1 General Definitions and Tools

1.1 Notations and Conventions

◆1.1.1 Metric etc.

Minkowski Metric : $\eta^{\mu\nu} = \mathrm{diag}(+,-,-,-)$; $\epsilon^{0123}_{0123} := \pm 1$

Coordinates : $x^\mu = (t,x,y,z)$; Therefore $\partial_\mu = \left(\frac{\partial}{\partial t}, \nabla\right)$

Gamma Matrices : $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$; $\gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{-\mathrm{i}}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$

Gamma Combinations : $1, \{\gamma^\mu\}, \{\sigma^{\mu\nu}\}, \{\gamma^\mu\gamma_5\}, \gamma_5$; $\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu] = 0 \Big/ \mathrm{i}\gamma^\mu\gamma^\nu$

◆1.1.2 Fields

Klein-Gordon Equation : $|\partial_\mu\phi|^2 - m^2|\phi|^2 = 0$

Klein-Gordon Field : $\phi(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol{p}}}}\left[a_{\boldsymbol{p}}\mathrm{e}^{-\mathrm{i}px} + b^\dagger_{\boldsymbol{p}}\mathrm{e}^{\mathrm{i}px}\right]$

Dirac Equation : $(i\partial\!\!\!/ - m)\psi(x) = 0$

Dirac Field : $\psi(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol{p}}}}\sum_{s=1,2}\left[a^s_{\boldsymbol{p}}u^s(p)\mathrm{e}^{-\mathrm{i}px} + b^{s\dagger}_{\boldsymbol{p}}v^s(p)\mathrm{e}^{\mathrm{i}px}\right]$

Gauge Boson : $(\partial^2 + m^2)A^\mu(x) = 0$ (Real Klein-Gordon Equation)

(Before Gauge Fixing) : $A^\mu(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol{p}}}}\sum_{r=0,1,2,3}\left[a^r_{\boldsymbol{p}}\epsilon^r(p)\mathrm{e}^{-\mathrm{i}px} + a^{r\dagger}_{\boldsymbol{p}}\epsilon^{r*}(p)\mathrm{e}^{\mathrm{i}px}\right]$

南部-Goldstone Boson : **TODO: .**

Chiral Notation

Gamma Matrices : $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}$; $\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Dirac Field : $\psi = \begin{pmatrix} \psi_\mathrm{L} \\ \psi_\mathrm{R} \end{pmatrix}$; $\bar\psi = \psi^\dagger\gamma^0 = \begin{pmatrix} \psi^\dagger_\mathrm{R} & \psi^\dagger_\mathrm{L} \end{pmatrix}$

: $u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar\sigma}\xi^s \end{pmatrix}$; $v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\eta^s \\ -\sqrt{p\cdot\bar\sigma}\eta^s \end{pmatrix}$

: $\left[\eta^s = \xi^{-s} := -\mathrm{i}\sigma^2(\xi^s)^* = (\xi^2, -\xi^1)\right]$

Weyl Equations : $\mathrm{i}\bar\sigma\cdot\partial\psi_L = m\psi_R$; $\mathrm{i}\sigma\cdot\partial\psi_L = m\psi_L$

CPT transf. : $P\psi(t,\boldsymbol{x})P = \eta\gamma^0\psi(t,-\boldsymbol{x})$ $(|\eta|^2 = 1)$

: $T\psi(t,\boldsymbol{x})T = \gamma^1\gamma^3\psi(-t,\boldsymbol{x})$ (ignoring intrinsic phase)

: $C\psi(t,\boldsymbol{x})C = -\mathrm{i}\gamma^2\psi^*(t,\boldsymbol{x}) = -\mathrm{i}(\bar\psi\gamma^0\gamma^2)^\mathrm{T}$ (〃)

: $\bar\psi \longrightarrow P : \eta^*\bar\psi\gamma^0$ $T : -\bar\psi\gamma^1\gamma^3$ $C : \mathrm{i}\bar\psi^*\gamma^2 = -\mathrm{i}(\gamma^0\gamma^2\psi)^\mathrm{T}$

Electromagnetic Fields : $A^\mu = (\phi, \boldsymbol{A})$ 【We can invert the signs, but cannot lower the index.】

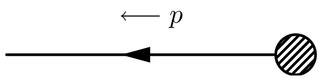
: $F_{\mu\nu} = \begin{pmatrix} 0 & & \boldsymbol{E} & \\ & 0 & -B_3 & B_2 \\ -\boldsymbol{E} & B_3 & 0 & -B_1 \\ & -B_2 & B_1 & 0 \end{pmatrix}$; $F_{\mu\nu}F^{\mu\nu} = -2\left(\|\boldsymbol{E}\|^2 - \|\boldsymbol{B}\|^2\right)$

◆1.1.3 CPT Table

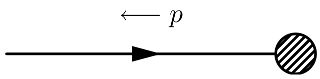
	ϕ	A^μ	$\bar{\psi}\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$	$\bar{\psi}\gamma^\mu\gamma_5\psi$	$\mathrm{i}\bar{\psi}\gamma_5\psi$	∂_μ
P	$\eta\phi$	$\eta{-}{+}{+}{+}A^\mu$	$+$	$+---$	$(+---)(+---)$	$-+++$	$-$	$+---$
T	$\zeta\phi$	$\zeta{+}{-}{-}{-}A^\mu$	$+$	$+---$	$-(+---)(+---)$	$+---$	$-$	$-+++$
C	$\xi\phi^*$	$\xi{+}A^{\mu *}$	$+$	$-$	$-$	$+$	$+$	$+$

($\eta\zeta\xi = 1$; especially, photon A^μ is $(\eta, \zeta, \xi) = (-, +, -)$.)

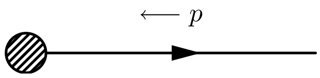
1.2 Feynman Rules

$\contraction{}{\psi}{|}{\boldsymbol{p}}\psi|\boldsymbol{p},s\rangle =$ 

hoge

$\contraction{}{\psi}{|}{\boldsymbol{p}}\psi|\boldsymbol{p},s\rangle =$ 

end

$\contraction{}{\psi}{|}{\boldsymbol{p}}\psi|\boldsymbol{p},s\rangle =$ 

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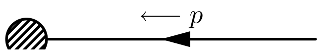
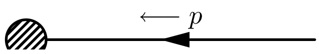
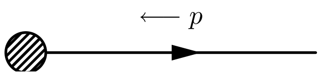
$\contraction{}{\psi}{|}{\boldsymbol{p}}\psi|\boldsymbol{p},s\rangle =$ 

end

Initial state

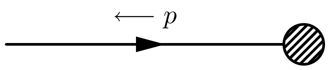
$\contraction{}{\psi}{\ |}{\boldsymbol{p}}\psi\ |\boldsymbol{p},s\rangle =$ 

$= u^s(p)$

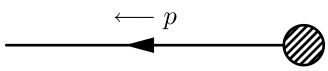
$\contraction{}{\overline{\psi}}{\ |}{\boldsymbol{p}}\overline{\psi}\ |\boldsymbol{p},s\rangle =$ 

$= \overline{v}^s(p)$

Final state

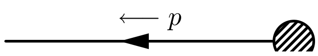
$\contraction{\langle\boldsymbol{p},}{s}{|\ }{\overline{\psi}}\langle\boldsymbol{p},s|\ \overline{\psi} =$ 

$= \overline{u}^s(p)$

$\contraction{\langle\boldsymbol{p},}{s}{|\ }{\psi}\langle\boldsymbol{p},s|\ \psi =$ 

$= v^s(p)$

Propagator

$\contraction{}{\psi}{(x)\ }{\overline{\psi}}\psi(x)\ \overline{\psi}(y) =$ 

$= v^s(p)$