1. Kinematics

Decay rate and cross section (Note: $\mathcal{M}$ has a mass dimension of $4 - N_\mathrm{i} - N_\mathrm{f}$.)

$$\text{decay rate (rest frame; } \sqrt{s} = M_0) : \quad \mathrm{d}\Gamma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2M_0} \left| \mathcal{M}(M_0 \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\}) \right|^2, \tag{1.1}$$

$$\text{cross section (Lorentz invariant)} : \quad \mathrm{d}\sigma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2E_A 2E_B\, v_{\mathrm{M\o l}}} \left| \mathcal{M}(p_A, p_B \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\}) \right|^2, \tag{1.2}$$

where $\overline{\mathrm{d}\Pi^n}$ is n-particle Lorentz-invariant phase space with momentum conservation

$$\overline{\mathrm{d}\Pi^n} := \mathrm{d}\Pi_1\, \mathrm{d}\Pi_2 \cdots \mathrm{d}\Pi_n\, (2\pi)^4\, \delta^{(4)} \left(P_0 - \sum p_n \right); \quad \mathrm{d}\Pi := \frac{\mathrm{d}^3 \boldsymbol{p}}{(2\pi)^3} \frac{1}{2E_{\boldsymbol{p}}}. \tag{1.3}$$

At the CM frame, two-body phase-space are characterized by the final momentum $\|\boldsymbol{p}\|$ and given by

$$\overline{\mathrm{d}\Pi^2} = \frac{\|\boldsymbol{p}\|}{4\pi\sqrt{s}} \frac{\mathrm{d}\Omega}{4\pi} = \frac{\|\boldsymbol{p}\|}{8\pi\sqrt{s}}\, \mathrm{d}\cos\theta = \frac{1}{16\pi} \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}}\, \mathrm{d}\cos\theta \tag{1.4}$$

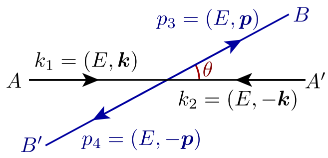
with $\sqrt{s} = M_0$ or E_CM, θ is the angle between initial and final motion, and

$$\|\boldsymbol{p}\| = \frac{\sqrt{s}}{2}\, \lambda^{1/2} \left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s} \right), \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad p_1 \cdot p_2 = \frac{s - (m_1^2 + m_2^2)}{2}.$$

Mandelstam variables For $(k_1, k_2) \to (p_3, p_4)$ collision,

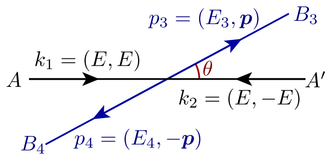
$$s = (k_1 + k_2)^2 = (p_3 + p_4)^2, \qquad t = (p_3 - k_1)^2 = (p_4 - k_2)^2, \qquad u = (p_3 - k_2)^2 = (p_4 - k_1)^2;$$
$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

If the collision is with the "same mass" $(m_A, m_A) \to (m_B, m_B)$,



$$\begin{aligned}
t &= m_A^2 + m_B^2 - s/2 + 2kp\cos\theta, & (k_1 - k_2)^2 &= 4m_A^2 - s, \\
u &= m_A^2 + m_B^2 - s/2 - 2kp\cos\theta, & (p_3 - p_4)^2 &= 4m_B^2 - s, \\
k &= \frac{\sqrt{s - 4m_A^2}}{2}, \quad k_1 \cdot k_2 = \frac{s}{2} - m_A^2, & k_1 \cdot p_3 &= k_2 \cdot p_4 = \frac{m_A^2 + m_B^2 - t}{2}, \\
p &= \frac{\sqrt{s - 4m_B^2}}{2}, \quad p_3 \cdot p_4 = \frac{s}{2} - m_B^2, & k_1 \cdot p_4 &= k_2 \cdot p_3 = \frac{m_A^2 + m_B^2 - u}{2}.
\end{aligned}$$

Instead, if the collision is "initially massless" $(0, 0) \to (m_3, m_4)$,



$$\begin{aligned}
t &= (m_3^2 + m_4^2 - s)/2 + p\sqrt{s}\cos\theta, \\
u &= (m_3^2 + m_4^2 - s)/2 - p\sqrt{s}\cos\theta, \\
p &= (\sqrt{s}/2)\, \lambda^{1/2} \left(1; m_3^2/s, m_4^2/s \right).
\end{aligned}$$

1.1. Fundamentals

Lorentz-invariant phase space:

$$\int \mathrm{d}\Pi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\sqrt{m^2+\|\boldsymbol{p}\|^2}} = \int \frac{\mathrm{d}p_0\,\mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}(2\pi)\,\delta\left(p_0^2-\|\boldsymbol{p}\|^2-m^2\right)\Theta(p_0)$$

Källén function:

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zx = (x-y-z)^2-4yz;$$

$$\lambda(1;\alpha_1^2,\alpha_2^2) = (1-(\alpha_1+\alpha_2)^2)(1-(\alpha_1-\alpha_2)^2) = (1+\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)(1-\alpha_1+\alpha_2).$$

$$\lambda^{1/2}\left(s;m_1^2,m_2^2\right) = s\,\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right); \qquad \lambda^{1/2}\left(1;\frac{m^2}{s},\frac{m^2}{s}\right) = \sqrt{1-\frac{4m^2}{s}},$$

$$\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = \sqrt{1-\frac{2(m_1^2+m_2^2)}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}}, \qquad \lambda^{1/2}\left(1;\frac{m_1^2}{s},0\right) = \frac{s-m_1^2}{s}.$$

Two-body phase space If $f(p_1^\mu,p_2^\mu)$ is Lorentz invariant, $f\equiv f(p_1^2,p_2^2,p_1^\mu p_{2\mu})\equiv f(p_1,p_2,\cos\theta_{12})$. Meanwhile,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \int \frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{(4\pi)\,\mathrm{d}p_1\,p_1^2}{(2\pi)^3}\frac{(2\pi)\,\mathrm{d}p_2\,p_2^2\,\mathrm{d}\cos\theta_{12}}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{\mathrm{d}E_+\,\mathrm{d}E_-\,\mathrm{d}s}{128\pi^4}, \tag{1.5}$$

with the replacement of the variables

$$E_\pm = E_1\pm E_2, \qquad s=(p_1+p_2)^2 = m_1^2+m_2^2+2E_1E_2-2\|\boldsymbol{p}_1\|\|\boldsymbol{p}_2\|\cos\theta_{12};$$

$$\left|\frac{\mathrm{d}(E_+,E_-,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{4p_1^2p_2^2}{E_1E_2}, \qquad \left|\frac{\mathrm{d}(E_1,E_2,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{2p_1^2p_2^2}{E_1E_2}.$$

Therefore,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \frac{1}{128\pi^4}\int_{(m_1+m_2)^2}^\infty \mathrm{d}s\int_{\sqrt{s}}^\infty \mathrm{d}E_+\int_{\min}^{\max}\mathrm{d}E_-, \tag{1.6}$$

where the boundary of E_- is given by

$$\cos\theta_{12} = \frac{E_+^2-E_-^2+2\left(m_1^2+m_2^2-s\right)}{\sqrt{(E_++E_-)^2-4m_1^2}\sqrt{(E_+-E_-)^2-4m_2^2}} \in [-1,1]$$

$$\therefore \quad \left|E_- - \frac{m_1^2-m_2^2}{s}E_+\right| \le \sqrt{E_+^2-s}\cdot\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = 2p\sqrt{\frac{E_+^2-s}{s}}.$$

Two-body phase space with momentum conservation As a general representation in any frame,

$$\overline{\mathrm{d}\Pi^2} = \frac{\mathrm{d}p_1\,\mathrm{d}\Omega\,p_1^2}{16\pi^2}\frac{\delta(E_0-\sqrt{m_1^2+p_1^2}-\sqrt{m_2^2+\|\boldsymbol{P}_0-\boldsymbol{p}_1\|^2})}{E_1E_2} = \frac{1}{8\pi}\,\mathrm{d}\cos\theta_1\frac{p_1^2}{E_0p_1-P_0E_1\cos\theta_1}, \tag{1.7}$$

where the momentum p_1 is given by

$$p_1 = \frac{(E_0^2+m_1^2-m_2^2-P_0^2)P_0\cos\theta_1+E_0\sqrt{\lambda(E_0^2,m_1^2,m_2^2)+P_0^4-2P_0^2(E_0^2+m_1^2-2m_1^2\cos^2\theta_1-m_2^2)}}{2(E_0^2-P_0^2\cos^2\theta_1)}. \tag{1.8}$$

CM frame result is recovered by setting $E_0=\sqrt{s}$ and $P_0=0$.

1.2. Decay rate and Cross section

As $\langle\mathrm{out}|\mathrm{in}\rangle = (2\pi)^4\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f})\mathrm{i}\mathcal{M}$ (for in $\neq$ out) and $\langle\boldsymbol{p}|\boldsymbol{p}\rangle = 2E_{\boldsymbol{p}}(2\pi)^3\,\delta^{(3)}(\boldsymbol{0}) = 2E_{\boldsymbol{p}}V$ for one-particle state,

$$\frac{N_\mathrm{ev}}{\prod_\mathrm{in}N_\mathrm{particle}} = \int \mathrm{d}\Pi^\mathrm{out}\frac{|\langle\mathrm{out}|\mathrm{in}\rangle|^2}{\langle\mathrm{in}|\mathrm{in}\rangle} = \int \mathrm{d}\Pi^\mathrm{out}\frac{(2\pi)^8|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}\frac{VT}{(2\pi^4)}\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f}) = VT\int\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}. \tag{1.9}$$

Therefore, decay rate (at the rest frame) is given by

$$\mathrm{d}\Gamma := \frac{1}{T}\frac{\mathrm{d}N_\mathrm{ev}}{N_\mathrm{particle}} = \frac{1}{T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.10}$$

We also define Lorentz-invariant cross section σ by $N_\mathrm{ev} =: (\rho_Av_\mathrm{Møl}T\sigma)N_B = (\rho_Av_\mathrm{Møl}T\sigma)(\rho_BV)$, or

$$\mathrm{d}\sigma := \frac{\mathrm{d}N_\mathrm{ev}}{\rho_Av_\mathrm{Møl}TN_B} = \frac{V}{v_\mathrm{Møl}T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{2E_A2E_BV^2} = \frac{1}{2E_A2E_B\,v_\mathrm{Møl}}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.11}$$

where the Møller parameter $v_\mathrm{Møl}$ is equal to $v_\mathrm{rel}^\mathrm{NR} = \|\boldsymbol{v}_A-\boldsymbol{v}_B\|$ if $\boldsymbol{v}_A \mathbin{/\!/} \boldsymbol{v}_B$ (cf. Ref. [**?**]). Generally,

$$v_\mathrm{Møl} := \frac{\sqrt{(p_A\cdot p_B)^2-m_A^2m_B^2}}{E_AE_B} = \frac{p_A\cdot p_B}{E_AE_B}v_\mathrm{rel} = (1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B)v_\mathrm{rel}, \tag{1.12}$$

where v_rel is the actual relative velocity

$$v_\mathrm{rel} = \sqrt{1-\frac{(1-v_A^2)(1-v_B^2)}{1-(\boldsymbol{v}_A\cdot\boldsymbol{v}_B)^2}} = \frac{\sqrt{\|\boldsymbol{v}_A-\boldsymbol{v}_B\|^2-\|\boldsymbol{v}_A\times\boldsymbol{v}_B\|^2}}{1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B} = \frac{\lambda^{1/2}(s,m_A^2,m_B^2)}{s-(m_A^2+m_B^2)} \neq v_\mathrm{rel}^\mathrm{NR}. \tag{1.13}$$

(Note that $p_A\cdot p_B/E_AE_B = 1$ if $\boldsymbol{p}_A=0$ or $\boldsymbol{p}_B=0$. Also, Each of v_rel, VT, and $E_AE_Bv_\mathrm{Møl}$ is Lorentz invariant.)

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation adj. $= (\epsilon^a)^{bc}$.[*1]

$$T_a = \frac{1}{2}\sigma_a, \qquad \mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \qquad [T_a, T_b] = \mathrm{i}\epsilon^{abc}T^c, \qquad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\overline{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a \epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\overline{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \to \epsilon^{ab}[\exp\left(\mathrm{i}g\theta^\alpha T^\alpha\right)]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp\left(\mathrm{i}g\theta^\alpha T^\alpha\right)]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp\left(-\mathrm{i}g\theta^\alpha T^{\alpha*}\right)]^{ab}(\epsilon^{bc}\mathbf{2}^c). \tag{2.1}$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\overline{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation adj. $= \mathbf{8} = (f^a)^{bc}$.
Gell-Mann matrices:

$$\lambda_{1\text{–}8} = \begin{pmatrix} 0&1&0\\1&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&-\mathrm{i}&0\\ \mathrm{i}&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0\\0&-1&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&-\mathrm{i}\\0&0&0\\ \mathrm{i}&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&1\\0&1&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&-\mathrm{i}\\0&\mathrm{i}&0 \end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix} 1&0&0\\0&1&0\\0&0&-2 \end{pmatrix}. \tag{2.2}$$

$$\tau_a = \frac{1}{2}\lambda_a, \qquad \mathrm{Tr}(\tau_a\tau_b) = \frac{1}{2}\delta_{ab}, \qquad [\tau_a, \tau_b] = \mathrm{i}f^{abc}\tau^c, \qquad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\begin{aligned}
\mathbf{3}: \quad & \phi_a \to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi_b \simeq \phi_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi_b \\
& \phi^*_a \to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi^*_b \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi^*_b \\
& \quad = \phi^*_b[\exp(-\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ba} \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\phi^*_b\tau^\alpha_{ba}
\end{aligned}$$

$$\begin{aligned}
\overline{\mathbf{3}}: \quad & \phi_a \to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi_b \\
& \phi^*_a \to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi^*_b \simeq \phi^*_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi^*_b
\end{aligned}$$

[*1] We do not distinguish sub- and superscripts for gauge indices.

3. Spinors

$$(\overline{\psi_1}\psi_2)^* = (\psi_2)^\dagger(\overline{\psi}_1)^\dagger = \overline{\psi_2}\psi_1. \tag{3.1}$$

4. Standard Model

(summary page)

4.1. Particle content and convention

4.2. Lagrangian

4.3. Higgs mechanism

4.4. Lagrangian in mass eigenstates

4.5. CKM matrix and Yukawa convention

Our convention is, with $Y = UY^{\mathrm{diag}}V^\dagger$,

$$\begin{aligned}\mathcal{L} &\supset \overline{U}Y_uHP_\mathrm{L}Q - \overline{D}Y_dH^\dagger P_\mathrm{R}Q - \overline{E}Y_eH^\dagger P_\mathrm{R}L + \text{h.c.} && (4.1)\\ &= \overline{U_i}Y_{uij}\epsilon^{ab}H^aP_\mathrm{L}Q_j^b - \overline{D_i}Y_{dij}H^{a*}P_\mathrm{L}Q_j^a - \overline{E_i}Y_{eij}H^{a*}P_\mathrm{L}L_j^a + \text{h.c.} && (4.2)\\ &= \epsilon^{ab}\overline{U}U_uY_u^{\mathrm{diag}}H^aP_\mathrm{L}V_u^\dagger Q^b - \overline{D}U_dY_d^{\mathrm{diag}}H^{a*}P_\mathrm{R}V_d^\dagger Q^a - \overline{E}U_eY_e^{\mathrm{diag}}H^{a*}P_\mathrm{R}V_e^\dagger L + \text{h.c.} && (4.3)\\ &\rightsquigarrow -\frac{v}{\sqrt2}\overline{U}U_uY_u^{\mathrm{diag}}V_u^\dagger P_\mathrm{L}Q^1 - \frac{v}{\sqrt2}\overline{D}U_dY_d^{\mathrm{diag}}V_d^\dagger P_\mathrm{L}Q^2 - \frac{v}{\sqrt2}\overline{E}U_eY_e^{\mathrm{diag}}V_e^\dagger P_\mathrm{L}L^2 + \text{h.c.} && (4.4)\end{aligned}$$

Noting that $(\overline{\psi_A}P_\mathrm{L}\psi_B)^* = \overline{\psi_B}P_\mathrm{R}\psi_A$, this definition is equivalent to

$$\mathcal{L} \supset -\overline{Q^a}Y_u^\dagger\epsilon^{ab}H^{b*}P_\mathrm{R}U - \overline{Q^a}Y_d^\dagger H^aP_\mathrm{R}D - \overline{L^a}Y_e^\dagger H^aP_\mathrm{R}E + \text{h.c.} \tag{4.5}$$

Moving to the CKM basis, the weak interaction is amended as

$$\mathcal{L} \supset \overline{Q}\mathrm{i}\gamma^\mu(-\mathrm{i}g_2W_\mu)P_\mathrm{L}Q \supset \frac{g_2}{\sqrt2}\left[\overline{Q^1}\not{W}^+P_\mathrm{L}Q^2 + \overline{Q^2}\not{W}^-P_\mathrm{L}Q^1\right] = \frac{g_2}{\sqrt2}\left[\overline{\hat{Q}^1}V_u^\dagger\not{W}^+P_\mathrm{L}V_d\hat{Q}^2 + \overline{\hat{Q}^2}V_d^\dagger\not{W}^-P_\mathrm{L}V_u\hat{Q}^1\right], \tag{4.6}$$

where hatted fields are in the CKM basis, and we define, with $\mathrm{s}_{ij} > 0$ and $\mathrm{c}_{ij} > 0$,

$$\begin{aligned}V_\mathrm{CKM} = V_u^\dagger V_d &= \begin{pmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix} = \begin{pmatrix}1 & & \\ & \mathrm{c}_{23} & \mathrm{s}_{23}\\ & -\mathrm{s}_{23} & \mathrm{c}_{23}\end{pmatrix}\begin{pmatrix}\mathrm{c}_{13} & & \mathrm{s}_{13}\,\mathrm{e}^{-\mathrm{i}\delta}\\ & 1 & \\ -\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & & \mathrm{c}_{13}\end{pmatrix}\begin{pmatrix}\mathrm{c}_{12} & \mathrm{s}_{12} & \\ -\mathrm{s}_{12} & \mathrm{c}_{12} & \\ & & 1\end{pmatrix}\mathrm{e}^{\mathrm{i}\Theta}\\ &= \begin{pmatrix}\mathrm{c}_{12}\mathrm{c}_{13} & \mathrm{s}_{12}\mathrm{c}_{13} & \mathrm{s}_{13}\,\mathrm{e}^{-\mathrm{i}\delta}\\ -\mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{s}_{23}\mathrm{c}_{13}\\ \mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & -\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{23}\mathrm{c}_{13}\end{pmatrix}\mathrm{e}^{\mathrm{i}\Theta}.\end{aligned} \tag{4.7}$$

Here, a 3×3 unitary matrix has three angles and six phases, among which five phases are removed by the fermion rotation, but another phase Θ is introduced due to the Θ-term removal.♣TODO:link♣

PDG convention (consistent in 2006§11 and 2018§12)

$$\mathcal{L} \supset -Y_{ij}^d\overline{Q_{Li}^I}\phi d_{Rj}^I - Y_{ij}^u\overline{Q_{Li}^I}\epsilon\phi^* u_{Rj}^I, \quad Y^{\mathrm{diag}} = V_LYV_R^\dagger, \quad V_\mathrm{CKM} = V_L^uV_L^{d\dagger}. \tag{4.8}$$

So, $Y^u = Y_u^\dagger$, $Y^d = Y_d^\dagger$; $Y^{\mathrm{diag}} = V_RY^\dagger V_L^\dagger = V_RYV_L^\dagger$ leads $V_L = V^\dagger$, and $V_\mathrm{CKM} = V_u^\dagger V_d = V_\mathrm{CKM}$.

SLHA2 convention (0801.0045)

$$W \supset \epsilon_{ab}\left[(Y_E)_{ij}H_1^aL_i^b\bar{E}_j + (Y_D)_{ij}H_1^aQ_i^b\bar{D}_j + (Y_U)_{ij}H_2^bQ_i^a\bar{U}_j\right]; \tag{4.9}$$

$$\mathcal{L} \supset -\epsilon_{ab}\left[(Y_E)_{ij}H_1^a\psi_{Li}^b\psi_{\bar{E}j} + (Y_D)_{ij}H_1^a\psi_{Qi}^b\psi_{\bar{D}j} + (Y_U)_{ij}H_2^b\psi_{Qi}^a\psi_{\bar{U}j}\right] \tag{4.10}$$

$$\rightsquigarrow -\left[\psi_{\bar{E}}v_\mathrm{d}Y_E^\mathrm{T}\psi_L^2 + \psi_{\bar{D}}v_\mathrm{d}Y_D^\mathrm{T}\psi_Q^2 + \psi_{\bar{U}}v_\mathrm{u}Y_U^\mathrm{T}\psi_Q^1\right]; \quad Y^{\mathrm{diag}} = U^\dagger Y^\mathrm{T}V, \quad V_\mathrm{CKM} = V_u^\dagger V_d. \tag{4.11}$$

Hence, $Y_E = Y_e^\mathrm{T}$, $Y_D = Y_d^\mathrm{T}$, $Y_U = Y_u^\mathrm{T}$; $Y^{\mathrm{diag}} = U^\dagger YV$, $V = V$ and $V_\mathrm{CKM} = V_\mathrm{CKM}$.

Wolfenstein parameterization The CKM matrix is precisely written in terms of λ, A, and $\bar\rho + \mathrm{i}\bar\eta$.

$$\lambda := \mathrm{s}_{12} = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad A := \frac{\mathrm{s}_{23}}{\lambda^2} = \lambda^{-1}\left|\frac{V_{cb}}{V_{us}}\right|, \quad \bar\rho + \mathrm{i}\bar\eta := \frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \tag{4.12}$$

They are independent of the phase convention and used for SLHA2 input, i.e., `VCKMIN` should contain $(\lambda, A, \bar\rho, \bar\eta)$.

Also, $\bar\rho + \mathrm{i}\eta$ is approximately written by

$$R = \rho + \mathrm{i}\eta := \frac{s_{13}\,\mathrm{e}^{\mathrm{i}\delta}}{A\lambda^3} = \frac{V_{ub}^*}{A\lambda^3}\frac{V_{ud}}{|V_{ud}|} = \frac{(\bar\rho + \mathrm{i}\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}\,[1 - A^2\lambda^4(\bar\rho + \mathrm{i}\bar\eta)]} = (\bar\rho + \mathrm{i}\bar\eta)\left(1 + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4)\right), \tag{4.13}$$

with which

$$V_\mathrm{CKM} = \begin{pmatrix}1 - \lambda^2/2 & \lambda & A\lambda^3R^*\\ -\lambda & 1 - \lambda^2/2 & A\lambda^2\\ A\lambda^3(1-R) & -A\lambda^2 & 1\end{pmatrix}\mathrm{e}^{\mathrm{i}\Theta} + \begin{pmatrix}\mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^7) & 0\\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^8)\\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^4)\end{pmatrix}. \tag{4.14}$$

4.6. Values of SM parameters

5. Supersymmetry with $\eta = \mathrm{diag}(+,-,-,-)$

Convention Our convention follows DHM (except for D_μ):

$$\eta = \mathrm{diag}(1,-1,-1,-1); \quad \epsilon^{0123} = -\epsilon_{0123} = 1, \quad \epsilon^{12} = \epsilon_{21} = \epsilon^{\dot1\dot2} = \epsilon_{\dot2\dot1} = 1 \quad \left(\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma\right),$$
$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta}, \quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta};$$
$$\sigma^\mu_{\alpha\dot\alpha} := (\mathbf{1},\boldsymbol{\sigma})_{\alpha\dot\alpha}, \qquad \sigma^{\mu\nu}{}_\alpha{}^\beta := \frac{\mathrm{i}}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)_\alpha{}^\beta,^{*2} \qquad \left(\sigma^\mu_{\alpha\dot\beta} = \epsilon_{\alpha\delta}\epsilon_{\dot\beta\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \quad \bar\sigma^{\mu\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\delta}\epsilon^{\beta\gamma}\sigma^\mu_{\gamma\dot\delta}\right)$$
$$\bar\sigma^{\mu\dot\alpha\alpha} := (\mathbf{1},-\boldsymbol{\sigma})^{\dot\alpha\alpha}, \quad \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta} := \frac{\mathrm{i}}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta},^{*2}$$
$$(\psi\xi) := \psi^\alpha\xi_\alpha, \quad (\bar\psi\bar\chi) := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}; \qquad \frac{\mathrm{d}}{\mathrm{d}\theta^\alpha}(\theta\theta) := \theta_\alpha \quad \text{[left derivative]}.$$

Especially, spinor-index contraction is done as ${}^\alpha{}_\alpha$ and ${}_{\dot\alpha}{}^{\dot\alpha}$ except for ϵ_{ab} (which always comes from left). Noting that complex conjugate reverses spinor order: $(\psi^\alpha\xi^\beta)^* := (\xi^\beta)^*(\psi^\alpha)^*$,

$$\bar\psi^{\dot\alpha} := (\psi^\alpha)^*, \quad \epsilon^{\dot a\dot b} := (\epsilon^{ab})^*, \qquad (\psi\chi)^* = (\bar\psi\bar\chi),$$
$$\left(\sigma^\mu_{\alpha\dot\beta}\right)^* = \bar\sigma^\mu{}_{\dot\alpha\beta} = \epsilon_{\beta\delta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \qquad \left(\sigma^{\mu\nu}\right)^{\dagger\alpha}{}_\beta = \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}, \qquad \left(\sigma^{\mu\nu}{}_\alpha{}^\beta\right)^* = \bar\sigma^{\mu\nu\dot\beta}{}_{\dot\alpha} = \bar\sigma^{\mu\nu}{}_{\dot\alpha}{}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\delta},$$
$$\left(\bar\sigma^{\mu\dot\alpha\beta}\right)^* = \sigma^{\mu\alpha\dot\beta} = \epsilon^{\dot\beta\dot\delta}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\delta}, \qquad \left(\bar\sigma^{\mu\nu}\right)^\dagger{}_{\dot\alpha}{}^{\dot\beta} = \sigma^{\mu\nu}{}_\alpha{}^\beta, \qquad \left(\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\right)^* = \sigma^{\mu\nu}{}_\beta{}^\alpha = \bar\sigma^{\mu\nu\alpha}{}_\beta = \epsilon_{\beta\delta}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\delta.$$

Contraction formulae

$$\theta^\alpha\theta^\beta = -\tfrac12(\theta\theta)\epsilon^{\alpha\beta} \qquad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\epsilon^{\dot\alpha\dot\beta} \qquad (\theta\xi)(\theta\chi) = -\tfrac12(\theta\theta)(\xi\chi) \qquad (\theta\sigma^\nu\bar\theta)\theta^\alpha = \tfrac12(\theta\theta)(\bar\theta\bar\sigma^\nu)^\alpha$$
$$\theta_\alpha\theta_\beta = \tfrac12(\theta\theta)\epsilon_{\alpha\beta} \qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac12(\bar\theta\bar\theta)\epsilon_{\dot\alpha\dot\beta} \qquad (\bar\theta\bar\xi)(\bar\theta\bar\chi) = -\tfrac12(\bar\theta\bar\theta)(\bar\xi\bar\chi) \qquad (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\tfrac12(\theta\sigma^\nu)_{\dot\alpha}(\bar\theta\bar\theta)$$
$$\theta^\alpha\theta_\beta = \tfrac12(\theta\theta)\delta^\alpha_\beta \qquad \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\delta^{\dot\alpha}_{\dot\beta} \qquad (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \tfrac12(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu}$$
$$(\theta\sigma^\mu\bar\sigma^\nu\theta) = (\theta\theta)\eta^{\mu\nu} \qquad (\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta) = (\bar\theta\bar\theta)\eta^{\mu\nu} \qquad (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \tfrac12(\bar\theta\bar\theta)(\sigma^\mu\bar\sigma^\nu\theta)_\alpha$$

$$\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\sigma^{\mu\nu} \qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho + \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\left(\sigma^\mu\eta^{\rho\nu} - \sigma^\nu\eta^{\mu\rho} + \sigma^\rho\eta^{\mu\nu}\right)$$
$$\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\bar\sigma^{\mu\nu} \qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\mathrm{i}\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$\mathrm{Tr}\left(\sigma^\mu\bar\sigma^\nu\right) = \mathrm{Tr}\left(\bar\sigma^\mu\sigma^\nu\right) = 2\eta^{\mu\nu} \qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho + \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = 2\left(\bar\sigma^\mu\eta^{\rho\nu} - \bar\sigma^\nu\eta^{\mu\rho} + \bar\sigma^\rho\eta^{\mu\nu}\right)$$
$$\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu{}^{\dot\beta\beta} = 2\delta^{\dot\beta}_{\dot\alpha}\delta^\alpha_\beta \qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2\mathrm{i}\bar\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$\sigma_{\mu\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} \qquad \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\dot\beta\dot\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\beta}$$
$$\bar\sigma_\mu{}^{\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} \qquad \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\beta\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\dot\alpha\dot\gamma}\sigma^\mu_{\beta\dot\gamma}$$
$$\mathrm{Tr}\left(\sigma^{\mu\nu}\right) = \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\right) = 0 \qquad \mathrm{Tr}\left(\sigma^{\mu\nu}\sigma^{\rho\sigma}\right) = \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma}$$
$$\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu} \qquad \sigma^{\mu\nu} = -\sigma^{\nu\mu} \qquad \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma}\right) = \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma} + \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$
$$\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = -2\mathrm{i}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon_{\alpha\beta} - 2\mathrm{i}\sigma^{\mu\nu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}$$
$$\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon^{\dot\beta\dot\alpha} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 4\sigma^{\rho\mu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\sigma\nu\dot\gamma}{}_{\dot\beta}\eta_{\rho\sigma} + \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\eta^{\mu\nu}$$
$$\bar\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = -2\mathrm{i}\bar\sigma^{\mu\nu} \qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} - \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = -2\mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\epsilon^{\alpha\beta} - 2\mathrm{i}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon^{\dot\alpha\dot\beta}$$
$$\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 2\mathrm{i}\sigma^{\mu\nu} \qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} + \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 4\epsilon^{\alpha\gamma}\sigma^{\sigma\nu}{}_\gamma{}^\beta\bar\sigma^{\rho\mu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\eta_{\rho\sigma} + \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\eta^{\mu\nu}$$

$$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi \qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi = \bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi \qquad \xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi \qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho\chi = -\chi\sigma^\rho\bar\sigma^\nu\sigma^\mu\bar\xi$$
$$\left(\xi\sigma^\mu\bar\chi\right)^* = \chi\sigma^\mu\bar\xi \qquad \left(\bar\xi\bar\sigma^\mu\chi\right)^* = \bar\chi\bar\sigma^\mu\xi \qquad \left(\bar\chi\bar\sigma^\mu\sigma^\nu\bar\xi\right)^* = \xi\sigma^\nu\bar\sigma^\mu\chi \qquad (\xi[\sigma\mathrm{s}]\chi)^* = \bar\chi[\sigma\mathrm{s}_{\mathrm{reversed}}]\bar\xi$$
$$(\xi\chi)\psi^\alpha = -(\psi\xi)\chi^\alpha - (\psi\chi)\xi^\alpha \qquad (\xi\chi)\bar\psi_{\dot\alpha} = \tfrac12(\xi\sigma^\mu\bar\psi)(\chi\sigma_\mu)_{\dot\alpha}$$
$$\mathrm{i}\psi_i\sigma^\mu\partial_\mu\bar\psi_j = -\mathrm{i}\partial_\mu\bar\psi_j\bar\sigma^\mu\psi_i \equiv \mathrm{i}\bar\psi_j\bar\sigma^\mu\partial_\mu\psi_i = -\mathrm{i}\partial_\mu\psi_i\sigma^\mu\bar\psi_j$$

[*2] As the definition of $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are not unified in literature, they are not used in this CheatSheet except for this page.

Superfields

$$\Phi = \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{\theta^4}{4}\partial^2\phi(x), \tag{5.1}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{\theta^4}{4}\partial^2\phi^*(x), \tag{5.2}$$

$$V = (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{\theta^4}{2}D(x) \qquad \text{(in Wess-Zumino supergauge).} \tag{5.3}$$

Without gauge symmetries

$$\mathcal{L} = \Phi_i^*\Phi_i\Big|_{\theta^4} + \left(W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right); \tag{5.4}$$

$$\Phi_i^*\Phi_i\Big|_{\theta^4} = (\partial_\mu\phi_i^*)(\partial^\mu\phi_i) + \mathrm{i}\bar\psi_i\sigma^\mu\partial_\mu\psi_i + F_i^*F_i, \tag{5.5}$$

$$\begin{aligned} W(\Phi_i)\Big|_{\theta^2} &\rightsquigarrow \Big[\kappa_i\Phi_i + m_{ij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k\Big]_{\theta^2} \\ &= \kappa_iF_i + m_{ij}\left(-\psi_i\psi_j + F_i\phi_j + \phi_iF_j\right) \\ &\qquad + y_{ijk}\Big[-(\psi_i\psi_j\phi_k + \psi_i\phi_j\psi_k + \phi_i\psi_j\psi_k) + \phi_i\phi_jF_k + \phi_iF_j\phi_k + F_i\phi_j\phi_k\Big]. \end{aligned} \tag{5.6}$$

With a U(1) gauge symmetry [*3]

$$\mathcal{L} = \Phi_i^*\,\mathrm{e}^{2gVQ_i}\Phi_i\Big|_{\theta^4} + \left[\left(\frac14 - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right] + \Lambda_{\text{FI}}D; \tag{5.7}$$

$$\Phi_i\,\mathrm{e}^{2gQ_iV}\Phi_i\Big|_{\theta^4} \equiv \mathrm{D}^\mu\phi_i^*\,\mathrm{D}_\mu\phi_i + \mathrm{i}\bar\psi_i\bar\sigma^\mu\,\mathrm{D}_\mu\psi_i + F_i^*F_i - \sqrt{2}gQ_i\phi_i^*\lambda\psi_i - \sqrt{2}gQ_i\bar\psi_i\bar\lambda\phi_i + gQ_i\phi_i^*\phi_iD, \tag{5.8}$$

$$\begin{aligned} \left(\frac14 - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + \text{H.c.} &= \frac12\,\mathrm{Re}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} + \frac{g^2\Theta}{16\pi^2}\,\mathrm{Im}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} \\ &\equiv \mathrm{i}\bar\lambda\bar\sigma^\mu\,\mathrm{D}_\mu\lambda + \frac12DD - \frac14F_{\mu\nu}F^{\mu\nu} + \frac{g^2\Theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \end{aligned} \tag{5.9}$$

$$\begin{aligned} &\mathrm{D}_\mu\phi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\phi_i, \qquad \mathrm{D}_\mu\psi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\psi_i, \\ &\mathrm{D}^\mu\phi_i^* = (\partial^\mu + \mathrm{i}gQ_iA^\mu)\phi_i^*, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \mathrm{D}_\mu\lambda = \partial_\mu\lambda. \end{aligned}$$

$$\{\phi,\psi,F\} \xrightarrow{\text{gauge}} \mathrm{e}^{\mathrm{i}gQ_i\theta}\{\phi,\psi,F\}, \qquad A_\mu \xrightarrow{\text{gauge}} A_\mu + \partial_\mu\theta, \qquad \lambda \xrightarrow{\text{gauge}} \lambda, \qquad D \xrightarrow{\text{gauge}} D. \tag{5.10}$$

[*3] We use the convention with $V \ni \lambda(x)\theta\bar\theta^2$, which corresponds to $\lambda = \mathrm{i}\lambda_{\text{SLHA}}$. In SLHA convention, the scalar-fermion-gaugino interaction is replaced to

$$-\sqrt{2}g\mathrm{i}\lambda^a_{\text{SLHA}}(\phi^*t^a\psi) - \sqrt{2}g(-\mathrm{i}\bar\lambda^a_{\text{SLHA}})(\bar\psi t^a\phi).$$

With an SU(N) gauge symmetry

$$\mathcal{L} = \Phi^* \,\mathrm{e}^{2gV} \Phi\Big|_{\theta^4} + \left[\left(\frac{1}{2} - \frac{\mathrm{i}g^2\Theta}{16\pi^2}\right) \mathrm{Tr}\, \mathcal{W}^\alpha \mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi)\Big|_{\theta^2} + \text{H.c.}\right]; \tag{5.11}$$

$$\Phi^* \,\mathrm{e}^{2gV} \Phi\Big|_{\theta^4} := \Phi_i^* \left[\mathrm{e}^{2gV^a t^a_\Phi}\right]_{ij} \Phi_j\Big|_{\theta^4} \tag{5.12}$$

$$\begin{aligned} &= (\partial_\mu \phi_i^*)(\partial^\mu \phi_i) + \mathrm{i}\bar\psi_i \bar\sigma^\mu \partial_\mu \psi_i + F_i^* F_i - \sqrt{2} g \lambda^a (\phi^* t^a \psi) - \sqrt{2} g \bar\lambda^a (\bar\psi^* t^a \phi) \\ &\quad + g A^a_\mu \bar\psi \bar\sigma^\mu (t^a \psi) + 2\mathrm{i} g A^a_\mu \phi^* \partial_\mu (t^a \phi) + g^2 A^{a\mu} A^b_\mu (\phi^* t^a t^b \phi) + g D^a (\phi^* t^a \phi) \end{aligned} \tag{5.13}$$

$$= \mathrm{D}^\mu \phi^* \,\mathrm{D}_\mu \phi + \mathrm{i}\bar\psi_i \bar\sigma^\mu \,\mathrm{D}_\mu \psi_i + F^* F - \sqrt{2} g \lambda^a (\phi^* t^a \psi) - \sqrt{2} g \bar\lambda^a (\bar\psi t^a \phi) + g D^a (\phi^* t^a \phi) \tag{5.14}$$

$$\begin{aligned} \left(\frac{1}{2} - \frac{\mathrm{i}g^2\Theta}{16\pi^2}\right) \mathrm{Tr}\, \mathcal{W}^\alpha \mathcal{W}_\alpha\Big|_{\theta^2} + \text{H.c.} &= \mathrm{Re}\,\mathrm{Tr}\, \mathcal{W}\mathcal{W}\big|_{\theta^2} + \frac{g^2\Theta}{8\pi^2} \mathrm{Im}\,\mathrm{Tr}\, \mathcal{W}\mathcal{W}\big|_{\theta^2} \\ &= \mathrm{i}\lambda^a \sigma^\mu \,\mathrm{D}_\mu \bar\lambda^a + \frac{1}{2} D^a D^a - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{g^2\Theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}; \end{aligned} \tag{5.15}$$

$$\mathrm{D}_\mu \phi_i = \partial_\mu \phi_i - \mathrm{i} g A^a_\mu t^a_{ij} \phi_j, \qquad \mathrm{D}_\mu \psi_i = \partial_\mu \psi_i - \mathrm{i} g A^a_\mu t^a_{ij} \psi_j, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g A^b_\mu A^c_\nu f^{abc},$$
$$\mathrm{D}^\mu \phi_i^* = \partial^\mu \phi_i^* + \mathrm{i} g A^{a\mu} \phi_j^* t^a_{ji}, \qquad \mathrm{D}_\mu \lambda^a_\alpha = \partial_\mu \lambda^a_\alpha + g f^{abc} A^b_\mu \lambda^c_\alpha.$$

$$\{\phi, \psi, F\} \xrightarrow{\text{gauge}} \mathrm{e}^{\mathrm{i}g\theta^a t^a} \{\phi, \psi, F\},$$
$$A^a_\mu \xrightarrow{\text{gauge}} A^a_\mu + \partial_\mu \theta^a + g f^{abc} A^b_\mu \theta^c + \mathcal{O}(\theta^2), \qquad \lambda^a \xrightarrow{\text{gauge}} \lambda^a + g f^{abc} \lambda^b \theta^c + \mathcal{O}(\theta^2),$$
$$D^a \xrightarrow{\text{gauge}} D^a + g f^{abc} D^b \theta^c + \mathcal{O}(\theta^2), \qquad \bar\lambda^a \xrightarrow{\text{gauge}} \bar\lambda^a + g f^{abc} \bar\lambda^b \theta^c + \mathcal{O}(\theta^2).$$ [*4]

Auxiliary fields and Scalar potential In all of the above three theories,

$$\mathcal{L} \supset F_i^* F_i + F_i \frac{\partial W}{\partial \Phi_i}\Big|_{\text{scalar}} + F_i^* \frac{\partial W^*}{\partial \Phi_i^*}\Big|_{\text{scalar}} + \frac{1}{2} D^a D^a + g D^a (\phi^* t^a \phi); \tag{5.16}$$

$$\langle F_i^* \rangle = -\frac{\partial W}{\partial \Phi_i}\Big|_{\text{scalar}}, \qquad \langle D^a \rangle = -g \phi^* t^a \phi; \tag{5.17}$$

$$\mathcal{L} \supset -V_{\text{SUSY}} = -\left[\langle F_i^* \rangle \langle F_i \rangle + \frac{g^2}{2} (\phi^* t^a \phi)(\phi^* t^a \phi)\right]. \tag{5.18}$$

*4 ♣TODO: give in non-infinitesimal form♣

5.1. Lorentz symmetry as SU(2)×SU(2)

5.2. Supersymmetry algebra

We define the generators as

$$P_\mu := \mathrm{i}\partial_\mu,\quad \{\mathcal{Q}_\alpha,\bar{\mathcal{Q}}_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu,\quad \{\mathcal{Q}_\alpha,\mathcal{Q}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha},\bar{\mathcal{Q}}_{\dot\beta}\} = 0, \tag{5.19}$$

which is realized by

$$\mathcal{Q}_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu,\quad \bar{\mathcal{Q}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu,\quad \mathcal{Q}^\alpha = -\frac{\partial}{\partial\theta_\alpha} - \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu,\quad \bar{\mathcal{Q}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu,$$

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu,\quad \bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu,\quad \mathcal{D}^\alpha = -\frac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu,\quad \bar{\mathcal{D}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu;$$

$\mathcal{D}_\alpha$ etc. works as covariant derivatives because of the commutation relations

$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\alpha}\} = +2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu,\qquad \{\mathcal{Q}_\alpha,\mathcal{D}_\beta\} = \{\mathcal{Q}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\} = \{\bar{\mathcal{Q}}_{\dot\alpha},\mathcal{D}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\} = \{\mathcal{D}_\alpha,\mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\} = 0.$$

Derivative formulae

$$\begin{aligned}
&\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha} && \frac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha && \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = -2\delta_\alpha^\beta && \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = 2\delta_{\dot\alpha}^{\dot\beta}\\
&\epsilon_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} = -\frac{\partial}{\partial\theta^\alpha} && \frac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha && \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = 2\epsilon^{\alpha\beta} && \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = -2\epsilon^{\dot\alpha\dot\beta}\\
&\epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\beta}} = -\frac{\partial}{\partial\bar\theta_{\dot\alpha}} && \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} && \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = 2\delta^\alpha_\beta && \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = -2\delta^{\dot\alpha}_{\dot\beta}\\
&\epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\beta}} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} && \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} && \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = -2\epsilon_{\alpha\beta} && \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = 2\epsilon_{\dot\alpha\dot\beta}
\end{aligned}$$

In addition, we define

$$(y,\theta',\bar\theta') := (x - \mathrm{i}\theta\sigma^\mu\bar\theta, \theta, \bar\theta) : \tag{5.20}$$

$$\bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta'^{\dot\alpha}};\qquad \begin{pmatrix}\frac{\partial}{\partial x^\mu}\\ \frac{\partial}{\partial\theta^\alpha}\\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\end{pmatrix} = \begin{pmatrix}\delta^\nu_\mu & 0 & 0\\ -\mathrm{i}(\sigma^\nu\bar\theta)_\alpha & \delta_\alpha^\beta & 0\\ \mathrm{i}(\theta\sigma^\nu)_{\dot\alpha} & 0 & \delta_{\dot\alpha}^{\dot\beta}\end{pmatrix}\begin{pmatrix}\frac{\partial}{\partial y^\nu}\\ \frac{\partial}{\partial\theta'^\beta}\\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}}\end{pmatrix},\qquad \begin{pmatrix}\frac{\partial}{\partial y^\nu}\\ \frac{\partial}{\partial\theta'^\beta}\\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}}\end{pmatrix} = \begin{pmatrix}\delta^\mu_\nu & 0 & 0\\ \mathrm{i}(\sigma^\mu\bar\theta)_\beta & \delta_\beta^\alpha & 0\\ -\mathrm{i}(\theta\sigma^\mu)_{\dot\beta} & 0 & \delta_{\dot\beta}^{\dot\alpha}\end{pmatrix}\begin{pmatrix}\frac{\partial}{\partial x^\mu}\\ \frac{\partial}{\partial\theta^\alpha}\\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\end{pmatrix}, \tag{5.21}$$

and a function $f:\mathbb{C}^4\to\mathbb{C}$ (independent of θ' and $\bar\theta'$) is expanded as

$$f(y) = f(x-\mathrm{i}\theta\sigma\bar\theta) = f(x) - \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f(x) - \frac{1}{4}\theta^4\partial^2 f(x). \tag{5.22}$$

Note that we differentiate $[f(y)]^*$ and $f^*(y)$:

$$[f(y)]^* = f(x) + \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f^*(x) - \frac{1}{4}\theta^4\partial^2 f^*(x) = f^*(y+\mathrm{i}\theta\sigma\bar\theta) = f^*(y^*). \tag{5.23}$$

5.3. Superfields

SUSY-invariant Lagrangian SUSY transformation is induced by $\xi\mathcal{Q}+\bar\xi\bar{\mathcal{Q}} = \xi^\alpha\partial_\alpha + \bar\xi_{\dot\alpha}\partial^{\dot\alpha} + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu$. Therefore, for an object Ψ in the superspace,

$$\bigl[\Psi\bigr]_{\theta^4} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \xi^\alpha\partial_\alpha\Psi + \bar\xi_{\dot\alpha}\partial^{\dot\alpha}\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\Bigr]_{\theta^4} = \bigl[\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\bigr]_{\theta^4}, \tag{5.24}$$

which means $\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant up to total derivative, i.e., $\int \mathrm{d}^4x\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant action. Also,

$$\bigl[\Psi\bigr]_{\theta^2} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \bar\xi_{\dot\alpha}\Bigl(\partial^{\dot\alpha} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Bigr)\Psi\Bigr]_{\theta^2} = \Bigl[\Psi + \bar\xi_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\Psi + 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Psi\Bigr]_{\theta^2} \tag{5.25}$$

will be SUSY-invariant if $\bar{\mathcal{D}}_{\dot\alpha}\Psi = 0$, i.e., Ψ is a chiral superfield. Therefore, SUSY-invariant Lagrangian is given by

$$\mathcal{L} = \Bigl[(\text{any real superfield})\Bigr]_{\theta^4} + \Bigl[(\text{any chiral superfield})\Bigr]_{\theta^2} + \Bigl[(\text{any chiral superfield})^*\Bigr]_{\bar\theta^2}. \tag{5.26}$$

Chiral superfield A chiral superfield is a superfield that satisfies $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$, i.e., we find

$$\Phi = \phi(y) + \sqrt{2}\theta'\psi(y) + \theta'^2F(y) \tag{5.27}$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{1}{4}\partial^2\phi(x)\theta^4 \tag{5.28}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{1}{4}\partial^2\phi^*(x)\theta^4; \tag{5.29}$$

their product is expanded as

$$\begin{aligned}\Phi_i^*\Phi_j &= \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - \mathrm{i}\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2\\ &\quad + \left[\sqrt{2}\bar\psi_i\bar\theta F_j - \frac{\mathrm{i}\left(\partial_\mu\phi_i^*\cdot\psi_j\sigma^\mu\bar\theta - \phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)}{\sqrt{2}}\right]\theta^2 + \left[\sqrt{2}F_i^*\theta\psi_j + \frac{\mathrm{i}\left(\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j - \theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)}{\sqrt{2}}\right]\bar\theta^2\\ &\quad + \frac14\left(4F_i^*F_j - \phi_i^*\partial^2\phi_j - (\partial^2\phi_i^*)\phi_j + 2(\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + 2\mathrm{i}(\psi_j\sigma^\mu\partial_\mu\bar\psi_i) - 2\mathrm{i}(\partial_\mu\psi_j\sigma^\mu\bar\psi_i)\right)\theta^4\end{aligned} \tag{5.30}$$

$$\begin{aligned}&\equiv \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - 2\mathrm{i}\left(\phi_i^*\partial_\mu\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2\\ &\quad + \sqrt{2}\left(\bar\psi_i\bar\theta F_j + \mathrm{i}\phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)\theta^2 + \sqrt{2}\left(F_i^*\theta\psi_j - \mathrm{i}\theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)\bar\theta^2\\ &\quad + \left(F_i^*F_j + (\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right)\theta^4\end{aligned} \tag{5.31}$$

$$\Phi_i\Phi_j\Big|_{\theta^2} = -\psi_i\psi_j + F_i\phi_j + \phi_iF_j \tag{5.32}$$

$$\Phi_i\Phi_j\Phi_k\Big|_{\theta^2} = -(\psi_i\psi_j)\phi_k - (\psi_k\psi_i)\phi_j - (\psi_j\psi_k)\phi_i + \phi_i\phi_jF_k + \phi_k\phi_iF_j + \phi_j\phi_kF_i \tag{5.33}$$

$$\mathrm{e}^{k\Phi} = \mathrm{e}^{k\phi}\left[1 + \sqrt{2}k\theta\psi + \left(kF - \frac{k^2}{2}\psi\psi\right)\theta^2 - \mathrm{i}k\partial_\mu\phi(\theta\sigma^\mu\bar\theta) + \frac{\mathrm{i}k\left(\partial_\mu\psi + k\psi\partial_\mu\phi\right)\sigma^\mu\bar\theta\theta^2}{\sqrt{2}} - \frac{k}{4}\left(\partial^2\phi + k\partial_\mu\phi\partial^\mu\phi\right)\theta^4\right]; \tag{5.34}$$

note that $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$, and $\mathrm{e}^{k\Phi}$ are all chiral superfields.

Vector superfield A vector superfield is a superfield V that satisfies $V = V^*$. It is given by real fields $\{C, M, N, D, A_\mu\}$ and Grassmann fields $\{\chi, \lambda\}$ as[*5]

$$\begin{aligned}V(x,\theta,\bar\theta) &= C(x) + \mathrm{i}\theta\chi(x) - \mathrm{i}\bar\theta\bar\chi(x) + \frac12\left(M(x) + \mathrm{i}N(x)\right)\theta^2 + \frac12\left(M(x) - \mathrm{i}N(x)\right)\bar\theta^2 + (\bar\theta\bar\sigma^\mu\theta)A_\mu(x)\\ &\qquad \left(\lambda(x) + \frac12\partial_\mu\bar\chi(x)\bar\sigma^\mu\right)\theta\bar\theta^2 + \theta^2\bar\theta\left(\bar\lambda(x) + \frac12\bar\sigma^\mu\partial_\mu\chi(x)\right) + \frac12\left(D(x) - \frac12\partial^2C(x)\right)\theta^4.\end{aligned} \tag{5.35}$$

With this convention,

$$V \to V - \mathrm{i}\Phi + \mathrm{i}\Phi^* \iff \begin{cases} C \to C - \mathrm{i}\phi + \mathrm{i}\phi^*, & \chi \to \chi - \sqrt{2}\psi, \qquad \lambda \to \lambda,\\ M + \mathrm{i}N \to M + \mathrm{i}N - 2\mathrm{i}F, & A_\mu \to A_\mu + \partial_\mu(\phi + \phi^*), \qquad D \to D.\end{cases} \tag{5.36}$$

The exponential of a vector superfield is also a vector superfield:

$$\begin{aligned}\mathrm{e}^{kV} = \mathrm{e}^{kC}\Bigg\{&1 + \mathrm{i}k(\theta\chi - \bar\theta\bar\chi) + \left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\theta^2 + \left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\bar\theta^2 + (k^2\theta\chi\bar\theta\bar\chi - k\theta\sigma^\mu\bar\theta A_\mu)\\ &+ \left[k\bar\theta\bar\lambda - \mathrm{i}k\bar\theta\bar\chi\left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right) + \frac12 k\bar\theta\bar\sigma^\mu\left(\partial_\mu\chi - \mathrm{i}k\chi A_\mu\right)\right]\theta^2\\ &+ \left[k\theta\lambda + \mathrm{i}k\theta\chi\left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right) - \frac12 k\theta\sigma^\mu\left(\partial_\mu\bar\chi + \mathrm{i}k\bar\chi A_\mu\right)\right]\bar\theta^2\\ &+ \left[\frac{k}{2}\left(D - \frac12\partial^2C\right) - \frac12\mathrm{i}k^2(\lambda\chi - \bar\lambda\bar\chi) + \left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\right.\\ &\qquad \left. + \frac{k^3}{4}\bar\chi\bar\sigma^\mu\chi A_\mu + \frac{k^2}{4}\left(\mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - \mathrm{i}\partial_\mu\bar\chi\bar\sigma^\mu\chi + A^\mu A_\mu\right)\right]\theta^4\Bigg\}.\end{aligned} \tag{5.37}$$

Supergauge symmetry The gauge transformation $\phi(x) \to \mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\phi(x)$ is not closed in the chiral superfield; i.e., $\mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\Phi(x)$ is not a chiral superfield if the parameter $\theta(x)$ has x^μ-dependence. Hence, in supersymmetric theories, it is extended to *supergauge symmetry* parameterized by a chiral superfield $\Omega(x)$, which is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Omega^a(x)t^a}\Phi, \qquad \Phi^* \to \Phi^*\,\mathrm{e}^{-2\mathrm{i}g\Omega^{*a}(x)t^a} \tag{5.38}$$

for a chiral superfield Φ and an anti-chiral superfield Φ^*. The supergauge-invariant Lagrangian should be

$$\mathcal{L} \sim \Phi^*\cdot(\text{real superfield})\cdot\Phi; \tag{5.39}$$

we parameterize the "real superfield" as $\mathrm{e}^{2gV^a(x)t^a}$:

$$\mathcal{L} = \Big[\Phi^*\,\mathrm{e}^{2gV^a(x)t^a}\Phi\Big]_{\theta^4}; \qquad \mathrm{e}^{2gV^a(x)t^a} \to \mathrm{e}^{2\mathrm{i}g\Omega^{*a}(x)t^a}\,\mathrm{e}^{2gV^a(x)t^a}\,\mathrm{e}^{-2\mathrm{i}g\Omega^a(x)t^a}. \tag{5.40}$$

[*5]Different coordination of "i"s are found in literature. Take care, especially, $\lambda(\text{ours}) = \mathrm{i}\lambda(\text{Wess-Bagger}) = \mathrm{i}\lambda(\text{SLHA})$.

In Abelian case, t^a is replaced by the charge Q of Φ and

$$\mathcal{L} = \left[\Phi^* \,\mathrm{e}^{2gQV(x)}\Phi\right]_{\theta^4}; \qquad \Phi \to \mathrm{e}^{2\mathrm{i}gQ\Omega(x)}\Phi, \quad \Phi^* \to \Phi^* \,\mathrm{e}^{-2\mathrm{i}gQ\Omega^*(x)}, \tag{5.41}$$

$$\mathrm{e}^{2gQV(x)} \to \mathrm{e}^{2\mathrm{i}gQ\Omega^*(x)}\,\mathrm{e}^{2gQV(x)}\,\mathrm{e}^{-2\mathrm{i}gQ\Omega(x)} = \mathrm{e}^{2gQ\left(V-\mathrm{i}\Omega+\mathrm{i}\Omega^*\right)}. \tag{5.42}$$

The usual gauge transformation corresponds to the real part of the lowest component of Ω, i.e., $\theta \equiv 2\,\mathrm{Re}\,\phi = \phi + \phi^*$, and we use the other components to fix the supergauge so that C, M, N and χ are eliminated:

$$\text{supergauge fixing:} \quad V(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{1}{2}D(x) \qquad \text{(Wess-Zumino gauge);} \tag{5.43}$$

$$\mathrm{e}^{2gQV} \longrightarrow 1 + gQ\left(-2\theta\sigma^\mu\bar\theta A_\mu + 2\theta^2\bar\theta\bar\lambda + 2\bar\theta^2\theta\lambda + D\theta^4\right) + g^2Q^2A^\mu A_\mu\theta^4. \tag{5.44}$$

The gauge transformation is the remnant freedom: $\Theta = \phi(y) = \phi - \mathrm{i}\partial_\mu\phi(\theta\sigma^\mu\bar\theta) - \partial^2\phi\theta^4/4$ with ϕ being real;

$$\Phi_i \to \mathrm{e}^{2\mathrm{i}gQ\Theta}\Phi_i, \qquad \mathrm{e}^{2gQV} \to \mathrm{e}^{2gQ(V-\mathrm{i}\Theta+\mathrm{i}\Theta^*)}. \tag{5.45}$$

Rules for each component is obvious in $(y, \theta, \bar\theta)$-basis and given by

$$\{\phi, \psi, F\} \to \mathrm{e}^{\mathrm{i}gQ\theta}\{\phi, \psi, F\}, \qquad A_\mu \to A_\mu + \partial_\mu\theta, \qquad \lambda \to \lambda, \qquad D \to D. \tag{5.46}$$

For non-Abelian gauges, the supergauge transformation for the real field is evaluated as

$$\mathrm{e}^{2gV} \to \mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{2gV}\,\mathrm{e}^{-2\mathrm{i}g\Omega} \tag{5.47}$$

$$= \left(\mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{2gV}\,\mathrm{e}^{-2\mathrm{i}g\Omega^*}\right)\left(\mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{-2\mathrm{i}g\Omega}\right) \tag{5.48}$$

$$= \exp\left(\mathrm{e}^{[2\mathrm{i}g\Omega^*,}2gV\right)\mathrm{e}^{2\mathrm{i}g(\Omega^*-\Omega)} + \mathcal{O}(\Omega^2) \tag{5.49}$$

$$= \exp\left(2gV + [2\mathrm{i}g\Omega^*, 2gV]\right)\mathrm{e}^{2\mathrm{i}g(\Omega^*-\Omega)} + \mathcal{O}(\Omega^2); \tag{5.50}$$

$$= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[2gV,})2\mathrm{i}g(\Omega^*-\Omega)] + \mathcal{O}(\Omega^2)\right] \tag{5.51}$$

$$= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \sum_{n=0}^\infty \frac{B_n\left([2gV,)^n\right.}{n!}2\mathrm{i}g(\Omega^*-\Omega)]\right] + \mathcal{O}(\Omega^2) \tag{5.52}$$

$$= \exp\left[2g\left(V + \mathrm{i}(\Omega^*-\Omega) - [V, \mathrm{i}g(\Omega^*+\Omega)] + \sum_{n=2}^\infty \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Omega^*-\Omega)]\right) + \mathcal{O}(\Omega^2)\right]. \tag{5.53}$$

Here, again we can use the "non-gauge" component of Ω to eliminate the C-term etc., i.e., we fix $\mathrm{i}(\Omega^*-\Omega)$, the second term of the expansion, to remove those terms:

$$V - [V, \mathrm{i}g(\Omega^*+\Omega)] + \left(\mathrm{i} + \sum_{n=2}^\infty \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}\right)(\Omega^*-\Omega)] + \mathcal{O}(\Omega^2) = (\bar\theta\bar\sigma^\mu\theta)A_\mu + \bar\theta^2\theta\lambda + \theta^2\bar\theta\bar\lambda + \frac{1}{2}D; \tag{5.54}$$

this defines the Wess-Zumino gauge:

$$\text{supergauge fixing:} \quad V^a(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A^a_\mu(x) + \bar\theta^2\theta\lambda^a(x) + \theta^2\bar\theta\bar\lambda^a(x) + \frac{1}{2}D^a(x), \tag{5.55}$$

$$\mathrm{e}^{2gV^at^a} \longrightarrow 1 + g\left(-2\theta\sigma^\mu\bar\theta A^a_\mu + 2\theta^2\bar\theta\bar\lambda^a + 2\bar\theta^2\theta\lambda^a + D^a\theta^4\right)t^a + g^2A^{a\mu}A^b_\mu\theta^4t^at^b. \tag{5.56}$$

The gauge transformation is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Theta^at^a}\Phi, \quad \mathrm{e}^{2gV^at^a} \to \mathrm{e}^{2\mathrm{i}g\Theta^bt^b}\,\mathrm{e}^{2gV^at^a}\,\mathrm{e}^{-2\mathrm{i}g\Theta^ct^c}. \tag{5.57}$$

For components in chiral superfields,

$$\{\phi, \psi, F\} \to \mathrm{e}^{\mathrm{i}g\theta^at^a}\{\phi, \psi, F\}, \tag{5.58}$$

while for vector superfield we can express as infinitesimal transformation:

$$V \to V' \simeq V + \mathrm{i}(\Theta^*-\Theta) - [V, \mathrm{i}g(\Theta^*+\Theta)] + \sum_{n=2}^\infty \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Theta^*-\Theta)] \tag{5.59}$$

$$= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi - \left[V, \mathrm{i}g\left(2\phi - \frac{\theta^4}{2}\partial^2\phi\right)\right] + 2\sum_{n=2}^\infty \frac{B_n\left([2gV,)^n\right.}{n!}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi] \tag{5.60}$$

$$= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi + 2gf^{abc}V^b\phi^ct^a \qquad \text{(Wess-Zumino gauge)} \tag{5.61}$$

$$\begin{aligned} \therefore A^a_\mu &\to A^a_\mu + \partial_\mu\theta^a + gf^{abc}A^b_\mu\theta^c + \mathcal{O}(\theta^2), & \lambda^a &\to \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2), \\ D^a &\to D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), & \bar\lambda^a &\to \bar\lambda^a + gf^{abc}\bar\lambda^b\theta^c + \mathcal{O}(\theta^2). \end{aligned} \tag{5.62}$$

Gauge-field strength The real superfield e^V is gauge-invariant in Abelian case and a candidate in Lagrangian term, but this is not case in non-Abelian case. We thus define a chiral superfield from e^V:

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right); \tag{5.63}$$

$$\mathcal{W}_\alpha \xrightarrow{\text{gauge}} \mathrm{e}^{2\mathrm{i}g\Omega}\mathcal{W}_\alpha\,\mathrm{e}^{-2\mathrm{i}g\Omega} \quad \left(\mathcal{W}^a_\alpha \xrightarrow{\text{gauge}} [\mathrm{e}^{+2g\tilde{f}^c\Omega^c}]^{ab}W^b_\alpha \quad \text{with} \quad [\tilde{f}^c]_{ab} = f^{abc}\right);^{*6} \tag{5.64}$$

it is not supergauge- or Lorentz-invariatn, but $\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha) = \mathrm{Tr}(\epsilon^{\alpha\beta}\mathcal{W}_\beta\mathcal{W}_\alpha)$ is supergauge- and Lorentz-invariant, and its θ^2-term is SUSY-invariant, which becomes a candidate in SUSY Lagrangian with its Hermitian conjugate.

In Wess-Zumino gauge, it is given by

$$\mathcal{W}_\alpha = \left\{\lambda^a_\alpha(y) + \theta_\alpha D^a(y) + \frac{[\mathrm{i}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)\theta]_\alpha}{4}F^a_{\mu\nu}(y) + \theta^2\left[\mathrm{i}\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a(y^*)\right]_\alpha\right\}t^a \tag{5.65}$$

$$\begin{aligned} &= \Bigg[\lambda^a_\alpha + \theta_\alpha D^a + \frac{\mathrm{i}}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F^a_{\mu\nu} + \mathrm{i}\theta^2(\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a)_\alpha + \mathrm{i}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\lambda^a_\alpha - \frac{\theta^4}{4}\partial^2\lambda^a_\alpha \\ &\qquad + \frac{\mathrm{i}\theta^2(\sigma^\mu\bar\theta)_\alpha}{2}\left(\partial_\mu D^a + \mathrm{i}\partial^\nu F^a_{\mu\nu} - gf^{abc}\epsilon_{\mu\nu\rho\sigma}A^{\nu b}\partial^\rho A^{\sigma c}\right)\Bigg]T^a, \end{aligned} \tag{5.66}$$

where, as usual,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gA^b_\mu A^c_\nu f^{abc}, \qquad \mathrm{D}_\mu\lambda^a_\alpha = \partial_\mu\lambda^a_\alpha + gf^{abc}A^b_\mu\lambda^c_\alpha. \tag{5.67}$$

Also,

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^2} = \left[\mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^b + \mathrm{i}\lambda^b\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + D^aD^b - \frac{1}{4}\left(\mathrm{i}\epsilon^{\sigma\mu\nu\rho} + 2\eta^{\mu\rho}\eta^{\nu\sigma}\right)F^a_{\mu\nu}F^b_{\rho\sigma}\right]\mathrm{Tr}(t^at^b) \tag{5.68}$$

$$= \mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + \frac{1}{2}D^aD^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}, \tag{5.69}$$

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^4} = \frac{\theta^4}{4}\left(2(\partial^\mu\lambda^a)(\partial_\mu\lambda^b) - \lambda^a\partial^2\lambda^b - (\partial^2\lambda^a)\lambda^b\right)\mathrm{Tr}(t^at^b) = \frac{\theta^4}{4}\left((\partial^\mu\lambda^a)(\partial_\mu\lambda^a) - \lambda^a\partial^2\lambda^a\right). \tag{5.70}$$

For Abelian theory,

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right) = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}_\alpha(2gV), \tag{5.71}$$

$$\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} = 2\left(\mathrm{i}\lambda\sigma^\mu\,\mathrm{D}_\mu\bar\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right). \tag{5.72}$$

5.4. Lagrangian blocks

Lagrangian construction The supergauge transformation is summarized as

$$\Phi_i \to [U_\Phi]_{ij}\Phi_j, \qquad \tilde\Phi_j \to \tilde\Phi_i[U_\Phi^{-1}]_{ij}, \qquad \mathcal{W}_\alpha \to U_\mathcal{W}\mathcal{W}_\alpha U_\mathcal{W}^{-1}, \tag{5.73}$$

where

$$\tilde\Phi^*_j := \Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}]_{ij}, \qquad U_\Phi := \exp(2\mathrm{i}g\Omega^at^a_\Phi), \qquad U_\mathcal{W} := \exp(2\mathrm{i}g\Omega^at^a_\mathcal{W}), \tag{5.74}$$

t^a_Φ is the representation matrix or U(1) charge for the field Φ, and $t^a_\mathcal{W}$ is the representation matrix that is used to define $\mathcal{W}_\alpha$. To construct a Lagrangian, we should composite these ingredients in real and invariant under SUSY, supergauge, and Lorentz transformation. A sufficient condition for SUSY invariance is given by (5.26), so

$$\mathcal{L} = \left[K(\Phi_i,\tilde\Phi^*_j)\right]_{\theta^4} + \left\{\left[f_{ab}(\Phi_i)\mathcal{W}^a\mathcal{W}^b\right]_{\theta^2} + \text{H.c.}\right\} + \left\{\left[W(\Phi_i)\right]_{\theta^2} + \text{H.c.}\right\} + D \tag{5.75}$$

is one possible construction. The Kähler function K should be real and supergauge invariant, the gauge kinetic function f should be holomorphic and supergauge invariant with $\mathcal{W}^a\mathcal{W}^b$, and the superpotential W is holomorphic and supergauge invariant. The last term D (Fayet-Illiopoulos term) comes from V of an U(1) gauge boson; note that its supergauge invariance is due to the intentional definition of V.

One can construct more general Lagrangian; for example, one can introduce a vector superfield that is not associated to a gauge symmetry, but then the supergauge fixing is not available and one has to include C or M fields.

Renormalizable Lagrangian Since $[\Phi]_{\theta^4}$ is a total derivative, renormalizable Lagrangian is limited to

$$\mathcal{L} = \left[\Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}]_{ij}\Phi_j\right]_{\theta^4} + \left\{[\mathcal{W}^a\mathcal{W}^a]_{\theta^2} + [W(\Phi_i)]_{\theta^2} + \text{H.c.}\right\} + D \tag{5.76}$$

up to numeric coefficients. With multiple gauge groups, the Kähler part is extended as $\Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}\,\mathrm{e}^{2gV't'^a_\Phi}\cdots]_{ij}\Phi_j$, where the inner part is obviously commutable.

[*6] ♣TODO: This equivalence should be checked/explained in gauge-theory section; especially, the sign is not verified and might be opposite.♣

6. Minimal Supersymmetric Standard Model

Gauge symmetry: $\mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_{\mathrm{weak}} \times \mathrm{U}(1)_Y$

Particle content:

(a) Chiral superfields

	SU(3)	SU(2)	U(1)	B	L	scalar/spinor
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$		$\tilde{q}_{\mathrm{L}}, q_{\mathrm{L}}$ $[\to (u_{\mathrm{L}}, d_{\mathrm{L}})]$
L_i		$\mathbf{2}$	$-1/2$		1	$\tilde{l}_{\mathrm{L}}, l_{\mathrm{L}}$ $[\to (\nu_{\mathrm{L}}, l_{\mathrm{L}})]$
U_i^{c}	$\overline{\mathbf{3}}$		$-2/3$	$-1/3$		$\tilde{u}_{\mathrm{R}}^{\mathrm{c}}, u_{\mathrm{R}}^{\mathrm{c}}$
D_i^{c}	$\overline{\mathbf{3}}$		$1/3$	$-1/3$		$\tilde{d}_{\mathrm{R}}^{\mathrm{c}}, d_{\mathrm{R}}^{\mathrm{c}}$
E_i^{c}			1		-1	$\tilde{e}_{\mathrm{R}}^{\mathrm{c}}, e_{\mathrm{R}}^{\mathrm{c}}$
H_{u}		$\mathbf{2}$	$1/2$			$h_{\mathrm{u}}, \tilde{h}_{\mathrm{u}}$ $[\to (h_{\mathrm{u}}^+, h_{\mathrm{u}}^0)]$
H_{d}		$\mathbf{2}$	$-1/2$			$h_{\mathrm{d}}, \tilde{h}_{\mathrm{d}}$ $[\to (h_{\mathrm{d}}^0, h_{\mathrm{d}}^-)]$

(b) Vector superfields

	SU(3)	SU(2)	U(1)	ino/boson
g	adj.			$\tilde{g}, g_\mu$
W		adj.		$\tilde{w}, W_\mu$
B				$\tilde{b}, B_\mu$

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and SU(2) decomposition).

"c"-notation For scalars, $\tilde{\phi}_{\mathrm{R}}^{\mathrm{c}} := \phi_{\mathrm{R}}^* = \mathsf{C}\phi_{\mathrm{R}}\mathsf{C}$ (because the intrinsic phase for C is $+1$ for quarks and leptons.)

For matter spinors, $\psi_{\mathrm{R}}^{\mathrm{c}} := \bar{\psi}_{\mathrm{R}}$ (and $\psi_{\mathrm{R}} = \bar{\psi}_{\mathrm{R}}^{\mathrm{c}}$); Dirac spinors are thus

$$\psi_{\mathrm{L}} = \begin{pmatrix}\psi_{\mathrm{L}}\\ 0\end{pmatrix}, \quad \overline{\psi_{\mathrm{L}}} = \begin{pmatrix}0 & \bar{\psi}_{\mathrm{L}}\end{pmatrix}, \quad \psi_{\mathrm{R}}^{\mathrm{c}} := \begin{pmatrix}\psi_{\mathrm{R}}^{\mathrm{c}}\\ 0\end{pmatrix} = C\begin{pmatrix}0\\ \psi_{\mathrm{R}}\end{pmatrix} = C\psi_{\mathrm{R}}, \quad \overline{\psi_{\mathrm{R}}^{\mathrm{c}}} = \begin{pmatrix}0 & \psi_{\mathrm{R}}\end{pmatrix} = \begin{pmatrix}\bar{\psi}_{\mathrm{R}} & 0\end{pmatrix} C = \overline{\psi_{\mathrm{R}}}C.$$

Superpotential and $\cancel{\mathrm{SUSY}}$-terms

$$W_{\mathrm{RPC}} = \mu H_{\mathrm{u}} H_{\mathrm{d}} - y_{uij} U_i^{\mathrm{c}} H_{\mathrm{u}} Q_j + y_{dij} D_i^{\mathrm{c}} H_{\mathrm{d}} Q_j + y_{eij} E_i^{\mathrm{c}} H_{\mathrm{d}} L_j, \tag{6.1}$$

$$W_{\mathrm{RPV}} = -\kappa_i L_i H_{\mathrm{u}} + \frac{1}{2}\lambda_{ijk} L_i L_j E_k^{\mathrm{c}} + \lambda'_{ijk} L_i Q_j D_k^{\mathrm{c}} + \frac{1}{2}\lambda''_{ijk} U_i^{\mathrm{c}} D_j^{\mathrm{c}} D_k^{\mathrm{c}}, \tag{6.2}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\Bigl(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \mathrm{H.c.}\Bigr) - V_{\cancel{\mathrm{SUSY}}}; \tag{6.3}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^{\mathrm{RPC}} = {}& \Bigl(\tilde{q}_{\mathrm{L}}^* m_Q^2 \tilde{q}_{\mathrm{L}} + \tilde{l}_{\mathrm{L}}^* m_L^2 \tilde{l}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^* m_{U^{\mathrm{c}}}^2 \tilde{u}_{\mathrm{R}} + \tilde{d}_{\mathrm{R}}^* m_{D^{\mathrm{c}}}^2 \tilde{d}_{\mathrm{R}} + \tilde{e}_{\mathrm{R}}^* m_{E^{\mathrm{c}}}^2 \tilde{e}_{\mathrm{R}} + m_{H_{\mathrm{u}}}^2 |h_{\mathrm{u}}|^2 + m_{H_{\mathrm{d}}}^2 |h_{\mathrm{d}}|^2\Bigr) \\ & + \Bigl(-\tilde{u}_{\mathrm{R}}^* h_{\mathrm{u}} a_{\mathrm{u}} \tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* h_{\mathrm{d}} a_{\mathrm{d}} \tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^* h_{\mathrm{d}} a_{\mathrm{e}} \tilde{l}_{\mathrm{L}} + b H_{\mathrm{u}} H_{\mathrm{d}} + \mathrm{H.c.}\Bigr) \\ & + \Bigl(+\tilde{u}_{\mathrm{R}}^* h_{\mathrm{d}}^* c_{\mathrm{u}} \tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* h_{\mathrm{u}}^* c_{\mathrm{d}} \tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^* h_{\mathrm{u}}^* c_{\mathrm{e}} \tilde{l}_{\mathrm{L}} + \mathrm{H.c.}\Bigr), \end{aligned} \tag{6.4}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^{\mathrm{RPV}} = {}& \Bigl(-b_i \tilde{l}_{\mathrm{L}i} H_{\mathrm{u}} + \frac{1}{2} T_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{l}_{\mathrm{L}j} \tilde{e}_{\mathrm{R}k}^* + T'_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{q}_{\mathrm{L}j} \tilde{d}_{\mathrm{R}k}^* + \frac{1}{2} T''_{ijk} \tilde{u}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j}^* \tilde{d}_{\mathrm{R}k}^* + \tilde{l}_{\mathrm{L}i}^* M_{Li}^2 H_{\mathrm{d}} + \mathrm{H.c.}\Bigr) \\ & + \Bigl(C_{ijk}^1 \tilde{l}_{\mathrm{L}i}^* \tilde{q}_{\mathrm{L}j} \tilde{u}_{\mathrm{R}k}^* + C_i^2 h_{\mathrm{u}}^* h_{\mathrm{d}} \tilde{e}_{\mathrm{R}i}^* + C_{ijk}^3 \tilde{d}_{\mathrm{R}i} \tilde{u}_{\mathrm{R}j}^* \tilde{e}_{\mathrm{R}k}^* + \frac{1}{2} C_{ijk}^4 \tilde{d}_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} \tilde{q}_{\mathrm{L}k} + \mathrm{H.c.}\Bigr), \end{aligned} \tag{6.5}$$

($\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C_{ijk}^4 = C_{ikj}^4$.)

6.1. Notation

Our notation in this section (and the previous section) follows DHM [?, PhysRept] and Martin [?, v7] (but note that Martin uses $(-,+,+,+)$-metric) for RPC part and SLHA2 convention for RPV part. In particular, the sign of gauge bosons are fixed by $\mathrm{D}_\mu\phi = \partial_\mu\phi - \mathrm{i}gA^a_\mu t^a_{ij}\phi_j$, and the phase of gauginos are by $\mathcal{L} \ni \sqrt{2}g(\phi^* t^a\psi\lambda^a)$. Phases of ϕ and ψ in chiral superfields are not yet specified; they are later used to remove $F\tilde{F}$ terms and diagonalize Yukawa matrices.

6.2. Lagrangian construction

The most generic form of the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{super}} + \mathcal{L}_{\text{FI}} + \mathcal{L}_{\cancel{\text{SUSY}}}; \tag{6.6}$$

$$\mathcal{L}_{\text{matter}} = \Phi_Q^* \exp\left(2g_Y(\tfrac{1}{6})V_B + 2g_2V_W^aT^a + 2g_3V_g^a\tau^a\right)\Phi_Q\Big|_{\theta^4} + \cdots; \tag{6.7}$$

$$\mathcal{L}_{\text{gauge}} = \left[\frac{1}{4}\left(1 - \frac{\mathrm{i}g_Y^2\Theta_B}{8\pi^2}\right)\mathcal{W}_B\mathcal{W}_B + \frac{1}{4}\left(1 - \frac{\mathrm{i}g_2^2\Theta_W}{8\pi^2}\right)\mathcal{W}_W^a\mathcal{W}_W^a + \frac{1}{4}\left(1 - \frac{\mathrm{i}g_3^2\Theta_g}{8\pi^2}\right)\mathcal{W}_g^a\mathcal{W}_g^a\right]_{\theta^2} + \text{H.c.}; \tag{6.8}$$

$$\mathcal{L}_{\text{super}} = W(\Phi)\Big|_{\theta^2} + \text{H.c.}, \tag{6.9}$$

$$W(\Phi) = W_{\text{RPC}} + W_{\text{RPV}}, \tag{6.10}$$

$$W_{\text{RPC}} = \mu H_{\mathrm{u}}H_{\mathrm{d}} - y_{uij}U_i^{\mathrm{c}}H_{\mathrm{u}}Q_j + y_{dij}D_i^{\mathrm{c}}H_{\mathrm{d}}Q_j + y_{eij}E_i^{\mathrm{c}}H_{\mathrm{d}}L_j, \tag{6.11}$$

$$W_{\text{RPV}} = -\kappa_iL_iH_{\mathrm{u}} + \frac{1}{2}\lambda_{ijk}L_iL_jE_k^{\mathrm{c}} + \lambda'_{ijk}L_iQ_jD_k^{\mathrm{c}} + \frac{1}{2}\lambda''_{ijk}U_i^{\mathrm{c}}D_j^{\mathrm{c}}D_k^{\mathrm{c}}; \tag{6.12}$$

$$\mathcal{L}_{\text{FI}} = \Lambda_{\text{FI}}D_B; \tag{6.13}$$

$$\mathcal{L}_{\cancel{\text{SUSY}}} = -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V^{\text{RPC}}_{\cancel{\text{SUSY}}} + V^{\text{RPV}}_{\cancel{\text{SUSY}}}\right), \tag{6.14}$$

$$\begin{aligned} V^{\text{RPC}}_{\cancel{\text{SUSY}}} = &\left(\tilde{q}_{\mathrm{L}}^*m_Q^2\tilde{q}_{\mathrm{L}} + \tilde{l}_{\mathrm{L}}^*m_L^2\tilde{l}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^*m_{U^{\mathrm{c}}}^2\tilde{u}_{\mathrm{R}} + \tilde{d}_{\mathrm{R}}^*m_{D^{\mathrm{c}}}^2\tilde{d}_{\mathrm{R}} + \tilde{e}_{\mathrm{R}}^*m_{E^{\mathrm{c}}}^2\tilde{e}_{\mathrm{R}} + m_{H_{\mathrm{u}}}^2|h_{\mathrm{u}}|^2 + m_{H_{\mathrm{d}}}^2|h_{\mathrm{d}}|^2\right) \\ &+ \left(-\tilde{u}_{\mathrm{R}}^*h_{\mathrm{u}}a_{\mathrm{u}}\tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^*h_{\mathrm{d}}a_{\mathrm{d}}\tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^*h_{\mathrm{d}}a_{\mathrm{e}}\tilde{l}_{\mathrm{L}} + bH_{\mathrm{u}}H_{\mathrm{d}} + \text{H.c.}\right) \\ &+ \left(\tilde{u}_{\mathrm{R}}^*h_{\mathrm{d}}^*c_{\mathrm{u}}\tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^*h_{\mathrm{u}}^*c_{\mathrm{d}}\tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^*h_{\mathrm{u}}^*c_{\mathrm{e}}\tilde{l}_{\mathrm{L}} + \text{H.c.}\right), \end{aligned} \tag{6.15}$$

$$\begin{aligned} V^{\text{RPV}}_{\cancel{\text{SUSY}}} = &\left(-b_i\tilde{l}_{\mathrm{L}i}H_{\mathrm{u}} + \frac{1}{2}T_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{l}_{\mathrm{L}j}\tilde{e}_{\mathrm{R}k}^* + T'_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{q}_{\mathrm{L}j}\tilde{d}_{\mathrm{R}k}^* + \frac{1}{2}T''_{ijk}\tilde{u}_{\mathrm{R}i}^*\tilde{d}_{\mathrm{R}j}^*\tilde{d}_{\mathrm{R}k}^* + \tilde{l}_{\mathrm{L}i}^*M_{Li}^2H_{\mathrm{d}} + \text{H.c.}\right) \\ &+ \left(C^1_{ijk}\tilde{l}_{\mathrm{L}i}^*\tilde{q}_{\mathrm{L}j}\tilde{u}_{\mathrm{R}k}^* + C^2_ih_{\mathrm{u}}^*h_{\mathrm{d}}\tilde{e}_{\mathrm{R}i}^* + C^3_{ijk}\tilde{d}_{\mathrm{R}i}\tilde{u}_{\mathrm{R}j}^*\tilde{e}_{\mathrm{R}k}^* + \frac{1}{2}C^4_{ijk}\tilde{d}_{\mathrm{R}i}\tilde{q}_{\mathrm{L}j}\tilde{q}_{\mathrm{L}k} + \text{H.c.}\right). \end{aligned} \tag{6.16}$$

As usual, we remove Θ_W and Θ_B by rotating fermions[*7] , which is compatible with mass diagonalization (discussed later), and assume the absence of Fayet-Illiopoulos term: $\Lambda_{\text{FI}} = 0$. The SU(3) angle Θ_g forms QCD phase Θ_{QCD} together with the phases from Yukawa matrices. Then,

$$\mathcal{L}_{\text{matter}} = \sum_{\text{matters}}\left[\mathrm{D}^\mu\phi^*\,\mathrm{D}_\mu\phi + \mathrm{i}\bar{\psi}\bar{\sigma}^\mu\,\mathrm{D}_\mu\psi - \sqrt{2}\sum_{\text{gauge}}g\left(\lambda^a(\phi^*t^a\psi) + \bar{\lambda}^a(\bar{\psi}t^a\phi)\right)\right] + (F\text{-terms}), \tag{6.17}$$

$$\mathcal{L}_{\text{gauge}} = \sum_{\text{gauges}}\left(-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \mathrm{i}\bar{\lambda}^a\bar{\sigma}^\mu\,\mathrm{D}_\mu\lambda^a\right) + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} + (D\text{-terms}), \tag{6.18}$$

$$\begin{aligned} \mathcal{L}_{\text{super}} = \epsilon^{ab}\Big(&-\mu\tilde{h}_{\mathrm{u}}^a\tilde{h}_{\mathrm{d}}^b - y_{dij}h_{\mathrm{d}}^ad_{\mathrm{R}i}^{\mathrm{c}x}q_{\mathrm{L}j}^{bx} - y_{dij}\tilde{d}_{\mathrm{R}i}^{x*}\tilde{h}_{\mathrm{d}}^aq_{\mathrm{L}j}^{bx} + y_{dji}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{h}_{\mathrm{d}}^bd_{\mathrm{R}j}^{\mathrm{c}x} \\ &-y_{eij}\tilde{e}_{\mathrm{R}i}^*\tilde{h}_{\mathrm{d}}^al_{\mathrm{L}j}^b - y_{eij}h_{\mathrm{d}}^ae_{\mathrm{R}i}^{\mathrm{c}}l_{\mathrm{L}j}^b + y_{eji}\tilde{l}_{\mathrm{L}i}^a\tilde{h}_{\mathrm{d}}^be_{\mathrm{R}j}^{\mathrm{c}} + y_{uij}h_{\mathrm{u}}^au_{\mathrm{R}i}^{\mathrm{c}x}q_{\mathrm{L}j}^{bx} + y_{uij}\tilde{u}_{\mathrm{R}i}^{x*}\tilde{h}_{\mathrm{u}}^aq_{\mathrm{L}j}^{bx} - y_{uji}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{h}_{\mathrm{u}}^bu_{\mathrm{R}j}^{\mathrm{c}x} \\ &-\kappa_i\tilde{h}_{\mathrm{u}}^al_{\mathrm{L}i}^b - \lambda_{ikj}\tilde{l}_{\mathrm{L}i}^ae_{\mathrm{R}j}^{\mathrm{c}}l_{\mathrm{L}k}^b - \frac{1}{2}\lambda_{jki}\tilde{e}_{\mathrm{R}i}^*l_{\mathrm{L}j}^al_{\mathrm{L}k}^b - \lambda'_{ikj}\tilde{l}_{\mathrm{L}i}^ad_{\mathrm{R}j}^{\mathrm{c}x}q_{\mathrm{L}k}^{bx} + \lambda'_{kij}\tilde{q}_{\mathrm{L}i}^{ax}d_{\mathrm{R}j}^{\mathrm{c}x}l_{\mathrm{L}k}^b + \lambda'_{kji}\tilde{d}_{\mathrm{R}i}^{x*}q_{\mathrm{L}j}^{ax}l_{\mathrm{L}k}^b\Big) \\ &-\frac{1}{2}\epsilon^{xyz}\lambda''_{ijk}\tilde{u}_{\mathrm{R}i}^{x*}d_{\mathrm{R}j}^{\mathrm{c}y}d_{\mathrm{R}k}^{\mathrm{c}z} + \epsilon^{xyz}\lambda''_{jik}\tilde{d}_{\mathrm{R}i}^{x*}u_{\mathrm{R}j}^{\mathrm{c}y}d_{\mathrm{R}k}^{\mathrm{c}z} + \text{H.c.} + (F\text{-terms}), \end{aligned} \tag{6.19}$$

$$\mathcal{L}_{\cancel{\text{SUSY}}} = -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V^{\text{RPC}}_{\cancel{\text{SUSY}}} + V^{\text{RPV}}_{\cancel{\text{SUSY}}}\right), \tag{6.20}$$

and the F- and D-terms form the supersymmetric scalar potential

$$V_{\text{SUSY}} = F_i^*F_i + \frac{1}{2}D^aD^a; \qquad F_i = -W_i^* = -\frac{\delta W^*}{\delta\phi_i^*}, \qquad D^a = -g(\phi^*t^a\phi), \tag{6.21}$$

$$V = V_{\text{SUSY}} + V^{\text{RPC}}_{\cancel{\text{SUSY}}} + V^{\text{RPV}}_{\cancel{\text{SUSY}}}, \tag{6.22}$$

[*7] Fail-safe memo: The Θ-terms are total derivatives and relevant in non-perturbative discussion. Redefinition of chiral fermions generates those terms (Fujikawa method) as a non-perturbative effect, so we can remove Θ-terms as long as we have such freedoms. Note also that the absence of gauge anomaly means the corresponding gauge transformations do not induce additional Θ-terms.

where t_a corresponds to the gauge-symmetry generator relevant for each ϕ.

Each auxiliary term is given by

$$-F^*_{h^a_\mathrm{u}} = \epsilon^{ab}\left(-\tilde{u}^{x*}_\mathrm{R} y_u \tilde{q}^{bx}_\mathrm{L} + \mu h^b_\mathrm{d} + \kappa_i \tilde{l}^b_{\mathrm{L}i}\right), \tag{6.23}$$

$$-F^*_{h^a_\mathrm{d}} = \epsilon^{ab}\left(\tilde{e}^*_\mathrm{R} y_e \tilde{l}^b_\mathrm{L} + \tilde{d}^{x*}_\mathrm{R} y_d \tilde{q}^{bx}_\mathrm{L} - \mu h^b_\mathrm{u}\right), \tag{6.24}$$

$$-F^*_{\tilde{q}^{ax}_{\mathrm{L}i}} = \epsilon^{ab}\left(-y_{dji} h^b_\mathrm{d} \tilde{d}^{x*}_{\mathrm{R}j} + y_{uji} h^b_\mathrm{u} \tilde{u}^{x*}_{\mathrm{R}j} - \lambda'_{kij} \tilde{d}^{x*}_{\mathrm{R}j} \tilde{l}^b_{\mathrm{L}k}\right), \tag{6.25}$$

$$-F^*_{\tilde{u}^{x*}_{\mathrm{R}i}} = -y_{uij} h_\mathrm{u} \tilde{q}^x_{\mathrm{L}j} + \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk} \tilde{d}^{y*}_{\mathrm{R}j} \tilde{d}^{z*}_{\mathrm{R}k}, \tag{6.26}$$

$$-F^*_{\tilde{d}^{x*}_{\mathrm{R}i}} = y_{dij} h_\mathrm{d} \tilde{q}^x_{\mathrm{L}j} + \lambda'_{jki} \tilde{l}_{\mathrm{L}j} \tilde{q}^x_{\mathrm{L}k} - \lambda''_{jik}\epsilon^{xyz} \tilde{u}^{y*}_{\mathrm{R}j} \tilde{d}^{z*}_{\mathrm{R}k}, \tag{6.27}$$

$$-F^*_{\tilde{l}^a_{\mathrm{L}i}} = \epsilon^{ab}\left(-y_{eji} \tilde{e}^*_{\mathrm{R}j} h^b_\mathrm{d} - \kappa_i h^b_\mathrm{u} + \lambda_{ikj} \tilde{e}^*_{\mathrm{R}j} \tilde{l}^b_{\mathrm{L}k} + \lambda'_{ikj} \tilde{d}^{x*}_{\mathrm{R}j} \tilde{q}^{bx}_{\mathrm{L}k}\right), \tag{6.28}$$

$$-F^*_{\tilde{e}^*_{\mathrm{R}i}} = y_{eij} h_\mathrm{d} \tilde{l}_{\mathrm{L}j} + \frac{1}{2}\lambda_{jki} \tilde{l}_{\mathrm{L}j} \tilde{l}_{\mathrm{L}k}. \tag{6.29}$$

$$D^\alpha_{\mathrm{SU}(3)} = -g_3 \sum_{i=1}^3 \left(\sum_{a=1,2} \tilde{q}^{a*}_{\mathrm{L}i} \tau^\alpha \tilde{q}^a_{\mathrm{L}i} - \tilde{u}^*_{\mathrm{R}i} \tau^\alpha \tilde{u}_{\mathrm{R}i} - \tilde{d}^*_{\mathrm{R}i} \tau^\alpha \tilde{d}_{\mathrm{R}i}\right), \tag{6.30}$$

$$D^\alpha_{\mathrm{SU}(2)} = -g_2 \left[\sum_{i=1}^3 \left(\sum_{x=1}^3 \tilde{q}^{x*}_{\mathrm{L}i} T^\alpha \tilde{q}^x_{\mathrm{L}i} + \tilde{l}^*_{\mathrm{L}i} T^\alpha \tilde{l}_{\mathrm{L}i}\right) + h^*_\mathrm{u} T^\alpha h_\mathrm{u} + h^*_\mathrm{d} T^\alpha h_\mathrm{d}\right], \tag{6.31}$$

$$D_{\mathrm{U}(1)} = -g_1 \left(\frac{1}{6}|\tilde{q}_\mathrm{L}|^2 - \frac{1}{2}|\tilde{l}_\mathrm{L}|^2 - \frac{2}{3}|\tilde{u}_\mathrm{R}|^2 + \frac{1}{3}|\tilde{d}_\mathrm{R}|^2 + |\tilde{e}_\mathrm{R}|^2 + \frac{1}{2}|h_\mathrm{u}|^2 - \frac{1}{2}|h_\mathrm{d}|^2\right). \tag{6.32}$$

6.3. Full Lagrangian

Here the Lagrangian $\mathcal{L} = \mathcal{L}_\mathrm{vector} + \mathcal{L}_\mathrm{fermions} + \mathcal{L}_\mathrm{SFG} + \mathcal{L}_\mathrm{scalar}$ is explicitly given:

$$\mathcal{L}_\mathrm{vector} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}, \tag{6.33}$$

$$\mathcal{L}_\mathrm{fermions} = \mathrm{i}\bar{\psi}\bar{\sigma}^\mu \mathrm{D}_\mu\psi + \mathrm{i}\bar{\lambda}^a\bar{\sigma}^\mu \mathrm{D}_\mu\lambda^a - \frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \mathrm{H.c.}\right) + \mathcal{L}_\mathrm{super}\big|_{\text{no }F\text{-terms}}, \tag{6.34}$$

$$\mathcal{L}_\mathrm{SFG} = -\sqrt{2}g\lambda^a(\phi^* t^a\psi) - \sqrt{2}g\bar{\lambda}^a(\bar{\psi}t^a\phi), \tag{6.35}$$

$$\mathcal{L}_\mathrm{scalar} = \mathrm{D}^\mu\phi^*\,\mathrm{D}_\mu\phi - V. \tag{6.36}$$

6.3.1. Vector part

$$\begin{aligned}\mathcal{L}_\mathrm{vector} = &-\frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)\partial^\mu B^\nu - \frac{1}{2}(\partial_\mu g^a_\nu - \partial_\nu g^a_\mu)\partial^\mu g^{a\nu} - \frac{1}{2}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)\partial^\mu W^{a\nu}\\ &- g_2\epsilon^{abc}W^b_\mu W^c_\nu\partial^\mu W^{a\nu} - \frac{g_2^2}{4}\epsilon^{abe}\epsilon^{cde}W^a_\mu W^b_\nu W^{c\mu}W^{d\nu}\\ &- g_3 f^{abc}g^b_\mu g^c_\nu\partial^\mu g^{a\nu} - \frac{g_3^2}{4}f^{cde}f^{abe}g^a_\mu g^b_\nu g^{c\mu}g^{d\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma},\end{aligned} \tag{6.37}$$

$$\begin{aligned}= &\ (\text{gluons}) - \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial^\mu A^\nu - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)\partial_\mu W^{+\nu} - \frac{1}{2}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\partial^\mu Z^\nu\\ &+ \mathrm{i}g_2\mathrm{c_w}\left[(W^-_\mu Z_\nu - W^-_\nu Z_\mu)\partial^\mu W^{+\nu} - (W^+_\mu Z_\nu - W^+_\nu Z_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)\partial^\mu Z^\nu\right]\\ &+ \mathrm{i}|e|\left[(W^-_\mu A_\nu - W^-_\nu A_\mu)\partial^\mu W^{+\nu} - (W^+_\mu A_\nu - W^+_\nu A_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)\partial^\mu A^\nu\right]\\ &+ \frac{g_2^2}{2}W^{+\mu}W^+_\mu W^{-\nu}W^-_\nu - \frac{g_2^2}{2}W^{+\mu}W^{+\nu}W^-_\mu W^-_\nu - g_2^2 W^{+\mu}W^-_\mu Z^\nu Z_\nu + g_2^2 W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\ &- e^2 W^{+\mu}W^-_\mu A^\nu A_\nu + e^2 W^{+\mu}W^-_\mu Z^\nu Z_\nu + e^2 W^{+\mu}W^{-\nu}A_\mu A_\nu - e^2 W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\ &- 2g_2^2\mathrm{c_w s_w}W^{+\mu}W^-_\mu A^\nu Z_\nu + g_2^2\mathrm{c_w s_w}W^{+\mu}W^{-\nu}A_\mu Z_\nu + g_2^2\mathrm{c_w s_w}W^{+\mu}W^{-\nu}A_\nu Z_\mu,\end{aligned} \tag{6.38}$$

where

$$W^1_\mu = \frac{W^+_\mu + W^-_\mu}{\sqrt{2}}, \quad W^2_\mu = \frac{\mathrm{i}(W^+_\mu - W^-_\mu)}{\sqrt{2}}; \qquad W^\pm_\mu = \frac{W^1_\mu \mp \mathrm{i}W^2_\mu}{\sqrt{2}};$$

$$\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix} = \begin{pmatrix}\mathrm{c_w} & \mathrm{s_w}\\ -\mathrm{s_w} & \mathrm{c_w}\end{pmatrix}\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix}; \qquad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} = \begin{pmatrix}\mathrm{c_w} & -\mathrm{s_w}\\ \mathrm{s_w} & \mathrm{c_w}\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix};$$

$$|e| = g_2\mathrm{s_w} = g_Y\mathrm{c_w} = g_Z\mathrm{s_w c_w}, \quad g_Z = g_2/\mathrm{c_w} = g_Y/\mathrm{s_w}; \qquad g_Y = |e|/\mathrm{c_w} = g_Z\mathrm{s_w} = g_2\mathrm{t_w}, \quad g_2 = |e|/\mathrm{s_w} = g_Z\mathrm{c_w}.$$

6.3.2. Fermion part

$$
\begin{aligned}
&\mathcal{L}_{\text{fermions}} \\
&= \mathrm{i}\bar{q}_{\mathrm{L}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_3 g^a_{\mu}\tau^a - \mathrm{i}g_2 W^a_{\mu}T^a - \tfrac{1}{6}\mathrm{i}g_Y B_{\mu}\right)q_{\mathrm{L}} \\
&\quad + \mathrm{i}\bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\left(\partial_{\mu} + \mathrm{i}g_3 g^a_{\mu}\tau^{a*} + \tfrac{2}{3}\mathrm{i}g_Y B_{\mu}\right)u^{\mathrm{c}}_{\mathrm{R}} + \mathrm{i}\bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\left(\partial_{\mu} + \mathrm{i}g_3 g^a_{\mu}\tau^{a*} - \tfrac{1}{3}\mathrm{i}g_Y B_{\mu}\right)d^{\mathrm{c}}_{\mathrm{R}} \\
&\quad + \mathrm{i}\bar{l}_{\mathrm{L}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W^a_{\mu}T^a + \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right)l_{\mathrm{L}} + \mathrm{i}\bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_Y B_{\mu}\right)e^{\mathrm{c}}_{\mathrm{R}} \\
&\quad + \mathrm{i}\bar{\tilde{h}}_{\mathrm{u}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W^a_{\mu}T^a - \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right)\tilde{h}_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}_{\mathrm{d}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W^a_{\mu}T^a + \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right)\tilde{h}_{\mathrm{d}} \\
&\quad + \mathrm{i}\bar{\tilde{g}}^a\bar{\sigma}^{\mu}\left(\partial_{\mu}\tilde{g}^a + g_3 f^{abc}g^b_{\mu}\tilde{g}^c\right) + \mathrm{i}\bar{\tilde{w}}^a\bar{\sigma}^{\mu}\left(\partial_{\mu}\tilde{w}^a + g_2\epsilon^{abc}W^b_{\mu}\tilde{w}^c\right) + \mathrm{i}\bar{\tilde{b}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{b} \\
&\quad - \frac{1}{2}\left(M_3\tilde{g}^a\tilde{g}^a + M_2\tilde{w}^a\tilde{w}^a + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) + \mathcal{L}_{\text{super}}\big|_{\text{no }F\text{-terms}}
\end{aligned}
\tag{6.39}
$$

$$
\begin{aligned}
&= \mathrm{i}\bar{\tilde{b}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{b} - \frac{1}{2}\left(M_1\tilde{b}\tilde{b} + M_1^*\bar{\tilde{b}}^a\bar{\tilde{b}}^a\right) + \mathrm{i}\bar{\tilde{g}}^a\bar{\sigma}^{\mu}\partial_{\mu}\tilde{g}^a - \frac{1}{2}\left(M_3\tilde{g}^a\tilde{g}^a + M_3^*\bar{\tilde{g}}^a\bar{\tilde{g}}^a\right) - \mathrm{i}g_3 f^{abc}(\bar{\tilde{g}}^a\bar{\sigma}^{\mu}\tilde{g}^b)g^c_{\mu} \\
&\quad + \mathrm{i}\bar{\tilde{w}}^+\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^+ + \mathrm{i}\bar{\tilde{w}}^-\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^- + \mathrm{i}\bar{\tilde{w}}^3\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^3 - \left(M_2\tilde{w}^+\tilde{w}^- + M_2^*\bar{\tilde{w}}^+\bar{\tilde{w}}^-\right) - \frac{1}{2}\left(M_2\tilde{w}^3\tilde{w}^3 + M_2^*\bar{\tilde{w}}^3\bar{\tilde{w}}^3\right) \\
&\quad + g_2(\bar{\tilde{w}}^3\bar{\sigma}^{\mu}\tilde{w}^- - \bar{\tilde{w}}^+\bar{\sigma}^{\mu}\tilde{w}^3)W^+_{\mu} - g_2(\bar{\tilde{w}}^3\bar{\sigma}^{\mu}\tilde{w}^+ - \bar{\tilde{w}}^-\bar{\sigma}^{\mu}\tilde{w}^3)W^-_{\mu} + g_2(\bar{\tilde{w}}^+\bar{\sigma}^{\mu}\tilde{w}^+ - \bar{\tilde{w}}^-\bar{\sigma}^{\mu}\tilde{w}^-)(\mathrm{c_w}Z_{\mu} + \mathrm{s_w}A_{\mu}) \\
&\quad + \bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} + g_3\tau^a g^a_{\mu})u_{\mathrm{L}} + \bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} - g_3\tau^{a*}g^a_{\mu})u^{\mathrm{c}}_{\mathrm{R}} + \mathrm{i}\bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}\partial_{\mu}\nu_{\mathrm{L}} \\
&\quad + \bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} + g_3\tau^a g^a_{\mu})d_{\mathrm{L}} + \bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} - g_3\tau^{a*}g^a_{\mu})d^{\mathrm{c}}_{\mathrm{R}} + \mathrm{i}\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}\partial_{\mu}e_{\mathrm{L}} + \mathrm{i}\bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_{\mu}e^{\mathrm{c}}_{\mathrm{R}} \\
&\quad + \mathrm{i}\bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^-_{\mathrm{d}} + \mathrm{i}\bar{\tilde{h}}^0_{\mathrm{d}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^0_{\mathrm{d}} + \mathrm{i}\bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^+_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}^0_{\mathrm{u}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^0_{\mathrm{u}} \\
&\quad + \frac{g_2}{\sqrt{2}}\left(\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}} + \bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}} + \bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{u}} + \bar{\tilde{h}}^0_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^-_{\mathrm{d}}\right)W^+_{\mu} + \frac{g_2}{\sqrt{2}}\left(\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}} + \bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}\nu_{\mathrm{L}} + \bar{\tilde{h}}^0_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^+_{\mathrm{u}} + \bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{d}}\right)W^-_{\mu} \\
&\quad + \frac{g_Z(3-4\mathrm{s_w^2})}{6}\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}}Z_{\mu} + \frac{g_Z\mathrm{s_w^2}}{3}\bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}u^{\mathrm{c}}_{\mathrm{R}}Z_{\mu} + \frac{g_Z(2\mathrm{s_w^2}-3)}{6}\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}}Z_{\mu} - \frac{g_Z\mathrm{s_w^2}}{3}\bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}d^{\mathrm{c}}_{\mathrm{R}}Z_{\mu} \\
&\quad + \frac{g_Z}{2}\bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}\nu_{\mathrm{L}}Z_{\mu} + \frac{g_Z(2\mathrm{s_w^2}-1)}{2}\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}}Z_{\mu} - g_Z\mathrm{s_w^2}\bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}e^{\mathrm{c}}_{\mathrm{R}}Z_{\mu} \\
&\quad + \frac{g_Z(1-2\mathrm{s_w^2})}{2}\bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^+_{\mathrm{u}}Z_{\mu} + \frac{g_Z}{2}\bar{\tilde{h}}^0_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{u}}Z_{\mu} + \frac{g_Z(2\mathrm{s_w^2}-1)}{2}\bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^-_{\mathrm{d}}Z_{\mu} + \frac{g_Z}{2}\bar{\tilde{h}}^0_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{d}}Z_{\mu} \\
&\quad + \frac{2|e|}{3}(\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}} - \bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}u^{\mathrm{c}}_{\mathrm{R}})A_{\mu} - \frac{|e|}{3}(\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}} - \bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}d^{\mathrm{c}}_{\mathrm{R}})A_{\mu} - |e|(\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}} - \bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}e^{\mathrm{c}}_{\mathrm{R}})A_{\mu} \\
&\quad + |e|\bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^+_{\mathrm{u}}A_{\mu} - |e|\bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^-_{\mathrm{d}}A_{\mu} + \mathcal{L}_{\text{super}}\big|_{\text{no }F\text{-terms}};
\end{aligned}
\tag{6.40}
$$

here,

$$
\begin{aligned}
\mathcal{L}_{\text{super}}\big|_{\text{no F-terms}} = {}& -\mu\tilde{h}^+_{\mathrm{u}}\tilde{h}^-_{\mathrm{d}} + \mu\tilde{h}^0_{\mathrm{u}}\tilde{h}^0_{\mathrm{d}} + y_{uij}h^+_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}j} - y_{uij}h^0_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}j} + y_{uij}\tilde{d}_{\mathrm{L}j}\tilde{h}^+_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i} - y_{uij}\tilde{u}_{\mathrm{L}j}\tilde{h}^0_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i} \\
& + y_{uji}\tilde{u}^*_{\mathrm{R}j}\tilde{h}^+_{\mathrm{u}}d_{\mathrm{L}i} - y_{uji}\tilde{u}^*_{\mathrm{R}j}\tilde{h}^0_{\mathrm{u}}u_{\mathrm{L}i} + y_{dij}h^-_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}j} - y_{dij}h^0_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}j} - y_{dij}\tilde{d}_{\mathrm{L}j}\tilde{h}^0_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i} \\
& + y_{dij}\tilde{u}_{\mathrm{L}j}\tilde{h}^-_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i} + y_{dji}\tilde{d}^*_{\mathrm{R}j}\tilde{h}^-_{\mathrm{d}}u_{\mathrm{L}i} - y_{dji}\tilde{d}^*_{\mathrm{R}j}\tilde{h}^0_{\mathrm{d}}d_{\mathrm{L}i} + y_{eij}h^-_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i}\nu_{\mathrm{L}j} - y_{eij}h^0_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}j} \\
& - y_{eij}\tilde{e}_{\mathrm{L}j}\tilde{h}^0_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i} + y_{eij}\tilde{\nu}_{\mathrm{L}j}\tilde{h}^-_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i} + y_{eji}\tilde{e}^*_{\mathrm{R}j}\tilde{h}^-_{\mathrm{d}}\nu_{\mathrm{L}i} - y_{eji}\tilde{e}^*_{\mathrm{R}j}\tilde{h}^0_{\mathrm{d}}e_{\mathrm{L}i} \\
& - \kappa_i\tilde{h}^+_{\mathrm{u}}e_{\mathrm{L}i} + \kappa_i\tilde{h}^0_{\mathrm{u}}\nu_{\mathrm{L}i} - \lambda_{ijk}\tilde{e}^*_{\mathrm{R}k}\nu_{\mathrm{L}i}e_{\mathrm{L}j} - \lambda_{jki}\tilde{e}_{\mathrm{L}k}e^{\mathrm{c}}_{\mathrm{R}i}\nu_{\mathrm{L}j} + \lambda_{jki}\tilde{\nu}_{\mathrm{L}k}e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}j} \\
& - \lambda'_{jik}\tilde{d}^*_{\mathrm{R}k}d_{\mathrm{L}i}\nu_{\mathrm{L}j} + \lambda'_{jik}\tilde{d}^*_{\mathrm{R}k}u_{\mathrm{L}i}e_{\mathrm{L}j} - \lambda'_{jki}\tilde{d}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}\nu_{\mathrm{L}j} + \lambda'_{jki}\tilde{u}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}j} + \lambda'_{kji}\tilde{e}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}j} \\
& - \lambda'_{kji}\tilde{\nu}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}j} - \epsilon^{xyz}\lambda''_{ijk}\tilde{d}^{x*}_{\mathrm{R}k}u^{\mathrm{c}y}_{\mathrm{R}i}d^{\mathrm{c}z}_{\mathrm{R}j} - \frac{1}{2}\epsilon^{xyz}\lambda''_{kij}\tilde{u}^{x*}_{\mathrm{R}k}d^{\mathrm{c}y}_{\mathrm{R}i}d^{\mathrm{c}z}_{\mathrm{R}j} + \text{H.c.}
\end{aligned}
\tag{6.41}
$$

6.3.3. Scalar-fermion-gaugino interaction

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SFG}} = {}& -g_2\tilde{u}_{\mathrm{L}}^{*}d_{\mathrm{L}}\tilde{w}^{+} - g_2\tilde{u}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{w}}^{+} - g_2\tilde{d}_{\mathrm{L}}^{*}u_{\mathrm{L}}\tilde{w}^{-} - g_2\tilde{d}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{w}}^{-} \\
& - \sqrt{2}g_3\tilde{u}_{\mathrm{L}}^{*}\tau^{a}u_{\mathrm{L}}\tilde{g}^{a} + \sqrt{2}g_3\tilde{u}_{\mathrm{R}}^{*}\tau^{a}\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{g}}^{a} - \sqrt{2}g_3\tilde{u}_{\mathrm{L}}\tau^{a*}\bar{u}_{\mathrm{L}}\bar{\tilde{g}}^{a} + \sqrt{2}g_3\tilde{u}_{\mathrm{R}}\tau^{a*}u_{\mathrm{R}}^{\mathrm{c}}\tilde{g}^{a} \\
& - \frac{g_2}{\sqrt{2}}\tilde{u}_{\mathrm{L}}^{*}u_{\mathrm{L}}\tilde{w}^{3} - \frac{g_2}{\sqrt{2}}\tilde{u}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{w}}^{3} - \frac{g_Y}{3\sqrt{2}}\tilde{u}_{\mathrm{L}}^{*}u_{\mathrm{L}}\tilde{b} + \frac{2\sqrt{2}g_Y}{3}\tilde{u}_{\mathrm{R}}^{*}\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}}\tilde{u}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{b}} + \frac{2\sqrt{2}g_Y}{3}\tilde{u}_{\mathrm{R}}u_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
& - \sqrt{2}g_3\tilde{d}_{\mathrm{L}}^{*}\tau^{a}d_{\mathrm{L}}\tilde{g}^{a} + \sqrt{2}g_3\tilde{d}_{\mathrm{R}}^{*}\tau^{a}\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{g}}^{a} - \sqrt{2}g_3\tilde{d}_{\mathrm{L}}\tau^{a*}\bar{d}_{\mathrm{L}}\bar{\tilde{g}}^{a} + \sqrt{2}g_3\tilde{d}_{\mathrm{R}}\tau^{a*}d_{\mathrm{R}}^{\mathrm{c}}\tilde{g}^{a} \\
& + \frac{g_2}{\sqrt{2}}\tilde{d}_{\mathrm{L}}^{*}d_{\mathrm{L}}\tilde{w}^{3} + \frac{g_2}{\sqrt{2}}\tilde{d}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{w}}^{3} - \frac{g_Y}{3\sqrt{2}}\tilde{d}_{\mathrm{L}}^{*}d_{\mathrm{L}}\tilde{b} - \frac{\sqrt{2}g_Y}{3}\tilde{d}_{\mathrm{R}}^{*}\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}}\tilde{d}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{b}} - \frac{\sqrt{2}g_Y}{3}\tilde{d}_{\mathrm{R}}d_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
& - g_2\tilde{e}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{w}}^{-} - g_2\tilde{\nu}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{w}}^{+} - g_2\tilde{e}_{\mathrm{L}}^{*}\nu_{\mathrm{L}}\tilde{w}^{-} - g_2\tilde{\nu}_{\mathrm{L}}^{*}e_{\mathrm{L}}\tilde{w}^{+} \\
& - \frac{g_2}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}^{*}\nu_{\mathrm{L}}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}^{*}\nu_{\mathrm{L}}\tilde{b} - \frac{g_2}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{b}} \\
& + \frac{g_2}{\sqrt{2}}\tilde{e}_{\mathrm{L}}^{*}e_{\mathrm{L}}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{e}_{\mathrm{L}}^{*}e_{\mathrm{L}}\tilde{b} - \sqrt{2}g_Y\tilde{e}_{\mathrm{R}}^{*}\bar{e}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} + \frac{g_2}{\sqrt{2}}\tilde{e}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{e}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{b}} - \sqrt{2}g_Y\tilde{e}_{\mathrm{R}}e_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
& - g_2h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^{0}\tilde{w}^{+} - g_2h_{\mathrm{d}}^{0}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\tilde{w}}^{+} - g_2h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^{0}\tilde{w}^{-} - g_2h_{\mathrm{u}}^{0}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\tilde{w}}^{-} \\
& - g_2h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^{-}\tilde{w}^{+} - g_2h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^{+}\tilde{w}^{-} - g_2h_{\mathrm{u}}^{+}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\tilde{w}}^{+} - g_2h_{\mathrm{d}}^{-}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\tilde{w}}^{-} \\
& - \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{+}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\tilde{w}}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{+}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\tilde{b}} - \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^{+}\tilde{w}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^{+}\tilde{b} + \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^{-}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^{-}\tilde{b} \\
& + \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{-}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{-}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\tilde{b}} + \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^{0}\tilde{w}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^{0}\tilde{b} - \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^{0}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^{0}\tilde{b} \\
& + \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{0}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\tilde{w}}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{0}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\tilde{b}} - \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{0}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{0}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\tilde{b}}
\end{aligned}
\tag{6.42}
$$

6.3.4. Scalar part

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} = {} & (\partial_\mu \tilde{u}_{\rm L}^* + \mathrm{i} g_3 \tilde{u}_{\rm L}^* \tau^a g_\mu^a)(\partial^\mu \tilde{u}_{\rm L} - \mathrm{i} g_3 g^{b\mu} \tau^b \tilde{u}_{\rm L}) + (\partial_\mu \tilde{u}_{\rm R} - \mathrm{i} g_3 \tilde{u}_{\rm R} \tau^{a*} g_\mu^a)(\partial^\mu \tilde{u}_{\rm R}^* + \mathrm{i} g_3 g_\mu^b \tau^{b*} \tilde{u}_{\rm R}^*) \\
& + (\partial_\mu \tilde{d}_{\rm L}^* + \mathrm{i} g_3 \tilde{d}_{\rm L}^* \tau^a g_\mu^a)(\partial^\mu \tilde{d}_{\rm L} - \mathrm{i} g_3 g^{b\mu} \tau^b \tilde{d}_{\rm L}) + (\partial_\mu \tilde{d}_{\rm R} - \mathrm{i} g_3 \tilde{d}_{\rm R}^y \tau^{a*} g_\mu^a)(\partial^\mu \tilde{d}_{\rm R}^* + \mathrm{i} g_3 g_\mu^b \tau^{b*} \tilde{d}_{\rm R}^*) \\
& + \sqrt{2} g_2 g_3 \tilde{u}_{\rm L}^* \tau^a \tilde{d}_{\rm L} W^{+\mu} g_\mu^a + \sqrt{2} g_2 g_3 \tilde{d}_{\rm L}^* \tau^a \tilde{u}_{\rm L} W^{-\mu} g_\mu^a + \frac{4}{3} g_3 |e| (\tilde{u}_{\rm L}^* \tau^a \tilde{u}_{\rm L} + \tilde{u}_{\rm R}^* \tau^a \tilde{u}_{\rm R}) g^{a\mu} A_\mu \\
& - \frac{2}{3} g_3 |e| (\tilde{d}_{\rm L}^* \tau^a \tilde{d}_{\rm L} + \tilde{d}_{\rm R}^* \tau^a \tilde{d}_{\rm R}) g^{a\mu} A_\mu + \frac{(3 - 4 \mathrm{s}_{\rm w}^2) g_Z}{3} g_3 \tilde{u}_{\rm L}^* \tau^a \tilde{u}_{\rm L} g^{a\mu} Z_\mu - \frac{4 \mathrm{s}_{\rm w}^2 g_Z}{3} g_3 \tilde{u}_{\rm R}^* \tau^a \tilde{u}_{\rm R} g^{a\mu} Z_\mu \\
& + \frac{(2 \mathrm{s}_{\rm w}^2 - 3) g_Z}{3} g_3 \tilde{d}_{\rm L}^* \tau^a \tilde{d}_{\rm L} g^{a\mu} Z_\mu + \frac{2 \mathrm{s}_{\rm w}^2 g_Z}{3} g_3 \tilde{d}_{\rm R}^* \tau^a \tilde{d}_{\rm R} g^{a\mu} Z_\mu \\
& + \frac{\mathrm{i} g_2}{\sqrt{2}} W_\mu^+ \left(\tilde{u}_{\rm L}^* \partial^\mu \tilde{d}_{\rm L} - \tilde{d}_{\rm L} \partial^\mu \tilde{u}_{\rm L}^* \right) - \frac{\mathrm{i} g_2}{\sqrt{2}} W_\mu^- \left(\tilde{u}_{\rm L} \partial^\mu \tilde{d}_{\rm L}^* - \tilde{d}_{\rm L}^* \partial^\mu \tilde{u}_{\rm L} \right) \\
& + \frac{2\mathrm{i}}{3} |e| A_\mu \left(\tilde{u}_{\rm L}^* \partial^\mu \tilde{u}_{\rm L} - \tilde{u}_{\rm L} \partial^\mu \tilde{u}_{\rm L}^* + \tilde{u}_{\rm R}^* \partial^\mu \tilde{u}_{\rm R} - \tilde{u}_{\rm R} \partial^\mu \tilde{u}_{\rm R}^* \right) - \frac{\mathrm{i}}{3} |e| A_\mu \left(\tilde{d}_{\rm L}^* \partial^\mu \tilde{d}_{\rm L} - \tilde{d}_{\rm L} \partial^\mu \tilde{d}_{\rm L}^* + \tilde{d}_{\rm R}^* \partial^\mu \tilde{d}_{\rm R} - \tilde{d}_{\rm R} \partial^\mu \tilde{d}_{\rm R}^* \right) \\
& + \frac{\mathrm{i}(4 \mathrm{s}_{\rm w}^2 - 3) g_Z}{6} Z_\mu \left(\tilde{u}_{\rm L} \partial^\mu \tilde{u}_{\rm L}^* - \tilde{u}_{\rm L}^* \partial^\mu \tilde{u}_{\rm L} \right) + \frac{\mathrm{i}(2 \mathrm{s}_{\rm w}^2 - 3) g_Z}{6} Z_\mu \left(\tilde{d}_{\rm L}^* \partial^\mu \tilde{d}_{\rm L} - \tilde{d}_{\rm L} \partial^\mu \tilde{d}_{\rm L}^* \right) \\
& - \frac{2\mathrm{i} \mathrm{s}_{\rm w}^2 g_Z}{3} Z_\mu \left(\tilde{u}_{\rm R}^* \partial^\mu \tilde{u}_{\rm R} - \tilde{u}_{\rm R} \partial^\mu \tilde{u}_{\rm R}^* \right) + \frac{\mathrm{i} \mathrm{s}_{\rm w}^2 g_Z}{3} Z_\mu \left(\tilde{d}_{\rm R}^* \partial^\mu \tilde{d}_{\rm R} - \tilde{d}_{\rm R} \partial^\mu \tilde{d}_{\rm R}^* \right) \\
& + \frac{g_2^2}{2} \left(|\tilde{u}_{\rm L}|^2 + |\tilde{d}_{\rm L}|^2 \right) W^{+\mu} W_\mu^- - \frac{\mathrm{s}_{\rm w}^2 g_2 g_Z}{3\sqrt{2}} \tilde{u}_{\rm L}^* \tilde{d}_{\rm L} W^{+\mu} Z_\mu - \frac{\mathrm{s}_{\rm w}^2 g_2 g_Z}{3\sqrt{2}} \tilde{d}_{\rm L}^* \tilde{u}_{\rm L} W^{-\mu} Z_\mu \\
& + \frac{(3 - 4 \mathrm{s}_{\rm w}^2)^2 g_Z^2}{36} |\tilde{u}_{\rm L}|^2 Z^\mu Z_\mu + \frac{4 \mathrm{s}_{\rm w}^4 g_Z^2}{9} |\tilde{u}_{\rm R}|^2 Z^\mu Z_\mu + \frac{(3 - 2 \mathrm{s}_{\rm w}^2)^2 g_Z^2}{36} |\tilde{d}_{\rm L}|^2 Z^\mu Z_\mu + \frac{\mathrm{s}_{\rm w}^4 g_Z^2}{9} |\tilde{d}_{\rm R}|^2 Z^\mu Z_\mu \\
& + \frac{4}{9} e^2 \left(|\tilde{u}_{\rm L}|^2 + |\tilde{u}_{\rm R}|^2 \right) A^\mu A_\mu + \frac{1}{9} e^2 \left(|\tilde{d}_{\rm L}|^2 + |\tilde{d}_{\rm R}|^2 \right) A^\mu A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{u}_{\rm L}^* \tilde{d}_{\rm L} W^{+\mu} A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{d}_{\rm L}^* \tilde{u}_{\rm L} W^{-\mu} A_\mu \\
& + \frac{2(3 - 4 \mathrm{s}_{\rm w}^2) g_Z |e|}{9} |\tilde{u}_{\rm L}|^2 A^\mu Z_\mu - \frac{8 \mathrm{s}_{\rm w}^2 g_Z |e|}{9} |\tilde{u}_{\rm R}|^2 A^\mu Z_\mu + \frac{(3 - 2 \mathrm{s}_{\rm w}^2) g_Z |e|}{9} |\tilde{d}_{\rm L}|^2 A^\mu Z_\mu - \frac{2 \mathrm{s}_{\rm w}^2 g_Z |e|}{9} |\tilde{d}_{\rm R}|^2 A^\mu Z_\mu \\[1ex]
& + \partial_\mu \tilde{e}_{\rm R} \partial^\mu \tilde{e}_{\rm R}^* + \partial_\mu \tilde{e}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} + \partial_\mu \tilde{\nu}_{\rm L}^* \partial^\mu \tilde{\nu}_{\rm L} + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ (\tilde{\nu}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} - \tilde{e}_{\rm L} \partial^\mu \tilde{\nu}_{\rm L}^*) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- (\tilde{e}_{\rm L}^* \partial^\mu \tilde{\nu}_{\rm L} - \tilde{\nu}_{\rm L} \partial^\mu \tilde{e}_{\rm L}^*) \\
& - \frac{\mathrm{i}(1 - 2 \mathrm{s}_{\rm w}^2) g_Z}{2} Z_\mu (\tilde{e}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} - \tilde{e}_{\rm L} \partial^\mu \tilde{e}_{\rm L}^*) + \frac{\mathrm{i} g_Z}{2} Z_\mu (\tilde{\nu}_{\rm L}^* \partial^\mu \tilde{\nu}_{\rm L} - \tilde{\nu}_{\rm L} \partial^\mu \tilde{\nu}_{\rm L}^*) + \mathrm{i} \mathrm{s}_{\rm w}^2 g_Z Z_\mu (\tilde{e}_{\rm R}^* \partial^\mu \tilde{e}_{\rm R} - \tilde{e}_{\rm R} \partial^\mu \tilde{e}_{\rm R}^*) \\
& + \mathrm{i} |e| A_\mu (\tilde{e}_{\rm L} \partial^\mu \tilde{e}_{\rm L}^* - \tilde{e}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} + \tilde{e}_{\rm R} \partial^\mu \tilde{e}_{\rm R}^* - \tilde{e}_{\rm R}^* \partial^\mu \tilde{e}_{\rm R}) \\
& + \frac{g_2^2}{2} (|\tilde{\nu}_{\rm L}|^2 + |\tilde{e}_{\rm L}|^2) W^{+\mu} W_\mu^- + \frac{g_2 g_Z \mathrm{s}_{\rm w}^2}{\sqrt{2}} \left(\tilde{e}_{\rm L}^* \tilde{\nu}_{\rm L} W_\mu^- Z^\mu + \tilde{\nu}_{\rm L}^* \tilde{e}_{\rm L} W_\mu^+ Z^\mu \right) - \frac{g_2 |e|}{\sqrt{2}} \left(\tilde{\nu}_{\rm L}^* \tilde{e}_{\rm L} W_\mu^+ A^\mu + \tilde{e}_{\rm L}^* \tilde{\nu}_{\rm L} W_\mu^- A^\mu \right) \\
& + \frac{(1 - 2 \mathrm{s}_{\rm w}^2)^2 g_Z^2}{4} |\tilde{e}_{\rm L}|^2 Z^\mu Z_\mu + \frac{g_Z^2}{4} |\tilde{\nu}_{\rm L}|^2 Z^\mu Z_\mu + g_Z^2 \mathrm{s}_{\rm w}^4 |\tilde{e}_{\rm R}|^2 Z^\mu Z_\mu \\
& + e^2 (|\tilde{e}_{\rm L}|^2 + |\tilde{e}_{\rm R}|^2) A^\mu A_\mu + (1 - 2 \mathrm{s}_{\rm w}^2) |e| g_Z |\tilde{e}_{\rm L}|^2 A^\mu Z_\mu - 2 \mathrm{s}_{\rm w}^2 g_Z |e| |\tilde{e}_{\rm R}|^2 A^\mu Z_\mu \\[1ex]
& + \partial_\mu h_{\rm d}^{-*} \partial^\mu h_{\rm d}^- + \partial_\mu h_{\rm d}^{0*} \partial^\mu h_{\rm d}^0 + \partial_\mu h_{\rm u}^{+*} \partial^\mu h_{\rm u}^+ + \partial_\mu h_{\rm u}^{0*} \partial^\mu h_{\rm u}^0 + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_{\rm u}^{+*} \partial^\mu h_{\rm u}^0 - h_{\rm u}^0 \partial^\mu h_{\rm u}^{+*} \right) \\
& + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- \left(h_{\rm u}^{0*} \partial^\mu h_{\rm u}^+ - h_{\rm u}^+ \partial^\mu h_{\rm u}^{0*} \right) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_{\rm d}^{0*} \partial^\mu h_{\rm d}^- - h_{\rm d}^- \partial^\mu h_{\rm d}^{0*} \right) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- \left(h_{\rm d}^{-*} \partial^\mu h_{\rm d}^0 - h_{\rm d}^0 \partial^\mu h_{\rm d}^{-*} \right) \\
& + \frac{\mathrm{i}(1 - 2 \mathrm{s}_{\rm w}^2) g_Z}{2} Z_\mu \left(h_{\rm u}^{+*} \partial^\mu h_{\rm u}^+ - h_{\rm u}^+ \partial^\mu h_{\rm u}^{+*} \right) + \frac{\mathrm{i}(2 \mathrm{s}_{\rm w}^2 - 1) g_Z}{2} Z_\mu \left(h_{\rm d}^{-*} \partial^\mu h_{\rm d}^- - h_{\rm d}^- \partial^\mu h_{\rm d}^{-*} \right) \\
& - \frac{\mathrm{i} g_Z}{2} Z_\mu \left(h_{\rm u}^{0*} \partial^\mu h_{\rm u}^0 - h_{\rm u}^0 \partial^\mu h_{\rm u}^{0*} \right) + \frac{\mathrm{i} g_Z}{2} Z_\mu \left(h_{\rm d}^{0*} \partial^\mu h_{\rm d}^0 - h_{\rm d}^0 \partial^\mu h_{\rm d}^{0*} \right) \\
& - \mathrm{i} |e| A_\mu \left(h_{\rm d}^{-*} \partial^\mu h_{\rm d}^- - h_{\rm d}^- \partial^\mu h_{\rm d}^{-*} \right) + \mathrm{i} |e| A_\mu \left(h_{\rm u}^{+*} \partial^\mu h_{\rm u}^+ - h_{\rm u}^+ \partial^\mu h_{\rm u}^{+*} \right) \\
& + \frac{g_2^2}{2} \left(|h_{\rm u}^+|^2 + |h_{\rm u}^0|^2 + |h_{\rm d}^0|^2 + |h_{\rm d}^-|^2 \right) W^{+\mu} W_\mu^- + \frac{\mathrm{s}_{\rm w}^2 g_Z g_2}{\sqrt{2}} \left(h_{\rm d}^{0*} h_{\rm d}^- - h_{\rm u}^{+*} h_{\rm u}^0 \right) W_\mu^+ Z^\mu \\
& + \frac{\mathrm{s}_{\rm w}^2 g_Z g_2}{\sqrt{2}} \left(h_{\rm d}^{-*} h_{\rm d}^0 - h_{\rm u}^{0*} h_{\rm u}^+ \right) W_\mu^- Z^\mu + \frac{g_2 |e|}{\sqrt{2}} \left(h_{\rm u}^{+*} h_{\rm u}^0 - h_{\rm d}^{0*} h_{\rm d}^- \right) W_\mu^+ A^\mu + \frac{g_2 |e|}{\sqrt{2}} \left(h_{\rm u}^{0*} h_{\rm u}^+ - h_{\rm d}^{-*} h_{\rm d}^0 \right) W_\mu^- A^\mu \\
& + \frac{(1 - 2 \mathrm{s}_{\rm w}^2)^2 g_Z^2}{4} \left(|h_{\rm u}^+|^2 + |h_{\rm d}^-|^2 \right) Z^\mu Z_\mu + \frac{g_Z^2}{4} \left(|h_{\rm u}^0|^2 + |h_{\rm d}^0|^2 \right) Z^\mu Z_\mu + e^2 \left(|h_{\rm u}^+|^2 + |h_{\rm d}^-|^2 \right) A^\mu A_\mu \\
& + (1 - 2 \mathrm{s}_{\rm w}^2) |e| g_Z \left(|h_{\rm u}^+|^2 + |h_{\rm d}^-|^2 \right) A^\mu Z_\mu \\
& - (V_{\rm SUSY} + V_{\cancel{\rm SUSY}}) \,,
\end{aligned}
$$

(6.43)

where the scalar potential is given by

$$
\begin{aligned}
V_{\mathrm{SUSY}} = {} & |h_{\mathrm{u}}|^2\left(|\mu|^2 + \sum_i |\kappa_i|^2\right) + |\mu|^2|h_{\mathrm{d}}|^2 + \left(\kappa_i^*\mu\tilde{l}^*_{\mathrm{L}i}h_{\mathrm{d}} + \mathrm{H.c.}\right) + \kappa_i^*\kappa_j\tilde{l}^*_{\mathrm{L}i}\tilde{l}_{\mathrm{L}j} \\
& + \Big[- y_{uij}\mu^* h_{\mathrm{d}}^*\tilde{u}^*_{\mathrm{R}i}\tilde{q}_{\mathrm{L}j} - y_{uij}\kappa_k^*\tilde{u}^*_{\mathrm{R}i}\tilde{q}_{\mathrm{L}j}\tilde{l}^*_{\mathrm{L}k} - \left(y_{dij}\mu^* + \lambda'_{kji}\kappa_k^*\right)h_{\mathrm{u}}^*\tilde{d}^*_{\mathrm{R}i}\tilde{q}_{\mathrm{L}j} \\
& \qquad + y_{eij}\kappa_j^*\tilde{e}^*_{\mathrm{R}i}h_{\mathrm{u}}^*h_{\mathrm{d}} + \left(\lambda_{jki}\kappa_k^* - y_{eij}\mu^*\right)h_{\mathrm{u}}^*\tilde{e}^*_{\mathrm{R}i}\tilde{l}_{\mathrm{L}j} + \mathrm{H.c.}\Big] \\
& + \frac{1}{8}\left(g_2^2 + g_Y^2\right)|h_{\mathrm{d}}|^4 + \frac{1}{8}\left(g_2^2 + g_Y^2\right)|h_{\mathrm{u}}|^4 + \left(-\frac{g_2^2}{4}|h_{\mathrm{d}}|^2|h_{\mathrm{u}}|^2 - \frac{g_Y^2}{4}|h_{\mathrm{d}}|^2|h_{\mathrm{u}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{d}}^*h_{\mathrm{u}}|^2\right) \\
& + \left(-\frac{g_2^2}{4}|h_{\mathrm{u}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_Y^2}{12}|h_{\mathrm{u}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{u}}^b\tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_u^\dagger y_u)_{ji}h_{\mathrm{u}}^a h_{\mathrm{u}}^{b*}\tilde{q}^c_{\mathrm{L}i}\tilde{q}^{d*}_{\mathrm{L}j}\right) \\
& + \left(-\frac{g_2^2}{4}|h_{\mathrm{d}}|^2|\tilde{q}_{\mathrm{L}}|^2 - \frac{g_Y^2}{12}|h_{\mathrm{d}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{d}}^*\tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_d^\dagger y_d)_{ji}h_{\mathrm{d}}^a h_{\mathrm{d}}^{b*}\tilde{q}^c_{\mathrm{L}i}\tilde{q}^{d*}_{\mathrm{L}j}\right) \\
& + \left(-\frac{g_2^2}{4}|h_{\mathrm{u}}|^2|\tilde{l}_{\mathrm{L}}|^2 - \frac{g_Y^2}{4}|h_{\mathrm{u}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{u}}^*\tilde{l}_{\mathrm{L}i}|^2\right) \\
& + \left(-\frac{g_2^2}{4}|h_{\mathrm{d}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \frac{g_Y^2}{4}|h_{\mathrm{d}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{d}}^*\tilde{l}_{\mathrm{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_e^\dagger y_e)_{ji}h_{\mathrm{d}}^a h_{\mathrm{d}}^{b*}\tilde{l}^c_{\mathrm{L}i}\tilde{l}^{d*}_{\mathrm{L}j}\right) \\
& + |h_{\mathrm{u}}|^2\left(-\frac{g_Y^2}{3}|\tilde{u}_{\mathrm{R}}|^2 + (y_u y_u^\dagger)_{ij}\tilde{u}^*_{\mathrm{R}i}\tilde{u}_{\mathrm{R}j}\right) + \frac{g_Y^2}{3}|h_{\mathrm{d}}|^2|\tilde{u}_{\mathrm{R}}|^2 \\
& + \frac{g_Y^2}{6}|h_{\mathrm{u}}|^2|\tilde{d}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2\left(-\frac{g_Y^2}{6}|\tilde{d}_{\mathrm{R}}|^2 + (y_d y_d^\dagger)_{ij}\tilde{d}^*_{\mathrm{R}i}\tilde{d}_{\mathrm{R}j}\right) - \left[(y_u y_d^\dagger)_{ij}\tilde{u}^*_{\mathrm{R}i}\tilde{d}_{\mathrm{R}j}(h_{\mathrm{d}}^*h_{\mathrm{u}}) + \mathrm{H.c.}\right] \\
& + \frac{g_Y^2}{2}|h_{\mathrm{u}}|^2|\tilde{e}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2\left(-\frac{g_Y^2}{2}|\tilde{e}_{\mathrm{R}}|^2 + (y_e y_e^\dagger)_{ij}\tilde{e}^*_{\mathrm{R}i}\tilde{e}_{\mathrm{R}j}\right) \\
& + \Big[-\frac{1}{2}\epsilon^{ab}\epsilon^{xyz}y_{ulk}\lambda''^*_{lij}h_{\mathrm{u}}^a\tilde{d}^x_{\mathrm{R}i}\tilde{d}^y_{\mathrm{R}j}\tilde{q}^{bz}_{\mathrm{L}k} + \epsilon^{ab}\epsilon^{xyz}y_{dlk}\lambda''^*_{ijl}h_{\mathrm{d}}^a\tilde{u}^x_{\mathrm{R}i}\tilde{d}^y_{\mathrm{R}j}\tilde{q}^{bz}_{\mathrm{L}k} - y_{uil}\lambda'^*_{klj}h_{\mathrm{u}}^a\tilde{u}^*_{\mathrm{R}i}\tilde{d}_{\mathrm{R}j}\tilde{l}^{a*}_{\mathrm{L}k} \\
& \qquad + y_{dil}\lambda'^*_{klj}h_{\mathrm{d}}^a\tilde{d}^*_{\mathrm{R}i}\tilde{d}_{\mathrm{R}j}\tilde{l}^{a*}_{\mathrm{L}k} - \epsilon^{ab}\epsilon^{cd}y_{dli}\lambda'^*_{kjl}h_{\mathrm{d}}^a\tilde{q}^b_{\mathrm{L}i}\tilde{q}^{c*}_{\mathrm{L}j}\tilde{l}^{d*}_{\mathrm{L}k} + y_{eil}\lambda^*_{klj}h_{\mathrm{d}}^a\tilde{e}^*_{\mathrm{R}i}\tilde{e}_{\mathrm{R}j}\tilde{l}^{a*}_{\mathrm{L}k} \qquad (6.44) \\
& \qquad - y_{ejl}\lambda'^*_{lki}h_{\mathrm{d}}^a\tilde{e}^*_{\mathrm{R}j}\tilde{d}_{\mathrm{R}i}\tilde{q}^{a*}_{\mathrm{L}k} + \frac{1}{2}\epsilon^{ab}\epsilon^{cd}y_{eli}\lambda^*_{jkl}h_{\mathrm{d}}^a\tilde{l}^b_{\mathrm{L}i}\tilde{l}^{c*}_{\mathrm{L}j}\tilde{l}^{d*}_{\mathrm{L}k} + \mathrm{H.c.}\Big] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8}\right)|\tilde{q}_{\mathrm{L}}|^4 + \frac{g_2^2}{4}\tilde{q}^{ax}_{\mathrm{L}i}\tilde{q}^{by}_{\mathrm{L}j}\tilde{q}^{bx*}_{\mathrm{L}i}\tilde{q}^{ay*}_{\mathrm{L}j} + \frac{g_3^2}{4}\tilde{q}^{ax}_{\mathrm{L}i}\tilde{q}^{by}_{\mathrm{L}j}\tilde{q}^{ay*}_{\mathrm{L}i}\tilde{q}^{bx*}_{\mathrm{L}j}\right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{2g_Y^2}{9}\right)|\tilde{u}_{\mathrm{R}}|^4 + \frac{g_3^2}{4}\tilde{u}^{x*}_{\mathrm{R}i}\tilde{u}^{y*}_{\mathrm{R}j}\tilde{u}^y_{\mathrm{R}i}\tilde{u}^x_{\mathrm{R}j}\right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18}\right)|\tilde{d}_{\mathrm{R}}|^4 + \frac{g_3^2}{4}\tilde{d}^{x*}_{\mathrm{R}i}\tilde{d}^{y*}_{\mathrm{R}j}\tilde{d}^y_{\mathrm{R}i}\tilde{d}^x_{\mathrm{R}j} + \frac{1}{2}\lambda''_{mij}\lambda''^*_{mkl}\tilde{d}^{x*}_{\mathrm{R}i}\tilde{d}^{y*}_{\mathrm{R}j}\tilde{d}^x_{\mathrm{R}k}\tilde{d}^y_{\mathrm{R}l}\right] \\
& + \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9}\right)|\tilde{u}_{\mathrm{R}}|^2|\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2}\tilde{u}^{x*}_{\mathrm{R}i}\tilde{q}^{ax}_{\mathrm{L}j}\tilde{u}^y_{\mathrm{R}i}\tilde{q}^{ay*}_{\mathrm{L}j} + y_{uik}y^*_{ujl}\tilde{u}^{x*}_{\mathrm{R}i}\tilde{u}^y_{\mathrm{R}j}\tilde{q}^{ax}_{\mathrm{L}k}\tilde{q}^{ay*}_{\mathrm{L}l}\right] \\
& + \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18}\right)|\tilde{d}_{\mathrm{R}}|^2|\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2}\tilde{d}^{x*}_{\mathrm{R}i}\tilde{q}^{ax}_{\mathrm{L}j}\tilde{d}^y_{\mathrm{R}i}\tilde{q}^{ay*}_{\mathrm{L}j} + \left(y_{dik}y^*_{djl} + \lambda'_{mki}\lambda'^*_{mlj}\right)\tilde{d}^{x*}_{\mathrm{R}i}\tilde{q}^{ax}_{\mathrm{L}k}\tilde{d}^y_{\mathrm{R}j}\tilde{q}^{ay*}_{\mathrm{L}l}\right] \\
& + \left[-\left(\frac{g_3^2}{6} + \frac{2g_Y^2}{9}\right)|\tilde{d}_{\mathrm{R}}|^2|\tilde{u}_{\mathrm{R}}|^2 + \left(\frac{g_3^2}{2} - \lambda''_{ikm}\lambda''^*_{ljm}\right)\tilde{u}^{x*}_{\mathrm{R}i}\tilde{d}^x_{\mathrm{R}j}\tilde{d}^{y*}_{\mathrm{R}j}\tilde{u}^y_{\mathrm{R}i} + \lambda''_{ikm}\lambda''^*_{jlm}\tilde{u}^{x*}_{\mathrm{R}i}\tilde{u}^x_{\mathrm{R}j}\tilde{d}^{y*}_{\mathrm{R}k}\tilde{d}^y_{\mathrm{R}l}\right] \\
& + \left[-\left(\frac{g_2^2}{4} + \frac{g_Y^2}{12}\right)|\tilde{l}_{\mathrm{L}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}\tilde{q}^{ax}_{\mathrm{L}i}\tilde{q}^{bx*}_{\mathrm{L}i}\tilde{l}^b_{\mathrm{L}j}\tilde{l}^{a*}_{\mathrm{L}j} + \epsilon^{ac}\epsilon^{bd}\lambda'_{kim}\lambda'^*_{ljm}\tilde{q}^{ax}_{\mathrm{L}i}\tilde{q}^{bx*}_{\mathrm{L}j}\tilde{l}^c_{\mathrm{L}k}\tilde{l}^{d*}_{\mathrm{L}l}\right] + \frac{g_Y^2}{6}|\tilde{e}_{\mathrm{R}}|^2|\tilde{q}_{\mathrm{L}}|^2 \\
& + \left(-\frac{g_Y^2}{6}|\tilde{d}_{\mathrm{R}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \lambda'_{lmi}\lambda'^*_{kmj}\tilde{d}^*_{\mathrm{R}i}\tilde{d}_{\mathrm{R}j}\tilde{l}^*_{\mathrm{L}k}\tilde{l}_{\mathrm{L}l}\right) + \frac{g_Y^2}{3}|\tilde{u}_{\mathrm{R}}|^2|\tilde{l}_{\mathrm{L}}|^2 - \frac{2g_Y^2}{3}|\tilde{u}_{\mathrm{R}}|^2|\tilde{e}_{\mathrm{R}}|^2 + \frac{g_Y^2}{3}|\tilde{d}_{\mathrm{R}}|^2|\tilde{e}_{\mathrm{R}}|^2 \\
& + \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8}\right)|\tilde{l}_{\mathrm{L}}|^4 + \frac{g_2^2}{4}\tilde{l}^a_{\mathrm{L}i}\tilde{l}^b_{\mathrm{L}j}\tilde{l}^{b*}_{\mathrm{L}i}\tilde{l}^{a*}_{\mathrm{L}j} + \frac{1}{4}\epsilon^{ab}\epsilon^{cd}\lambda_{ijm}\lambda^*_{klm}\tilde{l}^a_{\mathrm{L}i}\tilde{l}^b_{\mathrm{L}j}\tilde{l}^{c*}_{\mathrm{L}k}\tilde{l}^{d*}_{\mathrm{L}l}\right] \\
& + \frac{g_Y^2}{2}|\tilde{e}_{\mathrm{R}}|^4 + \left[-\frac{g_Y^2}{2}|\tilde{e}_{\mathrm{R}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \left(y_{eik}y^*_{ejl} + \lambda_{kmi}\lambda^*_{lmj}\right)\tilde{e}^*_{\mathrm{R}i}\tilde{e}_{\mathrm{R}j}\tilde{l}_{\mathrm{L}k}\tilde{l}^*_{\mathrm{L}l}\right] \\
& + \left[\left(y_{dik}y^*_{ejl} - \lambda'_{mki}\lambda^*_{lmj}\right)\tilde{d}^{x*}_{\mathrm{R}i}\tilde{q}^{ax}_{\mathrm{L}k}\tilde{e}_{\mathrm{R}j}\tilde{l}^{a*}_{\mathrm{L}l} - \epsilon^{ab}\epsilon^{xyz}\lambda'_{lkm}\lambda''^*_{ijm}\tilde{l}^b_{\mathrm{L}l}\tilde{q}^{az}_{\mathrm{L}k}\tilde{d}^y_{\mathrm{R}j}\tilde{u}^x_{\mathrm{R}i} + \mathrm{H.c.}\right].
\end{aligned}
$$

6.4. Higgs mechanism and fermion composition

The scalar potential includes

$$V_{\text{SUSY}} \supset |h_{\text{u}}|^2\Bigl(|\mu|^2+\sum|\kappa_i|^2\Bigr)+|\mu|^2|h_{\text{d}}|^2+\frac{g_Z^2}{8}\left(|h_{\text{u}}|^2-|h_{\text{d}}|^2\right)^2+\frac{g_2^2}{2}|h_{\text{d}}^*h_{\text{u}}|^2 \\ +\Bigl(\kappa_i^*\mu\tilde{l}_{\text{L}i}^*h_{\text{d}}+\text{H.c.}\Bigr)+\kappa_i^*\kappa_j\tilde{l}_{\text{L}i}^*\tilde{l}_{\text{L}j} \tag{6.45}$$

$$V_{\cancel{\text{SUSY}}} \supset m_{H_{\text{u}}}^2|h_{\text{u}}|^2+m_{H_{\text{d}}}^2|h_{\text{d}}|^2+\epsilon^{ab}\left(bh_{\text{u}}^ah_{\text{d}}^b+b^*h_{\text{u}}^{a*}h_{\text{d}}^{b*}-b_i\tilde{l}_{\text{L}i}^ah_{\text{u}}^b-b_i^*\tilde{l}_{\text{L}i}^{a*}h_{\text{u}}^{b*}\right)+\tilde{l}_{\text{L}i}^*M_{Li}^2h_{\text{d}}+\tilde{l}_{\text{L}i}M_{Li}^{2*}h_{\text{d}}^*; \tag{6.46}$$

the Higgs mass term is given by

$$V \supset \begin{pmatrix}h_{\text{u}} & h_{\text{d}}^* & \tilde{l}_{\text{L}i}^*\end{pmatrix}\begin{pmatrix}|\mu|^2+m_{H_{\text{u}}}^2+\sum|\kappa_i|^2 & b & -b_j\\ b^* & |\mu|^2+m_{H_{\text{d}}}^2 & \kappa_j\mu^*+M_{Lj}^{2*}\\ -b_i^* & \kappa_i^*\mu+M_{Li}^2 & (m_L^2)_{ij}+\kappa_i^*\kappa_j\end{pmatrix}\begin{pmatrix}h_{\text{u}}^*\\ h_{\text{d}}\\ \tilde{l}_{\text{L}j}\end{pmatrix} \tag{6.47}$$

while corresponding fermion terms are

$$\mathcal{L} \supset \epsilon^{ab}\left(-\mu\tilde{h}_{\text{u}}^a\tilde{h}_{\text{d}}^b-\kappa_i\tilde{h}_{\text{u}}^al_{\text{L}i}^b\right). \tag{6.48}$$

If the R-parity is not conserved, we redefine (H_{d},L) superfields so that the mass matrix is block-diagonal, which corresponds to $\mathrm{U}(4)_{H_{\text{d}},L}\to\mathrm{U}(3)_L\times\mathrm{U}(1)_{H_{\text{d}}}$ (DOF counting: $16\to9+1$ to remove b_i'). Then lepton and $\tilde{h}_{\text{d}}$ are mixed.[*8] With R-parity conservation, we do not suffer from these mixings.

6.4.1. Higgs potential and induced mass in R-parity conserved case

We perform "SU(2)-notation fixing", i.e., use the freedom associated to T_1 and T_2 of SU(2), so that $\langle h_{\text{u}}^+\rangle=0$. Then $\langle h_{\text{d}}^-\rangle=0$ and effectively

$$V_{\text{pot}}=(|\mu|^2+m_{H_{\text{u}}}^2)|h_{\text{u}}^0|^2+(|\mu|^2+m_{H_{\text{d}}}^2)|h_{\text{d}}^0|^2+\frac{g_Z^2}{8}\left(|h_{\text{u}}^0|^2-|h_{\text{d}}^0|^2\right)^2-\left(bh_{\text{u}}^0h_{\text{d}}^0+\text{H.c.}\right). \tag{6.49}$$

We redefine H_{d} superfield so that $b>0$.[*9] Then $\arg\langle h_{\text{u}}^0\rangle=-\arg\langle h_{\text{d}}^0\rangle$ and, with T_3-rotation, $\langle h_{\text{u}}^0\rangle>0$ and $\langle h_{\text{d}}^0\rangle>0$:

$$\langle h_{\text{u}}^0\rangle=:v_{\text{u}}=:v\sin\beta,\qquad \langle h_{\text{d}}^0\rangle=:v_{\text{d}}=:v\cos\beta; \tag{6.50}$$

$$V_{\text{pot}}=(|\mu|^2+m_{H_{\text{u}}}^2)v^2\sin^2\beta+(|\mu|^2+m_{H_{\text{d}}}^2)v^2\cos^2\beta+\frac{g_Z^2}{8}v^4\cos^22\beta-v^2b\sin2\beta. \tag{6.51}$$

This potential can have two minima; one with $0<\beta\le\pi/4$ and the other with $\pi/4\le\beta<\pi/2$:

$$\tan\beta=\frac{B\mp\sqrt{B^2-4b^2}}{2b}\quad\left(\cos2\beta=\pm\frac{\sqrt{B^2-4b^2}}{B}\right),\qquad m_Z^2:=\frac{g_Z^2}{2}v^2=\left(\pm\frac{m_{H_{\text{d}}}^2-m_{H_{\text{u}}}^2}{\sqrt{B^2-4b^2}}-1\right)B, \tag{6.52}$$

where $B:=2|\mu|^2+m_{H_{\text{u}}}^2+m_{H_{\text{d}}}^2>2b>0$ and m_Z is the Z-boson tree-level mass. Also,

$$\sin2\beta=\frac{2b}{2|\mu|^2+m_{H_{\text{u}}}^2+m_{H_{\text{d}}}^2},\qquad m_Z^2=\frac{-(m_{H_{\text{d}}}^2-m_{H_{\text{u}}}^2)}{\cos2\beta}-\left(2|\mu|^2+m_{H_{\text{u}}}^2+m_{H_{\text{d}}}^2\right) \tag{6.53}$$

are satisfied in both solutions.

Higgs sector The Nambu-Goldstone–Higgs mixings and the mass terms for the charged Higgs bosons are given by

$$\mathcal{L} \supset \partial_\mu h_{\text{d}}^{-*}\partial^\mu h_{\text{d}}^-+\partial_\mu h_{\text{u}}^{+*}\partial^\mu h_{\text{u}}^++\left(-b-\frac{1}{2}g_2^2v_{\text{u}}v_{\text{d}}\right)\left(h_{\text{u}}^+h_{\text{d}}^-+h_{\text{u}}^{+*}h_{\text{d}}^{-*}\right) \\ +\left[\frac{g_Y^2(v_{\text{u}}^2-v_{\text{d}}^2)-g_2^2(v_{\text{u}}^2+v_{\text{d}}^2)}{4}-|\mu|^2-m_{H_{\text{d}}}^2\right]|h_{\text{d}}^-|^2+\left[\frac{g_Y^2(v_{\text{d}}^2-v_{\text{u}}^2)-g_2^2(v_{\text{u}}^2+v_{\text{d}}^2)}{4}-|\mu|^2-m_{H_{\text{u}}}^2\right]|h_{\text{u}}^+|^2 \\ +\frac{\mathrm{i}g_2}{\sqrt{2}}W_\mu^-\partial^\mu\left(v_{\text{u}}h_{\text{u}}^+-v_{\text{u}}h_{\text{u}}^{+*}-v_{\text{d}}h_{\text{d}}^{-*}+v_{\text{d}}h_{\text{d}}^-\right) \tag{6.54}$$

and those for the neutral Higgs bosons are

$$\mathcal{L} \supset \partial_\mu h_{\text{d}}^{0*}\partial^\mu h_{\text{d}}^0+\partial_\mu h_{\text{u}}^{0*}\partial^\mu h_{\text{u}}^0-\frac{g_Z^2v_{\text{d}}^2}{8}(h_{\text{d}}^0h_{\text{d}}^0+h_{\text{d}}^{0*}h_{\text{d}}^{0*})-\frac{g_Z^2v_{\text{u}}^2}{8}(h_{\text{u}}^0h_{\text{u}}^0+h_{\text{u}}^{0*}h_{\text{u}}^{0*}) \\ +\left(b+\frac{g_Z^2v_{\text{u}}v_{\text{d}}}{4}\right)(h_{\text{u}}^0h_{\text{d}}^0+h_{\text{u}}^{0*}h_{\text{d}}^{0*})+\frac{g_Z^2v_{\text{u}}v_{\text{d}}}{4}(h_{\text{u}}^0h_{\text{d}}^{0*}+h_{\text{u}}^{0*}h_{\text{d}}^0) \\ +\left(\frac{g_Z^2(v_{\text{u}}^2-2v_{\text{d}}^2)}{4}-|\mu|^2-m_{H_{\text{d}}}^2\right)|h_{\text{d}}^0|^2+\left(\frac{g_Z^2(v_{\text{d}}^2-2v_{\text{u}}^2)}{4}-|\mu|^2-m_{H_{\text{u}}}^2\right)|h_{\text{d}}^0|^2 \\ +\frac{\mathrm{i}g_Z}{2}Z_\mu\partial^\mu\left(v_{\text{d}}h_{\text{d}}^0-v_{\text{d}}h_{\text{d}}^{0*}-v_{\text{u}}h_{\text{u}}^0+v_{\text{u}}h_{\text{u}}^{0*}\right). \tag{6.55}$$

[*8]If we separated leptons and $\tilde{h}_{\text{d}}$ first, sleptons would acquire VEVs and lepton-gaugino mixings would be induced.
[*9]Note that T_3-rotation induces $h_{\text{u}}^0\to\mathrm{e}^{\mathrm{i}\theta/2}h_{\text{u}}^0$ and $h_{\text{d}}^0\to\mathrm{e}^{-\mathrm{i}\theta/2}h_{\text{d}}^0$; it cannot the remove phase of b.

Therefore, with $m_W := \mathrm{c_w} m_Z$ and

$$\begin{pmatrix} h_\mathrm{u}^+ \\ h_\mathrm{d}^{-*} \end{pmatrix} = \begin{pmatrix} \mathrm{s}_\beta & \mathrm{c}_\beta \\ -\mathrm{c}_\beta & \mathrm{s}_\beta \end{pmatrix} \begin{pmatrix} -\mathrm{i}G^+ \\ H^+ \end{pmatrix}, \qquad \begin{pmatrix} h_\mathrm{u}^0 \\ h_\mathrm{d}^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_\mathrm{u} \\ \phi_\mathrm{d} \end{pmatrix} + \frac{\mathrm{i}}{\sqrt{2}} \begin{pmatrix} \mathrm{s}_\beta & \mathrm{c}_\beta \\ -\mathrm{c}_\beta & \mathrm{s}_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \tag{6.56}$$

we have

$$\begin{aligned} \mathcal{L} \supset{}& \partial_\mu G^{+*} \partial^\mu G^+ + \partial_\mu H^{+*} \partial^\mu H^+ + m_W (W_\mu^- \partial^\mu G^+ + W_\mu^+ \partial^\mu G^{+*}) + \left(\frac{m_{H_\mathrm{d}}^2 - m_{H_\mathrm{u}}^2}{\cos 2\beta} + m_Z^2 \mathrm{s_w}^2 \right) |H^+|^2 \\ &+ \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \frac{1}{2}(\partial_\mu A^0)^2 + \frac{1}{2}(\partial_\mu G^0)^2 + m_Z Z_\mu \partial^\mu G^0 - \frac{B}{2} A_0^2 \\ &- \frac{1}{4} \left(B + m_Z^2 + (B - m_Z^2) \cos 2\beta \right) \phi_\mathrm{u}^2 . - \frac{1}{4} \left(B + m_Z^2 - (B - m_Z^2) \cos 2\beta \right) \phi_\mathrm{d}^2 + \frac{1}{2} (B + m_Z^2)(\sin 2\beta) \phi_\mathrm{u} \phi_\mathrm{d}. \end{aligned} \tag{6.57}$$

In particular, the tree-level masses are

$$m_{A_0}^2 = B = 2|\mu|^2 + m_{H_\mathrm{u}}^2 + m_{H_\mathrm{d}}^2, \tag{6.58}$$

$$m_{H+}^2 = m_{A_0}^2 + m_W^2, \tag{6.59}$$

$$m_{h,H} = \frac{1}{2} \left(m_{A_0}^2 + m_Z^2 \mp \sqrt{\left(m_{A_0}^2 - m_Z^2 \right)^2 + 4 m_{A_0}^2 m_Z^2 \sin^2 2\beta} \right) \tag{6.60}$$

with

$$\begin{pmatrix} \phi_\mathrm{d} \\ \phi_\mathrm{u} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \frac{m_{A_0}^2 + m_Z^2}{m_{A_0}^2 - m_Z^2}. \tag{6.61}$$

The mixing α is stored in `ALPHA` block of SLHA, while `HMIX` stores μ, $\tan\beta$, $\sqrt{2}v$ (~ 246) and $2b/\sin 2\beta$ at the scale specified. The above discussion holds even with CP-violation, but quantum corrections mix the three Higgs bosons; such information should be stored in `(IM)VCHMIX`. ♣`TODO:`discuss when needed♣

Mass terms in the Lagrangian The other mass terms are given by

$$\begin{aligned} \mathcal{L} \supset{}& m_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} M_3 \tilde{g}^a \tilde{g}^a - \frac{1}{2} M_3^* \bar{\tilde{g}}^a \bar{\tilde{g}}^a \\ &+ \left(-\frac{1}{2} M_1 \tilde{b}\tilde{b} - \frac{1}{2} M_2 \tilde{w}^3 \tilde{w}^3 + \mu \tilde{h}_\mathrm{u}^0 \tilde{h}_\mathrm{d}^0 + \mathrm{c}_\beta m_Z \mathrm{s_w} \tilde{h}_\mathrm{d}^0 \tilde{b} - \mathrm{c_w} \mathrm{c}_\beta m_Z \tilde{h}_\mathrm{d}^0 \tilde{w}^3 - m_Z \mathrm{s_w} \mathrm{s}_\beta \tilde{h}_\mathrm{u}^0 \tilde{b} + \mathrm{c_w} m_Z \mathrm{s}_\beta \tilde{h}_\mathrm{u}^0 \tilde{w}^3 + \text{h.c.} \right) \\ &- M_2 \tilde{w}^+ \tilde{w}^- - \mu \tilde{h}_\mathrm{u}^+ \tilde{h}_\mathrm{d}^- - M_2^* \bar{\tilde{w}}^+ \bar{\tilde{w}}^- - \mu^* \bar{\tilde{h}}_\mathrm{u}^+ \bar{\tilde{h}}_\mathrm{d}^- - \sqrt{2} m_W \left(\mathrm{c}_\beta \tilde{h}_\mathrm{d}^- \tilde{w}^+ + \mathrm{s}_\beta \tilde{h}_\mathrm{u}^+ \tilde{w}^- + \mathrm{c}_\beta \bar{\tilde{h}}_\mathrm{d}^- \bar{\tilde{w}}^+ + \mathrm{s}_\beta \bar{\tilde{h}}_\mathrm{u}^+ \bar{\tilde{w}}^- \right) \\ &- v_\mathrm{u} y_{uij} u_{\mathrm{R}i}^\mathrm{c} u_{\mathrm{L}j} - v_\mathrm{d} y_{dij} d_{\mathrm{R}i}^\mathrm{c} d_{\mathrm{L}j} - v_\mathrm{d} y_{eij} e_{\mathrm{R}i}^\mathrm{c} e_{\mathrm{L}j} - v_\mathrm{u} y_{uij}^* \bar{u}_{\mathrm{R}i}^\mathrm{c} \bar{u}_{\mathrm{L}j} - v_\mathrm{d} y_{dij}^* \bar{d}_{\mathrm{R}i}^\mathrm{c} \bar{d}_{\mathrm{L}j} - v_\mathrm{d} y_{eij}^* \bar{e}_{\mathrm{R}i}^\mathrm{c} \bar{e}_{\mathrm{L}j} \\ &- \tilde{u}_\mathrm{L}^* \left(m_Q^2 + v_\mathrm{u}^2 y_u^\dagger y_u + \frac{3 - 4\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{u}_\mathrm{L} - \tilde{u}_\mathrm{R}^* \left(m_{U^\mathrm{c}}^2 + v_\mathrm{u}^2 y_u y_u^\dagger + \frac{4\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{u}_\mathrm{R} \\ &- v_\mathrm{u} a_{\mathrm{u}ij} \tilde{u}_{\mathrm{R}i}^* \tilde{u}_{\mathrm{L}j} + v_\mathrm{d} \mu^* y_{uij} \tilde{u}_{\mathrm{R}i}^* \tilde{u}_{\mathrm{L}j} - v_\mathrm{u} a_{\mathrm{u}ij}^* \tilde{u}_{\mathrm{R}i} \tilde{u}_{\mathrm{L}j}^* + v_\mathrm{d} \mu y_{uij}^* \tilde{u}_{\mathrm{R}i} \tilde{u}_{\mathrm{L}j}^* \\ &- \tilde{d}_\mathrm{L}^* \left(m_Q^2 + v_\mathrm{d}^2 y_d^\dagger y_d + \frac{-3 + 2\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{d}_\mathrm{L} - \tilde{d}_\mathrm{R}^* \left(m_{D^\mathrm{c}}^2 + v_\mathrm{d}^2 y_d y_d^\dagger + \frac{-2\mathrm{s_w^2}}{6} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{d}_\mathrm{R} \\ &- v_\mathrm{d} a_{\mathrm{d}ij} \tilde{d}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{L}j} + v_\mathrm{u} \mu^* y_{dij} \tilde{d}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{L}j} - v_\mathrm{d} a_{\mathrm{d}ij}^* \tilde{d}_{\mathrm{R}i} \tilde{d}_{\mathrm{L}j}^* + v_\mathrm{u} \mu y_{dij}^* \tilde{d}_{\mathrm{R}i} \tilde{d}_{\mathrm{L}j}^* \\ &- \tilde{\nu}_\mathrm{L}^* \left(m_L^2 + \frac{1}{2} \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{\nu}_\mathrm{L} \\ &- \tilde{e}_\mathrm{L}^* \left(m_L^2 + v_\mathrm{d}^2 y_e^\dagger y_e + \frac{-1 + 2\mathrm{s_w^2}}{2} \mathrm{c}_{2\beta} m_Z^2 \right) - \tilde{e}_\mathrm{R}^* \left(m_{E^\mathrm{c}}^2 + v_\mathrm{d}^2 y_e y_e^\dagger + (-\mathrm{s_w^2}) \mathrm{c}_{2\beta} m_Z^2 \right) \tilde{e}_\mathrm{R} \\ &- v_\mathrm{d} a_{eij} \tilde{e}_{\mathrm{R}i}^* \tilde{e}_{\mathrm{L}j} + v_\mathrm{u} \mu^* y_{eij} \tilde{e}_{\mathrm{R}i}^* \tilde{e}_{\mathrm{L}j} - v_\mathrm{d} a_{eij}^* \tilde{e}_{\mathrm{R}i} \tilde{e}_{\mathrm{L}j}^* + v_\mathrm{u} \mu y_{eij}^* \tilde{e}_{\mathrm{R}i} \tilde{e}_{\mathrm{L}j}^*, \end{aligned} \tag{6.62}$$

where, at the tree level, the gauge boson mass m_W and m_Z, the gluino mass M_3, and matter-fermion masses $v_\mathrm{u} y_u$, $v_\mathrm{d} y_d$, and $v_\mathrm{d} y_e$ are given with the "correct" sign (as far as $M_3 > 0$, etc.).

Neutralinos and charginos The mass matrices for neutralinos and charginos are given by

$$\begin{aligned} -\mathcal{L} \supset{}& \frac{1}{2} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_\mathrm{d}^0 \\ \tilde{h}_\mathrm{u}^0 \end{pmatrix}^\mathrm{T} \begin{pmatrix} M_1 & 0 & -\mathrm{c}_\beta \mathrm{s_w} m_Z & +\mathrm{s}_\beta \mathrm{s_w} m_Z \\ 0 & M_2 & +\mathrm{c}_\beta \mathrm{c_w} m_Z & -\mathrm{s}_\beta \mathrm{c_w} m_Z \\ -\mathrm{c}_\beta \mathrm{s_w} m_Z & +\mathrm{c}_\beta \mathrm{c_w} m_Z & 0 & -\mu \\ +\mathrm{s}_\beta \mathrm{s_w} m_Z & -\mathrm{s}_\beta \mathrm{c_w} m_Z & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_\mathrm{d}^0 \\ \tilde{h}_\mathrm{u}^0 \end{pmatrix} + \text{h.c.} \\ &+ \begin{pmatrix} \tilde{w}^- & \tilde{h}_\mathrm{d}^- \end{pmatrix} \begin{pmatrix} M_2 & \sqrt{2} \mathrm{s}_\beta m_W \\ \sqrt{2} \mathrm{c}_\beta m_W & \mu \end{pmatrix} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_\mathrm{u}^+ \end{pmatrix} + \begin{pmatrix} \bar{\tilde{w}}^- & \bar{\tilde{h}}_\mathrm{d}^- \end{pmatrix} \begin{pmatrix} M_2^* & \sqrt{2} \mathrm{s}_\beta m_W \\ \sqrt{2} \mathrm{c}_\beta m_W & \mu^* \end{pmatrix} \begin{pmatrix} \bar{\tilde{w}}^+ \\ \bar{\tilde{h}}_\mathrm{u}^+ \end{pmatrix}. \end{aligned} \tag{6.63}$$

Note that the mass matrices themselves are the same as those in SLHA convention, $\mathcal{M}_{\tilde{\psi}^0}$ and $\mathcal{M}_{\tilde{\psi}^+}$, while the fields are in different convention. Therefore, we continue our discussion based only on the mass matrices so that the discussion is free from the choice of field convention.

As $\mathcal{M}_{\tilde\psi^0}$ is a complex symmetric matrix, there is a unitary matrix $\tilde N$ such that $M_{\tilde\psi^0} = \tilde N^* \mathcal{M}_{\tilde\psi^0} \tilde N^\dagger$, where $M_{\tilde\psi^0}$ is a *positive* diagonal matrix whose elements are (non-negative) singular values of $\mathcal{M}_{\tilde\psi^0}$ and in increasing order (Autonne-Takagi factorization). In SLHA2 convention with CP-violation, this matrix $\tilde N$ is stored as the `(IM)NMIX` blocks and the (positive) masses are stored in the `MASS` block. Meanwhile, if M_1, M_2 and μ are real, $\mathcal{M}_{\tilde\psi^0}$ is a real symmetric matrix and there is a real orthogonal matrix $\hat N$ such that $\hat M_{\tilde\psi^0} = \hat N^* \mathcal{M}_{\tilde\psi^0} \hat N^\dagger = \hat N \mathcal{M}_{\tilde\psi^0} \hat N^{\mathrm{T}}$, where $\hat M_{\tilde\psi^0}$ is a *real* diagonal matrix whose elements are the eigenvalues of $\mathcal{M}_{\tilde\psi^0}$ and in absolute-value-increasing order (spectral theorem). This matrix $\hat N$ is the `NMIX` block of SLHA convention and $\hat M_{ii}$ is stored in the `MASS` block, hence `MASS` block may have negative values for neutralinos.

The chargino mass matrix $\mathcal{M}_{\tilde\psi^+}$ is decomposed as $M_{\tilde\psi^+} = U^* \mathcal{M}_{\tilde\psi^+} V^\dagger$, where U and V are unitary matrices and the elements of the diagonal matrix $M_{\tilde\psi^+}$ are singular values of $\mathcal{M}_{\tilde\psi^+}$ (thus non-negative) and sorted in increasing order (singular value decomposition). These U and V are stored in `(IM)UMIX` and `(IM)VMIX`, and the singular values are stored in `MASS` block. Because the SVD theorem is closed in $\mathbb{R}$, if M_2 and μ are real, U and V can be real, and the `IM`-blocks are omitted.

In summary,

$$M_{\tilde\psi^0} = \tilde N^* \mathcal{M}_{\tilde\psi^0} \tilde N^\dagger, \qquad \tilde N = \texttt{(IM)NMIX}, \qquad (\texttt{MASS}) = [M_{\tilde\psi^0}]_{ii} \ge 0 \quad \text{(singular values)}; \tag{6.64}$$

$$\hat M_{\tilde\psi^0} = \hat N \mathcal{M}_{\tilde\psi^0} \hat N^{\mathrm{T}}, \qquad \hat N = \texttt{NMIX}, \qquad (\texttt{MASS}) = [\hat M_{\tilde\psi^0}]_{ii} \in \mathbb{R} \quad \text{(eigenvalues)}; \tag{6.65}$$

$$M_{\tilde\psi^+} = U^* \mathcal{M}_{\tilde\psi^+} V^\dagger, \qquad U = \texttt{(IM)UMIX}, \quad V = \texttt{(IM)VMIX}, \qquad (\texttt{MASS}) = [M_{\tilde\psi^+}]_{ii} \ge 0 \quad \text{(singular values)}. \tag{6.66}$$

Note that the singular values are equal to absolute values of the eigenvalues, which guarantees consistency of the two decomposition.

We then define matrix N by[*10]

$$N = \begin{cases} \tilde N \\ \mathrm{diag}(\varphi_i) \cdot \hat N \end{cases} = \mathrm{diag}(\varphi_i) \cdot \Big((\texttt{NMIX}) + \mathrm{i}(\texttt{IMNMIX}) \Big); \qquad \varphi_i = \begin{cases} 1 & \text{if } (\texttt{MASS})_i \ge 0, \\ \mathrm{i} & \text{if } (\texttt{MASS})_i < 0. \end{cases} \tag{6.67}$$

It gives the proper mass diagonalization in both of the NMIX convention:

$$N^* \mathcal{M}_{\tilde\psi^0} N^\dagger = \begin{cases} \tilde N^* \mathcal{M}_{\tilde\psi^0} \tilde N^\dagger = M_{\tilde\psi^0}, \\ \mathrm{diag}(\varphi_i^*) \hat N^* \mathcal{M}_{\tilde\psi^0} \hat N^\dagger \,\mathrm{diag}(\varphi_i^*) = \mathrm{diag}(\varphi_i^*) \hat M_{\tilde\psi^0} \,\mathrm{diag}(\varphi_i^*) \end{cases} = M_{\tilde\psi^0} \quad \text{(neutralino masses} \ge 0). \tag{6.68}$$

Noting that the discussion up here is irrelevant of the convention, we have the neutralino/chargino mass eigenstates,

$$\tilde\chi^0_i = N_{ij} \begin{pmatrix} \tilde b \\ \tilde w^3 \\ \tilde h^0_{\mathrm{d}} \\ \tilde h^0_{\mathrm{u}} \end{pmatrix}_j, \quad \tilde\chi^+_i = V_{ij} \begin{pmatrix} \tilde w^+ \\ \tilde h^+_{\mathrm{u}} \end{pmatrix}_j, \quad \tilde\chi^-_i = U_{ij} \begin{pmatrix} \tilde w^- \\ \tilde h^-_{\mathrm{d}} \end{pmatrix}_j, \tag{6.69}$$

in our convention and the mass terms are now

$$-\mathcal{L} \supset \frac{1}{2} (\tilde\chi^0)^{\mathrm{T}} M_{\tilde\psi^0} \tilde\chi^0 + (\tilde\chi^-)^{\mathrm{T}} M_{\tilde\psi^+} \tilde\chi^+ + \text{h.c.} \tag{6.70}$$

Quarks, leptons, and super-CKM basis We here take the super-CKM basis. In the "original" Lagrangian,

$$-\mathcal{L} \supset u^{\mathrm{c}}_{\mathrm{R}} (v_{\mathrm{u}} y_u) u_{\mathrm{L}} + d^{\mathrm{c}}_{\mathrm{R}} (v_{\mathrm{d}} y_d) d_{\mathrm{L}} + e^{\mathrm{c}}_{\mathrm{R}} (v_{\mathrm{d}} y_e) e_{\mathrm{L}} + \text{h.c.} \tag{6.71}$$

$$= u^{\mathrm{c}}_{\mathrm{R}} (v_{\mathrm{u}} U_u y_u^{\mathrm{diag}} V_u^\dagger) u_{\mathrm{L}} + d^{\mathrm{c}}_{\mathrm{R}} (v_{\mathrm{d}} U_d y_d^{\mathrm{diag}} V_d^\dagger) d_{\mathrm{L}} + e^{\mathrm{c}}_{\mathrm{R}} (v_{\mathrm{d}} U_e y_e^{\mathrm{diag}} V_e^\dagger) e_{\mathrm{L}} + \text{h.c.}, \tag{6.72}$$

so the super-CKM basis is given by

$$\left[Q^1, Q^2, L, U^{\mathrm{c}}, D^{\mathrm{c}}, E^{\mathrm{c}}\right]_{\text{super-CKM}} = \left[V_u^\dagger Q^1, V_d^\dagger Q^2, V_e^\dagger L, U^{\mathrm{c}} U_u, D^{\mathrm{c}} U_d, E^{\mathrm{c}} U_e\right]_{\text{“original”}}. \tag{6.73}$$

Then the CKM mixings appear as, for example,

$$\left[\bar u_{\mathrm{L}} \bar\sigma^\mu d_{\mathrm{L}} W^+_\mu + \bar d_{\mathrm{L}} \bar\sigma^\mu u_{\mathrm{L}} W^-_\mu\right]_{\text{“original”}} = \left[\bar u_{\mathrm{L}} V_u^\dagger V_d \bar\sigma^\mu d_{\mathrm{L}} W^+_\mu + \bar d_{\mathrm{L}} V_d^\dagger V_u \bar\sigma^\mu u_{\mathrm{L}} W^-_\mu\right]_{\text{super-CKM}}; \tag{6.74}$$

i.e., defining $V_{\mathrm{CKM}} = V_u^\dagger V_d$ as in Sec. 4.5, the Lagrangian is amended as, e.g., $\bar u_{\mathrm{L}} d_{\mathrm{L}} \to \bar u_{\mathrm{L}} V_{\mathrm{CKM}} d_{\mathrm{L}}$, $\tilde d^*_{\mathrm{L}} \tilde u_{\mathrm{L}} \to \tilde d^*_{\mathrm{L}} V^\dagger_{\mathrm{CKM}} \tilde u_{\mathrm{L}}$.

The Θ_B- and Θ_W-terms are now removable. To this end, we rotate the matter superfields as

$$Q^1 \to \mathrm{e}^{\mathrm{i}\theta} Q^1, \quad Q^2 \to \mathrm{e}^{\mathrm{i}\theta'} Q^2, \quad U^{\mathrm{c}} \to \mathrm{e}^{-\mathrm{i}\theta} U^{\mathrm{c}}, \quad D^{\mathrm{c}} \to \mathrm{e}^{-\mathrm{i}\theta'} D^{\mathrm{c}}, \tag{6.75}$$

which induces $\Delta\Theta_G = 0$, $\Delta\Theta_W \propto \theta + \theta'$, and $\Delta\Theta_B \propto 5\theta + \theta'$, while the Lagrangian is unchanged except for the overall phase of V_{CKM}. So we can remove Θ_B and Θ_W, or in other words, pass the CP-violation in Θ-terms to V_{CKM}.[*11] If one (or more) quarks were massless, we could rotate the corresponding right-handed quark to remove Θ_G as well.

[*10] The sign of φ_i is arbitrary and (should be) unphysical.

[*11] There are other possible transformations $(L, E^{\mathrm{c}}) \to \mathrm{e}^{(\pm)\mathrm{i}\theta''}(L, E^{\mathrm{c}})$ and in total we may have $\Delta\Theta_W \propto \theta + \theta' + 2\theta''$ and $\Delta\Theta_B \propto 5\theta + \theta' + 6\theta''$. To remove both Θ-terms, we have to take $\theta \ne \theta'$ and V_{CKM} is anyway modified. The angle may chosen generation-dependently, and then $\Delta\Theta$'s should be read as $\theta \to \sum \theta_i/3$, etc.

Squark masses in super-CKM basis Finally, the squark masses are given by

$$
\begin{aligned}
-\mathcal{L} \supset{}& \tilde{u}_\mathrm{L}^* \left(m_Q^2 + m_u^2 + \frac{3-4\mathrm{s}_\mathrm{w}^2}{6}\mathrm{c}_{2\beta}m_Z^2 \right)\tilde{u}_\mathrm{L} + \tilde{u}_\mathrm{R}^* \left(m_{U^\mathrm{c}}^2 + m_u^2 + \frac{4\mathrm{s}_\mathrm{w}^2}{6}\mathrm{c}_{2\beta}m_Z^2 \right)\tilde{u}_\mathrm{R} \\
&+ \tilde{u}_\mathrm{R}^*(v_\mathrm{u} a_\mathrm{u} - \mu^* m_u \cot\beta)\tilde{u}_\mathrm{L} + \tilde{u}_\mathrm{L}^*(v_\mathrm{u} a_\mathrm{u}^\dagger - \mu m_u \cot\beta)\tilde{u}_\mathrm{R} \\
&+ \tilde{d}_\mathrm{L}^* \left(V_d^\dagger (V_u m_Q^2 V_u^\dagger) V_d + m_d^2 + \frac{-3+2\mathrm{s}_\mathrm{w}^2}{6}\mathrm{c}_{2\beta}m_Z^2 \right)\tilde{d}_\mathrm{L} + \tilde{d}_\mathrm{R}^* \left(m_{D^\mathrm{c}}^2 + m_d^2 + \frac{-2\mathrm{s}_\mathrm{w}^2}{6}\mathrm{c}_{2\beta}m_Z^2 \right)\tilde{d}_\mathrm{R} \\
&+ \tilde{d}_\mathrm{R}^*(v_\mathrm{d} a_d - \mu^* m_d \tan\beta)\tilde{d}_\mathrm{L} + \tilde{d}_\mathrm{L}^*(v_\mathrm{d} a_d^\dagger - \mu m_d \tan\beta)\tilde{d}_\mathrm{R} \\
&+ \tilde{\nu}_\mathrm{L}^* \left(m_L^2 + \frac{1}{2}\mathrm{c}_{2\beta}m_Z^2 \right)\tilde{\nu}_\mathrm{L} \\
&+ \tilde{e}_\mathrm{L}^* \left(m_L^2 + m_e^2 + \frac{-1+2\mathrm{s}_\mathrm{w}^2}{2}\mathrm{c}_{2\beta}m_Z^2 \right) + \tilde{e}_\mathrm{R}^* \left(m_{E^\mathrm{c}}^2 + m_e^2 + (-\mathrm{s}_\mathrm{w}^2)\mathrm{c}_{2\beta}m_Z^2 \right)\tilde{e}_\mathrm{R} \\
&+ \tilde{e}_\mathrm{R}^*(v_\mathrm{d} a_e - \mu^* m_e \tan\beta)\tilde{e}_\mathrm{L} + \tilde{e}_\mathrm{L}^*(v_\mathrm{d} a_e^\dagger - \mu m_e \tan\beta)\tilde{e}_\mathrm{R},
\end{aligned}
\tag{6.76}
$$

where the sfermion soft masses, yukawas, and a-terms are rewritten in super-CKM basis:

$$
[m_Q^2, m_{U^\mathrm{c}}^2, m_{D^\mathrm{c}}^2, m_L^2, m_{E^\mathrm{c}}^2]_\text{super-CKM} = [V_u^\dagger m_Q^2 V_u, U_u^\dagger m_{U^\mathrm{c}}^2 U_u, U_d^\dagger m_{D^\mathrm{c}}^2 U_d, V_e^\dagger m_L^2 V_e, U_e^\dagger m_{E^\mathrm{c}}^2 U_e]_\text{“original”}, \tag{6.77}
$$

$$
[a_u, a_d, a_e]_\text{super-CKM} = [U_u^\dagger a_u V_u, U_d^\dagger a_d V_d, U_e^\dagger a_e V_e]_\text{“original”}. \tag{6.78}
$$

In matrix form,

$$
\begin{aligned}
-\mathcal{L} \supset{}& \begin{pmatrix} \tilde{u}_{\mathrm{L}i}^* & \tilde{u}_{\mathrm{R}i}^* \end{pmatrix} \begin{pmatrix} [m_Q^2]_{ij} + \left(m_u^2 + \frac{3-4\mathrm{s}_\mathrm{w}^2}{6}\mathrm{c}_{2\beta}m_Z^2 \right)\delta_{ij} & v_u[a_u^\dagger]_{ij} - (\mu m_u \cot\beta)\delta_{ij} \\ v_u[a_u]_{ij} - (\mu^* m_u \cot\beta)\delta_{ij} & [m_{U^\mathrm{c}}^2]_{ij} + \left(m_u^2 + \frac{2\mathrm{s}_\mathrm{w}^2}{3}\mathrm{c}_{2\beta}m_Z^2 \right)\delta_{ij} \end{pmatrix} \begin{pmatrix} \tilde{u}_{\mathrm{L}j} \\ \tilde{u}_{\mathrm{R}j} \end{pmatrix} \\
&+ \begin{pmatrix} \tilde{d}_{\mathrm{L}i}^* & \tilde{d}_{\mathrm{R}i}^* \end{pmatrix} \begin{pmatrix} [V_\mathrm{CKM}^\dagger m_Q^2 V_\mathrm{CKM}]_{ij} + \left(m_d^2 + \frac{-3+2\mathrm{s}_\mathrm{w}^2}{6}\mathrm{c}_{2\beta}m_Z^2 \right)\delta_{ij} & v_d[a_d^\dagger]_{ij} - (\mu m_d \tan\beta)\delta_{ij} \\ v_d[a_d]_{ij} - (\mu^* m_d \tan\beta)\delta_{ij} & [m_{D^\mathrm{c}}^2]_{ij} + \left(m_d^2 - \frac{\mathrm{s}_\mathrm{w}^2}{3}\mathrm{c}_{2\beta}m_Z^2 \right)\delta_{ij} \end{pmatrix} \begin{pmatrix} \tilde{d}_{\mathrm{L}j} \\ \tilde{d}_{\mathrm{R}j} \end{pmatrix} \\
&+ \tilde{\nu}_{\mathrm{L}i}^* \Big([m_L^2]_{ij} + \left(\tfrac{1}{2}\mathrm{c}_{2\beta}m_Z^2 \right)\delta_{ij} \Big) \tilde{\nu}_{\mathrm{L}j} \\
&+ \begin{pmatrix} \tilde{e}_{\mathrm{L}i}^* & \tilde{e}_{\mathrm{R}i}^* \end{pmatrix} \begin{pmatrix} [m_L^2]_{ij} + \left(m_e^2 + \frac{-1+2\mathrm{s}_\mathrm{w}^2}{2}\mathrm{c}_{2\beta}m_Z^2 \right)\delta_{ij} & v_d[a_e^\dagger]_{ij} - (\mu m_e \tan\beta)\delta_{ij} \\ v_d[a_e]_{ij} - (\mu^* m_e \tan\beta)\delta_{ij} & [m_{E^\mathrm{c}}^2]_{ij} + \left(m_e^2 - \mathrm{s}_\mathrm{w}^2\mathrm{c}_{2\beta}m_Z^2 \right)\delta_{ij} \end{pmatrix} \begin{pmatrix} \tilde{e}_{\mathrm{L}j} \\ \tilde{e}_{\mathrm{R}j} \end{pmatrix}
\end{aligned}
\tag{6.79}
$$

$$
= \begin{pmatrix} \tilde{u}_{\mathrm{L}i}^* & \tilde{u}_{\mathrm{R}i}^* \end{pmatrix} \mathcal{M}_u \begin{pmatrix} \tilde{u}_{\mathrm{L}j} \\ \tilde{u}_{\mathrm{R}j} \end{pmatrix} + \begin{pmatrix} \tilde{d}_{\mathrm{L}i}^* & \tilde{d}_{\mathrm{R}i}^* \end{pmatrix} \mathcal{M}_d \begin{pmatrix} \tilde{d}_{\mathrm{L}j} \\ \tilde{d}_{\mathrm{R}j} \end{pmatrix} + \tilde{\nu}_\mathrm{L}^* \mathcal{M}_\nu \tilde{\nu}_\mathrm{L} + \begin{pmatrix} \tilde{e}_{\mathrm{L}i}^* & \tilde{e}_{\mathrm{R}i}^* \end{pmatrix} \mathcal{M}_e \begin{pmatrix} \tilde{e}_{\mathrm{L}j} \\ \tilde{e}_{\mathrm{R}j} \end{pmatrix} \tag{6.80}
$$

6.4.2. Fermion composition

Now we show the fermion-related Lagrangian terms verbosely:

$$
\begin{aligned}
&\mathcal{L}_\text{fermions} + \mathcal{L}_\text{SFG} \\
&=
\end{aligned}
\tag{6.81}
$$

6.5. SLHA convention

The SLHA convention [?] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table (**magenta color** for objects in other conventions), while

$\mu, b, m^2_{Q,L,H_\mathrm{u},H_\mathrm{d}}$, RPV-trilinears ($\lambda$s and Ts)

are in common.

SLHA		our notation	Martin/DHM
(H_1, H_2)	$=$	$(H_\mathrm{d}, H_\mathrm{u})$	
$Y_{\mathrm{u,d,e}}$	$=$	$(y_{\mathrm{u,d,e}})^\mathrm{T}$	
$T_{\mathrm{u,d,e}}$	$=$	$(a_{\mathrm{u,d,e}})^\mathrm{T}$	
$A_{\mathrm{u,d,e}}$	$=$	$(A_{\mathrm{u,d,e}})^\mathrm{T}$	
$m^2_{U^\mathrm{c},D^\mathrm{c},E\mathrm{c}}$	$=$	$(m^2_{U^\mathrm{c},D^\mathrm{c},E^\mathrm{c}})^\dagger$	
$M_{1,2,3}$	$=$	$-M_{1,2,3}$	
m_3^2	$=$	b	
m_A^2	$=$	$m^2_{A_0}$ (tree)	
		κ_i	$=-\mu'_i$ (rarely used)
D_i	$=$	b_i	
$m^2_{\tilde{L}_iH_1}$	$=$	M^2_{Li}	

In particular, the chargino/neutralino mass terms in RPC case are given by

$$\mathcal{L} \supset \left[\frac{1}{2}M_1\tilde{b}\tilde{b} + \frac{1}{2}M_2\tilde{w}\tilde{w} - \mu\tilde{h}_\mathrm{u}\tilde{h}_\mathrm{d} - \frac{g_Y}{2\sqrt{2}}\left(h_\mathrm{u}^*\tilde{h}_\mathrm{u} - h_\mathrm{d}^*\tilde{h}_\mathrm{d}\right)\tilde{b} - \sqrt{2}g_2\left(h_\mathrm{u}^*T^a\tilde{h}_\mathrm{u} + h_\mathrm{d}^*T_a\tilde{h}_\mathrm{d}\right)\tilde{w}\right] + \text{H.c.} \tag{6.82}$$

$$\to \frac{1}{2}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_\mathrm{u}^0\\ h_\mathrm{d}^0\end{pmatrix}^\mathrm{T}\begin{pmatrix}-M_1 & 0 & -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{s}_\beta\mathrm{s}_w\\ 0 & -M_2 & m_Z\mathrm{c}_\beta\mathrm{c}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w\\ -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{c}_\beta\mathrm{c}_w & 0 & -\mu\\ m_Z\mathrm{s}_\beta\mathrm{s}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w & -\mu & 0\end{pmatrix}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_\mathrm{u}^0\\ h_\mathrm{d}^0\end{pmatrix} \tag{6.83}$$

A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [?]:

$$\mathrm{e}^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \quad \text{(converges for any } X\text{)}, \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(A-1)^m}{m} \quad \text{(conv. if } \|A-I\|<1\text{)}. \tag{A.1}$$

$$\mathrm{e}^{\log A} = A \quad (\text{if } \|A-I\|<1), \quad \log \mathrm{e}^X = X \text{ and } \|\mathrm{e}^X - 1\| < 1 \quad (\text{if } \|X\| < \log 2). \tag{A.2}$$

$$\text{Hilbert-Schmidt norm}: \ \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \operatorname{Tr} X^\dagger X. \tag{A.3}$$

Properties:

$$\mathrm{e}^{(X^{\mathrm{T}})} = (\mathrm{e}^X)^{\mathrm{T}}, \qquad \mathrm{e}^{(X^*)} = (\mathrm{e}^X)^*, \qquad (\mathrm{e}^X)^{-1} = \mathrm{e}^{-X}, \qquad \mathrm{e}^{YXY^{-1}} = Y\,\mathrm{e}^X Y^{-1},$$

$$\det \exp X = \exp \operatorname{Tr} X, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{tX} = X\,\mathrm{e}^{tX} = \mathrm{e}^{tX} X \qquad \mathrm{e}^{(\alpha+\beta)X} = \mathrm{e}^{\alpha X}\,\mathrm{e}^{\beta X} \text{ for } \alpha,\beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$\mathrm{e}^X Y\,\mathrm{e}^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots = \mathrm{e}^{[X,]}Y; \tag{A.4}$$

$$\mathrm{e}^X\,\mathrm{e}^Y\,\mathrm{e}^{-X} = \sum_{n=0}^{\infty} \frac{1}{n!}(\mathrm{e}^X Y\,\mathrm{e}^{-X})^n = \exp\left(\mathrm{e}^{[X,]}Y\right); \tag{A.5}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = X + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[X,}\,\mathrm{e}^{t[Y,})Y \qquad \left[g(z) = \frac{\log z}{1-z^{-1}} = 1 - \sum_{n=1}^{\infty}\frac{(1-z)^n}{n(n+1)}; \quad g(\mathrm{e}^y) = \sum_{n=0}^{\infty}\frac{B_n y^n}{n!}\right] \tag{A.6}$$

$$= X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots \quad \text{(Baker-Campbell-Hausdorff)}. \tag{A.7}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\sum_{m,n=0}^{\infty}\frac{X^m Y^n}{m!n!} - 1\right)^k = \sum_{k=1}^{\infty}\sum_{m_1+n_1>0}\cdots\sum_{m_k+n_k>0}\frac{(-1)^{k-1}}{k}\frac{X^{m_1}Y^{n_1}\cdots X^{m_k}Y^{n_k}}{m_1!n_1!\cdots m_k!n_k!} \tag{A.8}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = \sum_{k=1}^{\infty}\sum_{m_1+n_1>0}\cdots\sum_{m_k+n_k>0}\frac{(-1)^{k-1}}{k\sum_{i=1}^{k}(m_i+n_i)}\frac{\big([X,\big)^{m_1}\big([Y,\big)^{n_1}\cdots\big([X,\big)^{m_k}\big([Y,\big)^{n_k}]\cdots]}{m_1!n_1!\cdots m_k!n_k!} \tag{A.9}$$

with $[X] := X$ understood.

If matrices t^a satisfies $[t^a, t^b] = \mathrm{i} f^{abc} t^c$ with totally-antisymmetric $f^{abc} \in \mathbb{R}$,

$$\left[\mathrm{e}^{\theta^a t^a} t_b\,\mathrm{e}^{-\theta^c t^c}\right]_{ij} = \left[\mathrm{e}^{\theta^a [t^a,} t_b\right]_{ij} = \left[\mathrm{e}^{\mathrm{i}\theta^a f^a}\right]^{bc} t^c_{ij} \tag{A.10}$$

holds for $\theta^a \in \mathbb{C}$, where $[f^a]_{bc} = f^{abc}$. ♣TODO: needs verification, generalization/restriction, and a nice proof or reference.♣

A.2. General unitary matrix

$$U_2 = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\alpha} \end{pmatrix}\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} \mathrm{e}^{\mathrm{i}\beta} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\gamma} \end{pmatrix} = \begin{pmatrix} c_\theta\,\mathrm{e}^{\mathrm{i}\beta} & s_\theta\,\mathrm{e}^{\mathrm{i}\gamma} \\ -s_\theta\,\mathrm{e}^{\mathrm{i}(\alpha+\beta)} & c_\theta\,\mathrm{e}^{\mathrm{i}(\alpha+\gamma)} \end{pmatrix} \tag{A.11}$$

$$U_3 = \begin{pmatrix} 1 & & \\ & \mathrm{e}^{\mathrm{i}a} & \\ & & \mathrm{e}^{\mathrm{i}b} \end{pmatrix}\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & & s_{13}\,\mathrm{e}^{-\mathrm{i}\delta} \\ & 1 & \\ -s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & & c_{13} \end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}\begin{pmatrix} \mathrm{e}^{\mathrm{i}c} & & \\ & \mathrm{e}^{\mathrm{i}d} & \\ & & \mathrm{e}^{\mathrm{i}e} \end{pmatrix} \tag{A.12}$$

$$= \begin{pmatrix} 1 & & \\ & \mathrm{e}^{\mathrm{i}a} & \\ & & \mathrm{e}^{\mathrm{i}b} \end{pmatrix}\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,\mathrm{e}^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} \mathrm{e}^{\mathrm{i}c} & & \\ & \mathrm{e}^{\mathrm{i}d} & \\ & & \mathrm{e}^{\mathrm{i}e} \end{pmatrix} \tag{A.13}$$

(e.g., hep-ph/9708216)