1. Kinematics

Decay rate and cross section (Note: $\mathcal{M}$ has a mass dimension of $4 - N_\mathrm{i} - N_\mathrm{f}$.)

$$\text{decay rate (rest frame; } \sqrt{s} = M_0) : \quad \mathrm{d}\Gamma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2M_0} \left| \mathcal{M}(M_0 \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\}) \right|^2, \tag{1.1}$$

$$\text{cross section (Lorentz invariant)} : \quad \mathrm{d}\sigma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2E_A 2E_B\, v_{\text{Møl}}} \left| \mathcal{M}(p_A, p_B \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\}) \right|^2, \tag{1.2}$$

where $\overline{\mathrm{d}\Pi^n}$ is n-particle Lorentz-invariant phase space with momentum conservation

$$\overline{\mathrm{d}\Pi^n} := \mathrm{d}\Pi_1\, \mathrm{d}\Pi_2 \cdots \mathrm{d}\Pi_n\, (2\pi)^4\, \delta^{(4)} \left(P_0 - \sum p_n \right); \quad \mathrm{d}\Pi := \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{2E_{\boldsymbol{p}}}. \tag{1.3}$$

At the CM frame, two-body phase-space are characterized by the final momentum $\|\boldsymbol{p}\|$ and given by

$$\overline{\mathrm{d}\Pi^2} = \frac{\|\boldsymbol{p}\|}{4\pi\sqrt{s}} \frac{\mathrm{d}\Omega}{4\pi} = \frac{\|\boldsymbol{p}\|}{8\pi\sqrt{s}}\, \mathrm{d}\cos\theta = \frac{1}{16\pi} \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}}\, \mathrm{d}\cos\theta \tag{1.4}$$

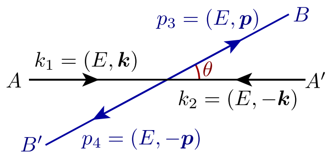
with $\sqrt{s} = M_0$ or E_CM, θ is the angle between initial and final motion, and

$$\|\boldsymbol{p}\| = \frac{\sqrt{s}}{2}\, \lambda^{1/2} \left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s} \right), \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad p_1 \cdot p_2 = \frac{s - (m_1^2 + m_2^2)}{2}.$$

Mandelstam variables For $(k_1, k_2) \to (p_3, p_4)$ collision,

$$s = (k_1 + k_2)^2 = (p_3 + p_4)^2, \qquad t = (p_3 - k_1)^2 = (p_4 - k_2)^2, \qquad u = (p_3 - k_2)^2 = (p_4 - k_1)^2;$$
$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

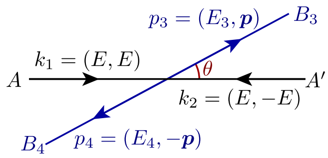
If the collision is with the "same mass" $(m_A, m_A) \to (m_B, m_B)$,



$$\begin{aligned} t &= m_A^2 + m_B^2 - s/2 + 2kp\cos\theta, & (k_1 - k_2)^2 &= 4m_A^2 - s, \\ u &= m_A^2 + m_B^2 - s/2 - 2kp\cos\theta, & (p_3 - p_4)^2 &= 4m_B^2 - s, \end{aligned}$$

$$k = \frac{\sqrt{s - 4m_A^2}}{2}, \qquad k_1 \cdot k_2 = \frac{s}{2} - m_A^2, \qquad k_1 \cdot p_3 = k_2 \cdot p_4 = \frac{m_A^2 + m_B^2 - t}{2},$$
$$p = \frac{\sqrt{s - 4m_B^2}}{2}, \qquad p_3 \cdot p_4 = \frac{s}{2} - m_B^2, \qquad k_1 \cdot p_4 = k_2 \cdot p_3 = \frac{m_A^2 + m_B^2 - u}{2}.$$

Instead, if the collision is "initially massless" $(0, 0) \to (m_3, m_4)$,



$$\begin{aligned} t &= (m_3^2 + m_4^2 - s)/2 + p\sqrt{s}\cos\theta, \\ u &= (m_3^2 + m_4^2 - s)/2 - p\sqrt{s}\cos\theta, \\ p &= (\sqrt{s}/2)\, \lambda^{1/2} \left(1; m_3^2/s, m_4^2/s \right). \end{aligned}$$

1.1. Fundamentals

Lorentz-invariant phase space:

$$\int \mathrm{d}\Pi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\sqrt{m^2+\|\boldsymbol{p}\|^2}} = \int \frac{\mathrm{d}p_0\,\mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}(2\pi)\,\delta\left(p_0^2-\|\boldsymbol{p}\|^2-m^2\right)\Theta(p_0)$$

Källén function:

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zx = (x-y-z)^2-4yz;$$

$$\lambda(1;\alpha_1^2,\alpha_2^2) = (1-(\alpha_1+\alpha_2)^2)(1-(\alpha_1-\alpha_2)^2) = (1+\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)(1-\alpha_1+\alpha_2).$$

$$\lambda^{1/2}\left(s;m_1^2,m_2^2\right) = s\,\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right); \qquad \lambda^{1/2}\left(1;\frac{m^2}{s},\frac{m^2}{s}\right) = \sqrt{1-\frac{4m^2}{s}},$$

$$\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = \sqrt{1-\frac{2(m_1^2+m_2^2)}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}}, \qquad \lambda^{1/2}\left(1;\frac{m_1^2}{s},0\right) = \frac{s-m_1^2}{s}.$$

Two-body phase space If $f(p_1^\mu,p_2^\mu)$ is Lorentz invariant, $f\equiv f(p_1^2,p_2^2,p_1^\mu p_{2\mu})\equiv f(p_1,p_2,\cos\theta_{12})$. Meanwhile,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \int \frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{(4\pi)\,\mathrm{d}p_1\,p_1^2}{(2\pi)^3}\frac{(2\pi)\,\mathrm{d}p_2\,p_2^2\,\mathrm{d}\cos\theta_{12}}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{\mathrm{d}E_+\,\mathrm{d}E_-\,\mathrm{d}s}{128\pi^4}, \tag{1.5}$$

with the replacement of the variables

$$E_\pm = E_1\pm E_2, \qquad s=(p_1+p_2)^2 = m_1^2+m_2^2+2E_1E_2-2\|\boldsymbol{p}_1\|\|\boldsymbol{p}_2\|\cos\theta_{12};$$

$$\left|\frac{\mathrm{d}(E_+,E_-,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{4p_1^2p_2^2}{E_1E_2}, \qquad \left|\frac{\mathrm{d}(E_1,E_2,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{2p_1^2p_2^2}{E_1E_2}.$$

Therefore,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \frac{1}{128\pi^4}\int_{(m_1+m_2)^2}^{\infty}\mathrm{d}s\int_{\sqrt{s}}^{\infty}\mathrm{d}E_+\int_{\min}^{\max}\mathrm{d}E_-, \tag{1.6}$$

where the boundary of E_- is given by

$$\cos\theta_{12} = \frac{E_+^2-E_-^2+2\left(m_1^2+m_2^2-s\right)}{\sqrt{(E_++E_-)^2-4m_1^2}\sqrt{(E_+-E_-)^2-4m_2^2}}\in[-1,1]$$

$$\therefore\quad \left|E_- - \frac{m_1^2-m_2^2}{s}E_+\right| \le \sqrt{E_+^2-s}\cdot\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = 2p\sqrt{\frac{E_+^2-s}{s}}.$$

Two-body phase space with momentum conservation As a general representation in any frame,

$$\overline{\mathrm{d}\Pi^2} = \frac{\mathrm{d}p_1\,\mathrm{d}\Omega\,p_1^2}{16\pi^2}\frac{\delta(E_0-\sqrt{m_1^2+p_1^2}-\sqrt{m_2^2+\|\boldsymbol{P}_0-\boldsymbol{p}_1\|^2})}{E_1E_2} = \frac{1}{8\pi}\,\mathrm{d}\cos\theta_1\frac{p_1^2}{E_0p_1-P_0E_1\cos\theta_1}, \tag{1.7}$$

where the momentum p_1 is given by

$$p_1 = \frac{(E_0^2+m_1^2-m_2^2-P_0^2)P_0\cos\theta_1+E_0\sqrt{\lambda(E_0^2,m_1^2,m_2^2)+P_0^4-2P_0^2(E_0^2+m_1^2-2m_1^2\cos^2\theta_1-m_2^2)}}{2(E_0^2-P_0^2\cos^2\theta_1)}. \tag{1.8}$$

CM frame result is recovered by setting $E_0=\sqrt{s}$ and $P_0=0$.

1.2. Decay rate and Cross section

As $\langle\mathrm{out}|\mathrm{in}\rangle = (2\pi)^4\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f})\mathrm{i}\mathcal{M}$ (for in $\neq$ out) and $\langle\boldsymbol{p}|\boldsymbol{p}\rangle = 2E_{\boldsymbol{p}}(2\pi)^3\,\delta^{(3)}(\boldsymbol{0}) = 2E_{\boldsymbol{p}}V$ for one-particle state,

$$\frac{N_\mathrm{ev}}{\prod_\mathrm{in}N_\mathrm{particle}} = \int \mathrm{d}\Pi^\mathrm{out}\frac{|\langle\mathrm{out}|\mathrm{in}\rangle|^2}{\langle\mathrm{in}|\mathrm{in}\rangle} = \int \mathrm{d}\Pi^\mathrm{out}\frac{(2\pi)^8|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}\frac{VT}{(2\pi^4)}\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f}) = VT\int\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}. \tag{1.9}$$

Therefore, decay rate (at the rest frame) is given by

$$\mathrm{d}\Gamma := \frac{1}{T}\frac{\mathrm{d}N_\mathrm{ev}}{N_\mathrm{particle}} = \frac{1}{T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.10}$$

We also define Lorentz-invariant cross section σ by $N_\mathrm{ev} =: (\rho_A v_\mathrm{Møl}T\sigma)N_B = (\rho_A v_\mathrm{Møl}T\sigma)(\rho_B V)$, or

$$\mathrm{d}\sigma := \frac{\mathrm{d}N_\mathrm{ev}}{\rho_A v_\mathrm{Møl}TN_B} = \frac{V}{v_\mathrm{Møl}T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{2E_A2E_BV^2} = \frac{1}{2E_A2E_B\,v_\mathrm{Møl}}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.11}$$

where the Møller parameter $v_\mathrm{Møl}$ is equal to $v_\mathrm{rel}^\mathrm{NR} = \|\boldsymbol{v}_A-\boldsymbol{v}_B\|$ if $\boldsymbol{v}_A \mathbin{/\!/} \boldsymbol{v}_B$ (cf. Ref. [?]). Generally,

$$v_\mathrm{Møl} := \frac{\sqrt{(p_A\cdot p_B)^2-m_A^2m_B^2}}{E_AE_B} = \frac{p_A\cdot p_B}{E_AE_B}v_\mathrm{rel} = (1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B)v_\mathrm{rel}, \tag{1.12}$$

where v_rel is the actual relative velocity

$$v_\mathrm{rel} = \sqrt{1-\frac{(1-v_A^2)(1-v_B^2)}{1-(\boldsymbol{v}_A\cdot\boldsymbol{v}_B)^2}} = \frac{\sqrt{\|\boldsymbol{v}_A-\boldsymbol{v}_B\|^2-\|\boldsymbol{v}_A\times\boldsymbol{v}_B\|^2}}{1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B} = \frac{\lambda^{1/2}(s,m_A^2,m_B^2)}{s-(m_A^2+m_B^2)} \neq v_\mathrm{rel}^\mathrm{NR}. \tag{1.13}$$

(Note that $p_A\cdot p_B/E_AE_B=1$ if $\boldsymbol{p}_A=0$ or $\boldsymbol{p}_B=0$. Also, Each of v_rel, VT, and $E_AE_Bv_\mathrm{Møl}$ is Lorentz invariant.)

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation adj. $= (\epsilon^a)^{bc}$.[*1]

$$T_a = \frac{1}{2}\sigma_a, \qquad \mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \qquad [T_a, T_b] = \mathrm{i}\epsilon^{abc}T^c, \qquad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\overline{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a \epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\overline{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \to \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp(-\mathrm{i}g\theta^\alpha T^{\alpha*})]^{ab}(\epsilon^{bc}\mathbf{2}^c). \tag{2.1}$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\overline{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation adj. $= \mathbf{8} = (f^a)^{bc}$.
Gell-Mann matrices:

$$\lambda_{1\text{–}8} = \begin{pmatrix} 0&1&0\\1&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&-\mathrm{i}&0\\ \mathrm{i}&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0\\0&-1&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&-\mathrm{i}\\0&0&0\\ \mathrm{i}&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&1\\0&1&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&-\mathrm{i}\\0&\mathrm{i}&0 \end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix} 1&0&0\\0&1&0\\0&0&-2 \end{pmatrix}. \tag{2.2}$$

$$\tau_a = \frac{1}{2}\lambda_a, \qquad \mathrm{Tr}(\tau_a\tau_b) = \frac{1}{2}\delta_{ab}, \qquad [\tau_a, \tau_b] = \mathrm{i}f^{abc}\tau^c, \qquad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\begin{aligned} \mathbf{3}: \quad \phi_a &\to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi_b \simeq \phi_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi_b \\ \phi^*_a &\to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi^*_b \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi^*_b \\ &= \phi^*_b[\exp(-\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ba} \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\phi^*_b\tau^\alpha_{ba} \end{aligned}$$

$$\begin{aligned} \overline{\mathbf{3}}: \quad \phi_a &\to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi_b \\ \phi^*_a &\to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi^*_b \simeq \phi^*_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi^*_b \end{aligned}$$

[*1] We do not distinguish sub- and superscripts for gauge indices.

3. Spinors

$$(\overline{\psi_1}\psi_2)^* = (\psi_2)^\dagger(\overline{\psi}_1)^\dagger = \overline{\psi_2}\psi_1. \tag{3.1}$$

4. Supersymmetry with $\eta = \mathrm{diag}(+,-,-,-)$

Convention Our convention follows DHM (except for D_μ):

$$\eta = \mathrm{diag}(1,-1,-1,-1); \quad \epsilon^{0123} = -\epsilon_{0123} = 1, \quad \epsilon^{12} = \epsilon_{21} = \epsilon^{\dot{1}\dot{2}} = \epsilon_{\dot{2}\dot{1}} = 1 \quad \left(\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma\right),$$
$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta}, \quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta};$$

$$\sigma^\mu_{\alpha\dot\alpha} := (\mathbf{1},\boldsymbol{\sigma})_{\alpha\dot\alpha}, \qquad \sigma^{\mu\nu}{}_\alpha{}^\beta := \frac{\mathrm{i}}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)_\alpha{}^\beta,^{*2} \qquad \left(\sigma^\mu_{\alpha\dot\beta} = \epsilon_{\alpha\delta}\epsilon_{\dot\beta\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \quad \bar\sigma^{\mu\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\delta}\epsilon^{\beta\gamma}\sigma^\mu_{\gamma\dot\delta}\right)$$
$$\bar\sigma^{\mu\dot\alpha\alpha} := (\mathbf{1},-\boldsymbol{\sigma})^{\dot\alpha\alpha}, \quad \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta} := \frac{\mathrm{i}}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta},^{*2}$$
$$(\psi\xi) := \psi^\alpha\xi_\alpha, \quad (\bar\psi\bar\chi) := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}; \qquad \frac{\mathrm{d}}{\mathrm{d}\theta^\alpha}(\theta\theta) := \theta_\alpha \quad \text{[left derivative]}.$$

Especially, spinor-index contraction is done as ${}^\alpha{}_\alpha$ and ${}_{\dot\alpha}{}^{\dot\alpha}$ except for ϵ_{ab} (which always comes from left). Noting that complex conjugate reverses spinor order: $(\psi^\alpha\xi^\beta)^* := (\xi^\beta)^*(\psi^\alpha)^*$,

$$\bar\psi^{\dot\alpha} := (\psi^\alpha)^*, \quad \epsilon^{\dot a\dot b} := (\epsilon^{ab})^*, \qquad \left(\psi\chi\right)^* = (\bar\psi\bar\chi),$$
$$\left(\sigma^\mu_{\alpha\dot\beta}\right)^* = \bar\sigma^\mu{}_{\dot\alpha\beta} = \epsilon_{\beta\delta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \qquad \left(\sigma^{\mu\nu}\right)^{\dagger\alpha}{}_\beta = \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}, \qquad \left(\sigma^{\mu\nu}{}_\alpha{}^\beta\right)^* = \bar\sigma^{\mu\nu\dot\beta}{}_{\dot\alpha} = \bar\sigma^{\mu\nu}{}_{\dot\alpha}{}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\delta},$$
$$\left(\bar\sigma^{\mu\dot\alpha\beta}\right)^* = \sigma^{\mu\alpha\dot\beta} = \epsilon^{\dot\beta\dot\delta}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\delta}, \qquad \left(\bar\sigma^{\mu\nu}\right)^\dagger{}_{\dot\alpha}{}^{\dot\beta} = \sigma^{\mu\nu}{}_\alpha{}^\beta, \qquad \left(\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\right)^* = \sigma^{\mu\nu}{}_\beta{}^\alpha = \bar\sigma^{\mu\nu\alpha}{}_\beta = \epsilon_{\beta\delta}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\delta.$$

Contraction formulae

$$\theta^\alpha\theta^\beta = -\tfrac12(\theta\theta)\epsilon^{\alpha\beta} \qquad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\epsilon^{\dot\alpha\dot\beta} \qquad (\theta\xi)(\theta\chi) = -\tfrac12(\theta\theta)(\xi\chi) \qquad (\theta\sigma^\nu\bar\theta)\theta^\alpha = \tfrac12(\theta\theta)(\bar\theta\bar\sigma^\nu)^\alpha$$
$$\theta_\alpha\theta_\beta = \tfrac12(\theta\theta)\epsilon_{\alpha\beta} \qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac12(\bar\theta\bar\theta)\epsilon_{\dot\alpha\dot\beta} \qquad (\bar\theta\bar\xi)(\bar\theta\bar\chi) = -\tfrac12(\bar\theta\bar\theta)(\bar\xi\bar\chi) \qquad (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\tfrac12(\theta\sigma^\nu)_{\dot\alpha}(\bar\theta\bar\theta)$$
$$\theta^\alpha\theta_\beta = \tfrac12(\theta\theta)\delta^\alpha_\beta \qquad \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\delta^{\dot\alpha}_{\dot\beta} \qquad (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \tfrac12(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu}$$
$$(\theta\sigma^\mu\bar\sigma^\nu\theta) = (\theta\theta)\eta^{\mu\nu} \qquad (\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta) = (\bar\theta\bar\theta)\eta^{\mu\nu} \qquad (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \tfrac12(\bar\theta\bar\theta)(\sigma^\mu\bar\sigma^\nu\theta)_\alpha$$

$$\begin{aligned}
&\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\sigma^{\mu\nu} && \sigma^\mu\bar\sigma^\nu\sigma^\rho + \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\left(\sigma^\mu\eta^{\rho\nu} - \sigma^\nu\eta^{\mu\rho} + \sigma^\rho\eta^{\mu\nu}\right)\\
&\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\bar\sigma^{\mu\nu} && \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\mathrm{i}\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}\\
&\mathrm{Tr}\left(\sigma^\mu\bar\sigma^\nu\right) = \mathrm{Tr}\left(\bar\sigma^\mu\sigma^\nu\right) = 2\eta^{\mu\nu} && \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho + \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = 2\left(\bar\sigma^\mu\eta^{\rho\nu} - \bar\sigma^\nu\eta^{\mu\rho} + \bar\sigma^\rho\eta^{\mu\nu}\right)\\
&\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu{}^{\dot\beta\beta} = 2\delta^{\dot\beta}_{\dot\alpha}\delta^\alpha_\beta && \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2\mathrm{i}\bar\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}\\
&\sigma_{\mu\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} && \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\dot\beta\dot\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\beta}\\
&\bar\sigma_\mu{}^{\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} && \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\beta\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\dot\alpha\dot\gamma}\sigma^\mu_{\beta\dot\gamma}\\
&\mathrm{Tr}\left(\sigma^{\mu\nu}\right) = \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\right) = 0 && \mathrm{Tr}\left(\sigma^{\mu\nu}\sigma^{\rho\sigma}\right) = \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma}\\
&\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu} \qquad \sigma^{\mu\nu} = -\sigma^{\nu\mu} && \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma}\right) = \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma} + \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)\\
&\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha} && \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = -2\mathrm{i}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon_{\alpha\beta} - 2\mathrm{i}\sigma^{\mu\nu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}\\
&\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon^{\dot\beta\dot\alpha} && \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 4\sigma^{\rho\mu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\sigma\nu\dot\gamma}{}_{\dot\beta}\eta_{\rho\sigma} + \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\eta^{\mu\nu}\\
&\bar\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = -2\mathrm{i}\bar\sigma^{\mu\nu} && \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} - \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = -2\mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\epsilon^{\alpha\beta} - 2\mathrm{i}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon^{\dot\alpha\dot\beta}\\
&\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 2\mathrm{i}\sigma^{\mu\nu} && \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} + \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 4\epsilon^{\alpha\gamma}\sigma^{\sigma\nu}{}_\gamma{}^\beta\bar\sigma^{\rho\mu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\eta_{\rho\sigma} + \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\eta^{\mu\nu}
\end{aligned}$$

$$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi \qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi = \bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi \qquad \xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi \qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho\chi = -\chi\sigma^\rho\bar\sigma^\nu\sigma^\mu\bar\xi$$
$$\left(\xi\sigma^\mu\bar\chi\right)^* = \chi\sigma^\mu\bar\xi \qquad \left(\bar\xi\bar\sigma^\mu\chi\right)^* = \bar\chi\bar\sigma^\mu\xi \qquad \left(\bar\chi\bar\sigma^\mu\sigma^\nu\bar\xi\right)^* = \xi\sigma^\nu\bar\sigma^\mu\chi \qquad (\xi[\sigma\mathrm{s}]\chi)^* = \bar\chi[\sigma\mathrm{s}_{\mathrm{reversed}}]\bar\xi$$
$$(\xi\chi)\psi^\alpha = -(\psi\xi)\chi^\alpha - (\psi\chi)\xi^\alpha \qquad (\xi\chi)\bar\psi_{\dot\alpha} = \tfrac12(\xi\sigma^\mu\bar\psi)(\chi\sigma_\mu)_{\dot\alpha}$$
$$\mathrm{i}\psi_i\sigma^\mu\partial_\mu\bar\psi_j = -\mathrm{i}\partial_\mu\bar\psi_j\bar\sigma^\mu\psi_i \equiv \mathrm{i}\bar\psi_j\bar\sigma^\mu\partial_\mu\psi_i = -\mathrm{i}\partial_\mu\psi_i\sigma^\mu\bar\psi_j$$

*2 As the definition of $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are not unified in literature, they are not used in this CheatSheet except for this page.

Superfields

$$\Phi = \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{\theta^4}{4}\partial^2\phi(x), \tag{4.1}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{\theta^4}{4}\partial^2\phi^*(x), \tag{4.2}$$

$$V = (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{\theta^4}{2}D(x) \qquad \text{(in Wess-Zumino supergauge).} \tag{4.3}$$

Without gauge symmetries

$$\mathcal{L} = \Phi_i^*\Phi_i\Big|_{\theta^4} + \left(W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right); \tag{4.4}$$

$$\Phi_i^*\Phi_i\Big|_{\theta^4} = (\partial_\mu\phi_i^*)(\partial^\mu\phi_i) + \mathrm{i}\bar\psi_i\sigma^\mu\partial_\mu\psi_i + F_i^*F_i, \tag{4.5}$$

$$\begin{aligned} W(\Phi_i)\Big|_{\theta^2} &\rightsquigarrow \Big[\kappa_i\Phi_i + m_{ij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k\Big]_{\theta^2} \\ &= \kappa_i F_i + m_{ij}\left(-\psi_i\psi_j + F_i\phi_j + \phi_i F_j\right) \\ &\qquad + y_{ijk}\Big[-(\psi_i\psi_j\phi_k + \psi_i\phi_j\psi_k + \phi_i\psi_j\psi_k) + \phi_i\phi_j F_k + \phi_i F_j\phi_k + F_i\phi_j\phi_k\Big]. \end{aligned} \tag{4.6}$$

With a U(1) gauge symmetry [*3]

$$\mathcal{L} = \Phi_i^*\,\mathrm{e}^{2gVQ_i}\Phi_i\Big|_{\theta^4} + \left[\left(\frac14 - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right] + \Lambda_{\text{FI}}D; \tag{4.7}$$

$$\Phi_i\,\mathrm{e}^{2gQ_iV}\Phi_i\Big|_{\theta^4} \equiv \mathrm{D}^\mu\phi_i^*\,\mathrm{D}_\mu\phi_i + \mathrm{i}\bar\psi_i\bar\sigma^\mu\,\mathrm{D}_\mu\psi_i + F_i^*F_i - \sqrt{2}gQ_i\phi_i^*\lambda\psi_i - \sqrt{2}gQ_i\bar\psi_i\bar\lambda\phi_i + gQ_i\phi_i^*\phi_i D, \tag{4.8}$$

$$\begin{aligned} \left(\frac14 - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + \text{H.c.} &= \frac12\operatorname{Re}\mathcal{W}\mathcal{W}\big|_{\theta^2} + \frac{g^2\Theta}{16\pi^2}\operatorname{Im}\mathcal{W}\mathcal{W}\big|_{\theta^2} \\ &\equiv \mathrm{i}\bar\lambda\bar\sigma^\mu\,\mathrm{D}_\mu\lambda + \frac12 DD - \frac14 F_{\mu\nu}F^{\mu\nu} + \frac{g^2\Theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \end{aligned} \tag{4.9}$$

$$\begin{aligned} &\mathrm{D}_\mu\phi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\phi_i, \qquad \mathrm{D}_\mu\psi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\psi_i, \\ &\mathrm{D}^\mu\phi_i^* = (\partial^\mu + \mathrm{i}gQ_iA^\mu)\phi_i^*, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \mathrm{D}_\mu\lambda = \partial_\mu\lambda. \end{aligned}$$

$$\{\phi,\psi,F\} \xrightarrow{\text{gauge}} \mathrm{e}^{\mathrm{i}gQ_i\theta}\{\phi,\psi,F\}, \qquad A_\mu \xrightarrow{\text{gauge}} A_\mu + \partial_\mu\theta, \qquad \lambda \xrightarrow{\text{gauge}} \lambda, \qquad D \xrightarrow{\text{gauge}} D. \tag{4.10}$$

[*3] We use the convention with $V \ni \lambda(x)\theta\bar\theta^2$, which corresponds to $\lambda = \mathrm{i}\lambda_{\text{SLHA}}$. In SLHA convention, the scalar-fermion-gaugino interaction is replaced to

$$-\sqrt{2}g\mathrm{i}\lambda^a_{\text{SLHA}}(\phi^*t^a\psi) - \sqrt{2}g(-\mathrm{i}\bar\lambda^a_{\text{SLHA}})(\bar\psi t^a\phi).$$

With an SU(N) gauge symmetry

$$\mathcal{L} = \Phi^* \,\mathrm{e}^{2gV}\Phi\Big|_{\theta^4} + \left[\left(\frac{1}{2} - \frac{\mathrm{i}g^2\Theta}{16\pi^2}\right)\mathrm{Tr}\,\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi)\Big|_{\theta^2} + \mathrm{H.c.}\right]; \tag{4.11}$$

$$\Phi^* \,\mathrm{e}^{2gV}\Phi\Big|_{\theta^4} := \Phi_i^*\left[\mathrm{e}^{2gV^a t^a_\Phi}\right]_{ij}\Phi_j\Big|_{\theta^4} \tag{4.12}$$

$$\begin{aligned} &= (\partial_\mu\phi_i^*)(\partial^\mu\phi_i) + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_i + F_i^*F_i - \sqrt{2}g\lambda^a(\phi^* t^a\psi) - \sqrt{2}g\bar\lambda^a(\bar\psi^* t^a\phi) \\ &\qquad + gA^a_\mu\bar\psi\bar\sigma^\mu(t^a\psi) + 2\mathrm{i}gA^a_\mu\phi^*\partial_\mu(t^a\phi) + g^2A^{a\mu}A^b_\mu(\phi^* t^a t^b\phi) + gD^a(\phi^* t^a\phi) \end{aligned} \tag{4.13}$$

$$= \mathrm{D}^\mu\phi^*\,\mathrm{D}_\mu\phi + \mathrm{i}\bar\psi_i\bar\sigma^\mu\,\mathrm{D}_\mu\psi_i + F^*F - \sqrt{2}g\lambda^a(\phi^* t^a\psi) - \sqrt{2}g\bar\lambda^a(\bar\psi t^a\phi) + gD^a(\phi^* t^a\phi) \tag{4.14}$$

$$\begin{aligned} \left(\frac{1}{2} - \frac{\mathrm{i}g^2\Theta}{16\pi^2}\right)\mathrm{Tr}\,\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + \mathrm{H.c.} &= \mathrm{Re}\,\mathrm{Tr}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} + \frac{g^2\Theta}{8\pi^2}\,\mathrm{Im}\,\mathrm{Tr}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} \\ &= \mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + \frac{1}{2}D^aD^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{g^2\Theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}; \end{aligned} \tag{4.15}$$

$$\mathrm{D}_\mu\phi_i = \partial_\mu\phi_i - \mathrm{i}gA^a_\mu t^a_{ij}\phi_j, \qquad \mathrm{D}_\mu\psi_i = \partial_\mu\psi_i - \mathrm{i}gA^a_\mu t^a_{ij}\psi_j, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gA^b_\mu A^c_\nu f^{abc},$$
$$\mathrm{D}^\mu\phi_i^* = \partial^\mu\phi_i^* + \mathrm{i}gA^{a\mu}\phi_j^* t^a_{ji}, \qquad \mathrm{D}_\mu\lambda^a_\alpha = \partial_\mu\lambda^a_\alpha + gf^{abc}A^b_\mu\lambda^c_\alpha.$$

$$\{\phi,\psi,F\} \xrightarrow{\text{gauge}} \mathrm{e}^{\mathrm{i}g\theta^a t^a}\{\phi,\psi,F\},$$
$$A^a_\mu \xrightarrow{\text{gauge}} A^a_\mu + \partial_\mu\theta^a + gf^{abc}A^b_\mu\theta^c + \mathcal{O}(\theta^2), \qquad \lambda^a \xrightarrow{\text{gauge}} \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2),$$
$$D^a \xrightarrow{\text{gauge}} D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), \qquad \bar\lambda^a \xrightarrow{\text{gauge}} \bar\lambda^a + gf^{abc}\bar\lambda^b\theta^c + \mathcal{O}(\theta^2).$$ [*4]

Auxiliary fields and Scalar potential In all of the above three theories,

$$\mathcal{L} \supset F_i^*F_i + F_i\frac{\partial W}{\partial\Phi_i}\Big|_{\text{scalar}} + F_i^*\frac{\partial W^*}{\partial\Phi_i^*}\Big|_{\text{scalar}} + \frac{1}{2}D^aD^a + gD^a(\phi^* t^a\phi); \tag{4.16}$$

$$\langle F_i^*\rangle = -\frac{\partial W}{\partial\Phi_i}\Big|_{\text{scalar}}, \qquad \langle D^a\rangle = -g\phi^* t^a\phi; \tag{4.17}$$

$$\mathcal{L} \supset -V_{\text{SUSY}} = -\left[\langle F_i^*\rangle\langle F_i\rangle + \frac{g^2}{2}(\phi^* t^a\phi)(\phi^* t^a\phi)\right]. \tag{4.18}$$

[*4] ♣TODO: give in non-infinitesimal form♣

4.1. Lorentz symmetry as SU(2)×SU(2)

4.2. Supersymmetry algebra

We define the generators as

$$P_\mu := \mathrm{i}\partial_\mu, \quad \{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \quad \{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{Q}}_{\dot\beta}\} = 0, \tag{4.19}$$

which is realized by

$$\mathcal{Q}_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{Q}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{Q}^\alpha = -\frac{\partial}{\partial\theta_\alpha} - \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{Q}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu,$$

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{D}^\alpha = -\frac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{D}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu;$$

$\mathcal{D}_\alpha$ etc. works as covariant derivatives because of the commutation relations

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = +2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \{\mathcal{Q}_\alpha, \mathcal{D}_\beta\} = \{\mathcal{Q}_\alpha, \bar{\mathcal{D}}_{\dot\beta}\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \mathcal{D}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 0.$$

Derivative formulae

$$\begin{array}{llll}
\epsilon^{\alpha\beta}\dfrac{\partial}{\partial\theta^\beta} = -\dfrac{\partial}{\partial\theta_\alpha} & \dfrac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha & \dfrac{\partial}{\partial\theta^\alpha}\dfrac{\partial}{\partial\theta_\beta}\theta\theta = -2\delta^\beta_\alpha & \dfrac{\partial}{\partial\bar\theta^{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = 2\delta^{\dot\beta}_{\dot\alpha} \\
\epsilon_{\alpha\beta}\dfrac{\partial}{\partial\theta_\beta} = -\dfrac{\partial}{\partial\theta^\alpha} & \dfrac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha & \dfrac{\partial}{\partial\theta_\alpha}\dfrac{\partial}{\partial\theta_\beta}\theta\theta = 2\epsilon^{\alpha\beta} & \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = -2\epsilon^{\dot\alpha\dot\beta} \\
\epsilon^{\dot\alpha\dot\beta}\dfrac{\partial}{\partial\bar\theta^{\dot\beta}} = -\dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} & \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} & \dfrac{\partial}{\partial\theta_\alpha}\dfrac{\partial}{\partial\theta^\beta}\theta\theta = 2\delta^\alpha_\beta & \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = -2\delta^{\dot\alpha}_{\dot\beta} \\
\epsilon_{\dot\alpha\dot\beta}\dfrac{\partial}{\partial\bar\theta_{\dot\beta}} = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} & \dfrac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} & \dfrac{\partial}{\partial\theta^\alpha}\dfrac{\partial}{\partial\theta^\beta}\theta\theta = -2\epsilon_{\alpha\beta} & \dfrac{\partial}{\partial\bar\theta^{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = 2\epsilon_{\dot\alpha\dot\beta}
\end{array}$$

In addition, we define

$$(y, \theta', \bar\theta') := (x - \mathrm{i}\theta\sigma^\mu\bar\theta, \theta, \bar\theta): \tag{4.20}$$

$$\bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta'^{\dot\alpha}}; \qquad \begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix} = \begin{pmatrix} \delta^\nu_\mu & 0 & 0 \\ -\mathrm{i}(\sigma^\nu\bar\theta)_\alpha & \delta^\beta_\alpha & 0 \\ \mathrm{i}(\theta\sigma^\nu)_{\dot\alpha} & 0 & \delta^{\dot\beta}_{\dot\alpha} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\beta} \\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}} \end{pmatrix}, \quad \begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\beta} \\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}} \end{pmatrix} = \begin{pmatrix} \delta^\mu_\nu & 0 & 0 \\ \mathrm{i}(\sigma^\mu\bar\theta)_\beta & \delta^\alpha_\beta & 0 \\ -\mathrm{i}(\theta\sigma^\mu)_{\dot\beta} & 0 & \delta^{\dot\alpha}_{\dot\beta} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix}, \tag{4.21}$$

and a function $f : \mathbb{C}^4 \to \mathbb{C}$ (independent of θ' and $\bar\theta'$) is expanded as

$$f(y) = f(x - \mathrm{i}\theta\sigma\bar\theta) = f(x) - \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f(x) - \frac{1}{4}\theta^4\partial^2 f(x). \tag{4.22}$$

Note that we differentiate $[f(y)]^*$ and $f^*(y)$:

$$[f(y)]^* = f(x) + \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f^*(x) - \frac{1}{4}\theta^4\partial^2 f^*(x) = f^*(y + \mathrm{i}\theta\sigma\bar\theta) = f^*(y^*). \tag{4.23}$$

4.3. Superfields

SUSY-invariant Lagrangian SUSY transformation is induced by $\xi\mathcal{Q} + \bar\xi\bar{\mathcal{Q}} = \xi^\alpha\partial_\alpha + \bar\xi_{\dot\alpha}\partial^{\dot\alpha} + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu$. Therefore, for an object Ψ in the superspace,

$$\bigl[\Psi\bigr]_{\theta^4} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \xi^\alpha\partial_\alpha\Psi + \bar\xi_{\dot\alpha}\partial^{\dot\alpha}\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\Bigr]_{\theta^4} = \bigl[\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\bigr]_{\theta^4}, \tag{4.24}$$

which means $\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant up to total derivative, i.e., $\int \mathrm{d}^4x\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant action. Also,

$$\bigl[\Psi\bigr]_{\theta^2} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \bar\xi_{\dot\alpha}\Bigl(\partial^{\dot\alpha} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Bigr)\Psi\Bigr]_{\theta^2} = \Bigl[\Psi + \bar\xi_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\Psi + 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Psi\Bigr]_{\theta^2} \tag{4.25}$$

will be SUSY-invariant if $\bar{\mathcal{D}}_{\dot\alpha}\Psi = 0$, i.e., Ψ is a chiral superfield. Therefore, SUSY-invariant Lagrangian is given by

$$\mathcal{L} = \Bigl[(\text{any real superfield})\Bigr]_{\theta^4} + \Bigl[(\text{any chiral superfield})\Bigr]_{\theta^2} + \Bigl[(\text{any chiral superfield})^*\Bigr]_{\bar\theta^2}. \tag{4.26}$$

Chiral superfield A chiral superfield is a superfield that satisfies $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$, i.e., we find

$$\Phi = \phi(y) + \sqrt{2}\theta'\psi(y) + \theta'^2F(y) \tag{4.27}$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{1}{4}\partial^2\phi(x)\theta^4 \tag{4.28}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{1}{4}\partial^2\phi^*(x)\theta^4; \tag{4.29}$$

their product is expanded as

$$\begin{aligned}\Phi_i^*\Phi_j &= \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - \mathrm{i}\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2 \\ &\quad + \left[\sqrt{2}\bar\psi_i\bar\theta F_j - \frac{\mathrm{i}\left(\partial_\mu\phi_i^*\cdot\psi_j\sigma^\mu\bar\theta - \phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)}{\sqrt{2}}\right]\theta^2 + \left[\sqrt{2}F_i^*\theta\psi_j + \frac{\mathrm{i}\left(\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j - \theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)}{\sqrt{2}}\right]\bar\theta^2 \\ &\quad + \frac{1}{4}\left(4F_i^*F_j - \phi_i^*\partial^2\phi_j - (\partial^2\phi_i^*)\phi_j + 2(\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + 2\mathrm{i}(\psi_j\sigma^\mu\partial_\mu\bar\psi_i) - 2\mathrm{i}(\partial_\mu\psi_j\sigma^\mu\bar\psi_i)\right)\theta^4\end{aligned} \tag{4.30}$$

$$\begin{aligned}&\equiv \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - 2\mathrm{i}\left(\phi_i^*\partial_\mu\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2 \\ &\quad + \sqrt{2}\left(\bar\psi_i\bar\theta F_j + \mathrm{i}\phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)\theta^2 + \sqrt{2}\left(F_i^*\theta\psi_j - \mathrm{i}\theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)\bar\theta^2 \\ &\quad + \left(F_i^*F_j + (\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right)\theta^4\end{aligned} \tag{4.31}$$

$$\Phi_i\Phi_j\Big|_{\theta^2} = -\psi_i\psi_j + F_i\phi_j + \phi_iF_j \tag{4.32}$$

$$\Phi_i\Phi_j\Phi_k\Big|_{\theta^2} = -(\psi_i\psi_j)\phi_k - (\psi_k\psi_i)\phi_j - (\psi_j\psi_k)\phi_i + \phi_i\phi_jF_k + \phi_k\phi_iF_j + \phi_j\phi_kF_i \tag{4.33}$$

$$\mathrm{e}^{k\Phi} = \mathrm{e}^{k\phi}\left[1 + \sqrt{2}k\theta\psi + \left(kF - \frac{k^2}{2}\psi\psi\right)\theta^2 - \mathrm{i}k\partial_\mu\phi(\theta\sigma^\mu\bar\theta) + \frac{\mathrm{i}k\left(\partial_\mu\psi + k\psi\partial_\mu\phi\right)\sigma^\mu\bar\theta\theta^2}{\sqrt{2}} - \frac{k}{4}\left(\partial^2\phi + k\partial_\mu\phi\partial^\mu\phi\right)\theta^4\right]; \tag{4.34}$$

note that $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$, and $\mathrm{e}^{k\Phi}$ are all chiral superfields.

Vector superfield A vector superfield is a superfield V that satisfies $V = V^*$. It is given by real fields $\{C, M, N, D, A_\mu\}$ and Grassmann fields $\{\chi, \lambda\}$ as[*5]

$$\begin{aligned}V(x,\theta,\bar\theta) &= C(x) + \mathrm{i}\theta\chi(x) - \mathrm{i}\bar\theta\bar\chi(x) + \frac{1}{2}\left(M(x) + \mathrm{i}N(x)\right)\theta^2 + \frac{1}{2}\left(M(x) - \mathrm{i}N(x)\right)\bar\theta^2 + (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) \\ &\quad \left(\lambda(x) + \frac{1}{2}\partial_\mu\bar\chi(x)\bar\sigma^\mu\right)\theta\bar\theta^2 + \theta^2\bar\theta\left(\bar\lambda(x) + \frac{1}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right) + \frac{1}{2}\left(D(x) - \frac{1}{2}\partial^2C(x)\right)\theta^4.\end{aligned} \tag{4.35}$$

With this convention,

$$V \to V - \mathrm{i}\Phi + \mathrm{i}\Phi^* \iff \begin{cases} C \to C - \mathrm{i}\phi + \mathrm{i}\phi^*, & \chi \to \chi - \sqrt{2}\psi, \qquad \lambda \to \lambda, \\ M + \mathrm{i}N \to M + \mathrm{i}N - 2\mathrm{i}F, & A_\mu \to A_\mu + \partial_\mu(\phi + \phi^*), \qquad D \to D. \end{cases} \tag{4.36}$$

The exponential of a vector superfield is also a vector superfield:

$$\begin{aligned}\mathrm{e}^{kV} = \mathrm{e}^{kC}\Bigg\{&1 + \mathrm{i}k(\theta\chi - \bar\theta\bar\chi) + \left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\theta^2 + \left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\bar\theta^2 + (k^2\theta\chi\bar\theta\bar\chi - k\theta\sigma^\mu\bar\theta A_\mu) \\ &+ \left[k\bar\theta\bar\lambda - \mathrm{i}k\bar\theta\bar\chi\left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right) + \frac{1}{2}k\bar\theta\bar\sigma^\mu\left(\partial_\mu\chi - \mathrm{i}k\chi A_\mu\right)\right]\theta^2 \\ &+ \left[k\theta\lambda + \mathrm{i}k\theta\chi\left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right) - \frac{1}{2}k\theta\sigma^\mu\left(\partial_\mu\bar\chi + \mathrm{i}k\bar\chi A_\mu\right)\right]\bar\theta^2 \\ &+ \left[\frac{k}{2}\left(D - \frac{1}{2}\partial^2C\right) - \frac{1}{2}\mathrm{i}k^2(\lambda\chi - \bar\lambda\bar\chi) + \left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\right. \\ &\qquad \left. + \frac{k^3}{4}\bar\chi\bar\sigma^\mu\chi A_\mu + \frac{k^2}{4}\left(\mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - \mathrm{i}\partial_\mu\bar\chi\bar\sigma^\mu\chi + A^\mu A_\mu\right)\right]\theta^4\Bigg\}.\end{aligned} \tag{4.37}$$

Supergauge symmetry The gauge transformation $\phi(x) \to \mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\phi(x)$ is not closed in the chiral superfield; i.e., $\mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\Phi(x)$ is not a chiral superfield if the parameter $\theta(x)$ has x^μ-dependence. Hence, in supersymmetric theories, it is extended to *supergauge symmetry* parameterized by a chiral superfield $\Omega(x)$, which is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Omega^a(x)t^a}\Phi, \qquad \Phi^* \to \Phi^*\,\mathrm{e}^{-2\mathrm{i}g\Omega^{*a}(x)t^a} \tag{4.38}$$

for a chiral superfield Φ and an anti-chiral superfield Φ^*. The supergauge-invariant Lagrangian should be

$$\mathcal{L} \sim \Phi^*\cdot(\text{real superfield})\cdot\Phi; \tag{4.39}$$

we parameterize the "real superfield" as $\mathrm{e}^{2gV^a(x)t^a}$:

$$\mathcal{L} = \left[\Phi^*\,\mathrm{e}^{2gV^a(x)t^a}\Phi\right]_{\theta^4}; \qquad \mathrm{e}^{2gV^a(x)t^a} \to \mathrm{e}^{2\mathrm{i}g\Omega^{*a}(x)t^a}\,\mathrm{e}^{2gV^a(x)t^a}\,\mathrm{e}^{-2\mathrm{i}g\Omega^a(x)t^a}. \tag{4.40}$$

[*5] Different coordination of "i"s are found in literature. Take care, especially, $\lambda(\text{ours}) = \mathrm{i}\lambda(\text{Wess-Bagger}) = \mathrm{i}\lambda(\text{SLHA})$.

In Abelian case, t^a is replaced by the charge Q of Φ and

$$\mathcal{L} = \left[\Phi^* \,\mathrm{e}^{2gQV(x)}\Phi\right]_{\theta^4}; \qquad \Phi \to \mathrm{e}^{2\mathrm{i}gQ\Omega(x)}\Phi, \quad \Phi^* \to \Phi^* \,\mathrm{e}^{-2\mathrm{i}gQ\Omega^*(x)}, \tag{4.41}$$

$$\mathrm{e}^{2gQV(x)} \to \mathrm{e}^{2\mathrm{i}gQ\Omega^*(x)}\,\mathrm{e}^{2gQV(x)}\,\mathrm{e}^{-2\mathrm{i}gQ\Omega(x)} = \mathrm{e}^{2gQ\left(V-\mathrm{i}\Omega+\mathrm{i}\Omega^*\right)}. \tag{4.42}$$

The usual gauge transformation corresponds to the real part of the lowest component of Ω, i.e., $\theta \equiv 2\,\mathrm{Re}\,\phi = \phi + \phi^*$, and we use the other components to fix the supergauge so that C, M, N and χ are eliminated:

$$\text{supergauge fixing:}\quad V(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{1}{2}D(x) \qquad \text{(Wess-Zumino gauge);} \tag{4.43}$$

$$\mathrm{e}^{2gQV} \longrightarrow 1 + gQ\left(-2\theta\sigma^\mu\bar\theta A_\mu + 2\theta^2\bar\theta\bar\lambda + 2\bar\theta^2\theta\lambda + D\theta^4\right) + g^2Q^2A^\mu A_\mu\theta^4. \tag{4.44}$$

The gauge transformation is the remnant freedom: $\Theta = \phi(y) = \phi - \mathrm{i}\partial_\mu\phi(\theta\sigma^\mu\bar\theta) - \partial^2\phi\theta^4/4$ with ϕ being real;

$$\Phi_i \to \mathrm{e}^{2\mathrm{i}gQ\Theta}\Phi_i, \qquad \mathrm{e}^{2gQV} \to \mathrm{e}^{2gQ(V-\mathrm{i}\Theta+\mathrm{i}\Theta^*)}. \tag{4.45}$$

Rules for each component is obvious in $(y,\theta,\bar\theta)$-basis and given by

$$\{\phi,\psi,F\} \to \mathrm{e}^{\mathrm{i}gQ\theta}\{\phi,\psi,F\}, \qquad A_\mu \to A_\mu + \partial_\mu\theta, \qquad \lambda \to \lambda, \qquad D \to D. \tag{4.46}$$

For non-Abelian gauges, the supergauge transformation for the real field is evaluated as

$$\begin{aligned}
\mathrm{e}^{2gV} &\to \mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{2gV}\,\mathrm{e}^{-2\mathrm{i}g\Omega} && (4.47)\\
&= \left(\mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{2gV}\,\mathrm{e}^{-2\mathrm{i}g\Omega^*}\right)\left(\mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{-2\mathrm{i}g\Omega}\right) && (4.48)\\
&= \exp\left(\mathrm{e}^{[2\mathrm{i}g\Omega^*,}2gV\right)\mathrm{e}^{2\mathrm{i}g(\Omega^*-\Omega)} + \mathcal{O}(\Omega^2) && (4.49)\\
&= \exp\left(2gV + [2\mathrm{i}g\Omega^*, 2gV]\right)\mathrm{e}^{2\mathrm{i}g(\Omega^*-\Omega)} + \mathcal{O}(\Omega^2); && (4.50)\\
&= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[2gV,})2\mathrm{i}g(\Omega^*-\Omega)] + \mathcal{O}(\Omega^2)\right] && (4.51)\\
&= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \sum_{n=0}^\infty \frac{B_n\left([2gV,)^n\right.}{n!}2\mathrm{i}g(\Omega^*-\Omega)]\right] + \mathcal{O}(\Omega^2) && (4.52)\\
&= \exp\left[2g\left(V + \mathrm{i}(\Omega^*-\Omega) - [V,\mathrm{i}g(\Omega^*+\Omega)] + \sum_{n=2}^\infty \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Omega^*-\Omega)]\right) + \mathcal{O}(\Omega^2)\right]. && (4.53)
\end{aligned}$$

Here, again we can use the "non-gauge" component of Ω to eliminate the C-term etc., i.e., we fix $\mathrm{i}(\Omega^*-\Omega)$, the second term of the expansion, to remove those terms:

$$V - [V,\mathrm{i}g(\Omega^*+\Omega)] + \left(\mathrm{i} + \sum_{n=2}^\infty \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}\right)(\Omega^*-\Omega)] + \mathcal{O}(\Omega^2) = (\bar\theta\bar\sigma^\mu\theta)A_\mu + \bar\theta^2\theta\lambda + \theta^2\bar\theta\bar\lambda + \frac{1}{2}D; \tag{4.54}$$

this defines the Wess-Zumino gauge:

$$\text{supergauge fixing:}\quad V^a(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A^a_\mu(x) + \bar\theta^2\theta\lambda^a(x) + \theta^2\bar\theta\bar\lambda^a(x) + \frac{1}{2}D^a(x), \tag{4.55}$$

$$\mathrm{e}^{2gV^at^a} \longrightarrow 1 + g\left(-2\theta\sigma^\mu\bar\theta A^a_\mu + 2\theta^2\bar\theta\bar\lambda^a + 2\bar\theta^2\theta\lambda^a + D^a\theta^4\right)t^a + g^2A^{a\mu}A^b_\mu\theta^4t^at^b. \tag{4.56}$$

The gauge transformation is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Theta^at^a}\Phi, \quad \mathrm{e}^{2gV^at^a} \to \mathrm{e}^{2\mathrm{i}g\Theta^bt^b}\,\mathrm{e}^{2gV^at^a}\,\mathrm{e}^{-2\mathrm{i}g\Theta^ct^c}. \tag{4.57}$$

For components in chiral superfields,

$$\{\phi,\psi,F\} \to \mathrm{e}^{\mathrm{i}g\theta^at^a}\{\phi,\psi,F\}, \tag{4.58}$$

while for vector superfield we can express as infinitesimal transformation:

$$\begin{aligned}
V \to V' &\simeq V + \mathrm{i}(\Theta^*-\Theta) - [V,\mathrm{i}g(\Theta^*+\Theta)] + \sum_{n=2}^\infty \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Theta^*-\Theta)] && (4.59)\\
&= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi - \left[V,\mathrm{i}g\left(2\phi - \frac{\theta^4}{2}\partial^2\phi\right)\right] + 2\sum_{n=2}^\infty \frac{B_n\left([2gV,)^n\right.}{n!}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi] && (4.60)\\
&= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi + 2gf^{abc}V^b\phi^ct^a \qquad \text{(Wess-Zumino gauge)} && (4.61)
\end{aligned}$$

$$\begin{aligned}
\therefore A^a_\mu &\to A^a_\mu + \partial_\mu\theta^a + gf^{abc}A^b_\mu\theta^c + \mathcal{O}(\theta^2), & \lambda^a &\to \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2),\\
D^a &\to D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), & \bar\lambda^a &\to \bar\lambda^a + gf^{abc}\bar\lambda^b\theta^c + \mathcal{O}(\theta^2).
\end{aligned} \tag{4.62}$$

Gauge-field strength The real superfield e^V is gauge-invariant in Abelian case and a candidate in Lagrangian term, but this is not case in non-Abelian case. We thus define a chiral superfield from e^V:

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right); \tag{4.63}$$

$$\mathcal{W}_\alpha \xrightarrow{\text{gauge}} \mathrm{e}^{2\mathrm{i}g\Omega}\mathcal{W}_\alpha\,\mathrm{e}^{-2\mathrm{i}g\Omega} \quad \left(\mathcal{W}^a_\alpha \xrightarrow{\text{gauge}} [\mathrm{e}^{+2g\tilde{f}^c\Omega^c}]^{ab}W^b_\alpha \quad \text{with} \quad [\tilde{f}^c]_{ab} = f^{abc}\right);^{*6} \tag{4.64}$$

it is not supergauge- or Lorentz-invariatn, but $\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha) = \mathrm{Tr}(\epsilon^{\alpha\beta}\mathcal{W}_\beta\mathcal{W}_\alpha)$ is supergauge- and Lorentz-invariant, and its θ^2-term is SUSY-invariant, which becomes a candidate in SUSY Lagrangian with its Hermitian conjugate.

In Wess-Zumino gauge, it is given by

$$\mathcal{W}_\alpha = \left\{\lambda^a_\alpha(y) + \theta_\alpha D^a(y) + \frac{[\mathrm{i}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)\theta]_\alpha}{4}F^a_{\mu\nu}(y) + \theta^2\left[\mathrm{i}\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a(y^*)\right]_\alpha\right\}t^a \tag{4.65}$$

$$\begin{aligned} &= \Bigg[\lambda^a_\alpha + \theta_\alpha D^a + \frac{\mathrm{i}}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F^a_{\mu\nu} + \mathrm{i}\theta^2(\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a)_\alpha + \mathrm{i}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\lambda^a_\alpha - \frac{\theta^4}{4}\partial^2\lambda^a_\alpha \\ &\qquad + \frac{\mathrm{i}\theta^2(\sigma^\mu\bar\theta)_\alpha}{2}\left(\partial_\mu D^a + \mathrm{i}\partial^\nu F^a_{\mu\nu} - gf^{abc}\epsilon_{\mu\nu\rho\sigma}A^{\nu b}\partial^\rho A^{\sigma c}\right)\Bigg]T^a, \end{aligned} \tag{4.66}$$

where, as usual,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gA^b_\mu A^c_\nu f^{abc}, \qquad \mathrm{D}_\mu\lambda^a_\alpha = \partial_\mu\lambda^a_\alpha + gf^{abc}A^b_\mu\lambda^c_\alpha. \tag{4.67}$$

Also,

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^2} = \left[\mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^b + \mathrm{i}\lambda^b\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + D^aD^b - \frac{1}{4}\left(\mathrm{i}\epsilon^{\sigma\mu\nu\rho} + 2\eta^{\mu\rho}\eta^{\nu\sigma}\right)F^a_{\mu\nu}F^b_{\rho\sigma}\right]\mathrm{Tr}(t^at^b) \tag{4.68}$$

$$= \mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + \frac{1}{2}D^aD^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}, \tag{4.69}$$

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^4} = \frac{\theta^4}{4}\left(2(\partial^\mu\lambda^a)(\partial_\mu\lambda^b) - \lambda^a\partial^2\lambda^b - (\partial^2\lambda^a)\lambda^b\right)\mathrm{Tr}(t^at^b) = \frac{\theta^4}{4}\left((\partial^\mu\lambda^a)(\partial_\mu\lambda^a) - \lambda^a\partial^2\lambda^a\right). \tag{4.70}$$

For Abelian theory,

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right) = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}_\alpha(2gV), \tag{4.71}$$

$$\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} = 2\left(\mathrm{i}\lambda\sigma^\mu\,\mathrm{D}_\mu\bar\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right). \tag{4.72}$$

4.4. Lagrangian blocks

Lagrangian construction The supergauge transformation is summarized as

$$\Phi_i \to [U_\Phi]_{ij}\Phi_j, \qquad \tilde\Phi_j \to \tilde\Phi_i[U_\Phi^{-1}]_{ij}, \qquad \mathcal{W}_\alpha \to U_{\mathcal{W}}\mathcal{W}_\alpha U_{\mathcal{W}}^{-1}, \tag{4.73}$$

where

$$\tilde\Phi^*_j := \Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}]_{ij}, \qquad U_\Phi := \exp(2\mathrm{i}g\Omega^at^a_\Phi), \qquad U_{\mathcal{W}} := \exp(2\mathrm{i}g\Omega^at^a_{\mathcal{W}}), \tag{4.74}$$

t^a_Φ is the representation matrix or U(1) charge for the field Φ, and $t^a_{\mathcal{W}}$ is the representation matrix that is used to define $\mathcal{W}_\alpha$. To construct a Lagrangian, we should composite these ingredients in real and invariant under SUSY, supergauge, and Lorentz transformation. A sufficient condition for SUSY invariance is given by (4.26), so

$$\mathcal{L} = \left[K(\Phi_i,\tilde\Phi^*_j)\right]_{\theta^4} + \left\{\left[f_{ab}(\Phi_i)\mathcal{W}^a\mathcal{W}^b\right]_{\theta^2} + \text{H.c.}\right\} + \left\{\left[W(\Phi_i)\right]_{\theta^2} + \text{H.c.}\right\} + D \tag{4.75}$$

is one possible construction. The Kähler function K should be real and supergauge invariant, the gauge kinetic function f should be holomorphic and supergauge invariant with $\mathcal{W}^a\mathcal{W}^b$, and the superpotential W is holomorphic and supergauge invariant. The last term D (Fayet-Illiopoulos term) comes from V of an U(1) gauge boson; note that its supergauge invariance is due to the intentional definition of V.

One can construct more general Lagrangian; for example, one can introduce a vector superfield that is not associated to a gauge symmetry, but then the supergauge fixing is not available and one has to include C or M fields.

Renormalizable Lagrangian Since $[\Phi]_{\theta^4}$ is a total derivative, renormalizable Lagrangian is limited to

$$\mathcal{L} = \left[\Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}]_{ij}\Phi_j\right]_{\theta^4} + \left\{[\mathcal{W}^a\mathcal{W}^a]_{\theta^2} + [W(\Phi_i)]_{\theta^2} + \text{H.c.}\right\} + D \tag{4.76}$$

up to numeric coefficients. With multiple gauge groups, the Kähler part is extended as $\Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}\,\mathrm{e}^{2gV't'^a_\Phi}\cdots]_{ij}\Phi_j$, where the inner part is obviously commutable.

*6 ♣TODO: This equivalence should be checked/explained in gauge-theory section; especially, the sign is not verified and might be opposite.♣

5. Minimal Supersymmetric Standard Model

Gauge symmetry: $\mathrm{SU}(3)_{\text{color}} \times \mathrm{SU}(2)_{\text{weak}} \times \mathrm{U}(1)_Y$

Particle content:

(a) Chiral superfields

	SU(3)	SU(2)	U(1)	B	L	scalar/spinor
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$		$\tilde{q}_\mathrm{L}\,, q_\mathrm{L}$ $[\to (u_\mathrm{L}, d_\mathrm{L})]$
L_i		$\mathbf{2}$	$-1/2$		1	$\tilde{l}_\mathrm{L}\,, l_\mathrm{L}$ $[\to (\nu_\mathrm{L}, l_\mathrm{L})]$
U_i^c	$\overline{\mathbf{3}}$		$-2/3$	$-1/3$		$\tilde{u}_\mathrm{R}^\mathrm{c}, u_\mathrm{R}^\mathrm{c}$
D_i^c	$\overline{\mathbf{3}}$		$1/3$	$-1/3$		$\tilde{d}_\mathrm{R}^\mathrm{c}, d_\mathrm{R}^\mathrm{c}$
E_i^c			1		-1	$\tilde{e}_\mathrm{R}^\mathrm{c}\,, e_\mathrm{R}^\mathrm{c}$
H_u		$\mathbf{2}$	$1/2$			$h_\mathrm{u}\,, \tilde{h}_\mathrm{u}$ $[\to (h_\mathrm{u}^+, h_\mathrm{u}^0)]$
H_d		$\mathbf{2}$	$-1/2$			$h_\mathrm{d}\,, \tilde{h}_\mathrm{d}$ $[\to (h_\mathrm{d}^0, h_\mathrm{d}^-)]$

(b) Vector superfields

	SU(3)	SU(2)	U(1)	ino/boson
g	adj.			$\tilde{g}, g_\mu$
W		adj.		$\tilde{w}, W_\mu$
B				$\tilde{b}, B_\mu$

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and SU(2) decomposition).

"c"-notation For scalars, $\tilde{\phi}_\mathrm{R}^\mathrm{c} := \phi_\mathrm{R}^* = \mathsf{C}\phi_\mathrm{R}\mathsf{C}$ (because the intrinsic phase for C is $+1$ for quarks and leptons.)

For matter spinors, $\psi_\mathrm{R}^\mathrm{c} := \bar{\psi}_\mathrm{R}$ (and $\psi_\mathrm{R} = \bar{\psi}_\mathrm{R}^\mathrm{c}$); Dirac spinors are thus

$$\psi_\mathrm{L} = \begin{pmatrix}\psi_\mathrm{L}\\0\end{pmatrix}, \quad \overline{\psi_\mathrm{L}} = \begin{pmatrix}0 & \bar{\psi}_\mathrm{L}\end{pmatrix}, \quad \psi_\mathrm{R}^\mathrm{c} := \begin{pmatrix}\psi_\mathrm{R}^\mathrm{c}\\0\end{pmatrix} = C\begin{pmatrix}0\\\psi_\mathrm{R}\end{pmatrix} = C\psi_\mathrm{R}, \quad \overline{\psi_\mathrm{R}^\mathrm{c}} = \begin{pmatrix}0 & \psi_\mathrm{R}\end{pmatrix} = \begin{pmatrix}\bar{\psi}_\mathrm{R} & 0\end{pmatrix}C = \overline{\psi_\mathrm{R}}C.$$

Superpotential and $\cancel{\text{SUSY}}$-terms

$$W_\mathrm{RPC} = \mu H_\mathrm{u} H_\mathrm{d} - y_{\mathrm{u}ij} U_i^\mathrm{c} H_\mathrm{u} Q_j + y_{\mathrm{d}ij} D_i^\mathrm{c} H_\mathrm{d} Q_j + y_{\mathrm{e}ij} E_i^\mathrm{c} H_\mathrm{d} L_j, \tag{5.1}$$

$$W_\mathrm{RPV} = -\kappa_i L_i H_\mathrm{u} + \frac{1}{2}\lambda_{ijk} L_i L_j E_k^\mathrm{c} + \lambda'_{ijk} L_i Q_j D_k^\mathrm{c} + \frac{1}{2}\lambda''_{ijk} U_i^\mathrm{c} D_j^\mathrm{c} D_k^\mathrm{c}, \tag{5.2}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) - V_{\cancel{\mathrm{SUSY}}}; \tag{5.3}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^\mathrm{RPC} = &\left(\tilde{q}_\mathrm{L}^* m_Q^2 \tilde{q}_\mathrm{L} + \tilde{l}_\mathrm{L}^* m_L^2 \tilde{l}_\mathrm{L} + \tilde{u}_\mathrm{R}^* m_{U^\mathrm{c}}^2 \tilde{u}_\mathrm{R} + \tilde{d}_\mathrm{R}^* m_{D^\mathrm{c}}^2 \tilde{d}_\mathrm{R} + \tilde{e}_\mathrm{R}^* m_{E^\mathrm{c}}^2 \tilde{e}_\mathrm{R} + m_{H_\mathrm{u}}^2 |h_\mathrm{u}|^2 + m_{H_\mathrm{d}}^2 |h_\mathrm{d}|^2\right) \\ &+ \left(-\tilde{u}_\mathrm{R}^* h_\mathrm{u} a_\mathrm{u} \tilde{q}_\mathrm{L} + \tilde{d}_\mathrm{R}^* h_\mathrm{d} a_\mathrm{d} \tilde{q}_\mathrm{L} + \tilde{e}_\mathrm{R}^* h_\mathrm{d} a_\mathrm{e} \tilde{l}_\mathrm{L} + b H_\mathrm{u} H_\mathrm{d} + \text{H.c.}\right) \\ &+ \left(+\tilde{u}_\mathrm{R}^* h_\mathrm{d}^* c_\mathrm{u} \tilde{q}_\mathrm{L} + \tilde{d}_\mathrm{R}^* h_\mathrm{u}^* c_\mathrm{d} \tilde{q}_\mathrm{L} + \tilde{e}_\mathrm{R}^* h_\mathrm{u}^* c_\mathrm{e} \tilde{l}_\mathrm{L} + \text{H.c.}\right), \end{aligned} \tag{5.4}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^\mathrm{RPV} = &\left(-b_i \tilde{l}_{\mathrm{L}i} H_\mathrm{u} + \frac{1}{2} T_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{l}_{\mathrm{L}j} \tilde{e}_{\mathrm{R}k}^* + T'_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{q}_{\mathrm{L}j} \tilde{d}_{\mathrm{R}k}^* + \frac{1}{2} T''_{ijk} \tilde{u}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j}^* \tilde{d}_{\mathrm{R}k}^* + \tilde{l}_{\mathrm{L}i}^* M_{Li}^2 H_\mathrm{d} + \text{H.c.}\right) \\ &+ \left(C_{ijk}^1 \tilde{l}_{\mathrm{L}i}^* \tilde{q}_{\mathrm{L}j} \tilde{u}_{\mathrm{R}k}^* + C_i^2 h_\mathrm{u}^* h_\mathrm{d} \tilde{e}_{\mathrm{R}i}^* + C_{ijk}^3 \tilde{d}_{\mathrm{R}i} \tilde{u}_{\mathrm{R}j}^* \tilde{e}_{\mathrm{R}k}^* + \frac{1}{2} C_{ijk}^4 \tilde{d}_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} \tilde{q}_{\mathrm{L}k} + \text{H.c.}\right), \end{aligned} \tag{5.5}$$

($\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C_{ijk}^4 = C_{ikj}^4$.)

5.1. Notation

Our notation in this section (and the previous section) follows DHM [?, PhysRept] and Martin [?, v7] (but note that Martin uses $(-,+,+,+)$-metric) for RPC part and SLHA2 convention for RPV part. In particular, the sign of gauge bosons are fixed by $\mathrm{D}_\mu\phi = \partial_\mu\phi - \mathrm{i}gA^a_\mu t^a_{ij}\phi_j$, and the phase of gauginos are by $\mathcal{L} \ni \sqrt{2}g(\phi^* t^a\psi\lambda^a)$. Phases of ϕ and ψ in chiral superfields are not yet specified; they are later used to remove $F\tilde{F}$ terms and diagonalize Yukawa matrices.

5.2. Lagrangian construction

The most generic form of the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{super}} + \mathcal{L}_{\text{FI}} + \mathcal{L}_{\cancel{\mathrm{SUSY}}}; \tag{5.6}$$

$$\mathcal{L}_{\text{matter}} = \Phi_Q^* \exp\left(2g_Y(\tfrac{1}{6})V_B + 2g_2 V_W^a T^a + 2g_3 V_g^a \tau^a\right)\Phi_Q\Big|_{\theta^4} + \cdots; \tag{5.7}$$

$$\mathcal{L}_{\text{gauge}} = \left[\frac{1}{4}\left(1 - \frac{\mathrm{i}g_Y^2\Theta_B}{8\pi^2}\right)\mathcal{W}_B\mathcal{W}_B + \frac{1}{4}\left(1 - \frac{\mathrm{i}g_2^2\Theta_W}{8\pi^2}\right)\mathcal{W}_W^a\mathcal{W}_W^a + \frac{1}{4}\left(1 - \frac{\mathrm{i}g_3^2\Theta_g}{8\pi^2}\right)\mathcal{W}_g^a\mathcal{W}_g^a\right]_{\theta^2} + \text{H.c.}; \tag{5.8}$$

$$\mathcal{L}_{\text{super}} = W(\Phi)\Big|_{\theta^2} + \text{H.c.}, \tag{5.9}$$

$$W(\Phi) = W_{\text{RPC}} + W_{\text{RPV}}, \tag{5.10}$$

$$W_{\text{RPC}} = \mu H_{\mathrm{u}}H_{\mathrm{d}} - y_{\mathrm{u}ij}U_i^{\mathrm{c}}H_{\mathrm{u}}Q_j + y_{\mathrm{d}ij}D_i^{\mathrm{c}}H_{\mathrm{d}}Q_j + y_{\mathrm{e}ij}E_i^{\mathrm{c}}H_{\mathrm{d}}L_j, \tag{5.11}$$

$$W_{\text{RPV}} = -\kappa_i L_i H_{\mathrm{u}} + \frac{1}{2}\lambda_{ijk}L_iL_jE_k^{\mathrm{c}} + \lambda'_{ijk}L_iQ_jD_k^{\mathrm{c}} + \frac{1}{2}\lambda''_{ijk}U_i^{\mathrm{c}}D_j^{\mathrm{c}}D_k^{\mathrm{c}}; \tag{5.12}$$

$$\mathcal{L}_{\text{FI}} = \Lambda_{\text{FI}}D_B; \tag{5.13}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V_{\cancel{\mathrm{SUSY}}}^{\text{RPC}} + V_{\cancel{\mathrm{SUSY}}}^{\text{RPV}}\right), \tag{5.14}$$

$$\begin{aligned}
V_{\cancel{\mathrm{SUSY}}}^{\text{RPC}} = {} & \left(\tilde{q}_{\mathrm{L}}^* m_Q^2 \tilde{q}_{\mathrm{L}} + \tilde{l}_{\mathrm{L}}^* m_L^2 \tilde{l}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^* m_{U^{\mathrm{c}}}^2 \tilde{u}_{\mathrm{R}} + \tilde{d}_{\mathrm{R}}^* m_{D^{\mathrm{c}}}^2 \tilde{d}_{\mathrm{R}} + \tilde{e}_{\mathrm{R}}^* m_{E^{\mathrm{c}}}^2 \tilde{e}_{\mathrm{R}} + m_{H_{\mathrm{u}}}^2|h_{\mathrm{u}}|^2 + m_{H_{\mathrm{d}}}^2|h_{\mathrm{d}}|^2\right)\\
& + \left(-\tilde{u}_{\mathrm{R}}^* h_{\mathrm{u}} a_{\mathrm{u}}\tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* h_{\mathrm{d}} a_{\mathrm{d}}\tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^* h_{\mathrm{d}} a_{\mathrm{e}}\tilde{l}_{\mathrm{L}} + bH_{\mathrm{u}}H_{\mathrm{d}} + \text{H.c.}\right)\\
& + \left(\tilde{u}_{\mathrm{R}}^* h_{\mathrm{d}}^* c_{\mathrm{u}}\tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* h_{\mathrm{u}}^* c_{\mathrm{d}}\tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^* h_{\mathrm{u}}^* c_{\mathrm{e}}\tilde{l}_{\mathrm{L}} + \text{H.c.}\right),
\end{aligned} \tag{5.15}$$

$$\begin{aligned}
V_{\cancel{\mathrm{SUSY}}}^{\text{RPV}} = {} & \left(-b_i\tilde{l}_{\mathrm{L}i}H_{\mathrm{u}} + \frac{1}{2}T_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{l}_{\mathrm{L}j}\tilde{e}_{\mathrm{R}k}^* + T'_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{q}_{\mathrm{L}j}\tilde{d}_{\mathrm{R}k}^* + \frac{1}{2}T''_{ijk}\tilde{u}_{\mathrm{R}i}^*\tilde{d}_{\mathrm{R}j}^*\tilde{d}_{\mathrm{R}k}^* + \tilde{l}_{\mathrm{L}i}^* M_{Li}^2 H_{\mathrm{d}} + \text{H.c.}\right)\\
& + \left(C_{ijk}^1\tilde{l}_{\mathrm{L}i}^*\tilde{q}_{\mathrm{L}j}\tilde{u}_{\mathrm{R}k}^* + C_i^2 h_{\mathrm{u}}^* h_{\mathrm{d}}\tilde{e}_{\mathrm{R}i}^* + C_{ijk}^3\tilde{d}_{\mathrm{R}i}\tilde{u}_{\mathrm{R}j}^*\tilde{e}_{\mathrm{R}k}^* + \frac{1}{2}C_{ijk}^4\tilde{d}_{\mathrm{R}i}\tilde{q}_{\mathrm{L}j}\tilde{q}_{\mathrm{L}k} + \text{H.c.}\right).
\end{aligned} \tag{5.16}$$

As usual, we remove Θ_W and Θ_B by rotating fermions, which is compatible with mass diagonalization, and assume the absence of Fayet-Illiopoulos term: $\Lambda_{\text{FI}} = 0$. The SU(3) angle Θ_g forms QCD phase Θ_{QCD} together with the phases from Yukawa matrices. Then,

$$\mathcal{L}_{\text{matter}} = \sum_{\text{matters}}\left[\mathrm{D}^\mu\phi^*\,\mathrm{D}_\mu\phi + \mathrm{i}\bar{\psi}\bar{\sigma}^\mu\,\mathrm{D}_\mu\psi - \sqrt{2}\sum_{\text{gauge}} g\left(\lambda^a(\phi^* t^a\psi) + \bar{\lambda}^a(\bar{\psi}t^a\phi)\right)\right] + (F\text{-terms}), \tag{5.17}$$

$$\mathcal{L}_{\text{gauge}} = \sum_{\text{gauges}}\left(-\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \mathrm{i}\bar{\lambda}^a\bar{\sigma}^\mu\,\mathrm{D}_\mu\lambda^a\right) + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^a G_{\rho\sigma}^a + (D\text{-terms}), \tag{5.18}$$

$$\begin{aligned}
\mathcal{L}_{\text{super}} = \epsilon^{ab} & \Big(-\mu\tilde{h}_{\mathrm{u}}^a\tilde{h}_{\mathrm{d}}^b - y_{\mathrm{d}ij}h_{\mathrm{d}}^a d_{\mathrm{R}i}^{\mathrm{c}x} q_{\mathrm{L}j}^{bx} - y_{\mathrm{d}ij}\tilde{d}_{\mathrm{R}i}^{x*}\tilde{h}_{\mathrm{d}}^a q_{\mathrm{L}j}^{bx} + y_{\mathrm{d}ji}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{h}_{\mathrm{d}}^b d_{\mathrm{R}j}^{\mathrm{c}x}\\
& -y_{\mathrm{e}ij}\tilde{e}_{\mathrm{R}i}^*\tilde{h}_{\mathrm{d}}^a l_{\mathrm{L}j}^b - y_{\mathrm{e}ij}h_{\mathrm{d}}^a e_{\mathrm{R}i}^{\mathrm{c}} l_{\mathrm{L}j}^b + y_{\mathrm{e}ji}\tilde{l}_{\mathrm{L}i}^a\tilde{h}_{\mathrm{d}}^b e_{\mathrm{R}j}^{\mathrm{c}} + y_{\mathrm{u}ij}h_{\mathrm{u}}^a u_{\mathrm{R}i}^{\mathrm{c}x} q_{\mathrm{L}j}^{bx} + y_{\mathrm{u}ij}\tilde{u}_{\mathrm{R}i}^{x*}\tilde{h}_{\mathrm{u}}^a q_{\mathrm{L}j}^{bx} - y_{\mathrm{u}ji}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{h}_{\mathrm{u}}^b u_{\mathrm{R}j}^{\mathrm{c}x}\\
& -\kappa_i\tilde{h}_{\mathrm{u}}^a l_{\mathrm{L}i}^b - \lambda_{ikj}\tilde{l}_{\mathrm{L}i}^a e_{\mathrm{R}j}^{\mathrm{c}} l_{\mathrm{L}k}^b - \frac{1}{2}\lambda_{jki}\tilde{e}_{\mathrm{R}i}^* l_{\mathrm{L}j}^a l_{\mathrm{L}k}^b - \lambda'_{ikj}\tilde{l}_{\mathrm{L}i}^a d_{\mathrm{R}j}^{\mathrm{c}x} q_{\mathrm{L}k}^{bx} + \lambda'_{kij}\tilde{q}_{\mathrm{L}i}^{ax} d_{\mathrm{R}j}^{\mathrm{c}x} l_{\mathrm{L}k}^b + \lambda'_{kji}\tilde{d}_{\mathrm{R}i}^{x*} q_{\mathrm{L}j}^{ax} l_{\mathrm{L}k}^b\Big)\\
& - \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk}\tilde{u}_{\mathrm{R}i}^{x*} d_{\mathrm{R}j}^{\mathrm{c}y} d_{\mathrm{R}k}^{\mathrm{c}z} + \epsilon^{xyz}\lambda''_{jik}\tilde{d}_{\mathrm{R}i}^{x*} u_{\mathrm{R}j}^{\mathrm{c}y} d_{\mathrm{R}k}^{\mathrm{c}z} + \text{H.c.} + (F\text{-terms}),
\end{aligned} \tag{5.19}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V_{\cancel{\mathrm{SUSY}}}^{\text{RPC}} + V_{\cancel{\mathrm{SUSY}}}^{\text{RPV}}\right), \tag{5.20}$$

and the F- and D-terms form the supersymmetric scalar potential

$$V_{\text{SUSY}} = F_i^* F_i + \frac{1}{2}D^a D^a; \qquad F_i = -W_i^* = -\frac{\delta W^*}{\delta\phi_i^*}, \qquad D^a = -g(\phi^* t^a\phi), \tag{5.21}$$

$$V = V_{\text{SUSY}} + V_{\cancel{\mathrm{SUSY}}}^{\text{RPC}} + V_{\cancel{\mathrm{SUSY}}}^{\text{RPV}}, \tag{5.22}$$

where t_a corresponds to the gauge-symmetry generator relevant for each ϕ.

Each auxiliary term is given by

$$-F^*_{h^a_{\rm u}} = \epsilon^{ab}\left(-\tilde u_{\rm R}^{x*} y_{\rm u}\tilde q_{\rm L}^{bx} + \mu h^b_{\rm d} + \kappa_i \tilde l^b_{{\rm L}i}\right), \tag{5.23}$$

$$-F^*_{h^a_{\rm d}} = \epsilon^{ab}\left(\tilde e^*_{\rm R} y_{\rm e}\tilde l^b_{\rm L} + \tilde d_{\rm R}^{x*} y_{\rm d}\tilde q_{\rm L}^{bx} - \mu h^b_{\rm u}\right), \tag{5.24}$$

$$-F^*_{\tilde q^{ax}_{{\rm L}i}} = \epsilon^{ab}\left(-y_{{\rm d}ji} h^b_{\rm d}\tilde d^{x*}_{{\rm R}j} + y_{{\rm u}ji} h^b_{\rm u}\tilde u^{x*}_{{\rm R}j} - \lambda'_{kij}\tilde d^{x*}_{{\rm R}j}\tilde l^b_{{\rm L}k}\right), \tag{5.25}$$

$$-F^*_{\tilde u^{x*}_{{\rm R}i}} = -y_{{\rm u}ij} h_{\rm u}\tilde q^x_{{\rm L}j} + \frac12\epsilon^{xyz}\lambda''_{ijk}\tilde d^{y*}_{{\rm R}j}\tilde d^{z*}_{{\rm R}k}, \tag{5.26}$$

$$-F^*_{\tilde d^{x*}_{{\rm R}i}} = y_{{\rm d}ij} h_{\rm d}\tilde q^x_{{\rm L}j} + \lambda'_{jki}\tilde l_{{\rm L}j}\tilde q^x_{{\rm L}k} - \lambda''_{jik}\epsilon^{xyz}\tilde u^{y*}_{{\rm R}j}\tilde d^{z*}_{{\rm R}k}, \tag{5.27}$$

$$-F^*_{\tilde l^a_{{\rm L}i}} = \epsilon^{ab}\left(-y_{{\rm e}ji}\tilde e^*_{{\rm R}j} h^b_{\rm d} - \kappa_i h^b_{\rm u} + \lambda_{ikj}\tilde e^*_{{\rm R}j}\tilde l^b_{{\rm L}k} + \lambda'_{ikj}\tilde d^{x*}_{{\rm R}j}\tilde q^{bx}_{{\rm L}k}\right), \tag{5.28}$$

$$-F^*_{\tilde e^*_{{\rm R}i}} = y_{{\rm e}ij} h_{\rm d}\tilde l_{{\rm L}j} + \frac12\lambda_{jki}\tilde l_{{\rm L}j}\tilde l_{{\rm L}k}. \tag{5.29}$$

$$D^\alpha_{\rm SU(3)} = -g_3\sum_{i=1}^3\left(\sum_{a=1,2}\tilde q^{a*}_{{\rm L}i}\tau^\alpha\tilde q^a_{{\rm L}i} - \tilde u^*_{{\rm R}i}\tau^\alpha\tilde u_{{\rm R}i} - \tilde d^*_{{\rm R}i}\tau^\alpha\tilde d_{{\rm R}i}\right), \tag{5.30}$$

$$D^\alpha_{\rm SU(2)} = -g_2\left[\sum_{i=1}^3\left(\sum_{x=1}^3\tilde q^{x*}_{{\rm L}i}T^\alpha\tilde q^x_{{\rm L}i} + \tilde l^*_{{\rm L}i}T^\alpha\tilde l_{{\rm L}i}\right) + h^*_{\rm u}T^\alpha h_{\rm u} + h^*_{\rm d}T^\alpha h_{\rm d}\right], \tag{5.31}$$

$$D_{\rm U(1)} = -g_1\left(\frac16|\tilde q_{\rm L}|^2 - \frac12|\tilde l_{\rm L}|^2 - \frac23|\tilde u_{\rm R}|^2 + \frac13|\tilde d_{\rm R}|^2 + |\tilde e_{\rm R}|^2 + \frac12|h_{\rm u}|^2 - \frac12|h_{\rm d}|^2\right). \tag{5.32}$$

5.3. Full Lagrangian

Here the Lagrangian $\mathcal L = \mathcal L_{\rm vector} + \mathcal L_{\rm fermions} + \mathcal L_{\rm SFG} + \mathcal L_{\rm scalar}$ is explicitly given:

$$\mathcal L_{\rm vector} = -\frac14 F^a_{\mu\nu}F^{a\mu\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}, \tag{5.33}$$

$$\mathcal L_{\rm fermions} = {\rm i}\bar\psi\bar\sigma^\mu\,{\rm D}_\mu\psi + {\rm i}\bar\lambda^a\bar\sigma^\mu\,{\rm D}_\mu\lambda^a - \frac12\left(M_3\tilde g\tilde g + M_2\tilde w\tilde w + M_1\tilde b\tilde b + \text{H.c.}\right) + \mathcal L_{\rm super}\big|_{\text{no }F\text{-terms}}, \tag{5.34}$$

$$\mathcal L_{\rm SFG} = -\sqrt2 g\lambda^a(\phi^* t^a\psi) - \sqrt2 g\bar\lambda^a(\bar\psi t^a\phi), \tag{5.35}$$

$$\mathcal L_{\rm scalar} = {\rm D}^\mu\phi^*\,{\rm D}_\mu\phi - V. \tag{5.36}$$

5.3.1. Vector part

$$\begin{aligned}\mathcal L_{\rm vector} = &-\frac12(\partial_\mu B_\nu - \partial_\nu B_\mu)\partial^\mu B^\nu - \frac12(\partial_\mu g^a_\nu - \partial_\nu g^a_\mu)\partial^\mu g^{a\nu} - \frac12(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)\partial^\mu W^{a\nu}\\ &- g_2\epsilon^{abc}W^b_\mu W^c_\nu\partial^\mu W^{a\nu} - \frac{g_2^2}{4}\epsilon^{abe}\epsilon^{cde}W^a_\mu W^b_\nu W^{c\mu}W^{d\nu}\\ &- g_3 f^{abc}g^b_\mu g^c_\nu\partial^\mu g^{a\nu} - \frac{g_3^2}{4}f^{cde}f^{abe}g^a_\mu g^b_\nu g^{c\mu}g^{d\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma},\end{aligned} \tag{5.37}$$

$$\begin{aligned}= &\,(\text{gluons}) - \frac12(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial^\mu A^\nu - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)\partial_\mu W^{+\nu} - \frac12(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\partial^\mu Z^\nu\\ &+ {\rm i}g_2{\rm c_w}\left[(W^-_\mu Z_\nu - W^-_\nu Z_\mu)\partial^\mu W^{+\nu} - (W^+_\mu Z_\nu - W^+_\nu Z_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)\partial^\mu Z^\nu\right]\\ &+ {\rm i}|e|\left[(W^-_\mu A_\nu - W^-_\nu A_\mu)\partial^\mu W^{+\nu} - (W^+_\mu A_\nu - W^+_\nu A_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)\partial^\mu A^\nu\right]\\ &+ \frac{g_2^2}{2}W^{+\mu}W^+_\mu W^{-\nu}W^-_\nu - \frac{g_2^2}{2}W^{+\mu}W^{+\nu}W^-_\mu W^-_\nu - g_2^2W^{+\mu}W^-_\mu Z^\nu Z_\nu + g_2^2W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\ &- e^2W^{+\mu}W^-_\mu A^\nu A_\nu + e^2W^{+\mu}W^-_\mu Z^\nu Z_\nu + e^2W^{+\mu}W^{-\nu}A_\mu A_\nu - e^2W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\ &- 2g_2^2{\rm c_w s_w}W^{+\mu}W^-_\mu A^\nu Z_\nu + g_2^2{\rm c_w s_w}W^{+\mu}W^{-\nu}A_\mu Z_\nu + g_2^2{\rm c_w s_w}W^{+\mu}W^{-\nu}A_\nu Z_\mu,\end{aligned} \tag{5.38}$$

where

$$W^1_\mu = \frac{W^+_\mu + W^-_\mu}{\sqrt2},\quad W^2_\mu = \frac{{\rm i}(W^+_\mu - W^-_\mu)}{\sqrt2};\qquad W^\pm_\mu = \frac{W^1_\mu \mp {\rm i}W^2_\mu}{\sqrt2};$$

$$\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix} = \begin{pmatrix}{\rm c_w} & {\rm s_w}\\ -{\rm s_w} & {\rm c_w}\end{pmatrix}\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix};\qquad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} = \begin{pmatrix}{\rm c_w} & -{\rm s_w}\\ {\rm s_w} & {\rm c_w}\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix};$$

$$|e| = g_2{\rm s_w} = g_Y{\rm c_w} = g_Z{\rm s_w c_w},\quad g_Z = g_2/{\rm c_w} = g_Y/{\rm s_w};\qquad g_Y = |e|/{\rm c_w} = g_Z{\rm s_w} = g_2{\rm t_w},\quad g_2 = |e|/{\rm s_w} = g_Z{\rm c_w}.$$

5.3.2. Fermion part

$$
\begin{aligned}
&\mathcal{L}_{\text{fermions}} \\
&= \mathrm{i}\bar{q}_{\mathrm{L}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_3 g^a_{\mu}\tau^a - \mathrm{i}g_2 W^a_{\mu}T^a - \tfrac{1}{6}\mathrm{i}g_Y B_{\mu}\right) q_{\mathrm{L}} \\
&\quad + \mathrm{i}\bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\left(\partial_{\mu} + \mathrm{i}g_3 g^a_{\mu}\tau^{a*} + \tfrac{2}{3}\mathrm{i}g_Y B_{\mu}\right) u^{\mathrm{c}}_{\mathrm{R}} + \mathrm{i}\bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\left(\partial_{\mu} + \mathrm{i}g_3 g^a_{\mu}\tau^{a*} - \tfrac{1}{3}\mathrm{i}g_Y B_{\mu}\right) d^{\mathrm{c}}_{\mathrm{R}} \\
&\quad + \mathrm{i}\bar{l}_{\mathrm{L}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W^a_{\mu}T^a + \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right) l_{\mathrm{L}} + \mathrm{i}\bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_Y B_{\mu}\right) e^{\mathrm{c}}_{\mathrm{R}} \\
&\quad + \mathrm{i}\bar{\tilde{h}}_{\mathrm{u}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W^a_{\mu}T^a - \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right) \tilde{h}_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}_{\mathrm{d}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W^a_{\mu}T^a + \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right) \tilde{h}_{\mathrm{d}} \\
&\quad + \mathrm{i}\bar{\tilde{g}}^a\bar{\sigma}^{\mu}\left(\partial_{\mu}\tilde{g}^a + g_3 f^{abc}g^b_{\mu}\tilde{g}^c\right) + \mathrm{i}\bar{\tilde{w}}^a\bar{\sigma}^{\mu}\left(\partial_{\mu}\tilde{w}^a + g_2\epsilon^{abc}W^b_{\mu}\tilde{w}^c\right) + \mathrm{i}\bar{\tilde{b}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{b} \\
&\quad - \frac{1}{2}\left(M_3\tilde{g}^a\tilde{g}^a + M_2\tilde{w}^a\tilde{w}^a + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) + \mathcal{L}_{\text{super}}\big|_{\text{no }F\text{-terms}}
\end{aligned}
\tag{5.39}
$$

$$
\begin{aligned}
&= \mathrm{i}\bar{\tilde{b}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{b} - \frac{1}{2}\left(M_1\tilde{b}\tilde{b} + M_1^*\bar{\tilde{b}}^a\bar{\tilde{b}}^a\right) + \mathrm{i}\bar{\tilde{g}}^a\bar{\sigma}^{\mu}\partial_{\mu}\tilde{g}^a - \frac{1}{2}\left(M_3\tilde{g}^a\tilde{g}^a + M_3^*\bar{\tilde{g}}^a\bar{\tilde{g}}^a\right) - \mathrm{i}g_3 f^{abc}(\bar{\tilde{g}}^a\bar{\sigma}^{\mu}\tilde{g}^b)g^c_{\mu} \\
&\quad + \mathrm{i}\bar{\tilde{w}}^+\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^+ + \mathrm{i}\bar{\tilde{w}}^-\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^- + \mathrm{i}\bar{\tilde{w}}^3\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^3 - \left(M_2\tilde{w}^+\tilde{w}^- + M_2^*\bar{\tilde{w}}^+\bar{\tilde{w}}^-\right) - \frac{1}{2}\left(M_2\tilde{w}^3\tilde{w}^3 + M_2^*\bar{\tilde{w}}^3\bar{\tilde{w}}^3\right) \\
&\quad + g_2(\bar{\tilde{w}}^3\bar{\sigma}^{\mu}\tilde{w}^- - \bar{\tilde{w}}^+\bar{\sigma}^{\mu}\tilde{w}^3)W^+_{\mu} - g_2(\bar{\tilde{w}}^3\bar{\sigma}^{\mu}\tilde{w}^+ - \bar{\tilde{w}}^-\bar{\sigma}^{\mu}\tilde{w}^3)W^-_{\mu} + g_2(\bar{\tilde{w}}^+\bar{\sigma}^{\mu}\tilde{w}^+ - \bar{\tilde{w}}^-\bar{\sigma}^{\mu}\tilde{w}^-)(\mathrm{c_w}Z_{\mu} + \mathrm{s_w}A_{\mu}) \\
&\quad + \bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} + g_3\tau^a g^a_{\mu})u_{\mathrm{L}} + \bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} - g_3\tau^{a*}g^a_{\mu})u^{\mathrm{c}}_{\mathrm{R}} + \mathrm{i}\bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}\partial_{\mu}\nu_{\mathrm{L}} \\
&\quad + \bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} + g_3\tau^a g^a_{\mu})d_{\mathrm{L}} + \bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} - g_3\tau^{a*}g^a_{\mu})d^{\mathrm{c}}_{\mathrm{R}} + \mathrm{i}\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}\partial_{\mu}e_{\mathrm{L}} + \mathrm{i}\bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_{\mu}e^{\mathrm{c}}_{\mathrm{R}} \\
&\quad + \mathrm{i}\bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^-_{\mathrm{d}} + \mathrm{i}\bar{\tilde{h}}^0_{\mathrm{d}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^0_{\mathrm{d}} + \mathrm{i}\bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^+_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}^0_{\mathrm{u}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}^0_{\mathrm{u}} \\
&\quad + \frac{g_2}{\sqrt{2}}\left(\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}} + \bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}} + \bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{u}} + \bar{\tilde{h}}^0_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^-_{\mathrm{d}}\right)W^+_{\mu} + \frac{g_2}{\sqrt{2}}\left(\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}} + \bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}\nu_{\mathrm{L}} + \bar{\tilde{h}}^0_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^+_{\mathrm{u}} + \bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{d}}\right)W^-_{\mu} \\
&\quad + \frac{g_Z(3-4\mathrm{s}^2_{\mathrm{w}})}{6}\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}}Z_{\mu} + \frac{g_Z\mathrm{s}^2_{\mathrm{w}}}{3}\bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}u^{\mathrm{c}}_{\mathrm{R}}Z_{\mu} + \frac{g_Z(2\mathrm{s}^2_{\mathrm{w}}-3)}{6}\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}}Z_{\mu} - \frac{g_Z\mathrm{s}^2_{\mathrm{w}}}{3}\bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}d^{\mathrm{c}}_{\mathrm{R}}Z_{\mu} \\
&\quad + \frac{g_Z}{2}\bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}\nu_{\mathrm{L}}Z_{\mu} + \frac{g_Z(2\mathrm{s}^2_{\mathrm{w}}-1)}{2}\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}}Z_{\mu} - g_Z\mathrm{s}^2_{\mathrm{w}}\bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}e^{\mathrm{c}}_{\mathrm{R}}Z_{\mu} \\
&\quad + \frac{g_Z(1-2\mathrm{s}^2_{\mathrm{w}})}{2}\bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^+_{\mathrm{u}}Z_{\mu} + \frac{g_Z}{2}\bar{\tilde{h}}^0_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{u}}Z_{\mu} + \frac{g_Z(2\mathrm{s}^2_{\mathrm{w}}-1)}{2}\bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^-_{\mathrm{d}}Z_{\mu} + \frac{g_Z}{2}\bar{\tilde{h}}^0_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^0_{\mathrm{d}}Z_{\mu} \\
&\quad + \frac{2|e|}{3}(\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}} - \bar{u}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}u^{\mathrm{c}}_{\mathrm{R}})A_{\mu} - \frac{|e|}{3}(\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}} - \bar{d}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}d^{\mathrm{c}}_{\mathrm{R}})A_{\mu} - |e|(\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}} - \bar{e}^{\mathrm{c}}_{\mathrm{R}}\bar{\sigma}^{\mu}e^{\mathrm{c}}_{\mathrm{R}})A_{\mu} \\
&\quad + |e|\bar{\tilde{h}}^+_{\mathrm{u}}\bar{\sigma}^{\mu}\tilde{h}^+_{\mathrm{u}}A_{\mu} - |e|\bar{\tilde{h}}^-_{\mathrm{d}}\bar{\sigma}^{\mu}\tilde{h}^-_{\mathrm{d}}A_{\mu} + \mathcal{L}_{\text{super}}\big|_{\text{no }F\text{-terms}};
\end{aligned}
\tag{5.40}
$$

here,

$$
\begin{aligned}
\mathcal{L}_{\text{super}}\big|_{\text{no F-terms}} = &-\mu\tilde{h}^+_{\mathrm{u}}\tilde{h}^-_{\mathrm{d}} + \mu\tilde{h}^0_{\mathrm{u}}\tilde{h}^0_{\mathrm{d}} + y_{\mathrm{u}ij}h^+_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}j} - y_{\mathrm{u}ij}h^0_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}j} + y_{\mathrm{u}ij}\tilde{d}_{\mathrm{L}j}\tilde{h}^+_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i} - y_{\mathrm{u}ij}\tilde{u}_{\mathrm{L}j}\tilde{h}^0_{\mathrm{u}}u^{\mathrm{c}}_{\mathrm{R}i} \\
&+ y_{\mathrm{u}ji}\tilde{u}^*_{\mathrm{R}j}\tilde{h}^+_{\mathrm{u}}d_{\mathrm{L}i} - y_{\mathrm{u}ji}\tilde{u}^*_{\mathrm{R}j}\tilde{h}^0_{\mathrm{u}}u_{\mathrm{L}i} + y_{\mathrm{d}ij}h^-_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}j} - y_{\mathrm{d}ij}h^0_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}j} - y_{\mathrm{d}ij}\tilde{d}_{\mathrm{L}j}\tilde{h}^0_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i} \\
&+ y_{\mathrm{d}ij}\tilde{u}_{\mathrm{L}j}\tilde{h}^-_{\mathrm{d}}d^{\mathrm{c}}_{\mathrm{R}i} + y_{\mathrm{d}ji}\tilde{d}^*_{\mathrm{R}j}\tilde{h}^-_{\mathrm{d}}u_{\mathrm{L}i} - y_{\mathrm{d}ji}\tilde{d}^*_{\mathrm{R}j}\tilde{h}^0_{\mathrm{d}}d_{\mathrm{L}i} + y_{\mathrm{e}ij}h^-_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i}\nu_{\mathrm{L}j} - y_{\mathrm{e}ij}h^0_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}j} \\
&- y_{\mathrm{e}ij}\tilde{e}_{\mathrm{L}j}\tilde{h}^0_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i} + y_{\mathrm{e}ij}\tilde{\nu}_{\mathrm{L}j}\tilde{h}^-_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i} + y_{\mathrm{e}ji}\tilde{e}^*_{\mathrm{R}j}\tilde{h}^-_{\mathrm{d}}\nu_{\mathrm{L}i} - y_{\mathrm{e}ji}\tilde{e}^*_{\mathrm{R}j}\tilde{h}^0_{\mathrm{d}}e_{\mathrm{L}i} \\
&- \kappa_i\tilde{h}^+_{\mathrm{u}}e_{\mathrm{L}i} + \kappa_i\tilde{h}^0_{\mathrm{u}}\nu_{\mathrm{L}i} - \lambda_{ijk}\tilde{e}^*_{\mathrm{R}k}\nu_{\mathrm{L}i}e_{\mathrm{L}j} - \lambda_{jki}\tilde{e}_{\mathrm{L}k}e^{\mathrm{c}}_{\mathrm{R}i}\nu_{\mathrm{L}j} + \lambda_{jki}\tilde{\nu}_{\mathrm{L}k}e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}j} \\
&- \lambda'_{jik}\tilde{d}^*_{\mathrm{R}k}d_{\mathrm{L}i}\nu_{\mathrm{L}j} + \lambda'_{jik}\tilde{d}^*_{\mathrm{R}k}u_{\mathrm{L}i}e_{\mathrm{L}j} - \lambda'_{jki}\tilde{d}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}\nu_{\mathrm{L}j} + \lambda'_{jki}\tilde{u}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}j} + \lambda'_{kji}\tilde{e}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}j} \\
&- \lambda'_{kji}\tilde{\nu}_{\mathrm{L}k}d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}j} - \epsilon^{xyz}\lambda''_{ijk}\tilde{d}^{x*}_{\mathrm{R}k}u^{\mathrm{c}y}_{\mathrm{R}i}d^{\mathrm{c}z}_{\mathrm{R}j} - \frac{1}{2}\epsilon^{xyz}\lambda''_{kij}\tilde{u}^{x*}_{\mathrm{R}k}d^{\mathrm{c}y}_{\mathrm{R}i}d^{\mathrm{c}z}_{\mathrm{R}j} + \text{H.c.}
\end{aligned}
\tag{5.41}
$$

5.3.3. Scalar-fermion-gaugino interaction

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SFG}} = &-g_2\tilde{u}_{\mathrm{L}}^* d_{\mathrm{L}}\tilde{w}^+ - g_2\tilde{u}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{w}}^+ - g_2\tilde{d}_{\mathrm{L}}^* u_{\mathrm{L}}\tilde{w}^- - g_2\tilde{d}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{w}}^- \\
&- \sqrt{2}g_3\tilde{u}_{\mathrm{L}}^*\tau^a u_{\mathrm{L}}\tilde{g}^a + \sqrt{2}g_3\tilde{u}_{\mathrm{R}}^*\tau^a\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{g}}^a - \sqrt{2}g_3\tilde{u}_{\mathrm{L}}\tau^{a*}\bar{u}_{\mathrm{L}}\bar{\tilde{g}}^a + \sqrt{2}g_3\tilde{u}_{\mathrm{R}}\tau^{a*}u_{\mathrm{R}}^{\mathrm{c}}\tilde{g}^a \\
&- \frac{g_2}{\sqrt{2}}\tilde{u}_{\mathrm{L}}^* u_{\mathrm{L}}\tilde{w}^3 - \frac{g_2}{\sqrt{2}}\tilde{u}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{w}}^3 - \frac{g_Y}{3\sqrt{2}}\tilde{u}_{\mathrm{L}}^* u_{\mathrm{L}}\tilde{b} + \frac{2\sqrt{2}g_Y}{3}\tilde{u}_{\mathrm{R}}^*\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}}\tilde{u}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{b}} + \frac{2\sqrt{2}g_Y}{3}\tilde{u}_{\mathrm{R}}u_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
&- \sqrt{2}g_3\tilde{d}_{\mathrm{L}}^*\tau^a d_{\mathrm{L}}\tilde{g}^a + \sqrt{2}g_3\tilde{d}_{\mathrm{R}}^*\tau^a\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{g}}^a - \sqrt{2}g_3\tilde{d}_{\mathrm{L}}\tau^{a*}\bar{d}_{\mathrm{L}}\bar{\tilde{g}}^a + \sqrt{2}g_3\tilde{d}_{\mathrm{R}}\tau^{a*}d_{\mathrm{R}}^{\mathrm{c}}\tilde{g}^a \\
&+ \frac{g_2}{\sqrt{2}}\tilde{d}_{\mathrm{L}}^* d_{\mathrm{L}}\tilde{w}^3 + \frac{g_2}{\sqrt{2}}\tilde{d}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{w}}^3 - \frac{g_Y}{3\sqrt{2}}\tilde{d}_{\mathrm{L}}^* d_{\mathrm{L}}\tilde{b} - \frac{\sqrt{2}g_Y}{3}\tilde{d}_{\mathrm{R}}^*\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}}\tilde{d}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{b}} - \frac{\sqrt{2}g_Y}{3}\tilde{d}_{\mathrm{R}}d_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
&- g_2\tilde{e}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{w}}^- - g_2\tilde{\nu}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{w}}^+ - g_2\tilde{e}_{\mathrm{L}}^*\nu_{\mathrm{L}}\tilde{w}^- - g_2\tilde{\nu}_{\mathrm{L}}^* e_{\mathrm{L}}\tilde{w}^+ \\
&- \frac{g_2}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}^*\nu_{\mathrm{L}}\tilde{w}^3 + \frac{g_Y}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}^*\nu_{\mathrm{L}}\tilde{b} - \frac{g_2}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{b}} \\
&+ \frac{g_2}{\sqrt{2}}\tilde{e}_{\mathrm{L}}^* e_{\mathrm{L}}\tilde{w}^3 + \frac{g_Y}{\sqrt{2}}\tilde{e}_{\mathrm{L}}^* e_{\mathrm{L}}\tilde{b} - \sqrt{2}g_Y\tilde{e}_{\mathrm{R}}^*\bar{e}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} + \frac{g_2}{\sqrt{2}}\tilde{e}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}}\tilde{e}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{b}} - \sqrt{2}g_Y\tilde{e}_{\mathrm{R}}e_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
&- g_2 h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^0\tilde{w}^+ - g_2 h_{\mathrm{d}}^0\bar{\tilde{h}}_{\mathrm{d}}^-\bar{\tilde{w}}^+ - g_2 h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^0\tilde{w}^- - g_2 h_{\mathrm{u}}^0\bar{\tilde{h}}_{\mathrm{u}}^+\bar{\tilde{w}}^- \\
&- g_2 h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^-\tilde{w}^+ - g_2 h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^+\tilde{w}^- - g_2 h_{\mathrm{u}}^+\bar{\tilde{h}}_{\mathrm{u}}^0\bar{\tilde{w}}^+ - g_2 h_{\mathrm{d}}^-\bar{\tilde{h}}_{\mathrm{d}}^0\bar{\tilde{w}}^- \\
&- \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^+\bar{\tilde{h}}_{\mathrm{u}}^+\bar{\tilde{w}}^3 - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^+\bar{\tilde{h}}_{\mathrm{u}}^+\bar{\tilde{b}} - \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^+\tilde{w}^3 - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^+\tilde{b} + \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^-\tilde{w}^3 + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^-\tilde{b} \\
&+ \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^-\bar{\tilde{h}}_{\mathrm{d}}^-\bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^-\bar{\tilde{h}}_{\mathrm{d}}^-\bar{\tilde{b}} + \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^0\tilde{w}^3 - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^0\tilde{b} - \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^0\tilde{w}^3 + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^0\tilde{b} \\
&+ \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^0\bar{\tilde{h}}_{\mathrm{u}}^0\bar{\tilde{w}}^3 - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^0\bar{\tilde{h}}_{\mathrm{u}}^0\bar{\tilde{b}} - \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^0\bar{\tilde{h}}_{\mathrm{d}}^0\bar{\tilde{w}}^3 + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^0\bar{\tilde{h}}_{\mathrm{d}}^0\bar{\tilde{b}}
\end{aligned}
\tag{5.42}
$$

5.4. Scalar part

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} = {} & (\partial_\mu \tilde{u}_\mathrm{L}^* + \mathrm{i} g_3 \tilde{u}_\mathrm{L}^* \tau^a g_\mu^a)(\partial^\mu \tilde{u}_\mathrm{L} - \mathrm{i} g_3 g^{b\mu} \tau^b \tilde{u}_\mathrm{L}) + (\partial_\mu \tilde{u}_\mathrm{R} - \mathrm{i} g_3 \tilde{u}_\mathrm{R} \tau^{a*} g_\mu^a)(\partial^\mu \tilde{u}_\mathrm{R}^* + \mathrm{i} g_3 g_\mu^b \tau^{b*} \tilde{u}_\mathrm{R}^*) \\
& + (\partial_\mu \tilde{d}_\mathrm{L}^* + \mathrm{i} g_3 \tilde{d}_\mathrm{L}^* \tau^a g_\mu^a)(\partial^\mu \tilde{d}_\mathrm{L} - \mathrm{i} g_3 g^{b\mu} \tau^b \tilde{d}_\mathrm{L}) + (\partial_\mu \tilde{d}_\mathrm{R} - \mathrm{i} g_3 \tilde{d}_\mathrm{R}^y \tau^{a*} g_\mu^a)(\partial^\mu \tilde{d}_\mathrm{R}^* + \mathrm{i} g_3 g_\mu^b \tau^{b*} \tilde{d}_\mathrm{R}^*) \\
& + \sqrt{2} g_2 g_3 \tilde{u}_\mathrm{L}^* \tau^a \tilde{d}_\mathrm{L} W^{+\mu} g_\mu^a + \sqrt{2} g_2 g_3 \tilde{d}_\mathrm{L}^* \tau^a \tilde{u}_\mathrm{L} W^{-\mu} g_\mu^a + \frac{4}{3} g_3 |e| (\tilde{u}_\mathrm{L}^* \tau^a \tilde{u}_\mathrm{L} + \tilde{u}_\mathrm{R}^* \tau^a \tilde{u}_\mathrm{R}) g^{a\mu} A_\mu \\
& - \frac{2}{3} g_3 |e| (\tilde{d}_\mathrm{L}^* \tau^a \tilde{d}_\mathrm{L} + \tilde{d}_\mathrm{R}^* \tau^a \tilde{d}_\mathrm{R}) g^{a\mu} A_\mu + \frac{(3 - 4\mathrm{s}_\mathrm{w}^2) g_Z}{3} g_3 \tilde{u}_\mathrm{L}^* \tau^a \tilde{u}_\mathrm{L} g^{a\mu} Z_\mu - \frac{4\mathrm{s}_\mathrm{w}^2 g_Z}{3} g_3 \tilde{u}_\mathrm{R}^* \tau^a \tilde{u}_\mathrm{R} g^{a\mu} Z_\mu \\
& + \frac{(2\mathrm{s}_\mathrm{w}^2 - 3) g_Z}{3} g_3 \tilde{d}_\mathrm{L}^* \tau^a \tilde{d}_\mathrm{L} g^{a\mu} Z_\mu + \frac{2\mathrm{s}_\mathrm{w}^2 g_Z}{3} g_3 \tilde{d}_\mathrm{R}^* \tau^a \tilde{d}_\mathrm{R} g^{a\mu} Z_\mu \\
& + \frac{\mathrm{i} g_2}{\sqrt{2}} W_\mu^+ \left(\tilde{u}_\mathrm{L}^* \partial^\mu \tilde{d}_\mathrm{L} - \tilde{d}_\mathrm{L} \partial^\mu \tilde{u}_\mathrm{L}^* \right) - \frac{\mathrm{i} g_2}{\sqrt{2}} W_\mu^- \left(\tilde{u}_\mathrm{L} \partial^\mu \tilde{d}_\mathrm{L}^* - \tilde{d}_\mathrm{L}^* \partial^\mu \tilde{u}_\mathrm{L} \right) \\
& + \frac{2\mathrm{i}}{3} |e| A_\mu \left(\tilde{u}_\mathrm{L}^* \partial^\mu \tilde{u}_\mathrm{L} - \tilde{u}_\mathrm{L} \partial^\mu \tilde{u}_\mathrm{L}^* + \tilde{u}_\mathrm{R}^* \partial^\mu \tilde{u}_\mathrm{R} - \tilde{u}_\mathrm{R} \partial^\mu \tilde{u}_\mathrm{R}^* \right) - \frac{\mathrm{i}}{3} |e| A_\mu \left(\tilde{d}_\mathrm{L}^* \partial^\mu \tilde{d}_\mathrm{L} - \tilde{d}_\mathrm{L} \partial^\mu \tilde{d}_\mathrm{L}^* + \tilde{d}_\mathrm{R}^* \partial^\mu \tilde{d}_\mathrm{R} - \tilde{d}_\mathrm{R} \partial^\mu \tilde{d}_\mathrm{R}^* \right) \\
& + \frac{\mathrm{i}(4\mathrm{s}_\mathrm{w}^2 - 3) g_Z}{6} Z_\mu \left(\tilde{u}_\mathrm{L} \partial^\mu \tilde{u}_\mathrm{L}^* - \tilde{u}_\mathrm{L}^* \partial^\mu \tilde{u}_\mathrm{L} \right) + \frac{\mathrm{i}(2\mathrm{s}_\mathrm{w}^2 - 3) g_Z}{6} Z_\mu \left(\tilde{d}_\mathrm{L}^* \partial^\mu \tilde{d}_\mathrm{L} - \tilde{d}_\mathrm{L} \partial^\mu \tilde{d}_\mathrm{L}^* \right) \\
& - \frac{2\mathrm{i}\mathrm{s}_\mathrm{w}^2 g_Z}{3} Z_\mu \left(\tilde{u}_\mathrm{R}^* \partial^\mu \tilde{u}_\mathrm{R} - \tilde{u}_\mathrm{R} \partial^\mu \tilde{u}_\mathrm{R}^* \right) + \frac{\mathrm{i}\mathrm{s}_\mathrm{w}^2 g_Z}{3} Z_\mu \left(\tilde{d}_\mathrm{R}^* \partial^\mu \tilde{d}_\mathrm{R} - \tilde{d}_\mathrm{R} \partial^\mu \tilde{d}_\mathrm{R}^* \right) \\
& + \frac{g_2^2}{2} \left(|\tilde{u}_\mathrm{L}|^2 + |\tilde{d}_\mathrm{L}|^2 \right) W^{+\mu} W_\mu^- - \frac{\mathrm{s}_\mathrm{w}^2 g_2 g_Z}{3\sqrt{2}} \tilde{u}_\mathrm{L}^* \tilde{d}_\mathrm{L} W^{+\mu} Z_\mu - \frac{\mathrm{s}_\mathrm{w}^2 g_2 g_Z}{3\sqrt{2}} \tilde{d}_\mathrm{L}^* \tilde{u}_\mathrm{L} W^{-\mu} Z_\mu \\
& + \frac{(3 - 4\mathrm{s}_\mathrm{w}^2)^2 g_Z^2}{36} |\tilde{u}_\mathrm{L}|^2 Z^\mu Z_\mu + \frac{4\mathrm{s}_\mathrm{w}^4 g_Z^2}{9} |\tilde{u}_\mathrm{R}|^2 Z^\mu Z_\mu + \frac{(3 - 2\mathrm{s}_\mathrm{w}^2)^2 g_Z^2}{36} |\tilde{d}_\mathrm{L}|^2 Z^\mu Z_\mu + \frac{\mathrm{s}_\mathrm{w}^4 g_Z^2}{9} |\tilde{d}_\mathrm{R}|^2 Z^\mu Z_\mu \\
& + \frac{4}{9} e^2 \left(|\tilde{u}_\mathrm{L}|^2 + |\tilde{u}_\mathrm{R}|^2 \right) A^\mu A_\mu + \frac{1}{9} e^2 \left(|\tilde{d}_\mathrm{L}|^2 + |\tilde{d}_\mathrm{R}|^2 \right) A^\mu A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{u}_\mathrm{L}^* \tilde{d}_\mathrm{L} W^{+\mu} A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{d}_\mathrm{L}^* \tilde{u}_\mathrm{L} W^{-\mu} A_\mu \\
& + \frac{2(3 - 4\mathrm{s}_\mathrm{w}^2) g_Z |e|}{9} |\tilde{u}_\mathrm{L}|^2 A^\mu Z_\mu - \frac{8\mathrm{s}_\mathrm{w}^2 g_Z |e|}{9} |\tilde{u}_\mathrm{R}|^2 A^\mu Z_\mu + \frac{(3 - 2\mathrm{s}_\mathrm{w}^2) g_Z |e|}{9} |\tilde{d}_\mathrm{L}|^2 A^\mu Z_\mu - \frac{2\mathrm{s}_\mathrm{w}^2 g_Z |e|}{9} |\tilde{d}_\mathrm{R}|^2 A^\mu Z_\mu \\[1ex]
& + \partial_\mu \tilde{e}_\mathrm{R} \partial^\mu \tilde{e}_\mathrm{R}^* + \partial_\mu \tilde{e}_\mathrm{L}^* \partial^\mu \tilde{e}_\mathrm{L} + \partial_\mu \tilde{\nu}_\mathrm{L}^* \partial^\mu \tilde{\nu}_\mathrm{L} + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ (\tilde{\nu}_\mathrm{L}^* \partial^\mu \tilde{e}_\mathrm{L} - \tilde{e}_\mathrm{L} \partial^\mu \tilde{\nu}_\mathrm{L}^*) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- (\tilde{e}_\mathrm{L}^* \partial^\mu \tilde{\nu}_\mathrm{L} - \tilde{\nu}_\mathrm{L} \partial^\mu \tilde{e}_\mathrm{L}^*) \\
& - \frac{\mathrm{i}(1 - 2\mathrm{s}_\mathrm{w}^2) g_Z}{2} Z_\mu (\tilde{e}_\mathrm{L}^* \partial^\mu \tilde{e}_\mathrm{L} - \tilde{e}_\mathrm{L} \partial^\mu \tilde{e}_\mathrm{L}^*) + \frac{\mathrm{i} g_Z}{2} Z_\mu (\tilde{\nu}_\mathrm{L}^* \partial^\mu \tilde{\nu}_\mathrm{L} - \tilde{\nu}_\mathrm{L} \partial^\mu \tilde{\nu}_\mathrm{L}^*) + \mathrm{i}\mathrm{s}_\mathrm{w}^2 g_Z Z_\mu (\tilde{e}_\mathrm{R}^* \partial^\mu \tilde{e}_\mathrm{R} - \tilde{e}_\mathrm{R} \partial^\mu \tilde{e}_\mathrm{R}^*) \\
& + \mathrm{i} |e| A_\mu (\tilde{e}_\mathrm{L} \partial^\mu \tilde{e}_\mathrm{L}^* - \tilde{e}_\mathrm{L}^* \partial^\mu \tilde{e}_\mathrm{L} + \tilde{e}_\mathrm{R} \partial^\mu \tilde{e}_\mathrm{R}^* - \tilde{e}_\mathrm{R}^* \partial^\mu \tilde{e}_\mathrm{R}) \\
& + \frac{g_2^2}{2} (|\tilde{\nu}_\mathrm{L}|^2 + |\tilde{e}_\mathrm{L}|^2) W^{+\mu} W_\mu^- + \frac{g_2 g_Z \mathrm{s}_\mathrm{w}^2}{\sqrt{2}} \left(\tilde{e}_\mathrm{L}^* \tilde{\nu}_\mathrm{L} W_\mu^- Z^\mu + \tilde{\nu}_\mathrm{L}^* \tilde{e}_\mathrm{L} W_\mu^+ Z^\mu \right) - \frac{g_2 |e|}{\sqrt{2}} \left(\tilde{\nu}_\mathrm{L}^* \tilde{e}_\mathrm{L} W_\mu^+ A^\mu + \tilde{e}_\mathrm{L}^* \tilde{\nu}_\mathrm{L} W_\mu^- A^\mu \right) \\
& + \frac{(1 - 2\mathrm{s}_\mathrm{w}^2)^2 g_Z^2}{4} |\tilde{e}_\mathrm{L}|^2 Z^\mu Z_\mu + \frac{g_Z^2}{4} |\tilde{\nu}_\mathrm{L}|^2 Z^\mu Z_\mu + g_Z^2 \mathrm{s}_\mathrm{w}^4 |\tilde{e}_\mathrm{R}|^2 Z^\mu Z_\mu \\
& + e^2 (|\tilde{e}_\mathrm{L}|^2 + |\tilde{e}_\mathrm{R}|^2) A^\mu A_\mu + (1 - 2\mathrm{s}_\mathrm{w}^2) |e| g_Z |\tilde{e}_\mathrm{L}|^2 A^\mu Z_\mu - 2\mathrm{s}_\mathrm{w}^2 g_Z |e| |\tilde{e}_\mathrm{R}|^2 A^\mu Z_\mu \\[1ex]
& + \partial_\mu h_\mathrm{d}^{-*} \partial^\mu h_\mathrm{d}^- + \partial_\mu h_\mathrm{d}^{0*} \partial^\mu h_\mathrm{d}^0 + \partial_\mu h_\mathrm{u}^{+*} \partial^\mu h_\mathrm{u}^+ + \partial_\mu h_\mathrm{u}^{0*} \partial^\mu h_\mathrm{u}^0 + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_\mathrm{u}^{+*} \partial^\mu h_\mathrm{u}^0 - h_\mathrm{u}^0 \partial^\mu h_\mathrm{u}^{+*} \right) \\
& + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- \left(h_\mathrm{u}^{0*} \partial^\mu h_\mathrm{u}^+ - h_\mathrm{u}^+ \partial^\mu h_\mathrm{u}^{0*} \right) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_\mathrm{d}^{0*} \partial^\mu h_\mathrm{d}^- - h_\mathrm{d}^- \partial^\mu h_\mathrm{d}^{0*} \right) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- \left(h_\mathrm{d}^{-*} \partial^\mu h_\mathrm{d}^0 - h_\mathrm{d}^0 \partial^\mu h_\mathrm{d}^{-*} \right) \\
& + \frac{\mathrm{i}(1 - 2\mathrm{s}_\mathrm{w}^2) g_Z}{2} Z_\mu \left(h_\mathrm{u}^{+*} \partial^\mu h_\mathrm{u}^+ - h_\mathrm{u}^+ \partial^\mu h_\mathrm{u}^{+*} \right) + \frac{\mathrm{i}(2\mathrm{s}_\mathrm{w}^2 - 1) g_Z}{2} Z_\mu \left(h_\mathrm{d}^{-*} \partial^\mu h_\mathrm{d}^- - h_\mathrm{d}^- \partial^\mu h_\mathrm{d}^{-*} \right) \\
& - \frac{\mathrm{i} g_Z}{2} Z_\mu \left(h_\mathrm{u}^{0*} \partial^\mu h_\mathrm{u}^0 - h_\mathrm{u}^0 \partial^\mu h_\mathrm{u}^{0*} \right) + \frac{\mathrm{i} g_Z}{2} Z_\mu \left(h_\mathrm{d}^{0*} \partial^\mu h_\mathrm{d}^0 - h_\mathrm{d}^0 \partial^\mu h_\mathrm{d}^{0*} \right) \\
& - \mathrm{i} |e| A_\mu \left(h_\mathrm{d}^{-*} \partial^\mu h_\mathrm{d}^- - h_\mathrm{d}^- \partial^\mu h_\mathrm{d}^{-*} \right) + \mathrm{i} |e| A_\mu \left(h_\mathrm{u}^{+*} \partial^\mu h_\mathrm{u}^+ - h_\mathrm{u}^+ \partial^\mu h_\mathrm{u}^{+*} \right) \\
& + \frac{g_2^2}{2} \left(|h_\mathrm{u}^+|^2 + |h_\mathrm{u}^0|^2 + |h_\mathrm{d}^0|^2 + |h_\mathrm{d}^-|^2 \right) W^{+\mu} W_\mu^- + \frac{\mathrm{s}_\mathrm{w}^2 g_Z g_2}{\sqrt{2}} \left(h_\mathrm{d}^{0*} h_\mathrm{d}^- - h_\mathrm{u}^{+*} h_\mathrm{u}^0 \right) W_\mu^+ Z^\mu \\
& + \frac{\mathrm{s}_\mathrm{w}^2 g_Z g_2}{\sqrt{2}} \left(h_\mathrm{d}^{-*} h_\mathrm{d}^0 - h_\mathrm{u}^{0*} h_\mathrm{u}^+ \right) W_\mu^- Z^\mu + \frac{g_2 |e|}{\sqrt{2}} \left(h_\mathrm{u}^{+*} h_\mathrm{u}^0 - h_\mathrm{d}^{0*} h_\mathrm{d}^- \right) W_\mu^+ A^\mu + \frac{g_2 |e|}{\sqrt{2}} \left(h_\mathrm{u}^{0*} h_\mathrm{u}^+ - h_\mathrm{d}^{-*} h_\mathrm{d}^0 \right) W_\mu^- A^\mu \\
& + \frac{(1 - 2\mathrm{s}_\mathrm{w}^2)^2 g_Z^2}{4} \left(|h_\mathrm{u}^+|^2 + |h_\mathrm{d}^-|^2 \right) Z^\mu Z_\mu + \frac{g_Z^2}{4} \left(|h_\mathrm{u}^0|^2 + |h_\mathrm{d}^0|^2 \right) Z^\mu Z_\mu + e^2 \left(|h_\mathrm{u}^+|^2 + |h_\mathrm{d}^-|^2 \right) A^\mu A_\mu \\
& + (1 - 2\mathrm{s}_\mathrm{w}^2) |e| g_Z \left(|h_\mathrm{u}^+|^2 + |h_\mathrm{d}^-|^2 \right) A^\mu Z_\mu \\
& - (V_\mathrm{SUSY} + V_{\cancel{\mathrm{SUSY}}}) \,,
\end{aligned}
$$

$$(5.43)$$

where the scalar potential is given by

$$
\begin{aligned}
V_{\text{SUSY}} = {} & |h_\text{u}|^2 \left(|\mu|^2 + \sum_i |\kappa_i|^2 \right) + |\mu|^2 |h_\text{d}|^2 + \left(\kappa_i^* \mu \tilde{l}_{\text{L}i}^* h_\text{d} + \text{H.c.} \right) + \kappa_i^* \kappa_j \tilde{l}_{\text{L}i}^* \tilde{l}_{\text{L}j} \\
& + \Big[- y_{\text{u}ij} \mu^* h_\text{d}^* \tilde{u}_{\text{R}i}^* \tilde{q}_{\text{L}j} - y_{\text{u}ij} \kappa_k^* \tilde{u}_{\text{R}i}^* \tilde{q}_{\text{L}j} \tilde{l}_{\text{L}k}^* - \left(y_{\text{d}ij} \mu^* + \lambda'_{kji} \kappa_k^* \right) h_\text{u}^* \tilde{d}_{\text{R}i}^* \tilde{q}_{\text{L}j} \\
& \qquad + y_{\text{e}ij} \kappa_j^* \tilde{e}_{\text{R}i}^* h_\text{u}^* h_\text{d} + \left(\lambda_{jki} \kappa_k^* - y_{\text{e}ij} \mu^* \right) h_\text{u}^* \tilde{e}_{\text{R}i}^* \tilde{l}_{\text{L}j} + \text{H.c.} \Big] \\
& + \frac{1}{8} \left(g_2^2 + g_Y^2 \right) |h_\text{d}|^4 + \frac{1}{8} \left(g_2^2 + g_Y^2 \right) |h_\text{u}|^4 + \left(-\frac{g_2^2}{4} |h_\text{d}|^2 |h_\text{u}|^2 - \frac{g_Y^2}{4} |h_\text{d}|^2 |h_\text{u}|^2 + \frac{g_2^2}{2} |h_\text{d}^* h_\text{u}|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_\text{u}|^2 |\tilde{q}_\text{L}|^2 + \frac{g_Y^2}{12} |h_\text{u}|^2 |\tilde{q}_\text{L}|^2 + \frac{g_2^2}{2} |h_\text{u}^b \tilde{q}_{\text{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_\text{u}^\dagger y_\text{u})_{ji} h_\text{u}^a h_\text{u}^{b*} \tilde{q}_{\text{L}i}^c \tilde{q}_{\text{L}j}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4} |h_\text{d}|^2 |\tilde{q}_\text{L}|^2 - \frac{g_Y^2}{12} |h_\text{d}|^2 |\tilde{q}_\text{L}|^2 + \frac{g_2^2}{2} |h_\text{d}^* \tilde{q}_{\text{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_\text{d}^\dagger y_\text{d})_{ji} h_\text{d}^a h_\text{d}^{b*} \tilde{q}_{\text{L}i}^c \tilde{q}_{\text{L}j}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4} |h_\text{u}|^2 |\tilde{l}_\text{L}|^2 - \frac{g_Y^2}{4} |h_\text{u}|^2 |\tilde{l}_\text{L}|^2 + \frac{g_2^2}{2} |h_\text{u}^* \tilde{l}_{\text{L}i}|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_\text{d}|^2 |\tilde{l}_\text{L}|^2 + \frac{g_Y^2}{4} |h_\text{d}|^2 |\tilde{l}_\text{L}|^2 + \frac{g_2^2}{2} |h_\text{d}^* \tilde{l}_{\text{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_\text{e}^\dagger y_\text{e})_{ji} h_\text{d}^a h_\text{d}^{b*} \tilde{l}_{\text{L}i}^c \tilde{l}_{\text{L}j}^{d*} \right) \\
& + |h_\text{u}|^2 \left(-\frac{g_Y^2}{3} |\tilde{u}_\text{R}|^2 + (y_\text{u} y_\text{u}^\dagger)_{ij} \tilde{u}_{\text{R}i}^* \tilde{u}_{\text{R}j} \right) + \frac{g_Y^2}{3} |h_\text{d}|^2 |\tilde{u}_\text{R}|^2 \\
& + \frac{g_Y^2}{6} |h_\text{u}|^2 |\tilde{d}_\text{R}|^2 + |h_\text{d}|^2 \left(-\frac{g_Y^2}{6} |\tilde{d}_\text{R}|^2 + (y_\text{d} y_\text{d}^\dagger)_{ij} \tilde{d}_{\text{R}i}^* \tilde{d}_{\text{R}j} \right) - \left[(y_\text{u} y_\text{d}^\dagger)_{ij} \tilde{u}_{\text{R}i}^* \tilde{d}_{\text{R}j} (h_\text{d}^* h_\text{u}) + \text{H.c.} \right] \\
& + \frac{g_Y^2}{2} |h_\text{u}|^2 |\tilde{e}_\text{R}|^2 + |h_\text{d}|^2 \left(-\frac{g_Y^2}{2} |\tilde{e}_\text{R}|^2 + (y_\text{e} y_\text{e}^\dagger)_{ij} \tilde{e}_{\text{R}i}^* \tilde{e}_{\text{R}j} \right) \\
& + \Big[-\frac{1}{2} \epsilon^{ab} \epsilon^{xyz} y_{\text{u}lk} \lambda''^*_{lij} h_\text{u}^a \tilde{d}_{\text{R}i}^x \tilde{d}_{\text{R}j}^y \tilde{q}_{\text{L}k}^{bz} + \epsilon^{ab} \epsilon^{xyz} y_{\text{d}lk} \lambda''^*_{ijl} h_\text{d}^a \tilde{u}_{\text{R}i}^x \tilde{d}_{\text{R}j}^y \tilde{q}_{\text{L}k}^{bz} - y_{\text{u}il} \lambda'^*_{klj} h_\text{u}^a \tilde{u}_{\text{R}i}^* \tilde{d}_{\text{R}j} \tilde{l}_{\text{L}k}^{a*} \\
& \quad + y_{\text{d}il} \lambda'^*_{klj} h_\text{d}^a \tilde{d}_{\text{R}i}^* \tilde{d}_{\text{R}j} \tilde{l}_{\text{L}k}^{a*} - \epsilon^{ab} \epsilon^{cd} y_{\text{d}li} \lambda'^*_{kjl} h_\text{d}^a \tilde{q}_{\text{L}i}^b \tilde{q}_{\text{L}j}^{c*} \tilde{l}_{\text{L}k}^{d*} + y_{\text{e}il} \lambda^*_{klj} h_\text{d}^a \tilde{e}_{\text{R}i}^* \tilde{e}_{\text{R}j} \tilde{l}_{\text{L}k}^{a*} \\
& \quad - y_{\text{e}jl} \lambda'^*_{lki} h_\text{d}^a \tilde{e}_{\text{R}j}^* \tilde{d}_{\text{R}i} \tilde{q}_{\text{L}k}^{a*} + \frac{1}{2} \epsilon^{ab} \epsilon^{cd} y_{\text{e}li} \lambda^*_{jkl} h_\text{d}^a \tilde{l}_{\text{L}i}^b \tilde{l}_{\text{L}j}^{c*} \tilde{l}_{\text{L}k}^{d*} + \text{H.c.} \Big] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8} \right) |\tilde{q}_\text{L}|^4 + \frac{g_2^2}{4} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}j}^{by} \tilde{q}_{\text{L}i}^{bx*} \tilde{q}_{\text{L}j}^{ay*} + \frac{g_3^2}{4} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}j}^{by} \tilde{q}_{\text{L}i}^{ay*} \tilde{q}_{\text{L}j}^{bx*} \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{2 g_Y^2}{9} \right) |\tilde{u}_\text{R}|^4 + \frac{g_3^2}{4} \tilde{u}_{\text{R}i}^{x*} \tilde{u}_{\text{R}j}^{y*} \tilde{u}_{\text{R}i}^y \tilde{u}_{\text{R}j}^x \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18} \right) |\tilde{d}_\text{R}|^4 + \frac{g_3^2}{4} \tilde{d}_{\text{R}i}^{x*} \tilde{d}_{\text{R}j}^{y*} \tilde{d}_{\text{R}i}^y \tilde{d}_{\text{R}j}^x + \frac{1}{2} \lambda''_{mij} \lambda''^*_{mkl} \tilde{d}_{\text{R}i}^{x*} \tilde{d}_{\text{R}j}^{y*} \tilde{d}_{\text{R}k}^x \tilde{d}_{\text{R}l}^y \right] \\
& + \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9} \right) |\tilde{u}_\text{R}|^2 |\tilde{q}_\text{L}|^2 - \frac{g_3^2}{2} \tilde{u}_{\text{R}i}^{x*} \tilde{q}_{\text{L}j}^{ax} \tilde{u}_{\text{R}i}^y \tilde{q}_{\text{L}j}^{ay*} + y_{\text{u}ik} y_{\text{u}jl}^* \tilde{u}_{\text{R}i}^{x*} \tilde{u}_{\text{R}j}^y \tilde{q}_{\text{L}k}^{ax} \tilde{q}_{\text{L}l}^{ay*} \right] \\
& + \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18} \right) |\tilde{d}_\text{R}|^2 |\tilde{q}_\text{L}|^2 - \frac{g_3^2}{2} \tilde{d}_{\text{R}i}^{x*} \tilde{q}_{\text{L}j}^{ax} \tilde{d}_{\text{R}i}^y \tilde{q}_{\text{L}j}^{ay*} + \left(y_{\text{d}ik} y_{\text{d}jl}^* + \lambda'_{mki} \lambda'^*_{mlj} \right) \tilde{d}_{\text{R}i}^{x*} \tilde{q}_{\text{L}k}^{ax} \tilde{d}_{\text{R}j}^y \tilde{q}_{\text{L}l}^{ay*} \right] \\
& + \left[-\left(\frac{g_3^2}{6} + \frac{2 g_Y^2}{9} \right) |\tilde{d}_\text{R}|^2 |\tilde{u}_\text{R}|^2 + \left(\frac{g_3^2}{2} - \lambda''_{ikm} \lambda''^*_{ljm} \right) \tilde{u}_{\text{R}i}^{x*} \tilde{d}_{\text{R}j}^x \tilde{d}_{\text{R}j}^{y*} \tilde{u}_{\text{R}i}^y + \lambda''_{ikm} \lambda''^*_{jlm} \tilde{u}_{\text{R}i}^{x*} \tilde{u}_{\text{R}j}^x \tilde{d}_{\text{R}k}^{y*} \tilde{d}_{\text{R}l}^y \right] \\
& + \left[-\left(\frac{g_2^2}{4} + \frac{g_Y^2}{12} \right) |\tilde{l}_\text{L}|^2 |\tilde{q}_\text{L}|^2 + \frac{g_2^2}{2} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}i}^{bx*} \tilde{l}_{\text{L}j}^b \tilde{l}_{\text{L}j}^{a*} + \epsilon^{ac} \epsilon^{bd} \lambda'_{kim} \lambda'^*_{ljm} \tilde{q}_{\text{L}i}^{ax} \tilde{q}_{\text{L}j}^{bx*} \tilde{l}_{\text{L}k}^c \tilde{l}_{\text{L}l}^{d*} \right] + \frac{g_Y^2}{6} |\tilde{e}_\text{R}|^2 |\tilde{q}_\text{L}|^2 \\
& + \left(-\frac{g_Y^2}{6} |\tilde{d}_\text{R}|^2 |\tilde{l}_\text{L}|^2 + \lambda'_{lmi} \lambda'^*_{kmj} \tilde{d}_{\text{R}i}^* \tilde{d}_{\text{R}j} \tilde{l}_{\text{L}k}^* \tilde{l}_{\text{L}l} \right) + \frac{g_Y^2}{3} |\tilde{u}_\text{R}|^2 |\tilde{l}_\text{L}|^2 - \frac{2 g_Y^2}{3} |\tilde{u}_\text{R}|^2 |\tilde{e}_\text{R}|^2 + \frac{g_Y^2}{3} |\tilde{d}_\text{R}|^2 |\tilde{e}_\text{R}|^2 \\
& + \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8} \right) |\tilde{l}_\text{L}|^4 + \frac{g_2^2}{4} \tilde{l}_{\text{L}i}^a \tilde{l}_{\text{L}j}^b \tilde{l}_{\text{L}i}^{b*} \tilde{l}_{\text{L}j}^{a*} + \frac{1}{4} \epsilon^{ab} \epsilon^{cd} \lambda_{ijm} \lambda^*_{klm} \tilde{l}_{\text{L}i}^a \tilde{l}_{\text{L}j}^b \tilde{l}_{\text{L}k}^{c*} \tilde{l}_{\text{L}l}^{d*} \right] \\
& + \frac{g_Y^2}{2} |\tilde{e}_\text{R}|^4 + \left[-\frac{g_Y^2}{2} |\tilde{e}_\text{R}|^2 |\tilde{l}_\text{L}|^2 + \left(y_{\text{e}ik} y_{\text{e}jl}^* + \lambda_{kmi} \lambda^*_{lmj} \right) \tilde{e}_{\text{R}i}^* \tilde{e}_{\text{R}j} \tilde{l}_{\text{L}k} \tilde{l}_{\text{L}l}^* \right] \\
& + \left[\left(y_{\text{d}ik} y_{\text{e}jl}^* - \lambda'_{mki} \lambda^*_{lmj} \right) \tilde{d}_{\text{R}i}^{x*} \tilde{q}_{\text{L}k}^{ax} \tilde{e}_{\text{R}j} \tilde{l}_{\text{L}l}^{a*} - \epsilon^{ab} \epsilon^{xyz} \lambda'_{lkm} \lambda''^*_{ijm} \tilde{l}_{\text{L}l}^b \tilde{q}_{\text{L}k}^{az} \tilde{d}_{\text{R}j}^y \tilde{u}_{\text{R}i}^x + \text{H.c.} \right] .
\end{aligned} \tag{5.44}
$$

5.5. Higgs mechanism and fermion composition

The scalar potential includes

$$V_{\rm SUSY} \supset |h_{\rm u}|^2\Big(|\mu|^2 + \sum|\kappa_i|^2\Big) + |\mu|^2|h_{\rm d}|^2 + \frac{g_Z^2}{8}\left(|h_{\rm u}|^2 - |h_{\rm d}|^2\right)^2 + \frac{g_2^2}{2}|h_{\rm d}^* h_{\rm u}|^2 \\ + \Big(\kappa_i^*\mu\tilde l_{{\rm L}i}^* h_{\rm d} + {\rm H.c.}\Big) + \kappa_i^*\kappa_j\tilde l_{{\rm L}i}^*\tilde l_{{\rm L}j} \tag{5.45}$$

$$V_{\cancel{\rm SUSY}} \supset m_{H_{\rm u}}^2|h_{\rm u}|^2 + m_{H_{\rm d}}^2|h_{\rm d}|^2 + \epsilon^{ab}\Big(bh_{\rm u}^a h_{\rm d}^b + b^* h_{\rm u}^{a*}h_{\rm d}^{b*} - b_i\tilde l_{{\rm L}i}^a h_{\rm u}^b - b_i^*\tilde l_{{\rm L}i}^{a*}h_{\rm u}^{b*}\Big) + \tilde l_{{\rm L}i}^* M_{Li}^2 h_{\rm d} + \tilde l_{{\rm L}i} M_{Li}^{2*} h_{\rm d}^*; \tag{5.46}$$

the Higgs mass term is given by

$$V \supset \begin{pmatrix} h_{\rm u} & h_{\rm d}^* & \tilde l_{{\rm L}i}^* \end{pmatrix} \begin{pmatrix} |\mu|^2 + m_{H_{\rm u}}^2 + \sum|\kappa_i|^2 & b & -b_j \\ b^* & |\mu|^2 + m_{H_{\rm d}}^2 & \kappa_j\mu^* + M_{Lj}^{2*} \\ -b_i^* & \kappa_i^*\mu + M_{Li}^2 & (m_L^2)_{ij} + \kappa_i^*\kappa_j \end{pmatrix} \begin{pmatrix} h_{\rm u}^* \\ h_{\rm d} \\ \tilde l_{{\rm L}j} \end{pmatrix} \tag{5.47}$$

while corresponding fermion terms are

$$\mathcal{L} \supset \epsilon^{ab}\Big(-\mu\tilde h_{\rm u}^a\tilde h_{\rm d}^b - \kappa_i\tilde h_{\rm u}^a l_{{\rm L}i}^b\Big). \tag{5.48}$$

We block-diagonalize the scalar mass matrix so that Higgses and $\tilde l_{\rm L}$ are separated; then in general lepton and Higgsinos are mixed. Also we rotate SU(2) symmetry so that $\langle h_{\rm u}^+\rangle = 0$; i.e., our SU(2) notation is fixed as such.

5.5.1. Higgs potential and induced mass in R-parity conserved case

With R-parity conservation (and SU(2) notation-fixing),

$$V_{\rm pot} = (|\mu|^2 + m_{H_{\rm u}}^2)|h_{\rm u}^0|^2 + (|\mu|^2 + m_{H_{\rm d}}^2)|h_{\rm d}^0|^2 + \frac{g_Z^2}{8}\left(|h_{\rm u}^0|^2 - |h_{\rm d}^0|^2\right)^2 - \left(bh_{\rm u}^0 h_{\rm d}^0 + {\rm H.c.}\right). \tag{5.49}$$

We also fix the U(1) notation so that $b > 0$, $\langle h_{\rm u}^0\rangle = v\sin\beta > 0$, and $\langle h_{\rm d}^0\rangle = v\cos\beta > 0$:

$$V_{\rm pot} = (|\mu|^2 + m_{H_{\rm u}}^2)v^2\sin^2\beta + (|\mu|^2 + m_{H_{\rm d}}^2)v^2\cos^2\beta + \frac{g_Z^2}{8}v^4\cos^2 2\beta - v^2\sin 2\beta, \tag{5.50}$$

where the solutions are

$$\sin 2\beta = \frac{2b}{2|\mu|^2 + m_{H_{\rm u}}^2 + m_{H_{\rm d}}^2}, \qquad \frac{g_Z^2 v^2}{2} = \left|\frac{m_{H_{\rm d}}^2 - m_{H_{\rm u}}^2}{\cos 2\beta}\right| - m_{H_{\rm u}}^2 - m_{H_{\rm d}}^2 - 2|\mu|^2. \tag{5.51}$$

The mass terms are summarized as, except for the neutral Higgs bosons,

$$\mathcal{L}_{\rm fermions} \supset -\Big(v{\rm s}_\beta\bar u_{\rm R}y_{\rm u}u_{\rm L} + v{\rm c}_\beta\bar d_{\rm R}y_{\rm d}d_{\rm L} + v{\rm c}_\beta\bar e_{\rm R}y_{\rm e}e_{\rm L} + {\rm H.c.}\Big) + (\text{gaugino masses}), \tag{5.52}$$

$$\mathcal{L}_{\rm SFG} \supset -g_2 v{\rm c}_\beta\bar{\tilde h}_{\rm d}^-\bar{\tilde w}^+ - g_2 v{\rm s}_\beta\bar{\tilde h}_{\rm u}^+\bar{\tilde w}^- + \frac{g_2 v{\rm s}_\beta}{\sqrt2}\tilde h_{\rm u}^0\tilde w^3 - \frac{g_Y v{\rm s}_\beta}{\sqrt2}\tilde h_{\rm u}^0\tilde b - \frac{g_2 v{\rm c}_\beta}{\sqrt2}\tilde h_{\rm d}^0\tilde w^3 + \frac{g_Y v{\rm c}_\beta}{\sqrt2}\tilde h_{\rm d}^0\tilde b + {\rm H.c.}, \tag{5.53}$$

$$\begin{aligned}\mathcal{L}_{\rm scalars} \supset{}& \frac{g_2^2 v^2}{2}W^{+\mu}W_\mu^- + \frac{g_Z^2 v^2}{4}Z^\mu Z_\mu - \Bigg\{ \\ & \left[|\mu|^2 + m_{H_{\rm u}}^2 + \frac{g_2^2 - g_Y^2{\rm c}_{2\beta}}{4}v^2\right]|h_{\rm u}^+|^2 + \left[|\mu|^2 + m_{H_{\rm d}}^2 + \frac{g_2^2 + g_Y^2{\rm c}_{2\beta}}{4}v^2\right]|h_{\rm d}^-|^2 \\ & + \frac{4b + g_2^2 v^2{\rm s}_{2\beta}}{4}(h_{\rm u}^+h_{\rm d}^- + h_{\rm u}^{+*}h_{\rm d}^{-*}) \\ & + \tilde u_{\rm L}^* m_Q^2\tilde u_{\rm L} + \tilde d_{\rm L}^* m_Q^2\tilde d_{\rm L} + \tilde\nu_{\rm L}^* m_L^2\tilde\nu_{\rm L} + \tilde e_{\rm L}^* m_L^2\tilde e_{\rm L} + \tilde u_{\rm R}^* m_{U^{\rm c}}^2\tilde u_{\rm R} + \tilde d_{\rm R}^* m_{D^{\rm c}}^2\tilde d_{\rm R} + \tilde e_{\rm R}^* m_{E^{\rm c}}^2\tilde e_{\rm R} \\ & + v^2{\rm s}_\beta^2(\tilde u_{\rm L}^* y_{\rm u}^\dagger y_{\rm u}\tilde u_{\rm L}) + v^2{\rm s}_\beta^2(\tilde u_{\rm R}^* y_{\rm u}y_{\rm u}^\dagger\tilde u_{\rm R}) + v^2{\rm c}_\beta^2(\tilde d_{\rm L}^* y_{\rm d}^\dagger y_{\rm d}\tilde d_{\rm L}) + v^2{\rm c}_\beta^2(\tilde d_{\rm R}^* y_{\rm d}y_{\rm d}^\dagger\tilde d_{\rm R}) + v^2{\rm c}_\beta^2(\tilde e_{\rm R}^* y_{\rm e}y_{\rm e}^\dagger\tilde e_{\rm R}) \\ & + v^2{\rm c}_\beta^2(\tilde e_{\rm L}^* y_{\rm e}^\dagger y_{\rm e}\tilde e_{\rm L}) - \Big[v{\rm s}_\beta\mu^*(\tilde d_{\rm R}^* y_{\rm d}\tilde d_{\rm L}) + v{\rm s}_\beta\mu^*(\tilde e_{\rm R}^* y_{\rm e}\tilde e_{\rm L}) + v{\rm c}_\beta\mu^*(\tilde u_{\rm R}^* y_{\rm u}\tilde u_{\rm L}) + {\rm H.c.}\Big] \\ & + \frac{(3g_2^2 - g_Y^2)v^2{\rm c}_{2\beta}}{12}|\tilde u_{\rm L}|^2 - \frac{(3g_2^2 + g_Y^2)v^2{\rm c}_{2\beta}}{12}|\tilde d_{\rm L}|^2 + \frac{g_Y^2 v^2{\rm c}_{2\beta}}{3}|\tilde u_{\rm R}|^2 - \frac{g_Y^2 v^2{\rm c}_{2\beta}}{6}|\tilde d_{\rm R}|^2 \\ & + \frac{g_Z^2 v^2{\rm c}_{2\beta}}{4}|\tilde\nu_{\rm L}|^2 - \frac{(g_2^2 - g_Y^2)v^2{\rm c}_{2\beta}}{4}|\tilde e_{\rm L}|^2 - \frac{g_Y^2 v^2{\rm c}_{2\beta}}{2}|\tilde e_{\rm R}|^2 \\ & + \Big(\tilde u_{\rm R}^* v{\rm s}_\beta a_{\rm u}\tilde u_{\rm L} + \tilde d_{\rm R}^* v{\rm c}_\beta a_{\rm d}\tilde d_{\rm L} + \tilde e_{\rm R}^* v{\rm c}_\beta a_{\rm e}\tilde e_{\rm L} + \tilde u_{\rm R}^* v{\rm c}_\beta c_{\rm u}\tilde u_{\rm L} + \tilde d_{\rm R}^* v{\rm s}_\beta c_{\rm d}\tilde d_{\rm L} + \tilde e_{\rm R}^* v{\rm s}_\beta c_{\rm e}\tilde e_{\rm L} + {\rm H.c.}\Big)\Bigg\}.\end{aligned} \tag{5.54}$$

5.6. SLHA convention

The SLHA convention [?] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table (**magenta color** for objects in other conventions), while

$$\mu, b, m^2_{Q,L,H_\mathrm{u},H_\mathrm{d}}, \text{RPV-trilinears } (\lambda\text{s and } T\text{s})$$

are in common.

SLHA		our notation	Martin/DHM
(H_1, H_2)	$=$	$(H_\mathrm{d}, H_\mathrm{u})$	
$Y_{\mathrm{u,d,e}}$	$=$	$(y_{\mathrm{u,d,e}})^\mathrm{T}$	
$T_{\mathrm{u,d,e}}$	$=$	$(a_{\mathrm{u,d,e}})^\mathrm{T}$	
$A_{\mathrm{u,d,e}}$	$=$	$(A_{\mathrm{u,d,e}})^\mathrm{T}$	
$m^2_{U^\mathrm{c},D^\mathrm{c},E\mathrm{c}}$	$=$	$(m^2_{U^\mathrm{c},D^\mathrm{c},E^\mathrm{c}})^\dagger$	
$M_{1,2,3}$	$=$	$-M_{1,2,3}$	
m^2_3	$=$	b	
m^2_A	$=$	$m^2_{A_0}$ (tree)	
		κ_i	$= -\mu'_i$ (rarely used)
D_i	$=$	b_i	
$m^2_{\tilde{L}_i H_1}$	$=$	M^2_{Li}	

In particular, the chargino/neutralino mass terms in RPC case are given by

$$\mathcal{L} \supset \left[\frac{1}{2}M_1\tilde{b}\tilde{b} + \frac{1}{2}M_2\tilde{w}\tilde{w} - \mu\tilde{h}_{\mathrm{u}}\tilde{h}_{\mathrm{d}} - \frac{g_Y}{2\sqrt{2}}\left(h_{\mathrm{u}}^*\tilde{h}_{\mathrm{u}} - h_{\mathrm{d}}^*\tilde{h}_{\mathrm{d}}\right)\tilde{b} - \sqrt{2}g_2\left(h_{\mathrm{u}}^*T^a\tilde{h}_{\mathrm{u}} + h_{\mathrm{d}}^*T_a\tilde{h}_{\mathrm{d}}\right)\tilde{w}\right] + \mathrm{H.c.} \tag{5.55}$$

$$\to \frac{1}{2}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_{\mathrm{u}}^0\\ h_{\mathrm{d}}^0\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}-M_1 & 0 & -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{s}_\beta\mathrm{s}_w\\ 0 & -M_2 & m_Z\mathrm{c}_\beta\mathrm{c}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w\\ -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{c}_\beta\mathrm{c}_w & 0 & -\mu\\ m_Z\mathrm{s}_\beta\mathrm{s}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w & -\mu & 0\end{pmatrix}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_{\mathrm{u}}^0\\ h_{\mathrm{d}}^0\end{pmatrix} \tag{5.56}$$

A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [?]:

$$e^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \quad \text{(converges for any } X\text{)}, \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(A-1)^m}{m} \quad \text{(conv. if } \|A-I\|<1\text{)}. \tag{A.1}$$

$$e^{\log A} = A \quad \text{(if } \|A-I\|<1\text{)}, \quad \log e^X = X \text{ and } \|e^X - 1\| < 1 \quad \text{(if } \|X\| < \log 2\text{)}. \tag{A.2}$$

$$\text{Hilbert-Schmidt norm}: \ \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \operatorname{Tr} X^\dagger X. \tag{A.3}$$

Properties:

$$e^{(X^{\mathrm{T}})} = (e^X)^{\mathrm{T}}, \qquad e^{(X^*)} = (e^X)^*, \qquad (e^X)^{-1} = e^{-X}, \qquad e^{YXY^{-1}} = Y e^X Y^{-1},$$

$$\det \exp X = \exp \operatorname{Tr} X, \qquad \frac{d}{dt} e^{tX} = X e^{tX} = e^{tX} X \qquad e^{(\alpha+\beta)X} = e^{\alpha X} e^{\beta X} \text{ for } \alpha, \beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$e^X Y e^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots = e^{[X,]} Y; \tag{A.4}$$

$$e^X e^Y e^{-X} = \sum_{n=0}^{\infty} \frac{1}{n!} (e^X Y e^{-X})^n = \exp\left(e^{[X,]} Y\right); \tag{A.5}$$

$$\log(e^X e^Y) = X + \int_0^1 dt\, g(e^{[X,} e^{t[Y,})Y \qquad \left[g(z) = \frac{\log z}{1 - z^{-1}} = 1 - \sum_{n=1}^{\infty} \frac{(1-z)^n}{n(n+1)}; \quad g(e^y) = \sum_{n=0}^{\infty} \frac{B_n y^n}{n!}\right] \tag{A.6}$$

$$= X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots \quad \text{(Baker-Campbell-Hausdorff)}. \tag{A.7}$$

$$\log(e^X e^Y) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \left(\sum_{m,n=0}^{\infty} \frac{X^m Y^n}{m! n!} - 1 \right)^k = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \cdots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k} \frac{X^{m_1} Y^{n_1} \cdots X^{m_k} Y^{n_k}}{m_1! n_1! \cdots m_k! n_k!} \tag{A.8}$$

$$\log(e^X e^Y) = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \cdots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k \sum_{i=1}^{k} (m_i + n_i)} \frac{\left([X,\right)^{m_1} \left([Y,\right)^{n_1} \cdots \left([X,\right)^{m_k} \left([Y,\right)^{n_k}] \cdots]}{m_1! n_1! \cdots m_k! n_k!} \tag{A.9}$$

with $[X] := X$ understood.

If matrices t^a satisfies $[t^a, t^b] = i f^{abc} t^c$ with totally-antisymmetric $f^{abc} \in \mathbb{R}$,

$$\left[e^{\theta^a t^a} t_b\, e^{-\theta^c t^c}\right]_{ij} = \left[e^{\theta^a [t^a,} t_b\right]_{ij} = \left[e^{i\theta^a f^a}\right]^{bc} t^c_{ij} \tag{A.10}$$

holds for $\theta^a \in \mathbb{C}$, where $[f^a]_{bc} = f^{abc}$. ♣TODO:needs verification, generalization/restriction, and a nice proof or reference.♣