alpha+\beta+1)}$$

$$Var(x) = E(x^2) - E(x)^2$$

$$Var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

BETA DAĞILIMI

Beta Dağılımı

Sıfır etrafındaki k . moment

$$E(X^k) = \int_0^1 x^k f(x) dx$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 x^{k+\alpha-1} (1-x)^{\beta-1} dx$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \frac{\Gamma(\alpha+k) \cdot \Gamma(\beta)}{\Gamma(\alpha+k+\beta)}$$

$$E(X^k) = \frac{\Gamma(\alpha+\beta) \Gamma(\alpha+k)}{\Gamma(\alpha) \Gamma(\alpha+k+\beta)}$$

Örnek

- Kimyasal bir maddenin maksimum bir yıl olan ömrü $\alpha=3$ $\beta=4$ parametreleri ile beta dağılımına uymaktadır. X tesadüfi değişkeni bu maddenin ömrünü gösterdiğine göre
- a) $F(x)$ 'i bulunuz
- b) Bu kimyasal maddenin ömrünün üç aydan az olma olasılığı nedir?
- c) $P(0,2 < X < 0,5) = ?$
- d) Bu maddenin beklenen ömrü kaç aydır?

BETA DAĞILIMI

Beta Dağılımı

c1) a) $X \sim \beta(3,4)$ $f(x) = \frac{1}{\beta(3,4)} \cdot x^{3-1} (1-x)^{4-1}$

\downarrow \downarrow
 3 4

$$f(x) = \begin{cases} \frac{1}{\beta(3,4)} x^2 \cdot (1-x)^3 \\ 0 & \text{d.d.} \end{cases}$$

$$\beta(3,4) = \frac{\Gamma(3) \Gamma(4)}{\Gamma(3+4)} = \frac{2! 3!}{6!} = \frac{1}{60}$$

$$f(x) = \begin{cases} 60 x^2 (1-x)^3 & 0 < x < 1 \\ 0 & \text{d.d.} \end{cases}$$

$$F(x) = \int_0^x 60 t^2 (1-t)^3 dt$$

$$= x^3 (36x^2 + 20) - x^4 (45 + 10x^2)$$

b) $\text{Say} = \frac{3}{12} = 0,25 \text{ y.1}$ $P(X < 0,25) = F(0,25) = 0,169433$

c) $P(0,2 < X < 0,5) = F(0,5) - F(0,2) = 0,65125 - 0,08882 = 0,56243$

d) $P(X < 0,25) = F(0,25) = 0,169433$ $E(X) = \frac{\alpha}{\alpha+\beta} = \frac{3}{3+4} \text{ y.1} = 0,42857$

STUDENT T DAĞILIMI

Student T Dağılımı

$X \sim N(0,1)$
 $Y \sim \chi^2_v$ X ve Y bağımsız

$$T = \frac{X}{\sqrt{Y/v}}$$

$$f(t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right) \sqrt{v\pi}} \left(1 + \frac{t^2}{v}\right)^{-\frac{v+1}{2}} \quad -\infty < t < \infty$$

t dağılımı v serbestlik derecesi t dağılımına sahiptir.

$$\lim_{v \rightarrow \infty} f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

$V=1$ olursa Cauchy dağılımı

$E(X)=0$ $\text{Var}(X) = \frac{v}{v-2} \quad v > 2$

$X_1, X_2, \dots, X_n \sim \text{Normal}(\mu, \sigma^2)$

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \rightarrow n-1 \text{ serbestlik derecesi}$$

T dağılımı

F DAĞILIMI

F Dağılımı

$X_1 \sim \chi^2_{v_1}$ ve $X_2 \sim \chi^2_{v_2}$

$$F = \frac{\chi^2_{v_1}/v_1}{\chi^2_{v_2}/v_2}$$

$$F(x, v_1, v_2) = \begin{cases} \frac{\Gamma\left[\frac{(v_1+v_2)}{2}\right]}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)} x^{\frac{v_1}{2}-1} (1-x)^{\frac{v_2}{2}-1} & 0 < x < 1 \\ 0 & \text{d.i.d} \end{cases}$$

simetrik değil v_1 ve $v_2 \uparrow$ normaldeyken

v_1 payda v_2 paydanın serbestlik derecesidir.

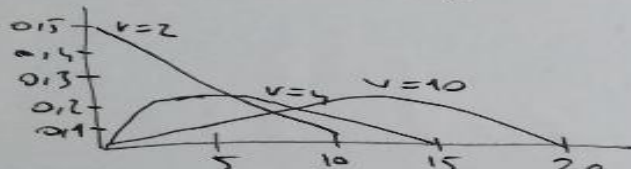
$$E(x) = \frac{v_2}{v_2-2} \quad \text{Var}(x) = \frac{2v_2^2 \left[1 + \frac{v_2-2}{v_1}\right]}{(v_2-2)^2 (v_2-4)}$$

KİKARE DAĞILIMI

Kikare Dağılımı

Gamma Dağılımı $\alpha = \frac{\nu}{2}$ $\beta = 2 \rightarrow$ Kikare Dağılımı

$$f(x^2) = \begin{cases} \frac{x^{\frac{\nu}{2}-1} \cdot e^{-\frac{x}{2}}}{\Gamma(\frac{1}{2}\nu) \cdot 2^{\nu/2}} & x \in [0, \infty) \\ 0 & , d.d. \end{cases}$$



Serbestlik derecesine göre
Kikare dağılımı

1. $y_i \in \text{Normal}(0,1)$ n tane değişken $X = \sum_{i=1}^n y_i^2 \sim \chi^2_n$
2. Normal dağılımlıyından n sayıda birim örnek \bar{X} ve s^2 bir örnekle alınırsa $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$
3. X değerler x_1, x_2, \dots, x_n olup C_1, C_2, \dots, C_k sınıfla ayrılabilir her bir sınıfın olasılıkları p_1, p_2, \dots, p_k olsun. Her sınıftaki bireylerin sayısı m_1, m_2, \dots, m_k birer birim değişkeni olup C_i sınıfına ait bireylerin, sayısı n_i ve ait olma olma olasılığı p_i dir. $n_i \sim B(n, p_i)$

KİKARE DAĞILIMI

Kikare Dağılımı

$$E(n_i) = n \cdot p_i \quad \text{Var}(n_i) = n \cdot p_i (1 - p_i) \approx n p_i$$

p_i yeterince küçük ise $1 - p_i$ 1'e yaklaşıyor

C_i 'ye ait eleman sayısının yeteri kadar büyük seçilmesiyle
Parektir. Binom \xrightarrow{n} Normal

$$z = \frac{n_i - np_i}{\sqrt{n \cdot p_i}}$$

$$\chi^2 = \sum_{i=1}^k z^2 = \frac{(n_i - np_i)^2}{np_i} = \sum_{i=1}^k \frac{(\text{Gözlenen Değer} - \text{Beklenen Değer})^2}{\text{Beklenen Değer}}$$

$$\chi^2 \approx k - 1$$

NORMAL DAĞILIM

Uygulamalı çalışmalarda en sık karşılaşılan dağılım normal dağılımdır. Örneğin insanların ağırlıkları, boyları, zekâ düzeyleri ya da bir sınavdan alınan notlar gibi birçok değişken normal dağılıma uygunluk gösterir.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right] \quad -\infty < x < +\infty; -\infty < \mu < +\infty; \sigma \geq 0$$

Dağılımın bilinmeyen parametreleri ortalama μ ve varyans σ^2 'dir.

Normal dağılıma sahip X rastgele değişkeni $X \sim N(\mu, \sigma^2)$ şeklinde gösterilir.

$X \sim N(50, 25)$ ise

X rastgele değişkeninin ortalaması 50 varyansı 25'tir.

NORMAL DAĞILIM

- **Normal Dağılımın Özellikleri**

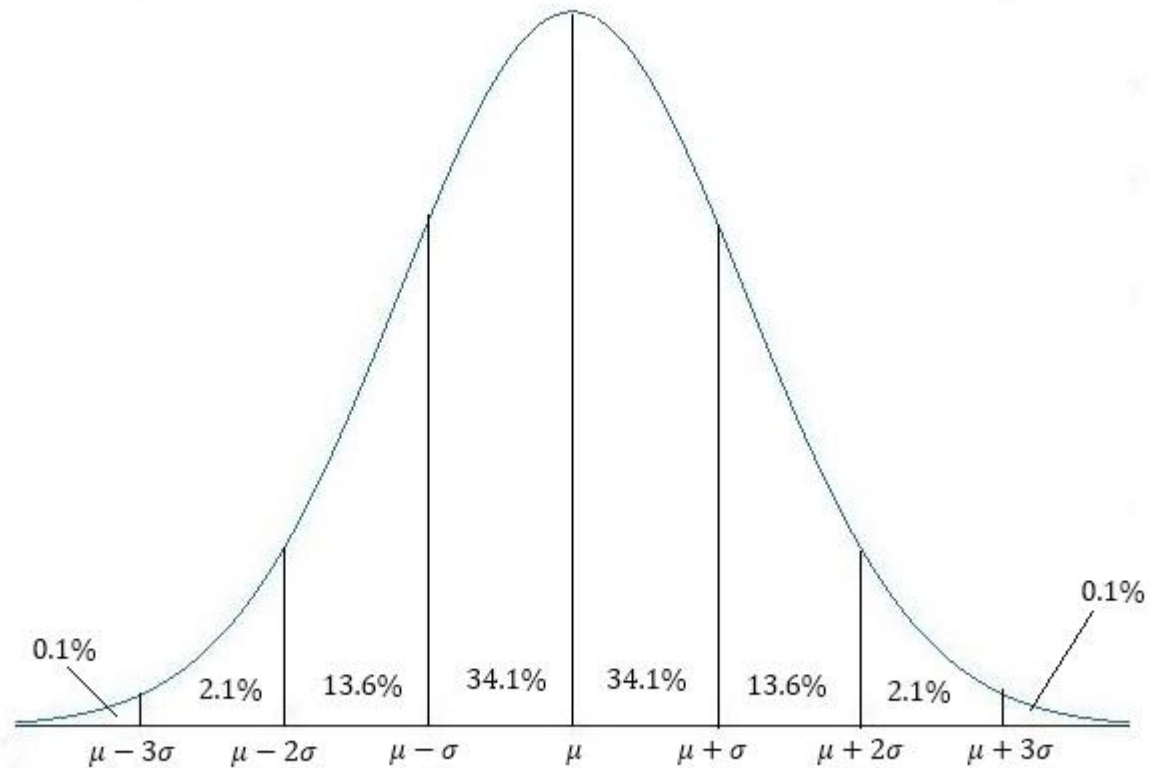
1. Ortalama, mod ve medyan eşittir.
2. Çan eğrisi şeklindedir ve ortalama etrafında simetriktir.
3. Eğrinin altında kalan alan toplamı 1'dir.
4. Normal eğri x eksenin ortalamadan uzaklaştıkça yakınlaşır fakat x eksenini asla kesmez.
5. $\mu - \sigma$ ve $\mu + \sigma$ eğrinin büküm noktalarıdır.
6. $x = \mu$ noktasında $f(x)$ fonksiyonu $1/\sqrt{2\pi}\sigma$ maksimum değerine sahiptir.

NORMAL DAĞILIM

- Olasılık yoğunluk fonksiyonunun tanım aralığındaki integralinin sonucu μ ve σ ne olursa olsun **1'dir**.

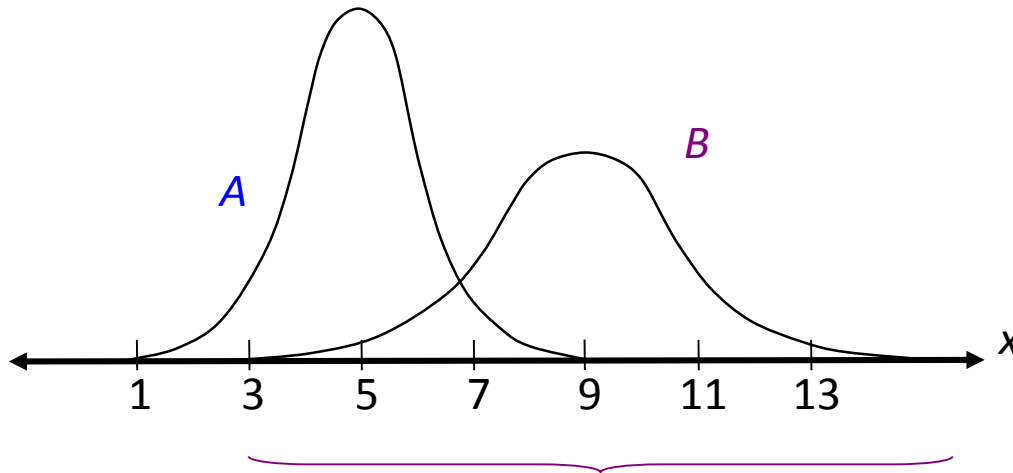
$$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1$$

NORMAL DAĞILIM

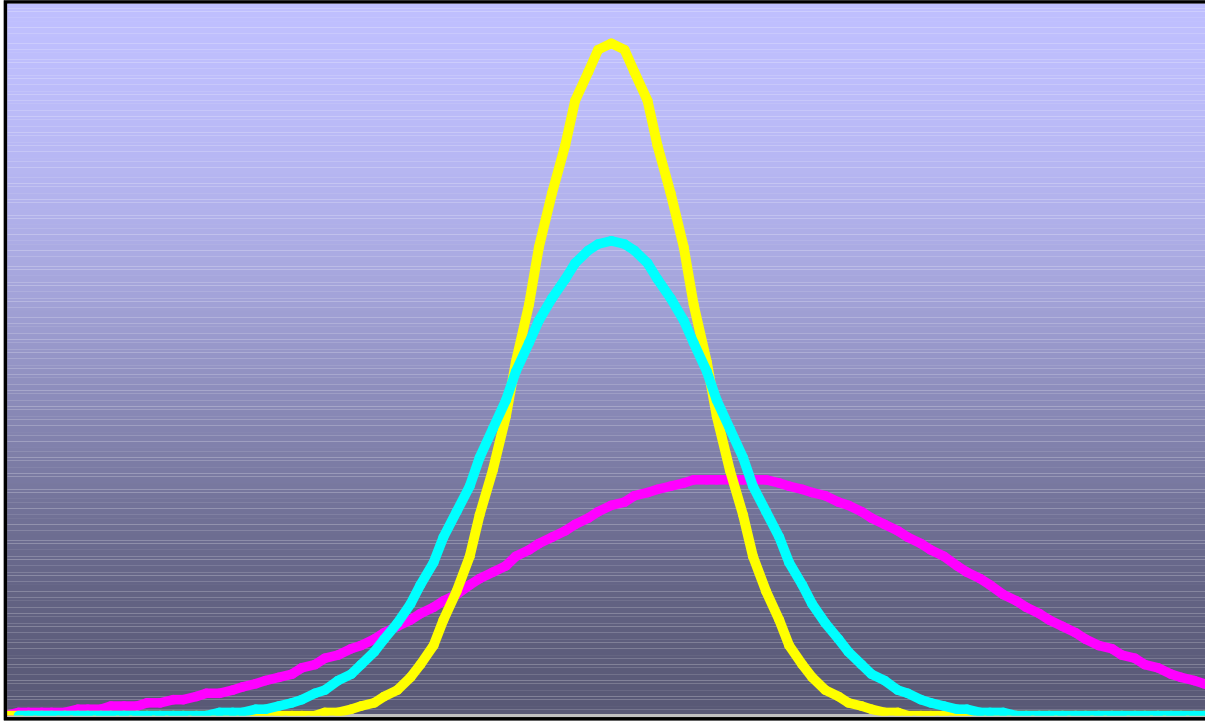


NORMAL DAĞILIM

- Örnek:
 1. Ortalama
 2. Standart sapma

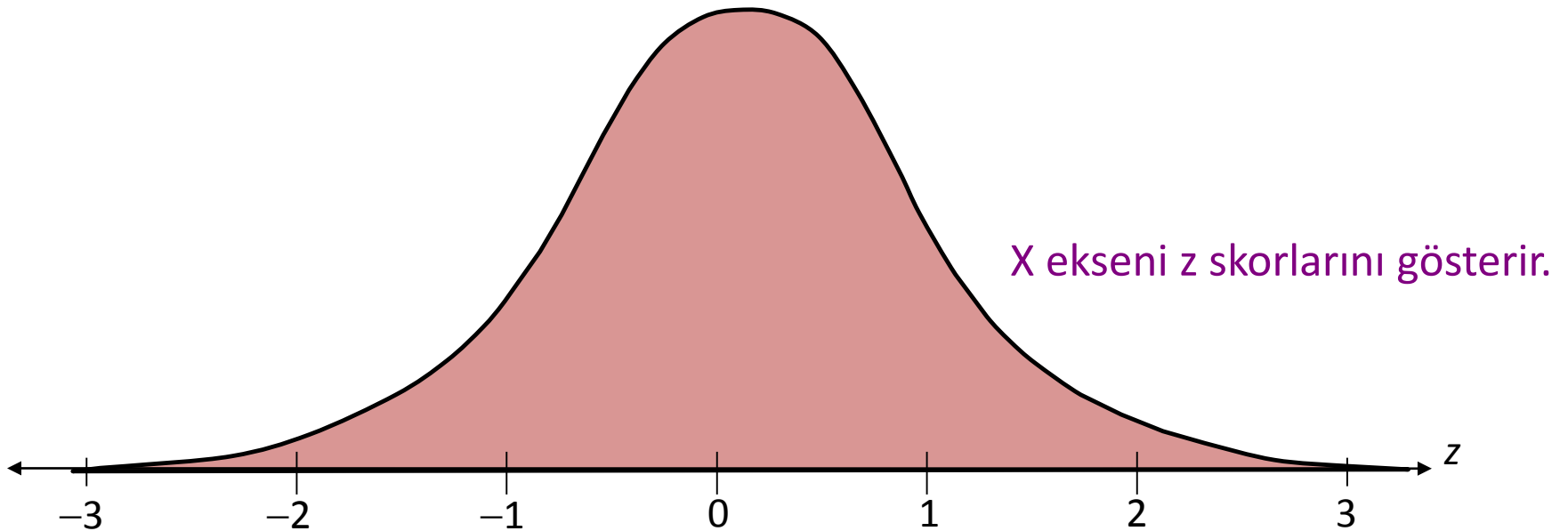


Farklılaşan μ ve σ değerleri farklı biçimlerde dağılımlar verir



The Standard Normal Distribution

Standart normal dağılım ortalaması 0 varyansı 1 olan normal dağılımdır.



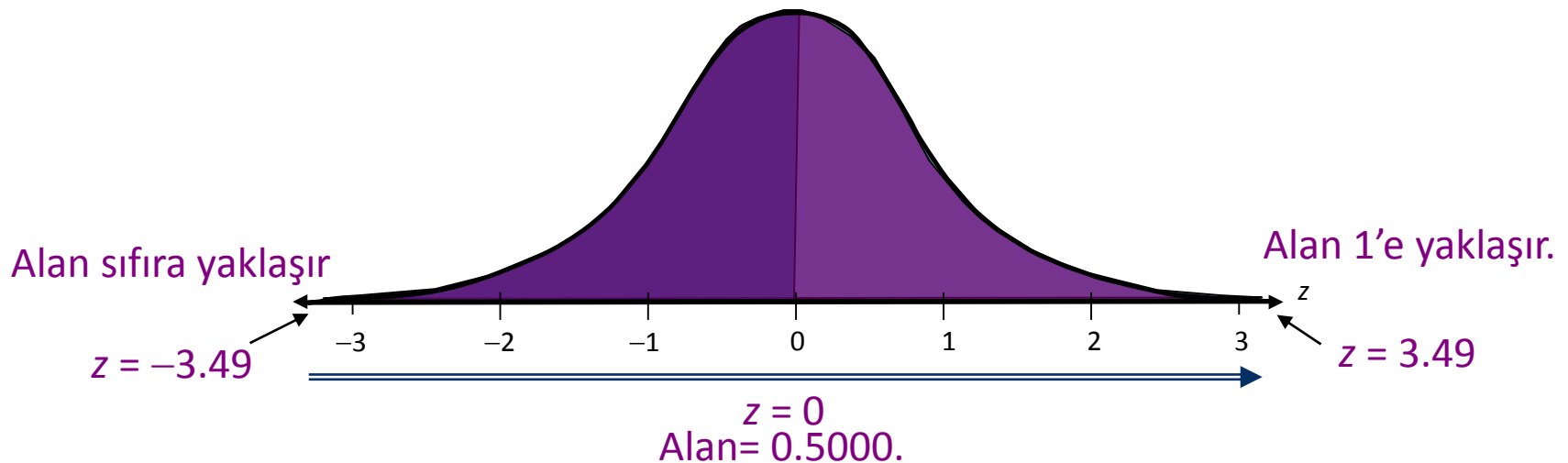
Tüm değerler aşağıdaki formül kullanılarak z skorlarına dönüştürülebilir.

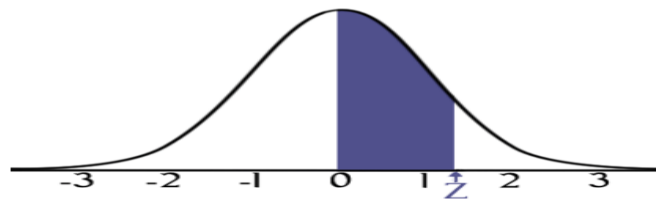
$$z = \frac{\text{Değer} - \text{Ortalama}}{\text{Standart Sapma}} = \frac{x - \mu}{\sigma}.$$

STANDART NORMAL DAĞILIM

Properties of the Standard Normal Distribution

1. $z = -3.49$ 'a yaklaştıkça kümülatif alan sifıra yaklaşıır.
2. z skoru arttıkça kümülatif alan artar.
3. $z = 0$ için kümülatif alan= 0.5000.
4. $z = 3.49$ 'a yaklaştıkça kümülatif alan bire yaklaşıır.





STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

[illegible]

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

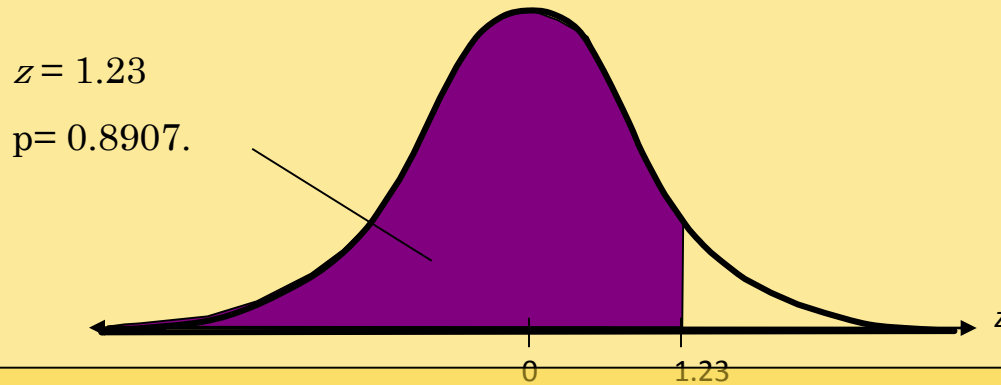
A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

[illegible]

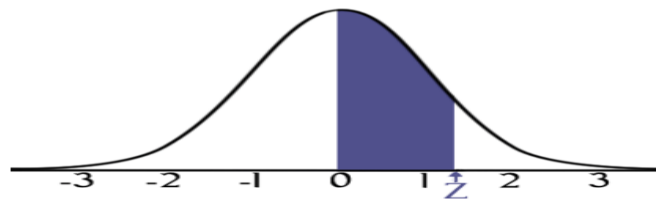
STANDART NORMAL DAĞILIM

Standart Normal Dağılımda Alan Hesabı

1. Standart normal eğriyi çizin ve eğrinin altındaki uygun alanı tarayın.
2. $z=1.23$ 'ün solunda kalan alanı bulun



Larson/Farber 4th ed



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

[illegible]

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

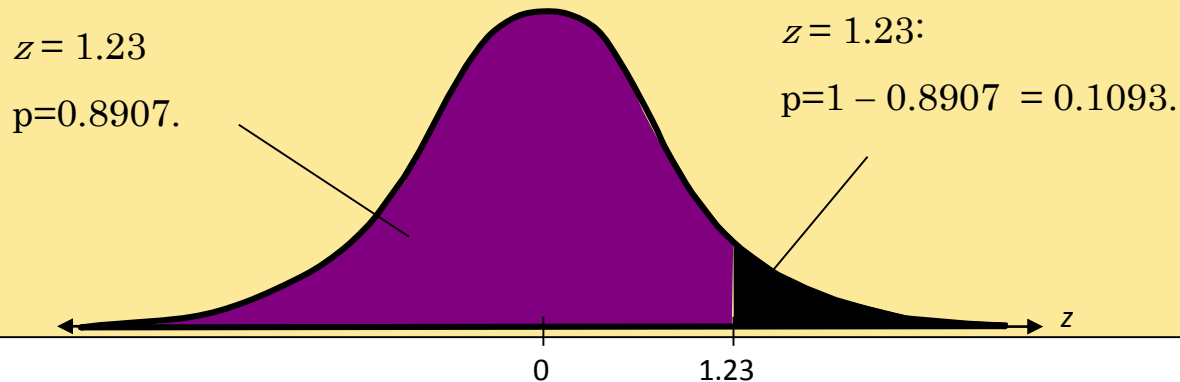
A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

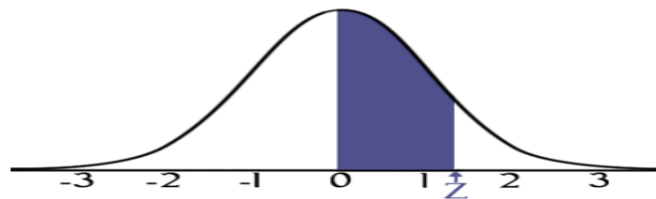
[illegible]

Guidelines for Finding Areas

Finding Areas Under the Standard Normal Curve

1.23'ün sağında kalan alan





STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

[illegible]

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

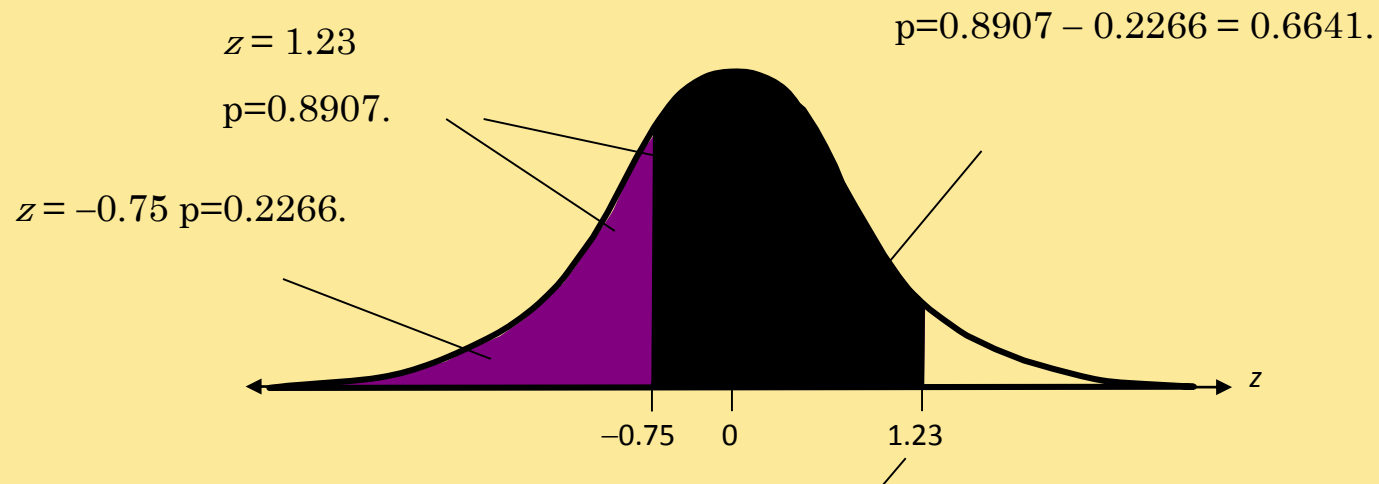
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
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-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown with the area to the left of a point z shaded. An arrow points to the shaded area with the label "Table entry".

[illegible]

Guidelines for Finding Areas

$z = -0.75$ ile $z = 1.23$ arasında kalan alan



Larson/Farber 4th ed

1. Use the table to find the area for the z -score.

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

[illegible]

ÖRNEKLER

- Rasgeler seçilen 560 öğrencinin boy ortalaması 166 cm standart sapması 30 cm'dir.
- 170 ile 182 cm arasında kaç öğrenci vardır?
- 175 cm'den uzun kaç öğrenci vardır?
- 170 cm'den kısa kaç öğrenci vardır?

$$z = \frac{x - \mu}{\sigma} \quad z = \frac{x - \mu}{\sigma} \quad z = \frac{x - \mu}{\sigma}$$
$$z = \frac{170 - 160}{30} \quad z = \frac{182 - 160}{30} \quad z = \frac{175 - 160}{30}$$
$$z = 0,13 \quad z = 0,53 \quad z = 0,3$$

$$p(0,13 \leq z \leq 0,53) = 0,7019 - 0,5517 = 0,1502 \dots 0,1502 * 560 = 84$$

$$p(0,3 \leq z) = 1 - 0,6179 = 0,3821 \dots 0,3821 * 560 = 214$$

$$p(z \leq 0,13) = 0,5517 \dots 0,5517 * 560 = 309$$

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

[illegible]

ÖRNEKLER

- X rastgele değişkeni normal dağılıma sahip olup ortalaması 5 varyansı 9'dur. Değerlerin %15.87'si hangi değerden daha düşüktür?

$$Z = \frac{X - \mu}{\sigma}$$

$$-1 = \frac{X - 5}{3}$$

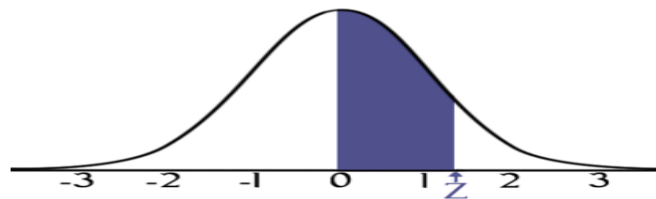
$$x = 2$$

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

[illegible]



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

[illegible]

NORMAL DAĞILIM-BİNOM DAĞILIMI

- Binom dağılımı için $E(X)=n.p$ $Var(X)=n.p.q$
- Bağımsız deneme sayısı arttıkça hesaplama zorlaşır
- n yeterince büyük ($n \geq 50$) olduğunda ve p ile q $\frac{1}{2}$ ye yaklaştıkça binom dağılımı normal dağılıma yakınsar.
- n 20 ile 50 arasında ise normal dağılıma yakınsama için süreklilik düzeltmesi yapılır.

NORMAL DAĞILIM-BİNOM DAĞILIMI

$$P(X \geq a) \cong P\left(\frac{a - \frac{1}{2} - n.p}{\sqrt{n.p.q}} \leq Z\right)$$

$$P(a < X) \cong P\left(Z \leq \frac{a + \frac{1}{2} - n.p}{\sqrt{n.p.q}}\right)$$

$$P(X \leq a) \cong P\left(Z \leq \frac{a + \frac{1}{2} - n.p}{\sqrt{n.p.q}}\right)$$

$N \geq 20$ ve $N < 50$

$$P(X < a) \cong P\left(Z \leq \frac{a - \frac{1}{2} - n.p}{\sqrt{n.p.q}}\right)$$

$$P(a \leq X \leq b) = P\left(\frac{a - \frac{1}{2} - n.p}{\sqrt{n.p.q}} \leq Z \leq \frac{b + \frac{1}{2} - n.p}{\sqrt{n.p.q}}\right)$$

NORMAL DAĞILIM-BİNOM DAĞILIMI

- Bir para 60 kez atılıyor paranın 22-40 arasında tura gelme olasılığı nedir?

$$\mu = n.p = 60 * 1 / 2 = 30$$

$$\sigma = \sqrt{n.p.q} = \sqrt{60 * 1 / 2 / 1 / 2} = \sqrt{15}$$

$$P(22 \leq X \leq 40) = P\left(\frac{22 - 30}{\sqrt{15}} \leq Z \leq \frac{40 - 30}{\sqrt{15}}\right)$$

$$P(-2,07 \leq X \leq 2,58) = 0,9951 - 0,0192 = 0,9759$$

A normal distribution curve is shown. The horizontal axis is labeled with z at a specific point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown with the area to the left of a point z shaded. An arrow points to the shaded area with the label "Table entry".

[illegible]

NORMAL DAĞILIM-BİNOM DAĞILIMI

- $n=38$ sınıf geçme olasılığı $p=0,70$ 27 ve daha az sayıda öğrencinin sınıf geçme olasılığı kaçtır?

$$\mu = n.p = 38 * 0,70 = 26,6$$

$$\sigma = \sqrt{n.p.q} = \sqrt{38 * 0,7 * 0,3} = 2,824$$

$$P(X \leq 27) = P(Z \leq \frac{27 + (1/2) - 26,6}{2,824})$$

$$P(Z \leq 0,32) = 0,6255$$

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

[illegible]

NORMAL DAĞILIM-POISSON DAĞILIMI

- Poisson dağılımı için $E(X)=\lambda$ ve $\text{Var}(X)=\lambda$
- λ çok büyük ise poisson dağılımı normal dağılıma yakınsar.

$$P(a \leq X \leq b) \cong P\left(\frac{a-\lambda}{\sqrt{\lambda}} \leq Z \leq \frac{b-\lambda}{\sqrt{\lambda}}\right)$$

$$P(a \leq X \leq b) \cong P\left(\frac{a-(1/2)-\lambda}{\sqrt{\lambda}} \leq Z \leq \frac{b+(1/2)-\lambda}{\sqrt{\lambda}}\right)$$

ÖRNEKLEME VE ÖRNEK

- Tamsayım
- Örneklem
 - Zaman
 - Maliyet
 - Fiziksel engeller
- Van Gölü'ndeki balıkların uzunlukları
- Bursa'da yaşayan ailelerin ortalama aylık harcamaları

ÖRNEK SEÇİMİ

- 2 5 6 10 14
- İadeli (Yerine koyarak)

$$N^n = 5^2 = 25$$

(2,2), (2,5), (2,6), (2,10), (2,14), (5,2), (5,5), (5,6), (5,10),
(5,14), (6,2), (6,5), (6,6), (6,10), (6,14), (10,2), (10,5),
(10,6), (10,10), (10,14), (14,2), (14,5), (14,6), (14,10), (14,14)

- İadesiz

$$\binom{N}{n} = \frac{N!}{n!(N-n)!} = \frac{5!}{2!(5-2)!} = 10$$

(2,5), (2,6), (2,10), (2,14), (5,6), (5,10), (5,14), (6,10),
(6,14), (10,14)

Örnekleme Hatası

- Parametre (Yığın ortalaması μ , Yığın standart sapması σ)
- İstatistik
- Örnekleme hatası
 - Parametre-İstatistik

ÖRNEKLEME DAĞILIM

- Örneklem dağılımı her bir mümkün örnek için hesaplanan ortalama, varyans, oran gibi istatistiklerin dağılımıdır.

ORTALAMANIN ÖRNEKLEME DAĞILIMI

Örnek		Örnek Ortalaması		Örnek		Örnek Ortalaması
1	2	1.5		4	5	4.5
1	3	2		4	6	5
1	4	2.5		4	7	5.5
1	5	3		4	8	6
1	6	3.5		4	9	6.5
1	7	4		4	10	7
1	8	4.5		5	6	5.5
1	9	5		5	7	6
1	10	5.5		5	8	6.5
2	3	2.5		5	9	7
2	4	3		5	10	7.5
2	5	3.5		6	7	6.5
2	6	4		6	8	7
2	7	4.5		6	9	7.5
2	8	5		6	10	8
2	9	5.5		7	8	7.5
2	10	6		7	9	8
3	4	3.5		7	10	8.5
3	5	4		8	9	8.5
3	6	4.5		8	10	9
3	7	5		9	10	9.5
3	8	5.5				
3	9	6				
3	10	6.5				

ORTALAMANIN ÖRNEKLEME DAĞILIMI

\bar{x}	$P(\bar{x})$	
Ortalama	f	p
1.5	1	1/45
2	1	1/45
2.5	2	2/45
3	2	2/45
3.5	3	1/15
4	3	1/15
4.5	4	4/45
5	4	4/45
5.5	5	1/9
6	4	4/45
6.5	4	4/45
7	3	1/15
7.5	3	1/15
8	2	2/45
8.5	2	2/45
9	1	1/45
9.5	1	1/45

ORTALAMANIN ÖRNEKLEME DAĞILIMI

- Aritmetik Ortalama Yansız Tahmin Edicidir

$$E(\bar{x}) = \sum_{i=1}^{45} \bar{x}_i \cdot P(\bar{x}_i) = \mu_{\bar{x}}$$

$$= (1,5) * 1/45 + 2 * 2/45 + (2,5) * 2/45 \dots + (9,5) * 1/45 = 5,5$$

$$\mu = \frac{1+2+3+4+5+6+7+8+9+10}{10} = 5,5$$

$$E(\bar{x}) = \mu$$

- Örnek ortalamasının standart sapması
Standart hata

$$\sigma_{\bar{x}}^2 = E(\bar{x}^2) - [E(\bar{x})]^2$$

$$E(\bar{x}^2) = (1,5)^2 * 1/45 + 2^2 * 1/45 + \dots + 9,5^2 * 1/45 = 33,92$$

$$E(\bar{x}) = \mu = 5,5$$

$$\sigma_{\bar{x}}^2 = 33,9 - (5,5)^2 = 3,65$$

$$\sigma_{\bar{x}} = 1,91$$

ORTALAMANIN ÖRNEKLEME DAĞILIMI

- Yığın varyansı σ_x^2 biliniyorken
- Örnek birimleri iadeli seçilmişse

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

- Örnek birimleri iadesiz seçilmişse

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} * \sqrt{\frac{N-n}{N-1}}$$

- N çok büyük n çok küçük olduğunda $\sqrt{\frac{N-n}{N-1}}$ ifadesi 1'e yaklaşır. Bu nedenle ihmal edilebilir.
- $n/N \leq 0,05$ olduğu durumlarda $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$ kullanılır.

ORTALAMANIN ÖRNEKLEME DAĞILIMI

- Örneklem birimleri yerine koyarak seçilmişse

$$\sigma_x^2 = \sum_{i=1}^N \frac{(x_i - \mu)^2 * f(x_i)}{N}$$

$$\sigma_x^2 = \frac{(1-5,5)^2 * 1 + (2-5,5)^2 * 1 + \dots + (10-5,5)^2 * 1}{10} = 8,25$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{\sqrt{n}} = \sqrt{\frac{8,25}{2}} = 4,125$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{\sqrt{n}} * \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{8,25}{2}} * \sqrt{\frac{10-2}{10-1}} = 1,91$$

MERKEZİ LİMİT TEOREMİ

- Örnek hacmi $n \geq 30$ olduğu durumlarda

$$\mu_{\bar{x}} = \mu \cdots \text{ve} \cdots \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

örneklem ortalamasının dağılımının şekli normal dağılıma yakınsar.

Örnek ortalamasının beklenen değeri anakütle ortalamasına eşit olur.

$n/N \leq 0,05$ olduğunda $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$

MERKEZİ LİMİT TEOREMİ

- 8 yaşından küçük çocukların televizyon izleme süre ortalaması 20 saat standart sapması 4 saattir. Rastgele seçilen 16 çocuğun televizyon seyretme saat ortalamasının 22 saatten çok olma olasılığı nedir?

$$\begin{aligned}P(\bar{X} > 22) &= 1 - P\left(\frac{22 - 20}{4 / \sqrt{16}}\right) \\&= 1 - P(Z < 2,00) = 1 - 0,9772 \\&= 0.0228\end{aligned}$$

A normal distribution curve is shown. The horizontal axis is labeled with z at a specific point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

[illegible]

Oranın Örnekleme Dağılımı

- N =Yığının birim sayısı n =Örnekteki birim sayısı
- X =Yığındaki belli özelliği taşıyan birim sayısı
- x =Örnekteki belli özelliği taşıyan birim sayısı

$$YığınOranı = \pi = \frac{X}{N}$$

$$ÖrnekOranı = p = \frac{x}{n}$$

Örnek

- A şehirde yaşayan 220000 ailenin 110000'sinin arabası vardır. 150 aile içeren bir örnek çekiliyor bu ailelerden 80'sinin arabası olduğu tespit ediliyor. Anakütle ve örnek oranlarını kıyaslayınız.

$$\pi = \frac{X}{N} = \frac{110000}{220000} = 0,5$$

$$p = \frac{x}{n} = \frac{80}{150} = 0,533$$

$$\text{Örnekleme hatası} = 0,533 - 0,5 = 0,033$$

Oranın Örneklem Dağılımı

- Merkezi limit teoremine göre $n \cdot \pi \geq 5$ ve $n \cdot (1 - \pi) \geq 5$ ise örnekteki birim sayısı normal dağılım için yeterince büyük kabul edilir.

$$\sigma_p = \sqrt{\frac{\pi * (1 - \pi)}{n}}$$

$n / N > 0,05 \dots$ ise

$$\sigma_p = \sqrt{\frac{\pi * (1 - \pi)}{n}} * \sqrt{\frac{N - n}{N - 1}}$$

ÖRNEK

- Ankara'da yapılan bir araştırmada devlet tiyatrolarına giden izleyicilerin %64'ünün kadın olduğu belirlenmiştir. 80 kişi seçiliyor %75'ten fazla kadın olma olasılığı nedir?

$$\mu_p = \pi = 0,64 \cdots \sigma_p = \sqrt{\frac{\pi * (1 - \pi)}{n}} = \sqrt{\frac{0,64 * (1 - 0,64)}{80}} = 0,054$$

$$n * \pi = 80 * 0,64 = 51,2$$

$$n * (1 - \pi) = 80 * 0,36 = 28,8$$

$$z = \frac{p - \mu_p}{\sigma_p} = \frac{0,75 - 0,64}{0,054} = 2,037$$

$$P(p > 0,75) = 1 - P(z < 2,037) = 1 - 0,9793 = 0,0207$$

A normal distribution curve is shown. The horizontal axis is labeled with z at a point. The area under the curve to the left of this point is shaded. An arrow points to this shaded area with the label "Table entry".

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded. An arrow points to this shaded area with the label "Table entry".

[illegible]

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