1 General Definitions and Tools

1.1 Notations and Conventions

◆1.1.1 Metric etc.

Minkowski Metric : $\eta^{\mu\nu} := \mathrm{diag}(+,-,-,-);\quad \epsilon^{0123}_{0123} := \pm 1$

Coordinates : $x^\mu := (t,x,y,z);$ therefore $\partial_\mu = \left(\frac{\partial}{\partial t}, \nabla\right)$.

Gamma Matrices : $\{\gamma^\mu,\gamma^\nu\} := 2\eta^{\mu\nu};\quad \gamma_5 := \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \dfrac{-\mathrm{i}}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$

: therefore $\{\gamma^\mu,\gamma_5\} = 0, (\gamma_5)^2 = 1$.

Gamma Combinations : $1, \{\gamma^\mu\}, \{\sigma^{\mu\nu}\}, \{\gamma^\mu\gamma_5\}, \gamma_5;\quad \sigma^{\mu\nu} := \frac{\mathrm{i}}{2}[\gamma^\mu,\gamma^\nu] = 0\big/\mathrm{i}\gamma^\mu\gamma^\nu$

Spinor ϵ and σ matrices : $\epsilon^{12} = \epsilon^{\dot 1\dot 2} = \epsilon_{21} = \epsilon_{\dot 2\dot 1} = 1$

: $(\sigma^\mu)_{\alpha\dot\beta} := (1,\boldsymbol{\sigma})_{\alpha\dot\beta},\quad (\bar\sigma^\mu)^{\dot\alpha\alpha} := \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}(\sigma^\mu)_{\beta\dot\beta} = (1,-\boldsymbol{\sigma})^{\dot\alpha\beta}$.

> Pauli Matrices : $\sigma_0 = \begin{pmatrix}1&0\\0&1\end{pmatrix},\quad \sigma_1 = \begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_2 = \begin{pmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{pmatrix},\quad \sigma_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix},$
>
> : $\sigma_+ = \frac{1}{2}(\sigma_1 + \mathrm{i}\sigma_2) = \begin{pmatrix}0&1\\0&0\end{pmatrix},\quad \sigma_- = \frac{1}{2}(\sigma_1 - \mathrm{i}\sigma_2) = \begin{pmatrix}0&0\\1&0\end{pmatrix},$
>
> : $\sigma^\mu := (1,\boldsymbol{\sigma}),\quad \bar\sigma^\mu := (1,-\boldsymbol{\sigma})$.

Fourier Transformation : $\widetilde{f}(k) := \displaystyle\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}kx} f(x);\qquad f(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\ \mathrm{e}^{-\mathrm{i}kx}\widetilde{f}(k)$.

◆1.1.2 Fields

Scalar : $(\partial^2 + m^2)\phi = 0;\quad \phi(x) = \displaystyle\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol p}}}\left[a_{\boldsymbol p}\mathrm{e}^{-\mathrm{i}px} + b^\dagger_{\boldsymbol p}\mathrm{e}^{\mathrm{i}px}\right]$

Dirac : $(i\partial\!\!\!/ - m)\psi = 0;\quad \psi(x) = \displaystyle\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol p}}}\sum_{s=1,2}\left[a^s_{\boldsymbol p}u^s(p)\mathrm{e}^{-\mathrm{i}px} + b^{s\dagger}_{\boldsymbol p}v^s(p)\mathrm{e}^{\mathrm{i}px}\right]$

Vector : $\partial^2 A^\mu = 0;\quad A^\mu(x) = \displaystyle\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol p}}}\sum_{r=0..3}\left[a^r_{\boldsymbol p}\epsilon^r(p)\mathrm{e}^{-\mathrm{i}px} + a^{r\dagger}_{\boldsymbol p}\epsilon^{r*}(p)\mathrm{e}^{\mathrm{i}px}\right]$

TODO: 南部–Goldstone; Gravitino

◆1.1.3 Electromagnetism

Electromagnetic Fields : $A^\mu = (\phi,\boldsymbol{A})$ 【We can invert the signs, but cannot lower the index.】

Maxwell Equations : $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu;\qquad \epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0,\quad \partial_\mu F^{\mu\nu} = ej^\nu$

Our Old Language : $\nabla\cdot\boldsymbol{B} = 0,\ \nabla\times\boldsymbol{E} + \dfrac{\partial}{\partial t}\boldsymbol{B} = 0;\quad \nabla\cdot\boldsymbol{E} = ej^0,\ (\nabla\times\boldsymbol{B})_i - \dfrac{\partial}{\partial t}E_i = ej^i$.

: $F_{\mu\nu} = \begin{pmatrix} 0 & & \boldsymbol{E} & \\ & 0 & -B_3 & B_2 \\ -\boldsymbol{E} & B_3 & 0 & -B_1 \\ & -B_2 & B_1 & 0 \end{pmatrix};\quad F_{\mu\nu}F^{\mu\nu} = -2\left(\|\boldsymbol{E}\|^2 - \|\boldsymbol{B}\|^2\right)$

1.2 Spinor Fields

Spinor : ξ_α, $\xi^\alpha := \epsilon^{\alpha\beta}\xi_\beta$; Lorentz tr. : $\xi_\alpha \mapsto \Lambda_\alpha{}^\beta \xi_\beta$, $\xi^\alpha \mapsto \xi^\beta \Lambda^{-1}{}_\beta{}^\alpha$;

: $\bar\eta^{\dot\alpha} := (\eta^\alpha)^*$ $\bar\eta_{\dot\alpha} := (\eta_\alpha)^*$: $\bar\eta^{\dot\alpha} \mapsto \Lambda^{\dagger -1\dot\alpha}{}_{\dot\beta}\bar\eta^{\dot\beta}$, $\bar\eta_{\dot\alpha} \mapsto \bar\eta_{\dot\beta}\Lambda^{\dagger\dot\beta}{}_{\dot\alpha}$.

Kinetic term : $\mathrm{i}\,\bar\xi\bar\sigma^\mu\partial_\mu\xi \quad (= \mathrm{i}\,\eta\sigma^\mu\partial_\mu\bar\eta\ )$

Mass term : [Majorana] $-\frac{1}{2}(m_{\mathrm{M}}\xi\xi + m_{\mathrm{M}}^*\bar\xi\bar\xi)$ [Dirac] $-(m_{\mathrm{D}}\xi\eta + m_{\mathrm{D}}^*\bar\xi\bar\eta)$

Dirac fermion : $\mathcal{L}_{\mathrm{Dirac}} = \mathrm{i}\,\bar\xi\bar\sigma^\mu\partial_\mu\xi + \mathrm{i}\,\eta\sigma^\mu\partial_\mu\bar\eta - m(\xi\eta+\bar\xi\bar\eta) = \overline{\psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi$

Majorana fermion : $\mathcal{L}_{\mathrm{Majorana}} = \mathrm{i}\,\bar\xi\bar\sigma^\mu\partial_\mu\xi - \frac{m}{2}(\xi\xi+\bar\xi\bar\xi) = \frac{1}{2}\overline{\psi}_{\mathrm{M}}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi_{\mathrm{M}}$

Charge conjugate : $\psi^{\mathrm{C}} := C(\overline{\psi})^{\mathrm{T}} \quad [(\psi_{\mathrm{M}})^{\mathrm{C}} = \psi_{\mathrm{M}}]$

◆1.2.1 Chiral Notation (Peskin)

Gamma Matrices : $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; $P_{\mathrm{L}}^{\mathrm{R}} = \dfrac{1\pm\gamma_5}{2}$.

Fields : $\psi = \begin{pmatrix} \xi_\alpha \\ \bar\eta^{\dot\alpha} \end{pmatrix} = \begin{pmatrix} \psi_{\mathrm{L}} \\ \psi_{\mathrm{R}} \end{pmatrix}$; $\overline{\psi} = \psi^\dagger\gamma^0 = \begin{pmatrix} \eta^\alpha & \bar\xi_{\dot\alpha} \end{pmatrix} = \begin{pmatrix} \psi_{\mathrm{R}}^\dagger & \psi_{\mathrm{L}}^\dagger \end{pmatrix}$; $\psi_{\mathrm{M}} = \begin{pmatrix} \xi_\alpha \\ \bar\xi^{\dot\alpha} \end{pmatrix}$.

: $u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar\sigma}\xi^s \end{pmatrix}$; $v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\eta^s \\ -\sqrt{p\cdot\bar\sigma}\eta^s \end{pmatrix}$

: $\Big[\eta^s = \xi^{-s} := -\mathrm{i}\sigma^2(\xi^s)^* = (\xi^2, -\xi^1)\Big]$.

Charge conj. : $C := -\mathrm{i}\gamma^2\gamma^0 = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$ $\begin{cases} -C = C^{-1} = C^\dagger = C^{\mathrm{T}} \\ C = C^* \end{cases}$, $C^{-1}\gamma^\mu C = -\gamma^{\mu\mathrm{T}}$.

: $\psi^{\mathrm{C}} = C(\overline{\psi})^{\mathrm{T}} = -\mathrm{i}\gamma^2\psi^* = \begin{pmatrix} \eta_\alpha \\ \bar\xi^{\dot\alpha} \end{pmatrix}$, $\overline{\psi}^{\mathrm{C}} = \psi^{\mathrm{T}}C = \mathrm{i}\overline{\psi}^*\gamma^2$

Weyl Equations : $\mathrm{i}\bar\sigma\cdot\partial\psi_{\mathrm{L}} = m\psi_{\mathrm{R}}$; $\mathrm{i}\sigma\cdot\partial\psi_{\mathrm{R}} = m\psi_{\mathrm{L}}$

z-boost limit : Halt: $u^s = \sqrt{m}\left(\begin{smallmatrix} \xi^s \\ \xi^s \end{smallmatrix}\right)$, $v^s = \sqrt{m}\left(\begin{smallmatrix} \eta^s \\ -\eta^s \end{smallmatrix}\right)$;

: Slow: $\sqrt{p\cdot\sigma} \simeq \sqrt{m}(1-\boldsymbol{v}\cdot\boldsymbol{\sigma}/2)$, $\sqrt{p\cdot\bar\sigma} \simeq \sqrt{m}(1+\boldsymbol{v}\cdot\boldsymbol{\sigma}/2)$;

: Extreme: $u^s = \sqrt{2E}\begin{pmatrix} \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)\xi^s \\ \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)\xi^s \end{pmatrix}$, $v^s = \sqrt{2E}\begin{pmatrix} \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)\eta^s \\ -\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)\eta^s \end{pmatrix}$.

◆1.2.2 Dirac Notation

Gamma Matrices : $\hat\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\hat\gamma^i = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}$, $\hat\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $\hat P_{\mathrm{L}}^{\mathrm{R}} = \dfrac{1\pm\gamma_5}{2}$.

: $\hat\sigma^{0i} = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$, $\hat\sigma^{ij} = \epsilon^{ijk}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$.

Fields : $\hat\psi = \begin{pmatrix} \psi_{\mathrm{A}} \\ \psi_{\mathrm{B}} \end{pmatrix} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\psi = \dfrac{1}{\sqrt{2}}\begin{pmatrix} \psi_{\mathrm{L}}+\psi_{\mathrm{R}} \\ -\psi_{\mathrm{L}}+\psi_{\mathrm{R}} \end{pmatrix}$; $\hat\psi_{\mathrm{M}} = \begin{pmatrix} \psi_{\mathrm{A}} \\ \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)\psi_{\mathrm{A}}^* \end{pmatrix}$.

: $\hat{\overline{\psi}} = \hat\psi^\dagger\gamma^0 = \begin{pmatrix} \psi_{\mathrm{A}}^\dagger & -\psi_{\mathrm{B}}^\dagger \end{pmatrix}$

: $\hat u^s(p) = \begin{pmatrix} \sqrt{p^0+m}\,\xi^s \\ \frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{\sqrt{p^0+m}}\xi^s \end{pmatrix}$; $\hat v^s(p) = \begin{pmatrix} -\frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{\sqrt{p^0+m}}\eta^s \\ -\sqrt{p^0+m}\,\eta^s \end{pmatrix}$

: $\Big[\eta^s = \xi^{-s} := -\mathrm{i}\sigma^2(\xi^s)^* = (\xi^2, -\xi^1)\Big]$

Charge conj. : $\hat{C} = -\mathrm{i}\hat{\gamma}^2\hat{\gamma}^0 = \begin{pmatrix} 0&0&0&1\\ 0&0&-1&0\\ 0&-1&0&0\\ 1&0&0&0 \end{pmatrix}$ with $C = -C^{-1} = -C^\dagger$, $C^{-1}\gamma^\mu C = -\gamma^{\mu\mathrm{T}}$.

z-boost limit : Halt: $\hat{u}^s = \sqrt{2m}\begin{pmatrix}\xi^s\\0\end{pmatrix}$, $\hat{v}^s = -\sqrt{2m}\begin{pmatrix}0\\\eta^s\end{pmatrix}$;

: Slow: $\sqrt{p^0+m} \simeq \sqrt{2m}(1+\frac{v^2}{8})$, $\frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{\sqrt{p^0+m}} \simeq \sqrt{\frac{m}{2}}(\boldsymbol{v}\cdot\boldsymbol{\sigma})$;

: Extreme: $\hat{u}^s = \sqrt{E}\begin{pmatrix}\xi^s\\ \begin{pmatrix}1&0\\0&-1\end{pmatrix}\xi^s\end{pmatrix}$, $\hat{v}^s = -\sqrt{E}\begin{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\eta^s\\ \eta^s\end{pmatrix}$

◆1.2.3 CPT transformations

【Note that these expressions are valid under the above frameworks.】

In the following, CP means "P, then C" **in algebraic sense**. Be careful to the order.

$$\psi(t,\boldsymbol{x}) \xrightarrow{P} \eta_P\gamma^0\psi(t,-\boldsymbol{x}) \qquad \overline{\psi} \xrightarrow{P} \eta_P^*\overline{\psi}\gamma^0$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{T} \eta_T C\gamma_5\psi(-t,\boldsymbol{x}) \qquad \overline{\psi} \xrightarrow{T} -\eta_T^*\overline{\psi}C\gamma_5$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{C} \eta_C C\overline{\psi}^{\mathrm{T}}(t,\boldsymbol{x}) = C\gamma^0\psi^* \qquad \overline{\psi} \xrightarrow{C} \eta_C^*\overline{\psi}^*\gamma^0 C = -\eta_C^*(C\psi)^{\mathrm{T}}$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{CP} \eta_{CP}(\overline{\psi}\gamma^0 C)^{\mathrm{T}} \qquad \overline{\psi} \xrightarrow{CP} \eta_{CP}^*(C\gamma^0\psi)^{\mathrm{T}}$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{CPT} (\overline{\psi}\gamma^0\gamma_5)^{\mathrm{T}} \qquad \overline{\psi} \xrightarrow{CPT} (\gamma^0\gamma_5\psi)^{\mathrm{T}}$$

Note that T-transformation is anti-unitary, and $\eta_{CPT} = 1$. Especially, photon is $(P,T,C) = (-,+,-)$.

	ϕ	A^μ	$\bar\psi\psi$	$\bar\psi\gamma^\mu\psi$	$\bar\psi\sigma^{\mu\nu}\psi$	$\bar\psi\gamma^\mu\gamma_5\psi$	$\mathrm{i}\bar\psi\gamma_5\psi$	∂_μ
P	ϕ	$-{+}{+}{+}A^\mu$	$+$	$+{-}{-}{-}$	$(+{-}{-}{-})(+{-}{-}{-})$	$-{+}{+}{+}$	$-$	$+{-}{-}{-}$
T	ϕ	$+{-}{-}{-}A^\mu$	$+$	$+{-}{-}{-}$	$-(+{-}{-}{-})(+{-}{-}{-})$	$+{-}{-}{-}$	$-$	$-{+}{+}{+}$
C	ϕ^*	$+A^{\mu *}$	$+$	$-$	$-$	$+$	$+$	$+$
CPT	ϕ^*	$-A^{\mu *}$	$+$	$-$	$+$	$-$	$+$	$-$

◆1.2.4 Noether current

Infinitesimal transformation : $\phi(x) \mapsto \phi'(x) := \phi(x) + \alpha\Delta\phi(x)$

Correspondent transformation : $\alpha\Delta\mathcal{L} = \alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi\right) + \alpha\cancel{\left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\right]}\Delta\phi$

: So, defining $\alpha\partial_\mu\mathcal{J}^\mu(x) := \mathcal{L}'(x) - \mathcal{L}(x)$,

Noether current : $j^\mu(x) := \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi - \mathcal{J}^\mu$; $\quad \partial_\mu j^\mu(x) = 0$

Noether charge : $Q := \int j^0\mathrm{d}^3x$

Energy-momentum tensor : $T^\mu{}_\nu = \partial_\mu\mathcal{L}(\partial_\mu\phi)\partial_\nu\phi - \mathcal{L}\delta^\mu_\nu$; $\quad \mathcal{H} = T^{00}$, $\quad \mathcal{P}^i = T^{0i}$.

: $T^\mu{}_\nu$ is the variation along μ in respect to the modification a^ν.

1.3 Feynman Rules

■Scalar Boson

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$\phi\ \phi = \qquad = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$$

$$\mathcal{L} \supset |\partial_\mu\phi|^2 - m^2|\phi|^2$$

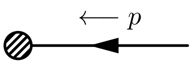
$$\phi^*\ \phi = \qquad = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$$

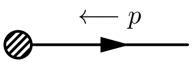
(External lines equal to 1 in both cases.)

■Dirac Fermion

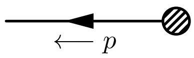
$$\mathcal{L} \supset \overline{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi$$
$$= \mathrm{i}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\xi + \mathrm{i}\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - m(\xi\chi + \bar{\xi}\bar{\chi})$$

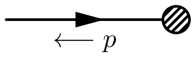
Initial state

$\psi\ |\boldsymbol{p}, s\rangle =$  $= u^s(p)$

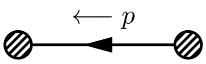
$\overline{\psi}\ |\boldsymbol{p}, s\rangle =$  $= \overline{v}^s(p)$

Final state

$\langle\boldsymbol{p}, s|\ \overline{\psi} =$  $= \overline{u}^s(p)$

$\langle\boldsymbol{p}, s|\ \psi =$  $= v^s(p)$

Propagator

$\psi\ \overline{\psi} =$  $= \dfrac{\mathrm{i}(p\!\!\!/ + m)}{p^2 - m^2 + \mathrm{i}\epsilon}$

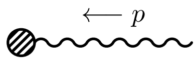
■Majorana Fermion

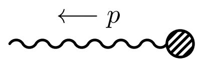
$$\mathcal{L} \supset \tfrac{1}{2}\overline{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi$$
$$= \mathrm{i}\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda - \tfrac{m}{2}(\lambda\lambda + \bar{\lambda}\bar{\lambda})$$

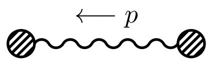
Initial state

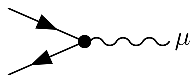
■Abelian Gauge Theory (Photon)

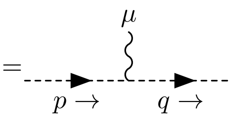
$$\mathcal{L} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\overline{\psi}D\!\!\!\!/\,\psi + |\mathrm{D}_\mu\phi|^2$$
$$(\mathrm{D}_\mu = \partial_\mu - \mathrm{i}QA_\mu)$$

$A_\mu\ |\boldsymbol{p}; あ\rangle =$  $= \epsilon^{あ}_\mu(p)$

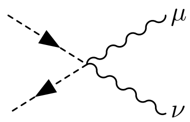
$\langle\boldsymbol{p}; あ|\ A_\mu =$  $= \epsilon^{あ*}_\mu(p)$

$A_\mu\ A_\nu =$  $= \dfrac{-\mathrm{i}\eta_{\mu\nu}}{p^2 + \mathrm{i}\epsilon}$

$Q\overline{\psi}A\!\!\!/\,\psi =$  $= \mathrm{i}Q\gamma^\mu$

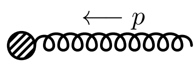
$\mathrm{i}QA^\mu\phi^*\partial_\mu\phi$ $+\mathrm{H.c.}$ $=$  $= \mathrm{i}Q(p^\mu + q^\mu)$

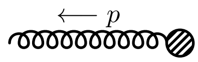
(Momentum must be taken along the arrow)

$Q^2A^2|\phi|^2 =$  $= 2\mathrm{i}Q^2\eta^{\mu\nu}$

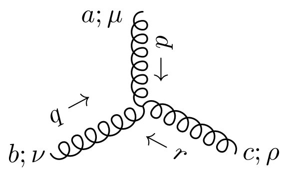
■Non-Abelian Gauge Theory (Gluon)

$$\mathcal{L} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\overline{\psi}D\!\!\!\!/\,\psi + |\mathrm{D}_\mu\phi|^2$$
$$(\mathrm{D}_\mu = \partial_\mu - \mathrm{i}gA_\mu)$$

$A^b_\mu\ |\boldsymbol{p}; あ, a\rangle =$  $= \epsilon^{あ}_\mu(p)\delta^{ab}$

$\langle\boldsymbol{p}; あ, a|\ A^b_\mu =$  $= \epsilon^{あ*}_\mu(p)\delta^{ab}$

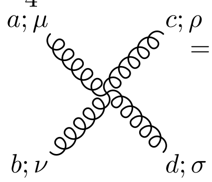
$$-gf^{abc}A^{\mu a}A^{\nu b}(\partial_\mu A^c_\nu) =$$



$$= gf^{abc}\big[\eta^{\mu\nu}(p-q)^\rho + \eta^{\nu\rho}(q-r)^\mu + \eta^{\rho\mu}(r-p)^\nu\big]$$

(Momentum are in incoming directions)

$$-\frac{1}{4}g^2(f^{abe}A^a_\mu A^b_\nu)(f^{cde}A^c_\rho A^d_\sigma) =$$



$$= -\mathrm{i}g^2\big[f^{abe}f^{cde}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) + f^{ace}f^{bde}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) + f^{ade}f^{bce}(\eta^{\mu\nu}\eta^{\sigma\rho} - \eta^{\mu\rho}\eta^{\nu\sigma})\big]$$

TODO: vertex は lagrangian の $(n!)$i 倍)

1.4 Field Calculation Techniques

◆1.4.1 Dirac Field Techniques

$$
\begin{array}{lll}
\text{Dirac Equations} & : & (\not{p}-m)u^s(p)=0; \quad (\not{p}+m)v^s(p)=0 \\
 & : & \bar{u}^s(p)(\not{p}-m)=0; \quad \bar{v}^s(p)(\not{p}+m)=0 \\
\text{Dirac Components} & : & u^{r\dagger}(p)u^s(p)=2E_{\boldsymbol{p}}\delta^{rs}; \quad v^{r\dagger}(p)v^s(p)=2E_{\boldsymbol{p}}\delta^{rs} \\
 & : & \bar{u}^r(p)u^s(p)=2m\delta^{rs}; \quad \bar{v}^r(p)v^s(p)=-2m\delta^{rs}; \quad \bar{u}^r(p)v^s(p)=\bar{v}^r(p)u^s(p)=0 \\
\text{Spin Sums} & : & \sum_{\text{spin}} u^s(p)\bar{u}^s(p)=\not{p}+m; \quad \sum_{\text{spin}} v^s(p)\bar{v}^s(p)=\not{p}-m \\
\text{Chage Conj.} & : & -C=C^{-1}=C^\dagger=C^{\mathrm{T}}, \quad C^{-1}\gamma^\mu C=-C\gamma^\mu C=-\gamma^{\mu\mathrm{T}}, \quad C^{-1}\gamma^0 C=-\gamma^0 \\
 & : & C=C^* \quad , \quad \psi^{\mathrm{C}}=C(\overline{\psi})^{\mathrm{T}}, \quad \overline{\psi}^{\mathrm{C}}=\psi^{\mathrm{T}}C \\
u \;\&\; v & : & u^*=-\mathrm{i}\gamma^2 v, \quad v^{\mathrm{T}}=-\mathrm{i}u^\dagger\gamma^2=\overline{u}C^{-1}, \quad v=C\overline{u}^{\mathrm{T}}; \qquad \overline{u}_{\mathrm{A}}P_{\mathrm{H}}u_{\mathrm{B}}=-\overline{v}_{\mathrm{B}}P_{\mathrm{H}}v_{\mathrm{A}} \\
 & : & v^*=-\mathrm{i}\gamma^2 u, \quad u^{\mathrm{T}}=-\mathrm{i}v^\dagger\gamma^2=\overline{v}C^{-1}, \quad u=C\overline{v}^{\mathrm{T}}; \qquad \overline{v}_{\mathrm{A}}P_{\mathrm{H}}u_{\mathrm{B}}=-\overline{v}_{\mathrm{B}}P_{\mathrm{H}}u_{\mathrm{A}}
\end{array}
$$

◆1.4.2 Polarization Sum

Single photon case $M=\epsilon^*_\mu(k)M^\mu$

When Ward identity $k_\mu M^\mu=0$ is valid,

$$\sum_{\text{pol.}}|M|^2=\sum_{\text{pol.}}\epsilon^*_\mu(k)\epsilon_\nu(k)M^\mu M^{\nu*}=\eta_{\mu\nu}M^\mu M^{\nu*}. \tag{1.1}$$

Double photons case $M=\epsilon^*_\mu(k)\epsilon'^*_\nu(k')M^{\mu\nu}$

When $k_\mu M^{\mu\nu}=k'_\nu M^{\mu\nu}=0$ is valid,

$$\sum_{\text{pol.}}|M|^2=\sum_{\text{pol.}}\epsilon^*_\mu(k)\epsilon_\rho(k)\epsilon'^*_\nu(k')\epsilon'_\sigma(k')M^{\mu\nu}M^{\rho\sigma*}=\eta_{\mu\rho}\eta_{\nu\sigma}M^{\mu\nu}M^{\rho\sigma*}. \tag{1.2}$$

【See Sec. ?? for verbose information.】

◆1.4.3 Fierz transformations

For Dirac spinors a,b,c,d,

$$
\begin{aligned}
S(a,b;c,d) &:= (\bar{a}b)(\bar{c}d);\\
V(a,b;c,d) &:= (\bar{a}\gamma^\mu b)(\bar{c}\gamma_\mu d);\\
T(a,b;c,d) &:= \frac{1}{2}(\bar{a}\sigma^{\mu\nu}b)(\bar{c}\sigma_{\mu\nu}d);\\
A(a,b;c,d) &:= (\bar{a}\gamma^\mu\gamma_5 b)(\bar{c}\gamma_\mu\gamma_5 d);\\
P(a,b;c,d) &:= (\bar{a}\gamma_5 b)(\bar{c}\gamma_5 d);
\end{aligned}
\qquad
\begin{pmatrix} S(a,b;c,d)\\ V(a,b;c,d)\\ T(a,b;c,d)\\ A(a,b;c,d)\\ P(a,b;c,d)\end{pmatrix}
=\frac{1}{4}
\begin{pmatrix} 1 & 1 & 1 & -1 & -1\\ 4 & -2 & 0 & -2 & 4\\ 6 & 0 & -2 & 0 & -6\\ -4 & -2 & 0 & -2 & -4\\ -1 & 1 & -1 & -1 & 1\end{pmatrix}
\begin{pmatrix} S(a,d;c,b)\\ V(a,d;c,b)\\ T(a,d;c,b)\\ A(a,d;c,b)\\ P(a,d;c,b)\end{pmatrix}
$$

Also defining $V_{\mathrm{LR}}(a,b;c,d):=(\bar{a}\gamma^\mu P_{\mathrm{L}}b)(\bar{c}\gamma_\mu P_{\mathrm{R}}d)$ and so on,

$$V_{\mathrm{LL}}(a,b;c,d)=-V_{\mathrm{LL}}(a,d;c,b) \qquad S_{\mathrm{RL}}(a,b;c,d)=\frac{1}{4}\left[V_{\mathrm{LR}}(a,d;b,c)-A_{\mathrm{LR}}(a,d;b,c)\right] \tag{1.3}$$

$$V_{\mathrm{RR}}(a,b;c,d)=-V_{\mathrm{RR}}(a,d;c,b) \qquad S_{\mathrm{LR}}(a,b;c,d)=\frac{1}{4}\left[V_{\mathrm{RL}}(a,d;b,c)-A_{\mathrm{RL}}(a,d;b,c)\right] \tag{1.4}$$

Here we can create another equations using

$$(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta}=2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}; \qquad (\bar{\sigma}^\mu)_{\alpha\beta}(\bar{\sigma}_\mu)_{\gamma\delta}=2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}. \tag{1.5}$$

◆1.4.4 Gordon identity

For $P := p' + p$ and $q := p' - p$,

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{P^\mu + \mathrm{i}\sigma^{\mu\nu}q_\nu}{2m}\right]u(p) \qquad \bar{u}(p')\gamma^\mu v(p) = \bar{u}(p')\left[\frac{q^\mu + \mathrm{i}\sigma^{\mu\nu}P_\nu}{2m}\right]v(p) \tag{1.6}$$

$$\bar{v}(p')\gamma^\mu v(p) = -\bar{v}(p')\left[\frac{P^\mu + \mathrm{i}\sigma^{\mu\nu}q_\nu}{2m}\right]v(p) \qquad \bar{v}(p')\gamma^\mu u(p) = -\bar{v}(p')\left[\frac{q^\mu + \mathrm{i}\sigma^{\mu\nu}P_\nu}{2m}\right]u(p) \tag{1.7}$$

◆1.4.5 Color Sum

(Here T^a is **3** of SU(3). For other representations or gauge groups, see Sec. 1.9.)

$$\mathrm{Tr}(T^aT^b) := \frac{1}{2}\delta^{ab} \qquad \text{(That is, } T^a\text{'s are } \tfrac{1}{2}\times\text{Gell-Mann matrices.)} \tag{1.8}$$

$$\sum_a T^aT^a = \frac{4}{3}\cdot\mathbf{1}, \qquad \sum_{c,d} f^{acd}f^{bcd} = 3\delta^{ab} \qquad \sum_a T^a_{ij}T^a_{kl} = \frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{6}\delta_{ij}\delta_{kl} \tag{1.9}$$

$$\sum_a T^aT^bT^a = -\frac{1}{6}T^b \qquad \sum_{b,c} f^{abc}T^bT^c = \frac{3\mathrm{i}}{2}T^a \qquad f^{Dab}f^{EDc}+f^{Dca}f^{EDb} + f^{Dbc}f^{EDa} = 0 \tag{1.10}$$

1.5 Miscellaneous Techniques

$(p\cdot\sigma)(p\cdot\bar{\sigma}) = p^2$

$\epsilon^{ab}\epsilon^{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}$

$\sqrt{p_\mu\sigma^\mu} = \dfrac{p_\mu\sigma^\mu + m}{\sqrt{2(m+p^0)}}$

$\sigma^i\sigma^j = \delta_{ij}\sigma^0 + \mathrm{i}\epsilon_{ijk}\sigma^k$

$\sigma^\mu\sigma^\nu = \mathrm{i}\epsilon^{0\mu\nu\rho}\sigma^\rho + \delta^\mu_0\sigma^\nu + \delta^\nu_0\sigma^\mu - \eta^{\mu\nu}\sigma^0$

$[\sigma^i,\sigma^j] = 2\mathrm{i}\epsilon_{ijk}\sigma^k$

$\sigma^i,\sigma^j = 2\delta_{ij}$

TODO: TODO:

- Majorana Ferminos
- Feynman Rules(A.1)

1.6 Dirac's Gamma Algebras

◆1.6.1 Traces

$$\begin{aligned} \mathrm{Tr}(\text{any odd } \# \text{ of } \gamma\text{'s}) &= 0 & (1.11)\\ \mathrm{Tr}(\gamma^\mu\gamma^\nu) &= 4\eta^{\mu\nu} & (1.12)\\ \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) & (1.13)\\ \mathrm{Tr}(\gamma_5 \text{ and any odd } \# \text{ of } \gamma\text{'s}) &= 0 & (1.14)\\ \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) &= 0 & (1.15)\\ \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) &= -4i\epsilon^{\mu\nu\rho\sigma} & (1.16) \end{aligned}$$

Generally, for some γ-matrices $A, B, C, \ldots,$

$$\begin{aligned} \mathrm{Tr}(ABCDEF\cdots) &= \eta^{AB}\,\mathrm{Tr}(CDEF\cdots) - \eta^{AC}\,\mathrm{Tr}(BDEF\cdots)\\ &\quad + \eta^{AD}\,\mathrm{Tr}(BCEF\cdots) - \eta^{AE}\,\mathrm{Tr}(BCDF\cdots) + \cdots, \end{aligned} \tag{1.17}$$

$$\mathrm{Tr}(ABCDEF\cdots\gamma_5) = \textbf{Not Established.} \tag{1.18}$$

To prove the second equation, we use following technique:

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\cdots) = \mathrm{Tr}(\cdots\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\mu); \qquad \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\cdots\gamma_5) = \mathrm{Tr}(\gamma_5\cdots\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\mu). \tag{1.19}$$

◆1.6.2 Contractions

$$\begin{aligned} \gamma^\mu\gamma_\mu &= 4 & (1.20)\\ \gamma^\mu\gamma^\nu\gamma_\mu &= -2\gamma^\nu & (1.21)\\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4\eta^{\nu\rho} & (1.22)\\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu & (1.23) \end{aligned}$$

Generally, for some γ-matrices $A, B, C, \ldots,$

$$\begin{aligned} \text{ODD } \# : \quad \gamma^\mu ABC\cdots\gamma_\mu &= -2(\cdots CBA), & (1.24)\\ \text{EVEN } \# : \quad \gamma^\mu ABC\cdots\gamma_\mu &= \mathrm{Tr}(ABC\cdots) - \mathrm{Tr}(ABC\cdots\gamma_5)\cdot\gamma_5. & (1.25) \end{aligned}$$

<u>Contractions in d-dimension</u>

$$\begin{aligned} \gamma^\mu\gamma_\mu &= d & (1.26)\\ \gamma^\mu\gamma^\nu\gamma_\mu &= -(d-2)\gamma^\nu & (1.27)\\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4\eta^{\nu\rho} - (4-d)\gamma^\nu\gamma^\rho & (1.28)\\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu + (4-d)\gamma^\nu\gamma^\rho\gamma^\sigma & (1.29) \end{aligned}$$

<u>Contractions of ϵ's</u>

$$\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\alpha\beta\gamma\delta} = -24; \quad \epsilon^{\alpha\beta\gamma\mu}\epsilon_{\alpha\beta\gamma\nu} = -6\delta^\mu_\nu; \quad \epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\rho\sigma} = -2(\delta^\mu_\rho\delta^\nu_\sigma - \delta^\mu_\sigma\delta^\nu_\rho) \tag{1.30}$$

$$\epsilon^{\mu\alpha\beta\gamma}\epsilon_{\mu\alpha'\beta'\gamma'} = -\left(\delta^\alpha_{\alpha'}\delta^\beta_{\beta'}\delta^\gamma_{\gamma'} + \delta^\alpha_{\beta'}\delta^\beta_{\gamma'}\delta^\gamma_{\alpha'} + \delta^\alpha_{\gamma'}\delta^\beta_{\alpha'}\delta^\gamma_{\beta'} - \delta^\alpha_{\alpha'}\delta^\beta_{\gamma'}\delta^\gamma_{\beta'} - \delta^\alpha_{\beta'}\delta^\beta_{\alpha'}\delta^\gamma_{\gamma'} - \delta^\alpha_{\gamma'}\delta^\beta_{\beta'}\delta^\gamma_{\alpha'}\right) \tag{1.31}$$

1.7 Loop Integrals and Dimensional Regularization

◆1.7.1 Feynman Parameters

$$\frac{1}{A_1 A_2 \cdots A_n} = \int_0^1 \mathrm{d}x_1 \cdots x_n \, \delta\left(\sum x_i - 1\right) \frac{(n-1)!}{[x_1 A_1 + x_2 A_2 + \cdots + x_n A_n]^n} \tag{1.32}$$

$$\frac{1}{A_1 A_2} = \int_0^1 \mathrm{d}x \frac{1}{[xA_1 + (1-x)A_2]^2} \tag{1.33}$$

◆1.7.2 d-dimensional integrals in Minkowski space

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{1}{(l^2 - \Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2}} \tag{1.34}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^2}{(l^2 - \Delta)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 1} \tag{1.35}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2 - \Delta)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{d/2}} \frac{\eta^{\mu\nu}}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 1} \tag{1.36}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{(l^2)^2}{(l^2 - \Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{d(d+2)}{4} \frac{\Gamma(n - \frac{d}{2} - 2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 2} \tag{1.37}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^\mu l^\nu l^\rho l^\sigma}{(l^2 - \Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2} - 2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 2} \frac{\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}}{4} \tag{1.38}$$

Here we can use following expansions: $(\gamma \simeq 0.5772)$

$$\left(\frac{1}{\Delta}\right)^{2 - \frac{d}{2}} = 1 - (d-4)\frac{\log \Delta}{2} + \mathrm{O}\left((d-4)^2\right) \quad \text{around } d = 4, \tag{1.39}$$

$$\Gamma(x) = \frac{1}{x} - \gamma + \mathrm{O}(x) \quad \text{around } x = 0, \tag{1.40}$$

$$\Gamma(x) = \frac{(-1)^n}{n!} \left[\frac{1}{x+n} - \gamma + \sum_{k=1}^{n} \frac{1}{k} + \mathrm{O}(x+n)\right] \quad \text{around } x = -n. \tag{1.41}$$

and we get following expansion:

$$\frac{\Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2}} \left(\frac{1}{\Delta}\right)^{2 - \frac{d}{2}} = \frac{1}{(4\pi)^2} \left[\left(\frac{2}{4-d} - \gamma + \log 4\pi\right) - \log \Delta + \mathrm{O}(4-d)\right]. \tag{1.42}$$

Usually this Δ is positive, but when Δ contains some timelike momenta, it becomes negative. Then these integrals acquire imaginary parts, which give the discontinuities of S-matrix elements. To compute the S-matrix in a physical region choose the correct branch

$$\left(\frac{1}{\Delta}\right)^{n - \frac{d}{2}} \to \left(\frac{1}{\Delta - \mathrm{i}\epsilon}\right)^{n - \frac{d}{2}}. \tag{1.43}$$

1.8 Cross Sections and Decay Rates

■General expression (The mass dimension of $\mathcal{M}$ is $2 - N_{\mathrm{f}}$ for $\mathrm{d}\sigma$ and $3 - N_{\mathrm{f}}$ for $\mathrm{d}\Gamma$.)

$$\mathrm{d}\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|}\left[\prod_f \frac{\mathrm{d}^3 p_f}{(2\pi)^3}\frac{1}{2E_f}\right]\left|\mathcal{M}(p_A, p_B \to \{p_f\})\right|^2 (2\pi)^4 \delta^{(4)}\left(p_A + p_B - \{p_f\}\right) \tag{1.44}$$

$$\mathrm{d}\Gamma = \frac{1}{2m_A}\left[\prod_f \frac{\mathrm{d}^3 p_f}{(2\pi)^3}\frac{1}{2E_f}\right]\left|\mathcal{M}(m_A \to \{p_f\})\right|^2 (2\pi)^4 \delta^{(4)}\left(m_A - \{p_f\}\right) \quad \text{(in A-rest frame.)} \tag{1.45}$$

■2-body phase space in center-of-mass frame

$$\int \Pi_2 := \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3}\int \frac{\mathrm{d}^3 p_2}{(2\pi)^3}\frac{1}{2E_1}\frac{1}{2E_2}(2\pi)^4\delta^{(4)}\left(E_{\mathrm{cm}} - (p_1 + p_2)\right) \qquad \text{(in center-of-mass frame)} \tag{1.46}$$

$$= \int \frac{\mathrm{d}\Omega}{4\pi}\frac{1}{8\pi}\frac{2\,\|\boldsymbol{p}_1\|}{E_{\mathrm{cm}}} \tag{1.47}$$

$$= \frac{1}{8\pi}\sqrt{1 - \frac{2({m_1}^2 + {m_2}^2)}{E_{\mathrm{cm}}^2} + \frac{({m_1}^2 - {m_2}^2)^2}{E_{\mathrm{cm}}^4}} \quad \xrightarrow{m_2 = 0} \quad \frac{1}{8\pi}\left(1 - \frac{{m_1}^2}{{E_{\mathrm{cm}}}^2}\right) \tag{1.48}$$

■Kinematics of Decay

$$K \to p_1 + p_2 \qquad \text{or} \qquad \begin{pmatrix} M \\ \boldsymbol{0} \end{pmatrix} \to \begin{pmatrix} \sqrt{p^2 + {m_1}^2} \\ \boldsymbol{p} \end{pmatrix} + \begin{pmatrix} \sqrt{p^2 + {m_2}^2} \\ -\boldsymbol{p} \end{pmatrix}; \tag{1.49}$$

$$\|\boldsymbol{p}\|^2 = \frac{1}{4}\left[M^2 - 2\left({m_1}^2 + {m_2}^2\right) + \frac{\left({m_1}^2 - {m_2}^2\right)^2}{M^2}\right] \approx \left(\frac{M^2 - {m_1}^2}{2M}\right)^2$$

$$E_1 = \frac{M^2 + {m_1}^2 - {m_2}^2}{2M}, \qquad E_2 = \frac{M^2 + {m_2}^2 - {m_1}^2}{2M};$$

$$K \cdot p_1 = \frac{M^2 + {m_1}^2 - {m_2}^2}{2}, \qquad p_1 \cdot p_2 = \frac{M^2 - \left({m_1}^2 + {m_2}^2\right)}{2}.$$

■Mandelstam Variables

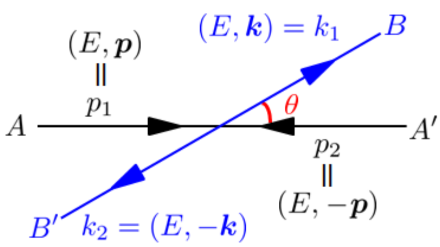
For $p_1 + p_2 \to k_1 + k_2$ collision,

$$s = (p_1 + p_2)^2 = (k_1 + k_2)^2,$$
$$t = (p_1 - k_1)^2 = (p_2 - k_2)^2,$$
$$u = (p_1 - k_2)^2 = (p_2 - k_1)^2,$$

and

$$s + t + u = p_1^2 + p_2^2 + k_1^2 + k_2^2 = \sum m^2.$$

■Kinematics of Collision (Same Mass)



$$\|\boldsymbol{p}\|^2 = E^2 - m_A^2 \qquad \boldsymbol{p}\cdot\boldsymbol{k} = \|\boldsymbol{p}\|\,\|\boldsymbol{k}\|\cos\theta$$
$$\|\boldsymbol{k}\|^2 = E^2 - m_B^2$$

$$p_1 \cdot p_2 = s/2 - m_A^2 \quad p_1 \cdot k_1 = p_2 \cdot k_2 = \tfrac{1}{2}(m_A^2 + m_B^2 - t);$$
$$k_1 \cdot k_2 = s/2 - m_B^2 \quad p_1 \cdot k_2 = p_1 \cdot k_2 = \tfrac{1}{2}(m_A^2 + m_B^2 - u);$$

$$s = 4E^2,$$
$$(p_1 - p_2)^2 = -4(E^2 - m_A^2) \qquad t = -(2E^2 - m_A^2 - m_B^2) + 2\boldsymbol{p}\cdot\boldsymbol{k}$$
$$(k_1 - k_2)^2 = -4(E^2 - m_B^2) \qquad u = -(2E^2 - m_A^2 - m_B^2) - 2\boldsymbol{p}\cdot\boldsymbol{k}$$

1.9 楊-Mills Theory (See App. ?? for verbose notes.)

◆1.9.1 Non-Abelian gauge theory

$$[T^a,T^b]=\mathrm{i}f^{ab}{}_cT^c,\qquad 0=f^D{}_{ab}f^E{}_{Dc}+f^D{}_{ca}f^E{}_{Db}+f^D{}_{bc}f^E{}_{Da},\qquad \mathrm{D}_\mu=\partial_\mu-\mathrm{i}gA_\mu$$

$$\operatorname{Tr}T^aT^b=\frac{1}{2}\delta^{ab},\qquad [\widetilde{T}^a]_i{}^j:=T^{\mathrm{ad}\ a}{}_i{}^j:=-\mathrm{i}f^{aij}\qquad [\widetilde{\mathrm{D}}_\mu]_i{}^j:=\delta_i^j\partial_\mu+gf^{iaj}A^a_\mu.$$

$$\begin{aligned}F_{\mu\nu}&=\frac{\mathrm{i}}{g}\left[\mathrm{D}_\mu,\mathrm{D}_\nu\right]\\&=\partial_\mu A_\nu-\partial_\nu A_\mu+\frac{g}{\mathrm{i}}\left[A_\mu,A_\nu\right]\\&=\Big[\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu\Big]T^a\end{aligned}\qquad\begin{aligned}&\mathrm{D}_\mu\phi=\partial_\mu\phi-\mathrm{i}gA^a_\mu(T^a_\phi\phi)\\&\mathrm{D}_\mu F_{\mu\nu}{}^a=\partial_\mu F^a_{\mu\nu}+gf^{abc}A^b_\mu F^c_{\mu\nu},\\&\Big(\mathrm{D}_\mu F_{\nu\rho}=\partial_\mu\lambda-\mathrm{i}g[A_\mu,F_{\nu\rho}]\Big).^{*1}\end{aligned}$$

$$\phi\mapsto V\phi:=\mathrm{e}^{\mathrm{i}g\theta}\phi\qquad A_\mu\mapsto V\left(A_\mu+\frac{\mathrm{i}}{g}\partial_\mu\right)V^{-1}\qquad F_{\mu\nu}\mapsto VF_{\mu\nu}V^{-1}$$

$$\phi^{a\prime}\simeq\phi+\mathrm{i}g\theta^aT^a\phi\qquad A^{a\prime}_\mu\simeq A^a_\mu+\partial_\mu\theta^a+gf^{abc}A^b_\mu\theta^c\qquad F^a_{\mu\nu}{}'\simeq F^a_{\mu\nu}+gf^{abc}F^b_{\mu\nu}\theta^c$$

$$\epsilon^{\mu\nu\rho\sigma}\left[\mathrm{D}_\nu,\left[\mathrm{D}_\rho,\mathrm{D}_\sigma\right]\right]=\epsilon^{\mu\nu\rho\sigma}\mathrm{D}_\nu F_{\rho\sigma}=0.$$

■Killing and Casimir Here we have two constants which **depend on representation** r.

$$\operatorname{Tr}(T^aT^b)=:C(r)\delta^{ab}\quad\text{(Killing form)},\qquad T^aT^a=:C_2(r)\cdot\mathbf{1}\quad\text{(quadratic Casimir operator)},\tag{1.50}$$

which satisfy

$$C(r)=\frac{d(r)}{d(\mathrm{ad})}C_2(r),\qquad T^aT^bT^a=\left[C_2(r)-\frac{1}{2}C_2(\mathrm{ad})\right]T^b,\tag{1.51}$$

$$f^{acd}f^{bcd}=C_2(\mathrm{ad})\delta^{ab},\qquad f^{abc}T^bT^c=\frac{\mathrm{i}}{2}C_2(\mathrm{ad})T^a.\tag{1.52}$$

For $\mathrm{SU}(N)$ For its fundamental representation N with definition $C(N):=\frac{1}{2}$, we have

$$C(N):=\frac{1}{2},\quad C_2(N)=\frac{N^2-1}{2N},\quad C(\mathrm{ad})=C_2(\mathrm{ad})=N;\quad (T^a)_{ij}(T^a)_{kl}=\frac{1}{2}\left(\delta_{il}\delta_{kj}-\frac{\delta_{ij}\delta_{kl}}{N}\right).$$

◆1.9.2 Abelian gauge theory

In Abelian gauge theory, V and fields are always commutative, and thus we have charge freedom (Q).

$$\mathrm{D}_\mu\phi=\left(\partial_\mu-\mathrm{i}gA_\mu Q\right)\phi\qquad\phi\mapsto\mathrm{e}^{igQ\theta}\phi\qquad F_{\mu\nu}=\frac{\mathrm{i}}{g}\left[\mathrm{D}_\mu,\mathrm{D}_\nu\right]=\partial_\mu A_\nu-\partial_\nu A_\mu$$

$$\mathrm{D}_\mu\lambda^a=\partial_\mu\lambda^a\qquad A_\mu\mapsto A_\mu+\partial_\mu\theta\qquad F_{\mu\nu}\mapsto F_{\mu\nu}$$

◆1.9.3 Lagrangian Block

$$\mathcal{L}\ni|\mathrm{D}_\mu\phi|^2-m^2|\phi|^2,\quad\overline{\psi}(\mathrm{i}\not{D}-m)\psi,\quad-\frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}\left(=-\frac{1}{2}\operatorname{Tr}F^{\mu\nu}F_{\mu\nu}\right),\quad\theta\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\tag{1.53}$$

$$-\frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}=-\frac{1}{2}\left[(\partial_\mu A^a_\nu)^2+A^a_\mu\partial^\mu\partial^\nu A^a_\nu\right]-gf^{abc}A^a_\mu A^b_\nu\partial^\mu A^{c\nu}-\frac{g^2}{4}f^{abc}f^{ade}A^b_\mu A^c_\nu A^{d\mu}A^{e\nu}\tag{1.54}$$

*1 Note that we can use any representation T^a but must the same ones for $A^a_\mu T^a$ and $\lambda^a T^a$.

2 Standard Model

Any representations assumed to be *normalized Hermitian*. Note that the $SU(2)$ **2** representation is

$$T^a = \frac{1}{2}\sigma^a; \qquad \left[T^a, T^b\right] = \mathrm{i}\epsilon^{abc}T^c; \qquad T^{\pm} := T^1 \pm \mathrm{i}T^2. \tag{2.1}$$

We use the following abridged notations:

$$(\partial A)_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad\qquad F^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu. \tag{2.2}$$

2.1 Symmetries and Fields

	$\mathrm{SU}(3)_{\mathrm{strong}}$	$\mathrm{SU}(2)_{\mathrm{weak}}$	$\mathrm{U}(1)_Y$
Matter Fields (Fermionic / Lorentz Spinor)			
$P_{\mathrm{L}}Q_i$: Left-handed quarks	**3**	**2**	1/6
$P_{\mathrm{L}}U_i$: Right-handed up-type quarks	**3**	**1**	2/3
$P_{\mathrm{R}}D_i$: Right-handed down-type quarks	**3**	**1**	−1/3
$P_{\mathrm{R}}L_i$: Left-handed leptons	**1**	**2**	−1/2
$P_{\mathrm{R}}E_i$: Right-handed leptons	**1**	**1**	−1
Higgs Field (Bosonic / Lorentz Scalar)			
H : Higgs	**1**	**2**	1/2
Gauge Fields (Bosonic / Lorentz Vector)			
G : Gluons	**8**	**1**	0
W : Weak bosons	**1**	**3**	0
B : B boson	**1**	**1**	0

■Full Lagrangian $\mathcal{L} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{matter}} + \mathcal{L}_{\text{湯川}}$

$$\text{where} \quad \mathcal{L}_{\mathrm{gauge}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} \tag{2.3}$$

$$\mathcal{L}_{\mathrm{Higgs}} = \left|\left(\partial_\mu - \mathrm{i}g_2W_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)H\right|^2 - V(H), \tag{2.4}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{matter}} = {} & \overline{Q}_i\mathrm{i}\gamma^\mu\left(\partial_\mu - \mathrm{i}g_3G_\mu - \mathrm{i}g_2W_\mu - \frac{1}{6}\mathrm{i}g_1B_\mu\right)P_{\mathrm{L}}Q_i \\ & + \overline{U}_i\mathrm{i}\gamma^\mu\left(\partial_\mu - \mathrm{i}g_3G_\mu - \frac{2}{3}\mathrm{i}g_1B_\mu\right)P_{\mathrm{R}}U_i \\ & + \overline{D}_i\mathrm{i}\gamma^\mu\left(\partial_\mu - \mathrm{i}g_3G_\mu + \frac{1}{3}\mathrm{i}g_1B_\mu\right)P_{\mathrm{R}}D_i \\ & + \overline{L}_i\mathrm{i}\gamma^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)P_{\mathrm{L}}L_i \\ & + \overline{E}_i\mathrm{i}\gamma^\mu\left(\partial_\mu + \mathrm{i}g_1B_\mu\right)P_{\mathrm{R}}E_i,\end{aligned} \tag{2.5}$$

$$\mathcal{L}_{\text{湯川}} = \overline{U}_i(y_u)_{ij}HP_{\mathrm{L}}Q_j - \overline{D}_i(y_d)_{ij}H^\dagger P_{\mathrm{L}}Q_j - \overline{E}_i(y_e)_{ij}H^\dagger P_{\mathrm{L}}L_j + \mathrm{H.c.} \tag{2.6}$$

We have no freedom to add other terms into this Lagrangian of the gauge theory. See Appendix **??**.

■Gauge Kinetic Terms

the gauge kinetic terms can be expanded as

$$\begin{aligned}\mathcal{L}_{\text{gauge}} =& -\frac{1}{4}(\partial B)(\partial B)\\ &-\frac{1}{4}(\partial W^a)(\partial W^a) - g_2\epsilon^{abc}(\partial_\mu W^a_\nu)W^{\mu b}W^{\nu c} - \frac{{g_2}^2}{4}\left(\epsilon^{eab}W^a_\mu W^b_\nu\right)\left(\epsilon^{ecd}W^{c\mu}W^{d\nu}\right)\\ &-\frac{1}{4}(\partial G^a)(\partial G^a) - g_3 f^{abc}(\partial_\mu G^a_\nu)G^{\mu b}G^{\nu c} - \frac{{g_3}^2}{4}\left(f^{eab}G^a_\mu G^b_\nu\right)\left(f^{ecd}G^{c\mu}G^{d\nu}\right).\end{aligned} \tag{2.7}$$

2.2 Higgs Mechanism

■Higgs Potential

The (renormalizable) Higgs potential must be

$$V(H) = -\mu^2(H^\dagger H) + \lambda\left(H^\dagger H\right)^2. \tag{2.8}$$

for the SU(2), and $\lambda > 0$ in order not to run away the VEVs, while μ^2 is positive for the EWSB.

To discuss this clearly, let us *redefine* the Higgs field as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + \mathrm{i}\phi_2\\ v + (h + \mathrm{i}\phi_3)\end{pmatrix}, \qquad \text{where} \quad v = \sqrt{\frac{\mu^2}{\lambda}}. \tag{2.9}$$

Here h is the "Higgs boson," and ϕ_i are 南部–Goldstone bosons.

The Higgs potential becomes

$$V(h) = \frac{\mu^2}{4v^2}h^4 + \frac{\mu^2}{v}h^3 + \mu^2 h^2, \tag{2.10}$$

and now we know the Higgs boson has acquired mass $m_h = \sqrt{2}\mu$. Also

$$\mathcal{L}_{\text{Higgs}} = \left|\left(\partial_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{2}\mathrm{i}g_1 B_\mu\right)H\right|^2 \tag{2.11}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{8}\left[{g_2}^2{W_1}^2 + {g_2}^2{W_2}^2 + (g_1 B - g_2 W_3)^2\right]. \tag{2.12}$$

Redefining the gauge fields (with concerning the norms) as

$$W^\pm_\mu := \frac{1}{\sqrt{2}}(W^1_\mu \mp \mathrm{i}W^2_\mu), \qquad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} := \begin{pmatrix}\cos\theta_{\mathrm{w}} & -\sin\theta_{\mathrm{w}}\\ \sin\theta_{\mathrm{w}} & \cos\theta_{\mathrm{w}}\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix}, \tag{2.13}$$

where

$$\tan\theta_{\mathrm{w}} := \frac{g_1}{g_2}, \qquad e := -\frac{g_1 g_2}{\sqrt{{g_1}^2 + {g_2}^2}}; \qquad g_Z := \sqrt{{g_1}^2 + {g_2}^2}; \tag{2.14}$$

$$g_1 = \frac{|e|}{\cos\theta_{\mathrm{w}}} = g_Z\sin\theta_{\mathrm{w}}, \qquad g_2 = \frac{|e|}{\sin\theta_{\mathrm{w}}} = g_Z\cos\theta_{\mathrm{w}}. \tag{2.15}$$

We obtain the following terms in $\mathcal{L}_{\text{Higgs}}$:

$$\mathcal{L}_{\text{Higgs}} \supset \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{4}\left[{g_2}^2 W^{+\mu}W^-_\mu + \frac{{g_Z}^2}{2}Z^\mu Z_\mu\right]. \tag{2.16}$$

Here we have omitted the 南部–Goldstone bosons.

Here we present another form:

$$g_1 B_\mu = |e| A_\mu - \tan\theta_\mathrm{w} Z_\mu, \tag{2.17}$$

$$g_2 W_\mu = \frac{g_2}{\sqrt{2}} \left(W^+_\mu T^+ + W^-_\mu T^- \right) + \left(\frac{|e|}{\tan\theta_\mathrm{w}} Z_\mu + |e| A_\mu \right) T^3, \tag{2.18}$$

$$Z^0_\mu := \frac{1}{\sqrt{{g_1}^2 + {g_2}^2}} (g_2 W^3_\mu - g_1 B_\mu), \quad A_\mu := \frac{1}{\sqrt{{g_1}^2 + {g_2}^2}} (g_1 W^3_\mu + g_2 B_\mu) \tag{2.19}$$

You can see the gauge bosons have acquired the masses

$$m_A = 0, \quad m_W := \frac{g_2}{2} v, \quad m_Z := \frac{g_Z}{2} v. \tag{2.20}$$

■Gauge Term The SU(2) gauge term is converted into

$$\begin{aligned} W^{a\mu\nu} W^a_{\mu\nu} = &(\partial W^3)(\partial W^3) + 2(\partial W^+)(\partial W^-) \\ &- 4\mathrm{i}g \left[(\partial W^3)^{\mu\nu} W^+_\mu W^-_\nu + (\partial W^+)^{\mu\nu} W^-_\mu W^3_\nu + (\partial W^-)^{\mu\nu} W^3_\mu W^+_\nu \right] \\ &- 2g^2 (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma}) \left(W^+_\mu W^+_\nu W^-_\rho W^-_\sigma - 2 W^3_\mu W^3_\nu W^+_\rho W^-_\sigma \right), \end{aligned}$$

and therefore the final expression is

$$\begin{aligned} \mathcal{L}_\text{gauge} := &-\frac{1}{4} \left[G^{a\mu\nu} G^a_{\mu\nu} + (\partial Z)^{\mu\nu} (\partial Z)_{\mu\nu} + (\partial A)^{\mu\nu} (\partial A)_{\mu\nu} + 2 (\partial W^+)^{\mu\nu} (\partial W^-)_{\mu\nu} \right] \\ &+ \frac{\mathrm{i}|e|}{\tan\theta_\mathrm{w}} \Big[(\partial W^+)^{\mu\nu} W^-_\mu Z_\nu + (\partial W^-)^{\mu\nu} Z_\mu W^+_\nu + (\partial Z)^{\mu\nu} W^+_\mu W^-_\nu \Big] \\ &+ \mathrm{i}|e| \Big[(\partial W^+)^{\mu\nu} W^-_\mu A_\nu + (\partial W^-)^{\mu\nu} A_\mu W^+_\nu + (\partial A)^{\mu\nu} W^+_\mu W^-_\nu \Big] \\ &+ (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma}) \Bigg[\frac{|e|^2}{2\sin^2\theta_\mathrm{w}} W^+_\mu W^+_\nu W^-_\rho W^-_\sigma + \frac{|e|^2}{\tan^2\theta_\mathrm{w}} W^+_\mu Z_\nu W^-_\rho Z_\sigma \\ &\qquad + \frac{|e|^2}{\tan\theta_\mathrm{w}} \left(W^+_\mu Z_\nu W^-_\rho A_\sigma + W^+_\mu A_\nu W^-_\rho Z_\sigma \right) + |e|^2 W^+_\mu A_\nu W^-_\rho A_\sigma \Bigg]. \end{aligned} \tag{2.21}$$

■湯川 Term

$$\begin{aligned} \mathcal{L}_\text{湯川} = &\overline{U} y_u H P_\mathrm{L} Q - \overline{D} y_d H^\dagger P_\mathrm{L} Q - \overline{E} y_e H^\dagger P_\mathrm{L} L + \text{H.c.} \\ = &\overline{U} y_u \epsilon^{\alpha\beta} H^\alpha P_\mathrm{L} Q^\beta - \overline{D} y_d {H^\dagger}^\alpha P_\mathrm{L} Q^\alpha - \overline{E} y_e {H^\dagger}^\alpha P_\mathrm{L} L^\alpha + \text{H.c.} \\ = &-\frac{v+h}{\sqrt{2}} \left(\overline{U} y_u P_\mathrm{L} Q^1 + \overline{D} y_d P_\mathrm{L} Q^2 + \overline{E} y_e P_\mathrm{L} L^2 \right) + \text{H.c.} \end{aligned} \tag{2.22}$$

2.3 Full Lagrangian After Higgs Mechanism

Now we have the following Lagrangian (with omitting P_{L} etc.):

$$\begin{aligned}
\mathcal{L} =& \mathcal{L}_{\text{gauge}} + {m_W}^2 W^+W^- + \frac{{m_Z}^2}{2}Z^2 \\
\text{【Higgs】}\quad & + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}{m_h}^2h^2 - \sqrt{\frac{\lambda}{2}}m_h h^3 - \frac{1}{4}\lambda h^4 \\
& + \frac{v{g_2}^2}{4}W^+W^-h + \frac{v({g_1}^2+{g_2}^2)}{8}Z^2h \\
& + \frac{{g_2}^2}{4}W^+W^-h^2 + \frac{{g_1}^2+{g_2}^2}{8}Z^2h^2 \\
& - \left(\frac{1}{\sqrt{2}}h\bar{U}y_uQ^1 + \frac{1}{\sqrt{2}}h\bar{D}y_dQ^2 + \frac{1}{\sqrt{2}}h\bar{E}y_eL^2 + \text{H.c.}\right) \\
\text{【SU(3)】}\quad & + \bar{Q}\left(\mathrm{i}\partial\!\!\!/ + g_3 G\!\!\!/\right)Q + \bar{U}\left(\mathrm{i}\partial\!\!\!/ + g_3 G\!\!\!/\right)U + \bar{D}\left(\mathrm{i}\partial\!\!\!/ + g_3 G\!\!\!/\right)D + \bar{L}\left(\mathrm{i}\partial\!\!\!/\right)L + \bar{E}\left(\mathrm{i}\partial\!\!\!/\right)E \\
\text{【}W\text{】}\quad & + \bar{Q}\frac{g_2}{\sqrt{2}}\left(W\!\!\!/^+T^+ + W\!\!\!/^-T^-\right)Q + \bar{L}\frac{g_2}{\sqrt{2}}\left(W\!\!\!/^+T^+ + W\!\!\!/^-T^-\right)L \\
\text{【}A\&Z^0\text{】}\quad & + \bar{Q}\left[\left(T^3+\frac{1}{6}\right)|e|A\!\!\!/ + \left(\frac{|e|c}{s}T^3 - \frac{|e|s}{6c}\right)Z\!\!\!/^0\right]Q \\
& + \bar{U}\left(\frac{2}{3}|e|A\!\!\!/ - \frac{2|e|s}{3c}Z\!\!\!/\right)U \\
& + \bar{D}\left(-\frac{1}{3}|e|A\!\!\!/ + \frac{|e|s}{3c}Z\!\!\!/\right)D \\
& + \bar{L}\left[\left(T^3-\frac{1}{2}\right)|e|A\!\!\!/ + \left(\frac{|e|c}{s}T^3 + \frac{|e|s}{2c}\right)Z\!\!\!/^0\right]L \\
& + \bar{E}\left(-|e|A\!\!\!/ + \frac{|e|s}{c}Z\!\!\!/\right)E \\
\text{【湯川項】}\quad & - \left(\frac{1}{\sqrt{2}}v\bar{U}y_uQ^1 + \frac{1}{\sqrt{2}}v\bar{D}y_dQ^2 + \frac{1}{\sqrt{2}}v\bar{E}y_eL^2 + \text{H.c.}\right)
\end{aligned} \tag{2.23}$$

2.4 Mass Eigenstates

Here we will obtain the mass eigenstates of the fermions, by diagonalizing the 湯川 matrices.

We use the singular value decomposition method to mass matrices $Y_\bullet := vy_\bullet/\sqrt{2}$. Generally, any matrices can be transformed with two unitary matrices Ψ and Φ as

$$Y = \Phi^\dagger \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \Psi =: \Phi^\dagger M \Psi \qquad (m_i \geq 0). \tag{2.24}$$

Using this Ψ and Φ, we can rotate the basis as

$$Q^1 \mapsto \Psi_u^\dagger Q^1, \quad Q^2 \mapsto \Psi_d^\dagger Q^2, \quad L \mapsto \Psi_e^\dagger L, \qquad U \mapsto \Phi_u^\dagger U, \quad D \mapsto \Phi_d^\dagger D, \quad E \mapsto \Phi_e^\dagger E, \tag{2.25}$$

and now we have the 湯川 terms in mass eigenstates as

$$\mathcal{L}_{湯川} = -\left(1+\frac{1}{v}h\right)\left[(m_u)_i\overline{U}_iP_{\mathrm{L}}Q_i^1 + (m_d)_i\overline{D}_iP_{\mathrm{L}}Q_i^2 + (m_e)_i\overline{E}_iP_{\mathrm{L}}L_i^2 + \text{H.c.}\right]. \tag{2.26}$$

In the transformation from the gauge eigenstates to the mass eigenstates, almost all the terms in the Lagrangian are not modified. However, only the terms of quark–quark–W interactions do change drastically, as

$$\mathcal{L} \supset \overline{Q}\mathrm{i}\gamma^\mu \left(-\mathrm{i}g_2 W_\mu - \frac{1}{6}\mathrm{i}g_1 B_\mu\right) P_\mathrm{L} Q \tag{2.27}$$

$$= \overline{Q}\frac{g_2}{\sqrt{2}}\left(\not{W}^+ T^+ + \not{W}^- T^-\right) P_\mathrm{L} Q \quad + \quad \text{(interaction terms with } Z \text{ and } A) \tag{2.28}$$

$$\mapsto \frac{g_2}{\sqrt{2}}\begin{pmatrix}\overline{Q}^1\Psi_u & \overline{Q}^2\Psi_d\end{pmatrix}\begin{pmatrix}0 & \not{W}^+ \\ \not{W}^- & 0\end{pmatrix} P_\mathrm{L}\begin{pmatrix}\Psi_u^\dagger Q^1 \\ \Psi_d^\dagger Q^2\end{pmatrix} + (\quad\cdots\quad) \tag{2.29}$$

$$= \frac{g_2}{\sqrt{2}}\left[\overline{Q}^2\not{W}^- X P_\mathrm{L} Q^1 + \overline{Q}^1\not{W}^+ X^\dagger P_\mathrm{L} Q^2\right] + (\quad\cdots\quad), \tag{2.30}$$

where $X := \Psi_d\Psi_u^\dagger$ is a matrix, so-called the Cabbibo–小林–益川 (CKM) matrix, which is *not* diagonal, and *not* real, generally. These terms violate the flavor symmetry of quarks, and even the CP-symmetry.

In our notation, CP-transformation of a spinor is described as

$$\mathscr{C}\mathscr{P}(\psi) = -\mathrm{i}\eta^*(\overline{\psi}\gamma^2)^\mathrm{T}, \quad \mathscr{C}\mathscr{P}(\overline{\psi}) = \mathrm{i}\eta(\gamma^2\psi)^\mathrm{T}, \tag{2.31}$$

where η is a complex phase ($|\eta| = 1$). Under this transformation, those terms are transformed as, e.g.,

$$\begin{aligned}\mathscr{C}\mathscr{P}\left(\overline{Q}^2\not{W}^- X P_\mathrm{L} Q^1\right) &= (\gamma^2 Q^2)^\mathrm{T}\mathscr{P}(-\not{W}^+) X P_\mathrm{L}(\overline{Q}^1\gamma^2)^\mathrm{T} \\ &= -W_\mu^{+P}(\gamma^2 Q^2)^\mathrm{T}(\overline{Q}^1 X^\mathrm{T}\gamma^2 P_\mathrm{L}\gamma^{\mu\mathrm{T}})^\mathrm{T} \\ &= (\overline{Q}^1\not{W}^+ X^\mathrm{T} P_\mathrm{L} Q^2).\end{aligned} \tag{2.32}$$

Therefore, we can see that the CP-symmetry is maintained if and only if $X^\mathrm{T} = X^\dagger$, that is, if and only if X is a real matrix.

以上より，標準模型の Lagrangian は

$$
\begin{aligned}
& \mathcal{L} = \mathcal{L}_{\text{gauge}} \\
\text{【質量項】} \quad & + \; {m_W}^2 W^+ W^- \; + \; \frac{{m_Z}^2}{2} Z^2 \\
& - \; \left(\bar{U} M_u P_{\mathrm{L}} Q^1 \; + \; \bar{D} M_d P_{\mathrm{L}} Q^2 \; + \; \bar{E} M_e P_{\mathrm{L}} L^2 \; + \; \text{H.c.} \right) \\
\text{【Higgs Field】} \quad & + \; \frac{1}{2} (\partial_\mu h)^2 \; - \frac{1}{2} {m_h}^2 h^2 - \sqrt{\frac{\lambda}{2}} m_h h^3 - \frac{1}{4} \lambda h^4 \\
\text{【Higgs との結合】} \quad & + \; \frac{v {g_2}^2}{4} W^+ W^- h \; + \; \frac{v({g_1}^2 + {g_2}^2)}{8} Z^2 h \\
& + \; \frac{{g_2}^2}{4} W^+ W^- h^2 \; + \; \frac{{g_1}^2 + {g_2}^2}{8} Z^2 h^2 \\
& - \; \left(\frac{1}{v} \bar{U} M_u P_{\mathrm{L}} Q^1 h + \frac{1}{v} \bar{D} M_d P_{\mathrm{L}} Q^2 h + \frac{1}{v} \bar{E} M_e P_{\mathrm{L}} L^2 h \; + \; \text{H.c.} \right) \\
\text{【SU(3) および微分項】} \quad & + \; \bar{Q} \left(\mathrm{i} \not{\partial} + g_3 \not{G} \right) P_{\mathrm{L}} Q \; + \; \bar{U} \left(\mathrm{i} \not{\partial} + g_3 \not{G} \right) P_{\mathrm{R}} U \; + \; \bar{D} \left(\mathrm{i} \not{\partial} + g_3 \not{G} \right) P_{\mathrm{R}} D \\
& + \; \bar{L} \left(\mathrm{i} \not{\partial} \right) P_{\mathrm{L}} L \; + \; \bar{E} \left(\mathrm{i} \not{\partial} \right) P_{\mathrm{R}} E \\
\text{【}W\text{ boson】} \quad & + \; \frac{g_2}{\sqrt{2}} \left[\bar{Q}^2 \not{W}^- X P_{\mathrm{L}} Q^1 + \bar{Q}^1 \not{W}^+ X^\dagger P_{\mathrm{L}} Q^2 \right] \qquad \text{【}\leftarrow CP\text{ and flavor violating!】} \\
& + \; \bar{L} \frac{g_2}{\sqrt{2}} \left(\not{W}^+ T^+ + \not{W}^- T^- \right) P_{\mathrm{L}} L \\
\text{【}A\&Z^0\text{ boson】} \quad & + \; \bar{Q} \left[\left(T^3 + \frac{1}{6} \right) |e| \not{A} + \left(\frac{|e|c}{s} T^3 - \frac{|e|s}{6c} \right) \not{Z}^0 \right] P_{\mathrm{L}} Q \\
& + \; \bar{U} \left(\frac{2}{3} |e| \not{A} - \frac{2|e|s}{3c} \not{Z} \right) P_{\mathrm{R}} U \\
& + \; \bar{D} \left(-\frac{1}{3} |e| \not{A} + \frac{|e|s}{3c} \not{Z} \right) P_{\mathrm{R}} D \\
& + \; \bar{L} \left[\left(T^3 - \frac{1}{2} \right) |e| \not{A} + \left(\frac{|e|c}{s} T^3 + \frac{|e|s}{2c} \right) \not{Z}^0 \right] P_{\mathrm{L}} L \\
& + \; \bar{E} \left(-|e| \not{A} + \frac{|e|s}{c} \not{Z} \right) P_{\mathrm{R}} E
\end{aligned}
\tag{2.33}
$$

となる。

2.5 Chiral Notation

以上の Lagrangian を chiral 表示で表すと，まず最初は

$$
\begin{aligned}
\mathcal{L} =&(\text{Higgs terms}) + (\text{Gauge fields strength})\\
&+ Q_\mathrm{L}^\dagger \mathrm{i}\bar\sigma^\mu\left(\partial_\mu - \mathrm{i}g_3 G_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{6}\mathrm{i}g_1 B_\mu\right)Q_\mathrm{L}\\
&+ U_\mathrm{R}^\dagger \mathrm{i}\sigma^\mu\left(\partial_\mu - \mathrm{i}g_3 G_\mu - \frac{2}{3}\mathrm{i}g_1 B_\mu\right)U_\mathrm{R}\\
&+ D_\mathrm{R}^\dagger \mathrm{i}\sigma^\mu\left(\partial_\mu - \mathrm{i}g_3 G_\mu + \frac{1}{3}\mathrm{i}g_1 B_\mu\right)D_\mathrm{R}\\
&+ L_\mathrm{L}^\dagger \mathrm{i}\bar\sigma^\mu\left(\partial_\mu - \mathrm{i}g_2 W_\mu + \frac{1}{2}\mathrm{i}g_1 B_\mu\right)L_\mathrm{L}\\
&+ E_\mathrm{R}^\dagger \mathrm{i}\sigma^\mu\left(\partial_\mu + \mathrm{i}g_1 B_\mu\right)E_\mathrm{R}\\
&- \left(U_\mathrm{R}^\dagger y_u H Q_\mathrm{L} + D_\mathrm{R}^\dagger y_d H^\dagger Q_\mathrm{L} + E_\mathrm{R}^\dagger y_e H^\dagger L_\mathrm{L} + \text{H.c.}\right)\\
=&(\text{Higgs terms}) + (\text{Gauge fields strength})\\
&+ \mathrm{i}Q_\mathrm{L}^\dagger\bar\sigma^\mu\partial_\mu Q_\mathrm{L} + \mathrm{i}U_\mathrm{R}\bar\sigma^\mu\partial_\mu U_\mathrm{R}^\dagger + \mathrm{i}D_\mathrm{R}\bar\sigma^\mu\partial_\mu D_\mathrm{R}^\dagger + \mathrm{i}L_\mathrm{L}^\dagger\bar\sigma^\mu\partial_\mu L_\mathrm{L} + \mathrm{i}E_\mathrm{R}\bar\sigma^\mu\partial_\mu E_\mathrm{R}^\dagger\\
&+ g_3\left(Q_\mathrm{L}^\dagger\bar\sigma^\mu G_\mu Q_\mathrm{L} + U_\mathrm{R}^\dagger\bar\sigma^\mu G_\mu U_\mathrm{R} + D_\mathrm{R}^\dagger\bar\sigma^\mu G_\mu D_\mathrm{R}\right)\\
&+ g_2\left(Q_\mathrm{L}^\dagger\bar\sigma^\mu W_\mu Q_\mathrm{L} + L_\mathrm{L}^\dagger\bar\sigma^\mu W_\mu L_\mathrm{L}\right)\\
&+ g_1\left(\frac{1}{6}Q_\mathrm{L}^\dagger\bar\sigma^\mu B_\mu Q_\mathrm{L} + \frac{2}{3}U_\mathrm{R}^\dagger\bar\sigma^\mu B_\mu U_\mathrm{R} - \frac{1}{3}D_\mathrm{R}^\dagger\bar\sigma^\mu B_\mu D_\mathrm{R} - \frac{1}{2}L_\mathrm{L}^\dagger\bar\sigma^\mu B_\mu L_\mathrm{L} - E_\mathrm{R}^\dagger\bar\sigma^\mu B_\mu E_\mathrm{R}\right)\\
&- \left(U_\mathrm{R}^\dagger y_u H Q_\mathrm{L} + D_\mathrm{R}^\dagger y_d H^\dagger Q_\mathrm{L} + E_\mathrm{R}^\dagger y_e H^\dagger L_\mathrm{L} + \text{H.c.}\right)
\end{aligned}
\tag{2.34}
$$

であり，そして最終的には

$$
\begin{aligned}
\mathcal{L} =&(\text{Gauge bosons and Higgs})\\
&+ \mathrm{i}Q_\mathrm{L}^\dagger\bar\sigma^\mu\partial_\mu Q_\mathrm{L} + \mathrm{i}U_\mathrm{R}\bar\sigma^\mu\partial_\mu U_\mathrm{R}^\dagger + \mathrm{i}D_\mathrm{R}\bar\sigma^\mu\partial_\mu D_\mathrm{R}^\dagger + \mathrm{i}L_\mathrm{L}^\dagger\bar\sigma^\mu\partial_\mu L_\mathrm{L} + \mathrm{i}E_\mathrm{R}\bar\sigma^\mu\partial_\mu E_\mathrm{R}^\dagger\\
&+ g_3\left(Q_\mathrm{L}^\dagger\bar\sigma^\mu G_\mu Q_\mathrm{L} + U_\mathrm{R}^\dagger\bar\sigma^\mu G_\mu U_\mathrm{R} + D_\mathrm{R}^\dagger\bar\sigma^\mu G_\mu D_\mathrm{R}\right)\\
&- m_u(u_\mathrm{R}^\dagger u_\mathrm{L} + u_\mathrm{L}^\dagger u_\mathrm{R}) - (\text{quarks}) - m_e(e_\mathrm{R}^\dagger e_\mathrm{L} + e_\mathrm{L}^\dagger e_\mathrm{R}) - (\text{leptons})\\
&- \frac{m_u}{v}(u_\mathrm{R}^\dagger u_\mathrm{L} + u_\mathrm{L}^\dagger u_\mathrm{R})h - (\text{quarks}) - \frac{m_e}{v}(e_\mathrm{R}^\dagger e_\mathrm{L} + e_\mathrm{L}^\dagger e_\mathrm{R})h - (\text{leptons})\\
&+ \frac{g_2}{\sqrt2}\left[\begin{pmatrix}d_\mathrm{L}^\dagger & s_\mathrm{L}^\dagger & b_\mathrm{L}^\dagger\end{pmatrix}\bar\sigma^\mu W_\mu^- X\begin{pmatrix}u_\mathrm{L}\\ c_\mathrm{L}\\ t_\mathrm{L}\end{pmatrix} + \begin{pmatrix}u_\mathrm{L}^\dagger & c_\mathrm{L}^\dagger & t_\mathrm{L}^\dagger\end{pmatrix}\bar\sigma^\mu W_\mu^+ X^\dagger\begin{pmatrix}d_\mathrm{L}\\ s_\mathrm{L}\\ b_\mathrm{L}\end{pmatrix}\right]\\
&+ \frac{g_2}{\sqrt2}\left[\nu_e^\dagger\bar\sigma^\mu W_\mu^+ e_\mathrm{L} + e_\mathrm{L}^\dagger\bar\sigma^\mu W_\mu^-\nu_e\right]\\
&+ |e|\left[\frac{2}{3}u_\mathrm{L}^\dagger\bar\sigma^\mu A_\mu u_\mathrm{L} - \frac{1}{3}d_\mathrm{L}^\dagger\bar\sigma^\mu A_\mu d_\mathrm{L} + \frac{2}{3}u_\mathrm{R}^\dagger\sigma^\mu A_\mu u_\mathrm{R} - \frac{1}{3}d_\mathrm{R}^\dagger\sigma^\mu A_\mu d_\mathrm{R} + (\text{quarks})\right.\\
&\qquad\left. - e_\mathrm{L}^\dagger\bar\sigma^\mu A_\mu e_\mathrm{L} - e_\mathrm{R}^\dagger\sigma^\mu A_\mu e_\mathrm{R} + (\text{leptons})\right]\\
&+ \frac{|e|s}{c}\left[\left(\frac{c^2}{2s^2} - \frac{1}{6}\right)u_\mathrm{L}^\dagger\bar\sigma^\mu Z_\mu u_\mathrm{L} - \left(\frac{c^2}{2s^2} + \frac{1}{6}\right)d_\mathrm{L}^\dagger\bar\sigma^\mu Z_\mu d_\mathrm{L} - \frac{2}{3}u_\mathrm{R}^\dagger\sigma^\mu Z_\mu u_\mathrm{R} + \frac{1}{3}d_\mathrm{R}^\dagger\sigma^\mu Z_\mu d_\mathrm{R}\right.\\
&\qquad\left. + \left(\frac{c^2}{2s^2} + \frac{1}{2}\right)\nu_e^\dagger\bar\sigma^\mu Z_\mu\nu_e - \left(\frac{c^2}{2s^2} - \frac{1}{2}\right)e_\mathrm{L}^\dagger\bar\sigma^\mu Z_\mu e_\mathrm{L} + e_\mathrm{R}^\dagger\sigma^\mu Z_\mu e_\mathrm{R} + (\text{others})\right]
\end{aligned}
\tag{2.35}
$$

となる。

2.6 Values of SM Parameters (Extracted from PDG 2010)

◆2.6.1 Experimental Values

Low energy values

$$\alpha_{\rm EM} = 1/137.035999679(94) \qquad G_{\rm F} = \frac{{g_2}^2}{4\sqrt{2}{m_W}^2} = \frac{1}{\sqrt{2}v^2} = 1.16637(1)\times 10^{-5}\,{\rm GeV}^{-2}$$

Electroweak scale 【These values are all in $\overline{\rm MS}$ scheme.】

$$\begin{aligned} \alpha_{\rm EM}^{-1}(m_Z) &= 127.925(16) & m_W(m_W) &= 80.399(23)\,{\rm GeV} \\ \alpha_{\rm EM}^{-1}(m_\tau) &= 133.452(16) & m_Z(m_Z) &= 91.1876(21)\,{\rm GeV} \\ \alpha_{\rm s}(m_Z) &= 0.1183(15) & \sin^2\theta_{\rm W}(m_Z) &= 0.23116(13) \\ \Gamma_{l^+l^-} &= 83.984(86)\,{\rm MeV} & \sin^2\theta_{\rm eff} &= 0.23146(12) \end{aligned}$$

Fundamental masses

$$\begin{array}{lll} e : 0.510998910(13)\,{\rm MeV} & u : 1.7 \text{ to } 3.3\,{\rm MeV} & d : 4.1 \text{ to } 5.8\,{\rm MeV} \\ \mu : 105.658367(4)\,{\rm MeV} & c : 1.27^{+0.07}_{-0.09}\,{\rm GeV} & s : 101^{+29}_{-21}\,{\rm MeV} \\ \tau : 1.77682(16)\,{\rm GeV} & t : 172.0_{\pm 2.2}\,{\rm GeV} & b : 4.19^{+0.18}_{-0.06}\,{\rm GeV} \end{array}$$

$$\begin{array}{lll} \pi^\pm : 139.57018(35)\,{\rm MeV} & K^\pm : 493.677(16)\,{\rm MeV} & p : 938.27203(8)\,{\rm MeV} \\ \pi^0 : 134.9766(6)\,{\rm MeV} & K^0 : 497.614(24)\,{\rm MeV} & n : 939.565346(23)\,{\rm MeV} \end{array}$$

Fundamental Lifetime (also $c\tau$ for some particles)

$$\begin{array}{lll} \mu : 2.197034(21)\,\mu{\rm s} \quad (659\,{\rm m}) & \pi^\pm : 2.6033(5)\times 10^{-8}\,{\rm s} & K^\pm : 1.2380(21)\times 10^{-8}\,{\rm s} \\ \tau : 2.906(10)\times 10^{-13}\,{\rm s} \quad (87\,\mu{\rm m}) & \pi^0 : 8.4(5)\times 10^{-17}\,{\rm s} & K^0_{\rm S} : 8.953(5)\times 10^{-11}\,{\rm s} \\ & & K^0_{\rm L} : 5.116(20)\times 10^{-8}\,{\rm s} \end{array}$$

CKM matrix

$$V_{\rm CKM} = \begin{pmatrix} 0.97428(15) & 0.2253(7) & 0.00347(16) \\ 0.2252(7) & 0.97345(16) & 0.0410(11) \\ 0.00862(26) & 0.0403(11) & 0.999152(45) \end{pmatrix} \sim \begin{pmatrix} 1-\epsilon^2 & \epsilon & \epsilon^4 \\ \epsilon & 1-\epsilon^2 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1-\epsilon^4 \end{pmatrix} \quad \text{for } \epsilon\sim 0.23 \tag{2.36}$$

◆2.6.2 Estimation of SM Parameters

For EW scale, we can estimate the values as

$$e\sim 0.313, \qquad g_1\sim 0.358, \qquad g_2 \sim 0.651, \qquad g_Z\sim 0.743; \qquad v=\sqrt{\frac{\mu^2}{\lambda}}\sim 246\,{\rm GeV} \tag{2.37}$$

Therefore 湯川 matrices are (after diagonalization), since $vy/\sqrt{2}=M$,

$$y_u \sim \begin{pmatrix} 10^{-5} & 0 & 0 \\ 0 & 0.007 & 0 \\ 0 & 0 & 0.98 \end{pmatrix}, \quad y_d \sim \begin{pmatrix} 3\times 10^{-5} & 0 & 0 \\ 0 & 0.0006 & 0 \\ 0 & 0 & 0.02 \end{pmatrix}, \quad y_e \sim \begin{pmatrix} 3\times 10^{-6} & 0 & 0 \\ 0 & 0.0006 & 0 \\ 0 & 0 & 0.01 \end{pmatrix}. \tag{2.38}$$

Also, for $m_h\sim 120\,{\rm GeV}$, we can estimate the Higgs potential as $\mu\sim 85\,{\rm GeV}$ and $\lambda\sim 0.12$.

3 Supersymmetry for $\eta = \mathrm{diag}(+,-,-,-)$

3.1 Spinor Convention

(See App. ?? for a verbose explanation.)

ϵ tensor : $\epsilon^{12} = \epsilon^{\dot 1\dot 2} = \epsilon_{21} = \epsilon_{\dot 2\dot 1} = 1$ (definition)

Sum Rule : ${}^{\alpha}{}_{\alpha}$ and ${}_{\dot\alpha}{}^{\dot\alpha}$, except for $\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta$, $\xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta$, $\xi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\xi^{\dot\beta}$, $\xi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\xi_{\dot\beta}$.

Lorentz 変換 : $\psi'_\alpha = \Lambda_\alpha{}^\beta\psi_\beta$, $\bar\psi'_{\dot\alpha} = \bar\psi_{\dot\beta}\Lambda^{\dagger\dot\beta}{}_{\dot\alpha}$, $\psi'^\alpha = \psi^\beta\Lambda^{-1}{}_\beta{}^\alpha$, $\bar\psi'^{\dot\alpha} = (\Lambda^{-1})^{\dagger\dot\alpha}{}_{\dot\beta}\bar\psi'^{\dot\beta}$.

σ matrices : $(\sigma^\mu)_{\alpha\dot\beta} := (1,\boldsymbol\sigma)_{\alpha\dot\beta}$, $(\bar\sigma^\mu)^{\dot\alpha\alpha} := \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}(\sigma^\mu)_{\beta\dot\beta} = (1,-\boldsymbol\sigma)^{\dot\alpha\beta}$.

3.2 Spinor Calculation Cheatsheet

$$\eta = (+,-,-,-),\qquad \epsilon^{0123} = -\epsilon_{0123} = 1;\qquad \textbf{Left Differential};$$
$$\epsilon^{12} = \epsilon_{21} = \epsilon^{\dot 1\dot 2} = \epsilon_{\dot 2\dot 1} = 1,\quad \xi^\alpha := \epsilon^{\alpha\beta}\xi_\beta,\quad \xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta,\quad \bar\xi^{\dot\alpha} := \epsilon^{\dot\alpha\dot\beta}\bar\xi_{\dot\beta},\quad \bar\xi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\xi^{\dot\beta}$$
$$\bar\sigma^{\mu\dot\alpha\alpha} := \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\sigma^\mu_{\beta\dot\beta}\qquad \sigma^\mu_{\alpha\dot\alpha} = \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\bar\sigma^{\mu\dot\beta\beta},\qquad \sigma^\mu := (1,\boldsymbol\sigma),\quad \bar\sigma^\mu := (1,-\boldsymbol\sigma)$$
$$(\sigma^{\mu\nu})_\alpha{}^\beta := \tfrac14\left(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu\right)_\alpha{}^\beta,\quad (\bar\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} := \tfrac14\left(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu\right)^{\dot\alpha}{}_{\dot\beta} = (\sigma^{\nu\mu})^{\dagger\dot\alpha}{}_{\dot\beta}.$$

$$\theta^\alpha\theta^\beta = -\tfrac12\epsilon^{\alpha\beta}\theta\theta \qquad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12\epsilon^{\dot\alpha\dot\beta}\bar\theta\bar\theta \qquad (\theta\phi)(\theta\psi) = -\tfrac12(\psi\phi)(\theta\theta) \qquad (\theta\sigma^\nu\bar\theta)\theta^\alpha = \tfrac12\theta\theta(\bar\theta\bar\sigma^\nu)^\alpha$$
$$\theta_\alpha\theta_\beta = \tfrac12\epsilon_{\alpha\beta}\theta\theta \qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac12\epsilon_{\dot\alpha\dot\beta}\bar\theta\bar\theta \qquad (\bar\theta\bar\phi)(\bar\theta\bar\psi) = -\tfrac12(\bar\psi\bar\phi)(\bar\theta\bar\theta) \qquad (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\tfrac12\bar\theta\bar\theta(\theta\sigma^\mu)_{\dot\alpha}$$
$$\theta^\alpha\theta_\beta = \tfrac12\delta^\alpha_\beta\theta\theta \qquad \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12\delta^{\dot\beta}_{\dot\alpha}\bar\theta\bar\theta \qquad (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \tfrac12\theta\theta\bar\theta\bar\theta\eta^{\mu\nu} \qquad (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \tfrac12(\sigma^\mu\bar\sigma^\nu\theta)_\alpha\bar\theta\bar\theta$$
$$\theta\sigma^\mu\bar\sigma^\nu\theta = \eta^{\mu\nu}\theta\theta \qquad \bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta = \eta^{\mu\nu}\bar\theta\bar\theta$$

$$\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} + 2\sigma^{\mu\nu} \qquad \sigma^\mu\bar\sigma^\rho\sigma^\nu + \sigma^\nu\bar\sigma^\rho\sigma^\mu = 2\left(\eta^{\mu\rho}\sigma^\nu + \eta^{\nu\rho}\sigma^\mu - \eta^{\mu\nu}\sigma^\rho\right)$$
$$\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} + 2\bar\sigma^{\mu\nu} \qquad \bar\sigma^\mu\sigma^\rho\bar\sigma^\nu + \bar\sigma^\nu\sigma^\rho\bar\sigma^\mu = 2\left(\eta^{\mu\rho}\bar\sigma^\nu + \eta^{\nu\rho}\bar\sigma^\mu - \eta^{\mu\nu}\bar\sigma^\rho\right)$$
$$\sigma^{\mu\nu} = -\sigma^{\nu\mu} \qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\mathrm{i}\epsilon^{\mu\nu\rho\sigma}\sigma_\sigma$$
$$\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu} \qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2\mathrm{i}\epsilon^{\mu\nu\rho\sigma}\bar\sigma_\sigma$$
$$\mathrm{Tr}\,\bar\sigma^\mu\sigma^\nu = \mathrm{Tr}\,\sigma^\mu\bar\sigma^\nu = 2\eta^{\mu\nu} \qquad \mathrm{Tr}\,\sigma^{\mu\nu}\sigma^{\rho\sigma} = -\tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) + \tfrac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho\sigma}$$
$$\mathrm{Tr}\,\sigma^{\mu\nu} = \mathrm{Tr}\,\bar\sigma^{\mu\nu} = 0 \qquad \mathrm{Tr}\,\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma} = -\tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \tfrac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho\sigma}$$
$$\sigma^\mu_{\alpha\dot\alpha}\bar\sigma^{\dot\beta\beta}_\mu = 2\delta^\beta_\alpha\delta^{\dot\beta}_{\dot\alpha} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\left[(\sigma^{\mu\nu}\epsilon)_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} + (\epsilon\bar\sigma^{\mu\nu})_{\dot\alpha\dot\beta}\epsilon_{\alpha\beta}\right]$$
$$\sigma^\mu_{\alpha\dot\alpha}\sigma_{\mu\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = \eta^{\mu\nu}\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} - 4\eta_{\rho\sigma}(\sigma^{\rho\mu}\epsilon)_{\alpha\beta}(\epsilon\bar\sigma^{\sigma\nu})_{\dot\alpha\dot\beta}$$
$$\bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\dot\beta\beta}_\mu = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} \qquad \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot\beta} \qquad \epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma} = 2\mathrm{i}\sigma^{\mu\nu}$$
$$\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha} \qquad \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon^{\dot\alpha\dot\beta}\sigma^\mu_{\beta\dot\beta} \qquad \epsilon^{\mu\nu\rho\sigma}\bar\sigma_{\rho\sigma} = -2\mathrm{i}\bar\sigma^{\mu\nu}$$

$$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi = (\bar\chi\bar\sigma^\mu\xi)^* = -(\xi\sigma^\mu\bar\chi)^* \qquad (\psi\phi)\chi_\alpha = -(\phi\chi)\psi_\alpha - (\chi\psi)\phi_\alpha$$
$$\xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi = (\bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi)^* = (\bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi)^* \qquad (\psi\phi)\bar\chi_{\dot\alpha} = \tfrac12(\phi\sigma^\mu\bar\chi)(\psi\sigma_\mu)_{\dot\alpha}$$

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha \qquad \epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\alpha}\theta\theta = 4$$
$$\epsilon_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} = -\frac{\partial}{\partial\theta^\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha \qquad \epsilon_{\alpha\beta}\frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\alpha}\theta\theta = -4$$
$$\epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\beta}} = -\frac{\partial}{\partial\bar\theta_{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} \qquad \epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 4$$
$$\epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\beta}} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} \qquad \epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -4$$

3.3 General Relations (Note: $P_\mu = \mathrm{i}\partial_\mu$ in our convention.)

$$Q_\alpha := \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \qquad D_\alpha := \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \qquad y := x - \mathrm{i}\theta\sigma\bar\theta,$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \qquad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \qquad y^\dagger = x + \mathrm{i}\theta\sigma\bar\theta$$

$$\{Q_\alpha, \bar Q_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \{D_\alpha, \bar D_{\dot\alpha}\} = 2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \text{(others)} = 0.$$

$$\begin{array}{llll}
 & < x\text{-basis} > & < y\text{-basis} > & < y^\dagger\text{-basis} > \\
D_\alpha & = \dfrac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu & = \dfrac{\partial}{\partial\theta^\alpha} - 2\mathrm{i}(\sigma^\mu\bar\theta)_\alpha\dfrac{\partial}{\partial y^\mu} & = \dfrac{\partial}{\partial\theta^\alpha} \qquad (3.1)\\
\bar D_{\dot\alpha} & = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu & = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} & = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2\mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\dfrac{\partial}{\partial(y^\dagger)^\mu} \qquad (3.2)\\
D^\alpha & = -\dfrac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu & = -\dfrac{\partial}{\partial\theta_\alpha} + 2\mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\dfrac{\partial}{\partial y^\mu} & = -\dfrac{\partial}{\partial\theta_\alpha} \qquad (3.3)\\
\bar D^{\dot\alpha} & = \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu & = \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} & = \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} - 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\dfrac{\partial}{\partial(y^\dagger)^\mu} \qquad (3.4)
\end{array}$$

$$\phi(y) = \phi(x) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(x) = \phi(y^\dagger) - 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y^\dagger) - \theta\theta\bar\theta\bar\theta\partial^2\phi(y^\dagger) \tag{3.5}$$

$$\phi(y^\dagger) = \phi(x) + \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(x) = \phi(y) + 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y) - \theta\theta\bar\theta\bar\theta\partial^2\phi(y) \tag{3.6}$$

$$\phi(x) = \phi(y) + \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(y) = \phi(y^\dagger) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y^\dagger) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(y^\dagger) \tag{3.7}$$

3.4 Chiral Superfields : $\bar{D}_{\dot\alpha}\Phi = 0$

Explicit Expression

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{3.8}$$
$$= \phi(x) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(x) + \sqrt{2}\theta\psi(x) - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x) + \theta\theta F(x) \tag{3.9}$$
$$= \phi(y^\dagger) - 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y^\dagger) - \theta\theta\bar\theta\bar\theta\partial^2\phi(y^\dagger) + \sqrt{2}\theta\psi(y^\dagger) - \sqrt{2}\mathrm{i}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(y^\dagger) + \theta\theta F(y^\dagger) \tag{3.10}$$
$$\Phi^\dagger = \phi^*(y^\dagger) + \sqrt{2}\bar\theta\bar\psi(y^\dagger) + \bar\theta\bar\theta F^*(y^\dagger) \tag{3.11}$$
$$= \phi^*(x) + \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi^*(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi^*(x) + \sqrt{2}\bar\theta\bar\psi(x) - \frac{\mathrm{i}}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\bar\psi(x) + \bar\theta\bar\theta F^*(x) \tag{3.12}$$
$$= \phi^*(y) + 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi^*(y) - \theta\theta\bar\theta\bar\theta\partial^2\phi^*(y) + \sqrt{2}\bar\theta\bar\psi(y) - \sqrt{2}\mathrm{i}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\bar\psi(y) + \bar\theta\bar\theta F^*(y) \tag{3.13}$$

Product of Chiral Superfields

$$\begin{aligned}
\Phi_i^\dagger\Phi_j(x,\theta,\bar\theta) = {}& \phi_i^*\phi_j + \sqrt{2}\phi_i^*\theta\psi_j + \sqrt{2}\bar\theta\bar\psi_i\phi_j + \theta\theta\phi_i^*F_j + \bar\theta\bar\theta F_i^*\phi_j \\
& - \mathrm{i}\theta\sigma^\mu\bar\theta\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right) + 2\bar\theta\bar\psi_i\theta\psi_j \\
& + \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\left(\phi_i^*\partial_\mu\psi_j - \partial_\mu\phi_i^*\psi_j\right)\sigma^\mu\bar\theta + \sqrt{2}\theta\theta\bar\theta\bar\psi_i F_j \\
& - \frac{\mathrm{i}}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\left(\partial_\mu\bar\psi_i\phi_j - \bar\psi_i\partial_\mu\phi_j\right) + \sqrt{2}\bar\theta\bar\theta F_i^*\theta\psi_j \\
& + \theta\theta\bar\theta\bar\theta\left[F_i^*F_j - \frac{1}{4}\phi_i^*\partial^2\phi_j - \frac{1}{4}\partial^2\phi_i^*\phi_j + \frac{1}{2}\partial_\mu\phi_i^*\partial_\mu\phi_j - \frac{\mathrm{i}}{2}\partial_\mu\bar\psi_i\bar\sigma^\mu\psi_j + \frac{\mathrm{i}}{2}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right]
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\rightsquigarrow {}& \phi_i^*\phi_j + \sqrt{2}\phi_i^*\theta\psi_j + \sqrt{2}\bar\theta\bar\psi_i\phi_j + \theta\theta\phi_i^*F_j + \bar\theta\bar\theta F_i^*\phi_j \\
& - 2\mathrm{i}(\theta\sigma^\mu\bar\theta)(\phi_i^*\partial_\mu\phi_j) + \sqrt{2}\mathrm{i}\theta\theta(\partial_\mu\phi_i^*)\bar\theta\bar\sigma^\mu\psi_j + \sqrt{2}\mathrm{i}\bar\theta\bar\theta\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j \\
& + 2\bar\theta\bar\psi_i\theta\psi_j + \sqrt{2}\theta\theta\bar\theta\bar\psi_i F_j + \sqrt{2}\bar\theta\bar\theta F_i^*\theta\psi_j \\
& + \theta\theta\bar\theta\bar\theta\left[F_i^*F_j + \partial^\mu\phi_i^*\partial_\mu\phi_j + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right]
\end{aligned} \tag{3.15}$$

$$\Phi_i\Phi_j(\text{in } y\text{-basis}) = \phi_i\phi_j + \sqrt{2}\theta\left[\psi_i\phi_j + \phi_i\psi_j\right] + \theta\theta\left[\phi_iF_j + F_i\phi_j - \psi_i\psi_j\right] \tag{3.16}$$
$$\begin{aligned}
\Phi_i\Phi_j\Phi_k(\text{in } y\text{-basis}) = {}& \phi_i\phi_j\phi_k + \sqrt{2}\theta\left[\psi_i\phi_j\phi_k + \phi_i\psi_j\phi_k + \phi_i\phi_j\psi_k\right] \\
& + \theta\theta\left[F_i\phi_j\phi_k + \phi_iF_j\phi_k + \phi_i\phi_jF_k - \psi_i\psi_j\phi_k - \psi_i\phi_j\psi_k - \phi_i\psi_j\psi_k\right]
\end{aligned} \tag{3.17}$$

(Products of chiral superfields are still chiral superfields.)

$$\mathrm{e}^{\mathrm{i}k\Phi} = \mathrm{e}^{\mathrm{i}k\phi(y)}\left[1 + \mathrm{i}k\left(\sqrt{2}\theta\psi(y) + \theta\theta F(y)\right) + \frac{k^2}{2}\theta\theta\psi(y)\psi(y)\right] \tag{3.18}$$

Lagrangian Blocks

$$\mathcal{L}_{\text{kin.}} = \Phi_i^\dagger\Phi_j\Big|_{\theta\theta\bar\theta\bar\theta} \rightsquigarrow F_i^*F_j + \partial^\mu\phi_i^*\partial_\mu\phi_j + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j \tag{3.19}$$
$$\begin{aligned}
\mathcal{L}_{\text{super}} = W\Big|_{\theta\theta} + W^*\Big|_{\bar\theta\bar\theta} &= \int \mathrm{d}^2\theta\left[\lambda_i\Phi_i + m_{ij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k\right] + \text{H.c.} \\
&= \lambda_iF_i + m_{ij}\left(\phi_iF_j + F_i\phi_j - \psi_i\psi_j\right) + y_{ijk}\Big[(F_i\phi_j\phi_k - \psi_i\psi_j\phi_k) + (jki \text{ and } kij \text{ terms})\Big]
\end{aligned} \tag{3.20}$$

3.5 Vector Superfields and Gauge Theory : $V = V^\dagger$

◆3.5.1 Abelian Case — Field Construction

Explicit Expression

$$\begin{aligned} V = &\ C(x) + \mathrm{i}\theta\chi(x) - \mathrm{i}\bar\theta\bar\chi(x) \\ &+ \frac{\mathrm{i}}{2}\theta\theta\left[M(x)+\mathrm{i}N(x)\right] - \frac{\mathrm{i}}{2}\bar\theta\bar\theta\left[M(x)-\mathrm{i}N(x)\right] - \theta\sigma^\mu\bar\theta A_\mu(x) \\ &+ \theta\theta\bar\theta\left[\bar\lambda(x) + \frac{1}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right] + \bar\theta\bar\theta\theta\left[\lambda(x) - \frac{1}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right] + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(x) - \frac{1}{2}\partial^2 C(x)\right] \end{aligned} \tag{3.21}$$

$$\begin{aligned} = &\ C(y) + \mathrm{i}\theta\chi(y) - \mathrm{i}\bar\theta\bar\chi(y) \\ &+ \frac{\mathrm{i}}{2}\theta\theta\left[M(y)+\mathrm{i}N(y)\right] - \frac{\mathrm{i}}{2}\bar\theta\bar\theta\left[M(y)-\mathrm{i}N(y)\right] - \theta\sigma^\mu\bar\theta\left[A_\mu(y) - \mathrm{i}\partial_\mu C(y)\right] \\ &+ \theta\theta\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\theta\left[\lambda(y) - \sigma^\mu\partial_\mu\bar\chi(y)\right] + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y) - \partial^2 C(y) - \mathrm{i}\partial_\mu A^\mu(y)\right] \end{aligned} \tag{3.22}$$

$$\begin{aligned} = &\ C(y^\dagger) + \mathrm{i}\theta\chi(y^\dagger) - \mathrm{i}\bar\theta\bar\chi(y^\dagger) \\ &+ \frac{\mathrm{i}}{2}\theta\theta\left[M(y^\dagger)+\mathrm{i}N(y^\dagger)\right] - \frac{\mathrm{i}}{2}\bar\theta\bar\theta\left[M(y^\dagger)-\mathrm{i}N(y^\dagger)\right] - \theta\sigma^\mu\bar\theta\left[A_\mu(y^\dagger) + \mathrm{i}\partial_\mu C(y^\dagger)\right] \\ &+ \theta\theta\bar\theta\left[\bar\lambda(y^\dagger) + \bar\sigma^\mu\partial_\mu\chi(y^\dagger)\right] + \bar\theta\bar\theta\theta\lambda(y^\dagger) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y^\dagger) - \partial^2 C(y^\dagger) + \mathrm{i}\partial_\mu A^\mu(y^\dagger)\right] \end{aligned} \tag{3.23}$$

Supersymmetric Gauge Transformation : $V \to V + \Phi + \Phi^\dagger$

$$\begin{aligned} C &\mapsto C + (\phi + \phi^*) & A_\mu &\mapsto A_\mu + \mathrm{i}\partial_\mu(\phi - \phi^*) \\ \chi &\mapsto \chi - \mathrm{i}\sqrt{2}\psi & \lambda &\mapsto \lambda \\ M + \mathrm{i}N &\mapsto M + \mathrm{i}N - 2\mathrm{i}F & D &\mapsto D \end{aligned} \tag{3.24}$$

Wess-Zumino Gauge $C = \chi = M = N = 0$

Fixing this gauge breaks SUSY, but still allows the usual gauge transformation

$$A_\mu \mapsto A_\mu + \partial_\mu\alpha, \quad \lambda \mapsto \lambda, \quad D \mapsto D. \tag{3.25}$$

$$\begin{aligned} V &= -\theta\sigma^\mu\bar\theta A_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x) \qquad & V^2 &= \frac{1}{2}\theta\theta\bar\theta\bar\theta A_\mu A^\mu \\ &= -\theta\sigma^\mu\bar\theta A_\mu(y) + \theta\theta\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\theta\lambda(y) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y) - \mathrm{i}\partial_\mu A^\mu(y)\right] & V^3 &= 0 \\ &= -\theta\sigma^\mu\bar\theta A_\mu(y^\dagger) + \theta\theta\bar\theta\bar\lambda(y^\dagger) + \bar\theta\bar\theta\theta\lambda(y^\dagger) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y^\dagger) + \mathrm{i}\partial_\mu A^\mu(y^\dagger)\right] \\ \mathrm{e}^{kV} &= 1 - k\theta\sigma^\mu\bar\theta A_\mu(x) + k\theta\theta\bar\theta\bar\lambda + k\bar\theta\bar\theta\theta\lambda + \theta\theta\bar\theta\bar\theta\left[\frac{k}{2}D + \frac{k^2}{4}A_\mu A^\mu\right] \end{aligned}$$

Field Strength

$$W_\alpha = -\frac{1}{4}\bar D\bar D D_\alpha V, \quad \bar W_{\dot\alpha} = -\frac{1}{4}DD\bar D_{\dot\alpha}V; \qquad W_\alpha \mapsto W_\alpha \quad \text{(gauge invariant)} \tag{3.26}$$

$$\bar D_{\dot\beta}W_\alpha = D_\beta\bar W_{\dot\alpha} = 0; \qquad D^\alpha W_\alpha = \bar D_{\dot\alpha}\bar W^{\dot\alpha} \tag{3.27}$$

$$W_\alpha = \lambda_\alpha(y) + \theta_\alpha D(y) + \mathrm{i}(\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}(y) + \mathrm{i}\theta\theta\left[\sigma^\mu\partial_\mu\bar\lambda(y)\right]_\alpha \tag{3.28}$$

$$\bar W_{\dot\alpha} = \bar\lambda_{\dot\alpha}(y^\dagger) + \bar\theta_{\dot\alpha}D(y^\dagger) + \mathrm{i}(\bar\theta\bar\sigma^{\mu\nu})_{\dot\alpha}F_{\mu\nu}(y^\dagger) - \mathrm{i}\bar\theta\bar\theta\left[\partial_\mu\lambda(y^\dagger)\sigma^\mu\right]_{\dot\alpha} \tag{3.29}$$

$$W^\alpha W_\alpha\big|_{\theta\theta} = -\frac{1}{4}\bar D\bar D W^\alpha D_\alpha V \rightsquigarrow -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{\mathrm{i}}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + 2\mathrm{i}\lambda\sigma^\mu\partial_\mu\bar\lambda + D^2 \tag{3.30}$$

Lagrangian Blocks ($\mathcal{L}_{\text{inv}}$ is SUSY- and gauge-invariant, while $\mathcal{L}_{\text{mass}}$ is not gauge-invariant.)

$$\begin{aligned}
\mathcal{L}_{\text{inv}} &= \frac{\tau}{4} W^\alpha W_\alpha\Big|_{\theta\theta} + \frac{\tau^*}{4}\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\Big|_{\bar\theta\bar\theta} \qquad \left(\text{with} \quad \tau := 1 + \frac{\mathrm{i}\theta}{8\pi^2}\right)\\
&\rightsquigarrow -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\lambda\sigma^\mu\partial_\mu\bar\lambda + \frac{1}{2}D^2 - \frac{\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \qquad (3.31)\\
\mathcal{L}_{\text{mass}} &= m^2 V^2\Big|_{\theta\theta\bar\theta\bar\theta}\\
&= m^2\left(\frac{1}{2}A^\mu A_\mu + \mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - (\lambda\chi + \bar\lambda\bar\chi) + \frac{1}{2}(M^2+N^2) + CD + \frac{1}{2}\partial_\mu C\partial^\mu C\right)\\
&= \frac{1}{2}\partial_\mu C'\partial^\mu C' + \mathrm{i}\bar\chi'\bar\sigma^\mu\partial_\mu\chi' + \frac{m^2}{2}A^\mu A_\mu - m(\lambda\chi' + \bar\lambda\bar\chi') + mC'D + \frac{m^2}{2}(M^2+N^2) \qquad (3.32)
\end{aligned}$$

◆3.5.2 Abelian Case — Gauge Theory

Here we turn on the coupling constant g. When $\Lambda = \mathrm{i}\lambda(y) + \sqrt{2}\theta\xi(y) + \theta\theta K(y)$,

$$\mathcal{L} \ni \Phi^\dagger \mathrm{e}^{2gqV}\Phi; \qquad \Phi \mapsto \mathrm{e}^{\mathrm{i}qg\Lambda}\Phi, \quad \Phi^\dagger \mapsto \Phi^\dagger \mathrm{e}^{-\mathrm{i}qg\Lambda^\dagger}; \qquad 2V \mapsto 2V - \mathrm{i}(\Lambda - \Lambda^\dagger) \qquad (3.33)$$

$$\begin{aligned}
&\phi \mapsto \mathrm{e}^{-qg\lambda}\phi && C \mapsto C + \operatorname{Re}\lambda && M + \mathrm{i}N \mapsto M + \mathrm{i}N - K\\
&\psi \mapsto \mathrm{e}^{-qg\lambda}\left(\psi + \mathrm{i}qg\phi\cdot\xi\right) && \chi \mapsto \chi - \tfrac{1}{\sqrt2}\xi && A_\mu \mapsto A_\mu - \partial_\mu(\operatorname{Im}\lambda)\\
&F \mapsto \mathrm{e}^{-qg\lambda}\left(F + \mathrm{i}qg\phi K - \mathrm{i}qg\xi\psi + \frac{(qg)^2}{2}\xi\xi\phi\right) && \lambda \mapsto \lambda && D \mapsto D
\end{aligned}$$

(Very similar to the gauge transformations in Sec. 1.9.2.)

Lagrangian block

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} &= \frac{1}{4}W^\alpha W_\alpha\Big|_{\theta\theta} + \text{H.c.} && \rightsquigarrow -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\lambda\sigma^\mu\partial_\mu\bar\lambda + \frac{1}{2}D^2\\
\mathcal{L}_{\text{chiral}} &= \Phi^\dagger \mathrm{e}^{2gqV}\Phi\Big|_{\theta\theta\bar\theta\bar\theta} && \rightsquigarrow F^*F + \mathrm{D}_\mu\phi^*\mathrm{D}^\mu\phi + \mathrm{i}\bar\psi\bar\sigma^\mu\mathrm{D}_\mu\psi + qgD\phi^*\phi - \sqrt2 gq\left(\phi^*\lambda\psi + \phi\bar\lambda\bar\psi\right)\\
\mathcal{L}_{\cancel{CP}} &= \frac{\mathrm{i}\theta}{32\pi^2}W^\alpha W_\alpha\Big|_{\theta\theta} + \text{H.c.} && \rightsquigarrow -\frac{\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\\
& && \text{with} \quad \mathrm{D}_\mu[\phi,\psi] = (\partial_\mu - \mathrm{i}gqA_\mu)[\phi,\psi], \qquad \mathrm{D}_\mu\lambda = \partial_\mu\lambda,
\end{aligned}$$

◆3.5.3 Non-Abelian Case

T^a : generators (Hermitian);
$\operatorname{Tr} T^aT^b = K\delta^{ab}$ $(K>0)$, $\qquad [T^a, T^b] = \mathrm{i}f^{abc}T^c$ (f is anti-symmetric)

Explicit Expression Same as the Abelian case.

Supersymmetric Gauge Transformation

$$\mathcal{L} \ni \Phi^\dagger \mathrm{e}^{2g\widetilde V}\Phi; \qquad \Phi \mapsto \mathrm{e}^{\mathrm{i}g\widetilde\Lambda}\Phi, \quad \Phi^\dagger \mapsto \Phi^\dagger\mathrm{e}^{-\mathrm{i}g\widetilde\Lambda^\dagger}; \qquad \mathrm{e}^{2g\widetilde V} \mapsto \mathrm{e}^{\mathrm{i}g\widetilde\Lambda^\dagger}\mathrm{e}^{2g\widetilde V}\mathrm{e}^{-\mathrm{i}g\widetilde\Lambda} \qquad (3.34)$$

$$\text{with } \widetilde V := V^aT^a, \quad \widetilde\Lambda := \Lambda^aT^a, \quad \widetilde\Lambda^\dagger := (\Lambda^a)^\dagger T^a$$

$$\begin{aligned}
2\widetilde V &\mapsto 2\widetilde V - \mathrm{i}(\widetilde\Lambda - \widetilde\Lambda^\dagger) - \frac{g}{2}\left([\widetilde\Lambda, \widetilde\Lambda^\dagger] + \mathrm{i}[2\widetilde V, \widetilde\Lambda + \widetilde\Lambda^\dagger]\right) + \cdots && (3.35)\\
&= \left[2V^a - \mathrm{i}(\Lambda^a - \Lambda^{\dagger a}) + \frac{g}{2}\left(-\mathrm{i}\Lambda^b\Lambda^{\dagger c} + 2V^b(\Lambda^c + \Lambda^{\dagger c})\right)f^{abc} + \cdots\right]T^a && (3.36)
\end{aligned}$$

We do not present the transformations of the components; note that λ and D are transformed in non-Abelian theories.

Wess-Zumino Gauge

$$V^a = -\theta\sigma^\mu\bar\theta A^a_\mu(x) + \theta\theta\bar\theta\bar\lambda^a(x) + \bar\theta\bar\theta\theta\lambda^a(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D^a(x) \tag{3.37}$$

$$\mathrm{e}^{kV^aT^a} = 1 + kV^aT^a + \frac{k^2}{4}\theta\theta\bar\theta\bar\theta A^a_\mu A^{b\mu}T^aT^b \tag{3.38}$$

Note that the lowest order term of the gauge transformation is independent of V, which guarantees that we can still take the Wess-Zumino gauge. The gauge transformation is restricted as $\mathrm{e}^{\mathrm{i}g\widetilde\Lambda}$, where $\Lambda^a = \xi^a(y) = \xi^a(x) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\xi^a(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\xi^a(x) : \xi \in \mathbb{R}$.

$$\begin{aligned} 2V^e \mapsto\ & 2\left(V^e - gf^{eab}\xi^aV^b + 6g^2f^{abc}f^{ade}V^b\xi^c\xi^d\right) \\ & + \theta\sigma^\mu\bar\theta\left(-2\partial_\mu\xi^e + gf^{eab}\xi^a\partial_\mu\xi^b + 4g^2f^{acd}f^{abe}\xi^b\xi^c\partial_\mu\xi^d\right) + \cdots \end{aligned} \tag{3.39}$$

$$\begin{aligned} A^e_\mu \mapsto\ & A^e_\mu + gf^{eab}A^a_\mu\xi^b + 6g^2f^{abc}f^{ade}A^b_\mu\xi^c\xi^d \\ & + \left(\partial_\mu\xi^e - \frac{g}{2}f^{eab}\xi^a\partial_\mu\xi^b - 2g^2f^{acd}f^{abe}\xi^b\xi^c\partial_\mu\xi^d\right) + \cdots \end{aligned} \tag{3.40}$$

$$\lambda^e \mapsto \lambda^e + gf^{eab}\lambda^a\xi^b + 6g^2f^{abc}f^{ade}\lambda^b\xi^c\xi^d + \cdots \tag{3.41}$$

$$D^e \mapsto D^e + gf^{eab}D^a\xi^b + 6g^2f^{abc}f^{ade}D^b\xi^c\xi^d + \cdots \tag{3.42}$$

Note that C, χ, M and N are kept invariant automatically, for now we are under Wess-Zumino gauge.

Field Strength [*2]

$$\widetilde{W}_\alpha = -\frac{1}{8g}\bar D\bar D\mathrm{e}^{-2g\widetilde V}D_\alpha\mathrm{e}^{2g\widetilde V} \qquad \bar D_{\dot\beta}W_\alpha = 0 \qquad W_\alpha \mapsto \mathrm{e}^{\mathrm{i}g\widetilde\Lambda}W_\alpha\mathrm{e}^{-\mathrm{i}g\Lambda^\dagger} \tag{3.43}$$

$$W^a_\alpha = \lambda^a_\alpha(y) + \theta_\alpha D^a(y) + \mathrm{i}(\sigma^{\mu\nu}\theta)_\alpha F^a_{\mu\nu}(y) + \mathrm{i}\theta\theta(\sigma^\mu\mathrm{D}_\mu\bar\lambda^a(y))_\alpha \tag{3.44}$$

$$\operatorname{Tr}\widetilde{W}^\alpha\widetilde{W}_\alpha\Big|_{\theta\theta} = \operatorname{Tr}\left[DD + \mathrm{i}\lambda\sigma^\mu\mathrm{D}_\mu\bar\lambda + \mathrm{i}\mathrm{D}_\mu\bar\lambda\bar\sigma^\mu\lambda - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{\mathrm{i}}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right] \tag{3.45}$$

$$= K\left[D^aD^a + \mathrm{i}\lambda^a\sigma^\mu\mathrm{D}_\mu\bar\lambda^a + \mathrm{i}\mathrm{D}_\mu\bar\lambda^a\bar\sigma^\mu\lambda^a - \frac{1}{2}F^{\mu\nu\,a}F^a_{\mu\nu} + \frac{\mathrm{i}}{4}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\right] \tag{3.46}$$

Lagrangian Block

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4K}\operatorname{Tr}\widetilde{W}^\alpha\widetilde{W}_\alpha\Big|_{\theta\theta} + \text{H.c.} \qquad \rightsquigarrow -\frac{1}{4}F^{\mu\nu\,a}F^a_{\mu\nu} + \mathrm{i}\bar\lambda^a\bar\sigma^\mu\mathrm{D}_\mu\lambda^a + \frac{1}{2}D^aD^a \tag{3.47}$$

$$\mathcal{L}_{\cancel{CP}} = \frac{\mathrm{i}}{32K\pi^2}\operatorname{Tr}\widetilde{W}^\alpha\widetilde{W}_\alpha\Big|_{\theta\theta} + \text{H.c.} \qquad \rightsquigarrow -\frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} \tag{3.48}$$

$$\begin{aligned} \mathcal{L}_{\text{matter}} = \Phi^\dagger_i[\mathrm{e}^{2gV^aT^a}]_{ij}\Phi_j\Big|_{\theta\theta\bar\theta\bar\theta} \qquad & \rightsquigarrow \mathrm{D}^\mu\phi^*_i\mathrm{D}_\mu\phi_i + \mathrm{i}\bar\psi_i\bar\sigma^\mu\mathrm{D}_\mu\psi_i + F^*_iF_i \\ & \quad + gD^a(\phi^*T^a\phi) - \sqrt{2}g\left(\phi^*T^a\psi\lambda + \bar\psi\bar\lambda T^a\phi\right) \end{aligned} \tag{3.49}$$

$$\begin{aligned} \mathrm{D}_\mu\phi_i &= \partial_\mu\phi - \mathrm{i}gA^a_\mu(T^a\phi)_i & F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g[A_\mu, A_\nu], \\ \mathrm{D}_\mu\phi^*_i &= \partial_\mu\phi + \mathrm{i}gA^a_\mu(\phi^*T^a)_i & \mathrm{D}_\mu\lambda^a &= \partial_\mu\lambda^a + gf^{abc}A^b_\mu\lambda^c, \\ \mathrm{D}_\mu\psi_i &= \partial_\mu\psi - \mathrm{i}gA^a_\mu(T^a\psi)_i & \mathrm{D}_\mu\bar\lambda^a &= \partial_\mu\bar\lambda^a + gf^{abc}A^b_\mu\bar\lambda^c, \end{aligned}$$

[*2] Note the signs. $\bar D\bar D\mathrm{e}^{2gV}D_\alpha\mathrm{e}^{-2gV}$ is not gauge invariant! Also the curvature tensor and the covariant derivative is well-known ones: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g[A_\mu, A_\nu]$, and $\mathrm{D}_\mu\bar\lambda = \partial_\mu\bar\lambda - \mathrm{i}g[A_\mu, \bar\lambda] = \partial_\mu\bar\lambda + gf^{abc}A^b_\mu\bar\lambda^cT^a$.

3.6 Minimal Supersymmetric Standard Model

◆3.6.1 Definitions

Gauge Group and Superfields [*3]

$$\mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_{\mathrm{weak}} \times \mathrm{U}(1)_{\mathrm{Y}} \quad (\times \mathrm{Z}_{2R} : R\text{-parity}); \tag{3.50}$$

Field	SU(3)	SU(2)	U(1)	B	L
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$	
L_i		$\mathbf{2}$	$-1/2$		1
$\bar{U}_i$	$\bar{\mathbf{3}}$		$-2/3$	$-1/3$	
$\bar{D}_i$	$\bar{\mathbf{3}}$		$1/3$	$-1/3$	
$\bar{E}_i$			1		-1
H_{u}		$\mathbf{2}$	$1/2$		
H_{d}		$\mathbf{2}$	$-1/2$		

Field	SU(3)	SU(2)	U(1)
g	$\mathbf{8}$		
W		$\mathbf{3}$	
B			

Superpotential and $\cancel{\text{SUSY}}$-terms

$$W_{\mathrm{RPC}} = \mu H_{\mathrm{u}} H_{\mathrm{d}} - y_{\mathrm{u}ij} \bar{U}_i H_{\mathrm{u}} Q_j + y_{\mathrm{d}ij} \bar{D}_i H_{\mathrm{d}} Q_j + y_{\mathrm{e}ij} \bar{E}_i H_{\mathrm{d}} L_j \tag{3.51}$$

$$W_{\mathrm{RPV}} = \mu_i H_{\mathrm{u}} L_i + \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k \tag{3.52}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{w}\widetilde{w} + M_1 \widetilde{b}\widetilde{b} + \mathrm{H.c.}\right) - V_{\cancel{\mathrm{SUSY}}}; \tag{3.53}$$

$$\begin{aligned} V^{\mathrm{RPC}}_{\cancel{\mathrm{SUSY}}} = &\left(\widetilde{q}^* m_Q^2 \widetilde{q} + \widetilde{l}^* m_L^2 \widetilde{l} + \widetilde{u}_{\mathrm{R}} m_{\bar{U}}^2 \widetilde{u}_{\mathrm{R}}^* + \widetilde{d}_{\mathrm{R}} m_{\bar{D}}^2 \widetilde{d}_{\mathrm{R}}^* + \widetilde{e}_{\mathrm{R}} m_{\bar{E}}^2 \widetilde{e}_{\mathrm{R}}^* + m_{H_{\mathrm{u}}}^2 |H_{\mathrm{u}}|^2 + m_{H_{\mathrm{d}}}^2 |H_{\mathrm{d}}|^2\right) \\ &+ \left(-\widetilde{u}_{\mathrm{R}}^* H_{\mathrm{u}} A^u \widetilde{q} + \widetilde{d}_{\mathrm{R}}^* H_{\mathrm{d}} A^d \widetilde{q} + \widetilde{e}_{\mathrm{R}}^* H_{\mathrm{d}} A^e \widetilde{l} + b H_{\mathrm{u}} H_{\mathrm{d}} + \mathrm{H.c.}\right) m \\ &+ \left(-\widetilde{u}_{\mathrm{R}}^* H_{\mathrm{d}}^* C^u \widetilde{q} + \widetilde{d}_{\mathrm{R}}^* H_{\mathrm{u}}^* C^d \widetilde{q} + \widetilde{e}_{\mathrm{R}}^* H_{\mathrm{u}}^* C^e \widetilde{l} + \mathrm{H.c.}\right) \end{aligned} \tag{3.54}$$

$$V^{\mathrm{RPV}}_{\cancel{\mathrm{SUSY}}} = \left(b_i H_{\mathrm{u}} \widetilde{l}_i + \frac{1}{2} A_{ijk} \widetilde{l}_i \widetilde{l}_j \widetilde{e}_{\mathrm{R}k}^* + A'_{ijk} \widetilde{l}_i \widetilde{q}_j \widetilde{d}_{\mathrm{R}k}^* + \frac{1}{2} A''_{ijk} \widetilde{u}_{\mathrm{R}i}^* \widetilde{d}_{\mathrm{R}j}^* \widetilde{d}_{\mathrm{R}k}^* + M_{Li}^2 H_{\mathrm{d}}^* \widetilde{l}_i + \mathrm{H.c.}\right) \tag{3.55}$$

$$+ \left(C^1_{ijk} \widetilde{l}_i^* \widetilde{q}_j \widetilde{u}_{\mathrm{R}k}^* + C^2_i H_{\mathrm{u}}^* H_{\mathrm{d}} \widetilde{e}_{\mathrm{R}i}^* + C^3_{ijk} \widetilde{d}_{\mathrm{R}i} \widetilde{u}_{\mathrm{R}j}^* \widetilde{e}_{\mathrm{R}k}^* + \frac{1}{2} C^4_{ijk} \widetilde{d}_{\mathrm{R}i} \widetilde{q}_j \widetilde{q}_k + \mathrm{H.c.}\right), \tag{3.56}$$

where we define $\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C^4_{ijk} = C^4_{ikj}$

[*3] For left-handed fermions, the superfield will be written as, e.g., Q, and it contains a scalar component $\widetilde{q}$ and a chiral fermion Q_α. Their complex conjugates will be shown as $\widetilde{q}^*$ and $\bar{Q}_{\dot{\alpha}}$ as is done in the previous section.

For right-handed fermions such as $\bar{U}$, the superfield will be written as $\bar{U}$. Its scalar component is written as (or, in other words, equivalent to) $\widetilde{u}_{\mathrm{R}}^*$, and its fermionic one is U^{c}_α. *c is just a label;* does not mean charge or complex conjugation. Their complex conjugates are $\widetilde{u}_{\mathrm{R}}$ and $\bar{U}^{\mathrm{c}}_{\dot{\alpha}}$.

They form a Dirac fermion as $U = \begin{pmatrix} u_\alpha \\ \bar{U}^{\mathrm{c}\dot{\alpha}} \end{pmatrix} =: \begin{pmatrix} U_{\mathrm{L}} \\ U_{\mathrm{R}} \end{pmatrix}$; its *charge* conjugate is $U^{\mathrm{C}} = \begin{pmatrix} U^{\mathrm{C}}_\alpha \\ \bar{u}^{\dot{\alpha}} \end{pmatrix}$.

Majorana fermions are written as, e.g., $\widetilde{b} = \widetilde{b}^{\mathrm{C}} = \begin{pmatrix} \widetilde{b}_\alpha \\ \bar{\widetilde{b}}^{\dot{\alpha}} \end{pmatrix}$. Here $\bar{\widetilde{b}}_{\dot{\alpha}}$ is the complex conjugate of $\widetilde{b}_\alpha$.

scalars : $\widetilde{q}\,(\widetilde{u}_{\mathrm{L}}, \widetilde{d}_{\mathrm{L}}), \widetilde{u}_{\mathrm{R}}^*, \widetilde{d}_{\mathrm{R}}^*, \widetilde{l}\,(\widetilde{e}_{\mathrm{L}}, \widetilde{\nu}), \widetilde{e}_{\mathrm{R}}^*, H_{\mathrm{u}}\,(H_{\mathrm{u}}^+, H_{\mathrm{u}}^0), H_{\mathrm{d}}\,(H_{\mathrm{d}}^0, H_{\mathrm{d}}^-)$

$\widetilde{q}^*\,(\widetilde{u}_{\mathrm{L}}^*, \widetilde{d}_{\mathrm{L}}^*), \widetilde{u}_{\mathrm{R}}, \widetilde{d}_{\mathrm{R}}, \widetilde{l}^*\,(\widetilde{e}_{\mathrm{L}}^*, \widetilde{\nu}^*), \widetilde{e}_{\mathrm{R}}, H_{\mathrm{u}}^*, H_{\mathrm{d}}^*$

Diracs : $U\,(U_{\mathrm{L}}, U_{\mathrm{R}}), D\,(D_{\mathrm{L}}, D_{\mathrm{R}}), E\,(E_{\mathrm{L}}, E_{\mathrm{R}}), \nu$

Weyls : $Q\,(u, d), U^{\mathrm{c}}, D^{\mathrm{c}}, L\,(\nu, e), E^{\mathrm{c}}$

$\bar{Q}\,(\bar{u}, \bar{d}), \bar{U}^{\mathrm{c}}, \bar{D}^{\mathrm{c}}, \bar{L}\,(\bar{\nu}, \bar{e}), \bar{E}^{\mathrm{c}}$

$\widetilde{h}_{\mathrm{u}}\,(\widetilde{h}_{\mathrm{u}}^+, \widetilde{h}_{\mathrm{u}}^0), \widetilde{h}_{\mathrm{d}}\,(\widetilde{h}_{\mathrm{d}}^0, \widetilde{h}_{\mathrm{d}}^-), \widetilde{b}, \widetilde{w}, \widetilde{g}$

$\bar{\widetilde{h}}_{\mathrm{u}}\,(\bar{\widetilde{h}}_{\mathrm{u}}^+, \bar{\widetilde{h}}_{\mathrm{u}}^0), \bar{\widetilde{h}}_{\mathrm{d}}\,(\bar{\widetilde{h}}_{\mathrm{d}}^0, \bar{\widetilde{h}}_{\mathrm{d}}^-), \bar{\widetilde{b}}, \bar{\widetilde{w}}, \bar{\widetilde{g}}$

◆3.6.2 Lagrangian Build Block

$$\mathcal{L}_{\mathrm{K;\,CP}} = -\frac{1}{4}F^{\mu\nu\,a}F^a_{\mu\nu} + \mathrm{D}^\mu\phi_i^*\mathrm{D}_\mu\phi_i + \mathrm{i}\bar{\psi}_i\bar{\sigma}^\mu\mathrm{D}_\mu\psi_i + \mathrm{i}\bar{\lambda}^a\bar{\sigma}^\mu\mathrm{D}_\mu\lambda^a - \sqrt{2}g\left(\phi^*T^a\psi\lambda + \bar{\psi}\bar{\lambda}T^a\phi\right) \tag{3.57}$$

$$\mathcal{L}^{\cancel{\mathrm{SUSY}}}_{\mathrm{gaugino}} = -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{w}\widetilde{w} + M_1\widetilde{b}\widetilde{b} + \mathrm{H.c.}\right) \tag{3.58}$$

$$\mathcal{L}_{\mathrm{scalar}} = -\left(\sum V^F + \sum V^D + \sum V_{\cancel{SUSY}}\right) \tag{3.59}$$

$$\mathcal{L}^{\mathrm{RPC}}_{\mathrm{S;\,fermi}} = -\left(\mu\widetilde{h}_{\mathrm{u}}\widetilde{h}_{\mathrm{d}} - y_{\mathrm{u}ij}U_i^{\mathrm{c}}H_{\mathrm{u}}Q_j + y_{\mathrm{d}ij}D_i^{\mathrm{c}}H_{\mathrm{d}}Q_j + y_{\mathrm{e}ij}E_i^{\mathrm{c}}H_{\mathrm{d}}L_j + \cdots + \mathrm{H.c.}\right) \tag{3.60}$$

$$\mathcal{L}^{\mathrm{RPV}}_{\mathrm{S;\,fermi}} = -\left(\mu_i\widetilde{h}_{\mathrm{u}}L_i + \frac{1}{2}\lambda_{ijk}L_iL_jE_k^{\mathrm{c}} + \lambda'_{ijk}L_iQ_jD_k^{\mathrm{c}} + \frac{1}{2}\lambda''_{ijk}U_i^{\mathrm{c}}D_j^{\mathrm{c}}D_k^{\mathrm{c}} + \cdots + \mathrm{H.c.}\right) \tag{3.61}$$

$$\mathcal{L}_{\mathrm{K;}\cancel{CP}} = -\frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} \tag{3.62}$$

The scalar potential is decomposed as

$$-F_{H_{\mathrm{u}}}{}^{a*} = \epsilon^{ab}\left(\mu H_{\mathrm{d}}^b - y_{\mathrm{u}ij}\bar{U}_i^xQ_j^{bx} + \mu_iL_i^b\right) \tag{3.63}$$

$$-F_{H_{\mathrm{d}}}{}^{a*} = \epsilon^{ab}\left(-\mu H_{\mathrm{u}}^b + y_{\mathrm{d}ij}\bar{D}_i^xQ_j^{bx} + y_{\mathrm{e}ij}\bar{E}_iL_j^b\right) \tag{3.64}$$

$$-F_{Q_i}{}^{ax*} = \epsilon^{ab}\left(y_{\mathrm{u}ji}H_{\mathrm{u}}^b\bar{U}_j^x - y_{\mathrm{d}ji}H_{\mathrm{d}}^b\bar{D}_j^x - \lambda'_{jik}L_i^b\bar{D}_j^x\right) \tag{3.65}$$

$$-F_{L_i}{}^{a*} = \epsilon^{ab}\left(-\mu_iH_{\mathrm{u}}^b - y_{\mathrm{e}ji}H_{\mathrm{d}}^b\bar{E}_j + \lambda_{ijk}L_j^b\bar{E}_k + \lambda'_{ijk}Q_j^{bx}\bar{D}_k^x\right) \tag{3.66}$$

$$-F_{\bar{U}_i}{}^{x*} = \left(-\epsilon^{ab}y_{\mathrm{u}ij}H_{\mathrm{u}}^aQ_j^{bx} + \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk}\bar{D}_j^y\bar{D}_k^z\right) \tag{3.67}$$

$$-F_{\bar{D}_i}{}^{x*} = \left(\epsilon^{ab}y_{\mathrm{d}ij}H_{\mathrm{d}}^aQ_j^{bx} + \epsilon^{ab}\lambda'_{jki}L_j^aQ_k^{bx} + \epsilon^{yzx}\lambda''_{jki}\bar{U}_j^y\bar{D}_k^z\right) \tag{3.68}$$

$$-F_{\bar{E}_i}{}^{*} = \left(\epsilon^{ab}y_{\mathrm{e}ij}H_{\mathrm{d}}^aL_j^b + \frac{1}{2}\epsilon^{ab}\lambda_{jki}L_j^aL_k^b\right) \tag{3.69}$$

and

$$D_g{}^\alpha = -g_3\sum_{i=1}^{3}\left[\sum_{a=1,2}Q_i^{ax*}(T^\alpha)_{xy}Q_i^{ay} - \bar{U}_i^{x*}(T^\alpha)_{xy}\bar{U}_i^y - \bar{D}_i^{x*}(T^\alpha)_{xy}\bar{D}_i^y\right] \tag{3.70}$$

$$D_W{}^\alpha = -g_2\left[\sum_{i=1}^{3}\sum_{x=1}^{3}Q_i^{ax*}(T^\alpha)_{ab}Q_i^{by} + \sum_{i=1}^{3}L_i^{a*}(T^\alpha)_{ab}L_i^b + H_{\mathrm{u}}^{a*}(T^\alpha)_{ab}H_{\mathrm{u}}^b + H_{\mathrm{d}}^{a*}(T^\alpha)_{ab}H_{\mathrm{d}}^b\right] \tag{3.71}$$

$$D_B = -g_1\left[\frac{1}{6}|Q_i^{ax}|^2 - \frac{1}{2}|L_i^a|^2 - \frac{2}{3}|\bar{U}_i^x|^2 + \frac{1}{3}|\bar{D}_i^x|^2 + |\bar{E}_i|^2 + \frac{1}{2}|H_{\mathrm{u}}^a|^2 - \frac{1}{2}|H_{\mathrm{d}}^a|^2\right]. \tag{3.72}$$

Here we use the superfield notation for simple appearance.

◆3.6.3 Scalar Potential (Verbose)

(with the superfield notation)

$$V^F_{H_\mathrm{u}} = |\mu|^2|H_\mathrm{d}|^2 + \sum_a\Big(|\bar{U}y^uQ^a|^2 + |\mu_iL^a_i|^2\Big) + \Big[\mu^*\mu_iH^*_\mathrm{d}L_i - \mu^*H^*_\mathrm{d}\bar{U}y^uQ - \mu^*_iL^*_i\bar{U}y^uQ + \text{H.c.}\Big] \tag{3.73}$$

$$V^F_{H_\mathrm{d}} = |\mu|^2|H_\mathrm{u}|^2 + \sum_a\Big(|\bar{D}y^dQ^a|^2 + |\bar{E}y^eL^a|^2\Big) + \Big[-\mu^*H^*_\mathrm{u}\bar{D}y^dQ - \mu^*H^*_\mathrm{u}\bar{E}y^eL + (\bar{D}y^dQ)^*(\bar{E}y^eL) + \text{H.c.}\Big] \tag{3.74}$$

$$V^F_Q = |H_\mathrm{u}|^2|\bar{U}_iy^u_{ij}|^2 + |H_\mathrm{d}|^2|\bar{D}_iy^d_{ij}|^2 + \lambda'^*_{jik}\lambda'_{lim}L^*_jL_l\bar{D}^*_k\bar{D}_m + \Big[-y^{u*}_{ji}y^d_{ki}H^*_\mathrm{u}H_\mathrm{d}\bar{U}^*_j\bar{D}_k - y^{u*}_{ji}\lambda'_{lim}H^*_\mathrm{u}L_l\bar{U}^*_j\bar{D}_m + y^{d*}_{ji}\lambda'_{lim}H^*_\mathrm{d}L_l\bar{D}^*_j\bar{D}_m + \text{H.c.}\Big] \tag{3.75}$$

$$\begin{aligned} V^F_L = {} & |\mu_i|^2|H_\mathrm{u}|^2 + |H_\mathrm{d}|^2(\bar{E}y^ey^{e\dagger}\bar{E}^*) + \lambda'_{ijk}\lambda'_{ilm}(Q^*_j\bar{D}^*_k)Q_l\bar{D}_m + \lambda^*_{ijk}\lambda_{ilm}L^*_jL_l\bar{E}^*_k\bar{E}_m \\ & + \Big[\mu^*_iy^e_{ji}\bar{E}_jH^*_\mathrm{u}H_\mathrm{d} - \mu^*_i\lambda'_{ijk}H^*_\mathrm{u}Q_j\bar{D}_k - \mu^*_i\lambda_{ijk}H^*_\mathrm{u}L_j\bar{E}_k \\ & \quad -y^{e*}_{ji}\lambda'_{ilm}\bar{E}^*_jH^*_\mathrm{d}Q_l\bar{D}_m - y^{e*}_{ji}\lambda_{ilm}\bar{E}^*_jH^*_\mathrm{d}L_l\bar{E}_m + \lambda'^*_{ijk}\lambda_{ilm}\bar{D}^*_kQ^*_jL_l\bar{E}_m + \text{H.c.}\Big] \end{aligned} \tag{3.76}$$

$$V^F_{\bar{U}} = y^{u*}_{ij}y^u_{ik}\epsilon^{ab}\epsilon^{cd}H^{a*}_\mathrm{u}H^c_\mathrm{u}Q^{b*}_jQ^d_k + \frac{1}{2}\lambda''^*_{ijk}\lambda''_{ilm}(\bar{D}^*_j\bar{D}_l)(\bar{D}^*_k\bar{D}_m) - \Big[y^{u*}_{il}\lambda''_{ijk}H^*_\mathrm{u}Q^*_l\bar{D}_j\bar{D}_k + \text{H.c.}\Big] \tag{3.77}$$

$$\begin{aligned} V^F_{\bar{D}} = {} & \epsilon^{ab}\epsilon^{cd}\left(y^d_{ij}H^a_\mathrm{d} + \lambda'_{kji}L^a_k\right)^*\left(y^d_{il}H^c_\mathrm{d} + \lambda'_{mli}L^c_m\right)Q^{b*}_jQ^d_l + \lambda''^*_{jki}\lambda''_{lmi}\left(\bar{U}^*_j\bar{U}_l\ \bar{D}^*_k\bar{D}_m - \bar{U}^*_j\bar{D}_m\ \bar{D}^*_k\bar{U}_l\right) \\ & + \Big[\lambda''_{lmi}(y^{d*}_{ij}H^*_\mathrm{d} + \lambda'^*_{kji}L^*_k)Q^*_j\bar{U}_l\bar{D}_m + \text{H.c.}\Big] \end{aligned} \tag{3.78}$$

$$V^F_{\bar{E}} = \epsilon^{ab}\epsilon^{cd}\left(y^e_{ij}H^a_\mathrm{d} + \frac{1}{2}\lambda_{kji}L^a_k\right)^*L^{*b}_j\left(y^e_{il}H^c_\mathrm{d} + \frac{1}{2}\lambda_{mli}L^c_m\right)L^d_l \tag{3.79}$$

$$V^D_g = \frac{g_3^2}{2}\left\{\sum_{\alpha=1}^{8}\sum_{i=1}^{3}\left[\sum_{a=1,2}Q^{a*}_i(t^\alpha)Q^a_i - \bar{U}^*_i(t^\alpha)\bar{U}_i - \bar{D}^*_i(t^\alpha)\bar{D}_i\right]\right\}^2 \tag{3.80}$$

$$V^D_W = \frac{g_2^2}{2}\left[\sum_{i=1}^{3}\sum_{x=1}^{3}Q^{x*}_i(T^\alpha)Q^x_i + \sum_{i=1}^{3}L^*_i(T^\alpha)L_i + H^*_\mathrm{u}(T^\alpha)H_\mathrm{u} + H^*_\mathrm{d}(T^\alpha)H_\mathrm{d}\right]^2 \tag{3.81}$$

$$V^D_B = \frac{g_1^2}{2}\left[\sum_i\left(\frac{1}{6}|Q_i|^2 - \frac{1}{2}|L_i|^2 - \frac{2}{3}|\bar{U}_i|^2 + \frac{1}{3}|\bar{D}_i|^2 + |\bar{E}_i|^2\right) + \frac{1}{2}|H_\mathrm{u}|^2 - \frac{1}{2}|H_\mathrm{d}|^2\right]^2 \tag{3.82}$$

$$\begin{aligned} V^\mathrm{RPC}_{\cancel{\mathrm{SUSY}}} = {} & \left(Q^*m^2_QQ + L^*m^2_LL + \bar{U}^*m^2_{\bar{U}}\bar{U} + \bar{D}^*m^2_{\bar{D}}\bar{D} + \bar{E}^*m^2_{\bar{E}}\bar{E} + m^2_{H_\mathrm{u}}|H_\mathrm{u}|^2 + m^2_{H_\mathrm{d}}|H_\mathrm{d}|^2\right) \\ & + \left(-\bar{U}H_\mathrm{u}A^uQ + \bar{D}H_\mathrm{d}A^dQ + \bar{E}H_\mathrm{d}A^eL + BH_\mathrm{u}H_\mathrm{d} + \text{H.c.}\right) \\ & + \left(-\bar{U}H^*_\mathrm{d}C^uQ + \bar{D}H^*_\mathrm{u}C^dQ + \bar{E}H^*_\mathrm{u}C^eL + \text{H.c.}\right) \end{aligned} \tag{3.83}$$

$$\begin{aligned} V^\mathrm{RPV}_{\cancel{\mathrm{SUSY}}} = {} & \left(B_iH_\mathrm{u}L_i + \frac{1}{2}A_{ijk}L_iL_j\bar{E}_k + A'_{ijk}L_iQ_j\bar{D}_k + \frac{1}{2}A''_{ijk}\bar{U}_i\bar{D}_j\bar{D}_k + \text{H.c.}\right) \\ & + \left(C^1_{ijk}L^*_iQ_j\bar{U}_k + C^2_iH^*_\mathrm{u}H_\mathrm{d}\bar{E}_i + C^3_{ijk}\bar{D}^*_i\bar{U}_j\bar{E}_k + \frac{1}{2}C^4_{ijk}\bar{D}^*_iQ_jQ_k + \text{H.c.}\right) \\ & + \left(M^2_{Li}H^*_\mathrm{d}L_i + \text{H.c.}\right) \end{aligned} \tag{3.84}$$

With R-parity

$$
\begin{aligned}
V_{\text{full}}^{\text{RPC}} &= \left(Q^* m_Q^2 Q + L^* m_L^2 L + \bar{U}^* m_{\bar{U}}^2 \bar{U} + \bar{D}^* m_{\bar{D}}^2 \bar{D} + \bar{E}^* m_{\bar{E}}^2 \bar{E}\right) \\
&+ \left(|\mu|^2 + m_{H_\text{u}}^2\right)|H_\text{u}|^2 + \left(|\mu|^2 + m_{H_\text{d}}^2\right)|H_\text{d}|^2 + (B H_\text{u} H_\text{d} + \text{H.c.}) \\
&+ \Big[(-\mu^* y^u H_\text{d}^* - A^u H_\text{u} - C^u H_\text{d}^*)_{ij}\, \bar{U}_i Q_j + \text{H.c.}\Big] \\
&+ \Big[\left(-\mu^* y^d H_\text{u}^* + A^d H_\text{d} + C^d H_\text{u}^*\right)_{ij} \bar{D}_i Q_j + \text{H.c.}\Big] \\
&+ \Big[(-\mu^* y^e H_\text{u}^* + A^e H_\text{d} + C^e H_\text{u}^*)_{ij}\, \bar{E}_i L_j + \text{H.c.}\Big] \\
&+ |H_\text{u}|^2 |\bar{U} y^u|^2 + |H_\text{d}|^2 |\bar{D} y^d|^2 + |H_\text{d}|^2 |\bar{E} y^e|^2 + |H_\text{u}|^2 |y^u Q|^2 + |H_\text{d}|^2 |y^d Q|^2 + |H_\text{d}|^2 |y^e L|^2 \\
&+ \sum_a \Big(|\bar{U} y^u Q^a|^2 + |\bar{D} y^d Q^a|^2 + |\bar{E} y^e L^a|^2\Big) \\
&+ \Big[(\bar{D} y^d Q)^* (\bar{E} y^e L) - y_{ji}^{u*} y_{ki}^d H_\text{u}^* H_\text{d} \bar{U}_j^* \bar{D}_k + \text{H.c.}\Big] \\
&- \left[y_{ki}^{u*} y_{kj}^u (H_\text{u}^* Q_j)(Q_i^* H_\text{u}) + y_{ki}^{d*} y_{kj}^d (H_\text{d}^* Q_j)(Q_i^* H_\text{d}) + y_{ki}^{e*} y_{kj}^e (H_\text{d}^* L_j)(L_i^* H_\text{d})\right] \\
&+ \frac{g_3^2}{2} \left\{\sum_{\alpha=1}^{8} \sum_{i=1}^{3} \left[\sum_{a=1,2} Q_i^{a*}(t^\alpha) Q_i^a - \bar{U}_i^*(t^\alpha)\bar{U}_i - \bar{D}_i^*(t^\alpha)\bar{D}_i\right]\right\}^2 \\
&+ \frac{g_2^2}{2} \left[\sum_{i=1}^{3} \sum_{x=1}^{3} Q_i^{x*}(T^\alpha) Q_i^x + \sum_{i=1}^{3} L_i^*(T^\alpha) L_i + H_\text{u}^*(T^\alpha) H_\text{u} + H_\text{d}^*(T^\alpha) H_\text{d}\right]^2 \\
&+ \frac{g_1^2}{2} \left[\sum_i \left(\frac{1}{6}|Q_i|^2 - \frac{1}{2}|L_i|^2 - \frac{2}{3}|\bar{U}_i|^2 + \frac{1}{3}|\bar{D}_i|^2 + |\bar{E}_i|^2\right) + \frac{1}{2}|H_\text{u}|^2 - \frac{1}{2}|H_\text{d}|^2\right]^2
\end{aligned}
\tag{3.85}
$$

With Bilinear R-parity Violation

$$V_{H_\text{u}}^F \mathrel{+}= \sum_a |\mu_i L_i^a|^2 + \Big[\mu^* \mu_i H_\text{d}^* L_i - \mu_i^* L_i^* \bar{U} y^u Q + \text{H.c.}\Big] \tag{3.86}$$

$$V_L^F \mathrel{+}= |\mu_i|^2 |H_\text{u}|^2 + \Big[\mu_i^* y_{ji}^e \bar{E}_j H_\text{u}^* H_\text{d} + \text{H.c.}\Big] \tag{3.87}$$

$$V_{\cancel{\text{SUSY}}}^{\text{RPV}} = \left(B_i H_\text{u} L_i + M_{Li}^2 H_\text{d}^* L_i + \text{H.c.}\right) \tag{3.88}$$

With Trilinear leptonic R-parity Violation

$$V_Q^F \mathrel{+}= |\lambda'_{jik} L_j \bar{D}_k|^2 + \Big[-y_{ji}^{u*} \lambda'_{lim} H_\text{u}^* L_l \bar{U}_j^* \bar{D}_m + y_{ji}^{d*} \lambda'_{lim} H_\text{d}^* L_l \bar{D}_j^* \bar{D}_m + \text{H.c.}\Big] \tag{3.89}$$

$$
\begin{aligned}
V_L^F \mathrel{+}= &|\lambda'_{ijk} Q_j \bar{D}_k|^2 + |\lambda_{ijk} L_j \bar{E}_k|^2 \\
&+ \Big[-y_{mi}^{e*} \lambda'_{ijk} \bar{E}_m^* H_\text{d}^* Q_j \bar{D}_k - y_{mi}^{e*} \lambda_{ijk} \bar{E}_m^* H_\text{d}^* L_j \bar{E}_k + \lambda'^*_{ijk} \lambda_{ilm} \bar{D}_k^* Q_j^* L_l \bar{E}_m + \text{H.c.}\Big]
\end{aligned}
\tag{3.90}
$$

$$
\begin{aligned}
V_{\bar{D}}^F \mathrel{+}= &\lambda'^*_{kji} \lambda'_{mli} \Big[(L_k^* L_m)(Q_j^* Q_l) - (L_k^* Q_l)(Q_j^* L_m)\Big] \\
&+ \Big\{y_{ij}^{d\,*} \lambda'_{mli} \left[(H_\text{d}^* L_m)(Q_j Q_l) - (H_\text{d}^* Q_l)(Q_j^* L_m)\right] + \text{H.c.}\Big\}
\end{aligned}
\tag{3.91}
$$

$$V_{\bar{E}}^F = \frac{1}{2} \lambda_{kji}^* \lambda_{mli} (L_k^* L_m)(L_j^* L_l) + \lambda_{kji}^* y_{ilp}^e \Big[(L_k^* H_\text{d})(L_j^* L_l) + \text{H.c.}\Big] \tag{3.92}$$

◆3.6.4 Full Lagrangian (in Gauge eigenstates)

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{\mu\nu\,a}W^a_{\mu\nu} - \frac{1}{4}G^{\mu\nu\,a}G^a_{\mu\nu} \quad {}^{*4} \tag{3.93}$$

$$\begin{aligned}\mathcal{L}_{\text{gaugino}} = &-\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{w}\widetilde{w} + M_1\widetilde{b}\widetilde{b} + \text{H.c.}\right)\\ &+ \mathrm{i}\bar{\widetilde{b}}^a\bar{\sigma}^\mu\partial_\mu\widetilde{b}^a + \mathrm{i}\bar{\widetilde{w}}^a\bar{\sigma}^\mu\partial_\mu\widetilde{w}^a + \mathrm{i}\bar{\widetilde{g}}^a\bar{\sigma}^\mu\partial_\mu\widetilde{g}^a + \mathrm{i}g_2\epsilon^{abc}\bar{\widetilde{w}}^a\bar{\sigma}^\mu W^b_\mu\widetilde{w}^c + \mathrm{i}g_3 f^{abc}\bar{\widetilde{g}}^a\bar{\sigma}^\mu G^b_\mu\widetilde{g}^c\end{aligned} \tag{3.94}$$

$$\mathcal{L}_{\cancel{CP}} = -\frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}B_{\rho\sigma} - \frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^a_{\rho\sigma} - \frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} \tag{3.95}$$

$$\begin{aligned}\mathcal{L}_{\text{scalar}} = &\Big[\big(\partial^\mu + \mathrm{i}g_3G^\mu + \mathrm{i}g_2W^\mu + \tfrac{1}{6}\mathrm{i}g_1B^\mu\big)\widetilde{q}_i^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_3G_\mu - \mathrm{i}g_2W_\mu - \tfrac{1}{6}\mathrm{i}g_1B_\mu\big)\widetilde{q}_i\Big]\\ &+ \Big[\big(\partial^\mu + \mathrm{i}g_3G^\mu + \tfrac{2}{3}\mathrm{i}g_1B^\mu\big)\widetilde{u}_{\mathrm{R}i}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_3G_\mu - \tfrac{2}{3}\mathrm{i}g_1B_\mu\big)\widetilde{u}_{\mathrm{R}i}\Big]\\ &+ \Big[\big(\partial^\mu + \mathrm{i}g_3G^\mu - \tfrac{1}{3}\mathrm{i}g_1B^\mu\big)\widetilde{d}_{\mathrm{R}i}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_3G_\mu + \tfrac{1}{3}\mathrm{i}g_1B_\mu\big)\widetilde{d}_{\mathrm{R}i}\Big]\\ &+ \Big[\big(\partial^\mu + \mathrm{i}g_2W^\mu - \tfrac{1}{2}\mathrm{i}g_1B^\mu\big)\widetilde{l}_i^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_2W_\mu + \tfrac{1}{2}\mathrm{i}g_1B_\mu\big)\widetilde{l}_i\Big]\\ &+ \Big[\big(\partial^\mu - \mathrm{i}g_1B^\mu\big)\widetilde{e}_{\mathrm{R}i}^*\Big]\Big[\big(\partial_\mu + \mathrm{i}g_1B_\mu\big)\widetilde{e}_{\mathrm{R}i}\Big]\\ &+ \Big[\big(\partial^\mu + \mathrm{i}g_2W^\mu + \tfrac{1}{2}\mathrm{i}g_1B^\mu\big)H_{\mathrm{u}}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_2W_\mu - \tfrac{1}{2}\mathrm{i}g_1B_\mu\big)H_{\mathrm{u}}\Big]\\ &+ \Big[\big(\partial^\mu + \mathrm{i}g_2W^\mu - \tfrac{1}{2}\mathrm{i}g_1B^\mu\big)H_{\mathrm{d}}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_2W_\mu + \tfrac{1}{2}\mathrm{i}g_1B_\mu\big)H_{\mathrm{d}}\Big],\end{aligned} \tag{3.96}$$

$$\begin{aligned}\mathcal{L}_{\text{fermion}} = &\,\mathrm{i}\bar{Q}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_3G_\mu - \mathrm{i}g_2W_\mu - \frac{1}{6}\mathrm{i}g_1B_\mu\right)Q_i\\ &+ \mathrm{i}\bar{U}^{\mathrm{c}}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_3[-G^{\mathrm{T}}_\mu] + \frac{2}{3}\mathrm{i}g_1B_\mu\right)U^{\mathrm{c}}_i + \mathrm{i}\bar{D}^{\mathrm{c}}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_3[-G^{\mathrm{T}}_\mu] - \frac{1}{3}\mathrm{i}g_1B_\mu\right)D^{\mathrm{c}}_i\\ &+ \mathrm{i}\bar{L}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)L_i + \mathrm{i}\bar{E}^{\mathrm{c}}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_1B_\mu\right)E^{\mathrm{c}}_i\\ &+ \mathrm{i}\bar{\widetilde{h}}_{\mathrm{u}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}_{\mathrm{u}} + \mathrm{i}\bar{\widetilde{h}}_{\mathrm{d}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}_{\mathrm{d}},\end{aligned} \tag{3.97}$$

$$\begin{aligned}\mathcal{L}_{\text{SFG}} = &-\sqrt{2}g_3\Big[\big(\widetilde{q}_i^*\tau^aQ_i + \widetilde{u}_{\mathrm{R}i}[-\tau^{a\,\mathrm{T}}]U^{\mathrm{c}}_i + \widetilde{d}_{\mathrm{R}i}[-\tau^{a\,\mathrm{T}}]D^{\mathrm{c}}_i\big)\widetilde{g}^a\\ &\qquad\qquad + \bar{\widetilde{g}}^a\big(\bar{Q}_i\tau^a\widetilde{q}_i + \bar{U}^{\mathrm{c}}_i[-\tau^{a\,\mathrm{T}}]\widetilde{u}^*_{\mathrm{R}i} + \bar{D}^{\mathrm{c}}_i[-\tau^{a\,\mathrm{T}}]\widetilde{d}^*_{\mathrm{R}i}\big)\Big]\\ &-\sqrt{2}g_2\Big[\big(\widetilde{q}_i^*T^aQ_i + \widetilde{l}_i^*T^aL_i + H_{\mathrm{u}}^*T^a\widetilde{h}_{\mathrm{u}} + H_{\mathrm{d}}^*T^a\widetilde{h}_{\mathrm{d}}\big)\widetilde{w}^a\\ &\qquad\qquad + \bar{\widetilde{w}}^a\big(\bar{Q}_iT^a\widetilde{q}_i + \bar{L}_iT^a\widetilde{l}_i + \bar{\widetilde{h}}_{\mathrm{u}}T^aH_{\mathrm{u}} + \bar{\widetilde{h}}_{\mathrm{d}}T^aH_{\mathrm{d}}\big)\Big]\\ &-\sqrt{2}g_1\Big[\big(\tfrac{1}{6}\widetilde{q}_i^*Q_i - \tfrac{2}{3}\widetilde{u}_{\mathrm{R}i}U^{\mathrm{c}}_i + \tfrac{1}{3}\widetilde{d}_{\mathrm{R}i}D^{\mathrm{c}}_i - \tfrac{1}{2}\widetilde{l}_i^*L_i + \widetilde{e}_{\mathrm{R}i}E^{\mathrm{c}}_i + \tfrac{1}{2}H_{\mathrm{u}}^*\widetilde{h}_{\mathrm{u}} - \tfrac{1}{2}H_{\mathrm{d}}^*\widetilde{h}_{\mathrm{d}}\big)\widetilde{b}\\ &\qquad\qquad + \bar{\widetilde{b}}\big(\tfrac{1}{6}\bar{Q}_i\widetilde{q}_i - \tfrac{2}{3}\bar{U}^{\mathrm{c}}_i\widetilde{u}^*_{\mathrm{R}i} + \tfrac{1}{3}\bar{D}^{\mathrm{c}}_i\widetilde{d}^*_{\mathrm{R}i} - \tfrac{1}{2}\bar{L}_i\widetilde{l}_i + \bar{E}^{\mathrm{c}}_i\widetilde{e}^*_{\mathrm{R}i} + \tfrac{1}{2}\bar{\widetilde{h}}_{\mathrm{u}}H_{\mathrm{u}} - \tfrac{1}{2}\bar{\widetilde{h}}_{\mathrm{d}}H_{\mathrm{d}}\big)\Big]\end{aligned} \tag{3.98}$$

$$\begin{aligned}\mathcal{L}^{\text{RPC}}_{\text{super}} = &-\mu\widetilde{h}_{\mathrm{u}}\widetilde{h}_{\mathrm{d}} + y_{\mathrm{u}ij}U^{\mathrm{c}}_iH_{\mathrm{u}}Q_j - y_{\mathrm{d}ij}D^{\mathrm{c}}_iH_{\mathrm{d}}Q_j - y_{\mathrm{e}ij}E^{\mathrm{c}}_iH_{\mathrm{d}}L_j\\ &+ y_{\mathrm{u}ij}U^{\mathrm{c}}_i\widetilde{h}_{\mathrm{u}}\widetilde{q}_j + y_{\mathrm{u}ij}\widetilde{u}^*_{\mathrm{R}i}\widetilde{h}_{\mathrm{u}}Q_j - y_{\mathrm{d}ij}D^{\mathrm{c}}_i\widetilde{h}_{\mathrm{d}}\widetilde{q}_j - y_{\mathrm{d}ij}\widetilde{d}^*_{\mathrm{R}i}\widetilde{h}_{\mathrm{d}}Q_j\\ &- y_{\mathrm{e}ij}E^{\mathrm{c}}_i\widetilde{h}_{\mathrm{d}}\widetilde{l}_j - y_{\mathrm{e}ij}\widetilde{e}^*_{\mathrm{R}i}\widetilde{h}_{\mathrm{d}}L_j + \text{H.c.}\end{aligned} \tag{3.99}$$

$$\mathcal{L}^{\text{RPC}}_{\text{pot.}} = -(3.85)\Big[V^{\text{RPC}}_{\text{full}}\Big] \tag{3.100}$$

*4 Further decomposed results are shown in *Standard Model* section, Eqs. (2.7) and (2.21).

Fermion Composition

$$\mathcal{L}_{\text{gaugino}} = \frac{1}{2}\overline{\widetilde{b}}\left(\mathrm{i}\partial\!\!\!/ - M_1\right)\widetilde{b} + \frac{1}{2}\overline{\widetilde{w}}\left(\mathrm{i}D\!\!\!\!/ - M_2\right)\widetilde{w} + \frac{1}{2}\overline{\widetilde{g}}\left(\mathrm{i}D\!\!\!\!/ - M_3\right)\widetilde{g} \tag{3.101}$$

$$\begin{aligned}
\mathcal{L}_{\text{fermion}} &= (2.5)\Big[\mathcal{L}^{\text{SM}}_{\text{matter}}\Big] \\
&\qquad - \left(\mu\widetilde{h}_{\text{u}}\widetilde{h}_{\text{d}} + \text{H.c.}\right) + \mathrm{i}\overline{\widetilde{h}}_{\text{u}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}_{\text{u}} + \mathrm{i}\overline{\widetilde{h}}_{\text{d}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}_{\text{d}} \\
&= \mathcal{L}^{\text{SM}}_{\text{matter}} - \Big[\mu\left(\widetilde{h}^+_{\text{u}}\widetilde{h}^-_{\text{d}} - \widetilde{h}^0_{\text{u}}\widetilde{h}^0_{\text{d}}\right) + \text{H.c.}\Big] \\
&\qquad + \mathrm{i}\overline{\widetilde{h}}{}^+_{\text{u}}\bar{\sigma}^\mu\left(\partial_\mu - \frac{1}{2}\mathrm{i}g_2W^3_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}^+_{\text{u}} + \mathrm{i}\overline{\widetilde{h}}{}^-_{\text{d}}\bar{\sigma}^\mu\left(\partial_\mu + \frac{1}{2}\mathrm{i}g_2W^3_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}^-_{\text{d}} \\
&\qquad + \mathrm{i}\overline{\widetilde{h}}{}^0_{\text{u}}\bar{\sigma}^\mu\left(\partial_\mu + \frac{1}{2}\mathrm{i}g_2W^3_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}^0_{\text{u}} + \mathrm{i}\overline{\widetilde{h}}{}^0_{\text{d}}\bar{\sigma}^\mu\left(\partial_\mu - \frac{1}{2}\mathrm{i}g_2W^3_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}^0_{\text{d}} \\
&\qquad + \frac{g_2}{\sqrt{2}}\left(\overline{\widetilde{h}}{}^0_{\text{u}}\bar{\sigma}^\mu W^-_\mu\widetilde{h}^+_{\text{u}} + \overline{\widetilde{h}}{}^0_{\text{d}}\bar{\sigma}^\mu W^+_\mu\widetilde{h}^-_{\text{d}} + \overline{\widetilde{h}}{}^+_{\text{u}}\bar{\sigma}^\mu W^+_\mu\widetilde{h}^0_{\text{u}} + \overline{\widetilde{h}}{}^-_{\text{d}}\bar{\sigma}^\mu W^-_\mu\widetilde{h}^0_{\text{d}}\right)
\end{aligned} \tag{3.102}$$

$$\begin{aligned}
\mathcal{L}_{\text{SFG}} = &-\sqrt{2}g_3\Big[\widetilde{u}^*_{\text{L}i}\tau^a\left(\overline{\widetilde{g}}{}^aP_{\text{L}}U_i\right) + \widetilde{d}^*_{\text{L}i}\tau^a\left(\overline{\widetilde{g}}{}^aP_{\text{L}}D_i\right) - \left(\overline{U}_iP_{\text{L}}\widetilde{g}^a\right)\tau^a\widetilde{u}_{\text{R}i} - \left(\overline{D}_iP_{\text{L}}\widetilde{g}^a\right)\tau^a\widetilde{d}_{\text{R}i} \\
&\qquad + \left(\overline{U}_iP_{\text{R}}\widetilde{g}^a\right)\tau^a\widetilde{u}_{\text{L}i} + \left(\overline{D}_iP_{\text{R}}\widetilde{g}^a\right)\tau^a\widetilde{d}_{\text{L}i} - \widetilde{u}^*_{\text{R}i}\tau^a\left(\overline{\widetilde{g}}{}^aP_{\text{R}}U\right) - \widetilde{d}^*_{\text{R}i}\tau^a\left(\overline{\widetilde{g}}{}^aP_{\text{R}}D\right)\Big] \\
&- g_2\Big[\widetilde{w}^+\left(d_i\widetilde{u}_{\text{L}i}{}^* + e_i\widetilde{\nu}^*_i + \widetilde{h}^0_{\text{u}}H^{+\,*}_{\text{u}} + \widetilde{h}^-_{\text{d}}H^{0*}_{\text{d}}\right) + \left(\widetilde{d}_{\text{L}i}\bar{U}_i + \widetilde{e}_{\text{L}i}\bar{\nu}_i + H^0_{\text{u}}\overline{\widetilde{h}}{}^+_{\text{u}} + H^-_{\text{d}}\overline{\widetilde{h}}{}^0_{\text{d}}\right)\overline{\widetilde{w}}{}^- \\
&\qquad + \widetilde{w}^-\left(u_i\widetilde{d}_{\text{L}i}{}^* + \nu_i\widetilde{e}_{\text{L}i}{}^* + \widetilde{h}^+_{\text{u}}H^{0*}_{\text{u}} + \widetilde{h}^0_{\text{d}}H^{-\,*}_{\text{d}}\right) + \left(\widetilde{u}_{\text{L}i}\bar{D}_i + \widetilde{\nu}_i\bar{E}_i + H^+_{\text{u}}\overline{\widetilde{h}}{}^0_{\text{u}} + H^0_{\text{d}}\overline{\widetilde{h}}{}^-_{\text{d}}\right)\overline{\widetilde{w}}{}^+\Big] \\
&- \sqrt{2}g_2\Big[\tfrac{1}{2}\big(\widetilde{u}_{\text{L}i}{}^*u_i + \widetilde{\nu}^*_i\nu_i + H^{+\,*}_{\text{u}}\widetilde{h}^+_{\text{u}} + H^{0*}_{\text{d}}\widetilde{h}^0_{\text{d}} - \widetilde{d}_{\text{L}i}{}^*d_i - \widetilde{e}_{\text{L}i}{}^*e_i - H^{0*}_{\text{u}}\widetilde{h}^0_{\text{u}} - H^{-\,*}_{\text{d}}\widetilde{h}^-_{\text{d}}\big)\widetilde{w}^3 \\
&\qquad\qquad + \tfrac{1}{2}\overline{\widetilde{w}}{}^a\big(\bar{U}_i\widetilde{u}_{\text{L}i} + \bar{\nu}_i\widetilde{\nu}_i + \overline{\widetilde{h}}{}^+_{\text{u}}H^+_{\text{u}} + \overline{\widetilde{h}}{}^0_{\text{d}}H^0_{\text{d}} - \bar{D}_i\widetilde{d}_{\text{L}i} - \bar{E}_i\widetilde{e}_{\text{L}i} - \overline{\widetilde{h}}{}^0_{\text{u}}H^0_{\text{u}} - \overline{\widetilde{h}}{}^-_{\text{d}}H^-_{\text{d}}\big)\Big] \\
&- \sqrt{2}g_1\Big[\tfrac{1}{6}\widetilde{q}^*_i\overline{\widetilde{b}}P_{\text{L}}Q_i - \tfrac{2}{3}\widetilde{u}_{\text{R}i}\overline{U}_iP_{\text{L}}\widetilde{b} + \tfrac{1}{3}\widetilde{d}_{\text{R}i}\overline{D}_iP_{\text{L}}\widetilde{b} - \tfrac{1}{2}\widetilde{l}^*_i\overline{\widetilde{b}}P_{\text{L}}L_i + \widetilde{e}_{\text{R}i}\overline{E}_iP_{\text{L}}\widetilde{b} \\
&\qquad\qquad + \tfrac{1}{6}\widetilde{q}_i\overline{Q}_iP_{\text{R}}\widetilde{b} - \tfrac{2}{3}\widetilde{u}^*_{\text{R}i}\overline{\widetilde{b}}P_{\text{R}}U_i + \tfrac{1}{3}\widetilde{d}^*_{\text{R}i}\overline{\widetilde{b}}P_{\text{R}}D_i - \tfrac{1}{2}\widetilde{l}_i\overline{L}_iP_{\text{R}}\widetilde{b} + \widetilde{e}^*_{\text{R}i}\overline{\widetilde{b}}P_{\text{R}}E_i \\
&\qquad\qquad + \tfrac{1}{2}H^*_{\text{u}}\widetilde{h}_{\text{u}}\widetilde{b} - \tfrac{1}{2}H^*_{\text{d}}\widetilde{h}_{\text{d}}\widetilde{b} + \tfrac{1}{2}H_{\text{u}}\overline{\widetilde{h}}_{\text{u}}\overline{\widetilde{b}} - \tfrac{1}{2}H_{\text{d}}\overline{\widetilde{h}}_{\text{d}}\overline{\widetilde{b}}\Big]
\end{aligned} \tag{3.103}$$

$$\begin{aligned}
\mathcal{L}_{\text{湯川}} = &\Big(-H^0_{\text{u}}\overline{U}y_{\text{u}}P_{\text{L}}U - H^0_{\text{d}}\overline{D}y_{\text{d}}P_{\text{L}}D - H^0_{\text{d}}\overline{E}y_{\text{e}}P_{\text{L}}E \\
&\qquad + H^+_{\text{u}}\overline{U}y_{\text{u}}P_{\text{L}}D + H^-_{\text{d}}\overline{D}y_{\text{d}}P_{\text{L}}U + H^-_{\text{d}}\overline{E}y_{\text{e}}P_{\text{L}}\nu + \text{H.c.}\Big) \\
&+ \widetilde{u}_{\text{L}j}\left(y_{\text{d}ij}D^{\text{c}}_i\widetilde{h}^-_{\text{d}} - y_{\text{u}ij}U^{\text{c}}_i\widetilde{h}^0_{\text{u}}\right) + \widetilde{d}_{\text{L}j}\left(y_{\text{u}ij}U^{\text{c}}_i\widetilde{h}^+_{\text{u}} - y_{\text{d}ij}D^{\text{c}}_i\widetilde{h}^0_{\text{d}}\right) + y_{\text{e}ij}E^{\text{c}}_i\widetilde{h}^-_{\text{d}}\widetilde{\nu}_j - y_{\text{e}ij}E^{\text{c}}_i\widetilde{h}^0_{\text{d}}\widetilde{e}_{\text{L}j} \\
&+ y_{\text{u}ij}\widetilde{u}^*_{\text{R}i}\left(\widetilde{h}^+_{\text{u}}d_j - \widetilde{h}^0_{\text{u}}u_j\right) + y_{\text{d}ij}\widetilde{d}^*_{\text{R}i}\left(\widetilde{h}^-_{\text{d}}u_j - \widetilde{h}^0_{\text{d}}d_j\right) + y_{\text{e}ij}\widetilde{e}^*_{\text{R}i}\left(\widetilde{h}^-_{\text{d}}\nu_j - \widetilde{h}^0_{\text{d}}e_j\right) \\
&+ \widetilde{u}^*_{\text{L}j}\left(y^*_{\text{d}ij}\bar{D}^{\text{c}}_i\overline{\widetilde{h}}{}^-_{\text{d}} - y^*_{\text{u}ij}\bar{U}^{\text{c}}_i\overline{\widetilde{h}}{}^0_{\text{u}}\right) + \widetilde{d}^*_{\text{L}j}\left(y^*_{\text{u}ij}\bar{U}^{\text{c}}_i\overline{\widetilde{h}}{}^+_{\text{u}} - y^*_{\text{d}ij}\bar{D}^{\text{c}}_i\overline{\widetilde{h}}{}^0_{\text{d}}\right) + \widetilde{\nu}^*{}_jy^*_{\text{e}\,ij}\bar{E}^{\text{c}}_i\overline{\widetilde{h}}{}^-_{\text{d}} - \widetilde{e}^*_{\text{L}j}y^*_{\text{e}\,ij}\bar{E}^{\text{c}}_i\overline{\widetilde{h}}{}^0_{\text{d}} \\
&+ y^*_{\text{u}ij}\widetilde{u}_{\text{R}i}\left(\overline{\widetilde{h}}{}^+_{\text{u}}\bar{d}_j - \overline{\widetilde{h}}{}^0_{\text{u}}\bar{u}_j\right) + y^*_{\text{d}ij}\widetilde{d}_{\text{R}i}\left(\overline{\widetilde{h}}{}^-_{\text{d}}\bar{u}_j - \overline{\widetilde{h}}{}^0_{\text{d}}\bar{d}_j\right) + y^*_{\text{e}\,ij}\widetilde{e}_{\text{R}i}\left(\overline{\widetilde{h}}{}^-_{\text{d}}\bar{\nu}_j - \overline{\widetilde{h}}{}^0_{\text{d}}\bar{e}_j\right)
\end{aligned} \tag{3.104}$$

and the rest part is $\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}^{\text{RPC}}_{\text{pot.}}$.