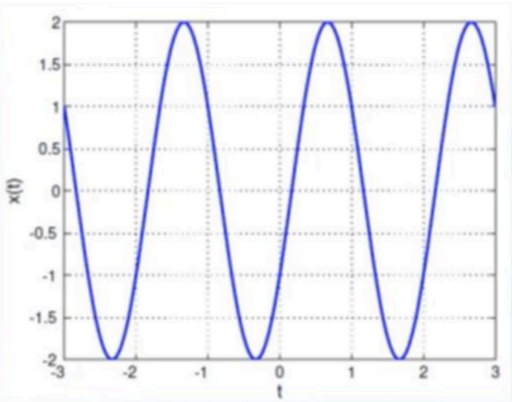
Exercise 1

Find the analytical representation and graph the signals

a)



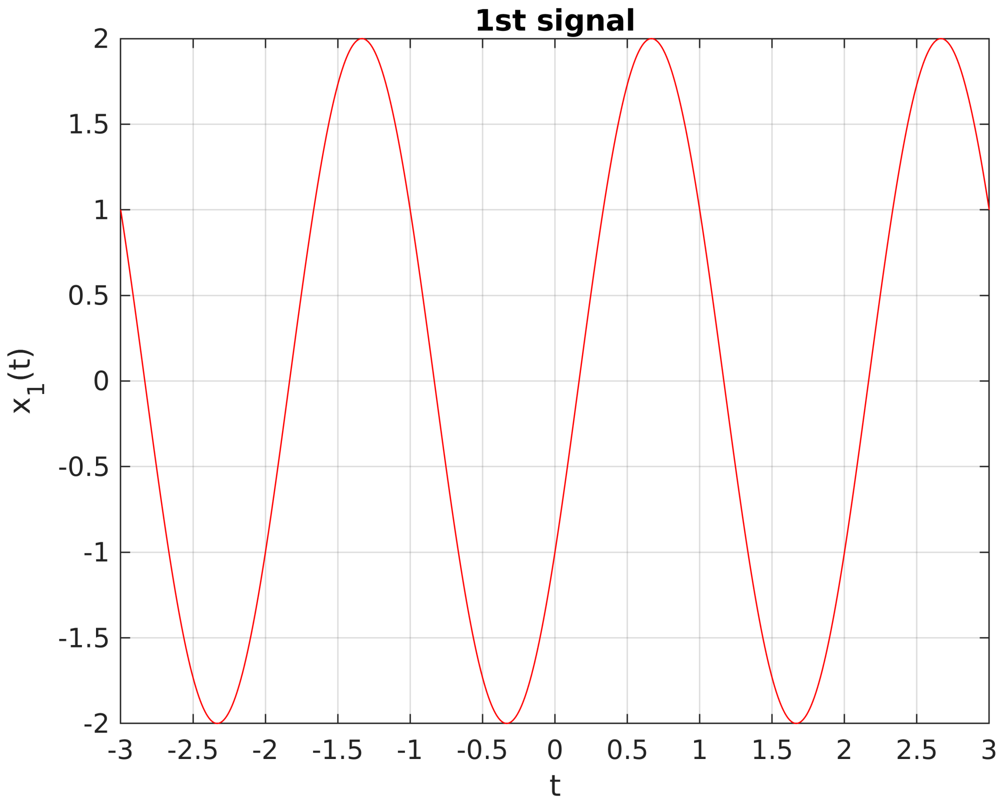
From the graph we can easily see that the signal has the following parameters:

- $A = 2$
- $T = 2$
- $v_{\text{offset}} = 0$
- $\varphi = \text{asin}(x(0))$

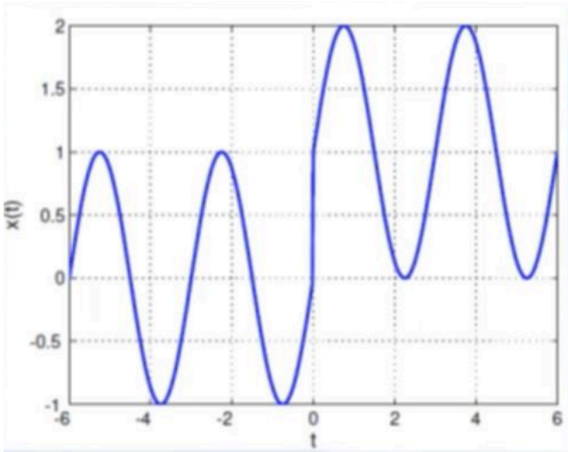
Hence, the equation of the signal will be:

$$x_1(t) = 2\sin\left(\pi t + \text{asin}\left(\frac{-1}{2}\right)\right)$$

```
cla;
clear all;
% Generate a linear space for (-3,3) with granularity 1/100
t = -3:0.01:3;
% Create the anonymous function that models the signal
x1 = @(t)(2*sin(pi.*t + asin(-1/2)));
% Plot the signal
plot(t,x1(t), '-r');
% Add a grid to the plot and increase the number of X ticks
grid;
xticks(-3:0.5:3);
title("1st signal")
xlabel('t');
ylabel('x_1(t)');
```



b)



This graph is a combination of two sine signals. Both signals have the same amplitude and period, though the second one has a vertical offset of 1.

The parameters of the signal as a whole are:

- $A = 1$
- $T = 3$
- $v_{\text{offset}} = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$

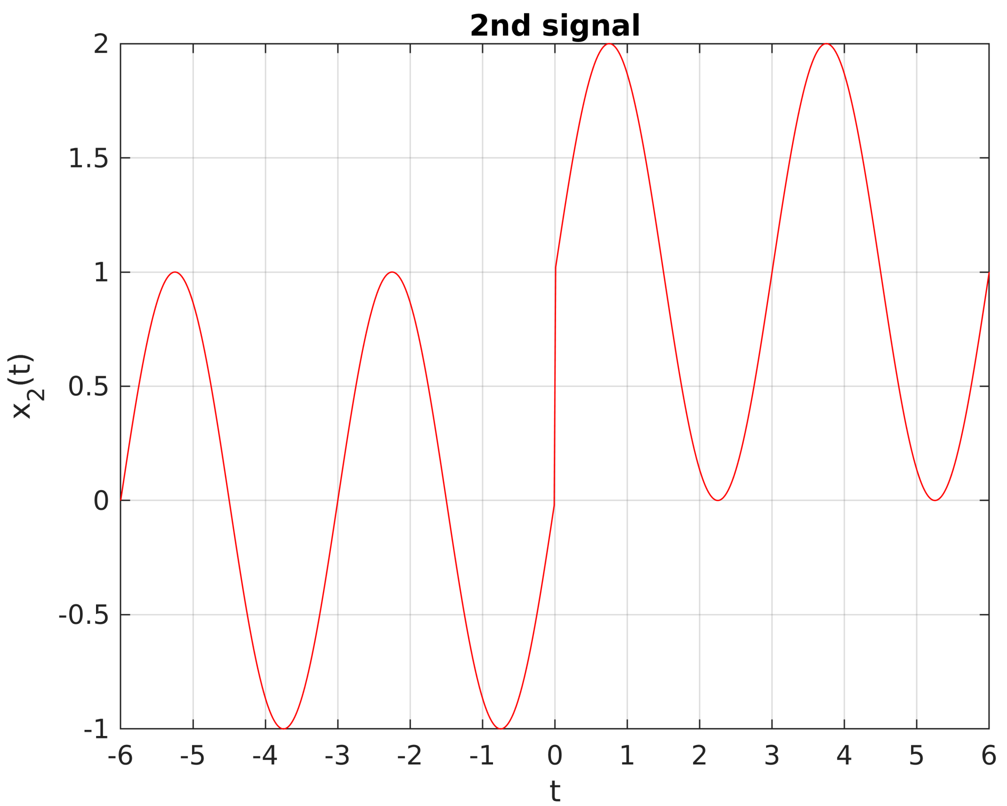
- $\varphi = 0$

The equation of the signal becomes:

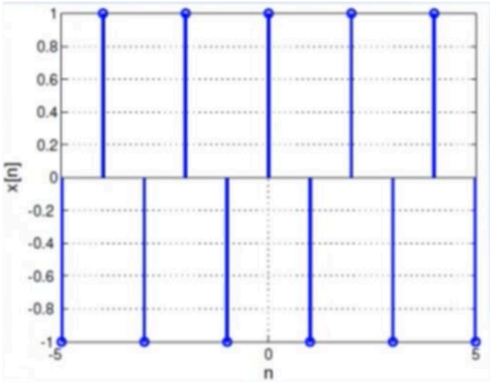
$$x_2(t) = \sin\left(\frac{2\pi t}{3}\right) + v_{\text{offset}}$$

where the vertical offset was previously defined.

```
clear all;
% Define the linear space to plot
t = -6:0.01:6;
% Define the function that describes the signal
% We model the vertical offset as a step function that takes the values 0
% or 1, depending on the sign of the input
x2 = @(t)(sin(2*pi.*t/3) + (sign(t)+1)/2);
% Plot the obtained signal
plot(t, x2(t), '-r');
% Add a grid and increase the X ticks
grid;
xticks(-6:1:6)
title("2nd signal")
xlabel('t');
ylabel('x_2(t)');
```



c)



This signal is a discrete one. It is a discrete sample of a cosine signal, having a sampling frequency of 1 second. The parameters of the signal are:

- $A = 1$
- $\varphi = 0$

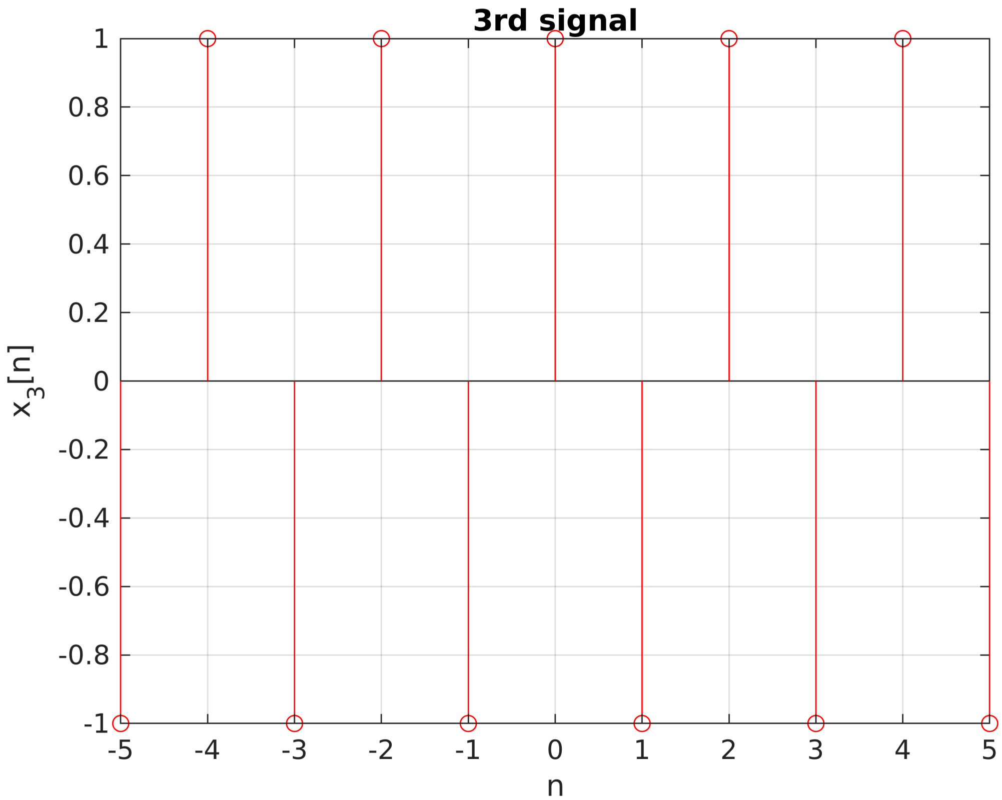
The period of the signal is 2, so the angular frequency is (according to the formula $\Omega_0 = \frac{2\pi t}{T}$):

- $\Omega_0 = \pi$

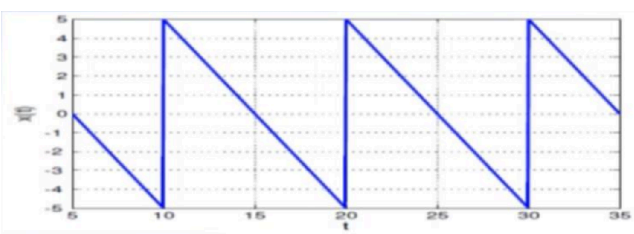
The formula of the signal is:

$x_3[n] = \cos(\pi n)$

```
clear all;
% Define the linear space to plot
t = -5:1:5;
% Define the function that describes the signal
% We model the vertical offset as a step function that takes the values 0
% or 1, depending on the sign of the input
x3 = @(n)(cos(pi*n));
% Plot the obtained signal
stem(t, x3(t), '-r');
% Add a grid and increase the X ticks
grid;
xticks(-5:1:5)
title("3rd signal");
xlabel('n');
ylabel('x_3[n]');
```



d)



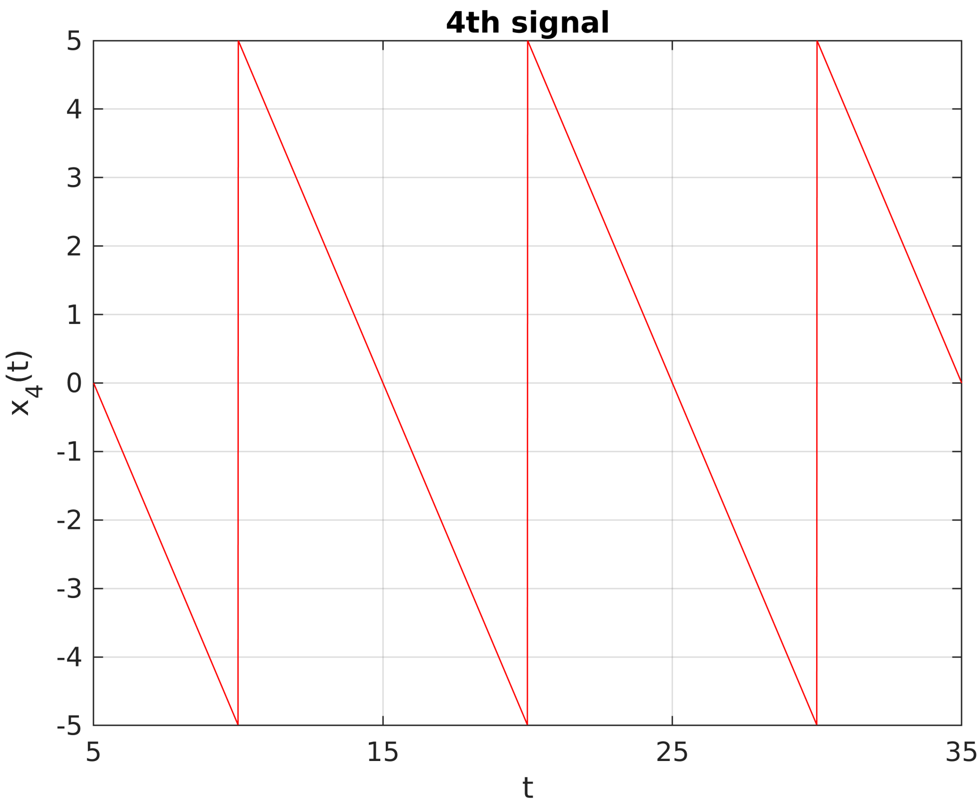
This signal can be represented using the mod function.

Formula:

$$x_4 = -\mathrm{mod}(t, 10) + 5$$

```
clear all;
% Define the linear space to plot
t = 5:0.01:35;
% Define the function that describes the signal
% We model the vertical offset as a step function that takes the values 0
% or 1, depending on the sign of the input
x4 = @(t)(-mod(t,10) + 5);
% Plot the obtained signal
plot(t, x4(t), '-r');
% Add a grid and increase the X ticks
grid;
xticks(5:10:35)
```

```
title("4th signal");
xlabel('t');
ylabel('x_4(t)');
```



Exercise 2

Determine which of the following signals are periodic. Plot the signals in Matlab.

$x(t) = 4\cos(5\pi t) - \pi/4$

$x(t) = 4u(t) + 2\sin(3t)$, $u(t)$ – unit step function

$x(t) = 3\cos(4t) + \sin(\pi t)$

$x(t) = \cos(2\pi t) + 2\cos(4\pi t) + \sin(\pi t)$

$x(t) = 4\cos(3\pi t + \pi/2) + 2\cos(8\pi t + \pi/2)$

$x(t) = 4\cos(3\pi t + \pi/2) + 4\cos(10\pi t - \pi/2)$

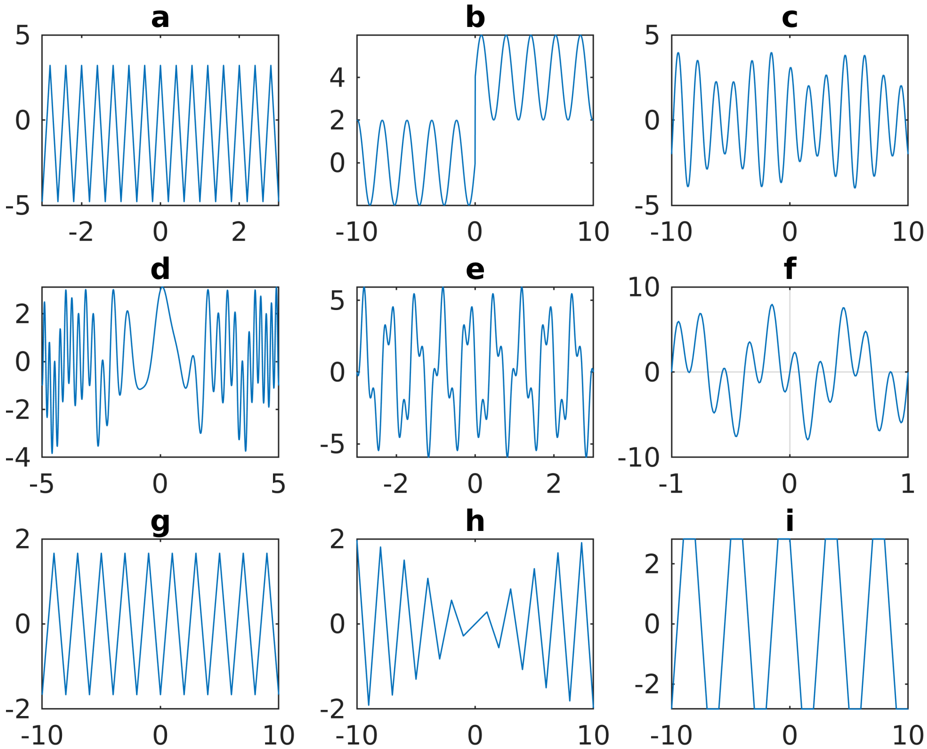
$x[n] = 4\cos(\pi n - 2)$

$x[n] = 2\sin(3n)$

$x[n] = 4\cos(0.5\pi n + \pi/4)$

```
cla;
clear all;
% Define the unit step function
ustep = @(x)((sign(x)+1)/2);
```

```
% Define a linear space
t = -10:0.01:10;
n = -10:1:10;

subplot(3,3,1);
t1 = -3:0.1:3;
x1=@(t)(4*cos(5*pi.*t) - pi/4);
plot(t1,x1(t1))
title("a")

subplot(3,3,2);
x2=@(t)(4.*ustep(t)+2*sin(3.*t));
plot(t,x2(t))
title("b")

subplot(3,3,3);
x3=@(t)(3*cos(4.*t) + sin(pi.*t));
plot(t,x3(t))
title("c")

subplot(3,3,4);
t4 = -5:0.01:5;
x4=@(t)(cos(2*pi.*t)+2*cos(t*pi.*t) + sin(pi.*t));
plot(t4,x4(t4))
title("d")

subplot(3,3,5);
t5 = -3:0.01:3;
x5=@(t)(4*cos(3*pi.*t+pi/2)+2*cos(8*pi.*t+pi/2));
plot(t5,x5(t5))
title("e")

subplot(3,3,6);
t6 = -1:0.01:1;
x6=@(t)(4*cos(3*pi.*t+pi/2)+4*cos(10*pi.*t - pi/2));
plot(t6,x6(t6));
title("f")
grid

subplot(3,3,7);
x7=@(t)(4*cos(pi.*t-2));
plot(n,x7(n))
title("g")

subplot(3,3,8);
x8=@(t)(2*sin(3.*t));
plot(n,x8(n))
title("h")

subplot(3,3,9);
x9=@(t)(4*cos(0.5*pi.*t+pi/4));
plot(n,x9(n))
title("i")
```


a
b
c
d
e
f
g
h
i

Exercise 2:

Determine the period of the periodic signals.

a) $x_1(t) = 4\cos(5\pi t) - \frac{\pi}{4}$

x_1 is a periodic ~~funct~~ signal with period $T = \frac{4}{10} = 0{,}4$

We know that the signal:

$\cos(2\pi t f + \varphi)$

has f as its frequency.

Thus:

$\cos(5\pi t) = \cos(2\pi t f + \varphi)$

$\Updownarrow$

$f = \frac{5}{2} \Rightarrow T = \frac{2}{5} = 0{,}4$

b) $x_2(t) = 4u(t) + 2\sin(3t)$ → unit step function

This signal is not periodic, because of the discontinuity inserted by $u(t)$. Despite this, on the intervals $(-\infty; 0)$, $(0, +\infty)$ the signal is periodic, with period $T = \frac{2\pi}{3}$.

c) $x_3(t) = 3\cos(4t) + \sin(\pi t)$

x_3 is a superposition of two periodic signals. The cosine has a period of $T_c = \frac{2\pi}{4} = \frac{\pi}{2}$, while the sine has the period $T_s = 2$.

The signal is a periodical one if there exists T such that $T = p \cdot T_c \pm q T_s$; $p, q \in \mathbb{Z}$

$$\frac{T_c}{T_s} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4} \notin \mathbb{Z}$$

d)

$$x_4(t) = \cos(2\pi t) + 2\cos(4\pi t) + \sin(\pi t)$$

$$T_1 = \frac{2\pi}{2\pi} = 1$$

$$T_2 = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$T_3 = \frac{2\pi}{\pi} = 2$$

$$T = pT_1 = qT_2 = rT_3$$

$$T = [1, \tfrac{1}{2}, 2] = 2$$

$$\left.\begin{aligned} pT_1 = 2 &\Leftrightarrow p = 2 \\ qT_2 = 2 &\Leftrightarrow q = 4 \\ rT_3 = 2 &\Leftrightarrow r = 1 \end{aligned}\right\} \in \mathbb{Z}$$

$\Rightarrow$ The signal is periodic for p, q, r equal to 2, 4 and 1, respectively.

The minimum period is $\boxed{T = 2}$

e) $x_5(t) = 4\cos(3\pi t + \frac{\pi}{2}) + 2\cos(8\pi t + \pi/2)$

$\downarrow$

composed from two signals, having frequencies:

$$T_1 = \frac{2}{3}$$

$$T_2 = \frac{1}{4}$$

$$\cancel{T = \text{lcm}\left[\frac{2}{3}, \frac{1}{4}\right] = 2}$$

$$\frac{T_1}{T_2} = \frac{\frac{2}{3}}{\frac{1}{4}} = \frac{2 \cdot 4}{3} = \frac{8}{3} \Rightarrow \begin{aligned} p &= 8 \\ q &= 3 \end{aligned}$$

$$T = qT_1 = pT_2 = 2$$

i) $x[n] = 4\cos(0.5\pi n + \pi/4)$

$\Omega_0 = 0{,}5\,\pi$

For $N=4$, we have $\Omega_0 \cdot N = 2\pi$. Thus, $N=4$ is the period of the signal