Operatör Metodu

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$\frac{d}{dx} = D$, $\frac{dy}{dx} = \overset{*}{D}y$, $\frac{d}{dt} = D$, $\frac{dy}{dt} = Dy$

$\frac{d^2}{dx^2} = D^2$, $\frac{d^2y}{dx^2} = D^2y$

$\frac{d^3}{dx^3} = D^3$, $\frac{d^3y}{dx^3} = D^3y$

$\frac{1}{D}$ integral, $\frac{d}{dx} = D$, $\frac{1}{D}x = \int x\,dx = \frac{x^2}{2} + c$

Ör $y' = x^2$, ddç.

$\frac{d}{dx} = D$, $Dy = x^2$, $y = \frac{1}{D}x^2 = \frac{x^3}{3} + c$ //

Ör $y'' - 6y' + 9y = x^{-3} \cdot e^{3x}$ ddç. ?

$y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$, $(r-3)^2 = 0$, $r_{1,2} = 3$

$$\boxed{y_h = (c_1 + xc_2)e^{3x}}$$

$y_p = ?$ $\frac{d}{dx} = D$ olmak üzere

$D^2y - 6Dy + 9y = x^{-3} e^{3x}$

$(D^2 - 6D + 9)y = x^{-3} e^{3x}$

$y_p = \frac{1}{D^2 - 6D + 9} x^{-3} e^{3x}$

$y_p = \frac{1}{(D-3)(D-3)} x^{-3} e^{3x}$ $u(x)$

$u(x) = \frac{1}{D-3} x^{-3} e^{3x}$

$(D-3)u(x) = x^{-3} e^{3x}$

$D-3$

$$(D-3)u(x) = x^{-3}e^{3x}$$

$$\frac{du}{dx} - 3u = x^{-3}e^{3x} \qquad (\text{lineer o.d.d})$$

$$e^{-3\int dx}.u = \int x^{-3}e^{3x}e^{-\int 3dx}dx$$

$$e^{-3x}.u = \int x^{-3}\underbrace{e^{3x}e^{-3x}}dx$$

$$e^{-3x}.u = -\frac{1}{2x^2} \Rightarrow \boxed{u(x) = -\frac{e^{3x}}{2x^2}}$$

$$y_p = \frac{1}{D-3}.u(x) = \frac{1}{D-3}.\left(-\frac{e^{3x}}{2x^2}\right)$$

$$(D-3)y = -\frac{e^{3x}}{2x^2}$$

$$\frac{dy}{dx} - 3y = -\frac{e^{3x}}{2x^2} \qquad (\text{lineer o.d.d})$$

$$e^{-\int 3dx}.y = -\int \frac{e^{3x}}{2x^2}e^{-\int 3dx}dx$$

$$e^{-3x}.y = -\int \frac{e^{3x}e^{-3x}}{2x^2}dx$$

$$e^{-3x}.y = \frac{1}{2x} \Rightarrow \boxed{y_p = \frac{e^{3x}}{2x}}$$

$$y = y_h + y_p = (c_1 + xc_2)e^{3x} + \frac{e^{3x}}{2x} = e^{3x}\left(c_1 + xc_2 + \frac{1}{2x}\right) //$$