

1. Kinematics

Decay rate and cross section (Note: \mathcal{M} has a mass dimension of $4 - N_i - N_f$.)

$$\text{decay rate (rest frame; } \sqrt{s} = M_0) : \quad d\Gamma = \frac{d\Pi^{N_f}}{2M_0} \left| \mathcal{M}(M_0 \rightarrow \{p_1, p_2, \dots, p_{N_f}\}) \right|^2, \quad (1.1)$$

$$\text{cross section (Lorentz invariant) :} \quad d\sigma = \frac{d\Pi^{N_f}}{2E_A 2E_B v_{\text{Mol}}} \left| \mathcal{M}(p_A, p_B \rightarrow \{p_1, p_2, \dots, p_{N_f}\}) \right|^2, \quad (1.2)$$

where $d\Pi^n$ is n -particle Lorentz-invariant phase space with momentum conservation

$$d\Pi^n := d\Pi_1 d\Pi_2 \dots d\Pi_n (2\pi)^4 \delta^{(4)} \left(P_0 - \sum p_n \right); \quad d\Pi := \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}}. \quad (1.3)$$

At the CM frame, two-body phase-space are characterized by the final momentum $\|\mathbf{p}\|$ and given by

$$d\Pi^2 = \frac{\|\mathbf{p}\|}{4\pi\sqrt{s}} \frac{d\Omega}{4\pi} = \frac{\|\mathbf{p}\|}{8\pi\sqrt{s}} d\cos\theta = \frac{1}{16\pi} \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}} d\cos\theta \quad (1.4)$$

with $\sqrt{s} = M_0$ or E_{CM} , θ is the angle between initial and final motion, and

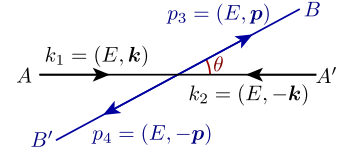
$$\|\mathbf{p}\| = \frac{\sqrt{s}}{2} \lambda^{1/2} \left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s} \right), \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad p_1 \cdot p_2 = \frac{s - (m_1^2 + m_2^2)}{2}.$$

Mandelstam variables For $(k_1, k_2) \rightarrow (p_3, p_4)$ collision,

$$s = (k_1 + k_2)^2 = (p_3 + p_4)^2, \quad t = (p_3 - k_1)^2 = (p_4 - k_2)^2, \quad u = (p_3 - k_2)^2 = (p_4 - k_1)^2; \\ s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

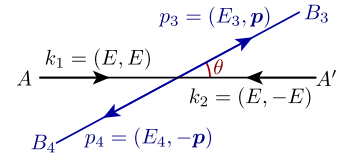
If the collision is with the “same mass” $(m_A, m_A) \rightarrow (m_B, m_B)$,

$$t = m_A^2 + m_B^2 - s/2 + 2kp \cos\theta, \quad (k_1 - k_2)^2 = 4m_A^2 - s, \\ u = m_A^2 + m_B^2 - s/2 - 2kp \cos\theta, \quad (p_3 - p_4)^2 = 4m_B^2 - s, \\ k = \frac{\sqrt{s - 4m_A^2}}{2}, \quad k_1 \cdot k_2 = \frac{s}{2} - m_A^2, \quad k_1 \cdot p_3 = k_2 \cdot p_4 = \frac{m_A^2 + m_B^2 - t}{2}, \\ p = \frac{\sqrt{s - 4m_B^2}}{2}, \quad p_3 \cdot p_4 = \frac{s}{2} - m_B^2, \quad k_1 \cdot p_4 = k_2 \cdot p_3 = \frac{m_A^2 + m_B^2 - u}{2}.$$



Instead, if the collision is “initially massless” $(0, 0) \rightarrow (m_3, m_4)$,

$$t = (m_3^2 + m_4^2 - s)/2 + p\sqrt{s} \cos\theta, \\ u = (m_3^2 + m_4^2 - s)/2 - p\sqrt{s} \cos\theta, \\ p = (\sqrt{s}/2) \lambda^{1/2} \left(1; m_3^2/s, m_4^2/s \right).$$



1.1. Fundamentals

Lorentz-invariant phase space:

$$\int d\Pi = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\sqrt{m^2 + \|\mathbf{p}\|^2}} = \int \frac{dp_0 d^3\mathbf{p}}{(2\pi)^4} (2\pi) \delta(p_0^2 - \|\mathbf{p}\|^2 - m^2) \Theta(p_0)$$

Källén function:

$$\begin{aligned} \lambda(x, y, z) &= x^2 + y^2 + z^2 - 2xy - 2yz - 2zx = (x - y - z)^2 - 4yz; \\ \lambda(1; \alpha_1^2, \alpha_2^2) &= (1 - (\alpha_1 + \alpha_2)^2)(1 - (\alpha_1 - \alpha_2)^2) = (1 + \alpha_1 + \alpha_2)(1 - \alpha_1 - \alpha_2)(1 + \alpha_1 - \alpha_2)(1 - \alpha_1 + \alpha_2). \\ \lambda^{1/2}(s; m_1^2, m_2^2) &= s \lambda^{1/2}\left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right); & \lambda^{1/2}\left(1; \frac{m^2}{s}, \frac{m^2}{s}\right) &= \sqrt{1 - \frac{4m^2}{s}}, \\ \lambda^{1/2}\left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) &= \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}}, & \lambda^{1/2}\left(1; \frac{m_1^2}{s}, 0\right) &= \frac{s - m_1^2}{s}. \end{aligned}$$

Two-body phase space If $f(p_1^\mu, p_2^\mu)$ is Lorentz invariant, $f \equiv f(p_1^2, p_2^2, p_1^\mu p_{2\mu}) \equiv f(p_1, p_2, \cos \theta_{12})$. Meanwhile,

$$\int d\Pi_1 d\Pi_2 = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} \frac{1}{2E_1 2E_2} = \int \frac{(4\pi) dp_1 p_1^2 (2\pi) dp_2 p_2^2 d\cos \theta_{12}}{(2\pi)^3} \frac{1}{2E_1 2E_2} = \int \frac{dE_+ dE_- ds}{128\pi^4}, \quad (1.5)$$

with the replacement of the variables

$$E_{\pm} = E_1 \pm E_2, \quad s = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2E_1 E_2 - 2\|\mathbf{p}_1\| \|\mathbf{p}_2\| \cos \theta_{12};$$

$$\left| \frac{d(E_+, E_-, s)}{d(p_1, p_2, \cos \theta_{12})} \right| = \frac{4p_1^2 p_2^2}{E_1 E_2}, \quad \left| \frac{d(E_1, E_2, s)}{d(p_1, p_2, \cos \theta_{12})} \right| = \frac{2p_1^2 p_2^2}{E_1 E_2}.$$

Therefore,

$$\int d\Pi_1 d\Pi_2 = \frac{1}{128\pi^4} \int_{(m_1+m_2)^2}^{\infty} ds \int_{\sqrt{s}}^{\infty} dE_+ \int_{\min}^{\max} dE_-, \quad (1.6)$$

where the boundary of E_- is given by

$$\begin{aligned} \cos \theta_{12} &= \frac{E_+^2 - E_-^2 + 2(m_1^2 + m_2^2 - s)}{\sqrt{(E_+ + E_-)^2 - 4m_1^2} \sqrt{(E_+ - E_-)^2 - 4m_2^2}} \in [-1, 1] \\ \therefore \left| E_- - \frac{m_1^2 - m_2^2}{s} E_+ \right| &\leq \sqrt{E_+^2 - s} \cdot \lambda^{1/2}\left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) = 2p \sqrt{\frac{E_+^2 - s}{s}}. \end{aligned}$$

Two-body phase space with momentum conservation As a general representation in any frame,

$$\overline{d\Pi^2} = \frac{dp_1 d\Omega p_1^2}{16\pi^2} \frac{\delta(E_0 - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + \|\mathbf{P}_0 - \mathbf{p}_1\|^2})}{E_1 E_2} = \frac{1}{8\pi} d\cos \theta_1 \frac{p_1^2}{E_0 p_1 - P_0 E_1 \cos \theta_1}, \quad (1.7)$$

where the momentum p_1 is given by

$$p_1 = \frac{(E_0^2 + m_1^2 - m_2^2 - P_0^2) P_0 \cos \theta_1 + E_0 \sqrt{\lambda(E_0^2, m_1^2, m_2^2) + P_0^4 - 2P_0^2(E_0^2 + m_1^2 - 2m_1^2 \cos^2 \theta_1 - m_2^2)}}{2(E_0^2 - P_0^2 \cos^2 \theta_1)}. \quad (1.8)$$

CM frame result is recovered by setting $E_0 = \sqrt{s}$ and $P_0 = 0$.

1.2. Decay rate and Cross section

As $\langle \text{out} | \text{in} \rangle = (2\pi)^4 \delta^{(4)}(p_i - p_f) i\mathcal{M}$ (for $\text{in} \neq \text{out}$) and $\langle \mathbf{p} | \mathbf{p} \rangle = 2E_{\mathbf{p}} (2\pi)^3 \delta^{(3)}(\mathbf{0}) = 2E_{\mathbf{p}} V$ for one-particle state,

$$\frac{N_{\text{ev}}}{\prod_{\text{in}} N_{\text{particle}}} = \int d\Pi^{\text{out}} \frac{|\langle \text{out} | \text{in} \rangle|^2}{\langle \text{in} | \text{in} \rangle} = \int d\Pi^{\text{out}} \frac{(2\pi)^8 |\mathcal{M}|^2}{\prod_{\text{in}} (2E)V (2\pi^4)} \delta^{(4)}(p_i - p_f) = VT \int \overline{d\Pi^{N_f}} \frac{|\mathcal{M}|^2}{\prod_{\text{in}} (2E)V}. \quad (1.9)$$

Therefore, decay rate (at the rest frame) is given by

$$d\Gamma := \frac{1}{T} \frac{dN_{\text{ev}}}{N_{\text{particle}}} = \frac{1}{T} VT \overline{d\Pi^{N_f}} \frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0} \overline{d\Pi^{N_f}} |\mathcal{M}|^2. \quad (1.10)$$

We also define Lorentz-invariant cross section σ by $N_{\text{ev}} =: (\rho_A v_{\text{Mø}} T \sigma) N_B = (\rho_A v_{\text{Mø}} T \sigma) (\rho_B V)$, or

$$d\sigma := \frac{dN_{\text{ev}}}{\rho_A v_{\text{Mø}} T N_B} = \frac{V}{v_{\text{Mø}} T} VT \overline{d\Pi^{N_f}} \frac{|\mathcal{M}|^2}{2E_A 2E_B V^2} = \frac{1}{2E_A 2E_B v_{\text{Mø}}} \overline{d\Pi^{N_f}} |\mathcal{M}|^2. \quad (1.11)$$

where the Møller parameter $v_{\text{Mø}}$ is equal to $v_{\text{rel}}^{\text{NR}} = \|\mathbf{v}_A - \mathbf{v}_B\|$ if $\mathbf{v}_A \parallel \mathbf{v}_B$ (cf. Ref. [?]). Generally,

$$v_{\text{Mø}} := \frac{\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}}{E_A E_B} = \frac{p_A \cdot p_B}{E_A E_B} v_{\text{rel}} = (1 - \mathbf{v}_A \cdot \mathbf{v}_B) v_{\text{rel}}, \quad (1.12)$$

where v_{rel} is the actual relative velocity

$$v_{\text{rel}} = \sqrt{1 - \frac{(1 - v_A^2)(1 - v_B^2)}{1 - (\mathbf{v}_A \cdot \mathbf{v}_B)^2}} = \frac{\sqrt{\|\mathbf{v}_A - \mathbf{v}_B\|^2 - \|\mathbf{v}_A \times \mathbf{v}_B\|^2}}{1 - \mathbf{v}_A \cdot \mathbf{v}_B} = \frac{\lambda^{1/2}(s, m_A^2, m_B^2)}{s - (m_A^2 + m_B^2)} \neq v_{\text{rel}}^{\text{NR}}. \quad (1.13)$$

(Note that $p_A \cdot p_B / E_A E_B = 1$ if $\mathbf{p}_A = 0$ or $\mathbf{p}_B = 0$. Also, Each of v_{rel} , VT , and $E_A E_B v_{\text{Mø}}$ is Lorentz invariant.)

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation $\text{adj.} = (\epsilon^a)^{bc}$.^{*1}

$$T_a = \frac{1}{2}\sigma_a, \quad \text{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \quad [T_a, T_b] = i\epsilon^{abc}T^c, \quad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\bar{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a \epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\bar{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \rightarrow \epsilon^{ab}[\exp(i\theta^\alpha T^\alpha)]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp(i\theta^\alpha T^\alpha)]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp(-i\theta^\alpha T^{\alpha*})]^{ab}(\epsilon^{bc}\mathbf{2}^c). \quad (2.1)$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\bar{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation $\text{adj.} = \mathbf{8} = (f^a)^{bc}$.
Gell-Mann matrices:

$$\lambda_{1-8} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (2.2)$$

$$\tau_a = \frac{1}{2}\lambda_a, \quad \text{Tr}(\tau_a \tau_b) = \frac{1}{2}\delta_{ab}, \quad [\tau_a, \tau_b] = if^{abc}\tau^c, \quad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\begin{aligned} \mathbf{3}: \quad & \phi_a \rightarrow [\exp(i\theta^\alpha \tau^\alpha)]_{ab}\phi_b \simeq \phi_a + i\theta^\alpha \tau_{ab}^\alpha \phi_b & \bar{\mathbf{3}}: \quad & \phi_a \rightarrow [\exp(-i\theta^\alpha \tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - i\theta^\alpha \tau_{ab}^{\alpha*} \phi_b \\ & \phi_a^* \rightarrow [\exp(-i\theta^\alpha \tau^{\alpha*})]_{ab}\phi_b^* \simeq \phi_a^* - i\theta^\alpha \tau_{ab}^{\alpha*} \phi_b^* & & \phi_a^* \rightarrow [\exp(i\theta^\alpha \tau^\alpha)]_{ab}\phi_b^* \simeq \phi_a^* + i\theta^\alpha \tau_{ab}^\alpha \phi_b^* \\ & = \phi_b^*[\exp(-i\theta^\alpha \tau^\alpha)]_{ba} \simeq \phi_a^* - i\theta^\alpha \phi_b^* \tau_{ba}^\alpha & & \end{aligned}$$

^{*1}We do not distinguish sub- and superscripts for gauge indices.

3. Spinors

$$(\overline{\psi_1}\psi_2)^* = (\psi_2)^\dagger(\overline{\psi_1})^\dagger = \overline{\psi_2}\psi_1. \quad (3.1)$$

4. Minimal Supersymmetric Standard Model

Gauge symmetry: $SU(3)_{\text{color}} \times SU(2)_{\text{weak}} \times U(1)_Y$

Particle content:

(a) Chiral superfields						(b) Vector superfields			
	SU(3)	SU(2)	U(1)	B	L	scalar/spinor	SU(3)	SU(2)	U(1) ino/boson
Q_i	3	2	1/6	1/3		$\tilde{q}_L, q_L \rightarrow (u_L, d_L)$	g	adj.	\tilde{g}, g_μ
L_i		2	-1/2		1	$\tilde{l}_L, l_L \rightarrow (\nu_L, l_L)$	W		\tilde{w}, W_μ
U_i^c	$\bar{\mathbf{3}}$		-2/3	-1/3		\tilde{u}_R^c, u_R^c	B	adj.	\tilde{b}, B_μ
D_i^c	$\bar{\mathbf{3}}$		1/3	-1/3		\tilde{d}_R^c, d_R^c			
E_i^c			1		-1	\tilde{e}_R^c, e_R^c			
H_u		2	1/2			$h_u, \tilde{h}_u \rightarrow (h_u^+, h_u^0)$			
H_d		2	-1/2			$h_d, \tilde{h}_d \rightarrow (h_d^0, h_d^-)$			

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and SU(2) decomposition).

“c”-notation For scalars, $\tilde{\phi}_R^c := \phi_R^* = C\phi_R C$ (because the intrinsic phase for C is +1 for quarks and leptons.)

For matter spinors, $\psi_R^c := \bar{\psi}_R$ (and $\psi_R = \bar{\psi}_R^c$); Dirac spinors are thus

$$\psi_L = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \bar{\psi}_L = (0 \quad \bar{\psi}_L), \quad \psi_R^c := \begin{pmatrix} \psi_R^c \\ 0 \end{pmatrix} = C \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} = C\psi_R, \quad \bar{\psi}_R^c = (0 \quad \psi_R) = (\bar{\psi}_R \quad 0) C = \bar{\psi}_R C.$$

Superpotential and SUSY-terms

$$W_{\text{RPC}} = \mu H_u H_d - y_{uij} U_i^c H_u Q_j + y_{dij} D_i^c H_d Q_j + y_{eij} E_i^c H_d L_j, \quad (4.1)$$

$$W_{\text{RPV}} = -\kappa_i L_i H_u + \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2} \lambda''_{ijk} U_i^c D_j^c D_k^c, \quad (4.2)$$

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{2} \left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{w}\tilde{w} + M_1 \tilde{b}\tilde{b} + \text{H.c.} \right) - V_{\text{SUSY}}, \quad (4.3)$$

$$\begin{aligned} V_{\text{SUSY}}^{\text{RPC}} = & \left(\tilde{q}_L^* m_Q^2 \tilde{q}_L + \tilde{l}_L^* m_L^2 \tilde{l}_L + \tilde{u}_R^* m_{U^c}^2 \tilde{u}_R + \tilde{d}_R^* m_{D^c}^2 \tilde{d}_R + \tilde{e}_R^* m_{E^c}^2 \tilde{e}_R + m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 \right) \\ & + \left(-\tilde{u}_R^* h_u a_u \tilde{q}_L + \tilde{d}_R^* h_d a_d \tilde{q}_L + \tilde{e}_R^* h_d a_e \tilde{l}_L + b H_u H_d + \text{H.c.} \right) \\ & + \left(-\tilde{u}_R^* h_u^* c_u \tilde{q}_L + \tilde{d}_R^* h_u^* c_d \tilde{q}_L + \tilde{e}_R^* h_u^* c_e \tilde{l}_L + \text{H.c.} \right), \end{aligned} \quad (4.4)$$

$$\begin{aligned} V_{\text{SUSY}}^{\text{RPV}} = & \left(-b_i \tilde{l}_{Li} H_u + \frac{1}{2} T_{ijk} \tilde{l}_{Li} \tilde{l}_{Lj} \tilde{e}_{Rk}^* + T'_{ijk} \tilde{l}_{Li} \tilde{q}_{Lj} \tilde{d}_{Rk}^* + \frac{1}{2} T''_{ijk} \tilde{u}_{Ri}^* \tilde{d}_{Rj}^* \tilde{d}_{Rk}^* + \tilde{l}_{Li}^* M_{Li}^2 H_d + \text{H.c.} \right) \\ & + \left(C_{ijk}^1 \tilde{l}_{Li}^* \tilde{q}_{Lj} \tilde{u}_{Rk}^* + C_i^2 h_u^* h_d \tilde{e}_{Ri}^* + C_{ijk}^3 \tilde{d}_{Ri} \tilde{u}_{Rj}^* \tilde{e}_{Rk}^* + \frac{1}{2} C_{ijk}^4 \tilde{d}_{Ri} \tilde{q}_{Lj} \tilde{q}_{Lk} + \text{H.c.} \right), \end{aligned} \quad (4.5)$$

$$(\lambda_{ijk} = -\lambda_{jik}, \lambda''_{ijk} = -\lambda''_{ikj}, \text{ and } C_{ijk}^4 = C_{ikj}^4.)$$

We follow the notation of DHM [?, PhysRept] and Martin [?, v7] (but note that Martin uses $(-, +, +, +)$ -metric) for RPC part and SLHA2 convention for RPV part.

4.1. Lagrangian construction

As in Ref. [?],

$$\mathcal{L} = \frac{1}{2} \int d^4\theta \left[K(e^{2gV} \Phi, \Phi^\dagger) + K(\Phi, \Phi^\dagger e^{2gV}) \right] + \left[\int d^2\theta \left(W(\Phi) + \sum_{\text{gauges}} \frac{f_{ab}(\Phi)}{4} \mathcal{W}^{\alpha a} \mathcal{W}_\alpha^b \right) + \text{H.c.} \right] \quad (4.6)$$

4.2. Scalar potential

The MSSM scalar potential has contributions from F -terms and D -terms:

$$V_{\text{SUSY}} = F_i^* F_i + \frac{1}{2} D^a D^a; \quad F_i = -W_i^* = -\frac{\delta W^*}{\delta \phi_i^*}, \quad D^a = -g(\phi^* T^a \phi), \quad (4.7)$$

where T_a corresponds to the gauge-symmetry generator relevant for each ϕ . They are given by

$$-F_{h_u^a}^* = \epsilon^{ab} \left(-\tilde{u}_R^{x*} y_u \tilde{q}_L^b + \mu h_d^b + \mu'_i \tilde{l}_{Li}^b \right), \quad (4.8)$$

$$-F_{h_d^a}^* = \epsilon^{ab} \left(\tilde{e}_R^{x*} y_e \tilde{l}_L^b + \tilde{d}_R^{x*} y_d \tilde{q}_L^b - \mu h_u^b \right), \quad (4.9)$$

$$-F_{\tilde{q}_{Li}^{ax}}^* = \epsilon^{ab} \left(-y_{dji} h_d^b \tilde{d}_{Rj}^{x*} + y_{uji} h_u^b \tilde{u}_{Rj}^{x*} - \lambda'_{kij} \tilde{d}_{Rj}^{x*} \tilde{l}_{Lk}^b \right), \quad (4.10)$$

$$-F_{\tilde{u}_{Ri}^{xx}}^* = -y_{uij} h_u \tilde{q}_{Lj}^x + \frac{1}{2} \epsilon^{xyz} \lambda''_{ijk} \tilde{d}_{Rj}^{y*} \tilde{d}_{Rk}^{z*}, \quad (4.11)$$

$$-F_{\tilde{d}_{Ri}^{xx}}^* = y_{dij} h_d \tilde{q}_{Lj}^x + \lambda'_{jki} \tilde{l}_{Lj}^x \tilde{q}_{Lk}^x - \lambda''_{jik} \epsilon^{xyz} \tilde{u}_{Rj}^{y*} \tilde{d}_{Rk}^{z*}, \quad (4.12)$$

$$-F_{\tilde{l}_{Li}^a}^* = \epsilon^{ab} \left(-y_{eji} \tilde{e}_{Rj}^{x*} h_d^b - \mu'_i h_u^b + \lambda_{ijk} \tilde{l}_{Lj}^b \tilde{e}_{Rk}^{x*} + \lambda'_{ijk} \tilde{q}_{Lj}^{bx} \tilde{d}_{Rk}^{x*} \right), \quad (4.13)$$

$$-F_{\tilde{e}_{Ri}^a}^* = y_{eij} h_d \tilde{l}_{Lj} + \frac{1}{2} \lambda_{jki} \tilde{l}_{Lj} \tilde{l}_{Lk}. \quad (4.14)$$

$$D_{\text{SU}(3)}^\alpha = -g_3 \sum_{i=1}^3 \left(\sum_{a=1,2} \tilde{q}_{Li}^{a*} \tau^\alpha \tilde{q}_{Li}^a - \tilde{u}_{Ri}^* \tau^\alpha \tilde{u}_{Ri} - \tilde{d}_{Ri}^* \tau^\alpha \tilde{d}_{Ri} \right), \quad (4.15)$$

$$D_{\text{SU}(2)}^\alpha = -g_2 \left[\sum_{i=1}^3 \left(\sum_{x=1}^3 \tilde{q}_{Li}^{x*} T^\alpha \tilde{q}_{Li}^x + \tilde{l}_{Li}^* T^\alpha \tilde{l}_{Li} \right) + h_u^* T^\alpha h_u + h_d^* T^\alpha h_d \right], \quad (4.16)$$

$$D_{\text{U}(1)} = -g_1 \left(\frac{1}{6} |\tilde{q}_L|^2 - \frac{1}{2} |\tilde{l}_L|^2 - \frac{2}{3} |\tilde{u}_R|^2 + \frac{1}{3} |\tilde{d}_R|^2 + |\tilde{e}_R|^2 + \frac{1}{2} |h_u|^2 - \frac{1}{2} |h_d|^2 \right). \quad (4.17)$$

Combining these, the full SUSY scalar potential is given by

$$\begin{aligned}
V &= V_{\text{SUSY}} + V_{\text{SUSY}}; \tag{4.18} \\
V_{\text{SUSY}} &= |h_u|^2 (|\mu|^2 + |\mu'|^2) + |\mu|^2 |h_d|^2 + \left(\mu_i^* \tilde{\mu}_{Li}^* h_d + \text{H.c.} \right) + \mu_i^* \mu_j' \tilde{\mu}_{Li}^* \tilde{L}_j \\
&+ \left[-y_{uij} \mu_i^* \tilde{u}_d^* \tilde{u}_{Ri}^* \tilde{q}_{Lj} - y_{uij} \mu_k' \tilde{u}_{Ri}^* \tilde{q}_{Lj} \tilde{L}_k^* - (y_{dij} \mu_i^* + \lambda_{kji}' \mu_k') h_u^* \tilde{d}_{Ri}^* \tilde{q}_{Lj} \right. \\
&\quad \left. + y_{eij} \mu_j' \tilde{e}_{Ri}^* h_u^* h_d + (\lambda_{jki} \mu_k' - y_{eij} \mu_i^*) h_u^* \tilde{e}_{Ri}^* \tilde{L}_j + \text{H.c.} \right] \\
&+ \frac{1}{8} (g_2^2 + g_Y^2) |h_d|^4 + \frac{1}{8} (g_2^2 + g_Y^2) |h_u|^4 + \left(-\frac{g_2^2}{4} |h_d|^2 |h_u|^2 - \frac{g_Y^2}{4} |h_d|^2 |h_u|^2 + \frac{g_2^2}{2} |h_d^* h_u|^2 \right) \\
&+ \left(-\frac{g_2^2}{4} |h_u|^2 |\tilde{q}_L|^2 + \frac{g_Y^2}{12} |h_u|^2 |\tilde{q}_L|^2 + \frac{g_2^2}{2} |h_u^b \tilde{q}_{Li}|^2 + \epsilon^{ac} \epsilon^{bd} (y_u^\dagger y_u)_{ji} h_u^a h_u^{b*} \tilde{q}_{Li}^c \tilde{q}_{Lj}^{d*} \right) \\
&+ \left(-\frac{g_2^2}{4} |h_d|^2 |\tilde{q}_L|^2 - \frac{g_Y^2}{12} |h_d|^2 |\tilde{q}_L|^2 + \frac{g_2^2}{2} |h_d^* \tilde{q}_{Li}|^2 + \epsilon^{ac} \epsilon^{bd} (y_d^\dagger y_d)_{ji} h_d^a h_d^{b*} \tilde{q}_{Li}^c \tilde{q}_{Lj}^{d*} \right) \\
&+ \left(-\frac{g_2^2}{4} |h_u|^2 |\tilde{L}_L|^2 - \frac{g_Y^2}{4} |h_u|^2 |\tilde{L}_L|^2 + \frac{g_2^2}{2} |h_u^* \tilde{L}_{Li}|^2 \right) \\
&+ \left(-\frac{g_2^2}{4} |h_d|^2 |\tilde{L}_L|^2 + \frac{g_Y^2}{4} |h_d|^2 |\tilde{L}_L|^2 + \frac{g_2^2}{2} |h_d^* \tilde{L}_{Li}|^2 + \epsilon^{ac} \epsilon^{bd} (y_e^\dagger y_e)_{ji} h_d^a h_d^{b*} \tilde{L}_{Li}^c \tilde{L}_{Lj}^{d*} \right) \\
&+ |h_u|^2 \left(-\frac{g_Y^2}{3} |\tilde{u}_R|^2 + (y_u y_u^\dagger)_{ij} \tilde{u}_{Ri}^* \tilde{u}_{Rj} \right) + \frac{g_Y^2}{3} |h_d|^2 |\tilde{u}_R|^2 \\
&+ \frac{g_Y^2}{6} |h_u|^2 |\tilde{d}_R|^2 + |h_d|^2 \left(-\frac{g_Y^2}{6} |\tilde{d}_R|^2 + (y_d y_d^\dagger)_{ij} \tilde{d}_{Ri}^* \tilde{d}_{Rj} \right) - \left[(y_u y_d^\dagger)_{ij} \tilde{u}_{Ri}^* \tilde{d}_{Rj} (h_d^* h_u) + \text{H.c.} \right] \\
&+ \frac{g_Y^2}{2} |h_u|^2 |\tilde{e}_R|^2 + |h_d|^2 \left(-\frac{g_Y^2}{2} |\tilde{e}_R|^2 + (y_e y_e^\dagger)_{ij} \tilde{e}_{Ri}^* \tilde{e}_{Rj} \right) \\
&+ \left[-\frac{1}{2} \epsilon^{ab} \epsilon^{xyz} y_{ulk} \lambda_{lij}'' h_u^a \tilde{d}_{Ri}^x \tilde{q}_{Rj}^y \tilde{q}_{Lk}^{bz} + \epsilon^{ab} \epsilon^{xyz} y_{dlk} \lambda_{ijl}'' h_d^a \tilde{u}_{Ri}^x \tilde{q}_{Rj}^y \tilde{q}_{Lk}^{bz} - y_{uil} \lambda_{klj}' h_u^a \tilde{u}_{Ri}^* \tilde{d}_{Rj} \tilde{L}_{Lk}^{a*} \right. \\
&\quad \left. + y_{dil} \lambda_{klj}' h_d^a \tilde{d}_{Ri}^* \tilde{d}_{Rj} \tilde{L}_{Lk}^{a*} - \epsilon^{ab} \epsilon^{cd} y_{dli} \lambda_{kjl}' h_d^a \tilde{q}_{Li}^b \tilde{q}_{Lj}^{c*} \tilde{L}_{Lk}^{d*} + y_{eil} \lambda_{klj}' h_d^a \tilde{e}_{Ri}^* \tilde{e}_{Rj} \tilde{L}_{Lk}^{a*} \right. \\
&\quad \left. - y_{ejl} \lambda_{kli}' h_d^a \tilde{e}_{Rj}^* \tilde{d}_{Ri} \tilde{q}_{Lk}^{a*} + \frac{1}{2} \epsilon^{ab} \epsilon^{cd} y_{eli} \lambda_{jkl}' h_d^a \tilde{L}_{Li}^b \tilde{L}_{Lj}^{c*} \tilde{L}_{Lk}^{d*} + \text{H.c.} \right] \\
&+ \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8} \right) |\tilde{q}_L|^4 + \frac{g_2^2}{4} \tilde{q}_{Li}^{ax} \tilde{q}_{Lj}^{by} \tilde{q}_{Li}^{bx*} \tilde{q}_{Lj}^{ay*} + \frac{g_3^2}{4} \tilde{q}_{Li}^{ax} \tilde{q}_{Lj}^{by} \tilde{q}_{Li}^{ay*} \tilde{q}_{Lj}^{bx*} \right] \\
&+ \left[\left(-\frac{g_3^2}{12} + \frac{2g_Y^2}{9} \right) |\tilde{u}_R|^4 + \frac{g_2^2}{4} \tilde{u}_{Ri}^{x*} \tilde{u}_{Rj}^{y*} \tilde{u}_{Ri}^y \tilde{u}_{Rj}^x \right] \\
&+ \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18} \right) |\tilde{d}_R|^4 + \frac{g_2^2}{4} \tilde{d}_{Ri}^{x*} \tilde{d}_{Rj}^{y*} \tilde{d}_{Ri}^y \tilde{d}_{Rj}^x + \frac{1}{2} \lambda_{mij}'' \lambda_{mkl}'' \tilde{d}_{Ri}^{x*} \tilde{d}_{Rj}^{y*} \tilde{d}_{Rk}^x \tilde{d}_{Rl}^y \right] \\
&+ \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9} \right) |\tilde{u}_R|^2 |\tilde{q}_L|^2 - \frac{g_2^2}{2} \tilde{u}_{Ri}^{x*} \tilde{q}_{Lj}^{ax} \tilde{u}_{Ri}^y \tilde{q}_{Lj}^{ay*} + y_{uik} y_{ujl} \tilde{u}_{Ri}^{x*} \tilde{u}_{Rj}^y \tilde{q}_{Lk}^{ax} \tilde{q}_{Ll}^{ay*} \right] \\
&+ \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18} \right) |\tilde{d}_R|^2 |\tilde{q}_L|^2 - \frac{g_2^2}{2} \tilde{d}_{Ri}^{x*} \tilde{q}_{Lj}^{ax} \tilde{d}_{Ri}^y \tilde{q}_{Lj}^{ay*} + (y_{dik} y_{djl}^* + \lambda_{mki}' \lambda_{mlj}') \tilde{d}_{Ri}^{x*} \tilde{q}_{Lk}^{ax} \tilde{d}_{Rj}^y \tilde{q}_{Ll}^{ay*} \right] \\
&+ \left[-\left(\frac{g_3^2}{6} + \frac{2g_Y^2}{9} \right) |\tilde{d}_R|^2 |\tilde{u}_R|^2 + \left(\frac{g_3^2}{2} - \lambda_{ikm}'' \lambda_{ljm}'' \right) \tilde{u}_{Ri}^{x*} \tilde{d}_{Rj}^x \tilde{d}_{Rj}^{y*} \tilde{u}_{Ri}^y + \lambda_{ikm}'' \lambda_{jlm}'' \tilde{u}_{Ri}^{x*} \tilde{u}_{Rj}^x \tilde{d}_{Rk}^{y*} \tilde{d}_{Rl}^y \right] \\
&+ \left[-\left(\frac{g_2^2}{4} + \frac{g_Y^2}{12} \right) |\tilde{L}_L|^2 |\tilde{q}_L|^2 + \frac{g_2^2}{2} \tilde{q}_{Li}^{ax} \tilde{q}_{Li}^{bx*} \tilde{L}_{Lj}^b \tilde{L}_{Lj}^{a*} + \epsilon^{ac} \epsilon^{bd} \lambda_{kim}' \lambda_{ljm}' \tilde{q}_{Li}^{ax} \tilde{q}_{Lj}^{bx*} \tilde{L}_{Lk}^c \tilde{L}_{Ll}^{d*} \right] + \frac{g_Y^2}{6} |\tilde{e}_R|^2 |\tilde{q}_L|^2 \\
&+ \left(-\frac{g_Y^2}{6} |\tilde{d}_R|^2 |\tilde{L}_L|^2 + \lambda_{lmi}' \lambda_{kmj}' \tilde{d}_{Ri}^* \tilde{d}_{Rj} \tilde{L}_{Lk}^* \tilde{L}_{Ll} \right) + \frac{g_Y^2}{3} |\tilde{u}_R|^2 |\tilde{L}_L|^2 - \frac{2g_Y^2}{3} |\tilde{u}_R|^2 |\tilde{e}_R|^2 + \frac{g_Y^2}{3} |\tilde{d}_R|^2 |\tilde{e}_R|^2 \\
&+ \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8} \right) |\tilde{L}_L|^4 + \frac{g_2^2}{4} \tilde{L}_{Li}^a \tilde{L}_{Lj}^b \tilde{L}_{Li}^{b*} \tilde{L}_{Lj}^{a*} + \frac{1}{4} \epsilon^{ab} \epsilon^{cd} \lambda_{ijm} \lambda_{klm}^* \tilde{L}_{Li}^a \tilde{L}_{Lj}^b \tilde{L}_{Lk}^{c*} \tilde{L}_{Ll}^{d*} \right] \\
&+ \frac{g_Y^2}{2} |\tilde{e}_R|^4 + \left[-\frac{g_Y^2}{2} |\tilde{e}_R|^2 |\tilde{L}_L|^2 + (y_{eik} y_{ejl}^* + \lambda_{kmi} \lambda_{lmj}^*) \tilde{e}_{Ri}^* \tilde{e}_{Rj} \tilde{L}_{Lk}^* \tilde{L}_{Ll} \right] \\
&+ \left[(y_{dik} y_{ejl}^* - \lambda_{mki}' \lambda_{lmj}') \tilde{d}_{Ri}^{x*} \tilde{q}_{Lk}^{ax} \tilde{e}_{Rj} \tilde{L}_{Ll}^{a*} - \epsilon^{ab} \epsilon^{xyz} \lambda_{lkm}' \lambda_{ijm}'' \tilde{L}_{Li}^b \tilde{q}_{Lk}^{az} \tilde{d}_{Rj}^y \tilde{u}_{Ri}^x + \text{H.c.} \right].
\end{aligned}
\tag{4.19}$$

4.3. SLHA convention

The SLHA convention [?] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table, while

$\mu, b, m_{Q,L,H_u,H_d}^2$, RPV-trilinears (λ s and T s) are in common.

SLHA	our notation	Martin/DHM
<hr/>		
(H_1, H_2)	$=$	(H_d, H_u)
<hr/>		
$Y_{u,d,e}$	$=$	$(y_{u,d,e})^T$
$T_{u,d,e}$	$=$	$(a_{u,d,e})^T$
$A_{u,d,e}$	$=$	$(A_{u,d,e})^T$
m_{U^c,D^c,E^c}^2	$=$	$(m_{U^c,D^c,E^c}^2)^\dagger$
$M_{1,2,3}$	$=$	$-M_{1,2,3}$
m_3^2	$=$	b
m_A^2	$=$	$m_{A_0}^2$ (tree)
<hr/>		
		$\kappa_i = -\mu'_i$ (rarely used)
D_i	$=$	b_i
$m_{\tilde{L}_i H_1}^2$	$=$	$M_{L_i}^2$
<hr/>		

A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [?]:

$$e^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \quad (\text{converges for any } X), \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(X-I)^m}{m} \quad (\text{conv. if } \|X-I\| < 1). \quad (\text{A.1})$$

$$e^{\log A} = A \quad (\text{if } \|A-I\| < 1), \quad \log e^X = X \text{ and } \|e^X - I\| < 1 \quad (\text{if } \|X\| < \log 2). \quad (\text{A.2})$$

$$\text{Hilbert-Schmidt norm: } \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \text{Tr } X^\dagger X. \quad (\text{A.3})$$

Properties:

$$e^{(X^T)} = (e^X)^T, \quad e^{(X^*)} = (e^X)^*, \quad (e^X)^{-1} = e^{-X}, \quad e^{YXY^{-1}} = Y e^X Y^{-1},$$

$$\det \exp X = \exp \text{Tr } X, \quad \frac{d}{dt} e^{tX} = X e^{tX} = e^{tX} X \quad e^{(\alpha+\beta)X} = e^{\alpha X} e^{\beta X} \text{ for } \alpha, \beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$e^X Y e^{-X} = Y + [X, Y] + \frac{1}{2!} [X, [X, Y]] + \frac{1}{3!} [X, [X, [X, Y]]] + \cdots; \quad (\text{A.4})$$

$$\log(e^X e^Y) = X + \int_0^1 dt g(e^{tX}, e^{tY}) Y \quad \left[g(z) = \frac{\log z}{1-z^{-1}} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (z-1)^n}{n(n+1)} \right] \quad (\text{A.5})$$

$$= X + Y + \frac{1}{2} [X, Y] + \frac{1}{12} [X, [X, Y]] - \frac{1}{12} [Y, [X, Y]] + \cdots \quad (\text{Baker-Campbell-Hausdorff}). \quad (\text{A.6})$$