Computational Astrophysics
Problem set 3

1 Simple Monte Carlo integration

In the lectures we had for the simple MC integration the formula

$$\int f dV \approx V \langle f \rangle \pm V \sqrt{\frac{\langle f^2 \rangle - \langle f \rangle^2}{N}}$$

where the arithmetic means are taken over N points:

$$\begin{aligned} \langle f \rangle &= \frac{1}{N} \sum_i f(x_i) \\ \langle f^2 \rangle &= \frac{1}{N} \sum_i f^2(x_i) \end{aligned}$$

In addition, we have discussed the rejection method, which can also be used to compute integrals with the MC method.

1.1 Problem

Use MC methods to integrate the function $y = \sqrt{1 - x^2}$ over x in the interval $0 \leq x \leq 1$.

1. use the simple MC integration method
2. use a rejection MC integration method
3. compute the MC values of the integral and the MC estimated errors for each method
4. compare the MC results to the analytic result
5. investigate the convergence properties of the MC results as function of the number of MC samples
6. use different open source random number generators and compare results
7. analyze 'randomness' of your favorite random number generator (optional)
8. use a Sobol sequence for the rejection method (optional)
9. convert your program to use openMP parallelism (C or F95, reall optional!)
10. convert your program to use OpenACC (C, C++, Fortran, really optional!)