

# 1 General Definitions and Tools

## 1.1 NOTATIONS AND CONVENTIONS

### 1.1.1 Metric etc.

$$\begin{aligned}
 \text{Minkowski Metric} & : \eta^{\mu\nu} = \text{diag}(+, -, -, -); \quad \epsilon_{0123}^{0123} := \pm 1 \\
 \text{Coordinates} & : x^\mu = (t, x, y, z); \quad \text{Therefore } \partial_\mu = \left( \frac{\partial}{\partial t}, \nabla \right) \\
 \text{Gamma Matrices} & : \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}; \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \\
 \text{Gamma Combinations} & : 1, \{\gamma^\mu\}, \{\sigma^{\mu\nu}\}, \{\gamma^\mu\gamma_5\}, \gamma_5; \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] = 0 / i\gamma^\mu\gamma^\nu
 \end{aligned}$$

### 1.1.2 Fields

$$\begin{aligned}
 \text{Klein-Gordon Equation} & : |\partial_\mu \phi|^2 - m^2 |\phi|^2 = 0 \\
 \text{Klein-Gordon Field} & : \phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[ a_p e^{-ipx} + b_p^\dagger e^{ipx} \right] \\
 \text{Dirac Equation} & : (i\not{\partial} - m)\psi(x) = 0 \\
 \text{Dirac Field} & : \psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1,2} \left[ a_p^s u^s(p) e^{-ipx} + b_p^{s\dagger} v^s(p) e^{ipx} \right] \\
 \text{Gauge Boson} & : (\partial^2 + m^2)A^\mu(x) = 0 \quad (\text{Real Klein-Gordon Equation}) \\
 (\text{Before Gauge Fixing}) & : A^\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{r=0,1,2,3} \left[ a_p^r \epsilon^r(p) e^{-ipx} + a_p^{r\dagger} \epsilon^{r*}(p) e^{ipx} \right] \\
 \text{南部-Goldstone Boson} & : \text{TODO: .}
 \end{aligned}$$

#### Chiral Notation

$$\begin{aligned}
 \text{Gamma Matrices} & : \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\
 \text{Dirac Field} & : \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}; \quad \bar{\psi} = \psi^\dagger \gamma^0 = \begin{pmatrix} \psi_R^\dagger & \psi_L^\dagger \end{pmatrix} \\
 & : u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix}; \quad v^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \eta^s \\ -\sqrt{p \cdot \bar{\sigma}} \eta^s \end{pmatrix} \\
 & : [\eta^s = \xi^{-s} := -i\sigma^2(\xi^s)^* = (\xi^2, -\xi^1)] \\
 \text{Weyl Equations} & : i\bar{\sigma} \cdot \partial \psi_L = m\psi_R; \quad i\sigma \cdot \partial \psi_L = m\psi_L \\
 \text{CPT transf.} & : P\psi(t, \mathbf{x})P = \eta\gamma^0\psi(t, -\mathbf{x}) \quad (|\eta|^2 = 1) \\
 & : T\psi(t, \mathbf{x})T = \gamma^1\gamma^3\psi(-t, \mathbf{x}) \quad (\text{ignoring intrinsic phase}) \\
 & : C\psi(t, \mathbf{x})C = -i\gamma^2\psi^*(t, \mathbf{x}) = -i(\bar{\psi}\gamma^0\gamma^2)^T \quad ( " ) \\
 & : \bar{\psi} \longrightarrow P : \eta^* \bar{\psi} \gamma^0 \quad T : -\bar{\psi} \gamma^1 \gamma^3 \quad C : i\bar{\psi}^* \gamma^2 = -i(\gamma^0 \gamma^2 \psi)^T
 \end{aligned}$$

$$\text{Electromagnetic Fields} : A^\mu = (\phi, \mathbf{A}) \quad \text{【We can invert the signs, but cannot lower the index.】}$$

$$: F_{\mu\nu} = \begin{pmatrix} 0 & \mathbf{E} \\ -\mathbf{E} & \begin{matrix} -B_3 & B_2 \\ B_3 & 0 & -B_1 \\ -B_2 & B_1 & 0 \end{matrix} \end{pmatrix}; \quad F_{\mu\nu}F^{\mu\nu} = -2 \left( \|\mathbf{E}\|^2 - \|\mathbf{B}\|^2 \right)$$

### 1.1.3 CPT Table

	$\phi$	$A^\mu$	$\bar{\psi}\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$	$\bar{\psi}\gamma^\mu\gamma_5\psi$	$i\bar{\psi}\gamma_5\psi$	$\partial_\mu$
$P$	$\eta\phi$	$\eta-++A^\mu$	+	----	(+----)(+----)	-+++	-	----
$T$	$\zeta\phi$	$\zeta+---A^\mu$	+	----	-(+----)(+----)	+----	-	-+++
$C$	$\xi\phi^*$	$\xi+A^{\mu*}$	+	-	-	+	+	+

( $\eta\zeta\xi = 1$ ; especially, photon  $A^\mu$  is  $(\eta, \zeta, \xi) = (-, +, -)$ . )

## 1.2 FEYNMAN RULES

$$\overline{\psi}|\mathbf{p}, s\rangle = \text{---}\blacktriangleleft\overset{\longleftarrow p}{\hspace{-0.8cm}}\bullet$$

$$\overline{\psi}[\boldsymbol{p}, s] = \text{---}\blacktriangleright\overset{\leftarrow p}{\text{---}}(\text{shaded circle})\text{end}$$

$$\overline{\psi}|\mathbf{p}, s\rangle = \text{hoge} \quad \text{---} \bullet \xleftarrow{p}$$

$$\overline{\psi}|\mathbf{p}, s\rangle = \text{---} \bullet \xleftarrow{p} \text{---} \text{end}$$

Initial state

$$\overbrace{\psi}^{\text{}} |p, s\rangle = \text{diagram of a shaded circle with an arrow pointing left labeled } p \text{ and a line extending to the right} = u^s(p)$$

$$\overline{\psi} |p, s\rangle = \text{diagram of a fermion line with a shaded circle at the start and an arrow pointing right labeled } p \text{ above it} = \overline{v}^s(p)$$

### Final state

$$\langle \overline{\mathbf{p}}, s | \overline{\psi} = \text{---} \overrightarrow{\hspace{0.5cm}} \overset{\longleftarrow p}{\hspace{0.5cm}} \bullet = \overline{u}^s(p)$$

$$\langle \overline{p}, s | \psi = \text{---} \blacktriangleleft \overset{\leftarrow p}{\text{---}} \bullet = v^s(p)$$

## Propagator

$$\overbrace{\psi(x) \psi(y)} = \begin{array}{c} \leftarrow p \\ \text{---} \blacktriangle \text{---} \text{shaded circle} \end{array} = v^s(p)$$