1. Kinematics

Decay rate and cross section   (Note: $\mathcal{M}$ has a mass dimension of $4 - N_i - N_f$.)

$$\text{decay rate (rest frame; } \sqrt{s} = M_0): \quad d\Gamma = \frac{\overline{d\Pi^{N_f}}}{2M_0} \left| \mathcal{M}(M_0 \to \{p_1, p_2, \cdots, p_{N_f}\}) \right|^2, \tag{1.1}$$

$$\text{cross section (Lorentz invariant):} \quad d\sigma = \frac{\overline{d\Pi^{N_f}}}{2E_A 2E_B \, v_{\text{Møl}}} \left| \mathcal{M}(p_A, p_B \to \{p_1, p_2, \cdots, p_{N_f}\}) \right|^2, \tag{1.2}$$

where $\overline{d\Pi^n}$ is n-particle Lorentz-invariant phase space with momentum conservation

$$\overline{d\Pi^n} := d\Pi_1 \, d\Pi_2 \cdots d\Pi_n \, (2\pi)^4 \, \delta^{(4)} \left(P_0 - \sum p_n \right); \quad d\Pi := \frac{d^3 \boldsymbol{p}}{(2\pi)^3} \frac{1}{2E_{\boldsymbol{p}}}. \tag{1.3}$$

At the CM frame, two-body phase-space are characterized by the final momentum $\|\boldsymbol{p}\|$ and given by

$$\overline{d\Pi^2} = \frac{\|\boldsymbol{p}\|}{4\pi\sqrt{s}} \frac{d\Omega}{4\pi} = \frac{\|\boldsymbol{p}\|}{8\pi\sqrt{s}} \, d\cos\theta = \frac{1}{16\pi} \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}} \, d\cos\theta \tag{1.4}$$

with $\sqrt{s} = M_0$ or E_{CM}, θ is the angle between initial and final motion, and

$$\|\boldsymbol{p}\| = \frac{\sqrt{s}}{2} \lambda^{1/2} \left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s} \right), \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad p_1 \cdot p_2 = \frac{s - (m_1^2 + m_2^2)}{2}.$$

**Mandelstam variables**   For $(k_1, k_2) \to (p_3, p_4)$ collision,

$$s = (k_1 + k_2)^2 = (p_3 + p_4)^2, \qquad t = (p_3 - k_1)^2 = (p_4 - k_2)^2, \qquad u = (p_3 - k_2)^2 = (p_4 - k_1)^2;$$
$$k_1 \cdot k_2 = (s - m_1^2 - m_2^2)/2, \qquad k_1 \cdot p_3 = (m_3^2 + m_1^2 - t)/2, \qquad s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2,$$
$$p_3 \cdot p_4 = (s - m_3^2 - m_4^2)/2, \qquad k_1 \cdot p_4 = (m_4^2 + m_1^2 - u)/2;$$
$$(k_1 - k_2)^2 = 2(m_1^2 + m_2^2) - s, \qquad (p_3 - p_4)^2 = 2(m_3^2 + m_4^2) - s.$$

If the collision is with the "same mass" $(m_A, m_A) \to (m_B, m_B)$,

$$t = m_A^2 + m_B^2 - s/2 + 2kp\cos\theta, \qquad k = \sqrt{s/4 - m_A^2},$$
$$u = m_A^2 + m_B^2 - s/2 - 2kp\cos\theta, \qquad p = \sqrt{s/4 - m_B^2}.$$

Instead, if the collision is "initially massless" $(0, 0) \to (m_3, m_4)$,

$$t = (m_3^2 + m_4^2 - s)/2 + p\sqrt{s}\cos\theta, \qquad p = (\sqrt{s}/2) \lambda^{1/2} \left(1; m_3^2/s, m_4^2/s \right),$$
$$u = (m_3^2 + m_4^2 - s)/2 - p\sqrt{s}\cos\theta.$$
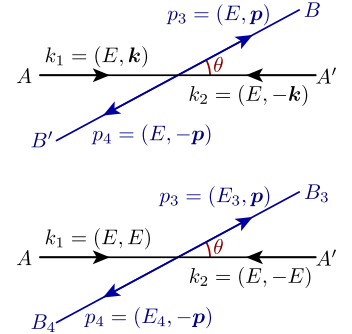
Three-body final state   Mandelstan variables can be defined, for $P \to (p_1, p_2, p_3)$, as

$$s_{ij} = (p_i + p_j)^2; \quad t_{0i} = (P - p_i)^2 = s_{jk}; \quad s_{12} + s_{23} + s_{31} = P^2 + p_1^2 + p_2^2 + p_3^2.$$

For spherically-symmetric processes, the phase-space integral is reduced to, at the center-of-mass frame,

$$\int \overline{d\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \boldsymbol{0}) = \frac{1}{128\pi^3} \frac{1}{s} \int_{(m_2 + m_3)^2}^{(\sqrt{s} - m_1)^2} ds_{23} \int ds_{13}; \tag{1.5}$$

$$(s_{13})_{\text{min}}^{\text{max}} = \frac{(s + m_3^2 - m_1^2 - m_2^2)^2}{4s_{23}} - \frac{1}{4s_{23}} \left[\lambda^{1/2}(s, m_1^2, s_{23}) \mp \lambda^{1/2}(s_{23}, m_2^2, m_3^2) \right]^2 \tag{1.6}$$

$$= (E_1^* + E_3^*)^2 - \left(\sqrt{E_1^{*2} - m_1^2} \mp \sqrt{E_3^{*2} - m_3^2} \right)^2, \tag{1.7}$$

where $E_1^* = \frac{s - s_{23} - m_1^2}{2\sqrt{s_{23}}}$, and $E_3^* = \frac{s_{23} - m_2^2 + m_3^2}{2\sqrt{s_{23}}}$.

1.1. Fundamentals

Lorentz-invariant phase space:
$$\int \mathrm{d}\Pi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\sqrt{m^2+\|\boldsymbol{p}\|^2}} = \int \frac{\mathrm{d}p_0\,\mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}(2\pi)\,\delta\left(p_0^2-\|\boldsymbol{p}\|^2-m^2\right)\Theta(p_0)$$

Källén function:
$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zx = (x-y-z)^2-4yz;$$
$$\lambda(1;\alpha_1^2,\alpha_2^2) = (1-(\alpha_1+\alpha_2)^2)(1-(\alpha_1-\alpha_2)^2) = (1+\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)(1-\alpha_1+\alpha_2).$$

$$\lambda^{1/2}\left(s;m_1^2,m_2^2\right) = s\,\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right); \qquad\qquad \lambda^{1/2}\left(1;\frac{m^2}{s},\frac{m^2}{s}\right) = \sqrt{1-\frac{4m^2}{s}},$$

$$\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = \sqrt{1-\frac{2(m_1^2+m_2^2)}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}}, \qquad \lambda^{1/2}\left(1;\frac{m_1^2}{s},0\right) = \frac{s-m_1^2}{s}.$$

Two-body phase space   If $f(p_1^\mu,p_2^\mu)$ is Lorentz invariant, $f \equiv f(p_1^2,p_2^2,p_1^\mu p_{2\mu}) \equiv f(p_1,p_2,\cos\theta_{12})$. Meanwhile,
$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \int \frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{(4\pi)\,\mathrm{d}p_1\,p_1^2}{(2\pi)^3}\frac{(2\pi)\,\mathrm{d}p_2\,p_2^2\,\mathrm{d}\cos\theta_{12}}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{\mathrm{d}E_+\,\mathrm{d}E_-\,\mathrm{d}s}{128\pi^4}, \quad (1.8)$$
with the replacement of the variables
$$E_\pm = E_1 \pm E_2, \qquad s = (p_1+p_2)^2 = m_1^2+m_2^2+2E_1 E_2-2\|\boldsymbol{p}_1\|\|\boldsymbol{p}_2\|\cos\theta_{12};$$
$$\left|\frac{\mathrm{d}(E_+,E_-,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{4p_1^2 p_2^2}{E_1 E_2}, \qquad \left|\frac{\mathrm{d}(E_1,E_2,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{2p_1^2 p_2^2}{E_1 E_2}.$$
Therefore,
$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \frac{1}{128\pi^4}\int_{(m_1+m_2)^2}^{\infty}\mathrm{d}s\int_{\sqrt{s}}^{\infty}\mathrm{d}E_+\int_{\min}^{\max}\mathrm{d}E_-, \tag{1.9}$$
where the boundary of E_- is given by
$$\cos\theta_{12} = \frac{E_+^2-E_-^2+2\left(m_1^2+m_2^2-s\right)}{\sqrt{(E_++E_-)^2-4m_1^2}\sqrt{(E_+-E_-)^2-4m_2^2}} \in [-1,1]$$
$$\therefore \quad \left|E_- - \frac{m_1^2-m_2^2}{s}E_+\right| \le \sqrt{E_+^2-s}\cdot\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = 2p\sqrt{\frac{E_+^2-s}{s}}.$$

Two-body phase space with momentum conservation   As a general representation in any frame,
$$\overline{\mathrm{d}\Pi^2} = \frac{\mathrm{d}p_1\,\mathrm{d}\Omega\,p_1^2}{16\pi^2}\frac{\delta(E_0-\sqrt{m_1^2+p_1^2}-\sqrt{m_2^2+\|\boldsymbol{P}_0-\boldsymbol{p}_1\|^2})}{E_1 E_2} = \frac{1}{8\pi}\,\mathrm{d}\cos\theta_1\frac{p_1^2}{E_0 p_1-P_0 E_1\cos\theta_1}, \tag{1.10}$$
where the momentum p_1 is given by
$$p_1 = \frac{(E_0^2+m_1^2-m_2^2-P_0^2)P_0\cos\theta_1+E_0\sqrt{\lambda(E_0^2-P_0^2,m_1^2,m_2^2)-4m_1^2 P_0^2\sin^2\theta_1}}{2(E_0^2-P_0^2\cos^2\theta_1)}. \tag{1.11}$$
CM frame result is recovered by setting $E_0=\sqrt{s}$ and $P_0=0$.

1.2. Decay rate and Cross section

As $\langle\text{out}|\text{in}\rangle = (2\pi)^4\,\delta^{(4)}(p_i-p_f)\mathrm{i}\mathcal{M}$ (for in $\ne$ out) and $\langle\boldsymbol{p}|\boldsymbol{p}\rangle = 2E_{\boldsymbol{p}}(2\pi)^3\,\delta^{(3)}(\boldsymbol{0}) = 2E_{\boldsymbol{p}}V$ for one-particle state,
$$\frac{N_{\text{ev}}}{\prod_{\text{in}}N_{\text{particle}}} = \int \mathrm{d}\Pi^{\text{out}}\frac{|\langle\text{out}|\text{in}\rangle|^2}{\langle\text{in}|\text{in}\rangle} = \int \mathrm{d}\Pi^{\text{out}}\frac{(2\pi)^8|\mathcal{M}|^2}{\prod_{\text{in}}(2E)V}\frac{VT}{(2\pi^4)}\delta^{(4)}(p_i-p_f) = VT\int \overline{\mathrm{d}\Pi^{N_f}}\frac{|\mathcal{M}|^2}{\prod_{\text{in}}(2E)V}. \quad (1.12)$$
Therefore, decay rate (at the rest frame) is given by
$$\mathrm{d}\Gamma := \frac{1}{T}\frac{\mathrm{d}N_{\text{ev}}}{N_{\text{particle}}} = \frac{1}{T}VT\overline{\mathrm{d}\Pi^{N_f}}\frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0}\overline{\mathrm{d}\Pi^{N_f}}|\mathcal{M}|^2. \tag{1.13}$$
We also define Lorentz-invariant cross section σ by $N_{\text{ev}} =: (\rho_A v_{\text{Møl}}T\sigma)N_B = (\rho_A v_{\text{Møl}}T\sigma)(\rho_B V)$, or
$$\mathrm{d}\sigma := \frac{\mathrm{d}N_{\text{ev}}}{\rho_A v_{\text{Møl}}TN_B} = \frac{V}{v_{\text{Møl}}T}VT\overline{\mathrm{d}\Pi^{N_f}}\frac{|\mathcal{M}|^2}{2E_A 2E_B V^2} = \frac{1}{2E_A 2E_B\,v_{\text{Møl}}}\overline{\mathrm{d}\Pi^{N_f}}|\mathcal{M}|^2. \tag{1.14}$$
where the Møller parameter $v_{\text{Møl}}$ is equal to $v_{\text{rel}}^{\text{NR}} = \|\boldsymbol{v}_A-\boldsymbol{v}_B\|$ if $\boldsymbol{v}_A /\!/ \boldsymbol{v}_B$ (cf. Ref. [1]). Generally,
$$v_{\text{Møl}} := \frac{\sqrt{(p_A\cdot p_B)^2-m_A^2 m_B^2}}{E_A E_B} = \frac{p_A\cdot p_B}{E_A E_B}v_{\text{rel}} = (1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B)v_{\text{rel}}, \tag{1.15}$$
where v_{rel} is the actual relative velocity
$$v_{\text{rel}} = \sqrt{1-\frac{(1-v_A^2)(1-v_B^2)}{(1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B)^2}} = \frac{\sqrt{\|\boldsymbol{v}_A-\boldsymbol{v}_B\|^2-\|\boldsymbol{v}_A\times\boldsymbol{v}_B\|^2}}{1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B} = \frac{\lambda^{1/2}(s,m_A^2,m_B^2)}{s-(m_A^2+m_B^2)} \ne v_{\text{rel}}^{\text{NR}}. \tag{1.16}$$
(Note that $p_A\cdot p_B/E_A E_B = 1$ if $\boldsymbol{p}_A=0$ or $\boldsymbol{p}_B=0$. Also, Each of v_{rel}, VT, and $E_A E_B v_{\text{Møl}}$ is Lorentz invariant.)

1.3. Three body phase space

The phase-space reduction utilizes the identity [2]

$$1 = \int \frac{\mathrm{d}^4 p_{ij}}{(2\pi)^4} (2\pi)^4 \, \delta^{(4)}(p_{ij} - p_i - p_j) \Theta(p_{ij}^0) \tag{1.17}$$

$$= \int \frac{\mathrm{d}^4 p_{ij}}{(2\pi)^4} \left[\int \frac{\mathrm{d}s}{2\pi} (2\pi) \, \delta(s - p_{ij}^2) \right] (2\pi)^4 \, \delta^{(4)}(p_{ij} - p_i - p_j) \Theta(p_{ij}^0) \tag{1.18}$$

$$= \int \frac{\mathrm{d}^3 \boldsymbol{p}_{ij}}{(2\pi)^3} \frac{\mathrm{d}s}{2\pi} \frac{1}{2p_{ij}^0} (2\pi)^4 \, \delta^{(4)}(p_{ij} - p_i - p_j) \Big|_{p_{ij}^0 = \sqrt{s + \|\boldsymbol{p}_{ij}\|^2}}. \tag{1.19}$$

For three-body phase space,

$$\overline{\mathrm{d}\Pi^3} = \int \mathrm{d}\Pi_1 \frac{\mathrm{d}^4 p_2 \, \mathrm{d}^4 p_3}{(2\pi)^8} (2\pi) \, \delta(p_2^2 - m_2^2)(2\pi) \, \delta(p_3^2 - m_3^2) \Theta(p_2^0) \Theta(p_3^0)(2\pi)^4 \, \delta^{(4)} \left(P - p_1 - p_2 - p_3 \right)$$
$$\times \frac{\mathrm{d}^3 \boldsymbol{p}_{23}}{(2\pi)^3} \frac{\mathrm{d}s_{23}}{2\pi} \frac{1}{2p_{23}^0} (2\pi)^4 \, \delta^{(4)}(p_{23} - p_2 - p_3) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\boldsymbol{p}_{23}\|^2}} \tag{1.20}$$

$$= \int \frac{\mathrm{d}s_{23}}{2\pi} \int \mathrm{d}\Pi_1 \frac{\mathrm{d}^3 \boldsymbol{p}_{23}}{(2\pi)^3} \frac{1}{2p_{23}^0} (2\pi)^4 \, \delta^{(4)}(P - p_1 - p_{23}) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\boldsymbol{p}_{23}\|^2}}$$
$$\times \frac{\mathrm{d}^4 p_2 \, \mathrm{d}^4 p_3}{(2\pi)^8} (2\pi) \, \delta(p_2^2 - m_2^2)(2\pi) \, \delta(p_3^2 - m_3^2) \Theta(p_2^0) \Theta(p_3^0)(2\pi)^4 \, \delta^{(4)}(p_{23} - p_2 - p_3). \tag{1.21}$$

$$= \int \frac{\mathrm{d}s_{23}}{2\pi} \int \mathrm{d}\Pi_1 \frac{\mathrm{d}^3 \boldsymbol{p}_{23}}{(2\pi)^3} \frac{1}{2p_{23}^0} (2\pi)^4 \, \delta^{(4)}(P - p_1 - p_{23}) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\boldsymbol{p}_{23}\|^2}} \times \overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23}) \tag{1.22}$$

and $\overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23})$ is given by Eq. (1.10); explicitly,

$$\overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23}) = \frac{\mathrm{d}\cos\theta_2}{8\pi} \frac{p_2^2}{p_{23}^0 p_2 - \|\boldsymbol{p}_{23}\|\sqrt{p_2^2 + m_2^2}\cos\theta_2}; \tag{1.23}$$

$$p_2 = \frac{(s_{23} + m_2^2 - m_3^2)\|\boldsymbol{p}_{23}\|\cos\theta_2 + p_{23}^0 \sqrt{\lambda(s_{23}, m_2^2, m_3^2) - 4m_2^2\|\boldsymbol{p}_{23}\|^2\sin^2\theta_2}}{2(s_{23} + \|\boldsymbol{p}_{23}\|^2\sin^2\theta_2)}, \tag{1.24}$$

where θ_2 is the angle between $\boldsymbol{p}_{23}$ and $\boldsymbol{p}_2$ (in the lab frame).

If the matrix element to integrate is spherically symmetric, so as $\overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23})|\mathcal{M}|^2$, i.e., it is independent of the angle of $\boldsymbol{p}_{23}$. Then one can simply evaluate $\int \mathrm{d}^3 \boldsymbol{p}_{23}$, which leads to, in the center-of-mass frame,

$$\overline{\mathrm{d}\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \mathbf{0}) = \frac{\mathrm{d}s_{23}\,\mathrm{d}\cos\theta_2}{64\pi^3} \frac{p_1}{\sqrt{s}} \frac{p_2^2}{p_2\sqrt{s_{23} + p_1^2} - p_1\sqrt{p_2^2 + m_2^2}\cos\theta_2} \Big|_{p_1^2 = \lambda(s, m_1^2, s_{23})/4s} = \frac{s}{128\pi^3} \, \mathrm{d}x_1 \, \mathrm{d}x_2, \tag{1.25}$$

where we defined $x_i := 2E_i/\sqrt{s}$. Noting that $s_{23} = s + m_1^2 - 2E_1\sqrt{s} = s(1 - x_1) + m_1^2$ etc.,

$$\overline{\mathrm{d}\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \mathbf{0}) = \frac{1}{128\pi^3} \frac{1}{s} \int_{(m_2 + m_3)^2}^{(\sqrt{s} - m_1)^2} \mathrm{d}s_{23} \int \mathrm{d}s_{13}; \tag{1.26}$$

$$(s_{13})_{\min}^{\max} = \frac{(s + m_3^2 - m_1^2 - m_2^2)^2}{4s_{23}} - \frac{1}{4s_{23}} \left[\lambda^{1/2}(s, m_1^2, s_{23}) \mp \lambda^{1/2}(s_{23}, m_2^2, m_3^2) \right]^2. \tag{1.27}$$

This is equal to the PDG-Eq. (47.23) [3].

3

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation adj. $= (\epsilon^a)^{bc}$.[*1]

$$T_a = \frac{1}{2}\sigma_a, \qquad \mathrm{Tr}(T_aT_b) = \frac{1}{2}\delta_{ab}, \qquad [T_a, T_b] = \mathrm{i}\epsilon^{abc}T^c, \qquad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\overline{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a\epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\overline{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \to \epsilon^{ab}[\exp{(\mathrm{i}g\theta^\alpha T^\alpha)}]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp{(\mathrm{i}g\theta^\alpha T^\alpha)}]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp{(-\mathrm{i}g\theta^\alpha T^{\alpha*})}]^{ab}(\epsilon^{bc}\mathbf{2}^c). \quad (2.1)$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\overline{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation adj. $= \mathbf{8} = (f^a)^{bc}$. Gell-Mann matrices:

$$\lambda_{1-8} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \qquad (2.2)$$

$$\tau_a = \frac{1}{2}\lambda_a, \qquad \mathrm{Tr}(\tau_a\tau_b) = \frac{1}{2}\delta_{ab}, \qquad [\tau_a, \tau_b] = \mathrm{i}f^{abc}\tau^c, \qquad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\mathbf{3}: \quad \phi_a \to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi_b \simeq \phi_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi_b \qquad \overline{\mathbf{3}}: \quad \phi_a \to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi_b$$

$$\phi_a^* \to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi_b^* \simeq \phi_a^* - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi_b^* \qquad \phi_a^* \to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi_b^* \simeq \phi_a^* + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi_b^*$$

$$= \phi_b^*[\exp(-\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ba} \simeq \phi_a^* - \mathrm{i}g\theta^\alpha\phi_b^*\tau^\alpha_{ba}$$

[*1]We do not distinguish sub- and superscripts for gauge indices.

3. Spinors

$$(\overline{\psi_1}\psi_2)^* = (\psi_2)^\dagger (\overline{\psi_1})^\dagger = \overline{\psi_2}\psi_1. \tag{3.1}$$

3.1. Convention ♣TODO:WIP!♣

First we prepare a vector x^μ and a symmetric matrix $\eta^{\mu\nu}$, which we call "contravariant vector" x^μ and the metric $\eta^{\mu\nu}$. Then we perform a Lorentz transformation on x^μ to obtain $(x')^\mu$, with which we can define a matrix $\Lambda(\boldsymbol{va}, \boldsymbol{\theta})^\mu_\nu$ through $\Lambda^\mu{}_\nu x^\nu = (x')^\mu$.

We then consider Λs for infinitesimal transformations and define $\boldsymbol{S}$, $\boldsymbol{J}$, and $\boldsymbol{K}$ by

$$\Lambda^\mu{}_\nu \simeq \delta^\mu_\nu - \mathrm{i}(\boldsymbol{\theta}\cdot\boldsymbol{J}^\mu{}_\nu + \boldsymbol{\beta}\cdot\boldsymbol{K}^\mu{}_\nu) \simeq \delta^\mu_\nu - \frac{\mathrm{i}}{2}\left[\Lambda^{\alpha\beta}S_{\alpha\beta}\right]^\mu{}_\nu \tag{3.2}$$

Imposing "Lorentz condition" (♣TODO:what?♣), we get the expression for $S = \mathrm{i}(\delta\cdots)$ and $[J^i, J^j] = \cdots$; further, we get $\Lambda^\mu_\nu = \exp(-\mathrm{i}\boldsymbol{\theta}\cdot\boldsymbol{J} - \mathrm{i}\boldsymbol{\xi}\cdot\boldsymbol{K})$, $\boldsymbol{\theta} = (\theta_{23}, \theta_{31}, \theta_{12})$, $\boldsymbol{\xi} = \hat{\boldsymbol{v}}\tanh^{-1}\|\boldsymbol{v}\| = (\theta^{10}, \theta^{20}, \theta^{30})$; $J = (S_{23}, S_{31}, S_{12})$, $K = (S^{01}, S^{02}, S^{03})....$?

Lorentz transformation with a rotation θ around an axis $\hat{\boldsymbol{\theta}}$ and a boost $\boldsymbol{v}$ are given by

$$\Lambda = \exp\left[-\mathrm{i}(\boldsymbol{\theta}\cdot\boldsymbol{J} + \boldsymbol{\beta}\cdot\boldsymbol{K})\right]; \qquad \boldsymbol{\theta} := \theta\hat{\boldsymbol{\theta}}, \quad \boldsymbol{\beta} := \hat{\boldsymbol{v}}\tanh^{-1}\|\boldsymbol{v}\|, \tag{3.3}$$

♣TODO:check!♣

Lorentz transformation (infinitesimal): $\Lambda = \begin{pmatrix} 0 & & \boldsymbol{\beta}^{\mathrm{T}} & \\ & 0 & -\theta_z & \theta_y \\ \boldsymbol{\beta} & \theta_z & 0 & -\theta_x \\ & -\theta_y & \theta_x & 0 \end{pmatrix}$

$[J_{\mu\nu}]^\alpha{}_\beta = \mathrm{i}(\delta^\alpha_\mu \eta_{\nu\beta} - \delta^\alpha_\nu \eta_{\mu\beta})$

Lorentz tensor $M^{\mu_1\mu_2\cdots\mu_n} \propto \bar{\sigma}^{\mu_1\dot{\beta}_1\alpha_1}\cdots M_{\alpha_1\cdots\dot{\beta}_1\cdots}$

Especially $V^\mu =: \frac{1}{2}\bar{\sigma}^{\mu\dot{\beta}\alpha}V_{\alpha\dot{\beta}}$, $V_{\alpha\dot{\beta}} = V^\mu \sigma_{\mu\alpha\dot{\beta}}$; hermite $V_{\alpha\dot{\beta}} \Leftrightarrow real V^\mu$.

$$(V^{\mathrm{T}})_{\alpha\dot{\beta}} = V_{\beta\dot{\alpha}}, \text{♣TODO:(correct? possibly wrong dot-positions?)♣} \tag{3.4}$$

$$(V^*)_{\dot{\alpha}\beta} := (V_{\alpha\dot{\beta}})^*, \tag{3.5}$$

$$(V^\dagger)_{\alpha\dot{\beta}} := (V_{\beta\dot{\alpha}})^* = (V^*)_{\dot{\beta}\alpha} \tag{3.6}$$

♣TODO:anyway not very sure about the reasoning; though my old note says like this...♣

In general, metric is symmetric.

$$(\Lambda^{-1})^\mu{}_\nu = \eta_{\nu\rho}\Lambda^\rho{}_\sigma(\eta^{-1})^{\sigma\mu} =: \Lambda_\nu{}^\mu \tag{3.7}$$

4. Loop calculation

Notation follows `LoopTools` [4]; capital Ms and Ps respectively denote squared masses and momenta.

Passarino–Veltman scalar integrals

$$A_0(M)/M = \Delta_\epsilon + \log \mu^2 + 1 - \log M, \tag{4.1}$$

$$B_0(P, M_0, M_1) = \Delta_\epsilon + \log \mu^2 - \int_0^1 \mathrm{d}x \, \log \left[-x(1-x)P + xM_1 + (1-x)M_0 \right] \tag{4.2}$$

$$C_0(P_1, P_2, P_3, M_1, M_2, M_3) = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \, \frac{x}{Q_1} \tag{4.3}$$

$$= \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{1}{Q_2}; \tag{4.4}$$

$$Q_1 = x(1-x)(1-y)P_2 + x^2 y(1-y)P_3 + x(1-x)yP_1 - xyM_1 - (1-x)M_2 - x(1-y)M_3,$$

$$Q_2 = -P_2 x^2 - P_1 y^2 + (P_1 + P_2 - P_3)xy + (P_2 - M_2 + M_3)x + (M_2 - M_1 + P_3 - P_2)y - M_3.$$

Kinematical invariance:

$$B_0(P, M_0, M_1) = B_0(P, M_1, M_0), \quad C_0(P_1, P_2, P_3, M_1, M_2, M_3) = C_0(P_2, P_3, P_1, M_2, M_3, M_1)$$
$$= C_0(P_1, P_3, P_2, M_2, M_1, M_3) \tag{4.5}$$

4.1. Passarino–Veltman scalar integrals

See `calculator/loop/PaVeAnalytic.wl` for validation. We use the notation [4,5]

$$\Delta_\epsilon = \frac{2}{4-d} - \gamma + \log 4\pi \equiv \texttt{GetDelta[]} \quad (= 0 \text{ in } \overline{\text{MS}}), \qquad\qquad \mu^2 \equiv \texttt{GetMudim[]}, \qquad\qquad (4.6)$$

where μ is introduced due to the different mass dimension of vector and spinor fields in d-dimensional theory:

$$[A_\mu] = 1 - \frac{4-d}{2}, \quad [\psi] = \frac{3}{2} - \frac{4-d}{2}, \quad [\text{gauge couplings}] = \frac{4-d}{2} \quad \Rightarrow \quad e = (e)_{\text{4-dim.}} \mu^{(4-d)/2}. \qquad (4.7)$$

The analytic form of scalar integrals are given in Refs. [5–7].

5. Cosmology

5.1. Cosmology formulae

Assuming that σ depends only on $s = (p_A + p_B)^2$, ♣TODO:(or always true?)♣

$$\int \frac{\mathrm{d}^3 \boldsymbol{p}_A}{(2\pi)^3} \frac{\mathrm{d}^3 \boldsymbol{p}_B}{(2\pi)^3} \, \mathrm{e}^{-E_A/T} \, \mathrm{e}^{-E_B/T} \sigma(s) v_{\mathrm{M\o l}} \tag{5.1}$$

$$= 4 \int \mathrm{d}\Pi_A \, \mathrm{d}\Pi_B \, \mathrm{e}^{-(E_A+E_B)/T} \sigma(s) \sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2} \tag{5.2}$$

$$= \frac{1}{32\pi^4} \int_{(m_A+m_B)^2}^{\infty} \mathrm{d}s \int_{\sqrt{s}}^{\infty} \mathrm{d}E_+ \left[\frac{2\sqrt{E_+^2 - s}}{s} \lambda^{1/2}(s, m_A^2, m_B^2) \right] \mathrm{e}^{-E_+/T} \sigma(s) \left[\frac{1}{2} \lambda^{1/2}(s, m_A^2, m_B^2) \right] \tag{5.3}$$

$$= \frac{T}{32\pi^4} \int_{(m_A+m_B)^2}^{\infty} \mathrm{d}s \, s^{3/2} \, K_1\left(\frac{\sqrt{s}}{T}\right) \lambda\left(1, \frac{m_A^2}{s}, \frac{m_B^2}{s}\right) \sigma(s). \tag{5.4}$$

6. Standard Model

(summary page)

6.1. Particle content and convention

6.2. Lagrangian

6.3. Higgs mechanism

A general expression for composing a Dirac fermion from $\psi_{\mathrm{L}}(T_{3\mathrm{L}}, Y_{\mathrm{L}})$ and $\psi_{\mathrm{R}}(T_{3\mathrm{R}}, Y_{\mathrm{R}})$ is given by

$$\left(g_2 \slashed{W}_3 T_{3\mathrm{L}} + g_Y \slashed{B} Y_{\mathrm{L}}\right) P_{\mathrm{L}} + \left(g_2 \slashed{W}_3 T_{3\mathrm{R}} + g_Y \slashed{B} Y_{\mathrm{R}}\right) P_{\mathrm{R}} \tag{6.1}$$

$$= \left[\left(|e|\slashed{A} + g_Z \mathrm{c}_{\mathrm{w}}^2 \slashed{Z}\right) T_{3\mathrm{L}} + \left(|e|\slashed{A} - g_Z \mathrm{s}_{\mathrm{w}}^2 \slashed{Z}\right) Y_{\mathrm{L}}\right] P_{\mathrm{L}} + (\text{right}) \tag{6.2}$$

$$= \frac{T_{3\mathrm{L}} + T_{3\mathrm{R}} + Y_{\mathrm{L}} + Y_{\mathrm{R}}}{2} |e|\slashed{A} + \frac{T_{3\mathrm{L}}\mathrm{c}_{\mathrm{w}}^2 - Y_{\mathrm{L}}\mathrm{s}_{\mathrm{w}}^2 + T_{3\mathrm{R}}\mathrm{c}_{\mathrm{w}}^2 - Y_{\mathrm{R}}\mathrm{s}_{\mathrm{w}}^2}{2} g_Z \slashed{Z}$$
$$+ \frac{-T_{3\mathrm{L}} - Y_{\mathrm{L}} + T_{3\mathrm{R}} + Y_{\mathrm{R}}}{2} |e|\slashed{A}\gamma_5 + \frac{-\mathrm{c}_{\mathrm{w}}^2 T_{3\mathrm{L}} + \mathrm{s}_{\mathrm{w}}^2 Y_{\mathrm{L}} + \mathrm{c}_{\mathrm{w}}^2 T_{3\mathrm{R}} - \mathrm{s}_{\mathrm{w}}^2 Y_{\mathrm{R}}}{2} g_Z \slashed{Z}\gamma_5. \tag{6.3}$$

In the SM, $T_{3\mathrm{L}} + Y_{\mathrm{L}} = Y_{\mathrm{R}} =: Q$ and $T_{3\mathrm{R}} = 0$ lead to

$$Q|e|\slashed{A} + g_Z \slashed{Z}\left(T_{3\mathrm{L}} P_{\mathrm{L}} - Q\mathrm{s}_{\mathrm{w}}^2\right). \tag{6.4}$$

6.4. Lagrangian in mass eigenstates

6.5. CKM matrix and Yukawa convention

Our convention is, with $Y = UY^{\mathrm{diag}}V^\dagger$,

$$\mathcal{L} \supset \overline{U}Y_u H P_\mathrm{L} Q - \overline{D}Y_d H^\dagger P_\mathrm{R} Q - \overline{E}Y_e H^\dagger P_\mathrm{R} L + \text{h.c.} \tag{6.5}$$

$$= \overline{U_i}Y_{uij}\epsilon^{ab}H^a P_\mathrm{L}Q_j^b - \overline{D_i}Y_{dij}H^{a*}P_\mathrm{L}Q_j^a - \overline{E_i}Y_{eij}H^{a*}P_\mathrm{L}L_j^a + \text{h.c.} \tag{6.6}$$

$$= \epsilon^{ab}\overline{U}U_u Y_u^{\mathrm{diag}}H^a P_\mathrm{L}V_u^\dagger Q^b - \overline{D}U_d Y_d^{\mathrm{diag}}H^{a*}P_\mathrm{R}V_d^\dagger Q^a - \overline{E}U_e Y_e^{\mathrm{diag}}H^{a*}P_\mathrm{R}V_e^\dagger L + \text{h.c.} \tag{6.7}$$

$$\rightsquigarrow -\frac{v}{\sqrt{2}}\overline{U}U_u Y_u^{\mathrm{diag}}V_u^\dagger P_\mathrm{L}Q^1 - \frac{v}{\sqrt{2}}\overline{D}U_d Y_d^{\mathrm{diag}}V_d^\dagger P_\mathrm{L}Q^2 - \frac{v}{\sqrt{2}}\overline{E}U_e Y_e^{\mathrm{diag}}V_e^\dagger P_\mathrm{L}L^2 + \text{h.c.} \tag{6.8}$$

Noting that $(\overline{\psi_A}P_\mathrm{L}\psi_B)^* = \overline{\psi_B}P_\mathrm{R}\psi_A$, this definition is equivalent to

$$\mathcal{L} \supset -\overline{Q^a}Y_u^\dagger\epsilon^{ab}H^{b*}P_\mathrm{R}U - \overline{Q^a}Y_d^\dagger H^a P_\mathrm{R}D - \overline{L^a}Y_e^\dagger H^a P_\mathrm{R}E + \text{h.c.} \tag{6.9}$$

Moving to the CKM basis, the weak interaction is amended as

$$\mathcal{L} \supset \overline{Q}\mathrm{i}\gamma^\mu(-\mathrm{i}g_2 W_\mu)P_\mathrm{L}Q \supset \frac{g_2}{\sqrt{2}}\left[\overline{Q^1}\slashed{W}^+ P_\mathrm{L}Q^2 + \overline{Q^2}\slashed{W}^- P_\mathrm{L}Q^1\right] = \frac{g_2}{\sqrt{2}}\left[\overline{\hat{Q}^1}V_u^\dagger\slashed{W}^+ P_\mathrm{L}V_d\hat{Q}^2 + \overline{\hat{Q}^2}V_d^\dagger\slashed{W}^- P_\mathrm{L}V_u\hat{Q}^1\right], \tag{6.10}$$

where hatted fields are in the CKM basis, and we define, with $\mathrm{s}_{ij} > 0$ and $\mathrm{c}_{ij} > 0$,

$$V_{\mathrm{CKM}} = V_u^\dagger V_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & & \\ & \mathrm{c}_{23} & \mathrm{s}_{23} \\ & -\mathrm{s}_{23} & \mathrm{c}_{23} \end{pmatrix}\begin{pmatrix} \mathrm{c}_{13} & & \mathrm{s}_{13}\,\mathrm{e}^{-\mathrm{i}\delta} \\ & 1 & \\ -\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & & \mathrm{c}_{13} \end{pmatrix}\begin{pmatrix} \mathrm{c}_{12} & \mathrm{s}_{12} & \\ -\mathrm{s}_{12} & \mathrm{c}_{12} & \\ & & 1 \end{pmatrix}\mathrm{e}^{\mathrm{i}\Theta}$$

$$= \begin{pmatrix} \mathrm{c}_{12}\mathrm{c}_{13} & \mathrm{s}_{12}\mathrm{c}_{13} & \mathrm{s}_{13}\,\mathrm{e}^{-\mathrm{i}\delta} \\ -\mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{s}_{23}\mathrm{c}_{13} \\ \mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & -\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{23}\mathrm{c}_{13} \end{pmatrix}\mathrm{e}^{\mathrm{i}\Theta}. \tag{6.11}$$

Here, a 3×3 unitary matrix has three angles and six phases, among which five phases are removed by the fermion rotation, but another phase Θ is introduced due to the Θ-term removal.♣TODO:link♣

PDG convention   (consistent in 2006§11 and 2018§12)

$$\mathcal{L} \supset -Y_{ij}^d\overline{Q_{Li}^I}\phi d_{Rj}^I - Y_{ij}^u\overline{Q_{Li}^I}\epsilon\phi^* u_{Rj}^I, \quad Y^{\mathrm{diag}} = V_L Y V_R^\dagger, \quad V_{\mathrm{CKM}} = V_L^u V_L^{d\dagger}. \tag{6.12}$$

So, $Y^u = Y_u^\dagger$, $Y^d = Y_d^\dagger$; $Y^{\mathrm{diag}} = V_R Y^\dagger V_L^\dagger = V_R Y V_L^\dagger$ leads $V_L = V^\dagger$, and $V_{\mathrm{CKM}} = V_u^\dagger V_d = V_{\mathrm{CKM}}$.

SLHA2 convention   (0801.0045)

$$W \supset \epsilon_{ab}\left[(Y_E)_{ij}H_1^a L_i^b \bar{E}_j + (Y_D)_{ij}H_1^a Q_i^b \bar{D}_j + (Y_U)_{ij}H_2^b Q_i^a \bar{U}_j\right]; \tag{6.13}$$

$$\mathcal{L} \supset -\epsilon_{ab}\left[(Y_E)_{ij}H_1^a \psi_{Li}^b \psi_{\bar{E}j} + (Y_D)_{ij}H_1^a \psi_{Qi}^b \bar{\psi}_{\bar{D}j} + (Y_U)_{ij}H_2^b \psi_{Qi}^a \bar{\psi}_{\bar{U}j}\right] \tag{6.14}$$

$$\rightsquigarrow -\left[\psi_{\bar{E}}v_\mathrm{d}Y_E^\mathrm{T}\psi_L^2 + \psi_{\bar{D}}v_\mathrm{d}Y_D^\mathrm{T}\psi_Q^2 + \psi_{\bar{U}}v_\mathrm{u}Y_U^\mathrm{T}\psi_Q^1\right]; \quad Y^{\mathrm{diag}} = U^\dagger Y^\mathrm{T}V, \quad V_{\mathrm{CKM}} = V_u^\dagger V_d. \tag{6.15}$$

Hence, $Y_E = Y_e^\mathrm{T}$, $Y_D = Y_d^\mathrm{T}$, $Y_U = Y_u^\mathrm{T}$; $Y^{\mathrm{diag}} = U^\dagger Y V$, $V = V$ and $V_{\mathrm{CKM}} = V_{\mathrm{CKM}}$.

Wolfenstein parameterization   The CKM matrix is precisely written in terms of λ, A, and $\bar{\rho} + \mathrm{i}\bar{\eta}$.

$$\lambda := \mathrm{s}_{12} = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad A := \frac{\mathrm{s}_{23}}{\lambda^2} = \lambda^{-1}\left|\frac{V_{cb}}{V_{us}}\right|, \quad \bar{\rho} + \mathrm{i}\bar{\eta} := \frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \tag{6.16}$$

They are independent of the phase convention and used for SLHA2 input, i.e., `VCKMIN` should contain $(\lambda, A, \bar{\rho}, \bar{\eta})$.

Also, $\bar{\rho} + \mathrm{i}\eta$ is approximately written by

$$R = \rho + \mathrm{i}\eta := \frac{s_{13}\,\mathrm{e}^{\mathrm{i}\delta}}{A\lambda^3} = \frac{V_{ub}^*}{A\lambda^3}\frac{V_{ud}}{|V_{ud}|} = \frac{(\bar{\rho} + \mathrm{i}\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1 - \lambda^2}\left[1 - A^2\lambda^4(\bar{\rho} + \mathrm{i}\bar{\eta})\right]} = (\bar{\rho} + \mathrm{i}\bar{\eta})\left(1 + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4)\right), \tag{6.17}$$

with which

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3 R^* \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - R) & -A\lambda^2 & 1 \end{pmatrix}\mathrm{e}^{\mathrm{i}\Theta} + \begin{pmatrix} \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^7) & 0 \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^8) \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^4) \end{pmatrix}. \tag{6.18}$$

6.6. General Higgs doublet and Nambu–Goldstone bosons

In linear parameterization,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathrm{i}\sqrt{2}\phi^+ \\ v + h + \mathrm{i}\phi_3 \end{pmatrix}, \qquad D_\mu H = \begin{pmatrix} \mathrm{i}\partial_\mu\phi^+ - \frac{\mathrm{i}g_2}{2}(v + h + \mathrm{i}\phi_3)W_\mu^+ + \left(|e|A_\mu + \frac{c_w^2 - s_w^2}{2}g_Z Z_\mu\right)\phi^+ \\ \partial_\mu(h + \mathrm{i}\phi_3)/\sqrt{2} + \frac{\mathrm{i}g_Z}{2}Z_\mu(v + h + \mathrm{i}\phi_3)/\sqrt{2} + g_2 W_\mu^- \phi^+/\sqrt{2} \end{pmatrix}; \tag{6.19}$$

$$
\begin{aligned}
|D_\mu H|^2 =\ & \frac{(\partial_\mu h)^2 + (\partial_\mu \phi_3)^2}{2} + \partial_\mu \phi^+ \partial^\mu \phi^- + \frac{(v + h)^2}{8}(2g_2^2 W^{+\mu} W_\mu^- + g_Z^2 Z^\mu Z_\mu) \\
& + \frac{\partial^\mu h}{2}\left[g_2 W_\mu^+ \phi^- + g_2 W_\mu^- \phi^+ - g_Z Z_\mu \phi_3\right] + \frac{\partial^\mu \phi_3}{2}\left[g_Z(v + h)Z_\mu + \mathrm{i}g_2(W_\mu^+ \phi^- - W_\mu^- \phi^+)\right] \\
& + \left\{\frac{\partial^\mu \phi^+}{2}\left[-g_2(v + h - \mathrm{i}\phi_3)W_\mu^- + (2|e|A_\mu + (c_w^2 - s_w^2)g_Z Z_\mu)\mathrm{i}\phi^-\right] + \mathrm{H.c.}\right\} \\
& + \frac{\mathrm{i}g_2(v + h)}{2}(|e|A^\mu - g_Z s_w^2 Z^\mu)(W_\mu^- \phi^+ - W_\mu^+ \phi^-) + \frac{g_Z \phi_3}{2}(|e|A^\mu - g_Z s_w^2 Z^\mu)(W_\mu^- \phi^+ + W_\mu^+ \phi^-) \\
& + \frac{\phi_3^2}{8}\left(2g_2^2 W^{+\mu} W_\mu^- + g_Z^2 Z^\mu Z_\mu\right) + \frac{\phi^+ \phi^-}{4}\left[2g_2^2 W^{+\mu} W_\mu^- + (2|e|A_\mu + g_Z(c_w^2 - s_w^2)Z_\mu)^2\right];
\end{aligned}
\tag{6.20}
$$

$$V = \lambda|H|^4 - \mu^2|H|^2 = \frac{\lambda}{4}h^4 + \lambda v h^3 + \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}v^4 + \frac{\lambda}{4}(2\phi^+ \phi^- + \phi_3^2)^2 + \frac{\lambda}{2}(h^2 + 2vh)(2\phi^+ \phi^- + \phi_3^2), \tag{6.21}$$

where $v = \mu/\sqrt{\lambda} \sim 246\,\mathrm{GeV}$, $\lambda \sim 0.13$, and $\mu \sim 89\,\mathrm{GeV}$. In exponential parameterization,

$$H = \frac{1}{\sqrt{2}}\exp\left(\frac{\mathrm{i}}{v}\sigma_i \varphi_i\right)\begin{pmatrix} 0 \\ v + h \end{pmatrix}, \tag{6.22}$$

$$D_\mu H = \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\sigma_i \varphi_i/v}\left[\mathrm{i}\sigma_i \partial_\mu \varphi_i \begin{pmatrix} 0 \\ 1 + h/v \end{pmatrix} + \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix}\right] + \frac{1}{\sqrt{2}}\frac{-\mathrm{i}}{2}(g_2 \sigma_i W_{i\mu} + g_Y B_\mu)\mathrm{e}^{\mathrm{i}\sigma_i \varphi_i/v}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{6.23}$$

$$= \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\sigma_i \varphi_i/v}\left[\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + \mathrm{i}\left(\sigma_i \partial_\mu \varphi_i - \frac{g_2 v}{2}\mathrm{e}^{-\mathrm{i}\sigma_j \varphi_j/v}\sigma_i \mathrm{e}^{\mathrm{i}\sigma_k \varphi_k/v}W_{i\mu} - \frac{g_Y v}{2}B_\mu\right)\begin{pmatrix} 0 \\ 1 + h/v \end{pmatrix}\right], \tag{6.24}$$

$$V = \lambda|H|^4 - \mu^2|H|^2 = \frac{\lambda}{4}h^4 + \lambda v h^3 + \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}v^4. \tag{6.25}$$

These expressions have gauge degeneracy (i.e., without gauge-fixing terms and ghost terms) and thus not ready for calculations. If we choose the unitarity gauge, $\phi_i(x) = 0$,

$$\mathcal{L}_H = |D_\mu H|^2 - V(H) = \frac{1}{2}(\partial_\mu h)^2 - \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}h^4 - \lambda v h^3 + \frac{(v + h)^2}{8}(2g_2^2 W^{+\mu} W_\mu^- + g_Z^2 Z^\mu Z_\mu) + \frac{\lambda}{4}v^4. \tag{6.26}$$

6.7. Values of SM parameters

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7. Supersymmetry with $\eta = \mathrm{diag}(+,-,-,-)$

Convention Our convention follows DHM (except for D_μ):

$$\eta = \mathrm{diag}(1,-1,-1,-1); \quad \epsilon^{0123} = -\epsilon_{0123} = 1, \quad \epsilon^{12} = \epsilon_{21} = \epsilon^{\dot1\dot2} = \epsilon_{\dot2\dot1} = 1 \quad \left(\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma\right),$$

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta}, \quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta};$$

$$\sigma^\mu_{\alpha\dot\alpha} := (\mathbf{1},\boldsymbol\sigma)_{\alpha\dot\alpha}, \quad \sigma^{\mu\nu}{}_\alpha{}^\beta := \frac{i}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)_\alpha{}^\beta, {}^{*2} \quad \left(\sigma^\mu_{\alpha\dot\beta} = \epsilon_{\alpha\delta}\epsilon_{\dot\beta\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \quad \bar\sigma^{\mu\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\delta}\epsilon^{\beta\gamma}\sigma^\mu_{\gamma\dot\delta}\right)$$

$$\bar\sigma^{\mu\dot\alpha\alpha} := (\mathbf{1},-\boldsymbol\sigma)^{\dot\alpha\alpha}, \quad \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta} := \frac{i}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta}, {}^{*2}$$

$$(\psi\xi) := \psi^\alpha\xi_\alpha, \quad (\bar\psi\bar\chi) := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}; \qquad \frac{\mathrm{d}}{\mathrm{d}\theta^\alpha}(\theta\theta) := \theta_\alpha \quad \text{[left derivative]}.$$

Especially, spinor-index contraction is done as ${}^\alpha{}_\alpha$ and ${}_{\dot\alpha}{}^{\dot\alpha}$ except for ϵ_{ab} (which always comes from left). Noting that complex conjugate reverses spinor order: $(\psi^\alpha\xi^\beta)^* := (\xi^\beta)^*(\psi^\alpha)^*$,

$$\bar\psi^{\dot\alpha} := (\psi^\alpha)^*, \quad \epsilon^{\dot a\dot b} := (\epsilon^{ab})^*, \quad (\psi\chi)^* = (\bar\psi\bar\chi),$$

$$(\sigma^\mu_{\alpha\dot\beta})^* = \bar\sigma^\mu_{\dot\alpha\beta} = \epsilon_{\beta\delta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \quad (\sigma^{\mu\nu})^{\dagger\alpha}{}_\beta = \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}, \quad (\sigma^{\mu\nu}{}_\alpha{}^\beta)^* = \bar\sigma^{\mu\nu\dot\beta}{}_{\dot\alpha} = \bar\sigma^{\mu\nu}{}_{\dot\alpha}{}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\delta},$$

$$(\bar\sigma^{\mu\dot\alpha\beta})^* = \sigma^{\mu\alpha\dot\beta} = \epsilon^{\dot\beta\dot\delta}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\delta}, \quad (\bar\sigma^{\mu\nu})^\dagger{}_{\dot\alpha}{}^{\dot\beta} = \sigma^{\mu\nu}{}_\alpha{}^\beta, \quad (\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta})^* = \sigma^{\mu\nu}{}_\beta{}^\alpha = \bar\sigma^{\mu\nu\alpha}{}_\beta = \epsilon_{\beta\delta}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\delta.$$

Contraction formulae

$$\theta^\alpha\theta^\beta = -\tfrac{1}{2}(\theta\theta)\epsilon^{\alpha\beta} \quad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac{1}{2}(\bar\theta\bar\theta)\epsilon^{\dot\alpha\dot\beta} \quad (\theta\xi)(\theta\chi) = -\tfrac{1}{2}(\theta\theta)(\xi\chi) \quad (\theta\sigma^\nu\bar\theta)\theta^\alpha = \tfrac{1}{2}(\theta\theta)(\bar\theta\bar\sigma^\nu)^\alpha$$

$$\theta_\alpha\theta_\beta = \tfrac{1}{2}(\theta\theta)\epsilon_{\alpha\beta} \quad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac{1}{2}(\bar\theta\bar\theta)\epsilon_{\dot\alpha\dot\beta} \quad (\bar\theta\bar\xi)(\bar\theta\bar\chi) = -\tfrac{1}{2}(\bar\theta\bar\theta)(\bar\xi\bar\chi) \quad (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\tfrac{1}{2}(\theta\sigma^\nu)_{\dot\alpha}(\bar\theta\bar\theta)$$

$$\theta^\alpha\theta_\beta = \tfrac{1}{2}(\theta\theta)\delta^\alpha_\beta \quad \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac{1}{2}(\bar\theta\bar\theta)\delta^{\dot\alpha}_{\dot\beta} \quad (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \tfrac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu}$$

$$(\theta\sigma^\mu\bar\sigma^\nu\theta) = (\theta\theta)\eta^{\mu\nu} \quad (\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta) = (\bar\theta\bar\theta)\eta^{\mu\nu} \quad (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \tfrac{1}{2}(\bar\theta\bar\theta)(\sigma^\mu\bar\sigma^\nu\theta)_\alpha$$

$$\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} - 2i\sigma^{\mu\nu} \qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho + \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\left(\sigma^\mu\eta^{\rho\nu} - \sigma^\nu\eta^{\mu\rho} + \sigma^\rho\eta^{\mu\nu}\right)$$

$$\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} - 2i\bar\sigma^{\mu\nu} \qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2i\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}\,(\sigma^\mu\bar\sigma^\nu) = \mathrm{Tr}\,(\bar\sigma^\mu\sigma^\nu) = 2\eta^{\mu\nu} \qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho + \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = 2\left(\bar\sigma^\mu\eta^{\rho\nu} - \bar\sigma^\nu\eta^{\mu\rho} + \bar\sigma^\rho\eta^{\mu\nu}\right)$$

$$\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu^{\dot\beta\beta} = 2\delta^{\dot\beta}_{\dot\alpha}\delta^\alpha_\beta \qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2i\bar\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$

$$\sigma_{\mu\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} \qquad \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\dot\beta\dot\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\beta}$$

$$\bar\sigma_\mu^{\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} \qquad \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\beta\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\dot\alpha\dot\gamma}\sigma^\mu_{\beta\dot\gamma}$$

$$\mathrm{Tr}\,(\sigma^{\mu\nu}) = \mathrm{Tr}\,(\bar\sigma^{\mu\nu}) = 0 \qquad \mathrm{Tr}\,(\sigma^{\mu\nu}\sigma^{\rho\sigma}) = \tfrac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \tfrac{1}{2}i\epsilon^{\mu\nu\rho\sigma}$$

$$\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu} \qquad \sigma^{\mu\nu} = -\sigma^{\nu\mu} \qquad \mathrm{Tr}\,(\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma}) = \tfrac{1}{2}i\epsilon^{\mu\nu\rho\sigma} + \tfrac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$

$$\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = -2i\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon_{\alpha\beta} - 2i\sigma^{\mu\nu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}$$

$$\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon^{\dot\beta\dot\alpha} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 4\sigma^{\rho\mu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\sigma\nu\dot\gamma}{}_{\dot\beta}\eta_{\rho\sigma} + \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\eta^{\mu\nu}$$

$$\bar\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = -2i\bar\sigma^{\mu\nu} \qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} - \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = -2i\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\epsilon^{\alpha\beta} - 2i\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon^{\dot\alpha\dot\beta}$$

$$\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 2i\sigma^{\mu\nu} \qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} + \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 4\epsilon^{\alpha\gamma}\sigma^{\sigma\nu}{}_\gamma{}^\beta\bar\sigma^{\rho\mu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\eta_{\rho\sigma} + \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\eta^{\mu\nu}$$

$$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi \quad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi = \bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi \quad \xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi \quad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho\chi = -\chi\sigma^\rho\bar\sigma^\nu\sigma^\mu\bar\xi$$

$$(\xi\sigma^\mu\bar\chi)^* = \chi\sigma^\mu\bar\xi \quad (\bar\xi\bar\sigma^\mu\chi)^* = \bar\chi\bar\sigma^\mu\xi \quad (\bar\chi\bar\sigma^\mu\sigma^\nu\bar\xi)^* = \xi\sigma^\nu\bar\sigma^\mu\chi \quad (\xi[\sigma s]\chi)^* = \bar\chi[\sigma s_{\text{reversed}}]\bar\xi$$

$$(\xi\chi)\psi^\alpha = -(\psi\xi)\chi^\alpha - (\psi\chi)\xi^\alpha \quad (\xi\chi)\bar\psi_{\dot\alpha} = \tfrac{1}{2}(\xi\sigma^\mu\bar\psi)(\chi\sigma_\mu)_{\dot\alpha}$$

$$i\psi_i\sigma^\mu\partial_\mu\bar\psi_j = -i\partial_\mu\bar\psi_j\bar\sigma^\mu\psi_i \equiv i\bar\psi_j\bar\sigma^\mu\partial_\mu\psi_i = -i\partial_\mu\psi_i\sigma^\mu\bar\psi_j$$

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^{*2}As the definition of $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are not unified in literature, they are not used in this CheatSheet except for this page.

Superfields

$$\Phi = \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar{\theta}) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar{\theta})\theta^2 - \frac{\theta^4}{4}\partial^2\phi(x),\tag{7.1}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar{\psi}(x)\bar{\theta} + F^*(x)\bar{\theta}^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar{\theta}) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar{\psi}(x)]\bar{\theta}^2 - \frac{\theta^4}{4}\partial^2\phi^*(x),\tag{7.2}$$

$$V = (\bar{\theta}\bar{\sigma}^\mu\theta)A_\mu(x) + \bar{\theta}^2\theta\lambda(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \frac{\theta^4}{2}D(x) \qquad \text{(in Wess-Zumino supergauge).}\tag{7.3}$$

Without gauge symmetries

$$\mathcal{L} = \Phi_i^*\Phi_i\Big|_{\theta^4} + \left(W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right);\tag{7.4}$$

$$\Phi_i^*\Phi_i\Big|_{\theta^4} = (\partial_\mu\phi_i^*)(\partial^\mu\phi_i) + \mathrm{i}\bar{\psi}_i\sigma^\mu\partial_\mu\psi_i + F_i^*F_i,\tag{7.5}$$

$$W(\Phi_i)\Big|_{\theta^2} \rightsquigarrow \left[\kappa_i\Phi_i + m_{ij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k\right]_{\theta^2}$$
$$= \kappa_iF_i + m_{ij}\left(-\psi_i\psi_j + F_i\phi_j + \phi_iF_j\right)$$
$$+ y_{ijk}\Big[-(\psi_i\psi_j\phi_k + \psi_i\phi_j\psi_k + \phi_i\psi_j\psi_k) + \phi_i\phi_jF_k + \phi_iF_j\phi_k + F_i\phi_j\phi_k\Big].\tag{7.6}$$

With a U(1) gauge symmetry [*3]

$$\mathcal{L} = \Phi_i^*\,\mathrm{e}^{2gVQ_i}\Phi_i\Big|_{\theta^4} + \left[\left(\frac{1}{4} - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right] + \Lambda_{\text{FI}}D;\tag{7.7}$$

$$\Phi_i\,\mathrm{e}^{2gQ_iV}\Phi_i\Big|_{\theta^4} \equiv \mathrm{D}^\mu\phi_i^*\,\mathrm{D}_\mu\phi_i + \mathrm{i}\bar{\psi}_i\bar{\sigma}^\mu\,\mathrm{D}_\mu\psi_i + F_i^*F_i - \sqrt{2}gQ_i\phi_i^*\lambda\psi_i - \sqrt{2}gQ_i\bar{\psi}_i\bar{\lambda}\phi_i + gQ_i\phi_i^*\phi_iD,\tag{7.8}$$

$$\left(\frac{1}{4} - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + \text{H.c.} = \frac{1}{2}\,\text{Re}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} + \frac{g^2\Theta}{16\pi^2}\,\text{Im}\,\mathcal{W}\mathcal{W}\big|_{\theta^2}$$
$$\equiv \mathrm{i}\bar{\lambda}\bar{\sigma}^\mu\,\mathrm{D}_\mu\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g^2\Theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma},\tag{7.9}$$

$$\mathrm{D}_\mu\phi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\phi_i, \qquad \mathrm{D}_\mu\psi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\psi_i,$$
$$\mathrm{D}^\mu\phi_i^* = (\partial^\mu + \mathrm{i}gQ_iA^\mu)\phi_i^*, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \mathrm{D}_\mu\lambda = \partial_\mu\lambda.$$

$$\{\phi, \psi, F\} \xrightarrow{\text{gauge}} \mathrm{e}^{\mathrm{i}gQ_i\theta}\{\phi, \psi, F\}, \qquad A_\mu \xrightarrow{\text{gauge}} A_\mu + \partial_\mu\theta, \qquad \lambda \xrightarrow{\text{gauge}} \lambda, \qquad D \xrightarrow{\text{gauge}} D.\tag{7.10}$$

[*3] We use the convention with $V \ni \lambda(x)\theta\bar{\theta}^2$, which corresponds to $\lambda = \mathrm{i}\lambda_{\text{SLHA}}$. In SLHA convention, the scalar-fermion-gaugino interaction is replaced to

$$-\sqrt{2}g\mathrm{i}\lambda_{\text{SLHA}}^a(\phi^*t^a\psi) - \sqrt{2}g(-\mathrm{i}\bar{\lambda}_{\text{SLHA}}^a)(\bar{\psi}t^a\phi).$$

With an SU(N) gauge symmetry

$$\mathcal{L} = \Phi^* \, e^{2gV} \Phi \Big|_{\theta^4} + \left[\left(\frac{1}{2} - \frac{ig^2 \Theta}{16\pi^2} \right) \operatorname{Tr} \mathcal{W}^\alpha \mathcal{W}_\alpha \Big|_{\theta^2} + W(\Phi) \Big|_{\theta^2} + \text{H.c.} \right]; \tag{7.11}$$

$$\Phi^* \, e^{2gV} \Phi \Big|_{\theta^4} := \Phi_i^* \left[e^{2gV^a t_\Phi^a} \right]_{ij} \Phi_j \Big|_{\theta^4} \tag{7.12}$$

$$\begin{aligned}
&= (\partial_\mu \phi_i^*)(\partial^\mu \phi_i) + i\bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_i + F_i^* F_i - \sqrt{2} g \lambda^a (\phi^* t^a \psi) - \sqrt{2} g \bar{\lambda}^a (\bar{\psi}^* t^a \phi) \\
&\quad + g A_\mu^a \bar{\psi} \bar{\sigma}^\mu (t^a \psi) + 2ig A_\mu^a \phi^* \partial_\mu (t^a \phi) + g^2 A^{a\mu} A_\mu^b (\phi^* t^a t^b \phi) + g D^a (\phi^* t^a \phi)
\end{aligned} \tag{7.13}$$

$$= D^\mu \phi^* \, D_\mu \phi + i\bar{\psi}_i \bar{\sigma}^\mu \, D_\mu \psi_i + F^* F - \sqrt{2} g \lambda^a (\phi^* t^a \psi) - \sqrt{2} g \bar{\lambda}^a (\bar{\psi} t^a \phi) + g D^a (\phi^* t^a \phi) \tag{7.14}$$

$$\left(\frac{1}{2} - \frac{ig^2 \Theta}{16\pi^2} \right) \operatorname{Tr} \mathcal{W}^\alpha \mathcal{W}_\alpha \Big|_{\theta^2} + \text{H.c.} = \operatorname{Re} \operatorname{Tr} \mathcal{W}\mathcal{W} \Big|_{\theta^2} + \frac{g^2 \Theta}{8\pi^2} \operatorname{Im} \operatorname{Tr} \mathcal{W}\mathcal{W} \Big|_{\theta^2}$$

$$= i\lambda^a \sigma^\mu \, D_\mu \bar{\lambda}^a + \frac{1}{2} D^a D^a - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{g^2 \Theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a; \tag{7.15}$$

$$D_\mu \phi_i = \partial_\mu \phi_i - ig A_\mu^a t_{ij}^a \phi_j, \qquad D_\mu \psi_i = \partial_\mu \psi_i - ig A_\mu^a t_{ij}^a \psi_j, \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g A_\mu^b A_\nu^c f^{abc},$$

$$D^\mu \phi_i^* = \partial^\mu \phi_i^* + ig A^{a\mu} \phi_j^* t_{ji}^a, \qquad D_\mu \lambda_\alpha^a = \partial_\mu \lambda_\alpha^a + g f^{abc} A_\mu^b \lambda_\alpha^c.$$

$$\{\phi, \psi, F\} \xrightarrow{\text{gauge}} e^{ig\theta^a t^a} \{\phi, \psi, F\},$$

$$A_\mu^a \xrightarrow{\text{gauge}} A_\mu^a + \partial_\mu \theta^a + g f^{abc} A_\mu^b \theta^c + \mathcal{O}(\theta^2), \qquad \lambda^a \xrightarrow{\text{gauge}} \lambda^a + g f^{abc} \lambda^b \theta^c + \mathcal{O}(\theta^2),$$

$$D^a \xrightarrow{\text{gauge}} D^a + g f^{abc} D^b \theta^c + \mathcal{O}(\theta^2), \qquad \bar{\lambda}^a \xrightarrow{\text{gauge}} \bar{\lambda}^a + g f^{abc} \bar{\lambda}^b \theta^c + \mathcal{O}(\theta^2).^{*4}$$

Auxiliary fields and Scalar potential In all of the above three theories,

$$\mathcal{L} \supset F_i^* F_i + F_i \frac{\partial W}{\partial \Phi_i} \Big|_{\text{scalar}} + F_i^* \frac{\partial W^*}{\partial \Phi_i^*} \Big|_{\text{scalar}} + \frac{1}{2} D^a D^a + g D^a (\phi^* t^a \phi); \tag{7.16}$$

$$\langle F_i^* \rangle = -\frac{\partial W}{\partial \Phi_i} \Big|_{\text{scalar}}, \qquad \langle D^a \rangle = -g \phi^* t^a \phi; \tag{7.17}$$

$$\mathcal{L} \supset -V_{\text{SUSY}} = -\left[\langle F_i^* \rangle \langle F_i \rangle + \frac{g^2}{2} (\phi^* t^a \phi)(\phi^* t^a \phi) \right]. \tag{7.18}$$

[*4] ♣TODO:give in non-infinitesimal form♣

7.1. Lorentz symmetry as SU(2)×SU(2)

7.2. Supersymmetry algebra

We define the generators as

$$P_\mu := \mathrm{i}\partial_\mu, \quad \{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \quad \{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{Q}}_{\dot\beta}\} = 0, \tag{7.19}$$

which is realized by

$$\mathcal{Q}_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{Q}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{Q}^\alpha = -\frac{\partial}{\partial\theta_\alpha} - \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{Q}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu,$$

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{D}^\alpha = -\frac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{D}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu;$$

$\mathcal{D}_\alpha$ etc. works as covariant derivatives because of the commutation relations

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = +2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \{\mathcal{Q}_\alpha, \mathcal{D}_\beta\} = \{\mathcal{Q}_\alpha, \bar{\mathcal{D}}_{\dot\beta}\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \mathcal{D}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 0.$$

Derivative formulae

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = -2\delta_\alpha^\beta \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = 2\delta_{\dot\alpha}^{\dot\beta}$$

$$\epsilon_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} = -\frac{\partial}{\partial\theta^\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = 2\epsilon^{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = -2\epsilon^{\dot\alpha\dot\beta}$$

$$\epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\beta}} = -\frac{\partial}{\partial\bar\theta_{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = 2\delta_\beta^\alpha \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = -2\delta_{\dot\beta}^{\dot\alpha}$$

$$\epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\beta}} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = -2\epsilon_{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = 2\epsilon_{\dot\alpha\dot\beta}$$

In addition, we define

$$(y, \theta', \bar\theta') := (x - \mathrm{i}\theta\sigma^\mu\bar\theta, \theta, \bar\theta): \tag{7.20}$$

$$\bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta'^{\dot\alpha}}; \quad \begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix} = \begin{pmatrix} \delta_\mu^\nu & 0 & 0 \\ -\mathrm{i}(\sigma^\nu\bar\theta)_\alpha & \delta_\alpha^\beta & 0 \\ \mathrm{i}(\theta\sigma^\nu)_{\dot\alpha} & 0 & \delta_{\dot\alpha}^{\dot\beta} \end{pmatrix}\begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\beta} \\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}} \end{pmatrix}, \quad \begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\alpha} \\ \frac{\partial}{\partial\bar\theta'^{\dot\alpha}} \end{pmatrix} = \begin{pmatrix} \delta_\nu^\mu & 0 & 0 \\ \mathrm{i}(\sigma^\mu\bar\theta)_\beta & \delta_\beta^\alpha & 0 \\ -\mathrm{i}(\theta\sigma^\mu)_{\dot\beta} & 0 & \delta_{\dot\beta}^{\dot\alpha} \end{pmatrix}\begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix}, \tag{7.21}$$

and a function $f: \mathbb{C}^4 \to \mathbb{C}$ (independent of θ' and $\bar\theta'$) is expanded as

$$f(y) = f(x - \mathrm{i}\theta\sigma\bar\theta) = f(x) - \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f(x) - \frac{1}{4}\theta^4\partial^2 f(x). \tag{7.22}$$

Note that we differentiate $[f(y)]^*$ and $f^*(y)$:

$$[f(y)]^* = f(x) + \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f^*(x) - \frac{1}{4}\theta^4\partial^2 f^*(x) = f^*(y + \mathrm{i}\theta\sigma\bar\theta) = f^*(y^*). \tag{7.23}$$

7.3. Superfields

SUSY-invariant Lagrangian SUSY transformation is induced by $\xi\mathcal{Q} + \bar\xi\bar{\mathcal{Q}} = \xi^\alpha\partial_\alpha + \bar\xi_{\dot\alpha}\partial^{\dot\alpha} + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu$. Therefore, for an object Ψ in the superspace,

$$\left[\Psi\right]_{\theta^4} \xrightarrow{\text{SUSY}} \left[\Psi + \xi^\alpha\partial_\alpha\Psi + \bar\xi_{\dot\alpha}\partial^{\dot\alpha}\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\right]_{\theta^4} = \left[\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\right]_{\theta^4}, \tag{7.24}$$

which means $\left[\Psi\right]_{\theta^4}$ is SUSY-invariant up to total derivative, i.e., $\int \mathrm{d}^4 x\left[\Psi\right]_{\theta^4}$ is SUSY-invariant action. Also,

$$\left[\Psi\right]_{\theta^2} \xrightarrow{\text{SUSY}} \left[\Psi + \bar\xi_{\dot\alpha}\left(\partial^{\dot\alpha} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\right)\Psi\right]_{\theta^2} = \left[\Psi + \bar\xi_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\Psi + 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Psi\right]_{\theta^2} \tag{7.25}$$

will be SUSY-invariant if $\bar{\mathcal{D}}_{\dot\alpha}\Psi = 0$, i.e., Ψ is a chiral superfield. Therefore, SUSY-invariant Lagrangian is given by

$$\mathcal{L} = \left[(\text{any real superfield})\right]_{\theta^4} + \left[(\text{any chiral superfield})\right]_{\theta^2} + \left[(\text{any chiral superfield})^*\right]_{\bar\theta^2}. \tag{7.26}$$

Chiral superfield A chiral superfield is a superfield that satisfies $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$, i.e., we find

$$\Phi = \phi(y) + \sqrt{2}\theta'\psi(y) + \theta'^2 F(y) \tag{7.27}$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{1}{4}\partial^2\phi(x)\theta^4 \tag{7.28}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{1}{4}\partial^2\phi^*(x)\theta^4; \tag{7.29}$$

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their product is expanded as

$$
\begin{aligned}
\Phi_i^* \Phi_j = {}& \phi_i^* \phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^* F_j \theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - \mathrm{i}\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2 \\
& + \left[\sqrt{2}\bar\psi_i\bar\theta F_j - \frac{\mathrm{i}\left(\partial_\mu\phi_i^*\cdot\psi_j\sigma^\mu\bar\theta - \phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)}{\sqrt{2}}\right]\theta^2 + \left[\sqrt{2}F_i^*\theta\psi_j + \frac{\mathrm{i}\left(\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j - \theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)}{\sqrt{2}}\right]\bar\theta^2 \\
& + \frac{1}{4}\left(4F_i^* F_j - \phi_i^*\partial^2\phi_j - (\partial^2\phi_i^*)\phi_j + 2(\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + 2\mathrm{i}(\psi_j\sigma^\mu\partial_\mu\bar\psi_i) - 2\mathrm{i}(\partial_\mu\psi_j\sigma^\mu\bar\psi_i)\right)\theta^4
\end{aligned}
\tag{7.30}
$$

$$
\begin{aligned}
\equiv {}& \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^* F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - 2\mathrm{i}\left(\phi_i^*\partial_\mu\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2 \\
& + \sqrt{2}\left(\bar\psi_i\bar\theta F_j + \mathrm{i}\phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)\theta^2 + \sqrt{2}\left(F_i^*\theta\psi_j - \mathrm{i}\theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)\bar\theta^2 \\
& + \left(F_i^* F_j + (\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right)\theta^4
\end{aligned}
\tag{7.31}
$$

$$
\Phi_i\Phi_j\big|_{\theta^2} = -\psi_i\psi_j + F_i\phi_j + \phi_i F_j
\tag{7.32}
$$

$$
\Phi_i\Phi_j\Phi_k\big|_{\theta^2} = -(\psi_i\psi_j)\phi_k - (\psi_k\psi_i)\phi_j - (\psi_j\psi_k)\phi_i + \phi_i\phi_j F_k + \phi_k\phi_i F_j + \phi_j\phi_k F_i
\tag{7.33}
$$

$$
\mathrm{e}^{k\Phi} = \mathrm{e}^{k\phi}\left[1 + \sqrt{2}k\theta\psi + \left(kF - \frac{k^2}{2}\psi\psi\right)\theta^2 - \mathrm{i}k\partial_\mu\phi(\theta\sigma^\mu\bar\theta) + \frac{\mathrm{i}k\left(\partial_\mu\psi + k\psi\partial_\mu\phi\right)\sigma^\mu\bar\theta\theta^2}{\sqrt{2}} - \frac{k}{4}\left(\partial^2\phi + k\partial_\mu\phi\partial^\mu\phi\right)\theta^4\right];
\tag{7.34}
$$

note that $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$, and $\mathrm{e}^{k\Phi}$ are all chiral superfields.

Vector superfield   A vector superfield is a superfield V that satisfies $V = V^*$. It is given by real fields $\{C, M, N, D, A_\mu\}$ and Grassmann fields $\{\chi, \lambda\}$ as[*5]

$$
\begin{aligned}
V(x,\theta,\bar\theta) = {}& C(x) + \mathrm{i}\theta\chi(x) - \mathrm{i}\bar\theta\bar\chi(x) + \frac{1}{2}\left(M(x) + \mathrm{i}N(x)\right)\theta^2 + \frac{1}{2}\left(M(x) - \mathrm{i}N(x)\right)\bar\theta^2 + (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) \\
& \left(\lambda(x) + \frac{1}{2}\partial_\mu\bar\chi(x)\bar\sigma^\mu\right)\theta\bar\theta^2 + \theta^2\bar\theta\left(\bar\lambda(x) + \frac{1}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right) + \frac{1}{2}\left(D(x) - \frac{1}{2}\partial^2 C(x)\right)\theta^4.
\end{aligned}
\tag{7.35}
$$

With this convention,

$$
V \to V - \mathrm{i}\Phi + \mathrm{i}\Phi^* \iff \begin{cases} C \to C - \mathrm{i}\phi + \mathrm{i}\phi^*, & \chi \to \chi - \sqrt{2}\psi, & \lambda \to \lambda, \\ M + \mathrm{i}N \to M + \mathrm{i}N - 2\mathrm{i}F, & A_\mu \to A_\mu + \partial_\mu(\phi + \phi^*), & D \to D. \end{cases}
\tag{7.36}
$$

The exponential of a vector superfield is also a vector superfield:

$$
\begin{aligned}
\mathrm{e}^{kV} = \mathrm{e}^{kC}\Bigg\{ {}& 1 + \mathrm{i}k(\theta\chi - \bar\theta\bar\chi) + \left(\frac{M + \mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\theta^2 + \left(\frac{M - \mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\bar\theta^2 + (k^2\theta\chi\bar\theta\bar\chi - k\theta\sigma^\mu\bar\theta A_\mu) \\
& + \left[k\bar\theta\bar\lambda - \mathrm{i}k\bar\theta\bar\chi\left(\frac{M + \mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right) + \frac{1}{2}k\bar\theta\bar\sigma^\mu\left(\partial_\mu\chi - \mathrm{i}k\chi A_\mu\right)\right]\theta^2 \\
& + \left[k\theta\lambda + \mathrm{i}k\theta\chi\left(\frac{M - \mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right) - \frac{1}{2}k\theta\sigma^\mu\left(\partial_\mu\bar\chi + \mathrm{i}k\bar\chi A_\mu\right)\right]\bar\theta^2 \\
& + \left[\frac{k}{2}\left(D - \frac{1}{2}\partial^2 C\right) - \frac{1}{2}\mathrm{i}k^2(\lambda\chi - \bar\lambda\bar\chi) + \left(\frac{M + \mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\left(\frac{M - \mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\right. \\
& \left. + \frac{k^3}{4}\bar\chi\bar\sigma^\mu\chi A_\mu + \frac{k^2}{4}\left(\mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - \mathrm{i}\partial_\mu\bar\chi\bar\sigma^\mu\chi + A^\mu A_\mu\right)\right]\theta^4\Bigg\}.
\end{aligned}
\tag{7.37}
$$

Supergauge symmetry   The gauge transformation $\phi(x) \to \mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\phi(x)$ is not closed in the chiral superfield; i.e., $\mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\Phi(x)$ is not a chiral superfield if the parameter $\theta(x)$ has x^μ-dependence. Hence, in supersymmetric theories, it is extended to *supergauge symmetry* parameterized by a chiral superfield $\Omega(x)$, which is given by

$$
\Phi \to \mathrm{e}^{2\mathrm{i}g\Omega^a(x)t^a}\Phi, \qquad \Phi^* \to \Phi^*\mathrm{e}^{-2\mathrm{i}g\Omega^{*a}(x)t^a}
\tag{7.38}
$$

for a chiral superfield Φ and an anti-chiral superfield Φ^*. The supergauge-invariant Lagrangian should be

$$
\mathcal{L} \sim \Phi^* \cdot (\text{real superfield}) \cdot \Phi;
\tag{7.39}
$$

we parameterize the "real superfield" as $\mathrm{e}^{2gV^a(x)t^a}$:

$$
\mathcal{L} = \left[\Phi^*\,\mathrm{e}^{2gV^a(x)t^a}\,\Phi\right]_{\theta^4}; \qquad \mathrm{e}^{2gV^a(x)t^a} \to \mathrm{e}^{2\mathrm{i}g\Omega^{*a}(x)t^a}\,\mathrm{e}^{2gV^a(x)t^a}\,\mathrm{e}^{-2\mathrm{i}g\Omega^a(x)t^a}.
\tag{7.40}
$$

[*5]Different coordination of "i"s are found in literature. Take care, especially, $\lambda(\text{ours}) = \mathrm{i}\lambda(\text{Wess-Bagger}) = \mathrm{i}\lambda(\text{SLHA})$.

In Abelian case, t^a is replaced by the charge Q of Φ and

$$\mathcal{L} = \left[\Phi^* \, e^{2gQV(x)} \Phi \right]_{\theta^4}; \qquad \Phi \to e^{2igQ\Omega(x)}\Phi, \quad \Phi^* \to \Phi^* \, e^{-2igQ\Omega^*(x)}, \tag{7.41}$$

$$e^{2gQV(x)} \to e^{2igQ\Omega^*(x)} \, e^{2gQV(x)} \, e^{-2igQ\Omega(x)} = e^{2gQ\left(V - i\Omega + i\Omega^*\right)}. \tag{7.42}$$

The usual gauge transformation corresponds to the real part of the lowest component of Ω, i.e., $\theta \equiv 2\,\mathrm{Re}\,\phi = \phi + \phi^*$, and we use the other components to fix the supergauge so that C, M, N and χ are eliminated:

$$\text{supergauge fixing:} \quad V(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{1}{2}D(x) \qquad \text{(Wess-Zumino gauge)}; \tag{7.43}$$

$$e^{2gQV} \longrightarrow 1 + gQ\left(-2\theta\sigma^\mu\bar\theta A_\mu + 2\bar\theta^2\theta\bar\lambda + 2\bar\theta^2\theta\lambda + D\theta^4\right) + g^2Q^2A^\mu A_\mu\theta^4. \tag{7.44}$$

The gauge transformation is the remnant freedom: $\Theta = \phi(y) = \phi - i\partial_\mu\phi(\theta\sigma^\mu\bar\theta) - \partial^2\phi\theta^4/4$ with ϕ being real;

$$\Phi_i \to e^{2igQ\Theta}\Phi_i, \qquad e^{2gQV} \to e^{2gQ(V - i\Theta + i\Theta^*)}. \tag{7.45}$$

Rules for each component is obvious in $(y, \theta, \bar\theta)$-basis and given by

$$\{\phi, \psi, F\} \to e^{igQ\theta}\{\phi, \psi, F\}, \qquad A_\mu \to A_\mu + \partial_\mu\theta, \qquad \lambda \to \lambda, \qquad D \to D. \tag{7.46}$$

For non-Abelian gauges, the supergauge transformation for the real field is evaluated as

$$e^{2gV} \to e^{2ig\Omega^*} \, e^{2gV} \, e^{-2ig\Omega} \tag{7.47}$$

$$= \left(e^{2ig\Omega^*} \, e^{2gV} \, e^{-2ig\Omega^*} \right) \left(e^{2ig\Omega^*} \, e^{-2ig\Omega} \right) \tag{7.48}$$

$$= \exp\left(e^{[2ig\Omega^*, \cdot} 2gV \right) e^{2ig(\Omega^* - \Omega)} + \mathcal{O}(\Omega^2) \tag{7.49}$$

$$= \exp\left(2gV + [2ig\Omega^*, 2gV] \right) e^{2ig(\Omega^* - \Omega)} + \mathcal{O}(\Omega^2); \tag{7.50}$$

$$= \exp\left[2gV + [2ig\Omega^*, 2gV] + \int_0^1 dt\, g(e^{[2gV, \cdot]}) 2ig(\Omega^* - \Omega)] + \mathcal{O}(\Omega^2) \right] \tag{7.51}$$

$$= \exp\left[2gV + [2ig\Omega^*, 2gV] + \sum_{n=0}^\infty \frac{B_n\left([2gV, \cdot]\right)^n}{n!} 2ig(\Omega^* - \Omega)] \right] + \mathcal{O}(\Omega^2) \tag{7.52}$$

$$= \exp\left[2g\left(V + i(\Omega^* - \Omega) - [V, ig(\Omega^* + \Omega)] + \sum_{n=2}^\infty \frac{iB_n\left([2gV, \cdot]\right)^n}{n!}(\Omega^* - \Omega)] \right) + \mathcal{O}(\Omega^2) \right]. \tag{7.53}$$

Here, again we can use the "non-gauge" component of Ω to eliminate the C-term etc., i.e., we fix $i(\Omega^* - \Omega)$, the second term of the expansion, to remove those terms:

$$V - [V, ig(\Omega^* + \Omega)] + \left(i + \sum_{n=2}^\infty \frac{iB_n\left([2gV, \cdot]\right)^n}{n!} \right)(\Omega^* - \Omega)] + \mathcal{O}(\Omega^2) = (\bar\theta\bar\sigma^\mu\theta)A_\mu + \bar\theta^2\theta\lambda + \theta^2\bar\theta\bar\lambda + \frac{1}{2}D; \tag{7.54}$$

this defines the Wess-Zumino gauge:

$$\text{supergauge fixing:} \quad V^a(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A_\mu^a(x) + \bar\theta^2\theta\lambda^a(x) + \theta^2\bar\theta\bar\lambda^a(x) + \frac{1}{2}D^a(x), \tag{7.55}$$

$$e^{2gV^a t^a} \longrightarrow 1 + g\left(-2\theta\sigma^\mu\bar\theta A_\mu^a + 2\bar\theta^2\theta\bar\lambda^a + 2\bar\theta^2\theta\lambda^a + D^a\theta^4\right)t^a + g^2 A^{a\mu}A_\mu^b\theta^4 t^a t^b. \tag{7.56}$$

The gauge transformation is given by

$$\Phi \to e^{2ig\Theta^a t^a}\Phi, \quad e^{2gV^a t^a} \to e^{2ig\Theta^b t^b} \, e^{2gV^a t^a} \, e^{-2ig\Theta^c t^c}. \tag{7.57}$$

For components in chiral superfields,

$$\{\phi, \psi, F\} \to e^{ig\theta^a t^a}\{\phi, \psi, F\}, \tag{7.58}$$

while for vector superfield we can express as infinitesimal transformation:

$$V \to V' \simeq V + i(\Theta^* - \Theta) - [V, ig(\Theta^* + \Theta)] + \sum_{n=2}^\infty \frac{iB_n\left([2gV, \cdot]\right)^n}{n!}(\Theta^* - \Theta)] \tag{7.59}$$

$$= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi - \left[V, ig\left(2\phi - \frac{\theta^4}{2}\partial^2\phi\right) \right] + 2\sum_{n=2}^\infty \frac{B_n\left([2gV, \cdot]\right)^n}{n!}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi] \tag{7.60}$$

$$= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi + 2gf^{abc}V^b\phi^c t^a \qquad \text{(Wess-Zumino gauge)} \tag{7.61}$$

$$\therefore A_\mu^a \to A_\mu^a + \partial_\mu\theta^a + gf^{abc}A_\mu^b\theta^c + \mathcal{O}(\theta^2), \quad \lambda^a \to \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2),$$
$$D^a \to D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), \qquad \bar\lambda^a \to \bar\lambda^a + gf^{abc}\bar\lambda^b\theta^c + \mathcal{O}(\theta^2). \tag{7.62}$$

Gauge-field strength   The real superfield e^V is gauge-invariant in Abelian case and a candidate in Lagrangian term, but this is not case in non-Abelian case. We thus define a chiral superfield from e^V:

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right); \tag{7.63}$$

$$\mathcal{W}_\alpha \xrightarrow{\text{gauge}} \mathrm{e}^{2\mathrm{i}g\Omega}\mathcal{W}_\alpha\,\mathrm{e}^{-2\mathrm{i}g\Omega} \quad \left(\mathcal{W}_\alpha^a \xrightarrow{\text{gauge}} [\mathrm{e}^{+2g\tilde{f}^c\Omega^c}]^{ab}W_\alpha^b \quad \text{with} \quad [\tilde{f}^c]_{ab} = f^{abc}\right);^{*6} \tag{7.64}$$

it is not supergauge- or Lorentz-invariatn, but $\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha) = \mathrm{Tr}(\epsilon^{\alpha\beta}\mathcal{W}_\beta\mathcal{W}_\alpha)$ is supergauge- and Lorentz-invariant, and its θ^2-term is SUSY-invariant, which becomes a candidate in SUSY Lagrangian with its Hermitian conjugate.

In Wess-Zumino gauge, it is given by

$$\mathcal{W}_\alpha = \left\{\lambda_\alpha^a(y) + \theta_\alpha D^a(y) + \frac{[\mathrm{i}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)\theta]_\alpha}{4}F_{\mu\nu}^a(y) + \theta^2\left[\mathrm{i}\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a(y^*)\right]_\alpha\right\}t^a \tag{7.65}$$

$$= \left[\lambda_\alpha^a + \theta_\alpha D^a + \frac{\mathrm{i}}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F_{\mu\nu}^a + \mathrm{i}\theta^2(\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a)_\alpha + \mathrm{i}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\lambda_\alpha^a - \frac{\theta^4}{4}\partial^2\lambda_\alpha^a\right.$$
$$\left. + \frac{\mathrm{i}\theta^2(\sigma^\mu\bar\theta)_\alpha}{2}\left(\partial_\mu D^a + \mathrm{i}\partial^\nu F_{\mu\nu}^a - gf^{abc}\epsilon_{\mu\nu\rho\sigma}A^{\nu b}\partial^\rho A^{\sigma c}\right)\right]T^a, \tag{7.66}$$

where, as usual,

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gA_\mu^b A_\nu^c f^{abc}, \qquad \mathrm{D}_\mu\lambda_\alpha^a = \partial_\mu\lambda_\alpha^a + gf^{abc}A_\mu^b\lambda_\alpha^c. \tag{7.67}$$

Also,

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^2} = \left[\mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^b + \mathrm{i}\lambda^b\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + D^aD^b - \frac{1}{4}\left(\mathrm{i}\epsilon^{\sigma\mu\nu\rho} + 2\eta^{\mu\rho}\eta^{\nu\sigma}\right)F_{\mu\nu}^a F_{\rho\sigma}^b\right]\mathrm{Tr}(t^a t^b) \tag{7.68}$$

$$= \mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + \frac{1}{2}D^aD^a - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^a F_{\rho\sigma}^a, \tag{7.69}$$

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^4} = \frac{\theta^4}{4}\left(2(\partial^\mu\lambda^a)(\partial_\mu\lambda^b) - \lambda^a\partial^2\lambda^b - (\partial^2\lambda^a)\lambda^b\right)\mathrm{Tr}(t^a t^b) = \frac{\theta^4}{4}\left((\partial^\mu\lambda^a)(\partial_\mu\lambda^a) - \lambda^a\partial^2\lambda^a\right). \tag{7.70}$$

For Abelian theory,

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right) = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}_\alpha(2gV), \tag{7.71}$$

$$\left.\mathcal{W}^\alpha\mathcal{W}_\alpha\right|_{\theta^2} = 2\left(\mathrm{i}\lambda\sigma^\mu\,\mathrm{D}_\mu\bar\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right). \tag{7.72}$$

7.4. Lagrangian blocks

Lagrangian construction   The supergauge transformation is summarized as

$$\Phi_i \to [U_\Phi]_{ij}\Phi_j, \qquad \tilde\Phi_j \to \tilde\Phi_i[U_\Phi^{-1}]_{ij}, \qquad \mathcal{W}_\alpha \to U_\mathcal{W}\mathcal{W}_\alpha U_\mathcal{W}^{-1}, \tag{7.73}$$

where

$$\tilde\Phi_j^* := \Phi_i^*[\mathrm{e}^{2gVt_\Phi^a}]_{ij}, \qquad U_\Phi := \exp(2\mathrm{i}g\Omega^a t_\Phi^a), \qquad U_\mathcal{W} := \exp(2\mathrm{i}g\Omega^a t_\mathcal{W}^a), \tag{7.74}$$

t_Φ^a is the representation matrix or U(1) charge for the field Φ, and $t_\mathcal{W}^a$ is the representation matrix that is used to define $\mathcal{W}_\alpha$. To construct a Lagrangian, we should composite these ingredients in real and invariant under SUSY, supergauge, and Lorentz transformation. A sufficient condition for SUSY invariance is given by (7.26), so

$$\mathcal{L} = \left[K(\Phi_i, \tilde\Phi_j^*)\right]_{\theta^4} + \left\{\left[f_{ab}(\Phi_i)\mathcal{W}^a\mathcal{W}^b\right]_{\theta^2} + \mathrm{H.c.}\right\} + \left\{\left[W(\Phi_i)\right]_{\theta^2} + \mathrm{H.c.}\right\} + D \tag{7.75}$$

is one possible construction. The Kähler function K should be real and supergauge invariant, the gauge kinetic function f should be holomorphic and supergauge invariant with $\mathcal{W}^a\mathcal{W}^b$, and the superpotential W is holomorphic and supergauge invariant. The last term D (Fayet-Illiopoulos term) comes from V of an U(1) gauge boson; note that its supergauge invariance is due to the intentional definition of V.

One can construct more general Lagrangian; for example, one can introduce a vector superfield that is not associated to a gauge symmetry, but then the supergauge fixing is not available and one has to include C or M fields.

Renormalizable Lagrangian   Since $[\Phi]_{\theta^4}$ is a total derivative, renormalizable Lagrangian is limited to

$$\mathcal{L} = \left[\Phi_i^*[\mathrm{e}^{2gVt_\Phi^a}]_{ij}\Phi_j\right]_{\theta^4} + \left\{[\mathcal{W}^a\mathcal{W}^a]_{\theta^2} + [W(\Phi_i)]_{\theta^2} + \mathrm{H.c.}\right\} + D \tag{7.76}$$

up to numeric coefficients. With multiple gauge groups, the Kähler part is extended as $\Phi_i^*[\mathrm{e}^{2gVt_\Phi^a}\,\mathrm{e}^{2gV't_\Phi'^a}\cdots]_{ij}\Phi_j$, where the inner part is obviously commutable.

[*6] ♣TODO:This equivalence should be checked/explained in gauge-theory section; especially, the sign is not verified and might be opposite.♣

8. Minimal Supersymmetric Standard Model

Gauge symmetry: $\mathrm{SU(3)_{color} \times SU(2)_{weak} \times U(1)_Y}$

Particle content:

(a) Chiral superfields

	SU(3)	SU(2)	U(1)	B	L	scalar/spinor
Q_i	**3**	**2**	1/6	1/3		$\tilde{q}_\mathrm{L}$, q_L $[\to (u_\mathrm{L}, d_\mathrm{L})]$
L_i		**2**	$-1/2$		1	$\tilde{l}_\mathrm{L}$, l_L $[\to (\nu_\mathrm{L}, l_\mathrm{L})]$
U_i^c	$\bar{\mathbf{3}}$		$-2/3$	$-1/3$		$\tilde{u}_\mathrm{R}^\mathrm{c}$, u_R^c
D_i^c	$\bar{\mathbf{3}}$		$1/3$	$-1/3$		$\tilde{d}_\mathrm{R}^\mathrm{c}$, d_R^c
E_i^c			1		-1	$\tilde{e}_\mathrm{R}^\mathrm{c}$, e_R^c
H_u		**2**	1/2			h_u, $\tilde{h}_\mathrm{u}$ $[\to (h_\mathrm{u}^+, h_\mathrm{u}^0)]$
H_d		**2**	$-1/2$			h_d, $\tilde{h}_\mathrm{d}$ $[\to (h_\mathrm{d}^0, h_\mathrm{d}^-)]$

(b) Vector superfields

	SU(3)	SU(2)	U(1)	ino/boson
g	adj.			$\tilde{g}, g_\mu$
W		adj.		$\tilde{w}, W_\mu$
B				$\tilde{b}, B_\mu$

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and SU(2) decomposition).

"c"-notation For scalars, $\tilde{\phi}_\mathrm{R}^\mathrm{c} := \phi_\mathrm{R}^* = \mathsf{C}\phi_\mathrm{R}\mathsf{C}$ (because the intrinsic phase for C is $+1$ for quarks and leptons.)

For matter spinors, $\psi_\mathrm{R}^\mathrm{c} := \bar{\psi}_\mathrm{R}$ (and $\psi_\mathrm{R} = \bar{\psi}_\mathrm{R}^\mathrm{c}$); Dirac spinors are thus

$$\psi_\mathrm{L} = \begin{pmatrix} \psi_\mathrm{L} \\ 0 \end{pmatrix}, \quad \overline{\psi_\mathrm{L}} = \begin{pmatrix} 0 & \bar{\psi}_\mathrm{L} \end{pmatrix}, \quad \psi_\mathrm{R}^\mathrm{c} := \begin{pmatrix} \psi_\mathrm{R}^\mathrm{c} \\ 0 \end{pmatrix} = C \begin{pmatrix} 0 \\ \psi_\mathrm{R} \end{pmatrix} = C\psi_\mathrm{R}, \quad \overline{\psi_\mathrm{R}^\mathrm{c}} = \begin{pmatrix} 0 & \psi_\mathrm{R} \end{pmatrix} C = \overline{\psi_\mathrm{R}}C.$$

Superpotential and ~~SUSY~~-terms

$$W_\mathrm{RPC} = \mu H_\mathrm{u} H_\mathrm{d} - y_{uij} U_i^\mathrm{c} H_\mathrm{u} Q_j + y_{dij} D_i^\mathrm{c} H_\mathrm{d} Q_j + y_{eij} E_i^\mathrm{c} H_\mathrm{d} L_j, \tag{8.1}$$

$$W_\mathrm{RPV} = -\kappa_i L_i H_\mathrm{u} + \frac{1}{2}\lambda_{ijk} L_i L_j E_k^\mathrm{c} + \lambda'_{ijk} L_i Q_j D_k^\mathrm{c} + \frac{1}{2}\lambda''_{ijk} U_i^\mathrm{c} D_j^\mathrm{c} D_k^\mathrm{c}, \tag{8.2}$$

$$\mathcal{L}_\mathrm{\not{SUSY}} = -\frac{1}{2}\left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w}\tilde{w} + M_1 \tilde{b}\tilde{b} + \mathrm{H.c.} \right) - V_\mathrm{\not{SUSY}}; \tag{8.3}$$

$$V_\mathrm{\not{SUSY}}^\mathrm{RPC} = \left(\tilde{q}_\mathrm{L}^* m_Q^2 \tilde{q}_\mathrm{L} + \tilde{l}_\mathrm{L}^* m_L^2 \tilde{l}_\mathrm{L} + \tilde{u}_\mathrm{R}^* m_{U^\mathrm{c}}^2 \tilde{u}_\mathrm{R} + \tilde{d}_\mathrm{R}^* m_{D^\mathrm{c}}^2 \tilde{d}_\mathrm{R} + \tilde{e}_\mathrm{R}^* m_{E^\mathrm{c}}^2 \tilde{e}_\mathrm{R} + m_{H_\mathrm{u}}^2 |h_\mathrm{u}|^2 + m_{H_\mathrm{d}}^2 |h_\mathrm{d}|^2 \right)$$
$$+ \left(-\tilde{u}_\mathrm{R}^* h_\mathrm{u} a_\mathrm{u} \tilde{q}_\mathrm{L} + \tilde{d}_\mathrm{R}^* h_\mathrm{d} a_\mathrm{d} \tilde{q}_\mathrm{L} + \tilde{e}_\mathrm{R}^* h_\mathrm{d} a_\mathrm{e} \tilde{l}_\mathrm{L} + b H_\mathrm{u} H_\mathrm{d} + \mathrm{H.c.} \right)$$
$$+ \left(+\tilde{u}_\mathrm{R}^* h_\mathrm{d}^* c_\mathrm{u} \tilde{q}_\mathrm{L} + \tilde{d}_\mathrm{R}^* h_\mathrm{u}^* c_\mathrm{d} \tilde{q}_\mathrm{L} + \tilde{e}_\mathrm{R}^* h_\mathrm{u}^* c_\mathrm{e} \tilde{l}_\mathrm{L} + \mathrm{H.c.} \right), \tag{8.4}$$

$$V_\mathrm{\not{SUSY}}^\mathrm{RPV} = \left(-b_i \tilde{l}_{\mathrm{L}i} H_\mathrm{u} + \frac{1}{2} T_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{l}_{\mathrm{L}j} \tilde{e}_{\mathrm{R}k}^* + T'_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{q}_{\mathrm{L}j} \tilde{d}_{\mathrm{R}k}^* + \frac{1}{2} T''_{ijk} \tilde{u}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j}^* \tilde{d}_{\mathrm{R}k}^* + \tilde{l}_{\mathrm{L}i}^* M_{Li}^2 H_\mathrm{d} + \mathrm{H.c.} \right)$$
$$+ \left(C_{ijk}^1 \tilde{l}_{\mathrm{L}i}^* \tilde{q}_{\mathrm{L}j} \tilde{u}_{\mathrm{R}k}^* + C_i^2 h_\mathrm{u}^* h_\mathrm{d} \tilde{e}_{\mathrm{R}i}^* + C_{ijk}^3 \tilde{d}_{\mathrm{R}i} \tilde{u}_{\mathrm{R}j}^* \tilde{e}_{\mathrm{R}k}^* + \frac{1}{2} C_{ijk}^4 \tilde{d}_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} \tilde{q}_{\mathrm{L}k} + \mathrm{H.c.} \right), \tag{8.5}$$

($\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C_{ijk}^4 = C_{ikj}^4$.)

8.1. Notation

Our notation in this section (and the previous section) follows DHM [8, PhysRept] and Martin [9, v7] (but note that Martin uses $(-,+,+,+)$-metric) for RPC part and SLHA2 convention for RPV part. In particular, the sign of gauge bosons are fixed by $D_\mu \phi = \partial_\mu \phi - ig A_\mu^a t_{ij}^a \phi_j$, and the phase of gauginos are by $\mathcal{L} \ni \sqrt{2} g (\phi^* t^a \psi \lambda^a)$. Phases of ϕ and ψ in chiral superfields are not yet specified; they are later used to remove $F\tilde{F}$ terms and diagonalize Yukawa matrices.

8.2. Lagrangian construction

The most generic form of the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{super}} + \mathcal{L}_{\text{FI}} + \mathcal{L}_{\text{SUSY}}; \tag{8.6}$$

$$\mathcal{L}_{\text{matter}} = \Phi_Q^* \exp\left(2g_Y(\tfrac{1}{6})V_B + 2g_2 V_W^a T^a + 2g_3 V_g^a \tau^a\right)\Phi_Q \Big|_{\theta^4} + \cdots; \tag{8.7}$$

$$\mathcal{L}_{\text{gauge}} = \left[\frac{1}{4}\left(1 - \frac{ig_Y^2 \Theta_B}{8\pi^2}\right)\mathcal{W}_B \mathcal{W}_B + \frac{1}{4}\left(1 - \frac{ig_2^2 \Theta_W}{8\pi^2}\right)\mathcal{W}_W^a \mathcal{W}_W^a + \frac{1}{4}\left(1 - \frac{ig_3^2 \Theta_g}{8\pi^2}\right)\mathcal{W}_g^a \mathcal{W}_g^a\right]_{\theta^2} + \text{H.c.}; \tag{8.8}$$

$$\mathcal{L}_{\text{super}} = W(\Phi)\Big|_{\theta^2} + \text{H.c.}, \tag{8.9}$$

$$W(\Phi) = W_{\text{RPC}} + W_{\text{RPV}}, \tag{8.10}$$

$$W_{\text{RPC}} = \mu H_u H_d - y_{uij} U_i^c H_u Q_j + y_{dij} D_i^c H_d Q_j + y_{eij} E_i^c H_d L_j, \tag{8.11}$$

$$W_{\text{RPV}} = -\kappa_i L_i H_u + \frac{1}{2}\lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2}\lambda''_{ijk} U_i^c D_j^c D_k^c; \tag{8.12}$$

$$\mathcal{L}_{\text{FI}} = \Lambda_{\text{FI}} D_B; \tag{8.13}$$

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{2}\left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w}\tilde{w} + M_1 \tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V_{\text{SUSY}}^{\text{RPC}} + V_{\text{SUSY}}^{\text{RPV}}\right), \tag{8.14}$$

$$V_{\text{SUSY}}^{\text{RPC}} = \left(\tilde{q}_L^* m_Q^2 \tilde{q}_L + \tilde{l}_L^* m_L^2 \tilde{l}_L + \tilde{u}_R^* m_{U^c}^2 \tilde{u}_R + \tilde{d}_R^* m_{D^c}^2 \tilde{d}_R + \tilde{e}_R^* m_{E^c}^2 \tilde{e}_R + m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2\right)$$
$$+ \left(-\tilde{u}_R^* h_u a_u \tilde{q}_L + \tilde{d}_R^* h_d a_d \tilde{q}_L + \tilde{e}_R^* h_d a_e \tilde{l}_L + b H_u H_d + \text{H.c.}\right)$$
$$+ \left(\tilde{u}_R^* h_d^* c_u \tilde{q}_L + \tilde{d}_R^* h_u^* c_d \tilde{q}_L + \tilde{e}_R^* h_u^* c_e \tilde{l}_L + \text{H.c.}\right), \tag{8.15}$$

$$V_{\text{SUSY}}^{\text{RPV}} = \left(-b_i \tilde{l}_{Li} H_u + \frac{1}{2}T_{ijk}\tilde{l}_{Li}\tilde{l}_{Lj}\tilde{e}_{Rk}^* + T'_{ijk}\tilde{l}_{Li}\tilde{q}_{Lj}\tilde{d}_{Rk}^* + \frac{1}{2}T''_{ijk}\tilde{u}_{Ri}^*\tilde{d}_{Rj}^*\tilde{d}_{Rk}^* + \tilde{l}_{Li}^* M_{Li}^2 H_d + \text{H.c.}\right)$$
$$+ \left(C_{ijk}^1 \tilde{l}_{Li}^* \tilde{q}_{Lj}\tilde{u}_{Rk}^* + C_i^2 h_u^* h_d \tilde{e}_{Ri}^* + C_{ijk}^3 \tilde{d}_{Ri}\tilde{u}_{Rj}^*\tilde{e}_{Rk}^* + \frac{1}{2}C_{ijk}^4 \tilde{d}_{Ri}\tilde{q}_{Lj}\tilde{q}_{Lk} + \text{H.c.}\right). \tag{8.16}$$

As usual, we remove Θ_W and Θ_B by rotating fermions[*7] , which is compatible with mass diagonalization (discussed later), and assume the absence of Fayet-Illiopoulos term: $\Lambda_{\text{FI}} = 0$. The SU(3) angle Θ_g forms QCD phase Θ_{QCD} together with the phases from Yukawa matrices. Then,

$$\mathcal{L}_{\text{matter}} = \sum_{\text{matters}}\left[D^\mu \phi^* D_\mu \phi + i\bar{\psi}\bar{\sigma}^\mu D_\mu \psi - \sqrt{2}\sum_{\text{gauge}} g\left(\lambda^a(\phi^* t^a \psi) + \bar{\lambda}^a(\bar{\psi}t^a \phi)\right)\right] + (F\text{-terms}), \tag{8.17}$$

$$\mathcal{L}_{\text{gauge}} = \sum_{\text{gauges}}\left(-\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a\right) + \frac{g_3^2 \Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^a G_{\rho\sigma}^a + (D\text{-terms}), \tag{8.18}$$

$$\mathcal{L}_{\text{super}} = \epsilon^{ab}\left(-\mu\tilde{h}_u^a \tilde{h}_d^b - y_{dij}h_d^a d_{Ri}^{cx}q_{Lj}^{bx} - y_{dij}\tilde{d}_{Ri}^{x*}\tilde{h}_d^a q_{Lj}^{bx} + y_{dji}\tilde{q}_{Li}^{ax}\tilde{h}_d^b d_{Rj}^{cx}\right.$$
$$-y_{eij}\tilde{e}_{Ri}^*\tilde{h}_d^a l_{Lj}^b - y_{eij}h_d^a e_{Ri}^c l_{Lj}^b + y_{eji}\tilde{l}_{Li}^a \tilde{h}_d^b e_{Rj}^c + y_{uij}h_u^a u_{Ri}^{cx}q_{Lj}^{bx} + y_{uij}\tilde{u}_{Ri}^{x*}\tilde{h}_u^a q_{Lj}^{bx} - y_{uji}\tilde{q}_{Li}^{ax}\tilde{h}_u^b u_{Rj}^{cx}$$
$$\left.-\kappa_i \tilde{h}_u^a l_{Li}^b - \lambda_{ikj}\tilde{l}_{Li}^a e_{Rj}^c l_{Lk}^b - \frac{1}{2}\lambda_{jki}\tilde{e}_{Ri}^* l_{Lj}^a l_{Lk}^b - \lambda'_{ikj}\tilde{l}_{Li}^a d_{Rj}^{cx}q_{Lk}^{bx} + \lambda'_{kij}\tilde{q}_{Li}^{ax}d_{Rj}^{cx}l_{Lk}^b + \lambda'_{kji}\tilde{d}_{Ri}^{x*}q_{Lj}^{ax}l_{Lk}^b\right) \tag{8.19}$$
$$-\frac{1}{2}\epsilon^{xyz}\lambda''_{ijk}\tilde{u}_{Ri}^{x*}d_{Rj}^{cy}d_{Rk}^{cz} + \epsilon^{xyz}\lambda''_{jik}\tilde{d}_{Ri}^{x*}u_{Rj}^{cy}d_{Rk}^{cz} + \text{H.c.} + (F\text{-terms}),$$

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{2}\left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w}\tilde{w} + M_1 \tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V_{\text{SUSY}}^{\text{RPC}} + V_{\text{SUSY}}^{\text{RPV}}\right), \tag{8.20}$$

and the F- and D-terms form the supersymmetric scalar potential

$$V_{\text{SUSY}} = F_i^* F_i + \frac{1}{2}D^a D^a; \qquad F_i = -W_i^* = -\frac{\delta W^*}{\delta \phi_i^*}, \qquad D^a = -g(\phi^* t^a \phi), \tag{8.21}$$

$$V = V_{\text{SUSY}} + V_{\text{SUSY}}^{\text{RPC}} + V_{\text{SUSY}}^{\text{RPV}}, \tag{8.22}$$

[*7]Fail-safe memo: The Θ-terms are total derivatives and relevant in non-perturbative discussion. Redefinition of chiral fermions generates those terms (Fujikawa method) as a non-perturbative effect, so we can remove Θ-terms as long as we have such freedoms. Note also that the absence of gauge anomaly means the corresponding gauge transformations do not induce additional Θ-terms.

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where t_a corresponds to the gauge-symmetry generator relevant for each ϕ.

Each auxiliary term is given by

$$-F^*_{h^a_u} = \epsilon^{ab}\left(-\tilde{u}^{x*}_R y_u \tilde{q}^{bx}_L + \mu h^b_d + \kappa_i \tilde{l}^b_{Li}\right),\tag{8.23}$$

$$-F^*_{h^a_d} = \epsilon^{ab}\left(\tilde{e}^*_R y_e \tilde{l}^b_L + \tilde{d}^{x*}_R y_d \tilde{q}^{bx}_L - \mu h^b_u\right),\tag{8.24}$$

$$-F^{a x*}_{\tilde{q}_{Li}} = \epsilon^{ab}\left(-y_{dji}h^b_d \tilde{d}^{x*}_{Rj} + y_{uji}h^b_u \tilde{u}^{x*}_{Rj} - \lambda'_{kij}\tilde{d}^{x*}_{Rj}\tilde{l}^b_{Lk}\right),\tag{8.25}$$

$$-F^*_{\tilde{u}^x_{Ri}} = -y_{uij}h_u \tilde{q}^x_{Lj} + \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk}\tilde{d}^{y*}_{Rj}\tilde{d}^{z*}_{Rk},\tag{8.26}$$

$$-F^*_{\tilde{d}^x_{Ri}} = y_{dij}h_d \tilde{q}^x_{Lj} + \lambda'_{jki}\tilde{l}_{Lj}\tilde{q}^x_{Lk} - \lambda''_{jik}\epsilon^{xyz}\tilde{u}^{y*}_{Rj}\tilde{d}^{z*}_{Rk},\tag{8.27}$$

$$-F^*_{\tilde{l}^a_{Li}} = \epsilon^{ab}\left(-y_{eji}\tilde{e}^*_{Rj}h^b_d - \kappa_i h^b_u + \lambda_{ikj}\tilde{e}^*_{Rj}\tilde{l}^b_{Lk} + \lambda'_{ikj}\tilde{d}^{x*}_{Rj}\tilde{q}^{bx}_{Lk}\right),\tag{8.28}$$

$$-F^*_{\tilde{e}^*_{Ri}} = y_{eij}h_d \tilde{l}_{Lj} + \frac{1}{2}\lambda_{jki}\tilde{l}_{Lj}\tilde{l}_{Lk}.\tag{8.29}$$

$$D^\alpha_{SU(3)} = -g_3 \sum_{i=1}^3 \left(\sum_{a=1,2}\tilde{q}^{a*}_{Li}\tau^\alpha \tilde{q}^a_{Li} - \tilde{u}^*_{Ri}\tau^\alpha \tilde{u}_{Ri} - \tilde{d}^*_{Ri}\tau^\alpha \tilde{d}_{Ri}\right),\tag{8.30}$$

$$D^\alpha_{SU(2)} = -g_2 \left[\sum_{i=1}^3 \left(\sum_{x=1}^3 \tilde{q}^{x*}_{Li}T^\alpha \tilde{q}^x_{Li} + \tilde{l}^*_{Li}T^\alpha \tilde{l}_{Li}\right) + h^*_u T^\alpha h_u + h^*_d T^\alpha h_d\right],\tag{8.31}$$

$$D_{U(1)} = -g_1 \left(\frac{1}{6}|\tilde{q}_L|^2 - \frac{1}{2}|\tilde{l}_L|^2 - \frac{2}{3}|\tilde{u}_R|^2 + \frac{1}{3}|\tilde{d}_R|^2 + |\tilde{e}_R|^2 + \frac{1}{2}|h_u|^2 - \frac{1}{2}|h_d|^2\right).\tag{8.32}$$

8.3. Full Lagrangian

Here the Lagrangian $\mathcal{L} = \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}} + \mathcal{L}_{\text{scalar}}$ is explicitly given:

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma},\tag{8.33}$$

$$\mathcal{L}_{\text{fermions}} = i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi + i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - \frac{1}{2}\left(M_3\tilde{g}_0\tilde{g}_0 + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) + \mathcal{L}_{\text{super}}\big|_{\text{no } F\text{-terms}},\tag{8.34}$$

$$\mathcal{L}_{\text{SFG}} = -\sqrt{2}g\lambda^a(\phi^* t^a \psi) - \sqrt{2}g\bar{\lambda}^a(\bar{\psi}t^a \phi),\tag{8.35}$$

$$\mathcal{L}_{\text{scalar}} = D^\mu\phi^* D_\mu\phi - V.\tag{8.36}$$

8.3.1. Vector part

$$\begin{aligned}
\mathcal{L}_{\text{vector}} = &-\frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)\partial^\mu B^\nu - \frac{1}{2}(\partial_\mu g^a_\nu - \partial_\nu g^a_\mu)\partial^\mu g^{a\nu} - \frac{1}{2}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)\partial^\mu W^{a\nu}\\
&- g_2\epsilon^{abc}W^b_\mu W^c_\nu \partial^\mu W^{a\nu} - \frac{g_2^2}{4}\epsilon^{abe}\epsilon^{cde}W^a_\mu W^b_\nu W^{c\mu}W^{d\nu}\\
&- g_3 f^{abc}g^b_\mu g^c_\nu \partial^\mu g^{a\nu} - \frac{g_3^2}{4}f^{cde}f^{abe}g^a_\mu g^b_\nu g^{c\mu}g^{d\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma},
\end{aligned}\tag{8.37}$$

$$\begin{aligned}
= &\,(\text{gluons}) - \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial^\mu A^\nu - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)\partial_\mu W^{+\nu} - \frac{1}{2}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\partial^\mu Z^\nu\\
&+ ig_2 c_w\left[(W^-_\mu Z_\nu - W^-_\nu Z_\mu)\partial^\mu W^{+\nu} - (W^+_\mu Z_\nu - W^+_\nu Z_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^-_\nu W^+_\mu)\partial^\mu Z^\nu\right]\\
&+ i|e|\left[(W^-_\mu A_\nu - W^-_\nu A_\mu)\partial^\mu W^{+\nu} - (W^+_\mu A_\nu - W^+_\nu A_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^-_\nu W^+_\mu)\partial^\mu A^\nu\right]\\
&+ \frac{g_2^2}{2}W^{+\mu}W^+_\mu W^{-\nu}W^-_\nu - \frac{g_2^2}{2}W^{+\mu}W^{+\nu}W^-_\mu W^-_\nu - g_2^2 W^{+\mu}W^-_\mu Z^\nu Z_\nu + g_2^2 W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\
&- e^2 W^{+\mu}W^-_\mu A^\nu A_\nu + e^2 W^{+\mu}W^-_\mu Z^\nu Z_\nu + e^2 W^{+\mu}W^{-\nu}A_\mu A_\nu - e^2 W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\
&- 2g_2^2 c_w s_w W^{+\mu}W^-_\mu A^\nu Z_\nu + g_2^2 c_w s_w W^{+\mu}W^{-\nu}A_\mu Z_\nu + g_2^2 c_w s_w W^{+\mu}W^{-\nu}A_\nu Z_\mu,
\end{aligned}\tag{8.38}$$

where

$$W^1_\mu = \frac{W^+_\mu + W^-_\mu}{\sqrt{2}}, \quad W^2_\mu = \frac{i(W^+_\mu - W^-_\mu)}{\sqrt{2}}; \qquad W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}};$$

$$\begin{pmatrix}W^3_\mu \\ B_\mu\end{pmatrix} = \begin{pmatrix}c_w & s_w \\ -s_w & c_w\end{pmatrix}\begin{pmatrix}Z_\mu \\ A_\mu\end{pmatrix}; \qquad \begin{pmatrix}Z_\mu \\ A_\mu\end{pmatrix} = \begin{pmatrix}c_w & -s_w \\ s_w & c_w\end{pmatrix}\begin{pmatrix}W^3_\mu \\ B_\mu\end{pmatrix};$$

$$|e| = g_2 s_w = g_Y c_w = g_Z s_w c_w, \quad g_Z = g_2/c_w = g_Y/s_w; \qquad g_Y = |e|/c_w = g_Z s_w = g_2 t_w, \quad g_2 = |e|/s_w = g_Z c_w.$$

8.3.2. Fermion part

$\mathcal{L}_{\text{fermions}}$

$$
\begin{aligned}
&= i\bar{q}_{\text{L}}\bar{\sigma}^\mu \left(\partial_\mu - ig_3 g_\mu^a \tau^a - ig_2 W_\mu^a T^a - \tfrac{1}{6} ig_Y B_\mu\right) q_{\text{L}} \\
&\quad + i\bar{u}_{\text{R}}^c \bar{\sigma}^\mu \left(\partial_\mu + ig_3 g_\mu^a \tau^{a*} + \tfrac{2}{3} ig_Y B_\mu\right) u_{\text{R}}^c + i\bar{d}_{\text{R}}^c \bar{\sigma}^\mu \left(\partial_\mu + ig_3 g_\mu^a \tau^{a*} - \tfrac{1}{3} ig_Y B_\mu\right) d_{\text{R}}^c \\
&\quad + i\bar{l}_{\text{L}}\bar{\sigma}^\mu \left(\partial_\mu - ig_2 W_\mu^a T^a + \tfrac{1}{2} ig_Y B_\mu\right) l_{\text{L}} + i\bar{e}_{\text{R}}^c \bar{\sigma}^\mu \left(\partial_\mu - ig_Y B_\mu\right) e_{\text{R}}^c \\
&\quad + i\bar{\tilde{h}}_{\text{u}}\bar{\sigma}^\mu \left(\partial_\mu - ig_2 W_\mu^a T^a - \tfrac{1}{2} ig_Y B_\mu\right) \tilde{h}_{\text{u}} + i\bar{\tilde{h}}_{\text{d}}\bar{\sigma}^\mu \left(\partial_\mu - ig_2 W_\mu^a T^a + \tfrac{1}{2} ig_Y B_\mu\right) \tilde{h}_{\text{d}} \\
&\quad + i\bar{\tilde{g}}_0^a \bar{\sigma}^\mu \left(\partial_\mu \tilde{g}_0^a + g_3 f^{abc} g_\mu^b \tilde{g}_0^c\right) + i\bar{\tilde{w}}^a \bar{\sigma}^\mu \left(\partial_\mu \tilde{w}^a + g_2 \epsilon^{abc} W_\mu^b \tilde{w}^c\right) + i\bar{\tilde{b}}\bar{\sigma}^\mu \partial_\mu \tilde{b} \\
&\quad - \frac{1}{2}\left(M_3 \tilde{g}_0^a \tilde{g}_0^a + M_2 \tilde{w}^a \tilde{w}^a + M_1 \tilde{b}\tilde{b} + \text{H.c.}\right) + \mathcal{L}_{\text{super}}\big|_{\text{no } F\text{-terms}} \qquad (8.39)
\end{aligned}
$$

$$
\begin{aligned}
&= i\bar{\tilde{b}}\bar{\sigma}^\mu \partial_\mu \tilde{b} - \frac{1}{2}\left(M_1 \tilde{b}\tilde{b} + M_1^* \bar{\tilde{b}}^a \bar{\tilde{b}}^a\right) + i\bar{\tilde{g}}_0^a \bar{\sigma}^\mu \partial_\mu \tilde{g}_0^a - \frac{1}{2}\left(M_3 \tilde{g}_0^a \tilde{g}_0^a + M_3^* \bar{\tilde{g}}_0^a \bar{\tilde{g}}_0^a\right) - ig_3 f^{abc}(\bar{\tilde{g}}_0^a \bar{\sigma}^\mu \tilde{g}_0^b) g_\mu^c \\
&\quad + i\bar{\tilde{w}}^+ \bar{\sigma}^\mu \partial_\mu \tilde{w}^+ + i\bar{\tilde{w}}^- \bar{\sigma}^\mu \partial_\mu \tilde{w}^- + i\bar{\tilde{w}}^3 \bar{\sigma}^\mu \partial_\mu \tilde{w}^3 - \left(M_2 \tilde{w}^+ \tilde{w}^- + M_2^* \bar{\tilde{w}}^+ \bar{\tilde{w}}^-\right) - \frac{1}{2}\left(M_2 \tilde{w}^3 \tilde{w}^3 + M_2^* \bar{\tilde{w}}^3 \bar{\tilde{w}}^3\right) \\
&\quad + g_2(\bar{\tilde{w}}^3 \bar{\sigma}^\mu \tilde{w}^- - \bar{\tilde{w}}^+ \bar{\sigma}^\mu \tilde{w}^3)W_\mu^+ - g_2(\bar{\tilde{w}}^3 \bar{\sigma}^\mu \tilde{w}^+ - \bar{\tilde{w}}^- \bar{\sigma}^\mu \tilde{w}^3)W_\mu^- + g_2(\bar{\tilde{w}}^+ \bar{\sigma}^\mu \tilde{w}^+ - \bar{\tilde{w}}^- \bar{\sigma}^\mu \tilde{w}^-)(c_{\text{w}} Z_\mu + s_{\text{w}} A_\mu) \\
&\quad + \bar{u}_{\text{L}}\bar{\sigma}^\mu(i\partial_\mu + g_3 \tau^a g_\mu^a)u_{\text{L}} + \bar{u}_{\text{R}}^c \bar{\sigma}^\mu(i\partial_\mu - g_3 \tau^{a*} g_\mu^a)u_{\text{R}}^c + i\bar{\nu}_{\text{L}}\bar{\sigma}^\mu \partial_\mu \nu_{\text{L}} \\
&\quad + \bar{d}_{\text{L}}\bar{\sigma}^\mu(i\partial_\mu + g_3 \tau^a g_\mu^a)d_{\text{L}} + \bar{d}_{\text{R}}^c \bar{\sigma}^\mu(i\partial_\mu - g_3 \tau^{a*} g_\mu^a)d_{\text{R}}^c + i\bar{e}_{\text{L}}\bar{\sigma}^\mu \partial_\mu e_{\text{L}} + i\bar{e}_{\text{R}}^c \bar{\sigma}^\mu \partial_\mu e_{\text{R}}^c \\
&\quad + i\bar{\tilde{h}}_{\text{d}}^- \bar{\sigma}^\mu \partial_\mu \tilde{h}_{\text{d}}^- + i\bar{\tilde{h}}_{\text{d}}^0 \bar{\sigma}^\mu \partial_\mu \tilde{h}_{\text{d}}^0 + i\bar{\tilde{h}}_{\text{u}}^+ \bar{\sigma}^\mu \partial_\mu \tilde{h}_{\text{u}}^+ + i\bar{\tilde{h}}_{\text{u}}^0 \bar{\sigma}^\mu \partial_\mu \tilde{h}_{\text{u}}^0 \\
&\quad + \frac{g_2}{\sqrt{2}}\left(\bar{u}_{\text{L}}\bar{\sigma}^\mu d_{\text{L}} + \bar{\nu}_{\text{L}}\bar{\sigma}^\mu e_{\text{L}} + \bar{\tilde{h}}_{\text{u}}^+ \bar{\sigma}^\mu \tilde{h}_{\text{u}}^0 + \bar{\tilde{h}}_{\text{d}}^0 \bar{\sigma}^\mu \tilde{h}_{\text{d}}^-\right)W_\mu^+ + \frac{g_2}{\sqrt{2}}\left(\bar{d}_{\text{L}}\bar{\sigma}^\mu u_{\text{L}} + \bar{e}_{\text{L}}\bar{\sigma}^\mu \nu_{\text{L}} + \bar{\tilde{h}}_{\text{u}}^0 \bar{\sigma}^\mu \tilde{h}_{\text{u}}^+ + \bar{\tilde{h}}_{\text{d}}^- \bar{\sigma}^\mu \tilde{h}_{\text{d}}^0\right)W_\mu^- \\
&\quad + \frac{g_Z(3 - 4s_{\text{w}}^2)}{6}\bar{u}_{\text{L}}\bar{\sigma}^\mu u_{\text{L}} Z_\mu + \frac{g_Z s_{\text{w}}^2}{3}\bar{u}_{\text{R}}^c \bar{\sigma}^\mu u_{\text{R}}^c Z_\mu + \frac{g_Z(2s_{\text{w}}^2 - 3)}{6}\bar{d}_{\text{L}}\bar{\sigma}^\mu d_{\text{L}} Z_\mu - \frac{g_Z s_{\text{w}}^2}{3}\bar{d}_{\text{R}}^c \bar{\sigma}^\mu d_{\text{R}}^c Z_\mu \\
&\quad + \frac{g_Z}{2}\bar{\nu}_{\text{L}}\bar{\sigma}^\mu \nu_{\text{L}} Z_\mu + \frac{g_Z(2s_{\text{w}}^2 - 1)}{2}\bar{e}_{\text{L}}\bar{\sigma}^\mu e_{\text{L}} Z_\mu - g_Z s_{\text{w}}^2 \bar{e}_{\text{R}}^c \bar{\sigma}^\mu e_{\text{R}}^c Z_\mu \\
&\quad + \frac{g_Z(1 - 2s_{\text{w}}^2)}{2}\bar{\tilde{h}}_{\text{u}}^+ \bar{\sigma}^\mu \tilde{h}_{\text{u}}^+ Z_\mu + \frac{g_Z}{2}\bar{\tilde{h}}_{\text{u}}^0 \bar{\sigma}^\mu \tilde{h}_{\text{u}}^0 Z_\mu + \frac{g_Z(2s_{\text{w}}^2 - 1)}{2}\bar{\tilde{h}}_{\text{d}}^- \bar{\sigma}^\mu \tilde{h}_{\text{d}}^- Z_\mu + \frac{g_Z}{2}\bar{\tilde{h}}_{\text{d}}^0 \bar{\sigma}^\mu \tilde{h}_{\text{d}}^0 Z_\mu \\
&\quad + \frac{2|e|}{3}(\bar{u}_{\text{L}}\bar{\sigma}^\mu u_{\text{L}} - \bar{u}_{\text{R}}^c \bar{\sigma}^\mu u_{\text{R}}^c)A_\mu - \frac{|e|}{3}(\bar{d}_{\text{L}}\bar{\sigma}^\mu d_{\text{L}} - \bar{d}_{\text{R}}^c \bar{\sigma}^\mu d_{\text{R}}^c)A_\mu - |e|(\bar{e}_{\text{L}}\bar{\sigma}^\mu e_{\text{L}} - \bar{e}_{\text{R}}^c \bar{\sigma}^\mu e_{\text{R}}^c)A_\mu \\
&\quad + |e|\bar{\tilde{h}}_{\text{u}}^+ \bar{\sigma}^\mu \tilde{h}_{\text{u}}^+ A_\mu - |e|\bar{\tilde{h}}_{\text{d}}^- \bar{\sigma}^\mu \tilde{h}_{\text{d}}^- A_\mu + \mathcal{L}_{\text{super}}\big|_{\text{no } F\text{-terms}};
\end{aligned}
$$

$$(8.40)$$

here,

$$
\begin{aligned}
\mathcal{L}_{\text{super}}\big|_{\text{no } F\text{-terms}} &= -\mu \tilde{h}_{\text{u}}^+ \tilde{h}_{\text{d}}^- + \mu \tilde{h}_{\text{u}}^0 \tilde{h}_{\text{d}}^0 + y_{uij} h_{\text{u}}^+ u_{\text{R}i}^c d_{\text{L}j} - y_{uij} h_{\text{u}}^0 u_{\text{R}i}^c u_{\text{L}j} + y_{uij}\tilde{d}_{\text{L}j}\tilde{h}_{\text{u}}^+ u_{\text{R}i}^c - y_{uij}\tilde{u}_{\text{L}j}\tilde{h}_{\text{u}}^0 u_{\text{R}i}^c \\
&\quad + y_{uji}\tilde{u}_{\text{R}j}^* \tilde{h}_{\text{u}}^+ d_{\text{L}i} - y_{uji}\tilde{u}_{\text{R}j}^* \tilde{h}_{\text{u}}^0 u_{\text{L}i} + y_{dij}h_{\text{d}}^- d_{\text{R}i}^c u_{\text{L}j} - y_{dij}h_{\text{d}}^0 d_{\text{R}i}^c d_{\text{L}j} - y_{dij}\tilde{d}_{\text{L}j}\tilde{h}_{\text{d}}^0 d_{\text{R}i}^c \\
&\quad + y_{dij}\tilde{u}_{\text{L}j}\tilde{h}_{\text{d}}^- d_{\text{R}i}^c + y_{dji}\tilde{d}_{\text{R}j}^* \tilde{h}_{\text{d}}^- u_{\text{L}i} - y_{dji}\tilde{d}_{\text{R}j}^* \tilde{h}_{\text{d}}^0 d_{\text{L}i} + y_{eij}h_{\text{d}}^- e_{\text{R}i}^c \nu_{\text{L}j} - y_{eij}h_{\text{d}}^0 e_{\text{R}i}^c e_{\text{L}j} \\
&\quad - y_{eij}\tilde{e}_{\text{L}j}\tilde{h}_{\text{d}}^0 e_{\text{R}i}^c + y_{eij}\tilde{\nu}_{\text{L}j}\tilde{h}_{\text{d}}^- e_{\text{R}i}^c + y_{eji}\tilde{e}_{\text{R}j}^* \tilde{h}_{\text{d}}^- \nu_{\text{L}i} - y_{eji}\tilde{e}_{\text{R}j}^* \tilde{h}_{\text{d}}^0 e_{\text{L}i} \\
&\quad - \kappa_i \tilde{h}_{\text{u}}^+ e_{\text{L}i} + \kappa_i \tilde{h}_{\text{u}}^0 \nu_{\text{L}i} - \lambda_{ijk}\tilde{e}_{\text{R}k}^* \nu_{\text{L}i} e_{\text{L}j} - \lambda_{jki}\tilde{e}_{\text{L}k}e_{\text{R}i}^c \nu_{\text{L}j} + \lambda_{jki}\tilde{\nu}_{\text{L}k}e_{\text{R}i}^c e_{\text{L}j} \\
&\quad - \lambda'_{jik}\tilde{d}_{\text{R}k}^* d_{\text{L}i}\nu_{\text{L}j} + \lambda'_{jik}\tilde{d}_{\text{R}k}^* u_{\text{L}i}e_{\text{L}j} - \lambda'_{jki}\tilde{d}_{\text{L}k}d_{\text{R}i}^c \nu_{\text{L}j} + \lambda'_{jki}\tilde{u}_{\text{L}k}d_{\text{R}i}^c e_{\text{L}j} + \lambda'_{kji}\tilde{e}_{\text{L}k}d_{\text{R}i}^c u_{\text{L}j} \\
&\quad - \lambda'_{kji}\tilde{\nu}_{\text{L}k}d_{\text{R}i}^c d_{\text{L}j} - \epsilon^{xyz}\lambda''_{ijk}\tilde{d}_{\text{R}k}^{x*} u_{\text{R}i}^{cy} d_{\text{R}j}^{cz} - \frac{1}{2}\epsilon^{xyz}\lambda''_{kij}\tilde{u}_{\text{R}k}^{x*} d_{\text{R}i}^{cy} d_{\text{R}j}^{cz} + \text{H.c.}
\end{aligned}
$$

$$(8.41)$$

8.3.3. Scalar-fermion-gaugino interaction

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SFG}} = {} & -g_2 \tilde{u}_{\mathrm{L}}^* d_{\mathrm{L}} \tilde{w}^+ - g_2 \tilde{u}_{\mathrm{L}} \bar{d}_{\mathrm{L}} \bar{\tilde{w}}^+ - g_2 \tilde{d}_{\mathrm{L}}^* u_{\mathrm{L}} \tilde{w}^- - g_2 \tilde{d}_{\mathrm{L}} \bar{u}_{\mathrm{L}} \bar{\tilde{w}}^- \\
& - \sqrt{2} g_3 \tilde{u}_{\mathrm{L}}^* \tau^a u_{\mathrm{L}} \tilde{g}_0^a + \sqrt{2} g_3 \tilde{u}_{\mathrm{R}}^* \tau^a \bar{u}_{\mathrm{R}}^{\mathrm{c}} \bar{\tilde{g}}_0^a - \sqrt{2} g_3 \tilde{u}_{\mathrm{L}} \tau^{a*} \bar{u}_{\mathrm{L}} \bar{\tilde{g}}_0^a + \sqrt{2} g_3 \tilde{u}_{\mathrm{R}} \tau^{a*} u_{\mathrm{R}}^{\mathrm{c}} \tilde{g}_0^a \\
& - \frac{g_2}{\sqrt{2}} \tilde{u}_{\mathrm{L}}^* u_{\mathrm{L}} \tilde{w}^3 - \frac{g_2}{\sqrt{2}} \tilde{u}_{\mathrm{L}} \bar{u}_{\mathrm{L}} \bar{\tilde{w}}^3 - \frac{g_Y}{3\sqrt{2}} \tilde{u}_{\mathrm{L}}^* u_{\mathrm{L}} \tilde{b} + \frac{2\sqrt{2} g_Y}{3} \tilde{u}_{\mathrm{R}}^* \bar{u}_{\mathrm{R}}^{\mathrm{c}} \bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}} \tilde{u}_{\mathrm{L}} \bar{u}_{\mathrm{L}} \bar{\tilde{b}} + \frac{2\sqrt{2} g_Y}{3} \tilde{u}_{\mathrm{R}} u_{\mathrm{R}}^{\mathrm{c}} \tilde{b} \\
& - \sqrt{2} g_3 \tilde{d}_{\mathrm{L}}^* \tau^a d_{\mathrm{L}} \tilde{g}_0^a + \sqrt{2} g_3 \tilde{d}_{\mathrm{R}}^* \tau^a \bar{d}_{\mathrm{R}}^{\mathrm{c}} \bar{\tilde{g}}_0^a - \sqrt{2} g_3 \tilde{d}_{\mathrm{L}} \tau^{a*} \bar{d}_{\mathrm{L}} \bar{\tilde{g}}_0^a + \sqrt{2} g_3 \tilde{d}_{\mathrm{R}} \tau^{a*} d_{\mathrm{R}}^{\mathrm{c}} \tilde{g}_0^a \\
& + \frac{g_2}{\sqrt{2}} \tilde{d}_{\mathrm{L}}^* d_{\mathrm{L}} \tilde{w}^3 + \frac{g_2}{\sqrt{2}} \tilde{d}_{\mathrm{L}} \bar{d}_{\mathrm{L}} \bar{\tilde{w}}^3 - \frac{g_Y}{3\sqrt{2}} \tilde{d}_{\mathrm{L}}^* d_{\mathrm{L}} \tilde{b} - \frac{\sqrt{2} g_Y}{3} \tilde{d}_{\mathrm{R}}^* \bar{d}_{\mathrm{R}}^{\mathrm{c}} \bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}} \tilde{d}_{\mathrm{L}} \bar{d}_{\mathrm{L}} \bar{\tilde{b}} - \frac{\sqrt{2} g_Y}{3} \tilde{d}_{\mathrm{R}} d_{\mathrm{R}}^{\mathrm{c}} \tilde{b} \\
& - g_2 \tilde{e}_{\mathrm{L}} \bar{\nu}_{\mathrm{L}} \bar{\tilde{w}}^- - g_2 \tilde{\nu}_{\mathrm{L}} \bar{e}_{\mathrm{L}} \bar{\tilde{w}}^+ - g_2 \tilde{e}_{\mathrm{L}}^* \nu_{\mathrm{L}} \tilde{w}^- - g_2 \tilde{\nu}_{\mathrm{L}}^* e_{\mathrm{L}} \tilde{w}^+ \\
& - \frac{g_2}{\sqrt{2}} \tilde{\nu}_{\mathrm{L}}^* \nu_{\mathrm{L}} \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} \tilde{\nu}_{\mathrm{L}}^* \nu_{\mathrm{L}} \tilde{b} - \frac{g_2}{\sqrt{2}} \tilde{\nu}_{\mathrm{L}} \bar{\nu}_{\mathrm{L}} \bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}} \tilde{\nu}_{\mathrm{L}} \bar{\nu}_{\mathrm{L}} \bar{\tilde{b}} \\
& + \frac{g_2}{\sqrt{2}} \tilde{e}_{\mathrm{L}}^* e_{\mathrm{L}} \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} \tilde{e}_{\mathrm{L}}^* e_{\mathrm{L}} \tilde{b} - \sqrt{2} g_Y \tilde{e}_{\mathrm{R}}^* \bar{e}_{\mathrm{R}}^{\mathrm{c}} \bar{\tilde{b}} + \frac{g_2}{\sqrt{2}} \tilde{e}_{\mathrm{L}} \bar{e}_{\mathrm{L}} \bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}} \tilde{e}_{\mathrm{L}} \bar{e}_{\mathrm{L}} \bar{\tilde{b}} - \sqrt{2} g_Y \tilde{e}_{\mathrm{R}} e_{\mathrm{R}}^{\mathrm{c}} \tilde{b} \\
& - g_2 h_{\mathrm{u}}^{+*} \tilde{h}_{\mathrm{u}}^0 \tilde{w}^+ - g_2 h_{\mathrm{d}}^0 \bar{\tilde{h}}_{\mathrm{d}}^- \bar{\tilde{w}}^+ - g_2 h_{\mathrm{d}}^{-*} \tilde{h}_{\mathrm{d}}^0 \tilde{w}^- - g_2 h_{\mathrm{u}}^0 \bar{\tilde{h}}_{\mathrm{u}}^+ \bar{\tilde{w}}^- \\
& - g_2 h_{\mathrm{d}}^{0*} \tilde{h}_{\mathrm{d}}^- \tilde{w}^+ - g_2 h_{\mathrm{u}}^{0*} \tilde{h}_{\mathrm{u}}^+ \tilde{w}^- - g_2 h_{\mathrm{u}}^+ \bar{\tilde{h}}_{\mathrm{u}}^0 \bar{\tilde{w}}^+ - g_2 h_{\mathrm{d}}^- \bar{\tilde{h}}_{\mathrm{d}}^0 \bar{\tilde{w}}^- \\
& - \frac{g_2}{\sqrt{2}} h_{\mathrm{u}}^+ \bar{\tilde{h}}_{\mathrm{u}}^+ \bar{\tilde{w}}^3 - \frac{g_Y}{\sqrt{2}} h_{\mathrm{u}}^+ \bar{\tilde{h}}_{\mathrm{u}}^+ \bar{\tilde{b}} - \frac{g_2}{\sqrt{2}} h_{\mathrm{u}}^{+*} \tilde{h}_{\mathrm{u}}^+ \tilde{w}^3 - \frac{g_Y}{\sqrt{2}} h_{\mathrm{u}}^{+*} \tilde{h}_{\mathrm{u}}^+ \tilde{b} + \frac{g_2}{\sqrt{2}} h_{\mathrm{d}}^{-*} \tilde{h}_{\mathrm{d}}^- \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} h_{\mathrm{d}}^{-*} \tilde{h}_{\mathrm{d}}^- \tilde{b} \\
& + \frac{g_2}{\sqrt{2}} h_{\mathrm{d}}^- \bar{\tilde{h}}_{\mathrm{d}}^- \bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}} h_{\mathrm{d}}^- \bar{\tilde{h}}_{\mathrm{d}}^- \bar{\tilde{b}} + \frac{g_2}{\sqrt{2}} h_{\mathrm{u}}^{0*} \tilde{h}_{\mathrm{u}}^0 \tilde{w}^3 - \frac{g_Y}{\sqrt{2}} h_{\mathrm{u}}^{0*} \tilde{h}_{\mathrm{u}}^0 \tilde{b} - \frac{g_2}{\sqrt{2}} h_{\mathrm{d}}^{0*} \tilde{h}_{\mathrm{d}}^0 \tilde{w}^3 + \frac{g_Y}{\sqrt{2}} h_{\mathrm{d}}^{0*} \tilde{h}_{\mathrm{d}}^0 \tilde{b} \\
& + \frac{g_2}{\sqrt{2}} h_{\mathrm{u}}^0 \bar{\tilde{h}}_{\mathrm{u}}^0 \bar{\tilde{w}}^3 - \frac{g_Y}{\sqrt{2}} h_{\mathrm{u}}^0 \bar{\tilde{h}}_{\mathrm{u}}^0 \bar{\tilde{b}} - \frac{g_2}{\sqrt{2}} h_{\mathrm{d}}^0 \bar{\tilde{h}}_{\mathrm{d}}^0 \bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}} h_{\mathrm{d}}^0 \bar{\tilde{h}}_{\mathrm{d}}^0 \bar{\tilde{b}}
\end{aligned}
\tag{8.42}
$$

8.3.4. Scalar part

$$\mathcal{L}_{\text{scalar}} = (\partial_\mu \tilde{u}_{\text{L}}^* + \mathrm{i}g_3 \tilde{u}_{\text{L}}^* \tau^a g_\mu^a)(\partial^\mu \tilde{u}_{\text{L}} - \mathrm{i}g_3 g^{b\mu} \tau^b \tilde{u}_{\text{L}}) + (\partial_\mu \tilde{u}_{\text{R}} - \mathrm{i}g_3 \tilde{u}_{\text{R}} \tau^{a*} g_\mu^a)(\partial^\mu \tilde{u}_{\text{R}}^* + \mathrm{i}g_3 g_\mu^b \tau^{b*} \tilde{u}_{\text{R}}^*)$$

$$+ (\partial_\mu \tilde{d}_{\text{L}}^* + \mathrm{i}g_3 \tilde{d}_{\text{L}}^* \tau^a g_\mu^a)(\partial^\mu \tilde{d}_{\text{L}} - \mathrm{i}g_3 g^{b\mu} \tau^b \tilde{d}_{\text{L}}) + (\partial_\mu \tilde{d}_{\text{R}} - \mathrm{i}g_3 \tilde{d}_{\text{R}}^y \tau^{a*} g_\mu^a)(\partial^\mu \tilde{d}_{\text{R}}^* + \mathrm{i}g_3 g_\mu^b \tau^{b*} \tilde{d}_{\text{R}}^*)$$

$$+ \sqrt{2}g_2 g_3 \tilde{u}_{\text{L}}^* \tau^a \tilde{d}_{\text{L}} W^{+\mu} g_\mu^a + \sqrt{2}g_2 g_3 \tilde{d}_{\text{L}}^* \tau^a \tilde{u}_{\text{L}} W^{-\mu} g_\mu^a + \frac{4}{3}g_3|e|(\tilde{u}_{\text{L}}^* \tau^a \tilde{u}_{\text{L}} + \tilde{u}_{\text{R}}^* \tau^a \tilde{u}_{\text{R}})g^{a\mu} A_\mu$$

$$- \frac{2}{3}g_3|e|(\tilde{d}_{\text{L}}^* \tau^a \tilde{d}_{\text{L}} + \tilde{d}_{\text{R}}^* \tau^a \tilde{d}_{\text{R}})g^{a\mu} A_\mu + \frac{(3 - 4\mathrm{s_w^2})g_Z}{3} g_3 \tilde{u}_{\text{L}}^* \tau^a \tilde{u}_{\text{L}} g^{a\mu} Z_\mu - \frac{4\mathrm{s_w^2}g_Z}{3} g_3 \tilde{u}_{\text{R}}^* \tau^a \tilde{u}_{\text{R}} g^{a\mu} Z_\mu$$

$$+ \frac{(2\mathrm{s_w^2} - 3)g_Z}{3} g_3 \tilde{d}_{\text{L}}^* \tau^a \tilde{d}_{\text{L}} g^{a\mu} Z_\mu + \frac{2\mathrm{s_w^2}g_Z}{3} g_3 \tilde{d}_{\text{R}}^* \tau^a \tilde{d}_{\text{R}} g^{a\mu} Z_\mu$$

$$+ \frac{\mathrm{i}g_2}{\sqrt{2}} W_\mu^+ \left(\tilde{u}_{\text{L}}^* \partial^\mu \tilde{d}_{\text{L}} - \tilde{d}_{\text{L}} \partial^\mu \tilde{u}_{\text{L}}^*\right) - \frac{\mathrm{i}g_2}{\sqrt{2}} W_\mu^- \left(\tilde{u}_{\text{L}} \partial^\mu \tilde{d}_{\text{L}}^* - \tilde{d}_{\text{L}}^* \partial^\mu \tilde{u}_{\text{L}}\right)$$

$$+ \frac{2\mathrm{i}}{3}|e|A_\mu \left(\tilde{u}_{\text{L}}^* \partial^\mu \tilde{u}_{\text{L}} - \tilde{u}_{\text{L}} \partial^\mu \tilde{u}_{\text{L}}^* + \tilde{u}_{\text{R}}^* \partial^\mu \tilde{u}_{\text{R}} - \tilde{u}_{\text{R}} \partial^\mu \tilde{u}_{\text{R}}^*\right) - \frac{\mathrm{i}}{3}|e|A_\mu \left(\tilde{d}_{\text{L}}^* \partial^\mu \tilde{d}_{\text{L}} - \tilde{d}_{\text{L}} \partial^\mu \tilde{d}_{\text{L}}^* + \tilde{d}_{\text{R}}^* \partial^\mu \tilde{d}_{\text{R}} - \tilde{d}_{\text{R}} \partial^\mu \tilde{d}_{\text{R}}^*\right)$$

$$+ \frac{\mathrm{i}(4\mathrm{s_w^2} - 3)g_Z}{6} Z_\mu \left(\tilde{u}_{\text{L}} \partial^\mu \tilde{u}_{\text{L}}^* - \tilde{u}_{\text{L}}^* \partial^\mu \tilde{u}_{\text{L}}\right) + \frac{\mathrm{i}(2\mathrm{s_w^2} - 3)g_Z}{6} Z_\mu \left(\tilde{d}_{\text{L}}^* \partial^\mu \tilde{d}_{\text{L}} - \tilde{d}_{\text{L}} \partial^\mu \tilde{d}_{\text{L}}^*\right)$$

$$- \frac{2\mathrm{i s_w^2}g_Z}{3} Z_\mu \left(\tilde{u}_{\text{R}}^* \partial^\mu \tilde{u}_{\text{R}} - \tilde{u}_{\text{R}} \partial^\mu \tilde{u}_{\text{R}}^*\right) + \frac{\mathrm{i s_w^2}g_Z}{3} Z_\mu \left(\tilde{d}_{\text{R}}^* \partial^\mu \tilde{d}_{\text{R}} - \tilde{d}_{\text{R}} \partial^\mu \tilde{d}_{\text{R}}^*\right)$$

$$+ \frac{g_2^2}{2} \left(|\tilde{u}_{\text{L}}|^2 + |\tilde{d}_{\text{L}}|^2\right) W^{+\mu} W_\mu^- - \frac{\mathrm{s_w^2}g_2 g_Z}{3\sqrt{2}} \tilde{u}_{\text{L}}^* \tilde{d}_{\text{L}} W^{+\mu} Z_\mu - \frac{\mathrm{s_w^2}g_2 g_Z}{3\sqrt{2}} \tilde{d}_{\text{L}}^* \tilde{u}_{\text{L}} W^{-\mu} Z_\mu$$

$$+ \frac{(3 - 4\mathrm{s_w^2})^2 g_Z^2}{36}|\tilde{u}_{\text{L}}|^2 Z^\mu Z_\mu + \frac{4\mathrm{s_w^4}g_Z^2}{9}|\tilde{u}_{\text{R}}|^2 Z^\mu Z_\mu + \frac{(3 - 2\mathrm{s_w^2})^2 g_Z^2}{36}|\tilde{d}_{\text{L}}|^2 Z^\mu Z_\mu + \frac{\mathrm{s_w^4}g_Z^2}{9}|\tilde{d}_{\text{R}}|^2 Z^\mu Z_\mu$$

$$+ \frac{4}{9}e^2 \left(|\tilde{u}_{\text{L}}|^2 + |\tilde{u}_{\text{R}}|^2\right) A^\mu A_\mu + \frac{1}{9}e^2 \left(|\tilde{d}_{\text{L}}|^2 + |\tilde{d}_{\text{R}}|^2\right) A^\mu A_\mu + \frac{|e|g_2}{3\sqrt{2}} \tilde{u}_{\text{L}}^* \tilde{d}_{\text{L}} W^{+\mu} A_\mu + \frac{|e|g_2}{3\sqrt{2}} \tilde{d}_{\text{L}}^* \tilde{u}_{\text{L}} W^{-\mu} A_\mu$$

$$+ \frac{2(3 - 4\mathrm{s_w^2})g_Z|e|}{9}|\tilde{u}_{\text{L}}|^2 A^\mu Z_\mu - \frac{8\mathrm{s_w^2}g_Z|e|}{9}|\tilde{u}_{\text{R}}|^2 A^\mu Z_\mu + \frac{(3 - 2\mathrm{s_w^2})g_Z|e|}{9}|\tilde{d}_{\text{L}}|^2 A^\mu Z_\mu - \frac{2\mathrm{s_w^2}g_Z|e|}{9}|\tilde{d}_{\text{R}}|^2 A^\mu Z_\mu$$

$$+ \partial_\mu \tilde{e}_{\text{R}} \partial^\mu \tilde{e}_{\text{R}}^* + \partial_\mu \tilde{e}_{\text{L}}^* \partial^\mu \tilde{e}_{\text{L}} + \partial_\mu \tilde{\nu}_{\text{L}}^* \partial^\mu \tilde{\nu}_{\text{L}} + \mathrm{i}\frac{g_2}{\sqrt{2}} W_\mu^+ (\tilde{\nu}_{\text{L}}^* \partial^\mu \tilde{e}_{\text{L}} - \tilde{e}_{\text{L}} \partial^\mu \tilde{\nu}_{\text{L}}^*) + \mathrm{i}\frac{g_2}{\sqrt{2}} W_\mu^- (\tilde{e}_{\text{L}}^* \partial^\mu \tilde{\nu}_{\text{L}} - \tilde{\nu}_{\text{L}} \partial^\mu \tilde{e}_{\text{L}}^*)$$

$$- \frac{\mathrm{i}(1 - 2\mathrm{s_w^2})g_Z}{2} Z_\mu (\tilde{e}_{\text{L}}^* \partial^\mu \tilde{e}_{\text{L}} - \tilde{e}_{\text{L}} \partial^\mu \tilde{e}_{\text{L}}^*) + \frac{\mathrm{i}g_Z}{2} Z_\mu (\tilde{\nu}_{\text{L}}^* \partial^\mu \tilde{\nu}_{\text{L}} - \tilde{\nu}_{\text{L}} \partial^\mu \tilde{\nu}_{\text{L}}^*) + \mathrm{i s_w^2}g_Z Z_\mu (\tilde{e}_{\text{R}}^* \partial^\mu \tilde{e}_{\text{R}} - \tilde{e}_{\text{R}} \partial^\mu \tilde{e}_{\text{R}}^*)$$

$$+ \mathrm{i}|e|A_\mu (\tilde{e}_{\text{L}} \partial^\mu \tilde{e}_{\text{L}}^* - \tilde{e}_{\text{L}}^* \partial^\mu \tilde{e}_{\text{L}} + \tilde{e}_{\text{R}} \partial^\mu \tilde{e}_{\text{R}}^* - \tilde{e}_{\text{R}}^* \partial^\mu \tilde{e}_{\text{R}})$$

$$+ \frac{g_2^2}{2}(|\tilde{\nu}_{\text{L}}|^2 + |\tilde{e}_{\text{L}}|^2)W^{+\mu} W_\mu^- + \frac{g_2 g_Z \mathrm{s_w^2}}{\sqrt{2}} \left(\tilde{e}_{\text{L}}^* \tilde{\nu}_{\text{L}} W_\mu^- Z^\mu + \tilde{\nu}_{\text{L}}^* \tilde{e}_{\text{L}} W_\mu^+ Z^\mu\right) - \frac{g_2|e|}{\sqrt{2}} \left(\tilde{\nu}_{\text{L}}^* \tilde{e}_{\text{L}} W_\mu^+ A^\mu + \tilde{e}_{\text{L}}^* \tilde{\nu}_{\text{L}} W_\mu^- A^\mu\right)$$

$$+ \frac{(1 - 2\mathrm{s_w^2})^2 g_Z^2}{4}|\tilde{e}_{\text{L}}|^2 Z^\mu Z_\mu + \frac{g_Z^2}{4}|\tilde{\nu}_{\text{L}}|^2 Z^\mu Z_\mu + g_Z^2 \mathrm{s_w^4}|\tilde{e}_{\text{R}}|^2 Z^\mu Z_\mu$$

$$+ e^2(|\tilde{e}_{\text{L}}|^2 + |\tilde{e}_{\text{R}}|^2)A^\mu A_\mu + (1 - 2\mathrm{s_w^2})|e|g_Z|\tilde{e}_{\text{L}}|^2 A^\mu Z_\mu - 2\mathrm{s_w^2}g_Z|e||\tilde{e}_{\text{R}}|^2 A^\mu Z_\mu$$

$$+ \partial_\mu h_{\text{d}}^{-*} \partial^\mu h_{\text{d}}^- + \partial_\mu h_{\text{d}}^{0*} \partial^\mu h_{\text{d}}^0 + \partial_\mu h_{\text{u}}^{+*} \partial^\mu h_{\text{u}}^+ + \partial_\mu h_{\text{u}}^{0*} \partial^\mu h_{\text{u}}^0 + \mathrm{i}\frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_{\text{u}}^{+*} \partial^\mu h_{\text{u}}^0 - h_{\text{u}}^0 \partial^\mu h_{\text{u}}^{+*}\right)$$

$$+ \mathrm{i}\frac{g_2}{\sqrt{2}} W_\mu^- \left(h_{\text{u}}^{0*} \partial^\mu h_{\text{u}}^+ - h_{\text{u}}^+ \partial^\mu h_{\text{u}}^{0*}\right) + \mathrm{i}\frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_{\text{d}}^{0*} \partial^\mu h_{\text{d}}^- - h_{\text{d}}^- \partial^\mu h_{\text{d}}^{0*}\right) + \mathrm{i}\frac{g_2}{\sqrt{2}} W_\mu^- \left(h_{\text{d}}^{-*} \partial^\mu h_{\text{d}}^0 - h_{\text{d}}^0 \partial^\mu h_{\text{d}}^{-*}\right)$$

$$+ \frac{\mathrm{i}(1 - 2\mathrm{s_w^2})g_Z}{2} Z_\mu \left(h_{\text{u}}^{+*} \partial^\mu h_{\text{u}}^+ - h_{\text{u}}^+ \partial^\mu h_{\text{u}}^{+*}\right) + \frac{\mathrm{i}(2\mathrm{s_w^2} - 1)g_Z}{2} Z_\mu \left(h_{\text{d}}^{-*} \partial^\mu h_{\text{d}}^- - h_{\text{d}}^- \partial^\mu h_{\text{d}}^{-*}\right)$$

$$- \frac{\mathrm{i}g_Z}{2} Z_\mu \left(h_{\text{u}}^{0*} \partial^\mu h_{\text{u}}^0 - h_{\text{u}}^0 \partial^\mu h_{\text{u}}^{0*}\right) + \frac{\mathrm{i}g_Z}{2} Z_\mu \left(h_{\text{d}}^{0*} \partial^\mu h_{\text{d}}^0 - h_{\text{d}}^0 \partial^\mu h_{\text{d}}^{0*}\right)$$

$$- \mathrm{i}|e|A_\mu \left(h_{\text{d}}^{-*} \partial^\mu h_{\text{d}}^- - h_{\text{d}}^- \partial^\mu h_{\text{d}}^{-*}\right) + \mathrm{i}|e|A_\mu \left(h_{\text{u}}^{+*} \partial^\mu h_{\text{u}}^+ - h_{\text{u}}^+ \partial^\mu h_{\text{u}}^{+*}\right)$$

$$+ \frac{g_2^2}{2} \left(|h_{\text{u}}^+|^2 + |h_{\text{u}}^0|^2 + |h_{\text{d}}^0|^2 + |h_{\text{d}}^-|^2\right) W^{+\mu} W_\mu^- + \frac{\mathrm{s_w^2}g_Z g_2}{\sqrt{2}} \left(h_{\text{d}}^{0*} h_{\text{d}}^- - h_{\text{u}}^{+*} h_{\text{u}}^0\right) W_\mu^+ Z^\mu$$

$$+ \frac{\mathrm{s_w^2}g_Z g_2}{\sqrt{2}} \left(h_{\text{d}}^{-*} h_{\text{d}}^0 - h_{\text{u}}^{0*} h_{\text{u}}^+\right) W_\mu^- Z^\mu + \frac{g_2|e|}{\sqrt{2}} \left(h_{\text{u}}^{+*} h_{\text{u}}^0 - h_{\text{d}}^{0*} h_{\text{d}}^-\right) W_\mu^+ A^\mu + \frac{g_2|e|}{\sqrt{2}} \left(h_{\text{u}}^{0*} h_{\text{u}}^+ - h_{\text{d}}^{-*} h_{\text{d}}^0\right) W_\mu^- A^\mu$$

$$+ \frac{(1 - 2\mathrm{s_w^2})^2 g_Z^2}{4} \left(|h_{\text{u}}^+|^2 + |h_{\text{d}}^-|^2\right) Z^\mu Z_\mu + \frac{g_Z^2}{4} \left(|h_{\text{u}}^0|^2 + |h_{\text{d}}^0|^2\right) Z^\mu Z_\mu + e^2 \left(|h_{\text{u}}^+|^2 + |h_{\text{d}}^-|^2\right) A^\mu A_\mu$$

$$+ (1 - 2\mathrm{s_w^2})|e|g_Z \left(|h_{\text{u}}^+|^2 + |h_{\text{d}}^-|^2\right) A^\mu Z_\mu$$

$$- (V_{\text{SUSY}} + V_{\cancel{\text{SUSY}}}),$$

25

$$(8.43)$$

where the scalar potential is given by

$$
\begin{aligned}
V_{\text{SUSY}} =\ & |h_{\text{u}}|^2 \left(|\mu|^2 + \sum_i |\kappa_i|^2 \right) + |\mu|^2 |h_{\text{d}}|^2 + \left(\kappa_i^* \mu \tilde{l}_{\text{L}i}^* h_{\text{d}} + \text{H.c.} \right) + \kappa_i^* \kappa_j \tilde{l}_{\text{L}i}^* \tilde{l}_{\text{L}j} \\
& + \Big[- y_{uij}\mu^* h_{\text{d}}^* \tilde{u}_{\text{R}i}^* \tilde{q}_{\text{L}j} - y_{uij}\kappa_k^* \tilde{u}_{\text{R}i}^* \tilde{q}_{\text{L}j} \tilde{l}_{\text{L}k}^* - \left(y_{dij}\mu^* + \lambda'_{kji}\kappa_k^* \right) h_{\text{u}}^* \tilde{d}_{\text{R}i}^* \tilde{q}_{\text{L}j} \\
& \qquad + y_{eij}\kappa_j^* \tilde{e}_{\text{R}i}^* h_{\text{u}}^* h_{\text{d}} + \left(\lambda_{jki}\kappa_k^* - y_{eij}\mu^* \right) h_{\text{u}}^* \tilde{e}_{\text{R}i}^* \tilde{l}_{\text{L}j} + \text{H.c.} \Big] \\
& + \frac{1}{8} \left(g_2^2 + g_Y^2 \right) |h_{\text{d}}|^4 + \frac{1}{8} \left(g_2^2 + g_Y^2 \right) |h_{\text{u}}|^4 + \left(-\frac{g_2^2}{4}|h_{\text{d}}|^2|h_{\text{u}}|^2 - \frac{g_Y^2}{4}|h_{\text{d}}|^2|h_{\text{u}}|^2 + \frac{g_2^2}{2}|h_{\text{d}}^* h_{\text{u}}|^2 \right) \\
& + \left(-\frac{g_2^2}{4}|h_{\text{u}}|^2|\tilde{q}_{\text{L}}|^2 + \frac{g_Y^2}{12}|h_{\text{u}}|^2|\tilde{q}_{\text{L}}|^2 + \frac{g_2^2}{2}|h_{\text{u}}^b \tilde{q}_{\text{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_u^\dagger y_u)_{ji} h_{\text{u}}^a h_{\text{u}}^{b*} \tilde{q}_{\text{L}i}^c \tilde{q}_{\text{L}j}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4}|h_{\text{d}}|^2|\tilde{q}_{\text{L}}|^2 - \frac{g_Y^2}{12}|h_{\text{d}}|^2|\tilde{q}_{\text{L}}|^2 + \frac{g_2^2}{2}|h_{\text{d}}^* \tilde{q}_{\text{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_d^\dagger y_d)_{ji} h_{\text{d}}^a h_{\text{d}}^{b*} \tilde{q}_{\text{L}i}^c \tilde{q}_{\text{L}j}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4}|h_{\text{u}}|^2|\tilde{l}_{\text{L}}|^2 - \frac{g_Y^2}{4}|h_{\text{u}}|^2|\tilde{l}_{\text{L}}|^2 + \frac{g_2^2}{2}|h_{\text{u}}^* \tilde{l}_{\text{L}i}|^2 \right) \\
& + \left(-\frac{g_2^2}{4}|h_{\text{d}}|^2|\tilde{l}_{\text{L}}|^2 + \frac{g_Y^2}{4}|h_{\text{d}}|^2|\tilde{l}_{\text{L}}|^2 + \frac{g_2^2}{2}|h_{\text{d}}^* \tilde{l}_{\text{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_e^\dagger y_e)_{ji} h_{\text{d}}^a h_{\text{d}}^{b*} \tilde{l}_{\text{L}i}^c \tilde{l}_{\text{L}j}^{d*} \right) \\
& + |h_{\text{u}}|^2 \left(-\frac{g_Y^2}{3}|\tilde{u}_{\text{R}}|^2 + (y_u y_u^\dagger)_{ij} \tilde{u}_{\text{R}i}^* \tilde{u}_{\text{R}j} \right) + \frac{g_Y^2}{3}|h_{\text{d}}|^2|\tilde{u}_{\text{R}}|^2 \\
& + \frac{g_Y^2}{6}|h_{\text{u}}|^2|\tilde{d}_{\text{R}}|^2 + |h_{\text{d}}|^2 \left(-\frac{g_Y^2}{6}|\tilde{d}_{\text{R}}|^2 + (y_d y_d^\dagger)_{ij} \tilde{d}_{\text{R}i}^* \tilde{d}_{\text{R}j} \right) - \Big[(y_u y_d^\dagger)_{ij} \tilde{u}_{\text{R}i}^* \tilde{d}_{\text{R}j}(h_{\text{d}}^* h_{\text{u}}) + \text{H.c.} \Big] \\
& + \frac{g_Y^2}{2}|h_{\text{u}}|^2|\tilde{e}_{\text{R}}|^2 + |h_{\text{d}}|^2 \left(-\frac{g_Y^2}{2}|\tilde{e}_{\text{R}}|^2 + (y_e y_e^\dagger)_{ij} \tilde{e}_{\text{R}i}^* \tilde{e}_{\text{R}j} \right) \\
& + \Big[-\frac{1}{2}\epsilon^{ab}\epsilon^{xyz} y_{ulk}\lambda''^*_{lij} h_{\text{u}}^a \tilde{d}_{\text{R}i}^x \tilde{d}_{\text{R}j}^y \tilde{q}_{\text{L}k}^{bz} + \epsilon^{ab}\epsilon^{xyz} y_{dlk}\lambda''^*_{ijl} h_{\text{d}}^a \tilde{u}_{\text{R}i}^x \tilde{d}_{\text{R}j}^y \tilde{q}_{\text{L}k}^{bz} - y_{uil}\lambda'^*_{klj} h_{\text{u}}^a \tilde{u}_{\text{R}i}^* \tilde{d}_{\text{R}j} \tilde{l}_{\text{L}k}^{a*} \\
& \qquad + y_{dil}\lambda'^*_{klj} h_{\text{d}}^a \tilde{d}_{\text{R}i}^* \tilde{d}_{\text{R}j} \tilde{l}_{\text{L}k}^{a*} - \epsilon^{ab}\epsilon^{cd} y_{dli}\lambda'^*_{kjl} h_{\text{d}}^a \tilde{q}_{\text{L}i}^b \tilde{q}_{\text{L}j}^{c*} \tilde{l}_{\text{L}k}^{d*} + y_{eil}\lambda'^*_{klj} h_{\text{d}}^a \tilde{e}_{\text{R}i}^* \tilde{e}_{\text{R}j} \tilde{l}_{\text{L}k}^{a*} \\
& \qquad - y_{ejl}\lambda'^*_{lki} h_{\text{d}}^a \tilde{e}_{\text{R}j}^* \tilde{d}_{\text{R}i} \tilde{q}_{\text{L}k}^{a*} + \frac{1}{2}\epsilon^{ab}\epsilon^{cd} y_{eli}\lambda^*_{jkl} h_{\text{d}}^a \tilde{l}_{\text{L}i}^b \tilde{l}_{\text{L}j}^{c*} \tilde{l}_{\text{L}k}^{d*} + \text{H.c.} \Big] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8} \right) |\tilde{q}_{\text{L}}|^4 + \frac{g_2^2}{4} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}j}^{by} \tilde{q}_{\text{L}i}^{bx*} \tilde{q}_{\text{L}j}^{ay*} + \frac{g_3^2}{4} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}j}^{by} \tilde{q}_{\text{L}i}^{ay*} \tilde{q}_{\text{L}j}^{bx*} \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{2g_Y^2}{9} \right) |\tilde{u}_{\text{R}}|^4 + \frac{g_3^2}{4} \tilde{u}_{\text{R}i}^{x*} \tilde{u}_{\text{R}j}^{y*} \tilde{u}_{\text{R}i}^y \tilde{u}_{\text{R}j}^x \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18} \right) |\tilde{d}_{\text{R}}|^4 + \frac{g_3^2}{4} \tilde{d}_{\text{R}i}^{x*} \tilde{d}_{\text{R}j}^{y*} \tilde{d}_{\text{R}i}^y \tilde{d}_{\text{R}j}^x + \frac{1}{2}\lambda''_{mij}\lambda''^*_{mkl} \tilde{d}_{\text{R}i}^{x*} \tilde{d}_{\text{R}j}^{y*} \tilde{d}_{\text{R}k}^x \tilde{d}_{\text{R}l}^y \right] \\
& + \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9} \right) |\tilde{u}_{\text{R}}|^2|\tilde{q}_{\text{L}}|^2 - \frac{g_3^2}{2} \tilde{u}_{\text{R}i}^{x*} \tilde{q}_{\text{L}j}^{ax} \tilde{u}_{\text{R}i}^y \tilde{q}_{\text{L}j}^{ay*} + y_{uik} y_{ujl}^* \tilde{u}_{\text{R}i}^{x*} \tilde{u}_{\text{R}j}^y \tilde{q}_{\text{L}k}^{ax} \tilde{q}_{\text{L}l}^{ay*} \right] \\
& + \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18} \right) |\tilde{d}_{\text{R}}|^2|\tilde{q}_{\text{L}}|^2 - \frac{g_3^2}{2} \tilde{d}_{\text{R}i}^{x*} \tilde{q}_{\text{L}j}^{ax} \tilde{d}_{\text{R}i}^y \tilde{q}_{\text{L}j}^{ay*} + \left(y_{dik} y_{djl}^* + \lambda'_{mki} \lambda'^*_{mlj} \right) \tilde{d}_{\text{R}i}^{x*} \tilde{q}_{\text{L}k}^{ax} \tilde{d}_{\text{R}j}^y \tilde{q}_{\text{L}l}^{ay*} \right] \\
& + \left[-\left(\frac{g_3^2}{6} + \frac{2g_Y^2}{9} \right) |\tilde{d}_{\text{R}}|^2|\tilde{u}_{\text{R}}|^2 + \left(\frac{g_3^2}{2} - \lambda''_{ikm}\lambda''^*_{ljm} \right) \tilde{u}_{\text{R}i}^{x*} \tilde{d}_{\text{R}j}^x \tilde{d}_{\text{R}j}^{y*} \tilde{u}_{\text{R}i}^y + \lambda''_{ikm}\lambda''^*_{jlm} \tilde{u}_{\text{R}i}^{x*} \tilde{u}_{\text{R}j}^x \tilde{d}_{\text{R}k}^{y*} \tilde{d}_{\text{R}l}^y \right] \\
& + \left[-\left(\frac{g_2^2}{4} + \frac{g_Y^2}{12} \right) |\tilde{l}_{\text{L}}|^2|\tilde{q}_{\text{L}}|^2 + \frac{g_2^2}{2} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}i}^{bx*} \tilde{l}_{\text{L}j}^b \tilde{l}_{\text{L}j}^{a*} + \epsilon^{ac}\epsilon^{bd} \lambda'_{kim}\lambda'^*_{ljm} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}j}^{bx*} \tilde{l}_{\text{L}k}^c \tilde{l}_{\text{L}l}^d \right] + \frac{g_Y^2}{6} |\tilde{e}_{\text{R}}|^2|\tilde{q}_{\text{L}}|^2 \\
& + \left(-\frac{g_Y^2}{6} |\tilde{d}_{\text{R}}|^2|\tilde{l}_{\text{L}}|^2 + \lambda'_{lmi}\lambda'^*_{kmj} \tilde{d}_{\text{R}i}^* \tilde{d}_{\text{R}j} \tilde{l}_{\text{L}k}^* \tilde{l}_{\text{L}l} \right) + \frac{g_Y^2}{3} |\tilde{u}_{\text{R}}|^2|\tilde{l}_{\text{L}}|^2 - \frac{2g_Y^2}{3} |\tilde{u}_{\text{R}}|^2|\tilde{e}_{\text{R}}|^2 + \frac{g_Y^2}{3} |\tilde{d}_{\text{R}}|^2|\tilde{e}_{\text{R}}|^2 \\
& + \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8} \right) |\tilde{l}_{\text{L}}|^4 + \frac{g_2^2}{4} \tilde{l}_{\text{L}i}^a \tilde{l}_{\text{L}j}^b \tilde{l}_{\text{L}i}^{b*} \tilde{l}_{\text{L}j}^{a*} + \frac{1}{4}\epsilon^{ab}\epsilon^{cd} \lambda_{ijm}\lambda^*_{klm} \tilde{l}_{\text{L}i}^a \tilde{l}_{\text{L}j}^b \tilde{l}_{\text{L}k}^{c*} \tilde{l}_{\text{L}l}^{d*} \right] \\
& + \frac{g_Y^2}{2} |\tilde{e}_{\text{R}}|^4 + \left[-\frac{g_Y^2}{2} |\tilde{e}_{\text{R}}|^2|\tilde{l}_{\text{L}}|^2 + \left(y_{eik} y_{ejl}^* + \lambda_{kmi}\lambda^*_{lmj} \right) \tilde{e}_{\text{R}i}^* \tilde{e}_{\text{R}j} \tilde{l}_{\text{L}k} \tilde{l}_{\text{L}l}^* \right] \\
& + \left[\left(y_{dik} y_{ejl}^* - \lambda'_{mki}\lambda^*_{lmj} \right) \tilde{d}_{\text{R}i}^{x*} \tilde{q}_{\text{L}k}^{ax} \tilde{e}_{\text{R}j} \tilde{l}_{\text{L}l}^{a*} - \epsilon^{ab}\epsilon^{xyz} \lambda'_{lkm}\lambda''^*_{ijm} \tilde{l}_{\text{L}l}^b \tilde{q}_{\text{L}k}^{az} \tilde{d}_{\text{R}j}^y \tilde{u}_{\text{R}i}^x + \text{H.c.} \right].
\end{aligned}
$$

$$(8.44)$$

8.4. Higgs mechanism and fermion composition

The scalar potential includes

$$
\begin{aligned}
V_{\text{SUSY}} \supset{}& |h_{\text{u}}|^2\Big(|\mu|^2 + \sum|\kappa_i|^2\Big) + |\mu|^2|h_{\text{d}}|^2 + \frac{g_Z^2}{8}\left(|h_{\text{u}}|^2 - |h_{\text{d}}|^2\right)^2 + \frac{g_2^2}{2}|h_{\text{d}}^* h_{\text{u}}|^2 \\
& + \Big(\kappa_i^* \mu \tilde{l}_{\text{L}i}^* h_{\text{d}} + \text{H.c.}\Big) + \kappa_i^*\kappa_j \tilde{l}_{\text{L}i}^* \tilde{l}_{\text{L}j}
\end{aligned}
\tag{8.45}
$$

$$
V_{\text{SUSY}} \supset m_{H_{\text{u}}}^2|h_{\text{u}}|^2 + m_{H_{\text{d}}}^2|h_{\text{d}}|^2 + \epsilon^{ab}\Big(bh_{\text{u}}^a h_{\text{d}}^b + b^* h_{\text{u}}^{a*} h_{\text{d}}^{b*} - b_i \tilde{l}_{\text{L}i}^a h_{\text{u}}^b - b_i^* \tilde{l}_{\text{L}i}^{a*} h_{\text{u}}^{b*}\Big) + \tilde{l}_{\text{L}i}^* M_{\text{L}i}^2 h_{\text{d}} + \tilde{l}_{\text{L}i} M_{\text{L}i}^{2*} h_{\text{d}}^*;
\tag{8.46}
$$

the Higgs mass term is given by

$$
V \supset \begin{pmatrix} h_{\text{u}} & h_{\text{d}}^* & \tilde{l}_{\text{L}i}^* \end{pmatrix}
\begin{pmatrix}
|\mu|^2 + m_{H_{\text{u}}}^2 + \sum|\kappa_i|^2 & b & -b_j \\
b^* & |\mu|^2 + m_{H_{\text{d}}}^2 & \kappa_j\mu^* + M_{\text{L}j}^{2*} \\
-b_i^* & \kappa_i^*\mu + M_{\text{L}i}^2 & (m_L^2)_{ij} + \kappa_i^*\kappa_j
\end{pmatrix}
\begin{pmatrix} h_{\text{u}}^* \\ h_{\text{d}} \\ \tilde{l}_{\text{L}j} \end{pmatrix}
\tag{8.47}
$$

while corresponding fermion terms are

$$
\mathcal{L} \supset \epsilon^{ab}\Big(-\mu \tilde{h}_{\text{u}}^a \tilde{h}_{\text{d}}^b - \kappa_i \tilde{h}_{\text{u}}^a l_{\text{L}i}^b\Big).
\tag{8.48}
$$

If the R-parity is not conserved, we redefine (H_{d}, L) superfields so that the mass matrix is block-diagonal, which corresponds to $\text{U}(4)_{H_{\text{d}},L} \to \text{U}(3)_L \times \text{U}(1)_{H_{\text{d}}}$ (DOF counting: $16 \to 9 + 1$ to remove b_i'). Then lepton and $\tilde{h}_{\text{d}}$ are mixed.[*8] With R-parity conservation, we do not suffer from these mixings.

8.4.1. Higgs potential and induced mass in R-parity conserved case

We perform "SU(2)-notation fixing", i.e., use the freedom associated to T_1 and T_2 of SU(2), so that $\langle h_{\text{u}}^+\rangle = 0$. Then $\langle h_{\text{d}}^-\rangle = 0$ and effectively

$$
V_{\text{pot}} = (|\mu|^2 + m_{H_{\text{u}}}^2)|h_{\text{u}}^0|^2 + (|\mu|^2 + m_{H_{\text{d}}}^2)|h_{\text{d}}^0|^2 + \frac{g_Z^2}{8}\left(|h_{\text{u}}^0|^2 - |h_{\text{d}}^0|^2\right)^2 - \left(bh_{\text{u}}^0 h_{\text{d}}^0 + \text{H.c.}\right).
\tag{8.49}
$$

We redefine H_{d} superfield so that $b > 0$.[*9] Then $\arg\langle h_{\text{u}}^0\rangle = -\arg\langle h_{\text{d}}^0\rangle$ and, with T_3-rotation, $\langle h_{\text{u}}^0\rangle > 0$ and $\langle h_{\text{d}}^0\rangle > 0$:

$$
\langle h_{\text{u}}^0\rangle =: v_{\text{u}} =: \frac{v_{\text{SM}}}{\sqrt{2}}\sin\beta, \qquad \langle h_{\text{d}}^0\rangle =: v_{\text{d}} =: \frac{v_{\text{SM}}}{\sqrt{2}}\cos\beta;
\tag{8.50}
$$

$$
V_{\text{pot}} = \frac{1}{2}(|\mu|^2 + m_{H_{\text{u}}}^2)v_{\text{SM}}^2\sin^2\beta + \frac{1}{2}(|\mu|^2 + m_{H_{\text{d}}}^2)v_{\text{SM}}^2\cos^2\beta + \frac{g_Z^2}{32}v_{\text{SM}}^4\cos^2 2\beta - \frac{1}{2}v_{\text{SM}}^2 b\sin 2\beta.
\tag{8.51}
$$

This potential can have two minima; one with $0 < \beta \leq \pi/4$ and the other with $\pi/4 \leq \beta < \pi/2$:

$$
\tan\beta = \frac{B \mp \sqrt{B^2 - 4b^2}}{2b} \quad \left(\cos 2\beta = \pm\frac{\sqrt{B^2 - 4b^2}}{B}\right), \qquad m_Z^2 := \frac{g_Z^2}{4}v_{\text{SM}}^2 = \left(\pm\frac{m_{H_{\text{d}}}^2 - m_{H_{\text{u}}}^2}{\sqrt{B^2 - 4b^2}} - 1\right)B,
\tag{8.52}
$$

where $B := 2|\mu|^2 + m_{H_{\text{u}}}^2 + m_{H_{\text{d}}}^2 > 2b > 0$ and m_Z is the Z-boson tree-level mass. Also,

$$
\sin 2\beta = \frac{2b}{2|\mu|^2 + m_{H_{\text{u}}}^2 + m_{H_{\text{d}}}^2}, \qquad m_Z^2 = \frac{-(m_{H_{\text{d}}}^2 - m_{H_{\text{u}}}^2)}{\cos 2\beta} - \left(2|\mu|^2 + m_{H_{\text{u}}}^2 + m_{H_{\text{d}}}^2\right)
\tag{8.53}
$$

are satisfied in both solutions.

Higgs sector The Nambu-Goldstone–Higgs mixings and the mass terms for the charged Higgs bosons are given by

$$
\begin{aligned}
\mathcal{L} \supset{}& \partial_\mu h_{\text{d}}^{-*}\partial^\mu h_{\text{d}}^- + \partial_\mu h_{\text{u}}^{+*}\partial^\mu h_{\text{u}}^+ + \left(-b - \frac{1}{2}g_2^2 v_{\text{u}} v_{\text{d}}\right)(h_{\text{u}}^+ h_{\text{d}}^- + h_{\text{u}}^{+*} h_{\text{d}}^{-*}) \\
& + \left[\frac{g_Y^2(v_{\text{u}}^2 - v_{\text{d}}^2) - g_2^2(v_{\text{u}}^2 + v_{\text{d}}^2)}{4} - |\mu|^2 - m_{H_{\text{d}}}^2\right]|h_{\text{d}}^-|^2 + \left[\frac{g_Y^2(v_{\text{d}}^2 - v_{\text{u}}^2) - g_2^2(v_{\text{u}}^2 + v_{\text{d}}^2)}{4} - |\mu|^2 - m_{H_{\text{u}}}^2\right]|h_{\text{u}}^+|^2 \\
& + \frac{ig_2}{\sqrt{2}}W_\mu^- \partial^\mu\left(v_{\text{u}} h_{\text{u}}^+ - v_{\text{u}} h_{\text{u}}^{+*} - v_{\text{d}} h_{\text{d}}^{-*} + v_{\text{d}} h_{\text{d}}^-\right)
\end{aligned}
\tag{8.54}
$$

and those for the neutral Higgs bosons are

$$
\begin{aligned}
\mathcal{L} \supset{}& \partial_\mu h_{\text{d}}^{0*}\partial^\mu h_{\text{d}}^0 + \partial_\mu h_{\text{u}}^{0*}\partial^\mu h_{\text{u}}^0 - \frac{g_Z^2 v_{\text{d}}^2}{8}(h_{\text{d}}^0 h_{\text{d}}^0 + h_{\text{d}}^{0*} h_{\text{d}}^{0*}) - \frac{g_Z^2 v_{\text{u}}^2}{8}(h_{\text{u}}^0 h_{\text{u}}^0 + h_{\text{u}}^{0*} h_{\text{u}}^{0*}) \\
& + \left(b + \frac{g_Z^2 v_{\text{u}} v_{\text{d}}}{4}\right)(h_{\text{u}}^0 h_{\text{d}}^0 + h_{\text{u}}^{0*} h_{\text{d}}^{0*}) + \frac{g_Z^2 v_{\text{u}} v_{\text{d}}}{4}(h_{\text{u}}^0 h_{\text{d}}^{0*} + h_{\text{u}}^{0*} h_{\text{d}}^0) \\
& + \left(\frac{g_Z^2(v_{\text{u}}^2 - 2v_{\text{d}}^2)}{4} - |\mu|^2 - m_{H_{\text{d}}}^2\right)|h_{\text{d}}^0|^2 + \left(\frac{g_Z^2(v_{\text{d}}^2 - 2v_{\text{u}}^2)}{4} - |\mu|^2 - m_{H_{\text{u}}}^2\right)|h_{\text{d}}^0|^2 \\
& + \frac{ig_Z}{2}Z_\mu\partial^\mu\left(v_{\text{d}} h_{\text{d}}^0 - v_{\text{d}} h_{\text{d}}^{0*} - v_{\text{u}} h_{\text{u}}^0 + v_{\text{u}} h_{\text{u}}^{0*}\right).
\end{aligned}
\tag{8.55}
$$

[*8]If we separated leptons and $\tilde{h}_{\text{d}}$ first, sleptons would acquire VEVs and lepton-gaugino mixings would be induced.
[*9]Note that T_3-rotation induces $h_{\text{u}}^0 \to e^{i\theta/2}h_{\text{u}}^0$ and $h_{\text{d}}^0 \to e^{-i\theta/2}h_{\text{d}}^0$; it cannot the remove phase of b.

Therefore, with $m_W := \mathrm{c_w} m_Z$ and

$$\begin{pmatrix} h_\mathrm{u}^+ \\ h_\mathrm{d}^{-*} \end{pmatrix} = \begin{pmatrix} \mathrm{s}_\beta & \mathrm{c}_\beta \\ -\mathrm{c}_\beta & \mathrm{s}_\beta \end{pmatrix} \begin{pmatrix} -\mathrm{i}G^+ \\ H^+ \end{pmatrix}, \quad \begin{pmatrix} h_\mathrm{u}^0 \\ h_\mathrm{d}^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_\mathrm{u} \\ \phi_\mathrm{d} \end{pmatrix} + \frac{\mathrm{i}}{\sqrt{2}} \begin{pmatrix} \mathrm{s}_\beta & \mathrm{c}_\beta \\ -\mathrm{c}_\beta & \mathrm{s}_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \tag{8.56}$$

we have

$$\mathcal{L} \supset \partial_\mu G^{+*} \partial^\mu G^+ + \partial_\mu H^{+*} \partial^\mu H^+ + m_W(W_\mu^- \partial^\mu G^+ + W_\mu^+ \partial^\mu G^{+*}) + \left(\frac{m_{H_\mathrm{d}}^2 - m_{H_\mathrm{u}}^2}{\cos 2\beta} + m_Z^2 \mathrm{s_w^2} \right) |H^+|^2$$

$$+ \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \frac{1}{2}(\partial_\mu A^0)^2 + \frac{1}{2}(\partial_\mu G^0)^2 + m_Z Z_\mu \partial^\mu G^0 - \frac{B}{2} A_0^2 \tag{8.57}$$

$$- \frac{1}{4}\left(B + m_Z^2 + (B - m_Z^2)\cos 2\beta \right) \phi_\mathrm{u}^2 - \frac{1}{4}\left(B + m_Z^2 - (B - m_Z^2)\cos 2\beta \right) \phi_\mathrm{d}^2 + \frac{1}{2}(B + m_Z^2)(\sin 2\beta)\phi_\mathrm{u}\phi_\mathrm{d}.$$

In particular, the tree-level masses are

$$m_{A_0}^2 = B = 2|\mu|^2 + m_{H_\mathrm{u}}^2 + m_{H_\mathrm{d}}^2, \tag{8.58}$$

$$m_{H^+}^2 = m_{A_0}^2 + m_W^2, \tag{8.59}$$

$$m_{h,H} = \frac{1}{2}\left(m_{A_0}^2 + m_Z^2 \mp \sqrt{\left(m_{A_0}^2 - m_Z^2 \right)^2 + 4 m_{A_0}^2 m_Z^2 \sin^2 2\beta} \right) \tag{8.60}$$

with

$$\begin{pmatrix} \phi_\mathrm{d} \\ \phi_\mathrm{u} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \frac{m_{A_0}^2 + m_Z^2}{m_{A_0}^2 - m_Z^2}. \tag{8.61}$$

The mixing α is stored in `ALPHA` block of SLHA, while `HMIX` stores

$$\mu = \mu, \quad \tan\beta = \tan\beta, \quad v = v_\mathrm{SM}(\sim 246\,\mathrm{GeV}), \quad m_A^2 = \frac{2b}{\sin 2\beta}, \tag{8.62}$$

at the scale specified. The above discussion holds even with CP-violation, but quantum corrections mix the three Higgs bosons; such information should be stored in `(IM)VCHMIX`. ♣TODO:discuss when needed♣

Mass terms in the Lagrangian   The other mass terms are given by

$$\mathcal{L} \supset m_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} M_3 \tilde{g}_0^a \tilde{g}_0^a - \frac{1}{2} M_3^* \bar{\tilde{g}}_0^a \bar{\tilde{g}}_0^a$$

$$+ \left(-\frac{1}{2} M_1 \tilde{b}\tilde{b} - \frac{1}{2} M_2 \tilde{w}^3 \tilde{w}^3 + \mu \tilde{h}_\mathrm{u}^0 \tilde{h}_\mathrm{d}^0 + \mathrm{c}_\beta m_Z \mathrm{s_w} \tilde{h}_\mathrm{d}^0 \tilde{b} - \mathrm{c_w} \mathrm{c}_\beta m_Z \tilde{h}_\mathrm{d}^0 \tilde{w}^3 - m_Z \mathrm{s_w} \mathrm{s}_\beta \tilde{h}_\mathrm{u}^0 \tilde{b} + \mathrm{c_w} m_Z \mathrm{s}_\beta \tilde{h}_\mathrm{u}^0 \tilde{w}^3 + \mathrm{h.c.} \right)$$

$$- M_2 \tilde{w}^+ \tilde{w}^- - \mu \tilde{h}_\mathrm{u}^+ \tilde{h}_\mathrm{d}^- - M_2^* \bar{\tilde{w}}^+ \bar{\tilde{w}}^- - \mu^* \bar{\tilde{h}}_\mathrm{u}^+ \bar{\tilde{h}}_\mathrm{d}^- - \sqrt{2} m_W \left(\mathrm{c}_\beta \tilde{h}_\mathrm{d}^- \tilde{w}^+ + \mathrm{s}_\beta \tilde{h}_\mathrm{u}^+ \tilde{w}^- + \mathrm{c}_\beta \bar{\tilde{h}}_\mathrm{d}^- \bar{\tilde{w}}^+ + \mathrm{s}_\beta \bar{\tilde{h}}_\mathrm{u}^+ \bar{\tilde{w}}^- \right)$$

$$- v_\mathrm{u} y_{uij} u_{\mathrm{R}i}^\mathrm{c} u_{\mathrm{L}j} - v_\mathrm{d} y_{dij} d_{\mathrm{R}i}^\mathrm{c} d_{\mathrm{L}j} - v_\mathrm{d} y_{eij} e_{\mathrm{R}i}^\mathrm{c} e_{\mathrm{L}j} - v_\mathrm{u} y_{uij}^* \bar{u}_{\mathrm{R}i}^\mathrm{c} \bar{u}_{\mathrm{L}j} - v_\mathrm{d} y_{dij}^* \bar{d}_{\mathrm{R}i}^\mathrm{c} \bar{d}_{\mathrm{L}j} - v_\mathrm{d} y_{eij}^* \bar{e}_{\mathrm{R}i}^\mathrm{c} \bar{e}_{\mathrm{L}j}$$

$$- \tilde{u}_\mathrm{L}^* \left(m_Q^2 + v_\mathrm{u}^2 y_u^\dagger y_u + \frac{3 - 4\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{u}_\mathrm{L} - \tilde{u}_\mathrm{R}^* \left(m_{U^c}^2 + v_\mathrm{u}^2 y_u y_u^\dagger + \frac{4\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{u}_\mathrm{R}$$

$$- v_\mathrm{u} a_{uij} \tilde{u}_{\mathrm{R}i}^* \tilde{u}_{\mathrm{L}j} + v_\mathrm{d} \mu^* y_{uij} \tilde{u}_{\mathrm{R}i}^* \tilde{u}_{\mathrm{L}j} - v_\mathrm{u} a_{uij}^* \tilde{u}_{\mathrm{R}i} \tilde{u}_{\mathrm{L}j}^* + v_\mathrm{d} \mu y_{uij}^* \tilde{u}_{\mathrm{R}i} \tilde{u}_{\mathrm{L}j}^* \tag{8.63}$$

$$- \tilde{d}_\mathrm{L}^* \left(m_Q^2 + v_\mathrm{d}^2 y_d^\dagger y_d + \frac{-3 + 2\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{d}_\mathrm{L} - \tilde{d}_\mathrm{R}^* \left(m_{D^c}^2 + v_\mathrm{d}^2 y_d y_d^\dagger + \frac{-2\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{d}_\mathrm{R}$$

$$- v_\mathrm{d} a_{dij} \tilde{d}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{L}j} + v_\mathrm{u} \mu^* y_{dij} \tilde{d}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{L}j} - v_\mathrm{d} a_{dij}^* \tilde{d}_{\mathrm{R}i} \tilde{d}_{\mathrm{L}j}^* + v_\mathrm{u} \mu y_{dij}^* \tilde{d}_{\mathrm{R}i} \tilde{d}_{\mathrm{L}j}^*$$

$$- \tilde{\nu}_\mathrm{L}^* \left(m_L^2 + \frac{1}{2} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{\nu}_\mathrm{L}$$

$$- \tilde{e}_\mathrm{L}^* \left(m_L^2 + v_\mathrm{d}^2 y_e^\dagger y_e + \frac{-1 + 2\mathrm{s_w^2}}{2} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{e}_\mathrm{L} - \tilde{e}_\mathrm{R}^* \left(m_{E^c}^2 + v_\mathrm{d}^2 y_e y_e^\dagger + (-\mathrm{s_w^2})\mathrm{c}_{2\beta} m_Z^2 \right) \tilde{e}_\mathrm{R}$$

$$- v_\mathrm{d} a_{eij} \tilde{e}_{\mathrm{R}i}^* \tilde{e}_{\mathrm{L}j} + v_\mathrm{u} \mu^* y_{eij} \tilde{e}_{\mathrm{R}i}^* \tilde{e}_{\mathrm{L}j} - v_\mathrm{d} a_{eij}^* \tilde{e}_{\mathrm{R}i} \tilde{e}_{\mathrm{L}j}^* + v_\mathrm{u} \mu y_{eij}^* \tilde{e}_{\mathrm{R}i} \tilde{e}_{\mathrm{L}j}^*,$$

where, at the tree level, the gauge boson mass m_W and m_Z, the gluino mass M_3, and matter-fermion masses $v_\mathrm{u} y_u$, $v_\mathrm{d} y_d$, and $v_\mathrm{d} y_e$ are given with the "correct" sign (as far as $M_3 > 0$, etc.).

Neutralinos and charginos   The mass matrices for neutralinos and charginos are given by

$$-\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_\mathrm{d}^0 \\ \tilde{h}_\mathrm{u}^0 \end{pmatrix}^\mathrm{T} \begin{pmatrix} M_1 & 0 & -\mathrm{c}_\beta \mathrm{s_w} m_Z & +\mathrm{s}_\beta \mathrm{s_w} m_Z \\ 0 & M_2 & +\mathrm{c}_\beta \mathrm{c_w} m_Z & -\mathrm{s}_\beta \mathrm{c_w} m_Z \\ -\mathrm{c}_\beta \mathrm{s_w} m_Z & +\mathrm{c}_\beta \mathrm{c_w} m_Z & 0 & -\mu \\ +\mathrm{s}_\beta \mathrm{s_w} m_Z & -\mathrm{s}_\beta \mathrm{c_w} m_Z & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_\mathrm{d}^0 \\ \tilde{h}_\mathrm{u}^0 \end{pmatrix} + \mathrm{h.c.}$$

$$+ \begin{pmatrix} \tilde{w}^- & \tilde{h}_\mathrm{d}^- \end{pmatrix} \begin{pmatrix} M_2 & \sqrt{2}\mathrm{s}_\beta m_W \\ \sqrt{2}\mathrm{c}_\beta m_W & \mu \end{pmatrix} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_\mathrm{u}^+ \end{pmatrix} + \begin{pmatrix} \bar{\tilde{w}}^- & \bar{\tilde{h}}_\mathrm{d}^- \end{pmatrix} \begin{pmatrix} M_2^* & \sqrt{2}\mathrm{s}_\beta m_W \\ \sqrt{2}\mathrm{c}_\beta m_W & \mu^* \end{pmatrix} \begin{pmatrix} \bar{\tilde{w}}^+ \\ \bar{\tilde{h}}_\mathrm{u}^+ \end{pmatrix}. \tag{8.64}$$

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Note that the mass matrices themselves are the same as those in SLHA convention, $\mathcal{M}_{\tilde{\psi}^0}$ and $\mathcal{M}_{\tilde{\psi}^+}$, while the fields are in different convention. Therefore, we continue our discussion based only on the mass matrices so that the discussion is free from the choice of field convention.

As $\mathcal{M}_{\tilde{\psi}^0}$ is a complex symmetric matrix, there is a unitary matrix $\tilde{N}$ such that $M_{\tilde{\psi}^0} = \tilde{N}^* \mathcal{M}_{\tilde{\psi}^0} \tilde{N}^\dagger$, where $M_{\tilde{\psi}^0}$ is a *positive* diagonal matrix whose elements are (non-negative) singular values of $\mathcal{M}_{\tilde{\psi}^0}$ and in increasing order (Autonne-Takagi factorization). In SLHA2 convention with CP-violation, this matrix $\tilde{N}$ is stored as the (IM)NMIX blocks and the (positive) masses are stored in the MASS block. Meanwhile, if M_1, M_2 and μ are real, $\mathcal{M}_{\tilde{\psi}^0}$ is a real symmetric matrix and there is a real orthogonal matrix $\hat{N}$ such that $\hat{M}_{\tilde{\psi}^0} = \hat{N}^* \mathcal{M}_{\tilde{\psi}^0} \hat{N}^\dagger = \hat{N} \mathcal{M}_{\tilde{\psi}^0} \hat{N}^{\mathrm{T}}$, where $\hat{M}_{\tilde{\psi}^0}$ is a *real* diagonal matrix whose elements are the eigenvalues of $\mathcal{M}_{\tilde{\psi}^0}$ and in absolute-value-increasing order (spectral theorem). This matrix $\hat{N}$ is the NMIX block of SLHA convention and $\hat{M}_{ii}$ is stored in the MASS block, hence MASS block may have negative values for neutralinos.

The chargino mass matrix $\mathcal{M}_{\tilde{\psi}^+}$ is decomposed as $M_{\tilde{\psi}^+} = U^* \mathcal{M}_{\tilde{\psi}^+} V^\dagger$, where U and V are unitary matrices and the elements of the diagonal matrix $M_{\tilde{\psi}^+}$ are singular values of $\mathcal{M}_{\tilde{\psi}^+}$ (thus non-negative) and sorted in increasing order (singular value decomposition). These U and V are stored in (IM)UMIX and (IM)VMIX, and the singular values are stored in MASS block. Because the SVD theorem is closed in $\mathbb{R}$, if M_2 and μ are real, U and V can be real, and the IM-blocks are omitted.

In summary,

$$M_{\tilde{\psi}^0} = \tilde{N}^* \mathcal{M}_{\tilde{\psi}^0} \tilde{N}^\dagger, \qquad \tilde{N} = \text{(IM)NMIX}, \qquad \text{(MASS)} = [M_{\tilde{\psi}^0}]_{ii} \geq 0 \quad \text{(singular values);} \tag{8.65}$$

$$\hat{M}_{\tilde{\psi}^0} = \hat{N} \mathcal{M}_{\tilde{\psi}^0} \hat{N}^{\mathrm{T}}, \qquad \hat{N} = \text{NMIX}, \qquad \text{(MASS)} = [\hat{M}_{\tilde{\psi}^0}]_{ii} \in \mathbb{R} \quad \text{(eigenvalues);} \tag{8.66}$$

$$M_{\tilde{\psi}^+} = U^* \mathcal{M}_{\tilde{\psi}^+} V^\dagger, \qquad U = \text{(IM)UMIX}, \quad V = \text{(IM)VMIX}, \quad \text{(MASS)} = [M_{\tilde{\psi}^+}]_{ii} \geq 0 \quad \text{(singular values).} \tag{8.67}$$

Note that the singular values are equal to absolute values of the eigenvalues, which guarantees consistency of the two decomposition.

We then define matrix N by[*10]

$$N = \begin{cases} \tilde{N} \\ \text{diag}(\varphi_i) \cdot \hat{N} \end{cases} = \text{diag}(\varphi_i) \cdot \Big(\text{(NMIX)} + \mathrm{i}\,\text{(IMNMIX)} \Big); \qquad \varphi_i = \begin{cases} 1 & \text{if } \text{(MASS)}_i \geq 0, \\ \mathrm{i} & \text{if } \text{(MASS)}_i < 0. \end{cases} \tag{8.68}$$

It gives the proper mass diagonalization in both of the NMIX convention:

$$N^* \mathcal{M}_{\tilde{\psi}^0} N^\dagger = \begin{cases} \tilde{N}^* \mathcal{M}_{\tilde{\psi}^0} \tilde{N}^\dagger = M_{\tilde{\psi}^0}, \\ \text{diag}(\varphi_i^*) \hat{N}^* \mathcal{M}_{\tilde{\psi}^0} \hat{N}^\dagger \, \text{diag}(\varphi_i^*) = \text{diag}(\varphi_i^*) \hat{M}_{\tilde{\psi}^0} \, \text{diag}(\varphi_i^*) \end{cases} = M_{\tilde{\psi}^0} \quad \text{(neutralino masses} \geq 0\text{).} \tag{8.69}$$

Noting that the discussion up here is irrelevant of the convention, we have the neutralino/chargino mass eigenstates,

$$\tilde{\chi}_i^0 = N_{ij} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_{\mathrm{d}}^0 \\ \tilde{h}_{\mathrm{u}}^0 \end{pmatrix}_j, \quad \tilde{\chi}_i^+ = V_{ij} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_{\mathrm{u}}^+ \end{pmatrix}_j, \quad \tilde{\chi}_i^- = U_{ij} \begin{pmatrix} \tilde{w}^- \\ \tilde{h}_{\mathrm{d}}^- \end{pmatrix}_j, \tag{8.70}$$

in our convention and the mass terms are now

$$-\mathcal{L} \supset \frac{1}{2} (\tilde{\chi}^0)^{\mathrm{T}} M_{\tilde{\psi}^0} \tilde{\chi}^0 + (\tilde{\chi}^-)^{\mathrm{T}} M_{\tilde{\psi}^+} \tilde{\chi}^+ + \text{h.c.} \tag{8.71}$$

Quarks, leptons, and super-CKM basis We here take the super-CKM basis. In the "original" Lagrangian,

$$-\mathcal{L} \supset u_{\mathrm{R}}^{\mathrm{c}} (v_{\mathrm{u}} y_u) u_{\mathrm{L}} + d_{\mathrm{R}}^{\mathrm{c}} (v_{\mathrm{d}} y_d) d_{\mathrm{L}} + e_{\mathrm{R}}^{\mathrm{c}} (v_{\mathrm{d}} y_e) e_{\mathrm{L}} + \text{h.c.} \tag{8.72}$$

$$= u_{\mathrm{R}}^{\mathrm{c}} (v_{\mathrm{u}} U_u y_u^{\mathrm{diag}} V_u^\dagger) u_{\mathrm{L}} + d_{\mathrm{R}}^{\mathrm{c}} (v_{\mathrm{d}} U_d y_d^{\mathrm{diag}} V_d^\dagger) d_{\mathrm{L}} + e_{\mathrm{R}}^{\mathrm{c}} (v_{\mathrm{d}} U_e y_e^{\mathrm{diag}} V_e^\dagger) e_{\mathrm{L}} + \text{h.c.}, \tag{8.73}$$

so the super-CKM basis is given by

$$\left[Q^1, Q^2, L, U^{\mathrm{c}}, D^{\mathrm{c}}, E^{\mathrm{c}} \right]_{\text{super-CKM}} = \left[V_u^\dagger Q^1, V_d^\dagger Q^2, V_e^\dagger L, U^{\mathrm{c}} U_u, D^{\mathrm{c}} U_d, E^{\mathrm{c}} U_e \right]_{\text{"original"}}. \tag{8.74}$$

Then the CKM mixings appear as, for example,

$$\left[\bar{u}_{\mathrm{L}} \bar{\sigma}^\mu d_{\mathrm{L}} W_\mu^+ + \bar{d}_{\mathrm{L}} \bar{\sigma}^\mu u_{\mathrm{L}} W_\mu^- \right]_{\text{"original"}} = \left[\bar{u}_{\mathrm{L}} V_u^\dagger V_d \bar{\sigma}^\mu d_{\mathrm{L}} W_\mu^+ + \bar{d}_{\mathrm{L}} V_d^\dagger V_u \bar{\sigma}^\mu u_{\mathrm{L}} W_\mu^- \right]_{\text{super-CKM}}; \tag{8.75}$$

i.e., defining $V_{\mathrm{CKM}} = V_u^\dagger V_d$ as in Sec. 6.5, the Lagrangian is amended as, e.g., $\bar{u}_{\mathrm{L}} d_{\mathrm{L}} \to \bar{u}_{\mathrm{L}} V_{\mathrm{CKM}} d_{\mathrm{L}}$, $\tilde{d}_{\mathrm{L}}^* \tilde{u}_{\mathrm{L}} \to \tilde{d}_{\mathrm{L}}^* V_{\mathrm{CKM}}^\dagger \tilde{u}_{\mathrm{L}}$.

The Θ_B- and Θ_W-terms are now removable. To this end, we rotate the matter superfields as

$$Q^1 \to \mathrm{e}^{\mathrm{i}\theta} Q^1, \quad Q^2 \to \mathrm{e}^{\mathrm{i}\theta'} Q^2, \quad U^{\mathrm{c}} \to \mathrm{e}^{-\mathrm{i}\theta} U^{\mathrm{c}}, \quad D^{\mathrm{c}} \to \mathrm{e}^{-\mathrm{i}\theta'} D^{\mathrm{c}}, \tag{8.76}$$

which induces $\Delta\Theta_G = 0$, $\Delta\Theta_W \propto \theta + \theta'$, and $\Delta\Theta_B \propto 5\theta + \theta'$, while the Lagrangian is unchanged except for the overall phase of V_{CKM}. So we can remove Θ_B and Θ_W, or in other words, pass the CP-violation in Θ-terms to V_{CKM}.[*11] If one (or more) quarks were massless, we could rotate the corresponding right-handed quark to remove Θ_G as well.

[*10]The sign of φ_i is arbitrary and (should be) unphysical.

[*11]There are other possible transformations $(L, E^{\mathrm{c}}) \to \mathrm{e}^{(\pm)\mathrm{i}\theta''}(L, E^{\mathrm{c}})$ and in total we may have $\Delta\Theta_W \propto \theta + \theta' + 2\theta''$ and $\Delta\Theta_B \propto 5\theta + \theta' + 6\theta''$. To remove both Θ-terms, we have to take $\theta \neq \theta'$ and V_{CKM} is anyway modified. The angle may chosen generation-dependently, and then $\Delta\Theta$'s should be read as $\theta \to \sum \theta_i/3$, etc.

Squark masses in super-CKM basis   Finally, the squark masses are given by

$$
\begin{aligned}
-\mathcal{L} \supset\ & \tilde{u}_{\mathrm{L}}^*\left(m_Q^2 + m_u^2 + \frac{3-4\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{u}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^*\left(m_{U^c}^2 + m_u^2 + \frac{4\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{u}_{\mathrm{R}} \\
& + \tilde{u}_{\mathrm{R}}^*(v_{\mathrm{u}}a_{\mathrm{u}} - \mu^* m_u\cot\beta)\tilde{u}_{\mathrm{L}} + \tilde{u}_{\mathrm{L}}^*(v_{\mathrm{u}}a_{\mathrm{u}}^\dagger - \mu m_u\cot\beta)\tilde{u}_{\mathrm{R}} \\
& + \tilde{d}_{\mathrm{L}}^*\left(V_d^\dagger(V_u m_Q^2 V_u^\dagger)V_d + m_d^2 + \frac{-3+2\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{d}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^*\left(m_{D^c}^2 + m_d^2 + \frac{-2\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{d}_{\mathrm{R}} \\
& + \tilde{d}_{\mathrm{R}}^*(v_{\mathrm{d}}a_d - \mu^* m_d\tan\beta)\tilde{d}_{\mathrm{L}} + \tilde{d}_{\mathrm{L}}^*(v_{\mathrm{d}}a_d^\dagger - \mu m_d\tan\beta)\tilde{d}_{\mathrm{R}} \\
& + \tilde{\nu}_{\mathrm{L}}^*\left(m_L^2 + \frac{1}{2}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{\nu}_{\mathrm{L}} \\
& + \tilde{e}_{\mathrm{L}}^*\left(m_L^2 + m_e^2 + \frac{-1+2\mathrm{s}_{\mathrm{w}}^2}{2}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{e}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^*\left(m_{E^c}^2 + m_e^2 + (-\mathrm{s}_{\mathrm{w}}^2)\mathrm{c}_{2\beta}m_Z^2\right)\tilde{e}_{\mathrm{R}} \\
& + \tilde{e}_{\mathrm{R}}^*(v_{\mathrm{d}}a_e - \mu^* m_e\tan\beta)\tilde{e}_{\mathrm{L}} + \tilde{e}_{\mathrm{L}}^*(v_{\mathrm{d}}a_e^\dagger - \mu m_e\tan\beta)\tilde{e}_{\mathrm{R}},
\end{aligned}
\tag{8.77}
$$

where the sfermion soft masses, yukawas, and a-terms are rewritten in super-CKM basis:

$$
[m_Q^2, m_{U^c}^2, m_{D^c}^2, m_L^2, m_{E^c}^2]_{\text{super-CKM}} = [V_d^\dagger m_Q^2 V_d, U_u^\dagger m_{U^c}^2 U_u, U_d^\dagger m_{D^c}^2 U_d, V_e^\dagger m_L^2 V_e, U_e^\dagger m_{E^c}^2 U_e]_{\text{“original”}},
\tag{8.78}
$$

$$
[a_u, a_d, a_e]_{\text{super-CKM}} = [U_u^\dagger a_u V_u, U_d^\dagger a_d V_d, U_e^\dagger a_e V_e]_{\text{“original”}}
\tag{8.79}
$$

(note that m_Q^2 is diagonalied for down-type; not for up-type). In matrix form,

$$
\begin{aligned}
-\mathcal{L} \supset\ & \begin{pmatrix}\tilde{u}_{\mathrm{L}i}^* & \tilde{u}_{\mathrm{R}i}^*\end{pmatrix}\begin{pmatrix}[V_{\mathrm{CKM}}m_Q^2 V_{\mathrm{CKM}}^\dagger]_{ij} + \left(m_u^2 + \frac{3-4\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij} & v_u[a_u^\dagger]_{ij} - (\mu m_u\cot\beta)\delta_{ij} \\ v_u[a_u]_{ij} - (\mu^* m_u\cot\beta)\delta_{ij} & [m_{U^c}^2]_{ij} + \left(m_u^2 + \frac{2\mathrm{s}_{\mathrm{w}}^2}{3}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\end{pmatrix}\begin{pmatrix}\tilde{u}_{\mathrm{L}j} \\ \tilde{u}_{\mathrm{R}j}\end{pmatrix} \\
& + \begin{pmatrix}\tilde{d}_{\mathrm{L}i}^* & \tilde{d}_{\mathrm{R}i}^*\end{pmatrix}\begin{pmatrix}[m_Q^2]_{ij} + \left(m_d^2 + \frac{-3+2\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij} & v_d[a_d^\dagger]_{ij} - (\mu m_d\tan\beta)\delta_{ij} \\ v_d[a_d]_{ij} - (\mu^* m_d\tan\beta)\delta_{ij} & [m_{D^c}^2]_{ij} + \left(m_d^2 - \frac{\mathrm{s}_{\mathrm{w}}^2}{3}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\end{pmatrix}\begin{pmatrix}\tilde{d}_{\mathrm{L}j} \\ \tilde{d}_{\mathrm{R}j}\end{pmatrix} \\
& + \tilde{\nu}_{\mathrm{L}i}^*\left([m_L^2]_{ij} + \left(\tfrac{1}{2}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\right)\tilde{\nu}_{\mathrm{L}j} \\
& + \begin{pmatrix}\tilde{e}_{\mathrm{L}i}^* & \tilde{e}_{\mathrm{R}i}^*\end{pmatrix}\begin{pmatrix}[m_L^2]_{ij} + \left(m_e^2 + \frac{-1+2\mathrm{s}_{\mathrm{w}}^2}{2}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij} & v_d[a_e^\dagger]_{ij} - (\mu m_e\tan\beta)\delta_{ij} \\ v_d[a_e]_{ij} - (\mu^* m_e\tan\beta)\delta_{ij} & [m_{E^c}^2]_{ij} + \left(m_e^2 - \mathrm{s}_{\mathrm{w}}^2\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\end{pmatrix}\begin{pmatrix}\tilde{e}_{\mathrm{L}j} \\ \tilde{e}_{\mathrm{R}j}\end{pmatrix}
\end{aligned}
\tag{8.80}
$$

$$
= \begin{pmatrix}\tilde{u}_{\mathrm{L}i}^* & \tilde{u}_{\mathrm{R}i}^*\end{pmatrix}\mathcal{M}_u\begin{pmatrix}\tilde{u}_{\mathrm{L}j} \\ \tilde{u}_{\mathrm{R}j}\end{pmatrix} + \begin{pmatrix}\tilde{d}_{\mathrm{L}i}^* & \tilde{d}_{\mathrm{R}i}^*\end{pmatrix}\mathcal{M}_d\begin{pmatrix}\tilde{d}_{\mathrm{L}j} \\ \tilde{d}_{\mathrm{R}j}\end{pmatrix} + \tilde{\nu}_{\mathrm{L}}^*\mathcal{M}_\nu\tilde{\nu}_{\mathrm{L}} + \begin{pmatrix}\tilde{e}_{\mathrm{L}i}^* & \tilde{e}_{\mathrm{R}i}^*\end{pmatrix}\mathcal{M}_e\begin{pmatrix}\tilde{e}_{\mathrm{L}j} \\ \tilde{e}_{\mathrm{R}j}\end{pmatrix}
\tag{8.81}
$$

The sfermion mass matrices are diagonalized by unitary matrices as

$$
\mathcal{M} = R\mathcal{M}^{\mathrm{diag}}R^\dagger; \quad \tilde{f}_i = R_{ij}\begin{pmatrix}\tilde{f}_{\mathrm{L}} \\ \tilde{f}_{\mathrm{R}}\end{pmatrix}_j = \begin{pmatrix}R_{ij}^{\mathrm{L}} & R_{ij}^{\mathrm{R}}\end{pmatrix}\begin{pmatrix}\tilde{f}_{\mathrm{L}j} \\ \tilde{f}_{\mathrm{R}j}\end{pmatrix}; \qquad \tilde{f}_{\mathrm{L}i} = [R^{\mathrm{L}\dagger}]_{ij}\tilde{f}_i, \quad \tilde{f}_{\mathrm{R}i} = [R^{\mathrm{R}\dagger}]_{ij}\tilde{f}_i.
\tag{8.82}
$$

where R_{ij} is 6×6 and $R_{ij}^{\mathrm{L,R}}$ are 3×6 matrices (except for sneutrinos).

These R-matrices are the same as `DSQMIX` etc. of SLHA2 format, but note that our notation for the other parameters is slightly different from SLHA's:

$$
m_{\tilde{Q},\tilde{L}}^2 = m_{Q,L}^2|_{\text{“orig”}}, \qquad m_{\tilde{u},\tilde{d},\tilde{e}}^2 = (m_{U^c,D^c,E^c}^2|_{\text{“orig”}})^{\mathrm{T}}, \qquad T_{U,D,E} = (a_{u,d,e}|_{\text{“orig”}})^{\mathrm{T}},
\tag{8.83}
$$

$$
\hat{m}_{\tilde{Q},\tilde{L}}^2 = m_{Q,L}^2|_{\text{sCKM}}, \qquad \hat{m}_{\tilde{u},\tilde{d},\tilde{e}}^2 = U^\dagger T^{\mathrm{T}} V = m_{U^c,D^c,E^c}^2|_{\text{sCKM}}, \qquad \hat{T}_{U,D,E} = U^\dagger T^{\mathrm{T}} V = a_{u,d,e}|_{\text{sCKM}},
\tag{8.84}
$$

together with $Y_{u,d,e} = (y_{u,d,e})^{\mathrm{T}}$. Anyway, the SLHA2 blocks corresponds to the variable in our convention as

$$
\texttt{(IM)MSX2(IN)} = m_{Q,L,U^c,D^c,E^c}^2|_{\text{sCKM};\overline{\mathrm{DR}}}, \qquad \texttt{(IM)TX(IN)} = a_{u,d,e}|_{\text{sCKM};\overline{\mathrm{DR}}}, \qquad \texttt{YX} = y_{u,d,e}|_{\text{sCKM};\overline{\mathrm{DR}}}.
\tag{8.85}
$$

The sfermion mass matrices above are in super-CKM basis, so their off-diagonal entries immediately induce flavor violation or sfermion left-right mixing. In old SLHA format, we assume that flavor- and CP-violation is absent and left-right mixing is ignorable except for third generation, which leads

$$
\mathcal{M}_d = \begin{pmatrix}\tilde{m}_{dL11}^2 & & & & & \\ & \tilde{m}_{dL22}^2 & & & & \\ & & \tilde{m}_{dL33}^2 & & & v_d a_{d33} - \mu m_b\tan\beta \\ & & & \tilde{m}_{dR11}^2 & & \\ & & & & \tilde{m}_{dR22}^2 & \\ & & v_d a_{d33} - \mu m_b\tan\beta & & & \tilde{m}_{dR33}^2\end{pmatrix}; \quad \begin{pmatrix}\tilde{d}_{\mathrm{L}} \\ \tilde{s}_{\mathrm{L}} \\ \tilde{b}_1 \\ \tilde{d}_{\mathrm{R}} \\ \tilde{s}_{\mathrm{R}} \\ \tilde{b}_2\end{pmatrix} = \begin{pmatrix}1 & & & & & \\ & 1 & & & & \\ & & F_{11} & & & F_{12} \\ & & & 1 & & \\ & & & & 1 & \\ & & F_{21} & & & F_{22}\end{pmatrix}\begin{pmatrix}\tilde{d}_{\mathrm{L}j} \\ \tilde{d}_{\mathrm{R}j}\end{pmatrix}
\tag{8.86}
$$

and these F_{ij} are stored in `SBOTMIX` etc.

Squark masses in super-CKM basis   The gluino mass and phase is given by

$$
m_{\tilde{g}} = |M_3|; \quad M_3 = m_{\tilde{g}}\,\mathrm{e}^{\mathrm{i}\theta_3} = m_{\tilde{g}}\varphi_{\tilde{g}}^{-2}; \quad \tilde{g}_0 = \varphi_{\tilde{g}}\tilde{g}.
\tag{8.87}
$$

8.4.2. Fermion composition

Now we show the fermion-related Lagrangian terms verbosely. In super-CKM basis, the interaction terms are

$$
\begin{aligned}
\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}} &= \mathrm{i}\bar{\tilde{\chi}}_i^0 \bar{\sigma}^\mu \partial_\mu \tilde{\chi}_i^0 + \mathrm{i}\bar{\tilde{\chi}}_i^- \bar{\sigma}^\mu \partial_\mu \tilde{\chi}_i^- + \mathrm{i}\bar{\tilde{\chi}}_i^+ \bar{\sigma}^\mu \partial_\mu \tilde{\chi}_i^+ + \mathrm{i}\bar{\tilde{g}}^a \bar{\sigma}^\mu \partial_\mu \tilde{g}^a + \mathrm{i}\bar{u}_{\mathrm{L}i} \bar{\sigma}^\mu \partial_\mu u_{\mathrm{L}i} + \mathrm{i}\bar{u}_{\mathrm{R}i}^{\mathrm{c}} \bar{\sigma}^\mu \partial_\mu u_{\mathrm{R}i}^{\mathrm{c}} \\
&\quad + \mathrm{i}\bar{d}_{\mathrm{L}i} \bar{\sigma}^\mu \partial_\mu d_{\mathrm{L}i} + \mathrm{i}\bar{d}_{\mathrm{R}i}^{\mathrm{c}} \bar{\sigma}^\mu \partial_\mu d_{\mathrm{R}i}^{\mathrm{c}} + \mathrm{i}\bar{\nu}_{\mathrm{L}i} \bar{\sigma}^\mu \partial_\mu \nu_{\mathrm{L}i} + \mathrm{i}\bar{e}_{\mathrm{L}i} \bar{\sigma}^\mu \partial_\mu e_{\mathrm{L}i} + \mathrm{i}\bar{e}_{\mathrm{R}i}^{\mathrm{c}} \bar{\sigma}^\mu \partial_\mu e_{\mathrm{R}i}^{\mathrm{c}} - \frac{m_{\tilde{g}}}{2}(\tilde{g}^a \tilde{g}^a + \text{h.c.}) \\
&\quad - m_{\tilde{\chi}_i^\pm}\left(\tilde{\chi}_i^- \tilde{\chi}_i^+ + \bar{\tilde{\chi}}_i^- \bar{\tilde{\chi}}_i^+\right) - m_{di}\left(d_{\mathrm{R}i}^{\mathrm{c}} d_{\mathrm{L}i} + \bar{d}_{\mathrm{R}i}^{\mathrm{c}} \bar{d}_{\mathrm{L}i}^x\right) - m_{ui}\left(u_{\mathrm{R}i}^{\mathrm{c}} u_{\mathrm{L}j} + \bar{u}_{\mathrm{R}i}^{\mathrm{c}} \bar{u}_{\mathrm{L}i}\right) - m_{ei}\left(e_{\mathrm{R}i}^{\mathrm{c}} e_{\mathrm{L}i} + \bar{e}_{\mathrm{R}i}^{\mathrm{c}} \bar{e}_{\mathrm{L}i}\right) \\
&\quad + \left[\sqrt{2}g_3 \varphi_{\tilde{g}} \tilde{g}^a \left(\tau^{yxa*} \tilde{d}_{\mathrm{R}}^y d_{\mathrm{R}}^{cx} + \tau^{yxa*} \tilde{u}_{\mathrm{R}}^y u_{\mathrm{R}}^{cx} - \tau^{yxa} \tilde{d}_{\mathrm{L}}^{y*} d_{\mathrm{L}}^x - \tau^{yxa} \tilde{u}_{\mathrm{L}}^{y*} u_{\mathrm{L}}^x\right) + \text{h.c.}\right] - \mathrm{i}g_3 f^{abc} g_\mu^a (\bar{\tilde{g}}^b \bar{\sigma}^\mu \tilde{g}^c) \\
&\quad + g_3 g_\mu^a \left(\bar{d}_{\mathrm{L}} \bar{\sigma}^\mu \tau^a d_{\mathrm{L}} + \bar{u}_{\mathrm{L}} \bar{\sigma}^\mu \tau^a u_{\mathrm{L}} - \bar{d}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu \tau^{a*} d_{\mathrm{R}}^{\mathrm{c}} - \bar{u}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu \tau^{a*} u_{\mathrm{R}}^{\mathrm{c}}\right) + A_\mu |e|\left[\frac{2}{3}\bar{u}_{\mathrm{L}} \bar{\sigma}^\mu u_{\mathrm{L}} - \frac{2}{3}\bar{u}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu u_{\mathrm{R}}^{\mathrm{c}} - \frac{1}{3}\bar{d}_{\mathrm{L}} \bar{\sigma}^\mu d_{\mathrm{L}}\right. \\
&\quad \left. + \frac{1}{3}\bar{d}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu d_{\mathrm{R}}^{\mathrm{c}} - \bar{e}_{\mathrm{L}} \bar{\sigma}^\mu e_{\mathrm{L}} + \bar{e}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu e_{\mathrm{R}}^{\mathrm{c}} + \bar{\tilde{\chi}}_i^+ \bar{\sigma}^\mu \tilde{\chi}_i^+ - \bar{\tilde{\chi}}_i^- \bar{\sigma}^\mu \tilde{\chi}_i^-\right] + g_Z Z_\mu \left[\frac{3 - 4\mathrm{s}_{\mathrm{w}}^2}{6}\bar{u}_{\mathrm{L}} \bar{\sigma}^\mu u_{\mathrm{L}} + \frac{2\mathrm{s}_{\mathrm{w}}^2}{3}\bar{u}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu u_{\mathrm{R}}^{\mathrm{c}}\right. \\
&\quad \left. - \frac{3 - 2\mathrm{s}_{\mathrm{w}}^2}{6}\bar{d}_{\mathrm{L}} \bar{\sigma}^\mu d_{\mathrm{L}} - \frac{\mathrm{s}_{\mathrm{w}}^2}{3}\bar{d}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu d_{\mathrm{R}}^{\mathrm{c}} + \frac{1}{2}\bar{\nu}_{\mathrm{L}} \bar{\sigma}^\mu \nu_{\mathrm{L}} + \frac{2\mathrm{s}_{\mathrm{w}}^2 - 1}{2}\bar{e}_{\mathrm{L}} \bar{\sigma}^\mu e_{\mathrm{L}} - \mathrm{s}_{\mathrm{w}}^2 \bar{e}_{\mathrm{R}}^{\mathrm{c}} \bar{\sigma}^\mu e_{\mathrm{R}}^{\mathrm{c}} + \frac{N_{j3}N_{i3}^* - N_{j4}N_{i4}^*}{2}\bar{\tilde{\chi}}_j^0 \bar{\sigma}^\mu \tilde{\chi}_i^0\right. \\
&\quad \left. + \left(\mathrm{c}_{\mathrm{w}}^2 V_{j1} V_{i1}^* + \frac{V_{j2}V_{i2}^*(\mathrm{c}_{\mathrm{w}}^2 - \mathrm{s}_{\mathrm{w}}^2)}{2}\right)\bar{\tilde{\chi}}_j^+ \bar{\sigma}^\mu \tilde{\chi}_i^+ + \left(\frac{U_{j2}U_{i2}^*(\mathrm{s}_{\mathrm{w}}^2 - \mathrm{c}_{\mathrm{w}}^2)}{2} - \mathrm{c}_{\mathrm{w}}^2 U_{j1} U_{i1}^*\right)\bar{\tilde{\chi}}_j^- \bar{\sigma}^\mu \tilde{\chi}_i^-\right] \\
&\quad + \frac{g_2}{\sqrt{2}}W_\mu^+ \left[\bar{u}_{\mathrm{L}} \bar{\sigma}^\mu V_{\mathrm{CKM}} d_{\mathrm{L}} + \bar{\nu}_{\mathrm{L}} \bar{\sigma}^\mu e_{\mathrm{L}} + \left(V_{j2}N_{i4}^* - \sqrt{2}V_{j1}N_{i2}^*\right)\bar{\tilde{\chi}}_j^+ \bar{\sigma}^\mu \tilde{\chi}_i^0 + \left(\sqrt{2}N_{j2}U_{i1}^* + N_{j3}U_{i2}^*\right)\bar{\tilde{\chi}}_j^0 \bar{\sigma}^\mu \tilde{\chi}_i^-\right] \\
&\quad + \frac{g_2}{\sqrt{2}}W_\mu^- \left[\bar{d}_{\mathrm{L}} \bar{\sigma}^\mu V_{\mathrm{CKM}}^\dagger u_{\mathrm{L}} + \bar{e}_{\mathrm{L}} \bar{\sigma}^\mu \nu_{\mathrm{L}} + \left(N_{j4}V_{i2}^* - \sqrt{2}N_{j2}V_{i1}^*\right)\bar{\tilde{\chi}}_j^0 \bar{\sigma}^\mu \tilde{\chi}_i^+ + \left(\sqrt{2}U_{j1}N_{i2}^* + U_{j2}N_{i3}^*\right)\bar{\tilde{\chi}}_j^- \bar{\sigma}^\mu \tilde{\chi}_i^0\right] \\
&\quad + \phi_{\mathrm{u}}\left(-\frac{y_{ui}}{\sqrt{2}}u_{\mathrm{R}i}^{\mathrm{c}} u_{\mathrm{L}i} + \frac{N_{i4}^*(g_2 N_{j2}^* - g_Y N_{j1}^*)}{2}\tilde{\chi}_i^0 \tilde{\chi}_j^0 - \frac{g_2 U_{i1}^* V_{j2}^*}{\sqrt{2}}\tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.}\right) \\
&\quad + \phi_{\mathrm{d}}\left(-\frac{y_{di}}{\sqrt{2}}d_{\mathrm{R}i}^{\mathrm{c}} d_{\mathrm{L}i} - \frac{y_{ei}}{\sqrt{2}}e_{\mathrm{R}i}^{\mathrm{c}} e_{\mathrm{L}i} + \frac{N_{i3}^*(g_Y N_{j1}^* - g_2 N_{j2}^*)}{2}\tilde{\chi}_i^0 \tilde{\chi}_j^0 - \frac{g_2 U_{i2}^* V_{j1}^*}{\sqrt{2}}\tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.}\right) \\
&\quad + \frac{A^0}{\sqrt{2}}\left[-\mathrm{i}\mathrm{c}_{\beta 0}y_{ui}u_{\mathrm{R}i}^{\mathrm{c}} u_{\mathrm{L}i} - \mathrm{i}\mathrm{s}_{\beta 0}y_{di}d_{\mathrm{R}i}^{\mathrm{c}} d_{\mathrm{L}i} - \mathrm{i}\mathrm{s}_{\beta 0}y_{ei}e_{\mathrm{R}i}^{\mathrm{c}} e_{\mathrm{L}i} - \frac{\mathrm{i}(\mathrm{s}_{\beta 0}N_{i3}^* - \mathrm{c}_{\beta 0}N_{i4}^*)(g_Y N_{j1}^* - g_2 N_{j2}^*)}{\sqrt{2}}\tilde{\chi}_i^0 \tilde{\chi}_j^0\right. \\
&\quad \left. + \mathrm{i}g_2\left(\mathrm{c}_{\beta 0}U_{i1}^* V_{j2}^* + \mathrm{s}_{\beta 0}U_{i2}^* V_{j1}^*\right)\tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.}\right] + \frac{G^0}{\sqrt{2}}\left[-\mathrm{i}\mathrm{s}_{\beta 0}y_{ui}u_{\mathrm{R}i}^{\mathrm{c}} u_{\mathrm{L}i} + \mathrm{i}\mathrm{c}_{\beta 0}y_{di}d_{\mathrm{R}i}^{\mathrm{c}} d_{\mathrm{L}i} + \mathrm{i}\mathrm{c}_{\beta 0}y_{ei}e_{\mathrm{R}i}^{\mathrm{c}} e_{\mathrm{L}i}\right. \\
&\quad \left. + \frac{\mathrm{i}(g_Y N_{j1}^* - g_2 N_{j2}^*)(\mathrm{c}_{\beta 0}N_{i3}^* + \mathrm{s}_{\beta 0}N_{i4}^*)}{\sqrt{2}}\tilde{\chi}_i^0 \tilde{\chi}_j^0 - \mathrm{i}g_2(\mathrm{c}_{\beta 0}U_{i2}^* V_{j1}^* - \mathrm{s}_{\beta 0}U_{i1}^* V_{j2}^*)\tilde{\chi}_i^- \tilde{\chi}_j^+ + \text{h.c.}\right] \\
&\quad + \left[\left(\frac{U_{i2}^*(g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} - g_2 U_{i1}^* N_{j3}^*\right)\mathrm{s}_{\beta +} H^+ \tilde{\chi}_i^- \tilde{\chi}_j^0 - \left(g_2 V_{i1}^* N_{j4}^* + \frac{V_{i2}^*\left(g_Y N_{j1}^* + g_2 N_{j2}^*\right)}{\sqrt{2}}\right)\mathrm{c}_{\beta +} H^- \tilde{\chi}_i^+ \tilde{\chi}_j^0\right. \\
&\quad \left. + \mathrm{c}_{\beta +}(u_{\mathrm{R}}^{\mathrm{c}} y_u V_{\mathrm{CKM}} d_{\mathrm{L}})H^+ + \mathrm{s}_{\beta +}(d_{\mathrm{R}}^{\mathrm{c}} y_d V_{\mathrm{CKM}}^\dagger u_{\mathrm{L}})H^- + \mathrm{s}_{\beta +}(e_{\mathrm{R}}^{\mathrm{c}} y_e \nu_{\mathrm{L}})H^- + \text{h.c.}\right] \\
&\quad + \left[\mathrm{i}G^+ \mathrm{c}_{\beta +}\left(\frac{U_{i2}^*(g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} - g_2 U_{i1}^* N_{j3}^*\right)\tilde{\chi}_i^- \tilde{\chi}_j^0 - \mathrm{i}G^- \mathrm{s}_{\beta +}\left(g_2 V_{i1}^* N_{j4}^* + \frac{V_{i2}^*(g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}}\right)\tilde{\chi}_i^+ \tilde{\chi}_j^0\right. \\
&\quad \left. - \mathrm{i}\mathrm{s}_{\beta +}(u_{\mathrm{R}}^{\mathrm{c}} y_u V_{\mathrm{CKM}} d_{\mathrm{L}})G^+ - \mathrm{i}\mathrm{c}_{\beta +}(d_{\mathrm{R}}^{\mathrm{c}} y_d V_{\mathrm{CKM}}^\dagger u_{\mathrm{L}})G^- - \mathrm{i}\mathrm{c}_{\beta +}(e_{\mathrm{R}}^{\mathrm{c}} y_e \nu_{\mathrm{L}})G^- + \text{h.c.}\right] \\
&\quad + \left[-\frac{\tilde{u}_{\mathrm{L}}^* u_{\mathrm{L}} \tilde{\chi}_j^0 \left(g_Y N_{j1}^* + 3g_2 N_{j2}^*\right)}{3\sqrt{2}} + \frac{2}{3}\sqrt{2}g_Y \tilde{u}_{\mathrm{R}} u_{\mathrm{R}}^{\mathrm{c}} \tilde{\chi}_j^0 N_{j1}^* - y_{ui}u_{\mathrm{R}i}^{\mathrm{c}} \tilde{u}_{\mathrm{L}k} \tilde{\chi}_j^0 N_{j4}^* - y_{ui}\tilde{u}_{\mathrm{R}i}^* u_{\mathrm{L}i} \tilde{\chi}_j^0 N_{j4}^*\right. \\
&\quad \left. - \frac{\tilde{d}_{\mathrm{L}i}^* d_{\mathrm{L}i} \tilde{\chi}_j^0 \left(g_Y N_{j1}^* - 3g_2 N_{j2}^*\right)}{3\sqrt{2}} - \frac{1}{3}\sqrt{2}g_Y \tilde{d}_{\mathrm{R}i} d_{\mathrm{R}i}^{\mathrm{c}} \tilde{\chi}_j^0 N_{j1}^* - y_{di}\tilde{d}_{\mathrm{L}i} d_{\mathrm{R}i}^{\mathrm{c}} \tilde{\chi}_j^0 N_{j3}^* - y_{di}\tilde{d}_{\mathrm{R}i}^* d_{\mathrm{L}i} \tilde{\chi}_j^0 N_{j3}^*\right. \\
&\quad \left. + \frac{\tilde{\nu}_{\mathrm{L}i}^* \nu_{\mathrm{L}i} \tilde{\chi}_j^0 \left(g_Y N_{j1}^* - g_2 N_{j2}^*\right)}{\sqrt{2}} - \sqrt{2}g_Y \tilde{e}_{\mathrm{R}i} e_{\mathrm{R}i}^{\mathrm{c}} \tilde{\chi}_j^0 N_{j1}^* + \frac{\tilde{e}_{\mathrm{L}i}^* e_{\mathrm{L}i} \tilde{\chi}_j^0 \left(g_Y N_{j1}^* + g_2 N_{j2}^*\right)}{\sqrt{2}}\right. \\
&\quad \left. - y_{ei}\tilde{e}_{\mathrm{L}i} e_{\mathrm{R}i}^{\mathrm{c}} \tilde{\chi}_j^0 N_{j3}^* - y_{ei}\tilde{e}_{\mathrm{R}i}^* e_{\mathrm{L}i} \tilde{\chi}_j^0 N_{j3}^* + \text{h.c.}\right] + \left[V_{i2}^*(u_{\mathrm{R}}^{\mathrm{c}} y_u V_{\mathrm{CKM}} \tilde{d}_{\mathrm{L}})\tilde{\chi}_i^+ + V_{i2}^*(\tilde{u}_{\mathrm{R}}^* y_u V_{\mathrm{CKM}} d_{\mathrm{L}})\tilde{\chi}_i^+\right. \\
&\quad \left. + U_{j2}^*(d_{\mathrm{R}}^{\mathrm{c}} y_d V_{\mathrm{CKM}}^\dagger \tilde{u}_{\mathrm{L}})\tilde{\chi}_j^- + U_{j2}^*(\tilde{d}_{\mathrm{R}}^* y_d V_{\mathrm{CKM}}^\dagger u_{\mathrm{L}})\tilde{\chi}_j^- - U_{j1}^* g_2(\tilde{d}_{\mathrm{L}}^* V_{\mathrm{CKM}}^\dagger u_{\mathrm{L}})\tilde{\chi}_j^- - V_{i1}^* g_2(\tilde{u}_{\mathrm{L}}^* V_{\mathrm{CKM}} d_{\mathrm{L}})\tilde{\chi}_i^+\right. \\
&\quad \left. - V_{i1}^* g_2 \tilde{\nu}_{\mathrm{L}}^* e_{\mathrm{L}} \tilde{\chi}_i^+ - U_{j1}^* g_2 \tilde{e}_{\mathrm{L}}^* \nu_{\mathrm{L}} \tilde{\chi}_j^- + U_{j2}^* y_{ei} \nu_{\mathrm{L}i} e_{\mathrm{R}i}^{\mathrm{c}} \tilde{\chi}_j^- + U_{j2}^* y_{ei} \tilde{e}_{\mathrm{R}i}^* \nu_{\mathrm{L}i} \tilde{\chi}_j^- + \text{h.c.}\right]. \\
&\quad - \epsilon^{xyz}\left(\lambda_{ijk}'' \tilde{d}_{\mathrm{R}k}^{x*} u_{\mathrm{R}i}^{cy} d_{\mathrm{R}j}^{cz} + \frac{1}{2}\lambda_{kij}'' \tilde{u}_{\mathrm{R}k}^{x*} d_{\mathrm{R}i}^{cy} d_{\mathrm{R}j}^{cz}\right) + \lambda_{ijk}\left(-\tilde{e}_{\mathrm{R}k}^* \nu_{\mathrm{L}i} e_{\mathrm{L}j} - \tilde{e}_{\mathrm{L}j} e_{\mathrm{R}k}^{\mathrm{c}} \nu_{\mathrm{L}i} + \tilde{\nu}_{\mathrm{L}j} e_{\mathrm{R}k}^{\mathrm{c}} e_{\mathrm{L}i}\right) \\
&\quad + \lambda_{ijk}'\left(-\tilde{d}_{\mathrm{R}k}^* d_{\mathrm{L}j} \nu_{\mathrm{L}i} + \tilde{d}_{\mathrm{R}k}^* u_{\mathrm{L}j} e_{\mathrm{L}i} - \tilde{d}_{\mathrm{L}j} d_{\mathrm{R}k}^{\mathrm{c}} \nu_{\mathrm{L}i} + \tilde{u}_{\mathrm{L}j} d_{\mathrm{R}k}^{\mathrm{c}} e_{\mathrm{L}i} + \tilde{e}_{\mathrm{L}i} d_{\mathrm{R}k}^{\mathrm{c}} u_{\mathrm{L}j} - \tilde{\nu}_{\mathrm{L}i} d_{\mathrm{R}k}^{\mathrm{c}} d_{\mathrm{L}j}\right) + \text{h.c.},
\end{aligned}
\tag{8.88}
$$

where R-parity violating terms are also shown as a reference; they are also redefined in super-CKM basis.

The full Lagrangian is given then by

$$\mathcal{L} = (\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}}) + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{scalar}}, \qquad (8.89)$$

where $\mathcal{L}_{\text{vector}}$ is given by Eq. (8.38), which should be amended with $m_W^2 W^{+\mu} W_\mu^- + (m_Z^2/2) Z^\mu Z_\mu$, mass and kinetic terms of $\mathcal{L}_{\text{scalar}}$ should be read from this section, and the interaction terms of $\mathcal{L}_{\text{scalar}}$ should be read from Eq. (8.43) and Eq. (8.44), replacing Higgs fields by VEVs, Higgs bosons, and Nambu-Goldstone bosons.

These Weyl fermions are combined to Dirac and Majorana fermions:

$$\tilde{\chi}_i^0 = \begin{pmatrix} \tilde{\chi}_i^0 \\ \bar{\tilde{\chi}}_i^0 \end{pmatrix}, \quad \tilde{\chi}_i^+ = \begin{pmatrix} \tilde{\chi}_i^+ \\ \bar{\tilde{\chi}}_i^- \end{pmatrix}, \quad f = \begin{pmatrix} f_{\text{L}} \\ f_{\text{R}}^{\text{c}} \end{pmatrix}; \quad \overline{\tilde{\chi}_i^0} = \begin{pmatrix} \tilde{\chi}_i^0 & \bar{\tilde{\chi}}_i^0 \end{pmatrix}, \quad \overline{\tilde{\chi}_i^+} = \begin{pmatrix} \tilde{\chi}_i^- & \bar{\tilde{\chi}}_i^+ \end{pmatrix}, \quad \overline{f} = \begin{pmatrix} f_{\text{R}}^{\text{c}} & \bar{f}_{\text{L}} \end{pmatrix}. \qquad (8.90)$$

For example,

$$\overline{u} P_{\text{L}} d = u_{\text{R}}^{\text{c}} d_{\text{L}}, \quad \overline{u} P_{\text{R}} d = \bar{u}_{\text{L}} \bar{d}_{\text{R}}^{\text{c}}, \quad u_{\text{R}}^{\text{c}} d_{\text{L}} + \text{h.c.} = \overline{u} P_{\text{L}} d + \overline{d} P_{\text{R}} u, \quad \overline{u} \gamma^\mu P_{\text{L}} d = \bar{u}_{\text{L}} \bar{\sigma}^\mu d_{\text{L}}, \quad \overline{u} \gamma^\mu P_{\text{R}} d = -\bar{d}_{\text{R}}^{\text{c}} \bar{\sigma}^\mu u_{\text{R}}^{\text{c}};$$

$$\tilde{\chi}^0 u_{\text{L}} + \text{h.c.} = \overline{\tilde{\chi}^0} P_{\text{L}} u + \overline{u} P_{\text{R}} \tilde{\chi}^0, \quad \tilde{\chi}^0 u_{\text{R}}^{\text{c}} + \text{h.c.} = \overline{u} P_{\text{L}} \tilde{\chi}^0 + \overline{\tilde{\chi}^0} P_{\text{R}} u,$$

$$\overline{u} \gamma^\mu P_{\{\text{L},\text{R}\}} \tilde{\chi}^0 = \{\bar{u}_{\text{L}} \bar{\sigma}^\mu \tilde{\chi}^0, -\bar{\tilde{\chi}}^0 \bar{\sigma}^\mu u_{\text{R}}^{\text{c}}\}, \quad \overline{\tilde{\chi}^0} \gamma^\mu P_{\{\text{L},\text{R}\}} u = \{\bar{\tilde{\chi}}^0 \bar{\sigma} u_{\text{L}}, -\bar{u}_{\text{R}}^{\text{c}} \bar{\sigma} \tilde{\chi}^0\},$$

$$\bar{\tilde{\chi}}_i^0 \bar{\sigma} \tilde{\chi}_j^0 = \overline{\tilde{\chi}_i^0} \gamma^\mu P_{\text{L}} \tilde{\chi}_j^0 = -\overline{\tilde{\chi}_j^0} \gamma^\mu P_{\text{R}} \tilde{\chi}_i^0.$$

With abbreviations

$$(\mathcal{N}_{\phi_{\text{u}}})_{ij} := N_{i4}^*(g_2 N_{j2}^* - g_Y N_{j1}^*) P_{\text{L}} + N_{i4}(g_2 N_{j2} - g_Y N_{j1}) P_{\text{R}},$$

$$(\mathcal{N}_{\phi_{\text{d}}})_{ij} := N_{i3}^*(g_Y N_{j1}^* - g_2 N_{j2}^*) P_{\text{L}} + N_{i3}(g_Y N_{j1} - g_2 N_{j2}) P_{\text{R}},$$

$$(\mathcal{N}_{A^0})_{ij} := -(\text{s}_{\beta 0} N_{i3}^* - \text{c}_{\beta 0} N_{i4}^*)(g_Y N_{j1}^* - g_2 N_{j2}^*) P_{\text{L}} + (\text{s}_{\beta 0} N_{i3} - \text{c}_{\beta 0} N_{i4})(g_Y N_{j1} - g_2 N_{j2}) P_{\text{R}},$$

$$(\mathcal{N}_{G^0})_{ij} := (g_Y N_{j1}^* - g_2 N_{j2}^*)(\text{c}_{\beta 0} N_{i3}^* + \text{s}_{\beta 0} N_{i4}^*) P_{\text{L}} - (g_Y N_{j1} - g_2 N_{j2})(\text{c}_{\beta 0} N_{i3} + \text{s}_{\beta 0} N_{i4}) P_{\text{R}},$$

$$(\mathcal{C}_{\phi_{\text{u}}})_{ij} := -U_{i1}^* V_{j2}^* P_{\text{L}} - U_{i1} V_{j2} P_{\text{R}},$$

$$(\mathcal{C}_{\phi_{\text{d}}})_{ij} := -U_{i2}^* V_{j1}^* P_{\text{L}} - U_{i2} V_{j1} P_{\text{R}},$$

$$(\mathcal{C}_{A^0})_{ij} := (\text{c}_{\beta 0} U_{i1}^* V_{j2}^* + \text{s}_{\beta 0} U_{i2}^* V_{j1}^*) P_{\text{L}} - (\text{c}_{\beta 0} U_{i1} V_{j2} + \text{s}_{\beta 0} U_{i2} V_{j1}) P_{\text{R}},$$

$$(\mathcal{C}_{G^0})_{ij} := -(\text{c}_{\beta 0} U_{i2}^* V_{j1}^* - \text{s}_{\beta 0} U_{i1}^* V_{j2}^*) P_{\text{L}} + (\text{c}_{\beta 0} U_{i2} V_{j1} - \text{s}_{\beta 0} U_{i1} V_{j2}) P_{\text{R}},$$

$$(\mathcal{C}_{H^-})_{ij} := g_2 V_{i1}^* N_{j4}^* + \frac{V_{i2}^*(g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}}$$

$$,(\mathcal{C}_{H^+})_{ij} := \frac{U_{i2}^*(g_Y N_{j1}^* + g_2 N_{j2}^*)}{\sqrt{2}} - g_2 U_{i1}^* N_{j3}^*$$

,

we have

$$
\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}}
$$

$$
= \frac{1}{2}\overline{\tilde{g}^a}\left[\left(\mathrm{i}\slashed{\partial} - m_{\tilde{g}}\right)\delta^{ab} - \mathrm{i}g_3 f^{cab}\slashed{g}^c\right]\tilde{g}^b + \frac{1}{2}\overline{\tilde{\chi}_i^0}\left[\left(\mathrm{i}\slashed{\partial} - m_{\tilde{\chi}_i^0}\right)\delta_{ij} + g_Z\left(N_{i3}N_{j3}^* - N_{i4}N_{j4}^*\right)\slashed{Z}P_{\mathrm{L}}\right]\tilde{\chi}_j^0
$$

$$
+ \overline{\tilde{\chi}_i^+}\left[\left(\mathrm{i}\slashed{\partial} - m_{\tilde{\chi}_i^\pm} + |e|\slashed{A}\right)\delta_{ij} + g_Z\slashed{Z}\left(\frac{2\mathrm{c}_{\mathrm{w}}^2 V_{i1}V_{j1}^* + \mathrm{c}_{2\mathrm{w}}V_{i2}V_{j2}^*}{2}P_{\mathrm{L}} + \frac{\mathrm{c}_{2\mathrm{w}}U_{i2}U_{j2}^* - 2\mathrm{c}_{\mathrm{w}}^2 U_{i1}U_{j1}^*}{2}P_{\mathrm{R}}\right)\right]\tilde{\chi}_j^+
$$

$$
+ \overline{u_i}\left[\mathrm{i}\slashed{\partial} - m_{ui} + g_3\tau^a\slashed{g}^a + \frac{2|e|}{3}\slashed{A} + g_Z\slashed{Z}\left(\frac{3 - 4\mathrm{s}_{\mathrm{w}}^2}{6}P_{\mathrm{L}} - \frac{2\mathrm{s}_{\mathrm{w}}^2}{3}P_{\mathrm{R}}\right)\right]u_i
$$

$$
+ \overline{d_i}\left[\mathrm{i}\slashed{\partial} - m_{di} + g_3\tau^a\slashed{g}^a - \frac{|e|}{3}\slashed{A} + g_Z\slashed{Z}\left(-\frac{3 - 2\mathrm{s}_{\mathrm{w}}^2}{6}P_{\mathrm{L}} + \frac{\mathrm{s}_{\mathrm{w}}^2}{3}P_{\mathrm{R}}\right)\right]d_i
$$

$$
+ \overline{\nu_i}\left(\mathrm{i}\slashed{\partial} + \frac{g_Z}{2}\slashed{Z}P_{\mathrm{L}}\right)\nu_i + \overline{e_i}\left[\mathrm{i}\slashed{\partial} - m_{ei} - |e|\slashed{A} + g_Z\slashed{Z}\left(\frac{2\mathrm{s}_{\mathrm{w}}^2 - 1}{2}P_{\mathrm{L}} + \mathrm{s}_{\mathrm{w}}^2 P_{\mathrm{R}}\right)\right]e_i
$$

$$
+ \frac{g_2}{\sqrt{2}}W_\mu^+\left[\overline{u}\gamma^\mu V_{\mathrm{CKM}}P_{\mathrm{L}}d + \overline{\nu}\gamma^\mu P_{\mathrm{L}}e + \overline{\tilde{\chi}_j^+}\gamma^\mu\left(\left(-\sqrt{2}V_{j1}N_{i2}^* + V_{j2}N_{i4}^*\right)P_{\mathrm{L}} + \left(-\sqrt{2}N_{i2}U_{j1}^* - N_{i3}U_{j2}^*\right)P_{\mathrm{R}}\right)\tilde{\chi}_i^0\right]
$$

$$
+ \frac{g_2}{\sqrt{2}}W_\mu^-\left[\overline{d}\gamma^\mu V_{\mathrm{CKM}}^\dagger P_{\mathrm{L}}u + \overline{e}\gamma^\mu P_{\mathrm{L}}\nu + \overline{\tilde{\chi}_i^0}\gamma^\mu\left(\left(-\sqrt{2}N_{i2}V_{j1}^* + N_{i4}V_{j2}^*\right)P_{\mathrm{L}} + \left(\sqrt{2}U_{j1}N_{i2}^* + U_{j2}N_{i3}^*\right)P_{\mathrm{R}}\right)\tilde{\chi}_j^+\right]
$$

$$
+ \sqrt{2}g_3\Big[\overline{\tilde{g}^a}(\varphi_{\tilde{g}}^*\tilde{u}_{\mathrm{R}}^* P_{\mathrm{R}} - \varphi_{\tilde{g}}\tilde{u}_{\mathrm{L}}^* P_{\mathrm{L}})\tau^a u + \overline{u}\tau^a(\varphi_{\tilde{g}}\tilde{u}_{\mathrm{R}}P_{\mathrm{L}} - \varphi_{\tilde{g}}^*\tilde{u}_{\mathrm{L}}P_{\mathrm{R}})\tilde{g}^a
$$

$$
+ \overline{d}\tau^a(\varphi_{\tilde{g}}\tilde{d}_{\mathrm{R}}P_{\mathrm{L}} - \varphi_{\tilde{g}}^*\tilde{d}_{\mathrm{L}}P_{\mathrm{R}})\tilde{g}^a + \overline{\tilde{g}^a}(\varphi_{\tilde{g}}^*\tilde{d}_{\mathrm{R}}^* P_{\mathrm{R}} - \varphi_{\tilde{g}}\tilde{d}_{\mathrm{L}}^* P_{\mathrm{L}})\tau^a d\Big] + \frac{y_{ui}}{\sqrt{2}}\overline{u_i}\left(-\phi_{\mathrm{u}} + \mathrm{i}\gamma_5\mathrm{c}_{\beta 0}A^0 + \mathrm{i}\gamma_5\mathrm{s}_{\beta 0}G^0\right)u_i
$$

$$
+ \frac{y_{di}}{\sqrt{2}}\overline{d_i}\left(-\phi_{\mathrm{d}} + \mathrm{i}\gamma_5\mathrm{s}_{\beta 0}A^0 - \mathrm{i}\gamma_5\mathrm{c}_{\beta 0}G^0\right)d_i + \frac{y_{ei}}{\sqrt{2}}\overline{e_i}\left(-\phi_{\mathrm{d}} + \mathrm{i}\gamma_5\mathrm{s}_{\beta 0}A^0 - \mathrm{i}\gamma_5\mathrm{c}_{\beta 0}G^0\right)e_i
$$

$$
+ \frac{1}{2}\overline{\tilde{\chi}_i^0}\left(\phi_{\mathrm{u}}\mathcal{N}_{\phi_{\mathrm{u}}} + \phi_{\mathrm{d}}\mathcal{N}_{\phi_{\mathrm{d}}} + \mathrm{i}A^0\mathcal{N}_{A^0} + \mathrm{i}G^0\mathcal{N}_{G^0}\right)_{ij}\tilde{\chi}_j^0 + \frac{g_2}{\sqrt{2}}\overline{\tilde{\chi}_i^+}\left(\phi_{\mathrm{u}}\mathcal{C}_{\phi_{\mathrm{u}}} + \phi_{\mathrm{d}}\mathcal{C}_{\phi_{\mathrm{d}}} + \mathrm{i}A_0\mathcal{C}_{A^0} + \mathrm{i}G_0\mathcal{C}_{G^0}\right)_{ij}\tilde{\chi}_j^+
$$

$$
- (H^-\mathrm{c}_{\beta+} + \mathrm{i}G^-\mathrm{s}_{\beta+})(\mathcal{C}_{H^-})_{ij}\overline{\tilde{\chi}_j^0}P_{\mathrm{L}}\tilde{\chi}_i^+ - (H^+\mathrm{c}_{\beta+} - \mathrm{i}G^+\mathrm{s}_{\beta+})(\mathcal{C}_{H^-})_{ij}^*\overline{\tilde{\chi}_i^+}P_{\mathrm{R}}\tilde{\chi}_j^0
$$

$$
+ (H^+\mathrm{s}_{\beta+} + \mathrm{i}G^+\mathrm{c}_{\beta+})(\mathcal{C}_{H^+})_{ij}\overline{\tilde{\chi}_i^+}P_{\mathrm{L}}\tilde{\chi}_j^0 + (H^-\mathrm{s}_{\beta+} - \mathrm{i}G^-\mathrm{c}_{\beta+})(\mathcal{C}_{H^+})_{ij}^*\overline{\tilde{\chi}_j^0}P_{\mathrm{R}}\tilde{\chi}_i^+
$$

$$
+ (H^-\mathrm{s}_{\beta+} - \mathrm{i}G^-\mathrm{c}_{\beta+})\overline{d}y_d V_{\mathrm{CKM}}^\dagger P_{\mathrm{L}}u + (H^-\mathrm{c}_{\beta+} + \mathrm{i}G^-\mathrm{s}_{\beta+})\overline{d}V_{\mathrm{CKM}}^\dagger y_u P_{\mathrm{R}}u + (H^-\mathrm{s}_{\beta+} - \mathrm{i}G^-\mathrm{c}_{\beta+})\overline{e}y_e P_{\mathrm{L}}\nu
$$

$$
+ (H^+\mathrm{c}_{\beta+} - \mathrm{i}G^+\mathrm{s}_{\beta+})\overline{u}y_u V_{\mathrm{CKM}}P_{\mathrm{L}}d + (H^+\mathrm{s}_{\beta+} + \mathrm{i}G^+\mathrm{c}_{\beta+})\overline{u}V_{\mathrm{CKM}}y_d P_{\mathrm{R}}d + (H^+\mathrm{s}_{\beta+} + \mathrm{i}G^+\mathrm{c}_{\beta+})\overline{\nu}y_e P_{\mathrm{R}}e
$$

$$
+ \left(-\frac{g_Y N_{j1}^* + 3g_2 N_{j2}^*}{3\sqrt{2}}\tilde{u}_{\mathrm{L}i}^* - y_{ui}N_{j4}^*\tilde{u}_{\mathrm{R}i}^*\right)\overline{\tilde{\chi}_j^0}P_{\mathrm{L}}u_i + \left(\frac{2\sqrt{2}g_Y}{3}N_{j1}\tilde{u}_{\mathrm{R}i}^* - y_{ui}N_{j4}\tilde{u}_{\mathrm{L}i}^*\right)\overline{\tilde{\chi}_j^0}P_{\mathrm{R}}u_i
$$

$$
+ \left(\frac{2\sqrt{2}g_Y}{3}N_{j1}^*\tilde{u}_{\mathrm{R}i} - y_{ui}N_{j4}^*\tilde{u}_{\mathrm{L}i}\right)\overline{u_i}P_{\mathrm{L}}\tilde{\chi}_j^0 + \left(-\frac{g_Y N_{j1} + 3g_2 N_{j2}}{3\sqrt{2}}\tilde{u}_{\mathrm{L}i} - y_{ui}N_{j4}\tilde{u}_{\mathrm{R}i}\right)\overline{u_i}P_{\mathrm{R}}\tilde{\chi}_j^0
$$

$$
+ \left(-\frac{g_Y N_{j1}^* - 3g_2 N_{j2}^*}{3\sqrt{2}}\tilde{d}_{\mathrm{L}i}^* - y_{di}N_{j3}^*\tilde{d}_{\mathrm{R}i}^*\right)\overline{\tilde{\chi}_j^0}P_{\mathrm{L}}d_i + \left(-\frac{\sqrt{2}g_Y}{3}N_{j1}\tilde{d}_{\mathrm{R}i}^* - y_{di}N_{j3}\tilde{d}_{\mathrm{L}i}^*\right)\overline{\tilde{\chi}_j^0}P_{\mathrm{R}}d_i
$$

$$
+ \left(-\frac{\sqrt{2}g_Y}{3}N_{j1}^*\tilde{d}_{\mathrm{R}i} - y_{di}N_{j3}^*\tilde{d}_{\mathrm{L}i}\right)\overline{d_i}P_{\mathrm{L}}\tilde{\chi}_j^0 + \left(-\frac{g_Y N_{j1} - 3g_2 N_{j2}}{3\sqrt{2}}\tilde{d}_{\mathrm{L}i} - y_{di}N_{j3}\tilde{d}_{\mathrm{R}i}\right)\overline{d_i}P_{\mathrm{R}}\tilde{\chi}_j^0
$$

$$
+ \left(\frac{g_Y N_{j1}^* + g_2 N_{j2}^*}{\sqrt{2}}\tilde{e}_{\mathrm{L}i}^* - y_{ei}N_{j3}^*\tilde{e}_{\mathrm{R}i}^*\right)\overline{\tilde{\chi}_j^0}P_{\mathrm{L}}e_i + \left(-\sqrt{2}g_Y N_{j1}\tilde{e}_{\mathrm{R}i}^* - y_{ei}N_{j3}\tilde{e}_{\mathrm{L}i}^*\right)\overline{\tilde{\chi}_j^0}P_{\mathrm{R}}e_i
$$

$$
+ \left(-\sqrt{2}g_Y N_{j1}^*\tilde{e}_{\mathrm{R}i} - y_{ei}N_{j3}^*\tilde{e}_{\mathrm{L}i}\right)\overline{e_i}P_{\mathrm{L}}\tilde{\chi}_j^0 + \left(+\frac{g_Y N_{j1} + g_2 N_{j2}}{\sqrt{2}}\tilde{e}_{\mathrm{L}i} - y_{ei}N_{j3}\tilde{e}_{\mathrm{R}i}\right)\overline{e_i}P_{\mathrm{R}}\tilde{\chi}_j^0
$$

$$
+ \frac{g_Y N_{j1} - g_2 N_{j2}}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}i}\overline{\nu_i}P_{\mathrm{R}}\tilde{\chi}_j^0 + \frac{g_Y N_{j1}^* - g_2 N_{j2}^*}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}i}^*\overline{\tilde{\chi}_j^0}P_{\mathrm{L}}\nu_i
$$

$$
+ \left(U_{i2}^*(\tilde{d}_{\mathrm{R}}^* y_d V_{\mathrm{CKM}}^\dagger)_j - U_{i1}^* g_2(\tilde{d}_{\mathrm{L}}^* V_{\mathrm{CKM}}^\dagger)_j\right)\overline{\tilde{\chi}_i^+}P_{\mathrm{L}}u_j + V_{i2}(\tilde{d}_{\mathrm{L}}^* V_{\mathrm{CKM}}^\dagger y_u)_j\overline{\tilde{\chi}_i^+}P_{\mathrm{R}}u_j
$$

$$
+ V_{i2}^*(y_u V_{\mathrm{CKM}}\tilde{d}_{\mathrm{L}})_j\overline{u_j}P_{\mathrm{L}}\tilde{\chi}_i^+ + \left(U_{i2}(V_{\mathrm{CKM}}y_d\tilde{d}_{\mathrm{R}})_j - U_{i1}g_2(V_{\mathrm{CKM}}\tilde{d}_{\mathrm{L}})_j\right)\overline{u_j}P_{\mathrm{R}}\tilde{\chi}_i^+
$$

$$
+ (-V_{i1}^* g_2(\tilde{u}_{\mathrm{L}}^* V_{\mathrm{CKM}})_j + V_{i2}^*(\tilde{u}_{\mathrm{R}}^* y_u V_{\mathrm{CKM}})_j)\overline{\tilde{\chi}_i^-}P_{\mathrm{L}}d_j + U_{i2}(\tilde{u}_{\mathrm{L}}^* V_{\mathrm{CKM}}y_d)_j\overline{\tilde{\chi}_i^-}P_{\mathrm{R}}d_j
$$

$$
+ U_{i2}^*(y_d V_{\mathrm{CKM}}^\dagger\tilde{u}_{\mathrm{L}})_j\overline{d_j}P_{\mathrm{L}}\tilde{\chi}_i^- + \left(-V_{i1}g_2(V_{\mathrm{CKM}}^\dagger\tilde{u}_{\mathrm{L}})_j + V_{i2}(V_{\mathrm{CKM}}^\dagger y_u\tilde{u}_{\mathrm{R}})_j\right)\overline{d_j}P_{\mathrm{R}}\tilde{\chi}_i^-
$$

$$
+ (-U_{i1}^* g_2\tilde{e}_{\mathrm{L}}^* + U_{i2}^* y_{ei}\tilde{e}_{\mathrm{R}i}^*)\overline{\tilde{\chi}_i^+}P_{\mathrm{L}}\nu + (-U_{i1}g_2\tilde{e}_{\mathrm{L}} + U_{i2}y_{ei}\tilde{e}_{\mathrm{R}i})\overline{\nu}P_{\mathrm{R}}\tilde{\chi}_i^+
$$

$$
+ \overline{\tilde{\chi}_i^-}\left(-V_{i1}^* g_2\tilde{\nu}_{\mathrm{L}}^* P_{\mathrm{L}} + U_{i2}y_{ej}\tilde{\nu}_{\mathrm{L}j}^* P_{\mathrm{R}}\right)e_j + \overline{e_j}\left(U_{i2}^* y_{ej}\tilde{\nu}_{\mathrm{L}j}P_{\mathrm{L}} - V_{i1}g_2\tilde{\nu}_{\mathrm{L}}P_{\mathrm{R}}\right)\tilde{\chi}_i^- + (\text{RPV part}).
$$

$$(8.91)$$

Note that squarks and sleptons are in super-CKM basis and not in mass eigenstates.

8.5. SLHA convention

The SLHA convention [10] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table (**magenta color** for objects in other conventions), while

$$\mu, b, m^2_{Q,L,H_\mathrm{u},H_\mathrm{d}}, \text{RPV-trilinears} \ (\lambda\text{s and } T\text{s})$$

are in common.

SLHA		our notation	Martin/DHM
(H_1, H_2)	$=$	$(H_\mathrm{d}, H_\mathrm{u})$	
$Y_{\mathrm{u,d,e}}$	$=$	$(y_{\mathrm{u,d,e}})^{\mathrm{T}}$	
$T_{\mathrm{u,d,e}}$	$=$	$(a_{\mathrm{u,d,e}})^{\mathrm{T}}$	
$A_{\mathrm{u,d,e}}$	$=$	$(A_{\mathrm{u,d,e}})^{\mathrm{T}}$	
$m^2_{U^\mathrm{c},D^\mathrm{c},E\mathrm{c}}$	$=$	$(m^2_{U^\mathrm{c},D^\mathrm{c},E^\mathrm{c}})^{\dagger}$	
$M_{1,2,3}$	$=$	$-M_{1,2,3}$	
m^2_3	$=$	b	
m^2_A	$=$	$m^2_{A_0}$ (tree)	
		κ_i	$=-\mu'_i$ (rarely used)
D_i	$=$	b_i	
$m^2_{\tilde{L}_i H_1}$	$=$	M^2_{Li}	

In particular, the chargino/neutralino mass terms in RPC case are given by

$$\mathcal{L} \supset \left[\frac{1}{2} M_1 \tilde{b}\tilde{b} + \frac{1}{2} M_2 \tilde{w}\tilde{w} - \mu \tilde{h}_\mathrm{u} \tilde{h}_\mathrm{d} - \frac{g_Y}{2\sqrt{2}} \left(h_\mathrm{u}^* \tilde{h}_\mathrm{u} - h_\mathrm{d}^* \tilde{h}_\mathrm{d} \right) \tilde{b} - \sqrt{2} g_2 \left(h_\mathrm{u}^* T^a \tilde{h}_\mathrm{u} + h_\mathrm{d}^* T_a \tilde{h}_\mathrm{d} \right) \tilde{w} \right] + \mathrm{H.c.} \tag{8.92}$$

$$\rightarrow \frac{1}{2} \begin{pmatrix} \tilde{b} \\ \tilde{w} \\ h_\mathrm{u}^0 \\ h_\mathrm{d}^0 \end{pmatrix}^\mathrm{T} \begin{pmatrix} -M_1 & 0 & -m_Z c_\beta s_w & m_Z s_\beta s_w \\ 0 & -M_2 & m_Z c_\beta c_w & -m_Z s_\beta c_w \\ -m_Z c_\beta s_w & m_Z c_\beta c_w & 0 & -\mu \\ m_Z s_\beta s_w & -m_Z s_\beta c_w & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{w} \\ h_\mathrm{u}^0 \\ h_\mathrm{d}^0 \end{pmatrix} \tag{8.93}$$

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8.6. GMSB formulae

$$\Gamma^{-1}(\tilde{l} \to l\tilde{G}) = \frac{1}{48\pi M^2} \frac{m_l^5}{m_{\tilde{G}}^2} \times (\text{phase space}) \approx 5.9 \times 10^{-7}\,\text{s}\,\frac{(m_{3/2}/\text{MeV})^2}{(m_{\tilde{l}}/\text{TeV})^5} \tag{8.94}$$

$$= \frac{3m_l^5}{48\pi F_{\text{tot}}^2} \times (\text{phase space}) \approx 3.3 \times 10^{-9}\,\text{s}\,\frac{(F_{\text{tot}}/10^{12}\,\text{GeV}^2)^2}{(m_{\tilde{l}}/100\,\text{GeV})^5} = 1.0\,\text{m}\,\frac{(F_{\text{tot}}/10^{12}\,\text{GeV}^2)^2}{(m_{\tilde{l}}/100\,\text{GeV})^5} \tag{8.95}$$

$$\Gamma^{-1}(\tilde{B} \to \gamma\tilde{G}) = \frac{c_{\text{w}}^2 m_{\tilde{B}}^5}{48\pi M^2 m_{\tilde{G}}^2}(1-r)^3(1+3r) \qquad \text{where} \quad r := \left(\frac{m_{\tilde{G}}}{m_{\tilde{B}}}\right)^2. \tag{8.96}$$

(cf. [11])

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A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [12]:

$$\mathrm{e}^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \ \ (\text{converges for any } X), \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(A-1)^m}{m} \ \ (\text{conv. if } \|A-I\| < 1). \tag{A.1}$$

$$\mathrm{e}^{\log A} = A \ \ (\text{if } \|A-I\| < 1), \quad \log \mathrm{e}^X = X \text{ and } \|\mathrm{e}^X - 1\| < 1 \ \ (\text{if } \|X\| < \log 2). \tag{A.2}$$

$$\text{Hilbert-Schmidt norm}: \ \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \operatorname{Tr} X^\dagger X. \tag{A.3}$$

Properties:

$$\mathrm{e}^{(X^{\mathrm{T}})} = (\mathrm{e}^X)^{\mathrm{T}}, \qquad \mathrm{e}^{(X^*)} = (\mathrm{e}^X)^*, \qquad (\mathrm{e}^X)^{-1} = \mathrm{e}^{-X}, \qquad \mathrm{e}^{YXY^{-1}} = Y \, \mathrm{e}^X Y^{-1},$$

$$\det \exp X = \exp \operatorname{Tr} X, \qquad \frac{\mathrm{d}}{\mathrm{d}t} \mathrm{e}^{tX} = X \, \mathrm{e}^{tX} = \mathrm{e}^{tX} X \qquad \mathrm{e}^{(\alpha+\beta)X} = \mathrm{e}^{\alpha X} \, \mathrm{e}^{\beta X} \text{ for } \alpha, \beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$\mathrm{e}^X Y \, \mathrm{e}^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots = \mathrm{e}^{[X,]} Y; \tag{A.4}$$

$$\mathrm{e}^X \, \mathrm{e}^Y \, \mathrm{e}^{-X} = \sum_{n=0}^{\infty} \frac{1}{n!} (\mathrm{e}^X Y \, \mathrm{e}^{-X})^n = \exp\left(\mathrm{e}^{[X,]} Y \right); \tag{A.5}$$

$$\log(\mathrm{e}^X \, \mathrm{e}^Y) = X + \int_0^1 \mathrm{d}t \, g(\mathrm{e}^{[X,} \mathrm{e}^{t[Y,}) Y \qquad \left[g(z) = \frac{\log z}{1 - z^{-1}} = 1 - \sum_{n=1}^{\infty} \frac{(1-z)^n}{n(n+1)}; \quad g(\mathrm{e}^y) = \sum_{n=0}^{\infty} \frac{B_n y^n}{n!} \right] \tag{A.6}$$

$$= X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots \quad (\text{Baker-Campbell-Hausdorff}). \tag{A.7}$$

$$\log(\mathrm{e}^X \, \mathrm{e}^Y) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \left(\sum_{m,n=0}^{\infty} \frac{X^m Y^n}{m!n!} - 1 \right)^k = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \cdots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k} \frac{X^{m_1} Y^{n_1} \cdots X^{m_k} Y^{n_k}}{m_1! n_1! \cdots m_k! n_k!} \tag{A.8}$$

$$\log(\mathrm{e}^X \, \mathrm{e}^Y) = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \cdots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k \sum_{i=1}^{k}(m_i+n_i)} \frac{\left([X,\right)^{m_1} \left([Y,\right)^{n_1} \cdots \left([X,\right)^{m_k} \left([Y,\right)^{n_k}] \cdots]}{m_1! n_1! \cdots m_k! n_k!} \tag{A.9}$$

with $[X] := X$ understood.

If matrices t^a satisfies $[t^a, t^b] = \mathrm{i} f^{abc} t^c$ with totally-antisymmetric $f^{abc} \in \mathbb{R}$,

$$\left[\mathrm{e}^{\theta^a t^a} t_b \, \mathrm{e}^{-\theta^c t^c} \right]_{ij} = \left[\mathrm{e}^{\theta^a [t^a,} t_b \right]_{ij} = \left[\mathrm{e}^{\mathrm{i}\theta^a f^a} \right]^{bc} t_{ij}^c \tag{A.10}$$

holds for $\theta^a \in \mathbb{C}$, where $[f^a]_{bc} = f^{abc}$. ♣TODO:needs verification, generalization/restriction, and a nice proof or reference.♣

A.2. General unitary matrix

$$U_2 = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\alpha} \end{pmatrix} \begin{pmatrix} \mathrm{c}_\theta & \mathrm{s}_\theta \\ -\mathrm{s}_\theta & \mathrm{c}_\theta \end{pmatrix} \begin{pmatrix} \mathrm{e}^{\mathrm{i}\beta} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\gamma} \end{pmatrix} = \begin{pmatrix} \mathrm{c}_\theta \, \mathrm{e}^{\mathrm{i}\beta} & \mathrm{s}_\theta \, \mathrm{e}^{\mathrm{i}\gamma} \\ -\mathrm{s}_\theta \, \mathrm{e}^{\mathrm{i}(\alpha+\beta)} & \mathrm{c}_\theta \, \mathrm{e}^{\mathrm{i}(\alpha+\gamma)} \end{pmatrix} \tag{A.11}$$

$$U_3 = \begin{pmatrix} 1 & & \\ & \mathrm{e}^{\mathrm{i}a} & \\ & & \mathrm{e}^{\mathrm{i}b} \end{pmatrix} \begin{pmatrix} 1 & & \\ & \mathrm{c}_{23} & \mathrm{s}_{23} \\ & -\mathrm{s}_{23} & \mathrm{c}_{23} \end{pmatrix} \begin{pmatrix} \mathrm{c}_{13} & & \mathrm{s}_{13} \, \mathrm{e}^{-\mathrm{i}\delta} \\ & 1 & \\ -\mathrm{s}_{13} \, \mathrm{e}^{\mathrm{i}\delta} & & \mathrm{c}_{13} \end{pmatrix} \begin{pmatrix} \mathrm{c}_{12} & \mathrm{s}_{12} & \\ -\mathrm{s}_{12} & \mathrm{c}_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \mathrm{e}^{\mathrm{i}c} & & \\ & \mathrm{e}^{\mathrm{i}d} & \\ & & \mathrm{e}^{\mathrm{i}e} \end{pmatrix} \tag{A.12}$$

$$= \begin{pmatrix} 1 & & \\ & \mathrm{e}^{\mathrm{i}a} & \\ & & \mathrm{e}^{\mathrm{i}b} \end{pmatrix} \begin{pmatrix} \mathrm{c}_{12}\mathrm{c}_{13} & \mathrm{s}_{12}\mathrm{c}_{13} & \mathrm{s}_{13} \, \mathrm{e}^{-\mathrm{i}\delta} \\ -\mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{23}\mathrm{s}_{13} \, \mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13} \, \mathrm{e}^{\mathrm{i}\delta} & \mathrm{s}_{23}\mathrm{c}_{13} \\ \mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13} \, \mathrm{e}^{\mathrm{i}\delta} & -\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{c}_{23}\mathrm{s}_{13} \, \mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{23}\mathrm{c}_{13} \end{pmatrix} \begin{pmatrix} \mathrm{e}^{\mathrm{i}c} & & \\ & \mathrm{e}^{\mathrm{i}d} & \\ & & \mathrm{e}^{\mathrm{i}e} \end{pmatrix} \tag{A.13}$$

(e.g., hep-ph/9708216)

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