

1. Kinematics

Decay rate and cross section (Note: \mathcal{M} has a mass dimension of $4 - N_i - N_f$.)

$$\text{decay rate (rest frame; } \sqrt{s} = M_0) : \quad d\Gamma = \frac{d\Pi^{N_f}}{2M_0} \left| \mathcal{M}(M_0 \rightarrow \{p_1, p_2, \dots, p_{N_f}\}) \right|^2, \quad (1.1)$$

$$\text{cross section (Lorentz invariant) :} \quad d\sigma = \frac{d\Pi^{N_f}}{2E_A 2E_B v_{\text{Mol}}} \left| \mathcal{M}(p_A, p_B \rightarrow \{p_1, p_2, \dots, p_{N_f}\}) \right|^2, \quad (1.2)$$

where $d\Pi^n$ is n -particle Lorentz-invariant phase space with momentum conservation

$$d\Pi^n := d\Pi_1 d\Pi_2 \dots d\Pi_n (2\pi)^4 \delta^{(4)} \left(P_0 - \sum p_n \right); \quad d\Pi := \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}}. \quad (1.3)$$

At the CM frame, two-body phase-space are characterized by the final momentum $\|\mathbf{p}\|$ and given by

$$d\Pi^2 = \frac{\|\mathbf{p}\|}{4\pi\sqrt{s}} \frac{d\Omega}{4\pi} = \frac{\|\mathbf{p}\|}{8\pi\sqrt{s}} d\cos\theta = \frac{1}{16\pi} \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}} d\cos\theta \quad (1.4)$$

with $\sqrt{s} = M_0$ or E_{CM} , θ is the angle between initial and final motion, and

$$\|\mathbf{p}\| = \frac{\sqrt{s}}{2} \lambda^{1/2} \left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s} \right), \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad p_1 \cdot p_2 = \frac{s - (m_1^2 + m_2^2)}{2}.$$

Mandelstam variables For $(k_1, k_2) \rightarrow (p_3, p_4)$ collision,

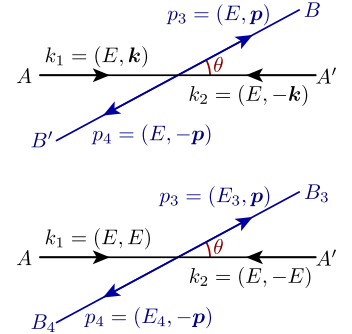
$$\begin{aligned} s &= (k_1 + k_2)^2 = (p_3 + p_4)^2, & t &= (p_3 - k_1)^2 = (p_4 - k_2)^2, & u &= (p_3 - k_2)^2 = (p_4 - k_1)^2; \\ k_1 \cdot k_2 &= (s - m_1^2 - m_2^2)/2, & k_1 \cdot p_3 &= (m_3^2 + m_1^2 - t)/2, & s + t + u &= m_1^2 + m_2^2 + m_3^2 + m_4^2, \\ p_3 \cdot p_4 &= (s - m_3^2 - m_4^2)/2, & k_1 \cdot p_4 &= (m_4^2 + m_1^2 - u)/2; \\ (k_1 - k_2)^2 &= 2(m_1^2 + m_2^2) - s, & (p_3 - p_4)^2 &= 2(m_3^2 + m_4^2) - s. \end{aligned}$$

If the collision is with the “same mass” $(m_A, m_A) \rightarrow (m_B, m_B)$,

$$\begin{aligned} t &= m_A^2 + m_B^2 - s/2 + 2kp \cos\theta, & k &= \sqrt{s/4 - m_A^2}, \\ u &= m_A^2 + m_B^2 - s/2 - 2kp \cos\theta, & p &= \sqrt{s/4 - m_B^2}. \end{aligned}$$

Instead, if the collision is “initially massless” $(0, 0) \rightarrow (m_3, m_4)$,

$$\begin{aligned} t &= (m_3^2 + m_4^2 - s)/2 + p\sqrt{s} \cos\theta, & p &= (\sqrt{s}/2) \lambda^{1/2} \left(1; m_3^2/s, m_4^2/s \right), \\ u &= (m_3^2 + m_4^2 - s)/2 - p\sqrt{s} \cos\theta. \end{aligned}$$



Three-body final state Mandelstam variables can be defined, for $P \rightarrow (p_1, p_2, p_3)$, as

$$s_{ij} = (p_i + p_j)^2; \quad t_{0i} = (P - p_i)^2 = s_{jk}; \quad s_{12} + s_{23} + s_{31} = P^2 + p_1^2 + p_2^2 + p_3^2.$$

For spherically-symmetric processes, the phase-space integral is reduced to, at the center-of-mass frame,

$$\int d\Pi^3_{\text{sph. sym.}}(\sqrt{s}, \mathbf{0}) = \frac{1}{128\pi^3} \frac{1}{s} \int_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} ds_{23} \int ds_{13}; \quad (1.5)$$

$$(s_{13})_{\min}^{\max} = \frac{(s + m_3^2 - m_1^2 - m_2^2)^2}{4s_{23}} - \frac{1}{4s_{23}} \left[\lambda^{1/2}(s, m_1^2, s_{23}) \mp \lambda^{1/2}(s_{23}, m_2^2, m_3^2) \right]^2 \quad (1.6)$$

$$= (E_1^* + E_3^*)^2 - \left(\sqrt{E_1^{*2} - m_1^2} \mp \sqrt{E_3^{*2} - m_3^2} \right)^2, \quad (1.7)$$

where $E_1^* = \frac{s - s_{23} - m_1^2}{2\sqrt{s_{23}}}$, and $E_3^* = \frac{s_{23} - m_2^2 + m_3^2}{2\sqrt{s_{23}}}$.

1.1. Fundamentals

Lorentz-invariant phase space:

$$\int d\Pi = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\sqrt{m^2 + \|\mathbf{p}\|^2}} = \int \frac{dp_0 d^3\mathbf{p}}{(2\pi)^4} (2\pi) \delta(p_0^2 - \|\mathbf{p}\|^2 - m^2) \Theta(p_0)$$

Källén function:

$$\begin{aligned} \lambda(x, y, z) &= x^2 + y^2 + z^2 - 2xy - 2yz - 2zx = (x - y - z)^2 - 4yz; \\ \lambda(1; \alpha_1^2, \alpha_2^2) &= (1 - (\alpha_1 + \alpha_2)^2)(1 - (\alpha_1 - \alpha_2)^2) = (1 + \alpha_1 + \alpha_2)(1 - \alpha_1 - \alpha_2)(1 + \alpha_1 - \alpha_2)(1 - \alpha_1 + \alpha_2). \\ \lambda^{1/2}(s; m_1^2, m_2^2) &= s \lambda^{1/2}\left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right); & \lambda^{1/2}\left(1; \frac{m^2}{s}, \frac{m^2}{s}\right) &= \sqrt{1 - \frac{4m^2}{s}}, \\ \lambda^{1/2}\left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) &= \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}}, & \lambda^{1/2}\left(1; \frac{m_1^2}{s}, 0\right) &= \frac{s - m_1^2}{s}. \end{aligned}$$

Two-body phase space If $f(p_1^\mu, p_2^\mu)$ is Lorentz invariant, $f \equiv f(p_1^2, p_2^2, p_1^\mu p_{2\mu}) \equiv f(p_1, p_2, \cos \theta_{12})$. Meanwhile,

$$\int d\Pi_1 d\Pi_2 = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} \frac{1}{2E_1 2E_2} = \int \frac{(4\pi) dp_1 p_1^2 (2\pi) dp_2 p_2^2 d\cos \theta_{12}}{(2\pi)^3} \frac{1}{2E_1 2E_2} = \int \frac{dE_+ dE_- ds}{128\pi^4}, \quad (1.8)$$

with the replacement of the variables

$$\begin{aligned} E_{\pm} &= E_1 \pm E_2, & s &= (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2E_1 E_2 - 2\|\mathbf{p}_1\| \|\mathbf{p}_2\| \cos \theta_{12}; \\ \left| \frac{d(E_+, E_-, s)}{d(p_1, p_2, \cos \theta_{12})} \right| &= \frac{4p_1^2 p_2^2}{E_1 E_2}, & \left| \frac{d(E_1, E_2, s)}{d(p_1, p_2, \cos \theta_{12})} \right| &= \frac{2p_1^2 p_2^2}{E_1 E_2}. \end{aligned}$$

Therefore,

$$\int d\Pi_1 d\Pi_2 = \frac{1}{128\pi^4} \int_{(m_1+m_2)^2}^{\infty} ds \int_{\sqrt{s}}^{\infty} dE_+ \int_{\min}^{\max} dE_-, \quad (1.9)$$

where the boundary of E_- is given by

$$\begin{aligned} \cos \theta_{12} &= \frac{E_+^2 - E_-^2 + 2(m_1^2 + m_2^2 - s)}{\sqrt{(E_+ + E_-)^2 - 4m_1^2} \sqrt{(E_+ - E_-)^2 - 4m_2^2}} \in [-1, 1] \\ \therefore \left| E_- - \frac{m_1^2 - m_2^2}{s} E_+ \right| &\leq \sqrt{E_+^2 - s} \cdot \lambda^{1/2}\left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) = 2p \sqrt{\frac{E_+^2 - s}{s}}. \end{aligned}$$

Two-body phase space with momentum conservation As a general representation in any frame,

$$\frac{d\Pi^2}{16\pi^2} = \frac{dp_1 d\Omega p_1^2 \delta(E_0 - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + \|\mathbf{P}_0 - \mathbf{p}_1\|^2})}{E_1 E_2} = \frac{1}{8\pi} d\cos \theta_1 \frac{p_1^2}{E_0 p_1 - P_0 E_1 \cos \theta_1}, \quad (1.10)$$

where the momentum p_1 is given by

$$p_1 = \frac{(E_0^2 + m_1^2 - m_2^2 - P_0^2) P_0 \cos \theta_1 + E_0 \sqrt{\lambda(E_0^2 - P_0^2, m_1^2, m_2^2) - 4m_1^2 P_0^2 \sin^2 \theta_1}}{2(E_0^2 - P_0^2 \cos^2 \theta_1)}. \quad (1.11)$$

CM frame result is recovered by setting $E_0 = \sqrt{s}$ and $P_0 = 0$.

1.2. Decay rate and Cross section

As $\langle \text{out} | \text{in} \rangle = (2\pi)^4 \delta^{(4)}(p_i - p_f) i\mathcal{M}$ (for $\text{in} \neq \text{out}$) and $\langle \mathbf{p} | \mathbf{p} \rangle = 2E_{\mathbf{p}} (2\pi)^3 \delta^{(3)}(\mathbf{0}) = 2E_{\mathbf{p}} V$ for one-particle state,

$$\frac{N_{\text{ev}}}{\prod_{\text{in}} N_{\text{particle}}} = \int d\Pi^{\text{out}} \frac{|\langle \text{out} | \text{in} \rangle|^2}{\langle \text{in} | \text{in} \rangle} = \int d\Pi^{\text{out}} \frac{(2\pi)^8 |\mathcal{M}|^2 VT}{\prod_{\text{in}} (2E)V (2\pi^4)} \delta^{(4)}(p_i - p_f) = VT \int d\Pi^{\text{Nf}} \frac{|\mathcal{M}|^2}{\prod_{\text{in}} (2E)V}. \quad (1.12)$$

Therefore, decay rate (at the rest frame) is given by

$$d\Gamma := \frac{1}{T} \frac{dN_{\text{ev}}}{N_{\text{particle}}} = \frac{1}{T} VT d\Pi^{\text{Nf}} \frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0} d\Pi^{\text{Nf}} |\mathcal{M}|^2. \quad (1.13)$$

We also define Lorentz-invariant cross section σ by $N_{\text{ev}} =: (\rho_A v_{\text{Møl}} T \sigma) N_B = (\rho_A v_{\text{Møl}} T \sigma) (\rho_B V)$, or

$$d\sigma := \frac{dN_{\text{ev}}}{\rho_A v_{\text{Møl}} T N_B} = \frac{V}{v_{\text{Møl}} T} VT d\Pi^{\text{Nf}} \frac{|\mathcal{M}|^2}{2E_A 2E_B V^2} = \frac{1}{2E_A 2E_B v_{\text{Møl}}} d\Pi^{\text{Nf}} |\mathcal{M}|^2. \quad (1.14)$$

where the Møller parameter $v_{\text{Møl}}$ is equal to $v_{\text{rel}}^{\text{NR}} = \|\mathbf{v}_A - \mathbf{v}_B\|$ if $\mathbf{v}_A \nparallel \mathbf{v}_B$ (cf. Ref. [1]). Generally,

$$v_{\text{Møl}} := \frac{\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}}{E_A E_B} = \frac{p_A \cdot p_B}{E_A E_B} v_{\text{rel}} = (1 - \mathbf{v}_A \cdot \mathbf{v}_B) v_{\text{rel}}, \quad (1.15)$$

where v_{rel} is the actual relative velocity

$$v_{\text{rel}} = \sqrt{1 - \frac{(1 - v_A^2)(1 - v_B^2)}{1 - (\mathbf{v}_A \cdot \mathbf{v}_B)^2}} = \frac{\sqrt{\|\mathbf{v}_A - \mathbf{v}_B\|^2 - \|\mathbf{v}_A \times \mathbf{v}_B\|^2}}{1 - \mathbf{v}_A \cdot \mathbf{v}_B} = \frac{\lambda^{1/2}(s, m_A^2, m_B^2)}{s - (m_A^2 + m_B^2)} \neq v_{\text{rel}}^{\text{NR}}. \quad (1.16)$$

(Note that $p_A \cdot p_B / E_A E_B = 1$ if $\mathbf{p}_A = 0$ or $\mathbf{p}_B = 0$. Also, Each of v_{rel} , VT , and $E_A E_B v_{\text{Møl}}$ is Lorentz invariant.)

1.3. Three body phase space

The phase-space reduction utilizes the identity [2]

$$1 = \int \frac{d^4 p_{ij}}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p_{ij} - p_i - p_j) \Theta(p_{ij}^0) \quad (1.17)$$

$$= \int \frac{d^4 p_{ij}}{(2\pi)^4} \left[\int \frac{ds}{2\pi} (2\pi) \delta(s - p_{ij}^2) \right] (2\pi)^4 \delta^{(4)}(p_{ij} - p_i - p_j) \Theta(p_{ij}^0) \quad (1.18)$$

$$= \int \frac{d^3 \mathbf{p}_{ij}}{(2\pi)^3} \frac{ds}{2\pi} \frac{1}{2p_{ij}^0} (2\pi)^4 \delta^{(4)}(p_{ij} - p_i - p_j) \Big|_{p_{ij}^0 = \sqrt{s + \|\mathbf{p}_{ij}\|^2}}. \quad (1.19)$$

For three-body phase space,

$$\begin{aligned} \overline{d\Pi^3} &= \int d\Pi_1 \frac{d^4 p_2 d^4 p_3}{(2\pi)^8} (2\pi) \delta(p_2^2 - m_2^2) (2\pi) \delta(p_3^2 - m_3^2) \Theta(p_2^0) \Theta(p_3^0) (2\pi)^4 \delta^{(4)}(P - p_1 - p_2 - p_3) \\ &\quad \times \frac{d^3 \mathbf{p}_{23}}{(2\pi)^3} \frac{ds_{23}}{2\pi} \frac{1}{2p_{23}^0} (2\pi)^4 \delta^{(4)}(p_{23} - p_2 - p_3) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\mathbf{p}_{23}\|^2}} \end{aligned} \quad (1.20)$$

$$\begin{aligned} &= \int \frac{ds_{23}}{2\pi} \int d\Pi_1 \frac{d^3 \mathbf{p}_{23}}{(2\pi)^3} \frac{1}{2p_{23}^0} (2\pi)^4 \delta^{(4)}(P - p_1 - p_{23}) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\mathbf{p}_{23}\|^2}} \\ &\quad \times \frac{d^4 p_2 d^4 p_3}{(2\pi)^8} (2\pi) \delta(p_2^2 - m_2^2) (2\pi) \delta(p_3^2 - m_3^2) \Theta(p_2^0) \Theta(p_3^0) (2\pi)^4 \delta^{(4)}(p_{23} - p_2 - p_3). \end{aligned} \quad (1.21)$$

$$= \int \frac{ds_{23}}{2\pi} \int d\Pi_1 \frac{d^3 \mathbf{p}_{23}}{(2\pi)^3} \frac{1}{2p_{23}^0} (2\pi)^4 \delta^{(4)}(P - p_1 - p_{23}) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\mathbf{p}_{23}\|^2}} \times \overline{d\Pi^2}(p_{23}^0, \mathbf{p}_{23}) \quad (1.22)$$

and $\overline{d\Pi^2}(p_{23}^0, \mathbf{p}_{23})$ is given by Eq. (1.10); explicitly,

$$\overline{d\Pi^2}(p_{23}^0, \mathbf{p}_{23}) = \frac{d \cos \theta_2}{8\pi} \frac{p_2^2}{p_{23}^0 p_2 - \|\mathbf{p}_{23}\| \sqrt{p_2^2 + m_2^2} \cos \theta_2}; \quad (1.23)$$

$$p_2 = \frac{(s_{23} + m_2^2 - m_3^2) \|\mathbf{p}_{23}\| \cos \theta_2 + p_{23}^0 \sqrt{\lambda(s_{23}, m_2^2, m_3^2)} - 4m_2^2 \|\mathbf{p}_{23}\|^2 \sin^2 \theta_2}{2(s_{23} + \|\mathbf{p}_{23}\|^2 \sin^2 \theta_2)}, \quad (1.24)$$

where θ_2 is the angle between \mathbf{p}_{23} and \mathbf{p}_2 (in the lab frame).

If the matrix element to integrate is spherically symmetric, so as $\overline{d\Pi^2}(p_{23}^0, \mathbf{p}_{23}) |\mathcal{M}|^2$, i.e., it is independent of the angle of \mathbf{p}_{23} . Then one can simply evaluate $\int d^3 \mathbf{p}_{23}$, which leads to, in the center-of-mass frame,

$$\overline{d\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \mathbf{0}) = \frac{ds_{23} d \cos \theta_2}{64\pi^3} \frac{p_1}{\sqrt{s}} \frac{p_2^2}{p_2 \sqrt{s_{23} + p_1^2} - p_1 \sqrt{p_2^2 + m_2^2} \cos \theta_2} \Big|_{p_1^2 = \lambda(s, m_1^2, s_{23})/4s} = \frac{s}{128\pi^3} dx_1 dx_2, \quad (1.25)$$

where we defined $x_i := 2E_i/\sqrt{s}$. Noting that $s_{23} = s + m_1^2 - 2E_1\sqrt{s} = s(1 - x_1) + m_1^2$ etc.,

$$\overline{d\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \mathbf{0}) = \frac{1}{128\pi^3} \frac{1}{s} \int_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} ds_{23} \int d\mathbf{s}_{13}; \quad (1.26)$$

$$(s_{13})_{\min}^{\max} = \frac{(s + m_3^2 - m_1^2 - m_2^2)^2}{4s_{23}} - \frac{1}{4s_{23}} \left[\lambda^{1/2}(s, m_1^2, s_{23}) \mp \lambda^{1/2}(s_{23}, m_2^2, m_3^2) \right]^2. \quad (1.27)$$

This is equal to the PDG-Eq. (47.23) [3].

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation $\text{adj.} = (\epsilon^a)^{bc}$.^{*1}

$$T_a = \frac{1}{2}\sigma_a, \quad \text{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \quad [T_a, T_b] = i\epsilon^{abc}T^c, \quad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\bar{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a \epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\bar{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \rightarrow \epsilon^{ab}[\exp(i\theta^\alpha T^\alpha)]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp(i\theta^\alpha T^\alpha)]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp(-i\theta^\alpha T^{\alpha*})]^{ab}(\epsilon^{bc}\mathbf{2}^c). \quad (2.1)$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\bar{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation $\text{adj.} = \mathbf{8} = (f^a)^{bc}$.
Gell-Mann matrices:

$$\lambda_{1-8} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (2.2)$$

$$\tau_a = \frac{1}{2}\lambda_a, \quad \text{Tr}(\tau_a \tau_b) = \frac{1}{2}\delta_{ab}, \quad [\tau_a, \tau_b] = i f^{abc}\tau^c, \quad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\begin{aligned} \mathbf{3}: \quad & \phi_a \rightarrow [\exp(i\theta^\alpha \tau^\alpha)]_{ab}\phi_b \simeq \phi_a + i\theta^\alpha \tau_{ab}^\alpha \phi_b & \bar{\mathbf{3}}: \quad & \phi_a \rightarrow [\exp(-i\theta^\alpha \tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - i\theta^\alpha \tau_{ab}^{\alpha*} \phi_b \\ & \phi_a^* \rightarrow [\exp(-i\theta^\alpha \tau^{\alpha*})]_{ab}\phi_b^* \simeq \phi_a^* - i\theta^\alpha \tau_{ab}^{\alpha*} \phi_b^* & & \phi_a^* \rightarrow [\exp(i\theta^\alpha \tau^\alpha)]_{ab}\phi_b^* \simeq \phi_a^* + i\theta^\alpha \tau_{ab}^\alpha \phi_b^* \\ & = \phi_b^*[\exp(-i\theta^\alpha \tau^\alpha)]_{ba} \simeq \phi_a^* - i\theta^\alpha \phi_b^* \tau_{ba}^\alpha & & \end{aligned}$$

^{*1}We do not distinguish sub- and superscripts for gauge indices.

3. Spinors

$$(\overline{\psi_1} \psi_2)^* = (\psi_2)^\dagger (\overline{\psi_1})^\dagger = \overline{\psi_2} \psi_1. \quad (3.1)$$

3.1. Convention ♣TODO:WIP!♣

First we prepare a vector x^μ and a symmetric matrix $\eta^{\mu\nu}$, which we call “contravariant vector” x^μ and the metric $\eta^{\mu\nu}$. Then we perform a Lorentz transformation on x^μ to obtain $(x')^\mu$, with which we can define a matrix $\Lambda(va, \theta)_\nu^\mu$ through $\Lambda^\mu{}_\nu x^\nu = (x')^\mu$.

We then consider Λ s for infinitesimal transformations and define \mathbf{S} , \mathbf{J} , and \mathbf{K} by

$$\Lambda^\mu{}_\nu \simeq \delta^\mu_\nu - i(\theta \cdot \mathbf{J}^\mu{}_\nu + \beta \cdot \mathbf{K}^\mu{}_\nu) \simeq \delta^\mu_\nu - \frac{i}{2} \left[\Lambda^{\alpha\beta} S_{\alpha\beta} \right]^\mu{}_\nu \quad (3.2)$$

Imposing “Lorentz condition” (♣TODO:what?♣), we get the expression for $S = i(\delta \dots)$ and $[J^i, J^j] = \dots$; further, we get $\Lambda^\mu{}_\nu = \exp(-i\theta \cdot \mathbf{J} - i\xi \cdot \mathbf{K})$, $\theta = (\theta_{23}, \theta_{31}, \theta_{12})$, $\xi = \hat{v} \tanh^{-1} \|v\| = (\theta^{10}, \theta^{20}, \theta^{30})$; $J = (S_{23}, S_{31}, S_{12})$, $K = (S^{01}, S^{02}, S^{03}) \dots$?

Lorentz transformation with a rotation θ around an axis $\hat{\theta}$ and a boost v are given by

$$\Lambda = \exp[-i(\theta \cdot \mathbf{J} + \beta \cdot \mathbf{K})]; \quad \theta := \theta \hat{\theta}, \quad \beta := \hat{v} \tanh^{-1} \|v\|, \quad (3.3)$$

♣TODO:check!♣

$$\text{Lorentz transformation (infinitesimal): } \Lambda = \begin{pmatrix} 0 & \beta^T & & \\ \beta & 0 & -\theta_z & \theta_y \\ & \theta_z & 0 & -\theta_x \\ & -\theta_y & \theta_x & 0 \end{pmatrix}$$

$$[J_{\mu\nu}]^\alpha{}_\beta = i(\delta_\mu^\alpha \eta_{\nu\beta} - \delta_\nu^\alpha \eta_{\mu\beta})$$

Lorentz tensor $M^{\mu_1 \mu_2 \dots \mu_n} \propto \bar{\sigma}^{\mu_1 \dot{\beta}_1 \alpha_1} \dots M_{\alpha_1 \dots \dot{\beta}_1 \dots}$

Especially $V^\mu =: \frac{1}{2} \bar{\sigma}^{\mu \dot{\beta} \alpha} V_{\alpha \dot{\beta}}$, $V_{\alpha \dot{\beta}} = V^\mu \sigma_{\mu \alpha \dot{\beta}}$; hermite $V_{\alpha \dot{\beta}} \Leftrightarrow \text{real} V^\mu$.

$$(V^T)_{\alpha \dot{\beta}} = V_{\beta \dot{\alpha}}, \quad \text{♣TODO:(correct? possibly wrong dot-positions?)♣} \quad (3.4)$$

$$(V^*)_{\dot{\alpha} \beta} := (V_{\alpha \dot{\beta}})^*, \quad (3.5)$$

$$(V^\dagger)_{\alpha \dot{\beta}} := (V_{\beta \dot{\alpha}})^* = (V^*)_{\dot{\beta} \alpha} \quad (3.6)$$

♣TODO: anyway not very sure about the reasoning; though my old note says like this...♣

In general, metric is symmetric.

$$(\Lambda^{-1})^\mu{}_\nu = \eta_{\nu\rho} \Lambda^\rho{}_\sigma (\eta^{-1})^{\sigma\mu} =: \Lambda_\nu{}^\mu \quad (3.7)$$

4. Loop calculation

Notation follows LoopTools [4]; capital M s and P s respectively denote squared masses and momenta.

Passarino–Veltman scalar integrals

$$A_0(M)/M = \Delta_\epsilon + \log \mu^2 + 1 - \log M, \quad (4.1)$$

$$B_0(P, M_0, M_1) = \Delta_\epsilon + \log \mu^2 - \int_0^1 dx \log [-x(1-x)P + xM_1 + (1-x)M_0] \quad (4.2)$$

$$C_0(P_1, P_2, P_3, M_1, M_2, M_3) = \int_0^1 dx \int_0^1 dy \frac{x}{Q_1} \quad (4.3)$$

$$= \int_0^1 dx \int_0^x dy \frac{1}{Q_2}; \quad (4.4)$$

$$Q_1 = x(1-x)(1-y)P_2 + x^2y(1-y)P_3 + x(1-x)yP_1 - xyM_1 - (1-x)M_2 - x(1-y)M_3,$$

$$Q_2 = -P_2x^2 - P_1y^2 + (P_1 + P_2 - P_3)xy + (P_2 - M_2 + M_3)x + (M_2 - M_1 + P_3 - P_2)y - M_3.$$

Kinematical invariance:

$$B_0(P, M_0, M_1) = B_0(P, M_1, M_0), \quad C_0(P_1, P_2, P_3, M_1, M_2, M_3) = C_0(P_2, P_3, P_1, M_2, M_3, M_1) \\ = C_0(P_1, P_3, P_2, M_2, M_1, M_3) \quad (4.5)$$

Special cases:

$$C_0(0, P, P, M, M, M') = \int_0^1 dx \int_0^x dy \frac{-1}{Px^2 - (P - M + M')x + M'}; \quad (4.6) \\ = \int_0^1 dx \frac{-x/P}{(x - \alpha)^2 - \lambda(P, M, M')/4P^2}; \quad \alpha = (P - M + M')/2P.$$

4.1. Passarino–Veltman scalar integrals

See `calculator/loop/PaVeAnalytic.wl` for validation. We use the notation [4,5]

$$\Delta_\epsilon = \frac{2}{4-d} - \gamma + \log 4\pi \equiv \text{GetDelta[]} \quad (= 0 \text{ in } \overline{\text{MS}}), \quad \mu^2 \equiv \text{GetMudim[]}, \quad (4.7)$$

where μ is introduced due to the different mass dimension of vector and spinor fields in d -dimensional theory:

$$[A_\mu] = 1 - \frac{4-d}{2}, \quad [\psi] = \frac{3}{2} - \frac{4-d}{2}, \quad [\text{gauge couplings}] = \frac{4-d}{2} \quad \Rightarrow \quad e = (e)_{4\text{-dim.}} \mu^{(4-d)/2}. \quad (4.8)$$

The analytic form of scalar integrals are given in Refs. [5–7].

5. Cosmology

5.1. Cosmology formulae

Assuming that σ depends only on $s = (p_A + p_B)^2$, ♣**TODO**:(or always true?)♣

$$\int \frac{d^3 \mathbf{p}_A}{(2\pi)^3} \frac{d^3 \mathbf{p}_B}{(2\pi)^3} e^{-E_A/T} e^{-E_B/T} \sigma(s) v_{\text{Mol}} \quad (5.1)$$

$$= 4 \int d\Pi_A d\Pi_B e^{-(E_A+E_B)/T} \sigma(s) \sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2} \quad (5.2)$$

$$= \frac{1}{32\pi^4} \int_{(m_A+m_B)^2}^{\infty} ds \int_{\sqrt{s}}^{\infty} dE_+ \left[\frac{2\sqrt{E_+^2 - s}}{s} \lambda^{1/2}(s, m_A^2, m_B^2) \right] e^{-E_+/T} \sigma(s) \left[\frac{1}{2} \lambda^{1/2}(s, m_A^2, m_B^2) \right] \quad (5.3)$$

$$= \frac{T}{32\pi^4} \int_{(m_A+m_B)^2}^{\infty} ds s^{3/2} K_1 \left(\frac{\sqrt{s}}{T} \right) \lambda \left(1, \frac{m_A^2}{s}, \frac{m_B^2}{s} \right) \sigma(s). \quad (5.4)$$

6. Standard Model

(summary page)

6.1. Particle content and convention

6.2. Lagrangian

6.3. Higgs mechanism

A general expression for composing a Dirac fermion from $\psi_L(T_{3L}, Y_L)$ and $\psi_R(T_{3R}, Y_R)$ is given by

$$(g_2 W_3 T_{3L} + g_Y B Y_L) P_L + (g_2 W_3 T_{3R} + g_Y B Y_R) P_R \quad (6.1)$$

$$= \left[(|e|A + g_Z c_w^2 \not{Z}) T_{3L} + (|e|A - g_Z s_w^2 \not{Z}) Y_L \right] P_L + (\text{right}) \quad (6.2)$$

$$= \frac{T_{3L} + T_{3R} + Y_L + Y_R}{2} |e|A + \frac{T_{3L} c_w^2 - Y_L s_w^2 + T_{3R} c_w^2 - Y_R s_w^2}{2} g_Z \not{Z} \\ + \frac{-T_{3L} - Y_L + T_{3R} + Y_R}{2} |e|A \gamma_5 + \frac{-c_w^2 T_{3L} + s_w^2 Y_L + c_w^2 T_{3R} - s_w^2 Y_R}{2} g_Z \not{Z} \gamma_5. \quad (6.3)$$

In the SM, $T_{3L} + Y_L = Y_R =: Q$ and $T_{3R} = 0$ lead to

$$Q |e|A + g_Z \not{Z} (T_{3L} P_L - Q s_w^2). \quad (6.4)$$

6.4. Lagrangian in mass eigenstates

6.5. CKM matrix and Yukawa convention

Our convention is, with $Y = UY^{\text{diag}}V^\dagger$,

$$\mathcal{L} \supset \bar{U}Y_u H P_L Q - \bar{D}Y_d H^\dagger P_R Q - \bar{E}Y_e H^\dagger P_R L + \text{h.c.} \quad (6.5)$$

$$= \bar{U}_i Y_{uij} \epsilon^{ab} H^a P_L Q_j^b - \bar{D}_i Y_{dij} H^{a*} P_L Q_j^a - \bar{E}_i Y_{eij} H^{a*} P_L L_j^a + \text{h.c.} \quad (6.6)$$

$$= \epsilon^{ab} \bar{U} U_u Y_u^{\text{diag}} H^a P_L V_u^\dagger Q^b - \bar{D} U_d Y_d^{\text{diag}} H^{a*} P_R V_d^\dagger Q^a - \bar{E} U_e Y_e^{\text{diag}} H^{a*} P_R V_e^\dagger L + \text{h.c.} \quad (6.7)$$

$$\rightsquigarrow -\frac{v}{\sqrt{2}} \bar{U} U_u Y_u^{\text{diag}} V_u^\dagger P_L Q^1 - \frac{v}{\sqrt{2}} \bar{D} U_d Y_d^{\text{diag}} V_d^\dagger P_L Q^2 - \frac{v}{\sqrt{2}} \bar{E} U_e Y_e^{\text{diag}} V_e^\dagger P_L L^2 + \text{h.c.} \quad (6.8)$$

Noting that $(\bar{\psi}_A P_L \psi_B)^* = \bar{\psi}_B P_R \psi_A$, this definition is equivalent to

$$\mathcal{L} \supset -\bar{Q}^a Y_u^\dagger \epsilon^{ab} H^{b*} P_R U - \bar{Q}^a Y_d^\dagger H^a P_R D - \bar{L}^a Y_e^\dagger H^a P_R E + \text{h.c.} \quad (6.9)$$

Moving to the CKM basis, the weak interaction is amended as

$$\mathcal{L} \supset \bar{Q} i \gamma^\mu (-ig_2 W_\mu) P_L Q \supset \frac{g_2}{\sqrt{2}} [\bar{Q}^1 W^+ P_L Q^2 + \bar{Q}^2 W^- P_L Q^1] = \frac{g_2}{\sqrt{2}} [\bar{Q}^1 V_u^\dagger W^+ P_L V_d \hat{Q}^2 + \bar{Q}^2 V_d^\dagger W^- P_L V_u \hat{Q}^1], \quad (6.10)$$

where hatted fields are in the CKM basis, and we define, with $s_{ij} > 0$ and $c_{ij} > 0$,

$$\begin{aligned} V_{\text{CKM}} = V_u^\dagger V_d &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13} e^{-i\delta} \\ & 1 & \\ -s_{13} e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \\ & & 1 \end{pmatrix} e^{i\Theta} \\ &= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} e^{i\Theta}. \end{aligned} \quad (6.11)$$

Here, a 3×3 unitary matrix has three angles and six phases, among which five phases are removed by the fermion rotation, but another phase Θ is introduced due to the Θ -term removal. ♣TODO:link♣

PDG convention (consistent in 2006§11 and 2018§12)

$$\mathcal{L} \supset -Y_{ij}^d \bar{Q}_{Li}^d \phi_L^j u_{Rj}^I - Y_{ij}^u \bar{Q}_{Li}^u \epsilon \phi_L^* u_{Rj}^I, \quad Y^{\text{diag}} = V_L Y V_R^\dagger, \quad V_{\text{CKM}} = V_L^u V_L^{d\dagger}. \quad (6.12)$$

So, $Y^u = Y_u^\dagger$, $Y^d = Y_d^\dagger$; $Y^{\text{diag}} = V_R Y^\dagger V_L^\dagger = V_R Y V_L^\dagger$ leads $V_L = V^\dagger$, and $V_{\text{CKM}} = V_u^\dagger V_d = V_{\text{CKM}}$.

SLHA2 convention (0801.0045)

$$W \supset \epsilon_{ab} [(Y_E)_{ij} H_1^a L_i^b \bar{E}_j + (Y_D)_{ij} H_1^a Q_i^b \bar{D}_j + (Y_U)_{ij} H_2^b Q_i^a \bar{U}_j]; \quad (6.13)$$

$$\mathcal{L} \supset -\epsilon_{ab} [(Y_E)_{ij} H_1^a \psi_{Li}^b \bar{\psi}_{Ej} + (Y_D)_{ij} H_1^a \psi_{Qi}^b \bar{\psi}_{Dj} + (Y_U)_{ij} H_2^b \psi_{Qi}^a \bar{\psi}_{Uj}] \quad (6.14)$$

$$\rightsquigarrow -[\psi_{Ej} v_d Y_E^T \psi_L^2 + \psi_{Dj} v_d Y_D^T \psi_Q^2 + \psi_{Uj} v_u Y_U^T \psi_Q^1]; \quad Y^{\text{diag}} = U^\dagger Y^T V, \quad V_{\text{CKM}} = V_u^\dagger V_d. \quad (6.15)$$

Hence, $Y_E = Y_e^T$, $Y_D = Y_d^T$, $Y_U = Y_u^T$; $Y^{\text{diag}} = U^\dagger Y V$, $V = V$ and $V_{\text{CKM}} = V_{\text{CKM}}$.

Wolfenstein parameterization The CKM matrix is precisely written in terms of λ , A , and $\bar{\rho} + i\bar{\eta}$.

$$\lambda := s_{12} = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad A := \frac{s_{23}}{\lambda^2} = \lambda^{-1} \left| \frac{V_{cb}}{V_{us}} \right|, \quad \bar{\rho} + i\bar{\eta} := \frac{-V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}. \quad (6.16)$$

They are independent of the phase convention and used for SLHA2 input, i.e., V_{CKMIN} should contain $(\lambda, A, \bar{\rho}, \bar{\eta})$.

Also, $\bar{\rho} + i\bar{\eta}$ is approximately written by

$$R = \rho + i\eta := \frac{s_{13} e^{i\delta}}{A \lambda^3} = \frac{V_{ub}^* V_{ud}}{A \lambda^3 |V_{ud}|} = \frac{(\bar{\rho} + i\bar{\eta}) \sqrt{1 - A^2 \lambda^4}}{\sqrt{1 - \lambda^2} [1 - A^2 \lambda^4 (\bar{\rho} + i\bar{\eta})]} = (\bar{\rho} + i\bar{\eta}) \left(1 + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) \right), \quad (6.17)$$

with which

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A \lambda^3 R^* \\ -\lambda & 1 - \lambda^2/2 & A \lambda^2 \\ A \lambda^3 (1 - R) & -A \lambda^2 & 1 \end{pmatrix} e^{i\Theta} + \begin{pmatrix} \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^7) & 0 \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^8) \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^4) \end{pmatrix}. \quad (6.18)$$

6.6. General Higgs doublet and Nambu–Goldstone bosons

In linear parameterization,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{2}\phi^+ \\ v + h + i\phi_3 \end{pmatrix}, \quad D_\mu H = \begin{pmatrix} i\partial_\mu\phi^+ - \frac{ig_2}{2}(v + h + i\phi_3)W_\mu^+ + \left(|e|A_\mu + \frac{c_w^2 - s_w^2}{2}g_Z Z_\mu\right)\phi^+ \\ \partial_\mu(h + i\phi_3)/\sqrt{2} + \frac{ig_Z}{2}Z_\mu(v + h + i\phi_3)/\sqrt{2} + g_2 W_\mu^- \phi^+/\sqrt{2} \end{pmatrix}; \quad (6.19)$$

$$\begin{aligned} |D_\mu H|^2 &= \frac{(\partial_\mu h)^2 + (\partial_\mu \phi_3)^2}{2} + \partial_\mu \phi^+ \partial^\mu \phi^- + \frac{(v + h)^2}{8} (2g_2^2 W^{+\mu} W_\mu^- + g_Z^2 Z^\mu Z_\mu) \\ &+ \frac{\partial^\mu h}{2} [g_2 W_\mu^+ \phi^- + g_2 W_\mu^- \phi^+ - g_Z Z_\mu \phi_3] + \frac{\partial^\mu \phi_3}{2} [g_Z(v + h)Z_\mu + ig_2(W_\mu^+ \phi^- - W_\mu^- \phi^+)] \\ &+ \left\{ \frac{\partial^\mu \phi^+}{2} [-g_2(v + h - i\phi_3)W_\mu^- + (2|e|A_\mu + (c_w^2 - s_w^2)g_Z Z_\mu)i\phi^-] + \text{H.c.} \right\} \\ &+ \frac{ig_2(v + h)}{2} (|e|A^\mu - g_Z s_w^2 Z^\mu)(W_\mu^- \phi^+ - W_\mu^+ \phi^-) + \frac{g_2 \phi_3}{2} (|e|A^\mu - g_Z s_w^2 Z^\mu)(W_\mu^- \phi^+ + W_\mu^+ \phi^-) \\ &+ \frac{\phi_3^2}{8} (2g_2^2 W^{+\mu} W_\mu^- + g_Z^2 Z^\mu Z_\mu) + \frac{\phi^+ \phi^-}{4} [2g_2^2 W^{+\mu} W_\mu^- + (2|e|A_\mu + g_Z(c_w^2 - s_w^2)Z_\mu)^2]; \end{aligned} \quad (6.20)$$

$$V = \lambda|H|^4 - \mu^2|H|^2 = \frac{\lambda}{4}h^4 + \lambda v h^3 + \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}v^4 + \frac{\lambda}{4}(2\phi^+ \phi^- + \phi_3^2)^2 + \frac{\lambda}{2}(h^2 + 2vh)(2\phi^+ \phi^- + \phi_3^2), \quad (6.21)$$

where $v = \mu/\sqrt{\lambda} \sim 246$ GeV, $\lambda \sim 0.13$, and $\mu \sim 89$ GeV. In exponential parameterization,

$$H = \frac{1}{\sqrt{2}} \exp\left(\frac{i}{v}\sigma_i \varphi_i\right) \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad (6.22)$$

$$D_\mu H = \frac{1}{\sqrt{2}} e^{i\sigma_i \varphi_i/v} \left[i\sigma_i \partial_\mu \varphi_i \begin{pmatrix} 0 \\ 1 + h/v \end{pmatrix} + \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} \right] + \frac{1}{\sqrt{2}} \frac{-i}{2} (g_2 \sigma_i W_{i\mu} + g_Y B_\mu) e^{i\sigma_i \varphi_i/v} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \quad (6.23)$$

$$= \frac{1}{\sqrt{2}} e^{i\sigma_i \varphi_i/v} \left[\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + i \left(\sigma_i \partial_\mu \varphi_i - \frac{g_2 v}{2} e^{-i\sigma_j \varphi_j/v} \sigma_i e^{i\sigma_k \varphi_k/v} W_{i\mu} - \frac{g_Y v}{2} B_\mu \right) \begin{pmatrix} 0 \\ 1 + h/v \end{pmatrix} \right], \quad (6.24)$$

$$V = \lambda|H|^4 - \mu^2|H|^2 = \frac{\lambda}{4}h^4 + \lambda v h^3 + \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}v^4. \quad (6.25)$$

These expressions have gauge degeneracy (i.e., without gauge-fixing terms and ghost terms) and thus not ready for calculations. If we choose the unitarity gauge, $\phi_i(x) = 0$,

$$\mathcal{L}_H = |D_\mu H|^2 - V(H) = \frac{1}{2}(\partial_\mu h)^2 - \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}h^4 - \lambda v h^3 + \frac{(v + h)^2}{8} (2g_2^2 W^{+\mu} W_\mu^- + g_Z^2 Z^\mu Z_\mu) + \frac{\lambda}{4}v^4. \quad (6.26)$$

6.7. Values of SM parameters

7. Values^{*2}

Mass and width

e : 0.510 998 9461(31) MeV	$m_{\nu;\text{tot}} < 0.2\text{--}0.3$ eV
μ : 105.658 3745(24) MeV, 2.196 9811(22) $\mu\text{s} = 659$ m	h : 125.10(14) GeV
τ : 1.776 86(12) GeV, $290.3(5) \times 10^{-15}$ s = 87.0 μm	W : 80.379(12) GeV, 2.085(42) GeV
t : 172.76(30) GeV ^{*3} , $1.42^{+0.19}_{-0.15}$ GeV	Z : 91.1876(21) GeV, 2.4952(23) GeV
$(u, d, s)_{2\text{GeV}}^{\overline{\text{MS}}} : (2.16^{+0.49}_{-0.26}, 4.67^{+0.48}_{-0.17}, 93^{+11}_{-5}) \text{ MeV}^{*4}$	c : 1.27(2) GeV $_{m_c}^{\overline{\text{MS}}}$ (1.67(7) GeV ^{pole})
$(\frac{u+d}{2}, \frac{u}{d}, \frac{2s}{u+d})_{2\text{GeV}}^{\overline{\text{MS}}} : (3.45^{+0.55}_{-0.15} \text{ MeV}, 0.47^{+0.06}_{-0.07}, 27.3^{+0.7}_{-1.3})$	b : 4.18 $^{+0.03}_{-0.02}$ GeV $_{m_b}^{\overline{\text{MS}}}$ (4.78(6) GeV ^{pole})
π^\pm : 139.570 39(18) MeV ρ_{770}^\pm : 775.11(34) MeV	$\eta_c(1S)$: 2983.9(5) MeV
π^0 : 134.9768(5) MeV ρ_{770}^0 : 775.26(25) MeV	$J/\psi(1S)$: 3096.900(6) MeV
η : 547.862(17) MeV ϕ_{1020} : 1019.461(16) MeV	$\eta_b(1S)$: 9398.7(20) MeV
η' : 957.78(6) MeV ω_{782} : 782.65(12) MeV	$\Upsilon(1S)$: 9460.30(26) MeV
K^\pm : 493.677(16) MeV $K_{892}^{*\pm}$: 891.66(26) MeV	$\Upsilon(2S)$: 10023.26(31) MeV
K^0 : 497.611(13) MeV K_{892}^{*0} : 895.55(20) MeV	$\Upsilon(3S)$: 10355.2(5) MeV
D^0 : 1864.83(5) MeV B^\pm : 5279.34(12) MeV	$\Upsilon(4S)$: 10579.4(12) MeV
D^\pm : 1869.65(5) MeV B^0 : 5279.65(12) MeV	p : 938.272 0813(58) MeV
D_s^\pm : 1968.34(7) MeV B_s : 5366.88(14) MeV	n : 939.565 413(6) MeV
	B_c^\pm : 6274.9(8) MeV
π^\pm : $2.6033(5) \times 10^{-8}$ s = 7.80 m	K^\pm : $1.2380(20) \times 10^{-8}$ s = 3.71 m $p \stackrel{\diamond}{>} 3.6 \times 10^{29}$ yr
π^0 : $8.52(18) \times 10^{-17}$ s = 0.0255 μm	K_S^0 : $0.8954(4) \times 10^{-10}$ s = 26.8 mm
n : 879.4(6) s = 2.64×10^8 km	K_L^0 : $5.116(21) \times 10^{-8}$ s = 15.3 m

Electric and magnetic moment, important branching ratios, and neutrino property

$a_e = 11\,596\,521.8091(26) \times 10^{-10}$	$\text{Br}(\tau \rightarrow e, \mu) \simeq 35.2\%$	$\Delta m_{21}^2/\text{eV}^2 = 7.53(18) \times 10^{-5}$
$a_\mu = 11\,659\,208.9(54)(33) \times 10^{-10}$	$\text{Br}(\tau \rightarrow \text{had}) \simeq 64.8\%$	$\Delta m_{32}^2/\text{eV}^2 =$
$\mu_p = 2.792\,847\,3446(8)\mu_N$	$\text{Br}(Z \rightarrow e, \mu, \tau) \simeq 10.10\%$	$2.453(34) \times 10^{-3}(\text{NH})$
$\mu_n = -1.913\,0427(5)\mu_N$	$\text{Br}(Z \rightarrow \text{had}) = 69.911(56)\%$	$-2.546^{+0.034}_{-0.040} \times 10^{-3}(\text{IH})$
$a_\tau \stackrel{\diamond}{\in} [-0.052, 0.013]$	$\text{Br}(\tau; 1\text{-prong}) = 85.24(6)\%$	$\sin^2 \theta_{12} = 0.307(13)$
$d_e \stackrel{\diamond}{<} 0.11 \times 10^{-28} e \text{ cm}$	$\text{Br}(\tau; 3\text{-prong}) = 14.55(6)\%$	$\sin^2 \theta_{13} = 0.0218(7)$
$d_\mu \stackrel{**}{<} 1.8 \times 10^{-19} e \text{ cm}$	$\text{Br}(W \rightarrow \text{had}) = 67.41(27)\%$	$\sin^2 \theta_{23}(\text{NH}) = 0.545(21)$
$d_p \stackrel{?}{<} 0.021 \times 10^{-23} e \text{ cm}$	$\text{Br}(Z \rightarrow \text{inv}) = 20.000(55)\%$	$\sin^2 \theta_{23}(\text{IH}) = 0.547(21)$
$d_n \stackrel{\diamond}{<} 0.18 \times 10^{-25} e \text{ cm}$	$\text{Br}(Z \rightarrow b\bar{b}) = 15.12(5)\%$	$\delta = 1.36(17)\pi$

CKM matrix

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97401(11) & 0.22650(48) & 0.00361^{(+11)}_{(-9)} \\ 0.22636(48) & 0.97320(11) & 0.04053^{(+83)}_{(-61)} \\ 0.00854^{(+23)}_{(-16)} & 0.03978^{(+82)}_{(-60)} & 0.0999172^{(+34)}_{(-35)} \end{pmatrix}; \quad J = 3.00^{(+15)}_{(-9)} \times 10^{-5}$$

$$(\lambda, A, \bar{\rho}, \bar{\eta}) = (0.22650(48), 0.790^{(+17)}_{(-12)}, 0.141^{(+16)}_{(-17)}, 0.357(11))$$

$$(\sin \theta_{12}, \sin \theta_{13}, \sin \theta_{23}, \delta) = (0.22650(48), 0.00361^{(+11)}_{(-9)}, 0.04053^{(+83)}_{(-61)}, 1.196^{(+45)}_{(-43)})$$

^{*2}Data source: **PDG2020**. Confidence levels are shown by the marks *, **, *** (1–3 σ), \diamond (90%), and $\diamond\diamond$ (99%).

^{*3}Cross section measurement gives $\overline{\text{MS}}$ top mass $162.5^{+2.1}_{-1.5}$ GeV, equivalent to 172.4(7) GeV.

^{*4} $m_{1\text{GeV}}^{\overline{\text{MS}}} = m_{2\text{GeV}}^{\overline{\text{MS}}} \times 1.35$.

Standard Model parameter fit ♣**TODO**: write down; the following is from the old note ♣

$$\begin{aligned}
 \alpha_{\text{EM}}^{-1}(0) &= 137.035999074(44) & G_{\text{F}} &= \frac{g_2^2}{4\sqrt{2}m_W^2} = \frac{1}{\sqrt{2}v^2} = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \\
 \alpha_{\text{EM}}^{-1}(m_Z) &= 127.944(14) & \alpha_{\text{EM}}^{-1}(m_\tau) &= 133.471(14) \\
 \alpha_s(m_Z) &= 0.1184(7) & \sin^2 \theta_{\text{W}}(m_Z) &= 0.23116(12) \\
 \sin^2 \theta_{\text{eff}} &= 0.23146(12)
 \end{aligned}$$

$$|e| \sim 0.313, \quad g_1 \sim 0.357, \quad g_2 \sim 0.652, \quad g_Z \sim 0.743; \quad v = \sqrt{\frac{\mu^2}{\lambda}} \sim 246 \text{ GeV} \quad (7.1)$$

$$y_u \sim \begin{pmatrix} 10^{-5} & 0 & 0 \\ 0 & 0.007 & 0 \\ 0 & 0 & 0.997 \end{pmatrix} \quad y_d \sim \begin{pmatrix} 3 \times 10^{-5} & 0 & 0 \\ 0 & 0.0005 & 0 \\ 0 & 0 & 0.02 \end{pmatrix} \quad y_e \sim \begin{pmatrix} 3 \times 10^{-6} & 0 & 0 \\ 0 & 0.0006 & 0 \\ 0 & 0 & 0.01 \end{pmatrix} \quad (7.2)$$

Also, for $m_h \sim 125 \text{ GeV}$, we can estimate the Higgs potential as $\mu \sim 88 \text{ GeV}$ and $\lambda \sim 0.13$

$n^{2s+1}l_J J^{PC}$	$I = 1$	$I = 1/2$	$I = 0$	$c\bar{c}$	$b\bar{b}$	charm		bottom		
$1^1S_0 \quad 0^{-+}$	π	K	η	η'_{958}	$\eta_c(1S)$	$\eta_b(1S)$	D	D_s	B	$B_s \quad B_c$
$1^1S_1 \quad 1^{--}$	ρ_{770}	K_{892}^*	ϕ_{1020}	ω_{782}	$J/\psi(1S)$	$\Upsilon(1S)$	D^*	D_s^*	B^*	B_s^*

♣**TODO**: define PMNS matrix and explain neutrino hierarchy. ♣

8. Supersymmetry with $\eta = \text{diag}(+, -, -, -)$

Convention Our convention follows DHM (except for D_μ):

$$\begin{aligned}\eta &= \text{diag}(1, -1, -1, -1); \quad \epsilon^{0123} = -\epsilon_{0123} = 1, \quad \epsilon^{12} = \epsilon_{21} = \epsilon^{\dot{1}\dot{2}} = \epsilon_{\dot{2}\dot{1}} = 1 \quad (\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta_\gamma^\alpha), \\ \psi^\alpha &= \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}}, \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}; \\ \sigma_{\alpha\dot{\alpha}}^\mu &:= (\mathbf{1}, \boldsymbol{\sigma})_{\alpha\dot{\alpha}}, \quad \sigma^{\mu\nu}{}_\alpha{}^\beta := \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)_\alpha{}^\beta,^{*5} \quad (\sigma_{\alpha\dot{\beta}}^\mu = \epsilon_{\alpha\delta}\epsilon_{\dot{\beta}\dot{\gamma}}\bar{\sigma}^{\mu\dot{\gamma}\delta}, \quad \bar{\sigma}^{\mu\dot{\alpha}\beta} = \epsilon^{\dot{\alpha}\dot{\delta}}\epsilon^{\beta\gamma}\sigma_{\gamma\dot{\delta}}^\mu) \\ \bar{\sigma}^{\mu\dot{\alpha}\alpha} &:= (\mathbf{1}, -\boldsymbol{\sigma})^{\dot{\alpha}\alpha}, \quad \bar{\sigma}^{\mu\nu}{}_{\dot{\beta}}{}^{\dot{\alpha}} := \frac{i}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)^{\dot{\alpha}}{}_{\dot{\beta}},^{*5} \\ (\psi\xi) &:= \psi^\alpha\xi_\alpha, \quad (\bar{\psi}\bar{\chi}) := \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}; \quad \frac{d}{d\theta^\alpha}(\theta\theta) := \theta_\alpha \quad [\text{left derivative}].\end{aligned}$$

Especially, spinor-index contraction is done as α_α and $\dot{\alpha}_{\dot{\alpha}}$ except for ϵ_{ab} (which always comes from left). Noting that complex conjugate reverses spinor order: $(\psi^\alpha\xi^\beta)^* := (\xi^\beta)^*(\psi^\alpha)^*$,

$$\begin{aligned}\bar{\psi}^{\dot{\alpha}} &:= (\psi^\alpha)^*, \quad \epsilon^{\dot{a}\dot{b}} := (\epsilon^{ab})^*, \quad (\psi\chi)^* = (\bar{\psi}\bar{\chi}), \\ (\sigma_{\alpha\dot{\beta}}^\mu)^* &= \bar{\sigma}^{\mu\dot{\alpha}\beta} = \epsilon_{\beta\delta}\epsilon_{\dot{\alpha}\dot{\gamma}}\bar{\sigma}^{\mu\dot{\gamma}\delta}, \quad (\sigma^{\mu\nu})_{\alpha}{}^\beta = \bar{\sigma}^{\mu\nu}{}_{\dot{\alpha}}{}^{\dot{\beta}}, \quad (\sigma^{\mu\nu}{}_\alpha{}^\beta)^* = \bar{\sigma}^{\mu\nu}{}_{\dot{\alpha}}{}^{\dot{\beta}} = \epsilon_{\dot{\alpha}\dot{\gamma}}\epsilon^{\dot{\beta}\delta}\bar{\sigma}^{\mu\dot{\gamma}\delta}, \\ (\bar{\sigma}^{\mu\dot{\alpha}\beta})^* &= \sigma^{\mu\alpha\dot{\beta}} = \epsilon^{\dot{\beta}\dot{\delta}}\epsilon^{\alpha\gamma}\sigma_{\gamma\dot{\delta}}^\mu, \quad (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} = \sigma^{\mu\nu}{}_\alpha{}^\beta, \quad (\bar{\sigma}^{\mu\nu}{}_{\dot{\beta}}{}^{\dot{\alpha}})^* = \sigma^{\mu\nu}{}_\beta{}^\alpha = \bar{\sigma}^{\mu\nu}{}_\beta{}^\alpha = \epsilon_{\beta\delta}\epsilon^{\alpha\gamma}\sigma_{\gamma\dot{\delta}}^\mu.\end{aligned}$$

Contraction formulae

$$\begin{aligned}\theta^\alpha\theta^\beta &= -\frac{1}{2}(\theta\theta)\epsilon^{\alpha\beta} & \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} &= \frac{1}{2}(\bar{\theta}\bar{\theta})\epsilon^{\dot{\alpha}\dot{\beta}} & (\theta\xi)(\theta\chi) &= -\frac{1}{2}(\theta\theta)(\xi\chi) & (\theta\sigma^\nu\bar{\theta})\theta^\alpha &= \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\sigma}^\nu)^\alpha \\ \theta_\alpha\theta_\beta &= \frac{1}{2}(\theta\theta)\epsilon_{\alpha\beta} & \bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} &= -\frac{1}{2}(\bar{\theta}\bar{\theta})\epsilon_{\dot{\alpha}\dot{\beta}} & (\bar{\theta}\xi)(\bar{\theta}\bar{\chi}) &= -\frac{1}{2}(\bar{\theta}\bar{\theta})(\bar{\xi}\bar{\chi}) & (\theta\sigma^\nu\bar{\theta})\bar{\theta}_{\dot{\alpha}} &= -\frac{1}{2}(\theta\sigma^\nu)_{\dot{\alpha}}(\bar{\theta}\bar{\theta}) \\ \theta^\alpha\theta_\beta &= \frac{1}{2}(\theta\theta)\delta_\beta^\alpha & \bar{\theta}^{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} &= \frac{1}{2}(\bar{\theta}\bar{\theta})\delta_{\dot{\beta}}^{\dot{\alpha}} & (\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta}) &= \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})\eta^{\mu\nu} \\ (\theta\sigma^\mu\bar{\sigma}^\nu\theta) &= (\theta\theta)\eta^{\mu\nu} & (\bar{\theta}\bar{\sigma}^\mu\sigma^\nu\bar{\theta}) &= (\bar{\theta}\bar{\theta})\eta^{\mu\nu} & (\sigma^\mu\bar{\theta})_\alpha(\theta\sigma^\nu\bar{\theta}) &= \frac{1}{2}(\bar{\theta}\bar{\theta})(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha\end{aligned}$$

$$\begin{aligned}\sigma^\mu\bar{\sigma}^\nu &= \eta^{\mu\nu} - 2i\sigma^{\mu\nu} & \sigma^\mu\bar{\sigma}^\nu\sigma^\rho + \sigma^\rho\bar{\sigma}^\nu\sigma^\mu &= 2(\sigma^\mu\eta^{\rho\nu} - \sigma^\nu\eta^{\mu\rho} + \sigma^\rho\eta^{\mu\nu}) \\ \bar{\sigma}^\mu\sigma^\nu &= \eta^{\mu\nu} - 2i\bar{\sigma}^{\mu\nu} & \sigma^\mu\bar{\sigma}^\nu\sigma^\rho - \sigma^\rho\bar{\sigma}^\nu\sigma^\mu &= 2i\sigma_\sigma\epsilon^{\mu\nu\rho\sigma} \\ \text{Tr}(\sigma^\mu\bar{\sigma}^\nu) &= \text{Tr}(\bar{\sigma}^\mu\sigma^\nu) = 2\eta^{\mu\nu} & \bar{\sigma}^\mu\sigma^\nu\bar{\sigma}^\rho + \bar{\sigma}^\rho\sigma^\nu\bar{\sigma}^\mu &= 2(\bar{\sigma}^\mu\eta^{\rho\nu} - \bar{\sigma}^\nu\eta^{\mu\rho} + \bar{\sigma}^\rho\eta^{\mu\nu}) \\ \sigma_{\alpha\dot{\alpha}}^\mu\bar{\sigma}_{\dot{\beta}}{}^\beta &= 2\delta_{\dot{\alpha}}^\beta\delta_{\alpha\dot{\beta}} & \bar{\sigma}^\mu\sigma^\nu\bar{\sigma}^\rho - \bar{\sigma}^\rho\sigma^\nu\bar{\sigma}^\mu &= -2i\bar{\sigma}_\sigma\epsilon^{\mu\nu\rho\sigma} \\ \sigma_{\mu\alpha\dot{\alpha}}\sigma_{\beta\dot{\beta}}^\mu &= 2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} & \epsilon_{\dot{\beta}\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha} &= \epsilon_{\dot{\beta}\dot{\alpha}}\epsilon^{\dot{\alpha}\dot{\gamma}}\epsilon^{\alpha\gamma}\sigma_{\gamma\dot{\gamma}}^\mu = \epsilon^{\alpha\gamma}\sigma_{\gamma\dot{\beta}}^\mu \\ \bar{\sigma}_{\mu}{}^{\dot{\alpha}\alpha}\bar{\sigma}^{\mu\dot{\beta}\beta} &= 2\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}} & \epsilon_{\beta\alpha}\bar{\sigma}^{\mu\dot{\alpha}\alpha} &= \epsilon_{\beta\alpha}\epsilon^{\dot{\alpha}\dot{\gamma}}\epsilon^{\alpha\gamma}\sigma_{\gamma\dot{\gamma}}^\mu = \epsilon^{\dot{\alpha}\dot{\gamma}}\sigma_{\beta\dot{\gamma}}^\mu \\ \text{Tr}(\sigma^{\mu\nu}) &= \text{Tr}(\bar{\sigma}^{\mu\nu}) = 0 & \text{Tr}(\sigma^{\mu\nu}\sigma^{\rho\sigma}) &= \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) - \frac{1}{2}i\epsilon^{\mu\nu\rho\sigma} \\ \bar{\sigma}^{\mu\nu} &= -\bar{\sigma}^{\nu\mu} \quad \sigma^{\mu\nu} = -\sigma^{\nu\mu} & \text{Tr}(\bar{\sigma}^{\mu\nu}\bar{\sigma}^{\rho\sigma}) &= \frac{1}{2}i\epsilon^{\mu\nu\rho\sigma} + \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) \\ \sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} &= \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha} & \sigma_{\alpha\dot{\alpha}}^\mu\sigma_{\beta\dot{\beta}}^\nu - \sigma_{\alpha\dot{\alpha}}^\nu\sigma_{\beta\dot{\beta}}^\mu &= -2i\epsilon_{\dot{\alpha}\dot{\gamma}}\bar{\sigma}^{\mu\nu\dot{\gamma}}{}_\beta\epsilon_{\alpha\beta} - 2i\sigma^{\mu\nu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot{\alpha}\dot{\beta}} \\ \bar{\sigma}^{\mu\nu}{}_{\dot{\beta}}{}^{\dot{\alpha}}\epsilon^{\dot{\beta}\dot{\gamma}} &= \bar{\sigma}^{\mu\nu\dot{\gamma}}{}_{\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}} & \sigma_{\alpha\dot{\alpha}}^\mu\sigma_{\beta\dot{\beta}}^\nu + \sigma_{\alpha\dot{\alpha}}^\nu\sigma_{\beta\dot{\beta}}^\mu &= 4\sigma^{\rho\mu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot{\alpha}\dot{\gamma}}\bar{\sigma}^{\nu\dot{\gamma}}{}_{\dot{\beta}}\eta_{\rho\sigma} + \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\eta^{\mu\nu} \\ \bar{\sigma}_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} &= -2i\bar{\sigma}^{\mu\nu} & \bar{\sigma}^{\mu\dot{\alpha}\alpha}\bar{\sigma}^{\nu\dot{\beta}\beta} - \bar{\sigma}^{\nu\dot{\alpha}\alpha}\bar{\sigma}^{\mu\dot{\beta}\beta} &= -2i\bar{\sigma}^{\mu\nu\dot{\gamma}}{}_\alpha\epsilon^{\dot{\gamma}\dot{\beta}}\epsilon^{\alpha\beta} - 2i\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\dot{\alpha}\dot{\beta}} \\ \sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} &= 2i\sigma^{\mu\nu} & \bar{\sigma}^{\mu\dot{\alpha}\alpha}\bar{\sigma}^{\nu\dot{\beta}\beta} + \bar{\sigma}^{\nu\dot{\alpha}\alpha}\bar{\sigma}^{\mu\dot{\beta}\beta} &= 4\epsilon^{\alpha\gamma}\sigma^{\sigma\nu}{}_\gamma{}^\beta\bar{\sigma}^{\rho\mu\dot{\gamma}}{}_\alpha\epsilon_{\dot{\gamma}\dot{\beta}}\eta_{\rho\sigma} + \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\eta^{\mu\nu}\end{aligned}$$

$$\begin{aligned}\bar{\xi}\bar{\sigma}^\mu\chi &= -\chi\sigma^\mu\bar{\xi} & \bar{\xi}\bar{\sigma}^\mu\sigma^\nu\bar{\chi} &= \bar{\chi}\bar{\sigma}^\nu\sigma^\mu\bar{\xi} & \xi\sigma^\mu\bar{\sigma}^\nu\chi &= \chi\sigma^\nu\bar{\sigma}^\mu\xi & \bar{\xi}\bar{\sigma}^\mu\sigma^\nu\bar{\sigma}^\rho\chi &= -\chi\sigma^\rho\bar{\sigma}^\nu\sigma^\mu\bar{\xi} \\ (\xi\sigma^\mu\bar{\chi})^* &= \chi\sigma^\mu\bar{\xi} & (\bar{\xi}\bar{\sigma}^\mu\chi)^* &= \bar{\chi}\bar{\sigma}^\mu\xi & (\bar{\chi}\bar{\sigma}^\mu\sigma^\nu\bar{\xi})^* &= \xi\sigma^\nu\bar{\sigma}^\mu\chi & (\xi[\sigma_s]\chi)^* &= \bar{\chi}[\sigma_{\text{sreversed}}]\bar{\xi} \\ (\xi\chi)\psi^\alpha &= -(\psi\xi)\chi^\alpha - (\psi\chi)\xi^\alpha & (\xi\chi)\bar{\psi}_{\dot{\alpha}} &= \frac{1}{2}(\xi\sigma^\mu\bar{\psi})(\chi\sigma_\mu)_{\dot{\alpha}} \\ i\psi_i\sigma^\mu\partial_\mu\bar{\psi}_j &= -i\partial_\mu\bar{\psi}_j\bar{\sigma}^\mu\psi_i \equiv i\bar{\psi}_j\bar{\sigma}^\mu\partial_\mu\psi_i = -i\partial_\mu\psi_i\sigma^\mu\bar{\psi}_j\end{aligned}$$

^{*5} As the definition of $\sigma^{\mu\nu}$ and $\bar{\sigma}^{\mu\nu}$ are not unified in literature, they are not used in this CheatSheet except for this page.

Superfields

$$\Phi = \phi(x) + \sqrt{2}\theta\psi(x) - i\partial_\mu\phi(x)(\theta\sigma^\mu\bar{\theta}) + F(x)\theta^2 + \frac{i}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar{\theta})\theta^2 - \frac{\theta^4}{4}\partial^2\phi(x), \quad (8.1)$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar{\psi}(x)\bar{\theta} + F^*(x)\bar{\theta}^2 + i\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar{\theta}) - \frac{i}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar{\psi}(x)]\bar{\theta}^2 - \frac{\theta^4}{4}\partial^2\phi^*(x), \quad (8.2)$$

$$V = (\bar{\theta}\bar{\sigma}^\mu\theta)A_\mu(x) + \bar{\theta}^2\theta\lambda(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \frac{\theta^4}{2}D(x) \quad (\text{in Wess-Zumino supergauge}). \quad (8.3)$$

Without gauge symmetries

$$\mathcal{L} = \Phi_i^*\Phi_i\Big|_{\theta^4} + \left(W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right); \quad (8.4)$$

$$\Phi_i^*\Phi_i\Big|_{\theta^4} = (\partial_\mu\phi_i^*)(\partial^\mu\phi_i) + i\bar{\psi}_i\sigma^\mu\partial_\mu\psi_i + F_i^*F_i, \quad (8.5)$$

$$\begin{aligned} W(\Phi_i)\Big|_{\theta^2} &\rightsquigarrow \left[\kappa_i\Phi_i + m_{ij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k\right]_{\theta^2} \\ &= \kappa_i F_i + m_{ij}(-\psi_i\psi_j + F_i\phi_j + \phi_i F_j) \\ &\quad + y_{ijk}\left[-(\psi_i\psi_j\phi_k + \psi_i\phi_j\psi_k + \phi_i\psi_j\psi_k) + \phi_i\phi_j F_k + \phi_i F_j\phi_k + F_i\phi_j\phi_k\right]. \end{aligned} \quad (8.6)$$

With a U(1) gauge symmetry ^{*6}

$$\mathcal{L} = \Phi_i^* e^{2gVQ_i}\Phi_i\Big|_{\theta^4} + \left[\left(\frac{1}{4} - \frac{ig^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right] + \Lambda_{\text{FI}}D; \quad (8.7)$$

$$\Phi_i e^{2gQ_i V}\Phi_i\Big|_{\theta^4} \equiv D^\mu\phi_i^* D_\mu\phi_i + i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i + F_i^*F_i - \sqrt{2}gQ_i\phi_i^*\lambda\psi_i - \sqrt{2}gQ_i\bar{\psi}_i\bar{\lambda}\phi_i + gQ_i\phi_i^*\phi_i D, \quad (8.8)$$

$$\begin{aligned} \left(\frac{1}{4} - \frac{ig^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + \text{H.c.} &= \frac{1}{2}\text{Re}\mathcal{W}\mathcal{W}\Big|_{\theta^2} + \frac{g^2\Theta}{16\pi^2}\text{Im}\mathcal{W}\mathcal{W}\Big|_{\theta^2} \\ &\equiv i\bar{\lambda}\bar{\sigma}^\mu D_\mu\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g^2\Theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \end{aligned} \quad (8.9)$$

$$\begin{aligned} D_\mu\phi_i &= (\partial_\mu - igQ_i A_\mu)\phi_i, & D_\mu\psi_i &= (\partial_\mu - igQ_i A_\mu)\psi_i, \\ D^\mu\phi_i^* &= (\partial^\mu + igQ_i A^\mu)\phi_i^*, & F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, & D_\mu\lambda &= \partial_\mu\lambda. \end{aligned}$$

$$\{\phi, \psi, F\} \xrightarrow{\text{gauge}} e^{igQ_i\theta}\{\phi, \psi, F\}, \quad A_\mu \xrightarrow{\text{gauge}} A_\mu + \partial_\mu\theta, \quad \lambda \xrightarrow{\text{gauge}} \lambda, \quad D \xrightarrow{\text{gauge}} D. \quad (8.10)$$

^{*6}We use the convention with $V \ni \lambda(x)\theta\bar{\theta}^2$, which corresponds to $\lambda = i\lambda_{\text{SLHA}}$. In SLHA convention, the scalar-fermion-gaugino interaction is replaced to

$$-\sqrt{2}g i\lambda_{\text{SLHA}}^g(\phi^* t^a \psi) - \sqrt{2}g(-i\bar{\lambda}_{\text{SLHA}}^g)(\bar{\psi} t^a \phi).$$

With an SU(N) gauge symmetry

$$\mathcal{L} = \Phi^* e^{2gV} \Phi \Big|_{\theta^4} + \left[\left(\frac{1}{2} - \frac{ig^2\Theta}{16\pi^2} \right) \text{Tr } \mathcal{W}^\alpha \mathcal{W}_\alpha \Big|_{\theta^2} + W(\Phi) \Big|_{\theta^2} + \text{H.c.} \right]; \quad (8.11)$$

$$\Phi^* e^{2gV} \Phi \Big|_{\theta^4} := \Phi_i^* \left[e^{2gV^a t_\Phi^a} \right]_{ij} \Phi_j \Big|_{\theta^4} \quad (8.12)$$

$$= (\partial_\mu \phi_i^*)(\partial^\mu \phi_i) + i\bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_i + F_i^* F_i - \sqrt{2}g\lambda^a(\phi^* t^a \psi) - \sqrt{2}g\bar{\lambda}^a(\bar{\psi} t^a \phi) \\ + gA_\mu^a \bar{\psi} \bar{\sigma}^\mu (t^a \psi) + 2igA_\mu^a \phi^* \partial_\mu (t^a \phi) + g^2 A^{a\mu} A_\mu^b (\phi^* t^a t^b \phi) + gD^a (\phi^* t^a \phi) \quad (8.13)$$

$$= D^\mu \phi^* D_\mu \phi + i\bar{\psi}_i \bar{\sigma}^\mu D_\mu \psi_i + F^* F - \sqrt{2}g\lambda^a(\phi^* t^a \psi) - \sqrt{2}g\bar{\lambda}^a(\bar{\psi} t^a \phi) + gD^a (\phi^* t^a \phi) \quad (8.14)$$

$$\left(\frac{1}{2} - \frac{ig^2\Theta}{16\pi^2} \right) \text{Tr } \mathcal{W}^\alpha \mathcal{W}_\alpha \Big|_{\theta^2} + \text{H.c.} = \text{Re Tr } \mathcal{W} \mathcal{W} \Big|_{\theta^2} + \frac{g^2\Theta}{8\pi^2} \text{Im Tr } \mathcal{W} \mathcal{W} \Big|_{\theta^2} \\ = i\lambda^a \sigma^\mu D_\mu \bar{\lambda}^a + \frac{1}{2} D^a D^a - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{g^2\Theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a; \quad (8.15)$$

$$D_\mu \phi_i = \partial_\mu \phi_i - igA_\mu^a t_{ij}^a \phi_j, \quad D_\mu \psi_i = \partial_\mu \psi_i - igA_\mu^a t_{ij}^a \psi_j, \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gA_\mu^b A_\nu^c f^{abc}, \\ D^\mu \phi_i^* = \partial^\mu \phi_i^* + igA^{a\mu} \phi_j^* t_{ji}^a, \quad D_\mu \lambda_\alpha^a = \partial_\mu \lambda_\alpha^a + gf^{abc} A_\mu^b \lambda_\alpha^c.$$

$$\{\phi, \psi, F\} \xrightarrow{\text{gauge}} e^{ig\theta^a t^a} \{\phi, \psi, F\}, \\ A_\mu^a \xrightarrow{\text{gauge}} A_\mu^a + \partial_\mu \theta^a + gf^{abc} A_\mu^b \theta^c + \mathcal{O}(\theta^2), \quad \lambda^a \xrightarrow{\text{gauge}} \lambda^a + gf^{abc} \lambda^b \theta^c + \mathcal{O}(\theta^2), \\ D^a \xrightarrow{\text{gauge}} D^a + gf^{abc} D^b \theta^c + \mathcal{O}(\theta^2), \quad \bar{\lambda}^a \xrightarrow{\text{gauge}} \bar{\lambda}^a + gf^{abc} \bar{\lambda}^b \theta^c + \mathcal{O}(\theta^2).^{*7}$$

Auxiliary fields and Scalar potential In all of the above three theories,

$$\mathcal{L} \supset F_i^* F_i + F_i \frac{\partial W}{\partial \Phi_i} \Big|_{\text{scalar}} + F_i^* \frac{\partial W^*}{\partial \Phi_i^*} \Big|_{\text{scalar}} + \frac{1}{2} D^a D^a + gD^a (\phi^* t^a \phi); \quad (8.16)$$

$$\langle F_i^* \rangle = - \frac{\partial W}{\partial \Phi_i} \Big|_{\text{scalar}}, \quad \langle D^a \rangle = -g\phi^* t^a \phi; \quad (8.17)$$

$$\mathcal{L} \supset -V_{\text{SUSY}} = - \left[\langle F_i^* \rangle \langle F_i \rangle + \frac{g^2}{2} (\phi^* t^a \phi) (\phi^* t^a \phi) \right]. \quad (8.18)$$

^{*7} ♣️ **TODO**: give in non-infinitesimal form ♣️

8.1. Lorentz symmetry as $SU(2) \times SU(2)$

8.2. Supersymmetry algebra

We define the generators as

$$P_\mu := i\partial_\mu, \quad \{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu = -2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu, \quad \{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = \{\bar{\mathcal{Q}}_{\dot{\alpha}}, \bar{\mathcal{Q}}_{\dot{\beta}}\} = 0, \quad (8.19)$$

which is realized by

$$\begin{aligned} \mathcal{Q}_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu, & \bar{\mathcal{Q}}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu, & \mathcal{Q}^\alpha &= -\frac{\partial}{\partial\theta_\alpha} - i(\bar{\theta}\bar{\sigma}^\mu)^\alpha\partial_\mu, & \bar{\mathcal{Q}}^{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu, \\ \mathcal{D}_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu, & \bar{\mathcal{D}}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu, & \mathcal{D}^\alpha &= -\frac{\partial}{\partial\theta_\alpha} + i(\bar{\theta}\bar{\sigma}^\mu)^\alpha\partial_\mu, & \bar{\mathcal{D}}^{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu; \end{aligned}$$

\mathcal{D}_α etc. works as covariant derivatives because of the commutation relations

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\} = +2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \quad \{\mathcal{Q}_\alpha, \mathcal{D}_\beta\} = \{\mathcal{Q}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} = \{\bar{\mathcal{Q}}_{\dot{\alpha}}, \mathcal{D}_\beta\} = \{\bar{\mathcal{Q}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0.$$

Derivative formulae

$$\begin{aligned} \epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} &= -\frac{\partial}{\partial\theta_\alpha} & \frac{\partial}{\partial\theta^\alpha}\theta\theta &= 2\theta_\alpha & \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta &= -2\delta_\alpha^\beta & \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}\bar{\theta}\bar{\theta} &= 2\delta_{\dot{\alpha}}^{\dot{\beta}} \\ \epsilon_{\alpha\beta}\frac{\partial}{\partial\theta^\beta} &= -\frac{\partial}{\partial\theta_\alpha} & \frac{\partial}{\partial\theta_\alpha}\theta\theta &= -2\theta^\alpha & \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta &= 2\epsilon^{\alpha\beta} & \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\beta}}}\bar{\theta}\bar{\theta} &= -2\epsilon^{\dot{\alpha}\dot{\beta}} \\ \epsilon^{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} &= -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} & \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\bar{\theta}\bar{\theta} &= 2\bar{\theta}^{\dot{\alpha}} & \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\beta}}}\bar{\theta}\bar{\theta} &= 2\delta_{\dot{\alpha}}^{\dot{\beta}} & \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\beta}}}\bar{\theta}\bar{\theta} &= -2\delta_{\dot{\alpha}}^{\dot{\beta}} \\ \epsilon_{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} &= -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} & \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\bar{\theta}\bar{\theta} &= -2\bar{\theta}^{\dot{\alpha}} & \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}\bar{\theta}\bar{\theta} &= -2\epsilon_{\dot{\alpha}\dot{\beta}} & \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}\bar{\theta}\bar{\theta} &= 2\epsilon_{\dot{\alpha}\dot{\beta}} \end{aligned}$$

In addition, we define

$$(y, \theta', \bar{\theta}') := (x - i\theta\sigma^\mu\bar{\theta}, \theta, \bar{\theta}) : \quad (8.20)$$

$$\bar{\mathcal{D}}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}'^{\dot{\alpha}}}; \quad \begin{pmatrix} \frac{\partial}{\partial\bar{\theta}'^{\dot{\alpha}}} \\ \frac{\partial}{\partial\theta'^\alpha} \end{pmatrix} = \begin{pmatrix} \delta_{\dot{\alpha}}^\nu & 0 & 0 \\ -i(\sigma^\nu\bar{\theta})_{\dot{\alpha}} & \delta_{\dot{\alpha}}^\alpha & 0 \\ i(\theta\sigma^\nu)_{\dot{\alpha}} & 0 & \delta_{\dot{\alpha}}^\alpha \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial\bar{\theta}^{\dot{\nu}}} \\ \frac{\partial}{\partial\theta'^\alpha} \\ \frac{\partial}{\partial\theta'^\beta} \end{pmatrix}, \quad \begin{pmatrix} \frac{\partial}{\partial\bar{\theta}'^{\dot{\alpha}}} \\ \frac{\partial}{\partial\theta'^\alpha} \end{pmatrix} = \begin{pmatrix} \delta_{\dot{\alpha}}^\mu & 0 & 0 \\ i(\sigma^\mu\bar{\theta})_{\dot{\alpha}} & \delta_{\dot{\alpha}}^\alpha & 0 \\ -i(\theta\sigma^\mu)_{\dot{\alpha}} & 0 & \delta_{\dot{\alpha}}^\alpha \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial\bar{\theta}^{\dot{\mu}}} \\ \frac{\partial}{\partial\theta'^\alpha} \\ \frac{\partial}{\partial\theta'^\beta} \end{pmatrix}, \quad (8.21)$$

and a function $f : \mathbb{C}^4 \rightarrow \mathbb{C}$ (independent of θ' and $\bar{\theta}'$) is expanded as

$$f(y) = f(x - i\theta\sigma\bar{\theta}) = f(x) - i(\theta\sigma^\mu\bar{\theta})\partial_\mu f(x) - \frac{1}{4}\theta^4\partial^2 f(x). \quad (8.22)$$

Note that we differentiate $[f(y)]^*$ and $f^*(y)$:

$$[f(y)]^* = f(x) + i(\theta\sigma^\mu\bar{\theta})\partial_\mu f^*(x) - \frac{1}{4}\theta^4\partial^2 f^*(x) = f^*(y + i\theta\sigma\bar{\theta}) = f^*(y^*). \quad (8.23)$$

8.3. Superfields

SUSY-invariant Lagrangian SUSY transformation is induced by $\xi Q + \bar{\xi}\bar{Q} = \xi^\alpha\partial_\alpha + \bar{\xi}_{\dot{\alpha}}\partial^{\dot{\alpha}} + i(\xi\sigma^\mu\bar{\theta} + \bar{\xi}\bar{\sigma}^\mu\theta)\partial_\mu$. Therefore, for an object Ψ in the superspace,

$$[\Psi]_{\theta^4} \xrightarrow{\text{SUSY}} [\Psi + \xi^\alpha\partial_\alpha\Psi + \bar{\xi}_{\dot{\alpha}}\partial^{\dot{\alpha}}\Psi + i(\xi\sigma^\mu\bar{\theta} + \bar{\xi}\bar{\sigma}^\mu\theta)\partial_\mu\Psi]_{\theta^4} = [\Psi + i(\xi\sigma^\mu\bar{\theta} + \bar{\xi}\bar{\sigma}^\mu\theta)\partial_\mu\Psi]_{\theta^4}, \quad (8.24)$$

which means $[\Psi]_{\theta^4}$ is SUSY-invariant up to total derivative, i.e., $\int d^4x [\Psi]_{\theta^4}$ is SUSY-invariant action. Also,

$$[\Psi]_{\theta^2} \xrightarrow{\text{SUSY}} [\Psi + \bar{\xi}_{\dot{\alpha}}(\partial^{\dot{\alpha}} + i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu)\Psi]_{\theta^2} = [\Psi + \bar{\xi}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}}\Psi + 2i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu\Psi]_{\theta^2} \quad (8.25)$$

will be SUSY-invariant if $\bar{\mathcal{D}}_{\dot{\alpha}}\Psi = 0$, i.e., Ψ is a chiral superfield. Therefore, SUSY-invariant Lagrangian is given by

$$\mathcal{L} = [(\text{any real superfield})]_{\theta^4} + [(\text{any chiral superfield})]_{\theta^2} + [(\text{any chiral superfield})^*]_{\bar{\theta}^2}. \quad (8.26)$$

Chiral superfield A chiral superfield is a superfield that satisfies $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$, i.e., we find

$$\Phi = \phi(y) + \sqrt{2}\theta'\psi(y) + \theta'^2 F(y) \quad (8.27)$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) - i\partial_\mu\phi(x)(\theta\sigma^\mu\bar{\theta}) + F(x)\theta^2 + \frac{i}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar{\theta})\theta^2 - \frac{1}{4}\partial^2\phi(x)\theta^4 \quad (8.28)$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar{\theta}\bar{\psi}(x) + F^*(x)\bar{\theta}^2 + i\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar{\theta}) - \frac{i}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar{\psi}(x)]\bar{\theta}^2 - \frac{1}{4}\partial^2\phi^*(x)\theta^4; \quad (8.29)$$

their product is expanded as

$$\begin{aligned}\Phi_i^* \Phi_j &= \phi_i^* \phi_j + \sqrt{2} \phi_i^* (\theta \psi_j) + \sqrt{2} (\bar{\psi}_i \bar{\theta}) \phi_j + \phi_i^* F_j \theta^2 + 2(\bar{\psi}_i \bar{\theta})(\theta \psi_j) - i(\phi_i^* \partial_\mu \phi_j - \partial_\mu \phi_i^* \phi_j)(\theta \sigma^\mu \bar{\theta}) + F_i^* \phi_j \bar{\theta}^2 \\ &+ \left[\sqrt{2} \bar{\psi}_i \bar{\theta} F_j - \frac{i(\partial_\mu \phi_i^* \cdot \psi_j \sigma^\mu \bar{\theta} - \phi_i^* \partial_\mu \psi_j \sigma^\mu \bar{\theta})}{\sqrt{2}} \right] \theta^2 + \left[\sqrt{2} F_i^* \theta \psi_j + \frac{i(\theta \sigma^\mu \bar{\psi}_i \partial_\mu \phi_j - \theta \sigma^\mu \partial_\mu \bar{\psi}_i \phi_j)}{\sqrt{2}} \right] \bar{\theta}^2 \\ &+ \frac{1}{4} (4F_i^* F_j - \phi_i^* \partial^2 \phi_j - (\partial^2 \phi_i^*) \phi_j + 2(\partial_\mu \phi_i^*)(\partial^\mu \phi_j) + 2i(\psi_j \sigma^\mu \partial_\mu \bar{\psi}_i) - 2i(\partial_\mu \psi_j \sigma^\mu \bar{\psi}_i)) \theta^4\end{aligned}\quad (8.30)$$

$$\begin{aligned}&\equiv \phi_i^* \phi_j + \sqrt{2} \phi_i^* (\theta \psi_j) + \sqrt{2} (\bar{\psi}_i \bar{\theta}) \phi_j + \phi_i^* F_j \theta^2 + 2(\bar{\psi}_i \bar{\theta})(\theta \psi_j) - 2i(\phi_i^* \partial_\mu \phi_j)(\theta \sigma^\mu \bar{\theta}) + F_i^* \phi_j \bar{\theta}^2 \\ &+ \sqrt{2} (\bar{\psi}_i \bar{\theta} F_j + i\phi_i^* \partial_\mu \psi_j \sigma^\mu \bar{\theta}) \theta^2 + \sqrt{2} (F_i^* \theta \psi_j - i\theta \sigma^\mu \partial_\mu \bar{\psi}_i \phi_j) \bar{\theta}^2 \\ &+ (F_i^* F_j + (\partial_\mu \phi_i^*)(\partial^\mu \phi_j) + i\bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_j) \theta^4\end{aligned}\quad (8.31)$$

$$\Phi_i \Phi_j \Big|_{\theta^2} = -\psi_i \psi_j + F_i \phi_j + \phi_i F_j \quad (8.32)$$

$$\Phi_i \Phi_j \Phi_k \Big|_{\theta^2} = -(\psi_i \psi_j) \phi_k - (\psi_k \psi_i) \phi_j - (\psi_j \psi_k) \phi_i + \phi_i \phi_j F_k + \phi_k \phi_i F_j + \phi_j \phi_k F_i \quad (8.33)$$

$$e^{k\Phi} = e^{k\phi} \left[1 + \sqrt{2} k \theta \psi + \left(kF - \frac{k^2}{2} \psi \psi \right) \theta^2 - i k \partial_\mu \phi (\theta \sigma^\mu \bar{\theta}) + \frac{i k (\partial_\mu \psi + k \psi \partial_\mu \phi) \sigma^\mu \bar{\theta} \theta^2}{\sqrt{2}} - \frac{k}{4} (\partial^2 \phi + k \partial_\mu \phi \partial^\mu \phi) \theta^4 \right]; \quad (8.34)$$

note that $\Phi_i \Phi_j$, $\Phi_i \Phi_j \Phi_k$, and $e^{k\Phi}$ are all chiral superfields.

Vector superfield A vector superfield is a superfield V that satisfies $V = V^*$. It is given by real fields $\{C, M, N, D, A_\mu\}$ and Grassmann fields $\{\chi, \lambda\}$ as^{*8}

$$\begin{aligned}V(x, \theta, \bar{\theta}) &= C(x) + i\theta \chi(x) - i\bar{\theta} \bar{\chi}(x) + \frac{1}{2} (M(x) + iN(x)) \theta^2 + \frac{1}{2} (M(x) - iN(x)) \bar{\theta}^2 + (\bar{\theta} \bar{\sigma}^\mu \theta) A_\mu(x) \\ &\quad \left(\lambda(x) + \frac{1}{2} \partial_\mu \bar{\chi}(x) \sigma^\mu \right) \theta \bar{\theta}^2 + \theta^2 \bar{\theta} \left(\bar{\lambda}(x) + \frac{1}{2} \bar{\sigma}^\mu \partial_\mu \chi(x) \right) + \frac{1}{2} \left(D(x) - \frac{1}{2} \partial^2 C(x) \right) \theta^4.\end{aligned}\quad (8.35)$$

With this convention,

$$V \rightarrow V - i\Phi + i\Phi^* \iff \begin{cases} C \rightarrow C - i\phi + i\phi^*, & \chi \rightarrow \chi - \sqrt{2}\psi, & \lambda \rightarrow \lambda, \\ M + iN \rightarrow M + iN - 2iF, & A_\mu \rightarrow A_\mu + \partial_\mu(\phi + \phi^*), & D \rightarrow D. \end{cases} \quad (8.36)$$

The exponential of a vector superfield is also a vector superfield:

$$\begin{aligned}e^{kV} &= e^{kC} \left\{ 1 + ik(\theta \chi - \bar{\theta} \bar{\chi}) + \left(\frac{M + iN}{2} k + \frac{\chi \chi}{4} k^2 \right) \theta^2 + \left(\frac{M - iN}{2} k + \frac{\bar{\chi} \bar{\chi}}{4} k^2 \right) \bar{\theta}^2 + (k^2 \theta \chi \bar{\theta} \bar{\chi} - k \theta \sigma^\mu \bar{\theta} A_\mu) \right. \\ &\quad + \left[k \bar{\theta} \bar{\lambda} - i k \bar{\theta} \bar{\chi} \left(\frac{M + iN}{2} k + \frac{\chi \chi}{4} k^2 \right) + \frac{1}{2} k \bar{\theta} \bar{\sigma}^\mu (\partial_\mu \chi - i k \chi A_\mu) \right] \theta^2 \\ &\quad + \left[k \theta \lambda + i k \theta \chi \left(\frac{M - iN}{2} k + \frac{\bar{\chi} \bar{\chi}}{4} k^2 \right) - \frac{1}{2} k \theta \sigma^\mu (\partial_\mu \bar{\chi} + i k \bar{\chi} A_\mu) \right] \bar{\theta}^2 \\ &\quad + \left[\frac{k}{2} \left(D - \frac{1}{2} \partial^2 C \right) - \frac{1}{2} i k^2 (\lambda \chi - \bar{\lambda} \bar{\chi}) + \left(\frac{M + iN}{2} k + \frac{\chi \chi}{4} k^2 \right) \left(\frac{M - iN}{2} k + \frac{\bar{\chi} \bar{\chi}}{4} k^2 \right) \right. \\ &\quad \left. \left. + \frac{k^3}{4} \bar{\chi} \bar{\sigma}^\mu \chi A_\mu + \frac{k^2}{4} (i \bar{\chi} \bar{\sigma}^\mu \partial_\mu \chi - i \partial_\mu \bar{\chi} \bar{\sigma}^\mu \chi + A^\mu A_\mu) \right] \theta^4 \right\}.\end{aligned}\quad (8.37)$$

Supergauge symmetry The gauge transformation $\phi(x) \rightarrow e^{ig\theta^a(x)t^a} \phi(x)$ is not closed in the chiral superfield; i.e., $e^{ig\theta^a(x)t^a} \Phi(x)$ is not a chiral superfield if the parameter $\theta(x)$ has x^μ -dependence. Hence, in supersymmetric theories, it is extended to *supergauge symmetry* parameterized by a chiral superfield $\Omega(x)$, which is given by

$$\Phi \rightarrow e^{2ig\Omega^a(x)t^a} \Phi, \quad \Phi^* \rightarrow \Phi^* e^{-2ig\Omega^{*a}(x)t^a} \quad (8.38)$$

for a chiral superfield Φ and an anti-chiral superfield Φ^* . The supergauge-invariant Lagrangian should be

$$\mathcal{L} \sim \Phi^* \cdot (\text{real superfield}) \cdot \Phi; \quad (8.39)$$

we parameterize the “real superfield” as $e^{2gV^a(x)t^a}$:

$$\mathcal{L} = \left[\Phi^* e^{2gV^a(x)t^a} \Phi \right]_{\theta^4}; \quad e^{2gV^a(x)t^a} \rightarrow e^{2ig\Omega^{*a}(x)t^a} e^{2gV^a(x)t^a} e^{-2ig\Omega^a(x)t^a}. \quad (8.40)$$

^{*8}Different coordination of “i”s are found in literature. Take care, especially, $\lambda(\text{ours}) = i\lambda(\text{Wess-Bagger}) = i\lambda(\text{SLHA})$.

In Abelian case, t^a is replaced by the charge Q of Φ and

$$\mathcal{L} = \left[\Phi^* e^{2gQV(x)} \Phi \right]_{\theta^4}; \quad \Phi \rightarrow e^{2igQ\Omega(x)} \Phi, \quad \Phi^* \rightarrow \Phi^* e^{-2igQ\Omega^*(x)}, \quad (8.41)$$

$$e^{2gQV(x)} \rightarrow e^{2igQ\Omega^*(x)} e^{2gQV(x)} e^{-2igQ\Omega(x)} = e^{2gQ(V-i\Omega+i\Omega^*)}. \quad (8.42)$$

The usual gauge transformation corresponds to the real part of the lowest component of Ω , i.e., $\theta \equiv 2 \operatorname{Re} \phi = \phi + \phi^*$, and we use the other components to fix the supergauge so that C , M , N and χ are eliminated:

$$\text{supergauge fixing: } V(x) \rightarrow (\bar{\theta}\bar{\sigma}^\mu\theta)A_\mu(x) + \bar{\theta}^2\theta\lambda(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \frac{1}{2}D(x) \quad (\text{Wess-Zumino gauge}); \quad (8.43)$$

$$e^{2gQV} \rightarrow 1 + gQ(-2\theta\sigma^\mu\bar{\theta}A_\mu + 2\theta^2\bar{\theta}\bar{\lambda} + 2\bar{\theta}^2\theta\lambda + D\theta^4) + g^2Q^2A^\mu A_\mu\theta^4. \quad (8.44)$$

The gauge transformation is the remnant freedom: $\Theta = \phi(y) = \phi - i\partial_\mu\phi(\theta\sigma^\mu\bar{\theta}) - \partial^2\phi\theta^4/4$ with ϕ being real;

$$\Phi_i \rightarrow e^{2igQ\Theta}\Phi_i, \quad e^{2gQV} \rightarrow e^{2gQ(V-i\Theta+i\Theta^*)}. \quad (8.45)$$

Rules for each component is obvious in $(y, \theta, \bar{\theta})$ -basis and given by

$$\{\phi, \psi, F\} \rightarrow e^{igQ\Theta}\{\phi, \psi, F\}, \quad A_\mu \rightarrow A_\mu + \partial_\mu\theta, \quad \lambda \rightarrow \lambda, \quad D \rightarrow D. \quad (8.46)$$

For non-Abelian gauges, the supergauge transformation for the real field is evaluated as

$$e^{2gV} \rightarrow e^{2ig\Omega^*} e^{2gV} e^{-2ig\Omega} \quad (8.47)$$

$$= \left(e^{2ig\Omega^*} e^{2gV} e^{-2ig\Omega^*} \right) \left(e^{2ig\Omega^*} e^{-2ig\Omega} \right) \quad (8.48)$$

$$= \exp \left(e^{[2ig\Omega^*, 2gV]} e^{2ig(\Omega^* - \Omega)} + \mathcal{O}(\Omega^2) \right) \quad (8.49)$$

$$= \exp \left(2gV + [2ig\Omega^*, 2gV] \right) e^{2ig(\Omega^* - \Omega)} + \mathcal{O}(\Omega^2); \quad (8.50)$$

$$= \exp \left[2gV + [2ig\Omega^*, 2gV] + \int_0^1 dt g(e^{[2gV, \cdot]} 2ig(\Omega^* - \Omega)) + \mathcal{O}(\Omega^2) \right] \quad (8.51)$$

$$= \exp \left[2gV + [2ig\Omega^*, 2gV] + \sum_{n=0}^{\infty} \frac{B_n([2gV, \cdot]^n)}{n!} 2ig(\Omega^* - \Omega) \right] + \mathcal{O}(\Omega^2) \quad (8.52)$$

$$= \exp \left[2g \left(V + i(\Omega^* - \Omega) - [V, ig(\Omega^* + \Omega)] + \sum_{n=2}^{\infty} \frac{iB_n([2gV, \cdot]^n)}{n!} (\Omega^* - \Omega) \right) + \mathcal{O}(\Omega^2) \right]. \quad (8.53)$$

Here, again we can use the “non-gauge” component of Ω to eliminate the C -term etc., i.e., we fix $i(\Omega^* - \Omega)$, the second term of the expansion, to remove those terms:

$$V - [V, ig(\Omega^* + \Omega)] + \left(i + \sum_{n=2}^{\infty} \frac{iB_n([2gV, \cdot]^n)}{n!} \right) (\Omega^* - \Omega) + \mathcal{O}(\Omega^2) = (\bar{\theta}\bar{\sigma}^\mu\theta)A_\mu + \bar{\theta}^2\theta\lambda + \theta^2\bar{\theta}\bar{\lambda} + \frac{1}{2}D; \quad (8.54)$$

this defines the Wess-Zumino gauge:

$$\text{supergauge fixing: } V^a(x) \rightarrow (\bar{\theta}\bar{\sigma}^\mu\theta)A_\mu^a(x) + \bar{\theta}^2\theta\lambda^a(x) + \theta^2\bar{\theta}\bar{\lambda}^a(x) + \frac{1}{2}D^a(x), \quad (8.55)$$

$$e^{2gV^a t^a} \rightarrow 1 + g(-2\theta\sigma^\mu\bar{\theta}A_\mu^a + 2\theta^2\bar{\theta}\bar{\lambda}^a + 2\bar{\theta}^2\theta\lambda^a + D^a\theta^4) t^a + g^2 A^{a\mu} A_\mu^b \theta^4 t^a t^b. \quad (8.56)$$

The gauge transformation is given by

$$\Phi \rightarrow e^{2ig\Theta^a t^a} \Phi, \quad e^{2gV^a t^a} \rightarrow e^{2ig\Theta^b t^b} e^{2gV^a t^a} e^{-2ig\Theta^c t^c}. \quad (8.57)$$

For components in chiral superfields,

$$\{\phi, \psi, F\} \rightarrow e^{ig\Theta^a t^a} \{\phi, \psi, F\}, \quad (8.58)$$

while for vector superfield we can express as infinitesimal transformation:

$$V \rightarrow V' \simeq V + i(\Theta^* - \Theta) - [V, ig(\Theta^* + \Theta)] + \sum_{n=2}^{\infty} \frac{iB_n([2gV, \cdot]^n)}{n!} (\Theta^* - \Theta) \quad (8.59)$$

$$= V + 2(\bar{\theta}\bar{\sigma}^\mu\theta)\partial_\mu\phi - \left[V, ig \left(2\phi - \frac{\theta^4}{2}\partial^2\phi \right) \right] + 2 \sum_{n=2}^{\infty} \frac{B_n([2gV, \cdot]^n)}{n!} (\bar{\theta}\bar{\sigma}^\mu\theta)\partial_\mu\phi \quad (8.60)$$

$$= V + 2(\bar{\theta}\bar{\sigma}^\mu\theta)\partial_\mu\phi + 2gf^{abc}V^b\phi^c t^a \quad (\text{Wess-Zumino gauge}) \quad (8.61)$$

$$\begin{aligned} \therefore A_\mu^a &\rightarrow A_\mu^a + \partial_\mu\theta^a + gf^{abc}A_\mu^b\theta^c + \mathcal{O}(\theta^2), & \lambda^a &\rightarrow \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2), \\ D^a &\rightarrow D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), & \bar{\lambda}^a &\rightarrow \bar{\lambda}^a + gf^{abc}\bar{\lambda}^b\theta^c + \mathcal{O}(\theta^2). \end{aligned} \quad (8.62)$$

Gauge-field strength The real superfield e^V is gauge-invariant in Abelian case and a candidate in Lagrangian term, but this is not case in non-Abelian case. We thus define a chiral superfield from e^V :

$$\mathcal{W}_\alpha = \frac{1}{4} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} \left(e^{-2gV} \mathcal{D}_\alpha e^{2gV} \right); \quad (8.63)$$

$$\mathcal{W}_\alpha \xrightarrow{\text{gauge}} e^{2ig\Omega} \mathcal{W}_\alpha e^{-2ig\Omega} \quad \left(\mathcal{W}_\alpha^a \xrightarrow{\text{gauge}} [e^{+2g\tilde{f}^c \Omega^c}]^{ab} W_\alpha^b \quad \text{with} \quad [\tilde{f}^c]_{ab} = f^{abc} \right);^{*9} \quad (8.64)$$

it is not supergauge- or Lorentz-invariant, but $\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) = \text{Tr}(\epsilon^{\alpha\beta} \mathcal{W}_\beta \mathcal{W}_\alpha)$ is supergauge- and Lorentz-invariant, and its θ^2 -term is SUSY-invariant, which becomes a candidate in SUSY Lagrangian with its Hermitian conjugate.

In Wess-Zumino gauge, it is given by

$$\mathcal{W}_\alpha = \left\{ \lambda_\alpha^a(y) + \theta_\alpha D^a(y) + \frac{[i(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu) \theta]_\alpha}{4} F_{\mu\nu}^a(y) + \theta^2 [i\sigma^\mu D_\mu \bar{\lambda}^a(y^*)]_\alpha \right\} t^a \quad (8.65)$$

$$= \left[\lambda_\alpha^a + \theta_\alpha D^a + \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha F_{\mu\nu}^a + i\theta^2 (\sigma^\mu D_\mu \bar{\lambda}^a)_\alpha + i(\bar{\theta} \bar{\sigma}^\mu \theta) \partial_\mu \lambda_\alpha^a - \frac{\theta^4}{4} \partial^2 \lambda_\alpha^a \right. \\ \left. + \frac{i\theta^2 (\sigma^\mu \bar{\theta})_\alpha}{2} \left(\partial_\mu D^a + i\partial^\nu F_{\mu\nu}^a - g f^{abc} \epsilon_{\mu\nu\rho\sigma} A^{\nu b} \partial^\rho A^{\sigma c} \right) \right] T^a, \quad (8.66)$$

where, as usual,

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g A_\mu^b A_\nu^c f^{abc}, \quad D_\mu \lambda_\alpha^a = \partial_\mu \lambda_\alpha^a + g f^{abc} A_\mu^b \lambda_\alpha^c. \quad (8.67)$$

Also,

$$[\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)]_{\theta^2} = \left[i\lambda^a \sigma^\mu D_\mu \bar{\lambda}^b + i\lambda^b \sigma^\mu D_\mu \bar{\lambda}^a + D^a D^b - \frac{1}{4} (i\epsilon^{\sigma\mu\nu\rho} + 2\eta^{\mu\rho} \eta^{\nu\sigma}) F_{\mu\nu}^a F_{\rho\sigma}^b \right] \text{Tr}(t^a t^b) \quad (8.68)$$

$$= i\lambda^a \sigma^\mu D_\mu \bar{\lambda}^a + \frac{1}{2} D^a D^a - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{i}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a, \quad (8.69)$$

$$[\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)]_{\theta^4} = \frac{\theta^4}{4} \left(2(\partial^\mu \lambda^a)(\partial_\mu \lambda^b) - \lambda^a \partial^2 \lambda^b - (\partial^2 \lambda^a) \lambda^b \right) \text{Tr}(t^a t^b) = \frac{\theta^4}{4} ((\partial^\mu \lambda^a)(\partial_\mu \lambda^a) - \lambda^a \partial^2 \lambda^a). \quad (8.70)$$

For Abelian theory,

$$\mathcal{W}_\alpha = \frac{1}{4} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} \left(e^{-2gV} \mathcal{D}_\alpha e^{2gV} \right) = \frac{1}{4} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} \mathcal{D}_\alpha (2gV), \quad (8.71)$$

$$\mathcal{W}^\alpha \mathcal{W}_\alpha \Big|_{\theta^2} = 2 \left(i\lambda \sigma^\mu D_\mu \bar{\lambda} + \frac{1}{2} D D - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right). \quad (8.72)$$

8.4. Lagrangian blocks

Lagrangian construction The supergauge transformation is summarized as

$$\Phi_i \rightarrow [U_\Phi]_{ij} \Phi_j, \quad \tilde{\Phi}_j \rightarrow \tilde{\Phi}_i [U_\Phi^{-1}]_{ij}, \quad \mathcal{W}_\alpha \rightarrow U_\mathcal{W} \mathcal{W}_\alpha U_\mathcal{W}^{-1}, \quad (8.73)$$

where

$$\tilde{\Phi}_j^* := \Phi_i^* [e^{2gV t_\Phi^a}]_{ij}, \quad U_\Phi := \exp(2ig\Omega^a t_\Phi^a), \quad U_\mathcal{W} := \exp(2ig\Omega^a t_\mathcal{W}^a), \quad (8.74)$$

t_Φ^a is the representation matrix or U(1) charge for the field Φ , and $t_\mathcal{W}^a$ is the representation matrix that is used to define \mathcal{W}_α . To construct a Lagrangian, we should composite these ingredients in real and invariant under SUSY, supergauge, and Lorentz transformation. A sufficient condition for SUSY invariance is given by (8.26), so

$$\mathcal{L} = \left[K(\Phi_i, \tilde{\Phi}_j^*) \right]_{\theta^4} + \left\{ \left[f_{ab}(\Phi_i) \mathcal{W}^a \mathcal{W}^b \right]_{\theta^2} + \text{H.c.} \right\} + \left\{ \left[W(\Phi_i) \right]_{\theta^2} + \text{H.c.} \right\} + D \quad (8.75)$$

is one possible construction. The Kähler function K should be real and supergauge invariant, the gauge kinetic function f should be holomorphic and supergauge invariant with $\mathcal{W}^a \mathcal{W}^b$, and the superpotential W is holomorphic and supergauge invariant. The last term D (Fayet-Iliopoulos term) comes from V of an U(1) gauge boson; note that its supergauge invariance is due to the intentional definition of V .

One can construct more general Lagrangian; for example, one can introduce a vector superfield that is not associated to a gauge symmetry, but then the supergauge fixing is not available and one has to include C or M fields.

Renormalizable Lagrangian Since $[\Phi]_{\theta^4}$ is a total derivative, renormalizable Lagrangian is limited to

$$\mathcal{L} = \left[\Phi_i^* [e^{2gV t_\Phi^a}]_{ij} \Phi_j \right]_{\theta^4} + \left\{ [\mathcal{W}^a \mathcal{W}^a]_{\theta^2} + [W(\Phi_i)]_{\theta^2} + \text{H.c.} \right\} + D \quad (8.76)$$

up to numeric coefficients. With multiple gauge groups, the Kähler part is extended as $\Phi_i^* [e^{2gV t_\Phi^a} e^{2gV' t_\Phi'^a} \dots]_{ij} \Phi_j$, where the inner part is obviously commutable.

^{*9} ♣ **TODO:** This equivalence should be checked/explained in gauge-theory section; especially, the sign is not verified and might be opposite. ♣

9. Minimal Supersymmetric Standard Model

Gauge symmetry: $SU(3)_{\text{color}} \times SU(2)_{\text{weak}} \times U(1)_Y$

Particle content:

(a) Chiral superfields						(b) Vector superfields			
	SU(3)	SU(2)	U(1)	B	L	scalar/spinor	SU(3)	SU(2)	U(1) ino/boson
Q_i	3	2	1/6	1/3		$\tilde{q}_L, q_L \rightarrow (u_L, d_L)$	g	adj.	\tilde{g}, g_μ
L_i		2	-1/2		1	$\tilde{l}_L, l_L \rightarrow (\nu_L, l_L)$	W		\tilde{w}, W_μ
U_i^c	$\bar{\mathbf{3}}$		-2/3	-1/3		\tilde{u}_R^c, u_R^c	B	adj.	\tilde{b}, B_μ
D_i^c	$\bar{\mathbf{3}}$		1/3	-1/3		\tilde{d}_R^c, d_R^c			
E_i^c			1		-1	\tilde{e}_R^c, e_R^c			
H_u		2	1/2			$h_u, \tilde{h}_u \rightarrow (h_u^+, h_u^0)$			
H_d		2	-1/2			$h_d, \tilde{h}_d \rightarrow (h_d^0, h_d^-)$			

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and SU(2) decomposition).

“c”-notation For scalars, $\tilde{\phi}_R^c := \phi_R^* = C\phi_R C$ (because the intrinsic phase for C is +1 for quarks and leptons.)

For matter spinors, $\psi_R^c := \bar{\psi}_R$ (and $\psi_R = \bar{\psi}_R^c$); Dirac spinors are thus

$$\psi_L = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \bar{\psi}_L = (0 \quad \bar{\psi}_L), \quad \psi_R^c := \begin{pmatrix} \psi_R^c \\ 0 \end{pmatrix} = C \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} = C\psi_R, \quad \bar{\psi}_R^c = (0 \quad \psi_R) = (\bar{\psi}_R \quad 0) C = \bar{\psi}_R C.$$

Superpotential and SUSY-terms

$$W_{\text{RPC}} = \mu H_u H_d - y_{uij} U_i^c H_u Q_j + y_{dij} D_i^c H_d Q_j + y_{eij} E_i^c H_d L_j, \quad (9.1)$$

$$W_{\text{RPV}} = -\kappa_i L_i H_u + \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2} \lambda''_{ijk} U_i^c D_j^c D_k^c, \quad (9.2)$$

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{2} \left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w} \tilde{w} + M_1 \tilde{b} \tilde{b} + \text{H.c.} \right) - V_{\text{SUSY}}, \quad (9.3)$$

$$\begin{aligned} V_{\text{SUSY}}^{\text{RPC}} = & \left(\tilde{q}_L^* m_Q^2 \tilde{q}_L + \tilde{l}_L^* m_L^2 \tilde{l}_L + \tilde{u}_R^* m_{U^c}^2 \tilde{u}_R + \tilde{d}_R^* m_{D^c}^2 \tilde{d}_R + \tilde{e}_R^* m_{E^c}^2 \tilde{e}_R + m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 \right) \\ & + \left(-\tilde{u}_R^* h_u a_u \tilde{q}_L + \tilde{d}_R^* h_d a_d \tilde{q}_L + \tilde{e}_R^* h_d a_e \tilde{l}_L + b H_u H_d + \text{H.c.} \right) \\ & + \left(+\tilde{u}_R^* h_u^* c_u \tilde{q}_L + \tilde{d}_R^* h_u^* c_d \tilde{q}_L + \tilde{e}_R^* h_u^* c_e \tilde{l}_L + \text{H.c.} \right), \end{aligned} \quad (9.4)$$

$$\begin{aligned} V_{\text{SUSY}}^{\text{RPV}} = & \left(-b_i \tilde{l}_{Li} H_u + \frac{1}{2} T_{ijk} \tilde{l}_{Li} \tilde{l}_{Lj} \tilde{e}_{Rk}^* + T'_{ijk} \tilde{l}_{Li} \tilde{q}_{Lj} \tilde{d}_{Rk}^* + \frac{1}{2} T''_{ijk} \tilde{u}_{Ri}^* \tilde{d}_{Rj}^* \tilde{d}_{Rk}^* + \tilde{l}_{Li}^* M_{Li}^2 H_d + \text{H.c.} \right) \\ & + \left(C_{ijk}^1 \tilde{l}_{Li}^* \tilde{q}_{Lj} \tilde{u}_{Rk}^* + C_i^2 h_u^* h_d \tilde{e}_{Ri}^* + C_{ijk}^3 \tilde{d}_{Ri} \tilde{u}_{Rj}^* \tilde{e}_{Rk}^* + \frac{1}{2} C_{ijk}^4 \tilde{d}_{Ri} \tilde{q}_{Lj} \tilde{q}_{Lk} + \text{H.c.} \right), \end{aligned} \quad (9.5)$$

$$(\lambda_{ijk} = -\lambda_{jik}, \lambda''_{ijk} = -\lambda''_{ikj}, \text{ and } C_{ijk}^4 = C_{ikj}^4.)$$

9.1. Notation

Our notation in this section (and the previous section) follows DHM [8, PhysRept] and Martin [9, v7] (but note that Martin uses $(-, +, +, +)$ -metric for RPC part and SLHA2 convention for RPV part. In particular, the sign of gauge bosons are fixed by $D_\mu \phi = \partial_\mu \phi - ig A_\mu^a t_{ij}^a \phi_j$, and the phase of gauginos are by $\mathcal{L} \ni \sqrt{2}g(\phi^* t^a \psi \lambda^a)$. Phases of ϕ and ψ in chiral superfields are not yet specified; they are later used to remove $F\tilde{F}$ terms and diagonalize Yukawa matrices.

9.2. Lagrangian construction

The most generic form of the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{super}} + \mathcal{L}_{\text{FI}} + \mathcal{L}_{\text{SUSY}}, \quad (9.6)$$

$$\mathcal{L}_{\text{matter}} = \Phi_Q^* \exp(2g_Y(\frac{1}{6})V_B + 2g_2 V_W^a T^a + 2g_3 V_g^a \tau^a) \Phi_Q \Big|_{\theta^4} + \dots; \quad (9.7)$$

$$\mathcal{L}_{\text{gauge}} = \left[\frac{1}{4} \left(1 - \frac{ig_Y^2 \Theta_B}{8\pi^2} \right) \mathcal{W}_B \mathcal{W}_B + \frac{1}{4} \left(1 - \frac{ig_2^2 \Theta_W}{8\pi^2} \right) \mathcal{W}_W^a \mathcal{W}_W^a + \frac{1}{4} \left(1 - \frac{ig_3^2 \Theta_g}{8\pi^2} \right) \mathcal{W}_g^a \mathcal{W}_g^a \right]_{\theta^2} + \text{H.c.}; \quad (9.8)$$

$$\mathcal{L}_{\text{super}} = W(\Phi) \Big|_{\theta^2} + \text{H.c.}, \quad (9.9)$$

$$W(\Phi) = W_{\text{RPC}} + W_{\text{RPV}}, \quad (9.10)$$

$$W_{\text{RPC}} = \mu H_u H_d - y_{uij} U_i^c H_u Q_j + y_{dij} D_i^c H_d Q_j + y_{eij} E_i^c H_d L_j, \quad (9.11)$$

$$W_{\text{RPV}} = -\kappa_i L_i H_u + \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2} \lambda''_{ijk} U_i^c D_j^c D_k^c; \quad (9.12)$$

$$\mathcal{L}_{\text{FI}} = \Lambda_{\text{FI}} D_B; \quad (9.13)$$

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{2} \left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w} \tilde{w} + M_1 \tilde{b} \tilde{b} + \text{H.c.} \right) - \left(V_{\text{SUSY}}^{\text{RPC}} + V_{\text{SUSY}}^{\text{RPV}} \right), \quad (9.14)$$

$$\begin{aligned} V_{\text{SUSY}}^{\text{RPC}} = & \left(\tilde{q}_L^* m_Q^2 \tilde{q}_L + \tilde{l}_L^* m_L^2 \tilde{l}_L + \tilde{u}_R^* m_{U^c}^2 \tilde{u}_R + \tilde{d}_R^* m_{D^c}^2 \tilde{d}_R + \tilde{e}_R^* m_{E^c}^2 \tilde{e}_R + m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 \right) \\ & + \left(-\tilde{u}_R^* h_u a_u \tilde{q}_L + \tilde{d}_R^* h_d a_d \tilde{q}_L + \tilde{e}_R^* h_d a_e \tilde{l}_L + b H_u H_d + \text{H.c.} \right) \\ & + \left(\tilde{u}_R^* h_d^* c_u \tilde{q}_L + \tilde{d}_R^* h_u^* c_d \tilde{q}_L + \tilde{e}_R^* h_u^* c_e \tilde{l}_L + \text{H.c.} \right), \end{aligned} \quad (9.15)$$

$$\begin{aligned} V_{\text{SUSY}}^{\text{RPV}} = & \left(-b_i \tilde{l}_{Li} H_u + \frac{1}{2} T_{ijk} \tilde{l}_{Li} \tilde{l}_{Lj} \tilde{e}_{Rk}^* + T'_{ijk} \tilde{l}_{Li} \tilde{q}_{Lj} \tilde{d}_{Rk}^* + \frac{1}{2} T''_{ijk} \tilde{u}_{Ri}^* \tilde{d}_{Rj}^* \tilde{d}_{Rk}^* + \tilde{l}_{Li}^* M_{Li}^2 H_d + \text{H.c.} \right) \\ & + \left(C_{ijk}^1 \tilde{l}_{Li}^* \tilde{q}_{Lj} \tilde{u}_{Rk}^* + C_i^2 h_u^* h_d \tilde{e}_{Ri}^* + C_{ijk}^3 \tilde{d}_{Ri} \tilde{u}_{Rj}^* \tilde{e}_{Rk}^* + \frac{1}{2} C_{ijk}^4 \tilde{d}_{Ri} \tilde{q}_{Lj} \tilde{q}_{Lk} + \text{H.c.} \right). \end{aligned} \quad (9.16)$$

As usual, we remove Θ_W and Θ_B by rotating fermions^{*10}, which is compatible with mass diagonalization (discussed later), and assume the absence of Fayet-Illiopoulos term: $\Lambda_{\text{FI}} = 0$. The SU(3) angle Θ_g forms QCD phase Θ_{QCD} together with the phases from Yukawa matrices. Then,

$$\mathcal{L}_{\text{matter}} = \sum_{\text{matters}} \left[D^\mu \phi^* D_\mu \phi + i \bar{\psi} \bar{\sigma}^\mu D_\mu \psi - \sqrt{2} \sum_{\text{gauge}} g (\lambda^a (\phi^* t^a \psi) + \bar{\lambda}^a (\bar{\psi} t^a \phi)) \right] + (F\text{-terms}), \quad (9.17)$$

$$\mathcal{L}_{\text{gauge}} = \sum_{\text{gauges}} \left(-\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + i \bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a \right) + \frac{g_3^2 \Theta_g}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a + (D\text{-terms}), \quad (9.18)$$

$$\begin{aligned} \mathcal{L}_{\text{super}} = & \epsilon^{ab} \left(-\mu \tilde{h}_u^a \tilde{h}_d^b - y_{dij} \tilde{h}_d^a d_{Ri}^{cx} \tilde{q}_{Lj}^{bx} - y_{dij} \tilde{d}_{Ri}^* \tilde{h}_d^a \tilde{q}_{Lj}^{bx} + y_{dji} \tilde{q}_{Li}^{ax} \tilde{h}_d^b d_{Rj}^{cx} \right. \\ & - y_{eij} \tilde{e}_{Ri}^* \tilde{h}_d^a \tilde{l}_{Lj}^b - y_{eij} \tilde{h}_d^a \tilde{e}_{Ri}^* \tilde{l}_{Lj}^b + y_{eji} \tilde{l}_{Li}^a \tilde{h}_d^b \tilde{e}_{Rj}^* + y_{uij} \tilde{h}_u^a u_{Ri}^{cx} \tilde{q}_{Lj}^{bx} + y_{uij} \tilde{u}_{Ri}^* \tilde{h}_u^a \tilde{q}_{Lj}^{bx} - y_{uji} \tilde{q}_{Li}^{ax} \tilde{h}_u^b u_{Rj}^{cx} \\ & - \kappa_i \tilde{h}_u^a \tilde{l}_{Li}^b - \lambda_{ikj} \tilde{l}_{Li}^a \tilde{e}_{Rj}^* \tilde{l}_{Lk}^b - \frac{1}{2} \lambda_{jki} \tilde{e}_{Ri}^* \tilde{l}_{Lj}^a \tilde{l}_{Lk}^b - \lambda'_{ikj} \tilde{l}_{Li}^a d_{Rj}^{cx} \tilde{l}_{Lk}^{bx} + \lambda'_{kij} \tilde{q}_{Li}^{ax} d_{Rj}^{cx} \tilde{l}_{Lk}^b + \lambda'_{kji} \tilde{d}_{Ri}^* \tilde{q}_{Lj}^{ax} \tilde{l}_{Lk}^b \Big) \\ & - \frac{1}{2} \epsilon^{xyz} \lambda_{ijk}'' \tilde{u}_{Ri}^* d_{Rj}^{cy} d_{Rk}^{cz} + \epsilon^{xyz} \lambda_{jik}'' \tilde{d}_{Ri}^* u_{Rj}^{cy} d_{Rk}^{cz} + \text{H.c.} + (F\text{-terms}), \end{aligned} \quad (9.19)$$

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{2} \left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w} \tilde{w} + M_1 \tilde{b} \tilde{b} + \text{H.c.} \right) - \left(V_{\text{SUSY}}^{\text{RPC}} + V_{\text{SUSY}}^{\text{RPV}} \right), \quad (9.20)$$

and the F - and D -terms form the supersymmetric scalar potential

$$V_{\text{SUSY}} = F_i^* F_i + \frac{1}{2} D^a D^a; \quad F_i = -W_i^* = -\frac{\delta W^*}{\delta \phi_i^*}, \quad D^a = -g(\phi^* t^a \phi), \quad (9.21)$$

$$V = V_{\text{SUSY}} + V_{\text{SUSY}}^{\text{RPC}} + V_{\text{SUSY}}^{\text{RPV}}, \quad (9.22)$$

^{*10}Fail-safe memo: The Θ -terms are total derivatives and relevant in non-perturbative discussion. Redefinition of chiral fermions generates those terms (Fujikawa method) as a non-perturbative effect, so we can remove Θ -terms as long as we have such freedoms. Note also that the absence of gauge anomaly means the corresponding gauge transformations do not induce additional Θ -terms.

where t_a corresponds to the gauge-symmetry generator relevant for each ϕ .

Each auxiliary term is given by

$$-F_{h_u^a}^* = \epsilon^{ab} \left(-\tilde{u}_R^{x*} y_u \tilde{q}_L^{bx} + \mu h_d^b + \kappa_i \tilde{l}_{Li}^b \right), \quad (9.23)$$

$$-F_{h_d^a}^* = \epsilon^{ab} \left(\tilde{e}_R^{x*} y_e \tilde{l}_L^b + \tilde{d}_R^{x*} y_d \tilde{q}_L^{bx} - \mu h_u^b \right), \quad (9.24)$$

$$-F_{\tilde{q}_{Li}^{ax}}^* = \epsilon^{ab} \left(-y_{dj} \tilde{h}_d^b \tilde{d}_{Rj}^{x*} + y_{uj} \tilde{h}_u^b \tilde{u}_{Rj}^{x*} - \lambda'_{kij} \tilde{d}_{Rj}^{x*} \tilde{l}_{Lk}^b \right), \quad (9.25)$$

$$-F_{\tilde{u}_{Ri}^{ax}}^* = -y_{uij} \tilde{h}_u \tilde{q}_{Lj}^x + \frac{1}{2} \epsilon^{xyz} \lambda'_{ijk} \tilde{d}_{Rj}^{y*} \tilde{d}_{Rk}^{z*}, \quad (9.26)$$

$$-F_{\tilde{d}_{Ri}^{ax}}^* = y_{dij} \tilde{h}_d \tilde{q}_{Lj}^x + \lambda'_{jki} \tilde{l}_{Lj} \tilde{q}_{Lk}^x - \lambda'_{jik} \epsilon^{xyz} \tilde{u}_{Rj}^{y*} \tilde{d}_{Rk}^{z*}, \quad (9.27)$$

$$-F_{\tilde{l}_{Li}^a}^* = \epsilon^{ab} \left(-y_{ej} \tilde{e}_R^{j*} \tilde{h}_d^b - \kappa_i \tilde{h}_u^b + \lambda_{ikj} \tilde{e}_R^{j*} \tilde{l}_{Lk}^b + \lambda'_{ikj} \tilde{d}_{Rj}^{x*} \tilde{q}_{Lk}^{bx} \right), \quad (9.28)$$

$$-F_{\tilde{e}_{Ri}^a}^* = y_{eij} \tilde{h}_d \tilde{l}_{Lj} + \frac{1}{2} \lambda_{jki} \tilde{l}_{Lj} \tilde{l}_{Lk}. \quad (9.29)$$

$$D_{\text{SU}(3)}^\alpha = -g_3 \sum_{i=1}^3 \left(\sum_{a=1,2} \tilde{q}_{Li}^{a*} \tau^\alpha \tilde{q}_{Li}^a - \tilde{u}_{Ri}^* \tau^\alpha \tilde{u}_{Ri} - \tilde{d}_{Ri}^* \tau^\alpha \tilde{d}_{Ri} \right), \quad (9.30)$$

$$D_{\text{SU}(2)}^\alpha = -g_2 \left[\sum_{i=1}^3 \left(\sum_{x=1}^3 \tilde{q}_{Li}^{x*} T^\alpha \tilde{q}_{Li}^x + \tilde{l}_{Li}^* T^\alpha \tilde{l}_{Li} \right) + h_u^* T^\alpha h_u + h_d^* T^\alpha h_d \right], \quad (9.31)$$

$$D_{\text{U}(1)} = -g_1 \left(\frac{1}{6} |\tilde{q}_L|^2 - \frac{1}{2} |\tilde{l}_L|^2 - \frac{2}{3} |\tilde{u}_R|^2 + \frac{1}{3} |\tilde{d}_R|^2 + |\tilde{e}_R|^2 + \frac{1}{2} |h_u|^2 - \frac{1}{2} |h_d|^2 \right). \quad (9.32)$$

9.3. Full Lagrangian

Here the Lagrangian $\mathcal{L} = \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}} + \mathcal{L}_{\text{scalar}}$ is explicitly given:

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{g_3^2 \Theta_g}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a, \quad (9.33)$$

$$\mathcal{L}_{\text{fermions}} = i\bar{\psi} \bar{\sigma}^\mu D_\mu \psi + i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a - \frac{1}{2} \left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w} \tilde{w} + M_1 \tilde{b} \tilde{b} + \text{H.c.} \right) + \mathcal{L}_{\text{super}}|_{\text{no } F\text{-terms}}, \quad (9.34)$$

$$\mathcal{L}_{\text{SFG}} = -\sqrt{2} g \lambda^a (\phi^* t^a \psi) - \sqrt{2} g \bar{\lambda}^a (\bar{\psi} t^a \phi), \quad (9.35)$$

$$\mathcal{L}_{\text{scalar}} = D^\mu \phi^* D_\mu \phi - V. \quad (9.36)$$

9.3.1. Vector part

$$\begin{aligned} \mathcal{L}_{\text{vector}} &= -\frac{1}{2} (\partial_\mu B_\nu - \partial_\nu B_\mu) \partial^\mu B^\nu - \frac{1}{2} (\partial_\mu g_\nu^a - \partial_\nu g_\mu^a) \partial^\mu g^{a\nu} - \frac{1}{2} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) \partial^\mu W^{a\nu} \\ &\quad - g_2 \epsilon^{abc} W_\mu^b W_\nu^c \partial^\mu W^{a\nu} - \frac{g_2^2}{4} \epsilon^{abe} \epsilon^{cde} W_\mu^a W_\nu^b W^{c\mu} W^{d\nu} \\ &\quad - g_3 f^{abc} g_\mu^b g_\nu^c \partial^\mu g^{a\nu} - \frac{g_3^2}{4} f^{cde} f^{abe} g_\mu^a g_\nu^b g^{c\mu} g^{d\nu} + \frac{g_3^2 \Theta_g}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a, \\ &= (\text{gluons}) - \frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu) \partial^\mu A^\nu - (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) \partial^\mu W^{+\nu} - \frac{1}{2} (\partial_\mu Z_\nu - \partial_\nu Z_\mu) \partial^\mu Z^\nu \\ &\quad + i g_2 c_w [(W_\mu^- Z_\nu - W_\nu^- Z_\mu) \partial^\mu W^{+\nu} - (W_\mu^+ Z_\nu - W_\nu^+ Z_\mu) \partial^\mu W^{-\nu} + (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) \partial^\mu Z^\nu] \\ &\quad + i |e| [(W_\mu^+ A_\nu - W_\nu^+ A_\mu) \partial^\mu W^{-\nu} - (W_\mu^- A_\nu - W_\nu^- A_\mu) \partial^\mu W^{+\nu} + (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) \partial^\mu A^\nu] \\ &\quad + \frac{g_2^2}{2} W^{+\mu} W_\mu^+ W^{-\nu} W_\nu^- - \frac{g_2^2}{2} W^{+\mu} W^{+\nu} W_\mu^- W_\nu^- - g_2^2 W^{+\mu} W_\mu^- Z^\nu Z_\nu + g_2^2 W^{+\mu} W^{-\nu} Z_\mu Z_\nu \\ &\quad - e^2 W^{+\mu} W_\mu^- A^\nu A_\nu + e^2 W^{+\mu} W_\mu^- Z^\nu Z_\nu + e^2 W^{+\mu} W^{-\nu} A_\mu A_\nu - e^2 W^{+\mu} W^{-\nu} Z_\mu Z_\nu \\ &\quad - 2 g_2^2 c_w s_w W^{+\mu} W_\mu^- A^\nu Z_\nu + g_2^2 c_w s_w W^{+\mu} W^{-\nu} A_\mu Z_\nu + g_2^2 c_w s_w W^{+\mu} W^{-\nu} A_\nu Z_\mu, \end{aligned} \quad (9.37)$$

where

$$\begin{aligned} W_\mu^1 &= \frac{W_\mu^+ + W_\mu^-}{\sqrt{2}}, \quad W_\mu^2 = \frac{i(W_\mu^+ - W_\mu^-)}{\sqrt{2}}, \quad W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}, \\ \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \\ |e| &= g_2 s_w = g_Y c_w = g_Z s_w c_w, \quad g_Z = g_2 / c_w = g_Y / s_w, \quad g_Y = |e| / c_w = g_Z s_w = g_2 t_w, \quad g_2 = |e| / s_w = g_Z c_w. \end{aligned}$$

9.3.2. Fermion part

$$\begin{aligned}
& \mathcal{L}_{\text{fermions}} \\
&= i\bar{q}_L \bar{\sigma}^\mu (\partial_\mu - ig_3 g_\mu^a \tau^a - ig_2 W_\mu^a T^a - \frac{1}{6} ig_Y B_\mu) q_L \\
&\quad + i\bar{u}_R^c \bar{\sigma}^\mu (\partial_\mu + ig_3 g_\mu^a \tau^{a*} + \frac{2}{3} ig_Y B_\mu) u_R^c + i\bar{d}_R^c \bar{\sigma}^\mu (\partial_\mu + ig_3 g_\mu^a \tau^{a*} - \frac{1}{3} ig_Y B_\mu) d_R^c \\
&\quad + i\bar{l}_L \bar{\sigma}^\mu (\partial_\mu - ig_2 W_\mu^a T^a + \frac{1}{2} ig_Y B_\mu) l_L + i\bar{e}_R^c \bar{\sigma}^\mu (\partial_\mu - ig_Y B_\mu) e_R^c \\
&\quad + i\bar{h}_u \bar{\sigma}^\mu (\partial_\mu - ig_2 W_\mu^a T^a - \frac{1}{2} ig_Y B_\mu) \tilde{h}_u + i\bar{h}_d \bar{\sigma}^\mu (\partial_\mu - ig_2 W_\mu^a T^a + \frac{1}{2} ig_Y B_\mu) \tilde{h}_d \\
&\quad + i\bar{g}_0^a \bar{\sigma}^\mu (\partial_\mu \tilde{g}_0^a + g_3 f^{abc} g_\mu^b \tilde{g}_0^c) + i\bar{w}^a \bar{\sigma}^\mu (\partial_\mu \tilde{w}^a + g_2 \epsilon^{abc} W_\mu^b \tilde{w}^c) + i\bar{b} \bar{\sigma}^\mu \partial_\mu \tilde{b} \\
&\quad - \frac{1}{2} \left(M_3 \tilde{g}_0^a \tilde{g}_0^a + M_2 \tilde{w}^a \tilde{w}^a + M_1 \tilde{b} \tilde{b} + \text{H.c.} \right) + \mathcal{L}_{\text{super}}|_{\text{no } F\text{-terms}} \\
&= i\bar{b} \bar{\sigma}^\mu \partial_\mu \tilde{b} - \frac{1}{2} \left(M_1 \tilde{b} \tilde{b} + M_1^* \tilde{b}^a \tilde{b}^a \right) + i\bar{g}_0^a \bar{\sigma}^\mu \partial_\mu \tilde{g}_0^a - \frac{1}{2} \left(M_3 \tilde{g}_0^a \tilde{g}_0^a + M_3^* \tilde{g}_0^a \tilde{g}_0^a \right) - ig_3 f^{abc} (\tilde{g}_0^a \bar{\sigma}^\mu \tilde{g}_0^b) g_\mu^c \\
&\quad + i\bar{w}^+ \bar{\sigma}^\mu \partial_\mu \tilde{w}^+ + i\bar{w}^- \bar{\sigma}^\mu \partial_\mu \tilde{w}^- + i\bar{w}^3 \bar{\sigma}^\mu \partial_\mu \tilde{w}^3 - (M_2 \tilde{w}^+ \tilde{w}^- + M_2^* \tilde{w}^+ \tilde{w}^-) - \frac{1}{2} (M_2 \tilde{w}^3 \tilde{w}^3 + M_2^* \tilde{w}^3 \tilde{w}^3) \\
&\quad + g_2 (\tilde{w}^3 \bar{\sigma}^\mu \tilde{w}^- - \tilde{w}^+ \bar{\sigma}^\mu \tilde{w}^3) W_\mu^+ - g_2 (\tilde{w}^3 \bar{\sigma}^\mu \tilde{w}^+ - \tilde{w}^- \bar{\sigma}^\mu \tilde{w}^3) W_\mu^- + g_2 (\tilde{w}^+ \bar{\sigma}^\mu \tilde{w}^+ - \tilde{w}^- \bar{\sigma}^\mu \tilde{w}^-) (c_W Z_\mu + s_W A_\mu) \\
&\quad + \bar{u}_L \bar{\sigma}^\mu (i\partial_\mu + g_3 \tau^a g_\mu^a) u_L + \bar{u}_R^c \bar{\sigma}^\mu (i\partial_\mu - g_3 \tau^{a*} g_\mu^a) u_R^c + i\bar{\nu}_L \bar{\sigma}^\mu \partial_\mu \nu_L \\
&\quad + \bar{d}_L \bar{\sigma}^\mu (i\partial_\mu + g_3 \tau^a g_\mu^a) d_L + \bar{d}_R^c \bar{\sigma}^\mu (i\partial_\mu - g_3 \tau^{a*} g_\mu^a) d_R^c + i\bar{e}_L \bar{\sigma}^\mu \partial_\mu e_L + i\bar{e}_R^c \bar{\sigma}^\mu \partial_\mu e_R^c \\
&\quad + i\bar{h}_d^- \bar{\sigma}^\mu \partial_\mu \tilde{h}_d^- + i\bar{h}_d^0 \bar{\sigma}^\mu \partial_\mu \tilde{h}_d^0 + i\bar{h}_u^+ \bar{\sigma}^\mu \partial_\mu \tilde{h}_u^+ + i\bar{h}_u^0 \bar{\sigma}^\mu \partial_\mu \tilde{h}_u^0 \\
&\quad + \frac{g_2}{\sqrt{2}} \left(\bar{u}_L \bar{\sigma}^\mu d_L + \bar{\nu}_L \bar{\sigma}^\mu e_L + \tilde{h}_u^+ \bar{\sigma}^\mu \tilde{h}_u^0 + \tilde{h}_d^0 \bar{\sigma}^\mu \tilde{h}_d^- \right) W_\mu^+ + \frac{g_2}{\sqrt{2}} \left(\bar{d}_L \bar{\sigma}^\mu u_L + \bar{e}_L \bar{\sigma}^\mu \nu_L + \tilde{h}_u^0 \bar{\sigma}^\mu \tilde{h}_u^+ + \tilde{h}_d^- \bar{\sigma}^\mu \tilde{h}_d^0 \right) W_\mu^- \\
&\quad + \frac{g_Z(3-4s_W^2)}{6} \bar{u}_L \bar{\sigma}^\mu u_L Z_\mu + \frac{g_Z s_W^2}{3} \bar{u}_R^c \bar{\sigma}^\mu u_R^c Z_\mu + \frac{g_Z(2s_W^2-3)}{6} \bar{d}_L \bar{\sigma}^\mu d_L Z_\mu - \frac{g_Z s_W^2}{3} \bar{d}_R^c \bar{\sigma}^\mu d_R^c Z_\mu \\
&\quad + \frac{g_Z}{2} \bar{\nu}_L \bar{\sigma}^\mu \nu_L Z_\mu + \frac{g_Z(2s_W^2-1)}{2} \bar{e}_L \bar{\sigma}^\mu e_L Z_\mu - g_Z s_W^2 \bar{e}_R^c \bar{\sigma}^\mu e_R^c Z_\mu \\
&\quad + \frac{g_Z(1-2s_W^2)}{2} \tilde{h}_u^+ \bar{\sigma}^\mu \tilde{h}_u^+ Z_\mu + \frac{g_Z}{2} \tilde{h}_u^0 \bar{\sigma}^\mu \tilde{h}_u^0 Z_\mu + \frac{g_Z(2s_W^2-1)}{2} \tilde{h}_d^- \bar{\sigma}^\mu \tilde{h}_d^- Z_\mu + \frac{g_Z}{2} \tilde{h}_d^0 \bar{\sigma}^\mu \tilde{h}_d^0 Z_\mu \\
&\quad + \frac{2|e|}{3} (\bar{u}_L \bar{\sigma}^\mu u_L - \bar{u}_R^c \bar{\sigma}^\mu u_R^c) A_\mu - \frac{|e|}{3} (\bar{d}_L \bar{\sigma}^\mu d_L - \bar{d}_R^c \bar{\sigma}^\mu d_R^c) A_\mu - |e| (\bar{e}_L \bar{\sigma}^\mu e_L - \bar{e}_R^c \bar{\sigma}^\mu e_R^c) A_\mu \\
&\quad + |e| \tilde{h}_u^+ \bar{\sigma}^\mu \tilde{h}_u^+ A_\mu - |e| \tilde{h}_d^- \bar{\sigma}^\mu \tilde{h}_d^- A_\mu + \mathcal{L}_{\text{super}}|_{\text{no } F\text{-terms}};
\end{aligned} \tag{9.40}$$

here,

$$\begin{aligned}
\mathcal{L}_{\text{super}}|_{\text{no } F\text{-terms}} &= -\mu \tilde{h}_u^+ \tilde{h}_d^- + \mu \tilde{h}_u^0 \tilde{h}_d^0 + y_{uij} h_u^+ u_{Ri}^c d_{Lj} - y_{uij} h_u^0 u_{Ri}^c u_{Lj} + y_{uij} \tilde{d}_{Lj} \tilde{h}_u^+ u_{Ri}^c - y_{uij} \tilde{u}_{Lj} \tilde{h}_u^0 u_{Ri}^c \\
&\quad + y_{uji} \tilde{u}_{Rj}^* \tilde{h}_u^+ d_{Li} - y_{uji} \tilde{u}_{Rj}^* \tilde{h}_u^0 u_{Li} + y_{dij} h_d^- d_{Ri}^c u_{Lj} - y_{dij} h_d^0 d_{Ri}^c d_{Lj} - y_{dij} \tilde{d}_{Lj} \tilde{h}_d^0 d_{Ri}^c \\
&\quad + y_{dij} \tilde{u}_{Lj} \tilde{h}_d^- d_{Ri}^c + y_{dij} \tilde{d}_{Rj}^* \tilde{h}_d^- u_{Li} - y_{dij} \tilde{d}_{Rj}^* \tilde{h}_d^0 d_{Li} + y_{eij} h_e^- e_{Ri}^c \nu_{Lj} - y_{eij} h_e^0 e_{Ri}^c e_{Lj} \\
&\quad - y_{eij} \tilde{e}_{Lj} \tilde{h}_e^0 e_{Ri}^c + y_{eij} \tilde{\nu}_{Lj} \tilde{h}_e^- e_{Ri}^c + y_{eji} \tilde{e}_{Rj}^* \tilde{h}_e^- \nu_{Li} - y_{eji} \tilde{e}_{Rj}^* \tilde{h}_e^0 e_{Li} \\
&\quad - \kappa_i \tilde{h}_u^+ e_{Li} + \kappa_i \tilde{h}_u^0 \nu_{Li} - \lambda_{ijk} \tilde{e}_{Rk}^* \nu_{Li} e_{Lj} - \lambda_{jki} \tilde{e}_{Lk} e_{Ri}^c \nu_{Lj} + \lambda_{jki} \tilde{\nu}_{Lk} e_{Ri}^c e_{Lj} \\
&\quad - \lambda'_{jik} \tilde{d}_{Rk}^* d_{Li} \nu_{Lj} + \lambda'_{jik} \tilde{d}_{Rk}^* u_{Li} e_{Lj} - \lambda'_{jki} \tilde{d}_{Lk} d_{Ri}^c \nu_{Lj} + \lambda'_{jki} \tilde{u}_{Lk} d_{Ri}^c e_{Lj} + \lambda'_{kji} \tilde{e}_{Lk} d_{Ri}^c u_{Lj} \\
&\quad - \lambda'_{kji} \tilde{\nu}_{Lk} d_{Ri}^c d_{Lj} - \epsilon^{xyz} \lambda''_{ijk} \tilde{d}_{Rk}^* u_{Ri}^{cy} d_{Rj}^{cz} - \frac{1}{2} \epsilon^{xyz} \lambda''_{kij} \tilde{u}_{Rk}^{x*} d_{Ri}^{cy} d_{Rj}^{cz} + \text{H.c.}
\end{aligned} \tag{9.41}$$

9.3.3. Scalar-fermion-gaugino interaction

$$\begin{aligned}
\mathcal{L}_{\text{SFG}} = & -g_2 \tilde{u}_L^* d_L \tilde{w}^+ - g_2 \tilde{u}_L \bar{d}_L \tilde{w}^+ - g_2 \tilde{d}_L^* u_L \tilde{w}^- - g_2 \tilde{d}_L \bar{u}_L \tilde{w}^- \\
& - \sqrt{2} g_3 \tilde{u}_L^* \tau^a u_L \tilde{g}_0^a + \sqrt{2} g_3 \tilde{u}_R^* \tau^a \bar{u}_R \tilde{g}_0^a - \sqrt{2} g_3 \tilde{u}_L \tau^{a*} \bar{u}_L \tilde{g}_0^a + \sqrt{2} g_3 \tilde{u}_R \tau^{a*} u_R \tilde{g}_0^a \\
& - \frac{g_2}{\sqrt{2}} \tilde{u}_L^* u_L \tilde{w}^3 - \frac{g_2}{\sqrt{2}} \tilde{u}_L \bar{u}_L \tilde{w}^3 - \frac{g_Y}{3\sqrt{2}} \tilde{u}_L^* u_L \tilde{b} + \frac{2\sqrt{2}g_Y}{3} \tilde{u}_R^* \bar{u}_R \tilde{b} - \frac{g_Y}{3\sqrt{2}} \tilde{u}_L \bar{u}_L \tilde{b} + \frac{2\sqrt{2}g_Y}{3} \tilde{u}_R u_R \tilde{b} \\
& - \sqrt{2} g_3 \tilde{d}_L^* \tau^a d_L \tilde{g}_0^a + \sqrt{2} g_3 \tilde{d}_R^* \tau^a \bar{d}_R \tilde{g}_0^a - \sqrt{2} g_3 \tilde{d}_L \tau^{a*} \bar{d}_L \tilde{g}_0^a + \sqrt{2} g_3 \tilde{d}_R \tau^{a*} d_R \tilde{g}_0^a \\
& + \frac{g_2}{\sqrt{2}} \tilde{d}_L^* d_L \tilde{w}^3 + \frac{g_2}{\sqrt{2}} \tilde{d}_L \bar{d}_L \tilde{w}^3 - \frac{g_Y}{3\sqrt{2}} \tilde{d}_L^* d_L \tilde{b} - \frac{\sqrt{2}g_Y}{3} \tilde{d}_R^* \bar{d}_R \tilde{b} - \frac{g_Y}{3\sqrt{2}} \tilde{d}_L \bar{d}_L \tilde{b} - \frac{\sqrt{2}g_Y}{3} \tilde{d}_R d_R \tilde{b} \\
& - g_2 \tilde{e}_L \bar{\nu}_L \tilde{w}^- - g_2 \tilde{\nu}_L \bar{e}_L \tilde{w}^+ - g_2 \tilde{e}_L^* \nu_L \tilde{w}^- - g_2 \tilde{\nu}_L^* e_L \tilde{w}^+ \\
& - \frac{g_2}{\sqrt{2}} \tilde{\nu}_L^* \nu_L \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} \tilde{\nu}_L^* \nu_L \tilde{b} - \frac{g_2}{\sqrt{2}} \tilde{\nu}_L \bar{\nu}_L \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} \tilde{\nu}_L \bar{\nu}_L \tilde{b} \\
& + \frac{g_2}{\sqrt{2}} \tilde{e}_L^* e_L \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} \tilde{e}_L^* e_L \tilde{b} - \sqrt{2} g_Y \tilde{e}_R^* \bar{e}_R \tilde{b} + \frac{g_2}{\sqrt{2}} \tilde{e}_L \bar{e}_L \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} \tilde{e}_L \bar{e}_L \tilde{b} - \sqrt{2} g_Y \tilde{e}_R e_R \tilde{b} \\
& - g_2 h_u^{+*} \tilde{h}_u^0 \tilde{w}^+ - g_2 h_d^{0*} \tilde{h}_d^- \tilde{w}^+ - g_2 h_d^{-*} \tilde{h}_d^0 \tilde{w}^- - g_2 h_u^0 \tilde{h}_u^+ \tilde{w}^- \\
& - g_2 h_d^{0*} \tilde{h}_d^- \tilde{w}^+ - g_2 h_u^{0*} \tilde{h}_u^+ \tilde{w}^- - g_2 h_u^{+*} \tilde{h}_u^0 \tilde{w}^+ - g_2 h_d^{-*} \tilde{h}_d^0 \tilde{w}^- \\
& - \frac{g_2}{\sqrt{2}} h_u^{+*} \tilde{h}_u^+ \tilde{w}^3 - \frac{g_Y}{\sqrt{2}} h_u^{+*} \tilde{h}_u^+ \tilde{b} - \frac{g_2}{\sqrt{2}} h_u^{+*} \tilde{h}_u^+ \tilde{w}^3 - \frac{g_Y}{\sqrt{2}} h_u^{+*} \tilde{h}_u^+ \tilde{b} + \frac{g_2}{\sqrt{2}} h_d^{-*} \tilde{h}_d^- \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} h_d^{-*} \tilde{h}_d^- \tilde{b} \\
& + \frac{g_2}{\sqrt{2}} h_d^{-*} \tilde{h}_d^- \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} h_d^{-*} \tilde{h}_d^- \tilde{b} + \frac{g_2}{\sqrt{2}} h_u^{0*} \tilde{h}_u^0 \tilde{w}^3 - \frac{g_Y}{\sqrt{2}} h_u^{0*} \tilde{h}_u^0 \tilde{b} - \frac{g_2}{\sqrt{2}} h_d^{0*} \tilde{h}_d^0 \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} h_d^{0*} \tilde{h}_d^0 \tilde{b} \\
& + \frac{g_2}{\sqrt{2}} h_u^0 \tilde{h}_u^+ \tilde{w}^3 - \frac{g_Y}{\sqrt{2}} h_u^0 \tilde{h}_u^+ \tilde{b} - \frac{g_2}{\sqrt{2}} h_d^0 \tilde{h}_d^0 \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} h_d^0 \tilde{h}_d^0 \tilde{b}
\end{aligned} \tag{9.42}$$

9.3.4. Scalar part

$$\begin{aligned}
\mathcal{L}_{\text{scalar}} = & (\partial_\mu \tilde{u}_L^* + ig_3 \tilde{u}_L^* \tau^a g_\mu^a) (\partial^\mu \tilde{u}_L - ig_3 g^{b\mu} \tau^b \tilde{u}_L) + (\partial_\mu \tilde{u}_R - ig_3 \tilde{u}_R \tau^{a*} g_\mu^a) (\partial^\mu \tilde{u}_R^* + ig_3 g_\mu^b \tau^{b*} \tilde{u}_R^*) \\
& + (\partial_\mu \tilde{d}_L^* + ig_3 \tilde{d}_L^* \tau^a g_\mu^a) (\partial^\mu \tilde{d}_L - ig_3 g^{b\mu} \tau^b \tilde{d}_L) + (\partial_\mu \tilde{d}_R - ig_3 \tilde{d}_R^* \tau^{a*} g_\mu^a) (\partial^\mu \tilde{d}_R^* + ig_3 g_\mu^b \tau^{b*} \tilde{d}_R^*) \\
& + \sqrt{2} g_2 g_3 \tilde{u}_L^* \tau^a \tilde{d}_L W^{+\mu} g_\mu^a + \sqrt{2} g_2 g_3 \tilde{d}_L^* \tau^a \tilde{u}_L W^{-\mu} g_\mu^a + \frac{4}{3} g_3 |e| (\tilde{u}_L^* \tau^a \tilde{u}_L + \tilde{u}_R^* \tau^a \tilde{u}_R) g^{a\mu} A_\mu \\
& - \frac{2}{3} g_3 |e| (\tilde{d}_L^* \tau^a \tilde{d}_L + \tilde{d}_R^* \tau^a \tilde{d}_R) g^{a\mu} A_\mu + \frac{(3 - 4s_w^2) g_Z}{3} g_3 \tilde{u}_L^* \tau^a \tilde{u}_L g^{a\mu} Z_\mu - \frac{4s_w^2 g_Z}{3} g_3 \tilde{u}_R^* \tau^a \tilde{u}_R g^{a\mu} Z_\mu \\
& + \frac{(2s_w^2 - 3) g_Z}{3} g_3 \tilde{d}_L^* \tau^a \tilde{d}_L g^{a\mu} Z_\mu + \frac{2s_w^2 g_Z}{3} g_3 \tilde{d}_R^* \tau^a \tilde{d}_R g^{a\mu} Z_\mu \\
& + \frac{ig_2}{\sqrt{2}} W_\mu^+ (\tilde{u}_L^* \partial^\mu \tilde{d}_L - \tilde{d}_L \partial^\mu \tilde{u}_L^*) - \frac{ig_2}{\sqrt{2}} W_\mu^- (\tilde{u}_L \partial^\mu \tilde{d}_L^* - \tilde{d}_L^* \partial^\mu \tilde{u}_L) \\
& + \frac{2i}{3} |e| A_\mu (\tilde{u}_L^* \partial^\mu \tilde{u}_L - \tilde{u}_L \partial^\mu \tilde{u}_L^* + \tilde{u}_R^* \partial^\mu \tilde{u}_R - \tilde{u}_R \partial^\mu \tilde{u}_R^*) - \frac{i}{3} |e| A_\mu (\tilde{d}_L^* \partial^\mu \tilde{d}_L - \tilde{d}_L \partial^\mu \tilde{d}_L^* + \tilde{d}_R^* \partial^\mu \tilde{d}_R - \tilde{d}_R \partial^\mu \tilde{d}_R^*) \\
& + \frac{i(4s_w^2 - 3) g_Z}{6} Z_\mu (\tilde{u}_L \partial^\mu \tilde{u}_L^* - \tilde{u}_L^* \partial^\mu \tilde{u}_L) + \frac{i(2s_w^2 - 3) g_Z}{6} Z_\mu (\tilde{d}_L^* \partial^\mu \tilde{d}_L - \tilde{d}_L \partial^\mu \tilde{d}_L^*) \\
& - \frac{2is_w^2 g_Z}{3} Z_\mu (\tilde{u}_R^* \partial^\mu \tilde{u}_R - \tilde{u}_R \partial^\mu \tilde{u}_R^*) + \frac{is_w^2 g_Z}{3} Z_\mu (\tilde{d}_R^* \partial^\mu \tilde{d}_R - \tilde{d}_R \partial^\mu \tilde{d}_R^*) \\
& + \frac{g_2^2}{2} (|\tilde{u}_L|^2 + |\tilde{d}_L|^2) W^{+\mu} W_\mu^- - \frac{s_w^2 g_2 g_Z}{3\sqrt{2}} \tilde{u}_L^* \tilde{d}_L W^{+\mu} Z_\mu - \frac{s_w^2 g_2 g_Z}{3\sqrt{2}} \tilde{d}_L^* \tilde{u}_L W^{-\mu} Z_\mu \\
& + \frac{(3 - 4s_w^2)^2 g_Z^2}{36} |\tilde{u}_L|^2 Z^\mu Z_\mu + \frac{4s_w^4 g_Z^2}{9} |\tilde{u}_R|^2 Z^\mu Z_\mu + \frac{(3 - 2s_w^2)^2 g_Z^2}{36} |\tilde{d}_L|^2 Z^\mu Z_\mu + \frac{4s_w^4 g_Z^2}{9} |\tilde{d}_R|^2 Z^\mu Z_\mu \\
& + \frac{4}{9} e^2 (|\tilde{u}_L|^2 + |\tilde{u}_R|^2) A^\mu A_\mu + \frac{1}{9} e^2 (|\tilde{d}_L|^2 + |\tilde{d}_R|^2) A^\mu A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{u}_L^* \tilde{d}_L W^{+\mu} A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{d}_L^* \tilde{u}_L W^{-\mu} A_\mu \\
& + \frac{2(3 - 4s_w^2) g_Z |e|}{9} |\tilde{u}_L|^2 A^\mu Z_\mu - \frac{8s_w^2 g_Z |e|}{9} |\tilde{u}_R|^2 A^\mu Z_\mu + \frac{(3 - 2s_w^2) g_Z |e|}{9} |\tilde{d}_L|^2 A^\mu Z_\mu - \frac{2s_w^2 g_Z |e|}{9} |\tilde{d}_R|^2 A^\mu Z_\mu \\
& + \partial_\mu \tilde{e}_R \partial^\mu \tilde{e}_R^* + \partial_\mu \tilde{e}_L^* \partial^\mu \tilde{e}_L + \partial_\mu \tilde{\nu}_L^* \partial^\mu \tilde{\nu}_L + i \frac{g_2}{\sqrt{2}} W_\mu^+ (\tilde{\nu}_L^* \partial^\mu \tilde{e}_L - \tilde{e}_L \partial^\mu \tilde{\nu}_L^*) + i \frac{g_2}{\sqrt{2}} W_\mu^- (\tilde{e}_L^* \partial^\mu \tilde{\nu}_L - \tilde{\nu}_L \partial^\mu \tilde{e}_L^*) \\
& - \frac{i(1 - 2s_w^2) g_Z}{2} Z_\mu (\tilde{e}_L^* \partial^\mu \tilde{e}_L - \tilde{e}_L \partial^\mu \tilde{e}_L^*) + \frac{ig_Z}{2} Z_\mu (\tilde{\nu}_L^* \partial^\mu \tilde{\nu}_L - \tilde{\nu}_L \partial^\mu \tilde{\nu}_L^*) + is_w^2 g_Z Z_\mu (\tilde{e}_R^* \partial^\mu \tilde{e}_R - \tilde{e}_R \partial^\mu \tilde{e}_R^*) \\
& + i|e| A_\mu (\tilde{e}_L \partial^\mu \tilde{e}_L^* - \tilde{e}_L^* \partial^\mu \tilde{e}_L + \tilde{e}_R \partial^\mu \tilde{e}_R^* - \tilde{e}_R^* \partial^\mu \tilde{e}_R) \\
& + \frac{g_2^2}{2} (|\tilde{\nu}_L|^2 + |\tilde{e}_L|^2) W^{+\mu} W_\mu^- + \frac{g_2 g_Z s_w^2}{\sqrt{2}} (\tilde{e}_L^* \tilde{\nu}_L W_\mu^- Z^\mu + \tilde{\nu}_L^* \tilde{e}_L W_\mu^+ Z^\mu) - \frac{g_2 |e|}{\sqrt{2}} (\tilde{\nu}_L^* \tilde{e}_L W_\mu^+ A^\mu + \tilde{e}_L^* \tilde{\nu}_L W_\mu^- A^\mu) \\
& + \frac{(1 - 2s_w^2)^2 g_Z^2}{4} |\tilde{e}_L|^2 Z^\mu Z_\mu + \frac{g_Z^2}{4} |\tilde{\nu}_L|^2 Z^\mu Z_\mu + g_Z^2 s_w^4 |\tilde{e}_R|^2 Z^\mu Z_\mu \\
& + e^2 (|\tilde{e}_L|^2 + |\tilde{e}_R|^2) A^\mu A_\mu + (1 - 2s_w^2) |e| g_Z |\tilde{e}_L|^2 A^\mu Z_\mu - 2s_w^2 g_Z |e| |\tilde{e}_R|^2 A^\mu Z_\mu \\
& + \partial_\mu h_d^{-*} \partial^\mu h_d^- + \partial_\mu h_d^{0*} \partial^\mu h_d^0 + \partial_\mu h_u^{+*} \partial^\mu h_u^+ + \partial_\mu h_u^{0*} \partial^\mu h_u^0 + i \frac{g_2}{\sqrt{2}} W_\mu^+ (h_u^{+*} \partial^\mu h_u^0 - h_u^0 \partial^\mu h_u^{+*}) \\
& + i \frac{g_2}{\sqrt{2}} W_\mu^- (h_u^{0*} \partial^\mu h_u^+ - h_u^+ \partial^\mu h_u^{0*}) + i \frac{g_2}{\sqrt{2}} W_\mu^+ (h_d^{0*} \partial^\mu h_d^- - h_d^- \partial^\mu h_d^{0*}) + i \frac{g_2}{\sqrt{2}} W_\mu^- (h_d^{-*} \partial^\mu h_d^0 - h_d^0 \partial^\mu h_d^{-*}) \\
& + \frac{i(1 - 2s_w^2) g_Z}{2} Z_\mu (h_u^{+*} \partial^\mu h_u^+ - h_u^+ \partial^\mu h_u^{+*}) + \frac{i(2s_w^2 - 1) g_Z}{2} Z_\mu (h_d^{-*} \partial^\mu h_d^- - h_d^- \partial^\mu h_d^{-*}) \\
& - \frac{ig_Z}{2} Z_\mu (h_u^{0*} \partial^\mu h_u^0 - h_u^0 \partial^\mu h_u^{0*}) + \frac{ig_Z}{2} Z_\mu (h_d^{0*} \partial^\mu h_d^0 - h_d^0 \partial^\mu h_d^{0*}) \\
& - i|e| A_\mu (h_d^{-*} \partial^\mu h_d^- - h_d^- \partial^\mu h_d^{-*}) + i|e| A_\mu (h_u^{+*} \partial^\mu h_u^+ - h_u^+ \partial^\mu h_u^{+*}) \\
& + \frac{g_2^2}{2} (|h_u^+|^2 + |h_u^0|^2 + |h_d^0|^2 + |h_d^-|^2) W^{+\mu} W_\mu^- + \frac{s_w^2 g_Z g_2}{\sqrt{2}} (h_d^{0*} h_d^- - h_u^{+*} h_u^0) W_\mu^+ Z^\mu \\
& + \frac{s_w^2 g_Z g_2}{\sqrt{2}} (h_d^{-*} h_d^0 - h_u^{0*} h_u^+) W_\mu^- Z^\mu + \frac{g_2 |e|}{\sqrt{2}} (h_u^{+*} h_u^0 - h_d^{0*} h_d^-) W_\mu^+ A^\mu + \frac{g_2 |e|}{\sqrt{2}} (h_u^{0*} h_u^+ - h_d^{-*} h_d^0) W_\mu^- A^\mu \\
& + \frac{(1 - 2s_w^2)^2 g_Z^2}{4} (|h_u^+|^2 + |h_d^-|^2) Z^\mu Z_\mu + \frac{g_Z^2}{4} (|h_u^0|^2 + |h_d^0|^2) Z^\mu Z_\mu + e^2 (|h_u^+|^2 + |h_d^-|^2) A^\mu A_\mu \\
& + (1 - 2s_w^2) |e| g_Z (|h_u^+|^2 + |h_d^-|^2) A^\mu Z_\mu \\
& - (V_{\text{SUSY}} + V_{\text{SUSY}}),
\end{aligned}$$

(9.43)

where the scalar potential is given by

$$\begin{aligned}
V_{\text{SUSY}} = & |h_u|^2 \left(|\mu|^2 + \sum_i |\kappa_i|^2 \right) + |\mu|^2 |h_d|^2 + \left(\kappa_i^* \mu \tilde{t}_{Li}^* h_d + \text{H.c.} \right) + \kappa_i^* \kappa_j \tilde{t}_{Li}^* \tilde{t}_{Lj} \\
& + \left[-y_{uij} \mu^* h_d^* \tilde{u}_{Ri}^* \tilde{q}_{Lj} - y_{uij} \kappa_k^* \tilde{u}_{Ri}^* \tilde{q}_{Lj} \tilde{t}_{Lk}^* - (y_{dij} \mu^* + \lambda'_{kji} \kappa_k^*) h_u^* \tilde{d}_{Ri}^* \tilde{q}_{Lj} \right. \\
& \quad \left. + y_{eij} \kappa_j^* \tilde{e}_{Ri}^* h_u^* h_d + (\lambda_{jki} \kappa_k^* - y_{eij} \mu^*) h_u^* \tilde{e}_{Ri}^* \tilde{t}_{Lj} + \text{H.c.} \right] \\
& + \frac{1}{8} (g_2^2 + g_Y^2) |h_d|^4 + \frac{1}{8} (g_2^2 + g_Y^2) |h_u|^4 + \left(-\frac{g_2^2}{4} |h_d|^2 |h_u|^2 - \frac{g_Y^2}{4} |h_d|^2 |h_u|^2 + \frac{g_2^2}{2} |h_d^* h_u|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_u|^2 |\tilde{q}_L|^2 + \frac{g_Y^2}{12} |h_u|^2 |\tilde{q}_L|^2 + \frac{g_2^2}{2} |h_u^b \tilde{q}_{Li}|^2 + \epsilon^{ac} \epsilon^{bd} (y_u^\dagger y_u)_{ji} h_u^a h_u^{b*} \tilde{q}_{Li}^c \tilde{q}_{Lj}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4} |h_d|^2 |\tilde{q}_L|^2 - \frac{g_Y^2}{12} |h_d|^2 |\tilde{q}_L|^2 + \frac{g_2^2}{2} |h_d^* \tilde{q}_{Li}|^2 + \epsilon^{ac} \epsilon^{bd} (y_d^\dagger y_d)_{ji} h_d^a h_d^{b*} \tilde{q}_{Li}^c \tilde{q}_{Lj}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4} |h_u|^2 |\tilde{t}_L|^2 - \frac{g_Y^2}{4} |h_u|^2 |\tilde{t}_L|^2 + \frac{g_2^2}{2} |h_u^* \tilde{t}_{Li}|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_d|^2 |\tilde{t}_L|^2 + \frac{g_Y^2}{4} |h_d|^2 |\tilde{t}_L|^2 + \frac{g_2^2}{2} |h_d^* \tilde{t}_{Li}|^2 + \epsilon^{ac} \epsilon^{bd} (y_e^\dagger y_e)_{ji} h_d^a h_d^{b*} \tilde{t}_{Li}^c \tilde{t}_{Lj}^{d*} \right) \\
& + |h_u|^2 \left(-\frac{g_Y^2}{3} |\tilde{u}_R|^2 + (y_u y_u^\dagger)_{ij} \tilde{u}_{Ri}^* \tilde{u}_{Rj} \right) + \frac{g_Y^2}{3} |h_d|^2 |\tilde{u}_R|^2 \\
& + \frac{g_Y^2}{6} |h_u|^2 |\tilde{d}_R|^2 + |h_d|^2 \left(-\frac{g_Y^2}{6} |\tilde{d}_R|^2 + (y_d y_d^\dagger)_{ij} \tilde{d}_{Ri}^* \tilde{d}_{Rj} \right) - \left[(y_u y_d^\dagger)_{ij} \tilde{u}_{Ri}^* \tilde{d}_{Rj} (h_d^* h_u) + \text{H.c.} \right] \\
& + \frac{g_Y^2}{2} |h_u|^2 |\tilde{e}_R|^2 + |h_d|^2 \left(-\frac{g_Y^2}{2} |\tilde{e}_R|^2 + (y_e y_e^\dagger)_{ij} \tilde{e}_{Ri}^* \tilde{e}_{Rj} \right) \\
& + \left[-\frac{1}{2} \epsilon^{ab} \epsilon^{xyz} y_{ulk} \lambda_{lij}'' h_u^a \tilde{d}_{Ri}^* \tilde{d}_{Rj}^* \tilde{q}_{Lk}^{bz} + \epsilon^{ab} \epsilon^{xyz} y_{dlk} \lambda_{ijl}'' h_d^a \tilde{u}_{Ri}^* \tilde{d}_{Rj}^* \tilde{q}_{Lk}^{bz} - y_{uil} \lambda_{klj}^* h_u^a \tilde{u}_{Ri}^* \tilde{d}_{Rj}^* \tilde{t}_{Lk}^{a*} \right. \\
& \quad \left. + y_{dil} \lambda_{klj}^* h_d^a \tilde{d}_{Ri}^* \tilde{d}_{Rj}^* \tilde{t}_{Lk}^{a*} - \epsilon^{ab} \epsilon^{cd} y_{dli} \lambda_{kjl}' h_d^a \tilde{q}_{Li}^b \tilde{q}_{Lj}^c \tilde{t}_{Lk}^{d*} + y_{eil} \lambda_{klj}^* h_d^a \tilde{e}_{Ri}^* \tilde{e}_{Rj}^* \tilde{t}_{Lk}^{a*} \right. \\
& \quad \left. - y_{ejl} \lambda_{kli}' h_d^a \tilde{e}_{Ri}^* \tilde{d}_{Rj}^* \tilde{t}_{Lk}^{a*} + \frac{1}{2} \epsilon^{ab} \epsilon^{cd} y_{eli} \lambda_{jkl}' h_d^a \tilde{t}_{Li}^b \tilde{t}_{Lj}^c \tilde{t}_{Lk}^{d*} + \text{H.c.} \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8} \right) |\tilde{q}_L|^4 + \frac{g_2^2}{4} \tilde{q}_{Li}^{ax} \tilde{q}_{Lj}^{by} \tilde{q}_{Li}^{bx*} \tilde{q}_{Lj}^{ay*} + \frac{g_3^2}{4} \tilde{q}_{Li}^{ax} \tilde{q}_{Lj}^{by} \tilde{q}_{Li}^{ay*} \tilde{q}_{Lj}^{bx*} \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{2g_Y^2}{9} \right) |\tilde{u}_R|^4 + \frac{g_3^2}{4} \tilde{u}_{Ri}^{x*} \tilde{u}_{Rj}^{y*} \tilde{u}_{Ri}^y \tilde{u}_{Rj}^x \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18} \right) |\tilde{d}_R|^4 + \frac{g_3^2}{4} \tilde{d}_{Ri}^{x*} \tilde{d}_{Rj}^{y*} \tilde{d}_{Ri}^y \tilde{d}_{Rj}^x + \frac{1}{2} \lambda_{mij}'' \lambda_{mkl}'' \tilde{d}_{Ri}^{x*} \tilde{d}_{Rj}^{y*} \tilde{d}_{Rk}^x \tilde{d}_{Rl}^y \right] \\
& + \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9} \right) |\tilde{u}_R|^2 |\tilde{q}_L|^2 - \frac{g_3^2}{2} \tilde{u}_{Ri}^{x*} \tilde{q}_{Lj}^{ax} \tilde{u}_{Ri}^y \tilde{q}_{Lj}^{ay*} + y_{uik} y_{ujl} \tilde{u}_{Ri}^{x*} \tilde{u}_{Rj}^y \tilde{q}_{Lk}^{ax} \tilde{q}_{Ll}^{ay*} \right] \\
& + \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18} \right) |\tilde{d}_R|^2 |\tilde{q}_L|^2 - \frac{g_3^2}{2} \tilde{d}_{Ri}^{x*} \tilde{q}_{Lj}^{ax} \tilde{d}_{Ri}^y \tilde{q}_{Lj}^{ay*} + (y_{dik} y_{djl}^* + \lambda_{mki}' \lambda_{mlj}^*) \tilde{d}_{Ri}^{x*} \tilde{q}_{Lk}^{ax} \tilde{d}_{Rj}^y \tilde{q}_{Ll}^{ay*} \right] \\
& + \left[-\left(\frac{g_3^2}{6} + \frac{2g_Y^2}{9} \right) |\tilde{d}_R|^2 |\tilde{u}_R|^2 + \left(\frac{g_3^2}{2} - \lambda_{ikm}'' \lambda_{ljm}'' \right) \tilde{u}_{Ri}^{x*} \tilde{d}_{Rj}^x \tilde{d}_{Rj}^{y*} \tilde{u}_{Ri}^y + \lambda_{ikm}'' \lambda_{ljm}'' \tilde{u}_{Ri}^{x*} \tilde{u}_{Rj}^x \tilde{d}_{Rk}^{y*} \tilde{d}_{Rl}^y \right] \\
& + \left[-\left(\frac{g_2^2}{4} + \frac{g_Y^2}{12} \right) |\tilde{t}_L|^2 |\tilde{q}_L|^2 + \frac{g_2^2}{2} \tilde{q}_{Li}^{ax} \tilde{q}_{Li}^{bx*} \tilde{t}_{Lj}^b \tilde{t}_{Lj}^{a*} + \epsilon^{ac} \epsilon^{bd} \lambda_{kim}' \lambda_{ljm}^* \tilde{q}_{Li}^{ax} \tilde{q}_{Lj}^{bx*} \tilde{t}_{Lk}^c \tilde{t}_{Ll}^{d*} \right] + \frac{g_Y^2}{6} |\tilde{e}_R|^2 |\tilde{q}_L|^2 \\
& + \left(-\frac{g_Y^2}{6} |\tilde{d}_R|^2 |\tilde{t}_L|^2 + \lambda_{lmi}' \lambda_{kmj}^* \tilde{d}_{Ri}^* \tilde{d}_{Rj}^* \tilde{t}_{Lk}^* \tilde{t}_{Ll} \right) + \frac{g_Y^2}{3} |\tilde{u}_R|^2 |\tilde{t}_L|^2 - \frac{2g_Y^2}{3} |\tilde{u}_R|^2 |\tilde{e}_R|^2 + \frac{g_Y^2}{3} |\tilde{d}_R|^2 |\tilde{e}_R|^2 \\
& + \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8} \right) |\tilde{t}_L|^4 + \frac{g_2^2}{4} \tilde{t}_{Li}^a \tilde{t}_{Lj}^b \tilde{t}_{Li}^{b*} \tilde{t}_{Lj}^{a*} + \frac{1}{4} \epsilon^{ab} \epsilon^{cd} \lambda_{ijm} \lambda_{klm}^* \tilde{t}_{Li}^a \tilde{t}_{Lj}^b \tilde{t}_{Lk}^{c*} \tilde{t}_{Ll}^{d*} \right] \\
& + \frac{g_Y^2}{2} |\tilde{e}_R|^4 + \left[-\frac{g_Y^2}{2} |\tilde{e}_R|^2 |\tilde{t}_L|^2 + (y_{eik} y_{ejl}^* + \lambda_{kmi} \lambda_{lmj}^*) \tilde{e}_{Ri}^* \tilde{e}_{Rj}^* \tilde{t}_{Lk}^* \tilde{t}_{Ll} \right] \\
& + \left[(y_{dik} y_{ejl}^* - \lambda_{mki}' \lambda_{lmj}^*) \tilde{d}_{Ri}^{x*} \tilde{q}_{Lk}^{ax} \tilde{e}_{Rj}^* \tilde{t}_{Ll}^{a*} - \epsilon^{ab} \epsilon^{xyz} \lambda_{ikm}'' \lambda_{ijm}^* \tilde{t}_{Li}^b \tilde{q}_{Lk}^{ax} \tilde{d}_{Rj}^y \tilde{u}_{Ri}^x + \text{H.c.} \right].
\end{aligned} \tag{9.44}$$

9.4. Higgs mechanism and fermion composition

The scalar potential includes

$$V_{\text{SUSY}} \supset |h_u|^2 \left(|\mu|^2 + \sum |\kappa_i|^2 \right) + |\mu|^2 |h_d|^2 + \frac{g_Z^2}{8} (|h_u|^2 - |h_d|^2)^2 + \frac{g_2^2}{2} |h_d^* h_u|^2 + \left(\kappa_i^* \mu \tilde{L}_i^* h_d + \text{H.c.} \right) + \kappa_i^* \kappa_j \tilde{L}_i^* \tilde{L}_j \quad (9.45)$$

$$V_{\text{SUSY}} \supset m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 + \epsilon^{ab} \left(b h_u^a h_d^b + b^* h_u^{a*} h_d^{b*} - b_i \tilde{L}_i^a h_u^b - b_i^* \tilde{L}_i^{a*} h_u^{b*} \right) + \tilde{L}_i^* M_{L_i}^2 h_d + \tilde{L}_i M_{L_i}^{2*} h_d^*; \quad (9.46)$$

the Higgs mass term is given by

$$V \supset \begin{pmatrix} h_u & h_d^* & \tilde{L}_i^* \end{pmatrix} \begin{pmatrix} |\mu|^2 + m_{H_u}^2 + \sum |\kappa_i|^2 & b & -b_j \\ b^* & |\mu|^2 + m_{H_d}^2 & \kappa_j \mu^* + M_{L_j}^{2*} \\ -b_i^* & \kappa_i^* \mu + M_{L_i}^2 & (m_L^2)_{ij} + \kappa_i^* \kappa_j \end{pmatrix} \begin{pmatrix} h_u^* \\ h_d \\ \tilde{L}_j \end{pmatrix} \quad (9.47)$$

while corresponding fermion terms are

$$\mathcal{L} \supset \epsilon^{ab} \left(-\mu \tilde{h}_u^a \tilde{h}_d^b - \kappa_i \tilde{h}_u^a \tilde{L}_i^b \right). \quad (9.48)$$

If the R -parity is not conserved, we redefine (H_d, L) superfields so that the mass matrix is block-diagonal, which corresponds to $U(4)_{H_d, L} \rightarrow U(3)_L \times U(1)_{H_d}$ (DOF counting: $16 \rightarrow 9 + 1$ to remove b_i'). Then lepton and \tilde{h}_d are mixed.^{*11} With R -parity conservation, we do not suffer from these mixings.

9.4.1. Higgs potential and induced mass in R -parity conserved case

We perform “SU(2)-notation fixing”, i.e., use the freedom associated to T_1 and T_2 of SU(2), so that $\langle h_u^+ \rangle = 0$. Then $\langle h_d^- \rangle = 0$ and effectively

$$V_{\text{pot}} = (|\mu|^2 + m_{H_u}^2) |h_u^0|^2 + (|\mu|^2 + m_{H_d}^2) |h_d^0|^2 + \frac{g_Z^2}{8} (|h_u^0|^2 - |h_d^0|^2)^2 - (b h_u^0 h_d^0 + \text{H.c.}). \quad (9.49)$$

We redefine H_d superfield so that $b > 0$.^{*12} Then $\arg \langle h_u^0 \rangle = -\arg \langle h_d^0 \rangle$ and, with T_3 -rotation, $\langle h_u^0 \rangle > 0$ and $\langle h_d^0 \rangle > 0$:

$$\langle h_u^0 \rangle =: v_u =: \frac{v_{\text{SM}}}{\sqrt{2}} \sin \beta, \quad \langle h_d^0 \rangle =: v_d =: \frac{v_{\text{SM}}}{\sqrt{2}} \cos \beta; \quad (9.50)$$

$$V_{\text{pot}} = \frac{1}{2} (|\mu|^2 + m_{H_u}^2) v_{\text{SM}}^2 \sin^2 \beta + \frac{1}{2} (|\mu|^2 + m_{H_d}^2) v_{\text{SM}}^2 \cos^2 \beta + \frac{g_Z^2}{32} v_{\text{SM}}^4 \cos^2 2\beta - \frac{1}{2} v_{\text{SM}}^2 b \sin 2\beta. \quad (9.51)$$

This potential can have two minima; one with $0 < \beta \leq \pi/4$ and the other with $\pi/4 \leq \beta < \pi/2$:

$$\tan \beta = \frac{B \mp \sqrt{B^2 - 4b^2}}{2b} \quad \left(\cos 2\beta = \pm \frac{\sqrt{B^2 - 4b^2}}{B} \right), \quad m_Z^2 := \frac{g_Z^2}{4} v_{\text{SM}}^2 = \left(\pm \frac{m_{H_d}^2 - m_{H_u}^2}{\sqrt{B^2 - 4b^2}} - 1 \right) B, \quad (9.52)$$

where $B := 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 > 2b > 0$ and m_Z is the Z -boson tree-level mass. Also,

$$\sin 2\beta = \frac{2b}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}, \quad m_Z^2 = \frac{-(m_{H_d}^2 - m_{H_u}^2)}{\cos 2\beta} - (2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2) \quad (9.53)$$

are satisfied in both solutions.

Higgs sector The Nambu-Goldstone-Higgs mixings and the mass terms for the charged Higgs bosons are given by

$$\begin{aligned} \mathcal{L} \supset & \partial_\mu h_d^- \partial^\mu h_d^- + \partial_\mu h_u^{+*} \partial^\mu h_u^{+*} + \left(-b - \frac{1}{2} g_Z^2 v_u v_d \right) (h_u^{+*} h_d^- + h_u^{+*} h_d^{-*}) \\ & + \left[\frac{g_Y^2 (v_u^2 - v_d^2) - g_Z^2 (v_u^2 + v_d^2)}{4} - |\mu|^2 - m_{H_d}^2 \right] |h_d^-|^2 + \left[\frac{g_Y^2 (v_d^2 - v_u^2) - g_Z^2 (v_u^2 + v_d^2)}{4} - |\mu|^2 - m_{H_u}^2 \right] |h_u^{+*}|^2 \\ & + \frac{ig_Z}{\sqrt{2}} W_\mu^- \partial^\mu (v_u h_u^{+*} - v_u h_u^{+*} - v_d h_d^{-*} + v_d h_d^-) \end{aligned} \quad (9.54)$$

and those for the neutral Higgs bosons are

$$\begin{aligned} \mathcal{L} \supset & \partial_\mu h_d^{0*} \partial^\mu h_d^0 + \partial_\mu h_u^{0*} \partial^\mu h_u^0 - \frac{g_Z^2 v_d^2}{8} (h_d^0 h_d^0 + h_d^{0*} h_d^{0*}) - \frac{g_Z^2 v_u^2}{8} (h_u^0 h_u^0 + h_u^{0*} h_u^{0*}) \\ & + \left(b + \frac{g_Z^2 v_u v_d}{4} \right) (h_u^0 h_d^0 + h_u^{0*} h_d^{0*}) + \frac{g_Z^2 v_u v_d}{4} (h_u^0 h_d^{0*} + h_u^{0*} h_d^0) \\ & + \left(\frac{g_Z^2 (v_u^2 - 2v_d^2)}{4} - |\mu|^2 - m_{H_d}^2 \right) |h_d^0|^2 + \left(\frac{g_Z^2 (v_d^2 - 2v_u^2)}{4} - |\mu|^2 - m_{H_u}^2 \right) |h_u^0|^2 \\ & + \frac{ig_Z}{2} Z_\mu \partial^\mu (v_d h_d^0 - v_d h_d^{0*} - v_u h_u^0 + v_u h_u^{0*}). \end{aligned} \quad (9.55)$$

^{*11}If we separated leptons and \tilde{h}_d first, sleptons would acquire VEVs and lepton-gaugino mixings would be induced.

^{*12}Note that T_3 -rotation induces $h_u^0 \rightarrow e^{i\theta/2} h_u^0$ and $h_d^0 \rightarrow e^{-i\theta/2} h_d^0$; it cannot remove the phase of b .

Therefore, with $m_W := c_w m_Z$ and

$$\begin{pmatrix} h_u^+ \\ h_d^{*-} \end{pmatrix} = \begin{pmatrix} s_\beta & c_\beta \\ -c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} -iG^+ \\ H^+ \end{pmatrix}, \quad \begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} s_\beta & c_\beta \\ -c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \quad (9.56)$$

we have

$$\begin{aligned} \mathcal{L} \supset & \partial_\mu G^{+*} \partial^\mu G^+ + \partial_\mu H^{+*} \partial^\mu H^+ + m_W (W_\mu^- \partial^\mu G^+ + W_\mu^+ \partial^\mu G^{+*}) + \left(\frac{m_{H_d}^2 - m_{H_u}^2}{\cos 2\beta} + m_Z^2 s_w^2 \right) |H^+|^2 \\ & + \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 + \frac{1}{2} (\partial_\mu A^0)^2 + \frac{1}{2} (\partial_\mu G^0)^2 + m_Z Z_\mu \partial^\mu G^0 - \frac{B}{2} A_0^2 \\ & - \frac{1}{4} (B + m_Z^2 + (B - m_Z^2) \cos 2\beta) \phi_u^2 - \frac{1}{4} (B + m_Z^2 - (B - m_Z^2) \cos 2\beta) \phi_d^2 + \frac{1}{2} (B + m_Z^2) (\sin 2\beta) \phi_u \phi_d. \end{aligned} \quad (9.57)$$

In particular, the tree-level masses are

$$m_{A_0}^2 = B = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2, \quad (9.58)$$

$$m_{H^\pm}^2 = m_{A_0}^2 + m_W^2, \quad (9.59)$$

$$m_{h,H} = \frac{1}{2} \left(m_{A_0}^2 + m_Z^2 \mp \sqrt{(m_{A_0}^2 - m_Z^2)^2 + 4m_{A_0}^2 m_Z^2 \sin^2 2\beta} \right) \quad (9.60)$$

with

$$\begin{pmatrix} \phi_d \\ \phi_u \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \quad \frac{\tan 2\alpha}{\tan 2\beta} = \frac{m_{A_0}^2 + m_Z^2}{m_{A_0}^2 - m_Z^2}. \quad (9.61)$$

The mixing α is stored in **ALPHA** block of SLHA, while **HMIX** stores

$$\mu = \mu, \quad \tan \beta = \tan \beta, \quad v = v_{\text{SM}} (\sim 246 \text{ GeV}), \quad m_A^2 = \frac{2b}{\sin 2\beta}, \quad (9.62)$$

at the scale specified. The above discussion holds even with CP -violation, but quantum corrections mix the three Higgs bosons; such information should be stored in **(IM)VCHMIX**. ♣TODO: discuss when needed♣

Mass terms in the Lagrangian The other mass terms are given by

$$\begin{aligned} \mathcal{L} \supset & m_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} M_3 \tilde{g}_0^a \tilde{g}_0^a - \frac{1}{2} M_3^* \tilde{g}_0^a \tilde{g}_0^a \\ & + \left(-\frac{1}{2} M_1 \tilde{b} \tilde{b} - \frac{1}{2} M_2 \tilde{w}^3 \tilde{w}^3 + \mu \tilde{h}_u^0 \tilde{h}_d^0 + c_\beta m_Z s_w \tilde{h}_d^0 \tilde{b} - c_w c_\beta m_Z \tilde{h}_d^0 \tilde{w}^3 - m_Z s_w s_\beta \tilde{h}_u^0 \tilde{b} + c_w m_Z s_\beta \tilde{h}_u^0 \tilde{w}^3 + \text{h.c.} \right) \\ & - M_2 \tilde{w}^+ \tilde{w}^- - \mu \tilde{h}_u^+ \tilde{h}_d^- - M_2^* \tilde{w}^+ \tilde{w}^- - \mu^* \tilde{h}_u^+ \tilde{h}_d^- - \sqrt{2} m_W \left(c_\beta \tilde{h}_d^- \tilde{w}^+ + s_\beta \tilde{h}_u^+ \tilde{w}^- + c_\beta \tilde{h}_d^- \tilde{w}^+ + s_\beta \tilde{h}_u^+ \tilde{w}^- \right) \\ & - v_u y_{uij} u_{Ri}^c u_{Lj} - v_d y_{dij} d_{Ri}^c d_{Lj} - v_d y_{eij} e_{Ri}^c e_{Lj} - v_u y_{uij}^* \tilde{u}_{Ri}^c \tilde{u}_{Lj} - v_d y_{dij}^* \tilde{d}_{Ri}^c \tilde{d}_{Lj} - v_d y_{eij}^* \tilde{e}_{Ri}^c \tilde{e}_{Lj} \\ & - \tilde{u}_L^* \left(m_Q^2 + v_u^2 y_u^\dagger y_u + \frac{3 - 4s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{u}_L - \tilde{u}_R^* \left(m_{U^c}^2 + v_u^2 y_u y_u^\dagger + \frac{4s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{u}_R \\ & - v_u a_{uij} \tilde{u}_{Ri}^* \tilde{u}_{Lj} + v_d \mu^* y_{uij} \tilde{u}_{Ri}^* \tilde{u}_{Lj} - v_u a_{uij}^* \tilde{u}_{Ri} \tilde{u}_{Lj}^* + v_d \mu y_{uij}^* \tilde{u}_{Ri} \tilde{u}_{Lj}^* \\ & - \tilde{d}_L^* \left(m_Q^2 + v_d^2 y_d^\dagger y_d + \frac{-3 + 2s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{d}_L - \tilde{d}_R^* \left(m_{D^c}^2 + v_d^2 y_d y_d^\dagger + \frac{-2s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{d}_R \\ & - v_d a_{dij} \tilde{d}_{Ri}^* \tilde{d}_{Lj} + v_u \mu^* y_{dij} \tilde{d}_{Ri}^* \tilde{d}_{Lj} - v_d a_{dij}^* \tilde{d}_{Ri} \tilde{d}_{Lj}^* + v_u \mu y_{dij}^* \tilde{d}_{Ri} \tilde{d}_{Lj}^* \\ & - \tilde{\nu}_L^* \left(m_L^2 + \frac{1}{2} c_{2\beta} m_Z^2 \right) \tilde{\nu}_L \\ & - \tilde{e}_L^* \left(m_L^2 + v_d^2 y_e^\dagger y_e + \frac{-1 + 2s_w^2}{2} c_{2\beta} m_Z^2 \right) \tilde{e}_L - \tilde{e}_R^* \left(m_{E^c}^2 + v_d^2 y_e y_e^\dagger + (-s_w^2) c_{2\beta} m_Z^2 \right) \tilde{e}_R \\ & - v_d a_{eij} \tilde{e}_{Ri}^* \tilde{e}_{Lj} + v_u \mu^* y_{eij} \tilde{e}_{Ri}^* \tilde{e}_{Lj} - v_d a_{eij}^* \tilde{e}_{Ri} \tilde{e}_{Lj}^* + v_u \mu y_{eij}^* \tilde{e}_{Ri} \tilde{e}_{Lj}^*, \end{aligned} \quad (9.63)$$

where, at the tree level, the gauge boson mass m_W and m_Z , the gluino mass M_3 , and matter-fermion masses $v_u y_u$, $v_d y_d$, and $v_d y_e$ are given with the “correct” sign (as far as $M_3 > 0$, etc.).

Neutralinos and charginos The mass matrices for neutralinos and charginos are given by

$$\begin{aligned} -\mathcal{L} \supset & \frac{1}{2} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \end{pmatrix}^T \begin{pmatrix} M_1 & 0 & -c_\beta s_w m_Z & +s_\beta s_w m_Z \\ 0 & M_2 & +c_\beta c_w m_Z & -s_\beta c_w m_Z \\ -c_\beta s_w m_Z & +c_\beta c_w m_Z & 0 & -\mu \\ +s_\beta s_w m_Z & -s_\beta c_w m_Z & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \end{pmatrix} + \text{h.c.} \\ & + (\tilde{w}^- \quad \tilde{h}_d^-) \begin{pmatrix} M_2 & \sqrt{2} s_\beta m_W \\ \sqrt{2} c_\beta m_W & \mu \end{pmatrix} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_u^+ \end{pmatrix} + (\tilde{w}^- \quad \tilde{h}_d^-) \begin{pmatrix} M_2^* & \sqrt{2} s_\beta m_W \\ \sqrt{2} c_\beta m_W & \mu^* \end{pmatrix} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_u^+ \end{pmatrix}. \end{aligned} \quad (9.64)$$

Note that the mass matrices themselves are the same as those in SLHA convention, $\mathcal{M}_{\tilde{\psi}0}$ and $\mathcal{M}_{\tilde{\psi}+}$, while the fields are in different convention. Therefore, we continue our discussion based only on the mass matrices so that the discussion is free from the choice of field convention.

As $\mathcal{M}_{\tilde{\psi}0}$ is a complex symmetric matrix, there is a unitary matrix \tilde{N} such that $M_{\tilde{\psi}0} = \tilde{N}^* \mathcal{M}_{\tilde{\psi}0} \tilde{N}^\dagger$, where $M_{\tilde{\psi}0}$ is a *positive* diagonal matrix whose elements are (non-negative) singular values of $\mathcal{M}_{\tilde{\psi}0}$ and in increasing order (Autonne-Takagi factorization). In SLHA2 convention with CP -violation, this matrix \tilde{N} is stored as the (IM)NMIX blocks and the (positive) masses are stored in the MASS block. Meanwhile, if M_1 , M_2 and μ are real, $\mathcal{M}_{\tilde{\psi}0}$ is a real symmetric matrix and there is a real orthogonal matrix \hat{N} such that $\hat{M}_{\tilde{\psi}0} = \hat{N}^* \mathcal{M}_{\tilde{\psi}0} \hat{N}^\dagger = \hat{N} \mathcal{M}_{\tilde{\psi}0} \hat{N}^T$, where $\hat{M}_{\tilde{\psi}0}$ is a *real* diagonal matrix whose elements are the eigenvalues of $\mathcal{M}_{\tilde{\psi}0}$ and in absolute-value-increasing order (spectral theorem). This matrix \hat{N} is the NMIX block of SLHA convention and \hat{M}_{ii} is stored in the MASS block, hence MASS block may have negative values for neutralinos.

The chargino mass matrix $\mathcal{M}_{\tilde{\psi}+}$ is decomposed as $M_{\tilde{\psi}+} = U^* \mathcal{M}_{\tilde{\psi}+} V^\dagger$, where U and V are unitary matrices and the elements of the diagonal matrix $M_{\tilde{\psi}+}$ are singular values of $\mathcal{M}_{\tilde{\psi}+}$ (thus non-negative) and sorted in increasing order (singular value decomposition). These U and V are stored in (IM)UMIX and (IM)VMIX, and the singular values are stored in MASS block. Because the SVD theorem is closed in \mathbb{R} , if M_2 and μ are real, U and V can be real, and the IM-blocks are omitted.

In summary,

$$M_{\tilde{\psi}0} = \tilde{N}^* \mathcal{M}_{\tilde{\psi}0} \tilde{N}^\dagger, \quad \tilde{N} = (\text{IM})\text{NMIX}, \quad (\text{MASS}) = [M_{\tilde{\psi}0}]_{ii} \geq 0 \quad (\text{singular values}); \quad (9.65)$$

$$\hat{M}_{\tilde{\psi}0} = \hat{N} \mathcal{M}_{\tilde{\psi}0} \hat{N}^T, \quad \hat{N} = \text{NMIX}, \quad (\text{MASS}) = [\hat{M}_{\tilde{\psi}0}]_{ii} \in \mathbb{R} \quad (\text{eigenvalues}); \quad (9.66)$$

$$M_{\tilde{\psi}+} = U^* \mathcal{M}_{\tilde{\psi}+} V^\dagger, \quad U = (\text{IM})\text{UMIX}, \quad V = (\text{IM})\text{VMIX}, \quad (\text{MASS}) = [M_{\tilde{\psi}+}]_{ii} \geq 0 \quad (\text{singular values}). \quad (9.67)$$

Note that the singular values are equal to absolute values of the eigenvalues, which guarantees consistency of the two decomposition.

We then define matrix N by^{*13}

$$N = \begin{cases} \tilde{N} \\ \text{diag}(\varphi_i) \cdot \tilde{N} \end{cases} = \text{diag}(\varphi_i) \cdot ((\text{NMIX}) + i(\text{IMNMIX})); \quad \varphi_i = \begin{cases} 1 & \text{if } (\text{MASS})_i \geq 0, \\ i & \text{if } (\text{MASS})_i < 0. \end{cases} \quad (9.68)$$

It gives the proper mass diagonalization in both of the NMIX convention:

$$N^* \mathcal{M}_{\tilde{\psi}0} N^\dagger = \begin{cases} \tilde{N}^* \mathcal{M}_{\tilde{\psi}0} \tilde{N}^\dagger = M_{\tilde{\psi}0}, \\ \text{diag}(\varphi_i^*) \hat{N}^* \mathcal{M}_{\tilde{\psi}0} \hat{N}^\dagger \text{diag}(\varphi_i^*) = \text{diag}(\varphi_i^*) \hat{M}_{\tilde{\psi}0} \text{diag}(\varphi_i^*) \end{cases} = M_{\tilde{\psi}0} \quad (\text{neutralino masses} \geq 0). \quad (9.69)$$

Noting that the discussion up here is irrelevant of the convention, we have the neutralino/chargino mass eigenstates,

$$\tilde{\chi}_i^0 = N_{ij} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \end{pmatrix}_j, \quad \tilde{\chi}_i^+ = V_{ij} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_u^+ \end{pmatrix}_j, \quad \tilde{\chi}_i^- = U_{ij} \begin{pmatrix} \tilde{w}^- \\ \tilde{h}_d^- \end{pmatrix}_j, \quad (9.70)$$

in our convention and the mass terms are now

$$-\mathcal{L} \supset \frac{1}{2} (\tilde{\chi}^0)^T M_{\tilde{\psi}0} \tilde{\chi}^0 + (\tilde{\chi}^-)^T M_{\tilde{\psi}+} \tilde{\chi}^+ + \text{h.c.} \quad (9.71)$$

Quarks, leptons, and super-CKM basis We here take the super-CKM basis. In the “original” Lagrangian,

$$-\mathcal{L} \supset u_R^c (v_u y_u) u_L + d_R^c (v_d y_d) d_L + e_R^c (v_e y_e) e_L + \text{h.c.} \quad (9.72)$$

$$= u_R^c (v_u U_u y_u^{\text{diag}} V_u^\dagger) u_L + d_R^c (v_d U_d y_d^{\text{diag}} V_d^\dagger) d_L + e_R^c (v_e U_e y_e^{\text{diag}} V_e^\dagger) e_L + \text{h.c.}, \quad (9.73)$$

so the super-CKM basis is given by

$$[Q^1, Q^2, L, U^c, D^c, E^c]_{\text{super-CKM}} = [V_u^\dagger Q^1, V_d^\dagger Q^2, V_e^\dagger L, U^c U_u, D^c U_d, E^c U_e]_{\text{“original”}}. \quad (9.74)$$

Then the CKM mixings appear as, for example,

$$[\bar{u}_L \bar{\sigma}^\mu d_L W_\mu^+ + \bar{d}_L \bar{\sigma}^\mu u_L W_\mu^-]_{\text{“original”}} = [\bar{u}_L V_u^\dagger V_d \bar{\sigma}^\mu d_L W_\mu^+ + \bar{d}_L V_d^\dagger V_u \bar{\sigma}^\mu u_L W_\mu^-]_{\text{super-CKM}}; \quad (9.75)$$

i.e., defining $V_{\text{CKM}} = V_u^\dagger V_d$ as in Sec. 6.5, the Lagrangian is amended as, e.g., $\bar{u}_L d_L \rightarrow \bar{u}_L V_{\text{CKM}} d_L$, $\bar{d}_L^* \tilde{u}_L \rightarrow \bar{d}_L^* V_{\text{CKM}}^\dagger \tilde{u}_L$.

The Θ_B - and Θ_W -terms are now removable. To this end, we rotate the matter superfields as

$$Q^1 \rightarrow e^{i\theta} Q^1, \quad Q^2 \rightarrow e^{i\theta'} Q^2, \quad U^c \rightarrow e^{-i\theta} U^c, \quad D^c \rightarrow e^{-i\theta'} D^c, \quad (9.76)$$

which induces $\Delta\Theta_G = 0$, $\Delta\Theta_W \propto \theta + \theta'$, and $\Delta\Theta_B \propto 5\theta + \theta'$, while the Lagrangian is unchanged except for the overall phase of V_{CKM} . So we can remove Θ_B and Θ_W , or in other words, pass the CP -violation in Θ -terms to V_{CKM} .^{*14} If one (or more) quarks were massless, we could rotate the corresponding right-handed quark to remove Θ_G as well.

^{*13}The sign of φ_i is arbitrary and (should be) unphysical.

^{*14}There are other possible transformations $(L, E^c) \rightarrow e^{(\pm)i\theta''} (L, E^c)$ and in total we may have $\Delta\Theta_W \propto \theta + \theta' + 2\theta''$ and $\Delta\Theta_B \propto 5\theta + \theta' + 6\theta''$. To remove both Θ -terms, we have to take $\theta \neq \theta'$ and V_{CKM} is anyway modified. The angle may chosen generation-dependently, and then $\Delta\Theta$'s should be read as $\theta \rightarrow \sum \theta_i/3$, etc.

Squark masses in super-CKM basis Finally, the squark masses are given by

$$\begin{aligned}
-\mathcal{L} \supset & \tilde{u}_L^* \left(m_Q^2 + m_u^2 + \frac{3-4s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{u}_L + \tilde{u}_R^* \left(m_{U^c}^2 + m_u^2 + \frac{4s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{u}_R \\
& + \tilde{u}_R^* (v_u a_u - \mu^* m_u \cot \beta) \tilde{u}_L + \tilde{u}_L^* (v_u a_u^\dagger - \mu m_u \cot \beta) \tilde{u}_R \\
& + \tilde{d}_L^* \left(V_d^\dagger (V_u m_Q^2 V_u^\dagger) V_d + m_d^2 + \frac{-3+2s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{d}_L + \tilde{d}_R^* \left(m_{D^c}^2 + m_d^2 + \frac{-2s_w^2}{6} c_{2\beta} m_Z^2 \right) \tilde{d}_R \\
& + \tilde{d}_R^* (v_d a_d - \mu^* m_d \tan \beta) \tilde{d}_L + \tilde{d}_L^* (v_d a_d^\dagger - \mu m_d \tan \beta) \tilde{d}_R \\
& + \tilde{\nu}_L^* \left(m_L^2 + \frac{1}{2} c_{2\beta} m_Z^2 \right) \tilde{\nu}_L \\
& + \tilde{e}_L^* \left(m_L^2 + m_e^2 + \frac{-1+2s_w^2}{2} c_{2\beta} m_Z^2 \right) + \tilde{e}_R^* (m_{E^c}^2 + m_e^2 + (-s_w^2) c_{2\beta} m_Z^2) \tilde{e}_R \\
& + \tilde{e}_R^* (v_d a_e - \mu^* m_e \tan \beta) \tilde{e}_L + \tilde{e}_L^* (v_d a_e^\dagger - \mu m_e \tan \beta) \tilde{e}_R,
\end{aligned} \tag{9.77}$$

where the sfermion soft masses, yukawas, and a -terms are rewritten in super-CKM basis:

$$[m_Q^2, m_{U^c}^2, m_{D^c}^2, m_L^2, m_{E^c}^2]_{\text{super-CKM}} = [V_d^\dagger m_Q^2 V_d, U_u^\dagger m_{U^c}^2 U_u, U_d^\dagger m_{D^c}^2 U_d, V_e^\dagger m_L^2 V_e, U_e^\dagger m_{E^c}^2 U_e]_{\text{“original”}}, \tag{9.78}$$

$$[a_u, a_d, a_e]_{\text{super-CKM}} = [U_u^\dagger a_u V_u, U_d^\dagger a_d V_d, U_e^\dagger a_e V_e]_{\text{“original”}} \tag{9.79}$$

(note that m_Q^2 is diagonalized for down-type; not for up-type). In matrix form,

$$\begin{aligned}
-\mathcal{L} \supset & \begin{pmatrix} \tilde{u}_{Li}^* & \tilde{u}_{Ri}^* \end{pmatrix} \begin{pmatrix} [V_{\text{CKM}} m_Q^2 V_{\text{CKM}}^\dagger]_{ij} + \left(m_u^2 + \frac{3-4s_w^2}{6} c_{2\beta} m_Z^2 \right) \delta_{ij} & v_u [a_u^\dagger]_{ij} - (\mu m_u \cot \beta) \delta_{ij} \\ v_u [a_u]_{ij} - (\mu^* m_u \cot \beta) \delta_{ij} & [m_{U^c}^2]_{ij} + \left(m_u^2 + \frac{2s_w^2}{3} c_{2\beta} m_Z^2 \right) \delta_{ij} \end{pmatrix} \begin{pmatrix} \tilde{u}_{Lj} \\ \tilde{u}_{Rj} \end{pmatrix} \\
& + \begin{pmatrix} \tilde{d}_{Li}^* & \tilde{d}_{Ri}^* \end{pmatrix} \begin{pmatrix} [m_Q^2]_{ij} + \left(m_d^2 + \frac{-3+2s_w^2}{6} c_{2\beta} m_Z^2 \right) \delta_{ij} & v_d [a_d^\dagger]_{ij} - (\mu m_d \tan \beta) \delta_{ij} \\ v_d [a_d]_{ij} - (\mu^* m_d \tan \beta) \delta_{ij} & [m_{D^c}^2]_{ij} + \left(m_d^2 - \frac{s_w^2}{3} c_{2\beta} m_Z^2 \right) \delta_{ij} \end{pmatrix} \begin{pmatrix} \tilde{d}_{Lj} \\ \tilde{d}_{Rj} \end{pmatrix} \\
& + \tilde{\nu}_{Li}^* \left([m_L^2]_{ij} + \left(\frac{1}{2} c_{2\beta} m_Z^2 \right) \delta_{ij} \right) \tilde{\nu}_{Lj} \\
& + \begin{pmatrix} \tilde{e}_{Li}^* & \tilde{e}_{Ri}^* \end{pmatrix} \begin{pmatrix} [m_L^2]_{ij} + \left(m_e^2 + \frac{-1+2s_w^2}{2} c_{2\beta} m_Z^2 \right) \delta_{ij} & v_d [a_e^\dagger]_{ij} - (\mu m_e \tan \beta) \delta_{ij} \\ v_d [a_e]_{ij} - (\mu^* m_e \tan \beta) \delta_{ij} & [m_{E^c}^2]_{ij} + \left(m_e^2 - s_w^2 c_{2\beta} m_Z^2 \right) \delta_{ij} \end{pmatrix} \begin{pmatrix} \tilde{e}_{Lj} \\ \tilde{e}_{Rj} \end{pmatrix}
\end{aligned} \tag{9.80}$$

$$= \begin{pmatrix} \tilde{u}_{Li}^* & \tilde{u}_{Ri}^* \end{pmatrix} \mathcal{M}_u \begin{pmatrix} \tilde{u}_{Lj} \\ \tilde{u}_{Rj} \end{pmatrix} + \begin{pmatrix} \tilde{d}_{Li}^* & \tilde{d}_{Ri}^* \end{pmatrix} \mathcal{M}_d \begin{pmatrix} \tilde{d}_{Lj} \\ \tilde{d}_{Rj} \end{pmatrix} + \tilde{\nu}_L^* \mathcal{M}_\nu \tilde{\nu}_L + \begin{pmatrix} \tilde{e}_{Li}^* & \tilde{e}_{Ri}^* \end{pmatrix} \mathcal{M}_e \begin{pmatrix} \tilde{e}_{Lj} \\ \tilde{e}_{Rj} \end{pmatrix} \tag{9.81}$$

The sfermion mass matrices are diagonalized by unitary matrices as

$$\mathcal{M} = R \mathcal{M}^{\text{diag}} R^\dagger; \quad \tilde{f}_i = R_{ij} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}_j = (R_{ij}^L \quad R_{ij}^R) \begin{pmatrix} \tilde{f}_{Lj} \\ \tilde{f}_{Rj} \end{pmatrix}; \quad \tilde{f}_{Li} = [R^{L\dagger}]_{ij} \tilde{f}_i, \quad \tilde{f}_{Ri} = [R^{R\dagger}]_{ij} \tilde{f}_i. \tag{9.82}$$

where R_{ij} is 6×6 and $R_{ij}^{L,R}$ are 3×6 matrices (except for sneutrinos).

These R -matrices are the same as **DSQMIX** etc. of **SLHA2** format, but note that our notation for the other parameters is slightly different from **SLHA**'s:

$$\hat{m}_{\tilde{Q},\tilde{L}}^2 = m_{\tilde{Q},L}^2|_{\text{“orig”}}, \quad \hat{m}_{\tilde{u},\tilde{d},\tilde{e}}^2 = (m_{U^c,D^c,E^c}^2|_{\text{“orig”}})^T, \quad \hat{T}_{U,D,E} = (a_{u,d,e}|_{\text{“orig”}})^T, \tag{9.83}$$

$$\hat{m}_{\tilde{Q},\tilde{L}}^2 = m_{\tilde{Q},L}^2|_{\text{sCKM}}, \quad \hat{m}_{\tilde{u},\tilde{d},\tilde{e}}^2 = U^\dagger T^T V = m_{U^c,D^c,E^c}^2|_{\text{sCKM}}, \quad \hat{T}_{U,D,E} = U^\dagger T^T V = a_{u,d,e}|_{\text{sCKM}}, \tag{9.84}$$

together with $\mathbf{Y}_{u,d,e} = (y_{u,d,e})^T$. Anyway, the **SLHA2** blocks corresponds to the variable in our convention as

$$(\mathbf{IM})\mathbf{MSX2}(\mathbf{IN}) = m_{\tilde{Q},L,U^c,D^c,E^c}^2|_{\text{sCKM};\overline{\text{DR}}}, \quad (\mathbf{IM})\mathbf{TX}(\mathbf{IN}) = a_{u,d,e}|_{\text{sCKM};\overline{\text{DR}}}, \quad \mathbf{YX} = y_{u,d,e}|_{\text{sCKM};\overline{\text{DR}}}. \tag{9.85}$$

The sfermion mass matrices above are in super-CKM basis, so their off-diagonal entries immediately induce flavor violation or sfermion left-right mixing. In old **SLHA** format, we assume that flavor- and CP -violation is absent and left-right mixing is ignorable except for third generation, which leads

$$\mathcal{M}_d = \begin{pmatrix} \tilde{m}_{dL11}^2 & & & & \\ & \tilde{m}_{dL22}^2 & & & \\ & & \tilde{m}_{dL33}^2 & & \\ & & & \tilde{m}_{dR11}^2 & v_d a_{d33} - \mu m_b \tan \beta \\ & & & & \tilde{m}_{dR22}^2 \\ & & & & & \tilde{m}_{dR33}^2 \end{pmatrix}; \quad \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{b}_1 \\ \tilde{d}_R \\ \tilde{s}_R \\ \tilde{b}_2 \end{pmatrix} = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & F_{11} & & & \\ & & & 1 & & \\ & & & & F_{21} & \\ & & & & & F_{22} \end{pmatrix} \begin{pmatrix} \tilde{d}_{Lj} \\ \tilde{d}_{Rj} \end{pmatrix} \tag{9.86}$$

and these F_{ij} are stored in **SBOTMIX** etc.

Squark masses in super-CKM basis The gluino mass and phase is given by

$$m_{\tilde{g}} = |M_3|; \quad M_3 = m_{\tilde{g}} e^{i\theta_3} = m_{\tilde{g}} \varphi_{\tilde{g}}^{-2}; \quad \tilde{g}_0 = \varphi_{\tilde{g}} \tilde{g}. \tag{9.87}$$

9.4.2. Fermion composition

Now we show the fermion-related Lagrangian terms verbosely. In super-CKM basis, the interaction terms are

$$\begin{aligned}
\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}} = & i\bar{\chi}_i^0 \bar{\sigma}^\mu \partial_\mu \tilde{\chi}_i^0 + i\bar{\chi}_i^- \bar{\sigma}^\mu \partial_\mu \tilde{\chi}_i^- + i\bar{\chi}_i^+ \bar{\sigma}^\mu \partial_\mu \tilde{\chi}_i^+ + i\bar{g}^a \bar{\sigma}^\mu \partial_\mu \tilde{g}^a + i\bar{u}_{Li} \bar{\sigma}^\mu \partial_\mu u_{Li} + i\bar{u}_{Ri}^c \bar{\sigma}^\mu \partial_\mu u_{Ri}^c \\
& + i\bar{d}_{Li} \bar{\sigma}^\mu \partial_\mu d_{Li} + i\bar{d}_{Ri}^c \bar{\sigma}^\mu \partial_\mu d_{Ri}^c + i\bar{\nu}_{Li} \bar{\sigma}^\mu \partial_\mu \nu_{Li} + i\bar{e}_{Li} \bar{\sigma}^\mu \partial_\mu e_{Li} + i\bar{e}_{Ri}^c \bar{\sigma}^\mu \partial_\mu e_{Ri}^c - \frac{m_{\tilde{g}}}{2} (\tilde{g}^a \tilde{g}^a + \text{h.c.}) \\
& - m_{\tilde{\chi}_i^\pm} (\tilde{\chi}_i^- \tilde{\chi}_i^+ + \tilde{\chi}_i^- \tilde{\chi}_i^+) - m_{di} (d_{Ri}^c d_{Li} + \bar{d}_{Ri}^c \bar{d}_{Li}) - m_{ui} (u_{Ri}^c u_{Lj} + \bar{u}_{Ri}^c \bar{u}_{Li}) - m_{ei} (e_{Ri}^c e_{Li} + \bar{e}_{Ri}^c \bar{e}_{Li}) \\
& + \left[\sqrt{2} g_3 \varphi_{\tilde{g}} \tilde{g}^a \left(\tau^{yxa*} \tilde{d}_{Ri}^{cy} + \tau^{yxa*} \tilde{u}_{Ri}^{cy} - \tau^{yxa} \tilde{d}_{Li}^{yx} - \tau^{yxa} \tilde{u}_{Li}^{yx} \right) + \text{h.c.} \right] - i g_3 f^{abc} g_\mu^a (\bar{g}^b \bar{\sigma}^\mu \tilde{g}^c) \\
& + g_3 g_\mu^a (\bar{d}_L \bar{\sigma}^\mu \tau^a d_L + \bar{u}_L \bar{\sigma}^\mu \tau^a u_L - \bar{d}_R^c \bar{\sigma}^\mu \tau^{a*} d_R^c - \bar{u}_R^c \bar{\sigma}^\mu \tau^{a*} u_R^c) + A_\mu |e| \left[\frac{2}{3} \bar{u}_L \bar{\sigma}^\mu u_L - \frac{2}{3} \bar{u}_R^c \bar{\sigma}^\mu u_R^c - \frac{1}{3} \bar{d}_L \bar{\sigma}^\mu d_L \right. \\
& + \frac{1}{3} \bar{d}_R^c \bar{\sigma}^\mu d_R^c - \bar{e}_L \bar{\sigma}^\mu e_L + \bar{e}_R^c \bar{\sigma}^\mu e_R^c + \bar{\chi}_i^+ \bar{\sigma}^\mu \tilde{\chi}_i^+ - \bar{\chi}_i^- \bar{\sigma}^\mu \tilde{\chi}_i^- \left. \right] + g_Z Z_\mu \left[\frac{3 - 4s_w^2}{6} \bar{u}_L \bar{\sigma}^\mu u_L + \frac{2s_w^2}{3} \bar{u}_R^c \bar{\sigma}^\mu u_R^c \right. \\
& - \frac{3 - 2s_w^2}{6} \bar{d}_L \bar{\sigma}^\mu d_L - \frac{s_w^2}{3} \bar{d}_R^c \bar{\sigma}^\mu d_R^c + \frac{1}{2} \bar{\nu}_L \bar{\sigma}^\mu \nu_L + \frac{2s_w^2 - 1}{2} \bar{e}_L \bar{\sigma}^\mu e_L - s_w^2 \bar{e}_R^c \bar{\sigma}^\mu e_R^c + \frac{N_{j3} N_{i3}^* - N_{j4} N_{i4}^*}{2} \bar{\chi}_j^0 \bar{\sigma}^\mu \tilde{\chi}_i^0 \\
& + \left(c_w^2 V_{j1} V_{i1}^* + \frac{V_{j2} V_{i2}^* (c_w^2 - s_w^2)}{2} \right) \bar{\chi}_j^+ \bar{\sigma}^\mu \tilde{\chi}_i^+ + \left(\frac{U_{j2} U_{i2}^* (s_w^2 - c_w^2)}{2} - c_w^2 U_{j1} U_{i1}^* \right) \bar{\chi}_j^- \bar{\sigma}^\mu \tilde{\chi}_i^- \left. \right] \\
& + \frac{g_2}{\sqrt{2}} W_\mu^+ \left[\bar{u}_L \bar{\sigma}^\mu V_{\text{CKM}} d_L + \bar{\nu}_L \bar{\sigma}^\mu e_L + \left(V_{j2} N_{i4}^* - \sqrt{2} V_{j1} N_{i2}^* \right) \bar{\chi}_j^+ \bar{\sigma}^\mu \tilde{\chi}_i^0 + \left(\sqrt{2} N_{j2} U_{i1}^* + N_{j3} U_{i2}^* \right) \bar{\chi}_j^0 \bar{\sigma}^\mu \tilde{\chi}_i^- \right] \\
& + \frac{g_2}{\sqrt{2}} W_\mu^- \left[\bar{d}_L \bar{\sigma}^\mu V_{\text{CKM}}^\dagger u_L + \bar{e}_L \bar{\sigma}^\mu \nu_L + \left(N_{j4} V_{i2}^* - \sqrt{2} N_{j2} V_{i1}^* \right) \bar{\chi}_j^0 \bar{\sigma}^\mu \tilde{\chi}_i^+ + \left(\sqrt{2} U_{j1} N_{i2}^* + U_{j2} N_{i3}^* \right) \bar{\chi}_j^- \bar{\sigma}^\mu \tilde{\chi}_i^0 \right] \\
& + \phi_u \left(-\frac{y_{ui}}{\sqrt{2}} u_{Ri}^c u_{Li} + \frac{N_{i4}^* (g_2 N_{j2}^* - g_Y N_{j1}^*)}{2} \tilde{\chi}_i^0 \tilde{\chi}_j^0 - \frac{g_2 U_{i1}^* V_{j2}^*}{\sqrt{2}} \tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.} \right) \\
& + \phi_d \left(-\frac{y_{di}}{\sqrt{2}} d_{Ri}^c d_{Li} - \frac{y_{ei}}{\sqrt{2}} e_{Ri}^c e_{Li} + \frac{N_{i3}^* (g_Y N_{j1}^* - g_2 N_{j2}^*)}{2} \tilde{\chi}_i^0 \tilde{\chi}_j^0 - \frac{g_2 U_{i2}^* V_{j1}^*}{\sqrt{2}} \tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.} \right) \\
& + \frac{A^0}{\sqrt{2}} \left[-i c_{\beta 0} y_{ui} u_{Ri}^c u_{Li} - i s_{\beta 0} y_{di} d_{Ri}^c d_{Li} - i s_{\beta 0} y_{ei} e_{Ri}^c e_{Li} - \frac{i (s_{\beta 0} N_{i3}^* - c_{\beta 0} N_{i4}^*) (g_Y N_{j1}^* - g_2 N_{j2}^*)}{\sqrt{2}} \tilde{\chi}_i^0 \tilde{\chi}_j^0 \right. \\
& + i g_2 (c_{\beta 0} U_{i1}^* V_{j2}^* + s_{\beta 0} U_{i2}^* V_{j1}^*) \tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.} \left. \right] + \frac{G^0}{\sqrt{2}} \left[-i s_{\beta 0} y_{ui} u_{Ri}^c u_{Li} + i c_{\beta 0} y_{di} d_{Ri}^c d_{Li} + i c_{\beta 0} y_{ei} e_{Ri}^c e_{Li} \right. \\
& + \frac{i (g_Y N_{j1}^* - g_2 N_{j2}^*) (c_{\beta 0} N_{i3}^* + s_{\beta 0} N_{i4}^*)}{\sqrt{2}} \tilde{\chi}_i^0 \tilde{\chi}_j^0 - i g_2 (c_{\beta 0} U_{i2}^* V_{j1}^* - s_{\beta 0} U_{i1}^* V_{j2}^*) \tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.} \left. \right] \\
& + \left[\left(\frac{U_{i2}^* (g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} - g_2 U_{i1}^* N_{j3}^* \right) s_{\beta+} H^+ \tilde{\chi}_i^- \tilde{\chi}_j^0 - \left(g_2 V_{i1}^* N_{j4}^* + \frac{V_{i2}^* (g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} \right) c_{\beta+} H^- \tilde{\chi}_i^+ \tilde{\chi}_j^0 \right. \\
& + c_{\beta+} (u_{Ri}^c y_u V_{\text{CKM}} d_L) H^+ + s_{\beta+} (d_{Ri}^c y_d V_{\text{CKM}}^\dagger u_L) H^- + s_{\beta+} (e_{Ri}^c y_e \nu_L) H^- + \text{h.c.} \left. \right] \\
& + \left[i G^+ c_{\beta+} \left(\frac{U_{i2}^* (g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} - g_2 U_{i1}^* N_{j3}^* \right) \tilde{\chi}_i^- \tilde{\chi}_j^0 - i G^- s_{\beta+} \left(g_2 V_{i1}^* N_{j4}^* + \frac{V_{i2}^* (g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} \right) \tilde{\chi}_i^+ \tilde{\chi}_j^0 \right. \\
& - i s_{\beta+} (u_{Ri}^c y_u V_{\text{CKM}} d_L) G^+ - i c_{\beta+} (d_{Ri}^c y_d V_{\text{CKM}}^\dagger u_L) G^- - i c_{\beta+} (e_{Ri}^c y_e \nu_L) G^- + \text{h.c.} \left. \right] \\
& + \left[-\frac{\tilde{u}_L^* u_L \tilde{\chi}_j^0 (g_Y N_{j1}^* + 3g_2 N_{j2}^*)}{3\sqrt{2}} + \frac{2}{3} \sqrt{2} g_Y \tilde{u}_R u_{Ri}^c \tilde{\chi}_j^0 N_{j1}^* - y_{ui} u_{Ri}^c \tilde{u}_L \tilde{\chi}_j^0 N_{j4}^* - y_{ui} \tilde{u}_R^* u_{Li} \tilde{\chi}_j^0 N_{j4}^* \right. \\
& - \frac{\tilde{d}_L^* d_L \tilde{\chi}_j^0 (g_Y N_{j1}^* - 3g_2 N_{j2}^*)}{3\sqrt{2}} - \frac{1}{3} \sqrt{2} g_Y \tilde{d}_R d_{Ri}^c \tilde{\chi}_j^0 N_{j1}^* - y_{di} \tilde{d}_L d_{Ri}^c \tilde{\chi}_j^0 N_{j3}^* - y_{di} \tilde{d}_R^* d_{Li} \tilde{\chi}_j^0 N_{j3}^* \\
& + \frac{\tilde{\nu}_L^* \nu_L \tilde{\chi}_j^0 (g_Y N_{j1}^* - g_2 N_{j2}^*)}{\sqrt{2}} - \sqrt{2} g_Y \tilde{\nu}_R e_{Ri}^c \tilde{\chi}_j^0 N_{j1}^* + \frac{\tilde{e}_L^* e_L \tilde{\chi}_j^0 (g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} \\
& - y_{ei} \tilde{e}_L e_{Ri}^c \tilde{\chi}_j^0 N_{j3}^* - y_{ei} \tilde{e}_R^* e_{Li} \tilde{\chi}_j^0 N_{j3}^* + \text{h.c.} \left. \right] + \left[V_{i2}^* (u_{Ri}^c y_u V_{\text{CKM}} \tilde{d}_L) \tilde{\chi}_i^+ + V_{i2}^* (\tilde{u}_R y_u V_{\text{CKM}} d_L) \tilde{\chi}_i^+ \right. \\
& + U_{j2}^* (d_{Ri}^c y_d V_{\text{CKM}}^\dagger \tilde{u}_L) \tilde{\chi}_j^- + U_{j2}^* (\tilde{d}_R y_d V_{\text{CKM}}^\dagger u_L) \tilde{\chi}_j^- - U_{j1}^* g_2 (\tilde{d}_L V_{\text{CKM}}^\dagger u_L) \tilde{\chi}_j^- - V_{i1}^* g_2 (\tilde{u}_L V_{\text{CKM}} d_L) \tilde{\chi}_i^+ \\
& - V_{i1}^* g_2 \tilde{\nu}_L e_{Li} \tilde{\chi}_i^+ - U_{j1}^* g_2 \tilde{e}_L^* \nu_L \tilde{\chi}_j^- + U_{j2}^* y_{ei} \tilde{\nu}_L e_{Ri}^c \tilde{\chi}_j^- + U_{j2}^* y_{ei} \tilde{e}_R^* \nu_{Li} \tilde{\chi}_j^- + \text{h.c.} \left. \right] \\
& - \epsilon^{xyz} \left(\lambda_{ijk}'' \tilde{d}_{Ri}^{x*} u_{Ri}^{cy} d_{Rj}^{cz} + \frac{1}{2} \lambda_{kij}'' \tilde{u}_{Ri}^{x*} d_{Ri}^{cy} d_{Rj}^{cz} \right) + \lambda_{ijk} (-\tilde{e}_{Ri}^* \nu_{Li} e_{Lj} - \tilde{e}_{Lj} e_{Ri}^c \nu_{Li} + \tilde{\nu}_{Lj} e_{Ri}^c e_{Li}) \\
& + \lambda_{ijk}' (-\tilde{d}_{Ri}^* d_{Lj} \nu_{Li} + \tilde{d}_{Ri}^* u_{Lj} e_{Li} - \tilde{d}_{Lj} d_{Ri}^c \nu_{Li} + \tilde{u}_{Lj} d_{Ri}^c e_{Li} + \tilde{e}_{Li} d_{Ri}^c u_{Lj} - \tilde{\nu}_{Lj} d_{Ri}^c d_{Lj}) + \text{h.c.},
\end{aligned} \tag{9.88}$$

where R -parity violating terms are also shown as a reference; they are also redefined in super-CKM basis.

The full Lagrangian is given then by

$$\mathcal{L} = (\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}}) + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{scalar}}, \quad (9.89)$$

where $\mathcal{L}_{\text{vector}}$ is given by Eq. (9.38), which should be amended with $m_W^2 W^{+\mu} W_\mu^- + (m_Z^2/2) Z^\mu Z_\mu$, mass and kinetic terms of $\mathcal{L}_{\text{scalar}}$ should be read from this section, and the interaction terms of $\mathcal{L}_{\text{scalar}}$ should be read from Eq. (9.43) and Eq. (9.44), replacing Higgs fields by VEVs, Higgs bosons, and Nambu-Goldstone bosons.

These Weyl fermions are combined to Dirac and Majorana fermions:

$$\tilde{\chi}_i^0 = \begin{pmatrix} \tilde{\chi}_i^0 \\ \tilde{\chi}_i^0 \end{pmatrix}, \quad \tilde{\chi}_i^\pm = \begin{pmatrix} \tilde{\chi}_i^\pm \\ \tilde{\chi}_i^\mp \end{pmatrix}, \quad f = \begin{pmatrix} f_L \\ f_R^c \end{pmatrix}; \quad \tilde{\chi}_i^0 = (\tilde{\chi}_i^0 \quad \tilde{\chi}_i^0), \quad \tilde{\chi}_i^\pm = (\tilde{\chi}_i^\pm \quad \tilde{\chi}_i^\pm), \quad \bar{f} = (f_R^c \quad \bar{f}_L). \quad (9.90)$$

For example,

$$\begin{aligned} \bar{u} P_L d &= u_R^c d_L, \quad \bar{u} P_R d = \bar{u}_L \bar{d}_R^c, \quad u_R^c d_L + \text{h.c.} = \bar{u} P_L d + \bar{d} P_R u, \quad \bar{u} \gamma^\mu P_L d = \bar{u}_L \bar{\sigma}^\mu d_L, \quad \bar{u} \gamma^\mu P_R d = -\bar{d}_R^c \bar{\sigma}^\mu u_R^c; \\ \tilde{\chi}^0 u_L + \text{h.c.} &= \bar{\tilde{\chi}}^0 P_L u + \bar{u} P_R \tilde{\chi}^0, \quad \tilde{\chi}^0 u_R^c + \text{h.c.} = \bar{u} P_L \tilde{\chi}^0 + \bar{\tilde{\chi}}^0 P_R u, \\ \bar{u} \gamma^\mu P_{\{L,R\}} \tilde{\chi}^0 &= \{\bar{u}_L \bar{\sigma}^\mu \tilde{\chi}^0, -\bar{\tilde{\chi}}^0 \bar{\sigma}^\mu u_R^c\}, \quad \bar{\tilde{\chi}}^0 \gamma^\mu P_{\{L,R\}} u = \{\bar{\tilde{\chi}}^0 \bar{\sigma} u_L, -\bar{u}_R^c \bar{\sigma} \tilde{\chi}^0\}, \\ \tilde{\chi}_i^0 \bar{\sigma} \tilde{\chi}_j^0 &= \bar{\tilde{\chi}}_i^0 \gamma^\mu P_L \tilde{\chi}_j^0 = -\bar{\tilde{\chi}}_j^0 \gamma^\mu P_R \tilde{\chi}_i^0. \end{aligned}$$

With abbreviations

$$\begin{aligned} (\mathcal{N}_{\phi_u})_{ij} &:= N_{i4}^* (g_2 N_{j2}^* - g_Y N_{j1}^*) P_L + N_{i4} (g_2 N_{j2} - g_Y N_{j1}) P_R, \\ (\mathcal{N}_{\phi_d})_{ij} &:= N_{i3}^* (g_Y N_{j1}^* - g_2 N_{j2}^*) P_L + N_{i3} (g_Y N_{j1} - g_2 N_{j2}) P_R, \\ (\mathcal{N}_{A^0})_{ij} &:= -(s_{\beta 0} N_{i3}^* - c_{\beta 0} N_{i4}^*) (g_Y N_{j1}^* - g_2 N_{j2}^*) P_L + (s_{\beta 0} N_{i3} - c_{\beta 0} N_{i4}) (g_Y N_{j1} - g_2 N_{j2}) P_R, \\ (\mathcal{N}_{G^0})_{ij} &:= (g_Y N_{j1}^* - g_2 N_{j2}^*) (c_{\beta 0} N_{i3}^* + s_{\beta 0} N_{i4}^*) P_L - (g_Y N_{j1} - g_2 N_{j2}) (c_{\beta 0} N_{i3} + s_{\beta 0} N_{i4}) P_R, \\ (\mathcal{C}_{\phi_u})_{ij} &:= -U_{i1}^* V_{j2}^* P_L - U_{i1} V_{j2} P_R, \\ (\mathcal{C}_{\phi_d})_{ij} &:= -U_{i2}^* V_{j1}^* P_L - U_{i2} V_{j1} P_R, \\ (\mathcal{C}_{A^0})_{ij} &:= (c_{\beta 0} U_{i1}^* V_{j2}^* + s_{\beta 0} U_{i2}^* V_{j1}^*) P_L - (c_{\beta 0} U_{i1} V_{j2} + s_{\beta 0} U_{i2} V_{j1}) P_R, \\ (\mathcal{C}_{G^0})_{ij} &:= -(c_{\beta 0} U_{i2}^* V_{j1}^* - s_{\beta 0} U_{i1}^* V_{j2}^*) P_L + (c_{\beta 0} U_{i2} V_{j1} - s_{\beta 0} U_{i1} V_{j2}) P_R, \\ (\mathcal{C}_{H^-})_{ij} &:= g_2 V_{i1}^* N_{j4}^* + \frac{V_{i2}^* (g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}}, \\ (\mathcal{C}_{H^+})_{ij} &:= \frac{U_{i2}^* (g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} - g_2 U_{i1}^* N_{j3}^* \end{aligned}$$

we have

$$\begin{aligned}
& \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}} \\
&= \frac{1}{2} \bar{g}^a \left[(i\not{\partial} - m_{\bar{g}}) \delta^{ab} - i g_3 f^{cab} \not{g}^c \right] \tilde{g}^b + \frac{1}{2} \bar{\chi}_i^0 \left[(i\not{\partial} - m_{\chi_i^0}) \delta_{ij} + g_Z (N_{i3} N_{j3}^* - N_{i4} N_{j4}^*) \not{Z} P_L \right] \tilde{\chi}_j^0 \\
&+ \bar{\chi}_i^+ \left[(i\not{\partial} - m_{\chi_i^+} + |e| \not{A}) \delta_{ij} + g_Z \not{Z} \left(\frac{2c_w^2 V_{i1} V_{j1}^* + c_{2w} V_{i2} V_{j2}^*}{2} P_L + \frac{c_{2w} U_{i2} U_{j2}^* - 2c_w^2 U_{i1} U_{j1}^*}{2} P_R \right) \right] \tilde{\chi}_j^+ \\
&+ \bar{u}_i \left[i\not{\partial} - m_{u_i} + g_3 \tau^a \not{g}^a + \frac{2|e|}{3} \not{A} + g_Z \not{Z} \left(\frac{3 - 4s_w^2}{6} P_L - \frac{2s_w^2}{3} P_R \right) \right] u_i \\
&+ \bar{d}_i \left[i\not{\partial} - m_{d_i} + g_3 \tau^a \not{g}^a - \frac{|e|}{3} \not{A} + g_Z \not{Z} \left(-\frac{3 - 2s_w^2}{6} P_L + \frac{s_w^2}{3} P_R \right) \right] d_i \\
&+ \bar{\nu}_i \left(i\not{\partial} + \frac{g_Z}{2} \not{Z} P_L \right) \nu_i + \bar{e}_i \left[i\not{\partial} - m_{e_i} - |e| \not{A} + g_Z \not{Z} \left(\frac{2s_w^2 - 1}{2} P_L + s_w^2 P_R \right) \right] e_i \\
&+ \frac{g_2}{\sqrt{2}} W_\mu^+ \left[\bar{u} \gamma^\mu V_{\text{CKM}} P_L d + \bar{\nu} \gamma^\mu P_L e + \bar{\chi}_j^+ \gamma^\mu \left((-\sqrt{2} V_{j1} N_{i2}^* + V_{j2} N_{i4}^*) P_L + (-\sqrt{2} N_{i2} U_{j1}^* - N_{i3} U_{j2}^*) P_R \right) \tilde{\chi}_i^0 \right] \\
&+ \frac{g_2}{\sqrt{2}} W_\mu^- \left[\bar{d} \gamma^\mu V_{\text{CKM}}^\dagger P_L u + \bar{e} \gamma^\mu P_L \nu + \bar{\chi}_i^0 \gamma^\mu \left((-\sqrt{2} N_{i2} V_{j1}^* + N_{i4} V_{j2}^*) P_L + (\sqrt{2} U_{j1} N_{i2}^* + U_{j2} N_{i3}^*) P_R \right) \tilde{\chi}_j^+ \right] \\
&+ \sqrt{2} g_3 \left[\bar{g}^a (\varphi_{\bar{g}}^* \tilde{u}_R^* P_R - \varphi_{\bar{g}} \tilde{u}_L^* P_L) \tau^a u + \bar{u} \tau^a (\varphi_{\bar{g}} \tilde{u}_R P_L - \varphi_{\bar{g}}^* \tilde{u}_L P_R) \tilde{g}^a \right. \\
&\quad \left. + \bar{d} \tau^a (\varphi_{\bar{g}} \tilde{d}_R P_L - \varphi_{\bar{g}}^* \tilde{d}_L P_R) \tilde{g}^a + \bar{g}^a (\varphi_{\bar{g}}^* \tilde{d}_R^* P_R - \varphi_{\bar{g}} \tilde{d}_L^* P_L) \tau^a d \right] + \frac{y_{ui}}{\sqrt{2}} \bar{u}_i (-\phi_u + i\gamma_5 c_{\beta 0} A^0 + i\gamma_5 s_{\beta 0} G^0) u_i \\
&+ \frac{y_{di}}{\sqrt{2}} \bar{d}_i (-\phi_d + i\gamma_5 s_{\beta 0} A^0 - i\gamma_5 c_{\beta 0} G^0) d_i + \frac{y_{ei}}{\sqrt{2}} \bar{e}_i (-\phi_d + i\gamma_5 s_{\beta 0} A^0 - i\gamma_5 c_{\beta 0} G^0) e_i \\
&+ \frac{1}{2} \bar{\chi}_i^0 (\phi_u \mathcal{N}_{\phi_u} + \phi_d \mathcal{N}_{\phi_d} + iA^0 \mathcal{N}_{A^0} + iG^0 \mathcal{N}_{G^0})_{ij} \tilde{\chi}_j^0 + \frac{g_2}{\sqrt{2}} \bar{\chi}_i^+ (\phi_u \mathcal{C}_{\phi_u} + \phi_d \mathcal{C}_{\phi_d} + iA^0 \mathcal{C}_{A^0} + iG^0 \mathcal{C}_{G^0})_{ij} \tilde{\chi}_j^+ \\
&- (H^- c_{\beta+} + iG^- s_{\beta+}) (\mathcal{C}_{H^-})_{ij} \bar{\chi}_j^0 P_L \tilde{\chi}_i^+ - (H^+ c_{\beta+} - iG^+ s_{\beta+}) (\mathcal{C}_{H^+})_{ij}^* \bar{\chi}_i^+ P_R \tilde{\chi}_j^0 \\
&+ (H^+ s_{\beta+} + iG^+ c_{\beta+}) (\mathcal{C}_{H^+})_{ij} \bar{\chi}_i^+ P_L \tilde{\chi}_j^0 + (H^- s_{\beta+} - iG^- c_{\beta+}) (\mathcal{C}_{H^-})_{ij}^* \bar{\chi}_j^0 P_R \tilde{\chi}_i^+ \\
&+ (H^- s_{\beta+} - iG^- c_{\beta+}) \bar{d} y_d V_{\text{CKM}}^\dagger P_L u + (H^- c_{\beta+} + iG^- s_{\beta+}) \bar{d} V_{\text{CKM}}^\dagger y_u P_R u + (H^- s_{\beta+} - iG^- c_{\beta+}) \bar{e} y_e P_L \nu \\
&+ (H^+ c_{\beta+} - iG^+ s_{\beta+}) \bar{u} y_u V_{\text{CKM}} P_L d + (H^+ s_{\beta+} + iG^+ c_{\beta+}) \bar{u} V_{\text{CKM}} y_d P_R d + (H^+ s_{\beta+} + iG^+ c_{\beta+}) \bar{\nu} y_e P_R e \\
&+ \left(-\frac{g_Y N_{j1}^* + 3g_2 N_{j2}^*}{3\sqrt{2}} \tilde{u}_{Li}^* - y_{ui} N_{j4}^* \tilde{u}_{Ri}^* \right) \bar{\chi}_j^0 P_L u_i + \left(\frac{2\sqrt{2} g_Y}{3} N_{j1} \tilde{u}_{Ri}^* - y_{ui} N_{j4} \tilde{u}_{Li}^* \right) \bar{\chi}_j^0 P_R u_i \\
&+ \left(\frac{2\sqrt{2} g_Y}{3} N_{j1}^* \tilde{u}_{Ri} - y_{ui} N_{j4}^* \tilde{u}_{Li} \right) \bar{u}_i P_L \tilde{\chi}_j^0 + \left(-\frac{g_Y N_{j1} + 3g_2 N_{j2}}{3\sqrt{2}} \tilde{u}_{Li} - y_{ui} N_{j4} \tilde{u}_{Ri} \right) \bar{u}_i P_R \tilde{\chi}_j^0 \\
&+ \left(-\frac{g_Y N_{j1}^* - 3g_2 N_{j2}^*}{3\sqrt{2}} \tilde{d}_{Li}^* - y_{di} N_{j3}^* \tilde{d}_{Ri}^* \right) \bar{\chi}_j^0 P_L d_i + \left(-\frac{\sqrt{2} g_Y}{3} N_{j1} \tilde{d}_{Ri}^* - y_{di} N_{j3} \tilde{d}_{Li}^* \right) \bar{\chi}_j^0 P_R d_i \\
&+ \left(-\frac{\sqrt{2} g_Y}{3} N_{j1}^* \tilde{d}_{Ri} - y_{di} N_{j3}^* \tilde{d}_{Li} \right) \bar{d}_i P_L \tilde{\chi}_j^0 + \left(-\frac{g_Y N_{j1} - 3g_2 N_{j2}}{3\sqrt{2}} \tilde{d}_{Li} - y_{di} N_{j3} \tilde{d}_{Ri} \right) \bar{d}_i P_R \tilde{\chi}_j^0 \\
&+ \left(\frac{g_Y N_{j1}^* + g_2 N_{j2}^*}{\sqrt{2}} \tilde{e}_{Li}^* - y_{ei} N_{j3}^* \tilde{e}_{Ri}^* \right) \bar{\chi}_j^0 P_L e_i + \left(-\sqrt{2} g_Y N_{j1} \tilde{e}_{Ri}^* - y_{ei} N_{j3} \tilde{e}_{Li}^* \right) \bar{\chi}_j^0 P_R e_i \\
&+ \left(-\sqrt{2} g_Y N_{j1}^* \tilde{e}_{Ri} - y_{ei} N_{j3}^* \tilde{e}_{Li} \right) \bar{e}_i P_L \tilde{\chi}_j^0 + \left(+\frac{g_Y N_{j1} + g_2 N_{j2}}{\sqrt{2}} \tilde{e}_{Li} - y_{ei} N_{j3} \tilde{e}_{Ri} \right) \bar{e}_i P_R \tilde{\chi}_j^0 \\
&+ \frac{g_Y N_{j1} - g_2 N_{j2}}{\sqrt{2}} \tilde{\nu}_{Li} \bar{\nu}_i P_R \tilde{\chi}_j^0 + \frac{g_Y N_{j1}^* - g_2 N_{j2}^*}{\sqrt{2}} \tilde{\nu}_{Li}^* \bar{\chi}_j^0 P_L \nu_i \\
&+ \left(U_{i2}^* (\tilde{d}_R^* y_d V_{\text{CKM}}^\dagger)_j - U_{i1}^* g_2 (\tilde{d}_L^* V_{\text{CKM}}^\dagger)_j \right) \bar{\chi}_i^+ P_L u_j + V_{i2} (\tilde{d}_L^* V_{\text{CKM}}^\dagger y_u)_j \bar{\chi}_i^+ P_R u_j \\
&+ V_{i2}^* (y_u V_{\text{CKM}} \tilde{d}_L)_j \bar{u}_j P_L \tilde{\chi}_i^+ + \left(U_{i2} (V_{\text{CKM}} y_d \tilde{d}_R)_j - U_{i1} g_2 (V_{\text{CKM}} \tilde{d}_L)_j \right) \bar{u}_j P_R \tilde{\chi}_i^+ \\
&+ (-V_{i1}^* g_2 (\tilde{u}_L^* V_{\text{CKM}})_j + V_{i2}^* (\tilde{u}_R^* y_u V_{\text{CKM}})_j) \bar{\chi}_i^- P_L d_j + U_{i2} (\tilde{u}_L^* V_{\text{CKM}} y_d)_j \bar{\chi}_i^- P_R d_j \\
&+ U_{i2}^* (y_d V_{\text{CKM}}^\dagger \tilde{u}_L)_j \bar{d}_j P_L \tilde{\chi}_i^- + \left(-V_{i1} g_2 (V_{\text{CKM}}^\dagger \tilde{u}_L)_j + V_{i2} (V_{\text{CKM}}^\dagger y_u \tilde{u}_R)_j \right) \bar{d}_j P_R \tilde{\chi}_i^- \\
&+ (-U_{i1}^* g_2 \tilde{e}_L^* + U_{i2}^* y_{ei} \tilde{e}_{Ri}^*) \bar{\chi}_i^+ P_L \nu + (-U_{i1} g_2 \tilde{e}_L + U_{i2} y_{ei} \tilde{e}_{Ri}) \bar{\nu} P_R \tilde{\chi}_i^+ \\
&+ \bar{\chi}_i^- (-V_{i1}^* g_2 \tilde{\nu}_L^* P_L + U_{i2} y_{ej} \tilde{\nu}_{Lj}^* P_R) e_j + \bar{e}_j (U_{i2}^* y_{ej} \tilde{\nu}_{Lj} P_L - V_{i1} g_2 \tilde{\nu}_L P_R) \tilde{\chi}_i^- + (\text{RPV part}).
\end{aligned}$$

(9.91)

Note that squarks and sleptons are in super-CKM basis and not in mass eigenstates.

9.5. SLHA convention

The SLHA convention [10] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table (**magenta color** for objects in other conventions), while

$\mu, b, m_{Q,L,H_u,H_d}^2$, RPV-trilinears (λ s and T s) are in common.

SLHA	our notation	Martin/DHM
(H_1, H_2)	(H_d, H_u)	
$Y_{u,d,e}$	$(y_{u,d,e})^T$	
$T_{u,d,e}$	$(a_{u,d,e})^T$	
$A_{u,d,e}$	$(A_{u,d,e})^T$	
m_{U^c, D^c, E^c}^2	$(m_{U^c, D^c, E^c}^2)^\dagger$	
$M_{1,2,3}$	$-M_{1,2,3}$	
m_3^2	b	
m_A^2	$m_{A_0}^2$ (tree)	
	κ_i	$= -\mu'_i$ (rarely used)
D_i	b_i	
$m_{L_i H_1}^2$	$M_{L_i}^2$	

In particular, the chargino/neutralino mass terms in RPC case are given by

$$\mathcal{L} \supset \left[\frac{1}{2} \mathbf{M}_1 \tilde{b} \tilde{b} + \frac{1}{2} \mathbf{M}_2 \tilde{w} \tilde{w} - \mu \tilde{h}_u \tilde{h}_d - \frac{g_Y}{2\sqrt{2}} \left(h_u^* \tilde{h}_u - h_d^* \tilde{h}_d \right) \tilde{b} - \sqrt{2} g_2 \left(h_u^* T^a \tilde{h}_u + h_d^* T^a \tilde{h}_d \right) \tilde{w} \right] + \text{H.c.} \quad (9.92)$$

$$\rightarrow \frac{1}{2} \begin{pmatrix} \tilde{b} \\ \tilde{w} \\ h_u^0 \\ h_d^0 \end{pmatrix}^T \begin{pmatrix} -M_1 & 0 & -m_{ZC\beta S_w} & m_{ZS\beta S_w} \\ 0 & -M_2 & m_{ZC\beta C_w} & -m_{ZS\beta C_w} \\ -m_{ZC\beta S_w} & m_{ZC\beta C_w} & 0 & -\mu \\ m_{ZS\beta S_w} & -m_{ZS\beta C_w} & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{w} \\ h_u^0 \\ h_d^0 \end{pmatrix} \quad (9.93)$$

9.6. GMSB formulae

$$\Gamma^{-1}(\tilde{l} \rightarrow l\tilde{G}) = \frac{1}{48\pi M^2} \frac{m_l^5}{m_{\tilde{G}}^2} \times (\text{phase space}) \approx 5.9 \times 10^{-7} \text{ s} \frac{(m_{3/2}/\text{TeV})^2}{(m_{\tilde{l}}/\text{TeV})^5} \quad (9.94)$$

$$= \frac{3m_l^5}{48\pi F_{\text{tot}}^2} \times (\text{phase space}) \approx 3.3 \times 10^{-9} \text{ s} \frac{(F_{\text{tot}}/10^{12} \text{ GeV}^2)^2}{(m_{\tilde{l}}/100 \text{ GeV})^5} = 1.0 \text{ m} \frac{(F_{\text{tot}}/10^{12} \text{ GeV}^2)^2}{(m_{\tilde{l}}/100 \text{ GeV})^5} \quad (9.95)$$

$$\Gamma^{-1}(\tilde{B} \rightarrow \gamma\tilde{G}) = \frac{c_w^2 m_{\tilde{B}}^5}{48\pi M^2 m_{\tilde{G}}^2} (1-r)^3 (1+3r) \quad \text{where} \quad r := \left(\frac{m_{\tilde{G}}}{m_{\tilde{B}}} \right)^2. \quad (9.96)$$

(cf. [11])

A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [12]:

$$e^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \quad (\text{converges for any } X), \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(X-I)^m}{m} \quad (\text{conv. if } \|X-I\| < 1). \quad (\text{A.1})$$

$$e^{\log A} = A \quad (\text{if } \|A-I\| < 1), \quad \log e^X = X \quad \text{and} \quad \|e^X - 1\| < 1 \quad (\text{if } \|X\| < \log 2). \quad (\text{A.2})$$

$$\text{Hilbert-Schmidt norm : } \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \text{Tr } X^\dagger X. \quad (\text{A.3})$$

Properties:

$$e^{(X^T)^T} = (e^X)^T, \quad e^{(X^*)^*} = (e^X)^*, \quad (e^X)^{-1} = e^{-X}, \quad e^{YXY^{-1}} = Y e^X Y^{-1},$$

$$\det \exp X = \exp \text{Tr } X, \quad e^{(\alpha+\beta)X} = e^{\alpha X} e^{\beta X} \quad \text{for } \alpha, \beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$e^X Y e^{-X} = Y + [X, Y] + \frac{1}{2!}[X, [X, Y]] + \frac{1}{3!}[X, [X, [X, Y]]] + \dots = e^{[X, \cdot]} Y; \quad (\text{A.4})$$

$$e^X e^Y e^{-X} = \sum_{n=0}^{\infty} \frac{1}{n!} (e^X Y e^{-X})^n = \exp(e^{[X, \cdot]} Y); \quad (\text{A.5})$$

$$\log(e^X e^Y) = X + \int_0^1 dt g(e^{[X, \cdot]} e^{t[Y, \cdot]}) Y \quad \left[g(z) = \frac{\log z}{1-z^{-1}} = 1 - \sum_{n=1}^{\infty} \frac{(1-z)^n}{n(n+1)}; \quad g(e^y) = \sum_{n=0}^{\infty} \frac{B_n y^n}{n!} \right] \quad (\text{A.6})$$

$$= X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X, [X, Y]] - \frac{1}{12}[Y, [X, Y]] + \dots \quad (\text{Baker-Campbell-Hausdorff}). \quad (\text{A.7})$$

$$\log(e^X e^Y) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \left(\sum_{m,n=0}^{\infty} \frac{X^m Y^n}{m!n!} - 1 \right)^k = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \dots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k} \frac{X^{m_1} Y^{n_1} \dots X^{m_k} Y^{n_k}}{m_1!n_1! \dots m_k!n_k!} \quad (\text{A.8})$$

$$\log(e^X e^Y) = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \dots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k \sum_{i=1}^k (m_i + n_i)} \frac{([X, \cdot]^{m_1} ([Y, \cdot]^{n_1} \dots ([X, \cdot]^{m_k} ([Y, \cdot]^{n_k} \dots]))}{m_1!n_1! \dots m_k!n_k!} \quad (\text{A.9})$$

with $[X, \cdot]$ being $X \cdot$.

Derivative:

$$\frac{d}{dt} e^{tX} = X e^{tX} = e^{tX} X \quad (\text{A.10})$$

$$e^{-X(t)} \left(\frac{d}{dt} e^{X(t)} \right) = \frac{I - e^{-\text{ad}_X}}{\text{ad}_X} \left(\frac{dX}{dt} \right) = X' + \frac{[-X, X']}{2!} + \frac{[-X, [-X, X']]}{3!} + \dots \quad (\text{A.11})$$

$$\left(\frac{d}{dt} e^{X(t)} \right) e^{-X(t)} = X' + \frac{[X, X']}{2!} + \frac{[X, [X, X']]}{3!} + \dots \quad (\text{A.12})$$

where $X' = dX/dt$ and $\text{ad}_X(Y) = [X, Y]$ is the adjoint action of a Lie algebra. Thus, explicitly,

$$\frac{d}{dt} e^{aX(t)} = e^{aX} \left\{ \sum_{n=0}^{\infty} \frac{a^{n+1}}{(n+1)!} ([X, \cdot]^n X') \right\} = \left\{ \sum_{n=0}^{\infty} \frac{a^{n+1}}{(n+1)!} ([X, \cdot]^n X') \right\} e^{aX} \quad (\text{A.13})$$

Component: If matrices t^a satisfies $[t^a, t^b] = i f^{abc} t^c$ with totally-antisymmetric $f^{abc} \in \mathbb{R}$,

$$\left[e^{\theta^a t^a} t_b e^{-\theta^c t^c} \right]_{ij} = \left[e^{\theta^a [t^a, t_b]} \right]_{ij} = \left[e^{i\theta^a f^a} \right]^{bc} t_{ij}^c \quad (\text{A.14})$$

holds for $\theta^a \in \mathbb{C}$, where $[f^a]_{bc} = f^{abc}$. ♣**TODO:needs verification, generalization/restriction, and a nice proof or reference.**♣

A.2. General unitary matrix

$$U_2 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} e^{i\beta} & 0 \\ 0 & e^{i\gamma} \end{pmatrix} = \begin{pmatrix} c_\theta e^{i\beta} & s_\theta e^{i\gamma} \\ -s_\theta e^{i(\alpha+\beta)} & c_\theta e^{i(\alpha+\gamma)} \end{pmatrix} \quad (\text{A.15})$$

$$U_3 = \begin{pmatrix} 1 & & \\ & e^{ia} & \\ & & e^{ib} \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13} e^{-i\delta} \\ & 1 & \\ -s_{13} e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \\ & & 1 \end{pmatrix} \begin{pmatrix} e^{ic} & & \\ & e^{id} & \\ & & e^{ie} \end{pmatrix} \quad (\text{A.16})$$

$$= \begin{pmatrix} 1 & & \\ & e^{ia} & \\ & & e^{ib} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{ic} & & \\ & e^{id} & \\ & & e^{ie} \end{pmatrix} \quad (\text{A.17})$$

(see, e.g., Ref. [13]).

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