1 General Definitions and Tools

1.1 Notations and Conventions

◆1.1.1 Metric etc.

Minkowski Metric : $\eta^{\mu\nu} := \mathrm{diag}(+,-,-,-)$; $\epsilon^{0123}_{0123} := \pm 1$

Coordinates : $x^\mu := (t,x,y,z)$; therefore $\partial_\mu = \left(\frac{\partial}{\partial t}, \nabla\right)$.

Gamma Matrices : $\{\gamma^\mu,\gamma^\nu\} := 2\eta^{\mu\nu}$; $\gamma_5 := \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \dfrac{-\mathrm{i}}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$

: therefore $\{\gamma^\mu,\gamma_5\} = 0, (\gamma_5)^2 = 1$.

Gamma Combinations : $1, \{\gamma^\mu\}, \{\sigma^{\mu\nu}\}, \{\gamma^\mu\gamma_5\}, \gamma_5$; $\sigma^{\mu\nu} := \frac{\mathrm{i}}{2}[\gamma^\mu,\gamma^\nu] = 0 \big/ \mathrm{i}\gamma^\mu\gamma^\nu$

Spinor ϵ and σ matrices : $\epsilon^{12} = \epsilon^{\dot 1\dot 2} = \epsilon_{21} = \epsilon_{\dot 2\dot 1} = 1$

: $(\sigma^\mu)_{\alpha\dot\beta} := (1,\boldsymbol{\sigma})_{\alpha\dot\beta}$, $(\bar\sigma^\mu)^{\dot\alpha\alpha} := \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}(\sigma^\mu)_{\beta\dot\beta} = (1,-\boldsymbol{\sigma})^{\dot\alpha\beta}$.

> Pauli Matrices : $\sigma_0 = \left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$, $\sigma_1 = \left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right)$, $\sigma_2 = \left(\begin{smallmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{smallmatrix}\right)$, $\sigma_3 = \left(\begin{smallmatrix}1&0\\0&-1\end{smallmatrix}\right)$,
>
> : $\sigma_+ = \frac{1}{2}(\sigma_1 + \mathrm{i}\sigma_2) = \left(\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right)$, $\sigma_- = \frac{1}{2}(\sigma_1 - \mathrm{i}\sigma_2) = \left(\begin{smallmatrix}0&0\\1&0\end{smallmatrix}\right)$,
>
> : $\sigma^\mu := (1,\boldsymbol{\sigma})$, $\bar\sigma^\mu := (1,-\boldsymbol{\sigma})$.

Fourier Transformation : $\widetilde{f}(k) := \displaystyle\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}kx} f(x)$; $f(x) = \displaystyle\int \frac{\mathrm{d}^4k}{(2\pi)^4}\ \mathrm{e}^{-\mathrm{i}kx}\widetilde{f}(k)$.

◆1.1.2 Fields

Scalar : $(\partial^2 + m^2)\phi = 0$; $\phi(x) = \displaystyle\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol p}}}\left[a_{\boldsymbol p}\mathrm{e}^{-\mathrm{i}px} + b^\dagger_{\boldsymbol p}\mathrm{e}^{\mathrm{i}px}\right]$

Dirac : $(\mathrm{i}\partial\!\!\!/ - m)\psi = 0$; $\psi(x) = \displaystyle\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol p}}}\sum_{s=1,2}\left[a^s_{\boldsymbol p}u^s(p)\mathrm{e}^{-\mathrm{i}px} + b^{s\dagger}_{\boldsymbol p}v^s(p)\mathrm{e}^{\mathrm{i}px}\right]$

Vector : $\partial^2 A^\mu = 0$; $A^\mu(x) = \displaystyle\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol p}}}\sum_{r=0..3}\left[a^r_{\boldsymbol p}\epsilon^r(p)\mathrm{e}^{-\mathrm{i}px} + a^{r\dagger}_{\boldsymbol p}\epsilon^{r*}(p)\mathrm{e}^{\mathrm{i}px}\right]$

TODO: 南部–Goldstone; Gravitino

◆1.1.3 Electromagnetism

Electromagnetic Fields : $A^\mu = (\phi, \boldsymbol{A})$ 【We can invert the signs, but cannot lower the index.】

Maxwell Equations : $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$; $\epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0$, $\partial_\mu F^{\mu\nu} = ej^\nu$

Our Old Language : $\nabla\cdot\boldsymbol{B} = 0$, $\nabla\times\boldsymbol{E} + \dfrac{\partial}{\partial t}\boldsymbol{B} = 0$; $\nabla\cdot\boldsymbol{E} = ej^0$, $(\nabla\times\boldsymbol{B})_i - \dfrac{\partial}{\partial t}E_i = ej^i$.

: $F_{\mu\nu} = \begin{pmatrix} 0 & & \boldsymbol{E} & \\ & 0 & -B_3 & B_2 \\ -\boldsymbol{E} & B_3 & 0 & -B_1 \\ & -B_2 & B_1 & 0 \end{pmatrix}$; $F_{\mu\nu}F^{\mu\nu} = -2\left(\|\boldsymbol{E}\|^2 - \|\boldsymbol{B}\|^2\right)$

1.2 Spinor Fields

Spinor : ξ_α, $\xi^\alpha := \epsilon^{\alpha\beta}\xi_\beta$; Lorentz tr. : $\xi_\alpha \mapsto \Lambda_\alpha{}^\beta \xi_\beta$, $\xi^\alpha \mapsto \xi^\beta \Lambda^{-1}{}_\beta{}^\alpha$;

: $\bar\eta^{\dot\alpha} := (\eta^\alpha)^*$ $\bar\eta_{\dot\alpha} := (\eta_\alpha)^*$: $\bar\eta^{\dot\alpha} \mapsto \Lambda^{\dagger -1\dot\alpha}{}_{\dot\beta}\bar\eta^{\dot\beta}$, $\bar\eta_{\dot\alpha} \mapsto \bar\eta_{\dot\beta}\Lambda^{\dagger\dot\beta}{}_{\dot\alpha}$.

Kinetic term : $\mathrm{i}\,\bar\xi\bar\sigma^\mu\partial_\mu\xi \quad (= \mathrm{i}\,\eta\sigma^\mu\partial_\mu\bar\eta\ )$

Mass term : [Majorana] $-\frac{1}{2}(m_{\mathrm{M}}\xi\xi + m_{\mathrm{M}}^*\bar\xi\bar\xi)$ [Dirac] $-(m_{\mathrm{D}}\xi\eta + m_{\mathrm{D}}^*\bar\xi\bar\eta)$

Dirac fermion : $\mathcal{L}_{\mathrm{Dirac}} = \mathrm{i}\,\bar\xi\bar\sigma^\mu\partial_\mu\xi + \mathrm{i}\,\eta\sigma^\mu\partial_\mu\bar\eta - m(\xi\eta + \bar\xi\bar\eta) = \overline{\psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi$

Majorana fermion : $\mathcal{L}_{\mathrm{Majorana}} = \mathrm{i}\,\bar\xi\bar\sigma^\mu\partial_\mu\xi - \frac{m}{2}(\xi\xi + \bar\xi\bar\xi) = \frac{1}{2}\overline{\psi}_{\mathrm{M}}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi_{\mathrm{M}}$

Charge conjugate : $\psi^{\mathrm{C}} := C(\overline{\psi})^{\mathrm{T}} \quad [(\psi_{\mathrm{M}})^{\mathrm{C}} = \psi_{\mathrm{M}}]$

◆1.2.1 Chiral Notation (Peskin)

Gamma Matrices : $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \quad P^{\mathrm{R}}_{\mathrm{L}} = \frac{1 \pm \gamma_5}{2}.$

Fields : $\psi = \begin{pmatrix} \xi_\alpha \\ \bar\eta^{\dot\alpha} \end{pmatrix} = \begin{pmatrix} \psi_{\mathrm{L}} \\ \psi_{\mathrm{R}} \end{pmatrix}; \quad \overline{\psi} = \psi^\dagger\gamma^0 = \begin{pmatrix} \eta^\alpha & \bar\xi_{\dot\alpha} \end{pmatrix} = \begin{pmatrix} \psi_{\mathrm{R}}^\dagger & \psi_{\mathrm{L}}^\dagger \end{pmatrix}; \quad \psi_{\mathrm{M}} = \begin{pmatrix} \xi_\alpha \\ \bar\xi^{\dot\alpha} \end{pmatrix}.$

: $u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar\sigma}\xi^s \end{pmatrix}; \; v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\eta^s \\ -\sqrt{p\cdot\bar\sigma}\eta^s \end{pmatrix}$

: $\left[\eta^s = \xi^{-s} := -\mathrm{i}\sigma^2(\xi^s)^* = (\xi^2, -\xi^1)\right].$

Charge conj. : $C := -\mathrm{i}\gamma^2\gamma^0 = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot\alpha\dot\beta} \end{pmatrix} \qquad \begin{cases} -C = C^{-1} = C^\dagger = C^{\mathrm{T}} \\ C = C^* \end{cases}, \; C^{-1}\gamma^\mu C = -\gamma^{\mu\mathrm{T}}.$

: $\psi^{\mathrm{C}} = C(\overline{\psi})^{\mathrm{T}} = -\mathrm{i}\gamma^2\psi^* = \begin{pmatrix} \eta_\alpha \\ \bar\xi^{\dot\alpha} \end{pmatrix}, \quad \overline{\psi}^{\mathrm{C}} = \psi^{\mathrm{T}}C = \mathrm{i}\overline{\psi}^*\gamma^2$

Weyl Equations : $\mathrm{i}\bar\sigma\cdot\partial\psi_{\mathrm{L}} = m\psi_{\mathrm{R}}; \quad \mathrm{i}\sigma\cdot\partial\psi_{\mathrm{R}} = m\psi_{\mathrm{L}}$

z-boost limit : Halt: $u^s = \sqrt{m}\left(\begin{smallmatrix} \xi^s \\ \xi^s \end{smallmatrix}\right)$, $v^s = \sqrt{m}\left(\begin{smallmatrix} \eta^s \\ -\eta^s \end{smallmatrix}\right)$;

: Slow: $\sqrt{p\cdot\sigma} \simeq \sqrt{m}(1 - \boldsymbol{v}\cdot\boldsymbol{\sigma}/2)$, $\sqrt{p\cdot\bar\sigma} \simeq \sqrt{m}(1 + \boldsymbol{v}\cdot\boldsymbol{\sigma}/2)$;

: Extreme: $u^s = \sqrt{2E}\begin{pmatrix} \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)\xi^s \\ \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)\xi^s \end{pmatrix}$, $v^s = \sqrt{2E}\begin{pmatrix} \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)\eta^s \\ -\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)\eta^s \end{pmatrix}$.

◆1.2.2 Dirac Notation

Gamma Matrices : $\hat\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat\gamma^i = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \hat\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \hat P^{\mathrm{R}}_{\mathrm{L}} = \frac{1 \pm \gamma_5}{2}.$

: $\hat\sigma^{0i} = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad \hat\sigma^{ij} = \epsilon^{ijk}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}.$

Fields : $\hat\psi = \begin{pmatrix} \psi_{\mathrm{A}} \\ \psi_{\mathrm{B}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi_{\mathrm{L}} + \psi_{\mathrm{R}} \\ -\psi_{\mathrm{L}} + \psi_{\mathrm{R}} \end{pmatrix}; \quad \hat\psi_{\mathrm{M}} = \begin{pmatrix} \psi_{\mathrm{A}} \\ \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)\psi_{\mathrm{A}}^* \end{pmatrix}.$

: $\overline{\hat\psi} = \hat\psi^\dagger\gamma^0 = \begin{pmatrix} \psi_{\mathrm{A}}^\dagger & -\psi_{\mathrm{B}}^\dagger \end{pmatrix}$

: $\hat u^s(p) = \begin{pmatrix} \sqrt{p^0 + m}\,\xi^s \\ \frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{\sqrt{p^0+m}}\xi^s \end{pmatrix}; \; \hat v^s(p) = \begin{pmatrix} -\frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{\sqrt{p^0+m}}\eta^s \\ -\sqrt{p^0 + m}\,\eta^s \end{pmatrix}$

: $\left[\eta^s = \xi^{-s} := -\mathrm{i}\sigma^2(\xi^s)^* = (\xi^2, -\xi^1)\right]$

Charge conj. : $\hat{C} = -\mathrm{i}\hat{\gamma}^2\hat{\gamma}^0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ with $C = -C^{-1} = -C^\dagger$, $C^{-1}\gamma^\mu C = -\gamma^{\mu\mathrm{T}}$.

z-boost limit : Halt: $\hat{u}^s = \sqrt{2m}\left(\begin{smallmatrix} \xi^s \\ 0 \end{smallmatrix}\right)$, $\hat{v}^s = -\sqrt{2m}\left(\begin{smallmatrix} 0 \\ \eta^s \end{smallmatrix}\right)$;

: Slow: $\sqrt{p^0+m} \simeq \sqrt{2m}(1+\frac{v^2}{8})$, $\frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{\sqrt{p^0+m}} \simeq \sqrt{\frac{m}{2}}(\boldsymbol{v}\cdot\boldsymbol{\sigma})$;

: Extreme: $\hat{u}^s = \sqrt{E}\begin{pmatrix} \xi^s \\ \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)\xi^s \end{pmatrix}$, $\hat{v}^s = -\sqrt{E}\begin{pmatrix} \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)\eta^s \\ \eta^s \end{pmatrix}$

◆1.2.3 CPT transformations

【Note that these expressions are valid under the above frameworks.】

In the following, CP means "P, then C" **in algebraic sense**. Be careful to the order.

$$\psi(t,\boldsymbol{x}) \xrightarrow{P} \eta_P\gamma^0\psi(t,-\boldsymbol{x}) \qquad \overline{\psi} \xrightarrow{P} \eta_P^*\overline{\psi}\gamma^0$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{T} \eta_T C\gamma_5\psi(-t,\boldsymbol{x}) \qquad \overline{\psi} \xrightarrow{T} -\eta_T^*\overline{\psi}C\gamma_5$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{C} \eta_C C\overline{\psi}^{\mathrm{T}}(t,\boldsymbol{x}) = C\gamma^0\psi^* \qquad \overline{\psi} \xrightarrow{C} \eta_C^*\overline{\psi}^*\gamma^0 C = -\eta_C^*(C\psi)^{\mathrm{T}}$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{CP} \eta_{CP}(\overline{\psi}\gamma^0 C)^{\mathrm{T}} \qquad \overline{\psi} \xrightarrow{CP} \eta_{CP}^*(C\gamma^0\psi)^{\mathrm{T}}$$

$$\psi(t,\boldsymbol{x}) \xrightarrow{CPT} (\overline{\psi}\gamma^0\gamma_5)^{\mathrm{T}} \qquad \overline{\psi} \xrightarrow{CPT} (\gamma^0\gamma_5\psi)^{\mathrm{T}}$$

Note that T-transformation is anti-unitary, and $\eta_{CPT} = 1$. Especially, photon is $(P,T,C) = (-,+,-)$.

	ϕ	A^μ	$\bar{\psi}\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$	$\bar{\psi}\gamma^\mu\gamma_5\psi$	$\mathrm{i}\bar{\psi}\gamma_5\psi$	∂_μ
P	ϕ	$-{+}{+}{+}A^\mu$	$+$	$+---$	$(+---)(+---)$	$-{+}{+}{+}$	$-$	$+---$
T	ϕ	$+---A^\mu$	$+$	$+---$	$-(+---)(+---)$	$+---$	$-$	$-{+}{+}{+}$
C	ϕ^*	$+A^{\mu*}$	$+$	$-$	$-$	$+$	$+$	$+$
CPT	ϕ^*	$-A^{\mu*}$	$+$	$-$	$+$	$-$	$+$	$-$

◆1.2.4 Noether current

Infinitesimal transformation : $\phi(x) \mapsto \phi'(x) := \phi(x) + \alpha\Delta\phi(x)$

Correspondent transformation : $\alpha\Delta\mathcal{L} = \alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi\right) + \alpha\cancel{\left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\right]}\Delta\phi$

: So, defining $\alpha\partial_\mu\mathcal{J}^\mu(x) := \mathcal{L}'(x) - \mathcal{L}(x)$,

Noether current : $j^\mu(x) := \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi - \mathcal{J}^\mu$; $\partial_\mu j^\mu(x) = 0$

Noether charge : $Q := \int j^0\mathrm{d}^3x$

Energy-momentum tensor : $T^\mu{}_\nu = \partial_\mu\mathcal{L}(\partial_\mu\phi)\partial_\nu\phi - \mathcal{L}\delta^\mu_\nu$; $\mathcal{H} = T^{00}$, $\mathcal{P}^i = T^{0i}$.

: $T^\mu{}_\nu$ is the variation along μ in respect to the modification a^ν.

1.3 Feynman Rules

■Scalar Boson

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$\wick{\c\phi\ \c\phi} = \qquad = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$$

$$\mathcal{L} \supset |\partial_\mu\phi|^2 - m^2|\phi|^2$$

$$\wick{\c\phi^*\ \c\phi} = \qquad = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$$

(External lines equal to 1 in both cases.)

■Dirac Fermion

$$\mathcal{L} \supset \overline{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi$$
$$= \mathrm{i}\bar\xi\bar\sigma^\mu\partial_\mu\xi + \mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - m(\xi\chi + \bar\xi\bar\chi)$$

Initial state

$$\wick{\c\psi\ |\c{\boldsymbol{p}}, s\rangle} = \qquad = u^s(p)$$

$$\wick{\c{\overline{\psi}}\ |\c{\boldsymbol{p}}, s\rangle} = \qquad = \overline{v}^s(p)$$

Final state

$$\wick{\langle\boldsymbol{p}, \c s|\ \c{\overline{\psi}}} = \qquad = \overline{u}^s(p)$$

$$\wick{\langle\boldsymbol{p}, \c s|\ \c\psi} = \qquad = v^s(p)$$

Propagator

$$\wick{\c\psi\ \c{\overline{\psi}}} = \qquad = \frac{\mathrm{i}(p\!\!\!/ + m)}{p^2 - m^2 + \mathrm{i}\epsilon}$$

■Majorana Fermion

$$\mathcal{L} \supset \tfrac{1}{2}\overline{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi$$
$$= \mathrm{i}\bar\lambda\bar\sigma^\mu\partial_\mu\lambda - \tfrac{m}{2}(\lambda\lambda + \bar\lambda\bar\lambda)$$

Initial state

■Abelian Gauge Theory (Photon)

$$\mathcal{L} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\overline{\psi}D\!\!\!\!/\,\psi + |\mathrm{D}_\mu\phi|^2$$
$$(\mathrm{D}_\mu = \partial_\mu - \mathrm{i}QA_\mu)$$

$$\wick{\c A_\mu\ |\c{\boldsymbol{p}}; あ\rangle} = \qquad = \epsilon_\mu^{あ}(p)$$

$$\wick{\langle\boldsymbol{p}; \c あ|\ \c A_\mu} = \qquad = \epsilon_\mu^{あ*}(p)$$

$$\wick{\c A_\mu\ \c A_\nu} = \qquad = \frac{-\mathrm{i}\eta_{\mu\nu}}{p^2 + \mathrm{i}\epsilon}$$

$$Q\overline{\psi}A\!\!\!/\psi = \qquad = \mathrm{i}Q\gamma^\mu$$

$$\begin{matrix}\mathrm{i}QA^\mu\phi^*\partial_\mu\phi \\ +\mathrm{H.c.}\end{matrix} = \qquad = \mathrm{i}Q(p^\mu + q^\mu)$$

(Momentum must be taken along the arrow)

$$Q^2A^2|\phi|^2 = \qquad = 2\mathrm{i}Q^2\eta^{\mu\nu}$$

■Non-Abelian Gauge Theory (Gluon)

$$\mathcal{L} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\overline{\psi}D\!\!\!\!/\,\psi + |\mathrm{D}_\mu\phi|^2$$
$$(\mathrm{D}_\mu = \partial_\mu - \mathrm{i}gA_\mu)$$

$$\wick{\c A_\mu^b\ |\c{\boldsymbol{p}}; あ, a\rangle} = \qquad = \epsilon_\mu^{あ}(p)\delta^{ab}$$

$$\wick{\langle\boldsymbol{p}; あ, \c a|\ \c A_\mu^b} = \qquad = \epsilon_\mu^{あ*}(p)\delta^{ab}$$

$$-gf^{abc}A^{\mu a}A^{\nu b}(\partial_\mu A_\nu^c) =$$
$$= gf^{abc}\big[\eta^{\mu\nu}(p-q)^\rho + \eta^{\nu\rho}(q-r)^\mu + \eta^{\rho\mu}(r-p)^\nu\big]$$

(Momentum are in incoming directions)

$$-\frac{1}{4}g^2(f^{abe}A_\mu^aA_\nu^b)(f^{cde}A_\rho^cA_\sigma^d) =$$
$$= -\mathrm{i}g^2\big[f^{abe}f^{cde}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) + f^{ace}f^{bde}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) + f^{ade}f^{bce}(\eta^{\mu\nu}\eta^{\sigma\rho} - \eta^{\mu\rho}\eta^{\nu\sigma})\big]$$

TODO: vertex は lagrangian の $(n!)\mathrm{i}$ 倍)

1.4 Field Calculation Techniques

◆1.4.1 Dirac Field Techniques

Dirac Equations	:	$(\not{p}-m)u^s(p)=0;\quad (\not{p}+m)v^s(p)=0$
	:	$\bar{u}^s(p)(\not{p}-m)=0;\quad \bar{v}^s(p)(\not{p}+m)=0$
Dirac Components	:	$u^{r\dagger}(p)u^s(p)=2E_{\boldsymbol{p}}\delta^{rs};\quad v^{r\dagger}(p)v^s(p)=2E_{\boldsymbol{p}}\delta^{rs}$
	:	$\bar{u}^r(p)u^s(p)=2m\delta^{rs};\quad \bar{v}^r(p)v^s(p)=-2m\delta^{rs};\quad \bar{u}^r(p)v^s(p)=\bar{v}^r(p)u^s(p)=0$
Spin Sums	:	$\sum_{\text{spin}} u^s(p)\bar{u}^s(p)=\not{p}+m;\quad \sum_{\text{spin}} v^s(p)\bar{v}^s(p)=\not{p}-m$
Chage Conj.	:	$-C=C^{-1}=C^\dagger=C^{\mathrm{T}},\quad C^{-1}\gamma^\mu C=-C\gamma^\mu C=-\gamma^{\mu\mathrm{T}},\quad C^{-1}\gamma^0 C=-\gamma^0$
	:	$C=C^*\quad ,\quad \psi^{\mathrm{C}}=C(\overline{\psi})^{\mathrm{T}},\quad \overline{\psi}^{\mathrm{C}}=\psi^{\mathrm{T}}C$
u & v	:	$u^*=-\mathrm{i}\gamma^2 v,\quad v^{\mathrm{T}}=-\mathrm{i}u^\dagger\gamma^2=\overline{u}C^{-1},\quad v=C\overline{u}^{\mathrm{T}};\qquad \overline{u}_{\mathrm{A}}P_{\mathrm{H}}u_{\mathrm{B}}=-\overline{v}_{\mathrm{B}}P_{\mathrm{H}}v_{\mathrm{A}}$
	:	$v^*=-\mathrm{i}\gamma^2 u,\quad u^{\mathrm{T}}=-\mathrm{i}v^\dagger\gamma^2=\overline{v}C^{-1},\quad u=C\overline{v}^{\mathrm{T}};\qquad \overline{v}_{\mathrm{A}}P_{\mathrm{H}}u_{\mathrm{B}}=-\overline{v}_{\mathrm{B}}P_{\mathrm{H}}u_{\mathrm{A}}$

◆1.4.2 Polarization Sum

Single photon case $M=\epsilon^*_\mu(k)M^\mu$

When Ward identity $k_\mu M^\mu=0$ is valid,

$$\sum_{\text{pol.}}|M|^2=\sum_{\text{pol.}}\epsilon^*_\mu(k)\epsilon_\nu(k)M^\mu M^{\nu*}=\eta_{\mu\nu}M^\mu M^{\nu*}. \tag{1.1}$$

Double photons case $M=\epsilon^*_\mu(k)\epsilon'^*_\nu(k')M^{\mu\nu}$

When $k_\mu M^{\mu\nu}=k'_\nu M^{\mu\nu}=0$ is valid,

$$\sum_{\text{pol.}}|M|^2=\sum_{\text{pol.}}\epsilon^*_\mu(k)\epsilon_\rho(k)\epsilon'^*_\nu(k')\epsilon'_\sigma(k')M^{\mu\nu}M^{\rho\sigma*}=\eta_{\mu\rho}\eta_{\nu\sigma}M^{\mu\nu}M^{\rho\sigma*}. \tag{1.2}$$

【See Sec. ?? for verbose information.】

◆1.4.3 Fierz transformations

For Dirac spinors a,b,c,d,

$$\begin{aligned}
S(a,b;c,d)&:=(\bar{a}b)(\bar{c}d);\\
V(a,b;c,d)&:=(\bar{a}\gamma^\mu b)(\bar{c}\gamma_\mu d);\\
T(a,b;c,d)&:=\frac{1}{2}(\bar{a}\sigma^{\mu\nu}b)(\bar{c}\sigma_{\mu\nu}d);\\
A(a,b;c,d)&:=(\bar{a}\gamma^\mu\gamma_5 b)(\bar{c}\gamma_\mu\gamma_5 d);\\
P(a,b;c,d)&:=(\bar{a}\gamma_5 b)(\bar{c}\gamma_5 d);
\end{aligned}
\qquad
\begin{pmatrix}S(a,b;c,d)\\V(a,b;c,d)\\T(a,b;c,d)\\A(a,b;c,d)\\P(a,b;c,d)\end{pmatrix}=\frac{1}{4}\begin{pmatrix}1&1&1&-1&-1\\4&-2&0&-2&4\\6&0&-2&0&-6\\-4&-2&0&-2&-4\\-1&1&-1&-1&1\end{pmatrix}\begin{pmatrix}S(a,d;c,b)\\V(a,d;c,b)\\T(a,d;c,b)\\A(a,d;c,b)\\P(a,d;c,b)\end{pmatrix}$$

Also defining $V_{\mathrm{LR}}(a,b;c,d):=(\bar{a}\gamma^\mu P_{\mathrm{L}}b)(\bar{c}\gamma_\mu P_{\mathrm{R}}d)$ and so on,

$$V_{\mathrm{LL}}(a,b;c,d)=-V_{\mathrm{LL}}(a,d;c,b)\qquad S_{\mathrm{RL}}(a,b;c,d)=\frac{1}{4}\left[V_{\mathrm{LR}}(a,d;b,c)-A_{\mathrm{LR}}(a,d;b,c)\right] \tag{1.3}$$

$$V_{\mathrm{RR}}(a,b;c,d)=-V_{\mathrm{RR}}(a,d;c,b)\qquad S_{\mathrm{LR}}(a,b;c,d)=\frac{1}{4}\left[V_{\mathrm{RL}}(a,d;b,c)-A_{\mathrm{RL}}(a,d;b,c)\right] \tag{1.4}$$

Here we can create another equations using

$$(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta}=2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta};\qquad (\bar{\sigma}^\mu)_{\alpha\beta}(\bar{\sigma}_\mu)_{\gamma\delta}=2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}. \tag{1.5}$$

◆1.4.4 Gordon identity

For $P := p' + p$ and $q := p' - p$,

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{P^\mu + \mathrm{i}\sigma^{\mu\nu}q_\nu}{2m}\right]u(p) \qquad \bar{u}(p')\gamma^\mu v(p) = \bar{u}(p')\left[\frac{q^\mu + \mathrm{i}\sigma^{\mu\nu}P_\nu}{2m}\right]v(p) \tag{1.6}$$

$$\bar{v}(p')\gamma^\mu v(p) = -\bar{v}(p')\left[\frac{P^\mu + \mathrm{i}\sigma^{\mu\nu}q_\nu}{2m}\right]v(p) \qquad \bar{v}(p')\gamma^\mu u(p) = -\bar{v}(p')\left[\frac{q^\mu + \mathrm{i}\sigma^{\mu\nu}P_\nu}{2m}\right]u(p) \tag{1.7}$$

◆1.4.5 Color Sum

(Here T^a is **3** of SU(3). For other representations or gauge groups, see Sec. 1.9.)

$$\mathrm{Tr}(T^aT^b) := \frac{1}{2}\delta^{ab} \qquad \text{(That is, } T^a\text{'s are } \tfrac{1}{2}\times\text{Gell-Mann matrices.)} \tag{1.8}$$

$$\sum_a T^aT^a = \frac{4}{3}\cdot\mathbf{1}, \qquad \sum_{c,d} f^{acd}f^{bcd} = 3\delta^{ab} \qquad \sum_a T^a_{ij}T^a_{kl} = \frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{6}\delta_{ij}\delta_{kl} \tag{1.9}$$

$$\sum_a T^aT^bT^a = -\frac{1}{6}T^b \qquad \sum_{b,c} f^{abc}T^bT^c = \frac{3\mathrm{i}}{2}T^a \qquad f^{Dab}f^{EDc} + f^{Dca}f^{EDb} + f^{Dbc}f^{EDa} = 0 \tag{1.10}$$

1.5 Miscellaneous Techniques

$(p\cdot\sigma)(p\cdot\bar{\sigma}) = p^2$

$\epsilon^{ab}\epsilon^{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}$

$\sqrt{p_\mu\sigma^\mu} = \dfrac{p_\mu\sigma^\mu + m}{\sqrt{2(m+p^0)}}$

$\sigma^i\sigma^j = \delta_{ij}\sigma^0 + \mathrm{i}\epsilon_{ijk}\sigma^k$

$\sigma^\mu\sigma^\nu = \mathrm{i}\epsilon^{0\mu\nu\rho}\sigma^\rho + \delta^\mu_0\sigma^\nu + \delta^\nu_0\sigma^\mu - \eta^{\mu\nu}\sigma^0$

$[\sigma^i, \sigma^j] = 2\mathrm{i}\epsilon_{ijk}\sigma^k$

$\sigma^i, \sigma^j = 2\delta_{ij}$

TODO: TODO:

- Majorana Ferminos
- Feynman Rules(A.1)

1.6 Dirac's Gamma Algebras

◆1.6.1 Traces

$$\mathrm{Tr}(\text{any odd } \# \text{ of } \gamma\text{'s}) = 0 \tag{1.11}$$
$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4\eta^{\mu\nu} \tag{1.12}$$
$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) \tag{1.13}$$
$$\mathrm{Tr}(\gamma_5 \text{ and any odd } \# \text{ of } \gamma\text{'s}) = 0 \tag{1.14}$$
$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = 0 \tag{1.15}$$
$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4i\epsilon^{\mu\nu\rho\sigma} \tag{1.16}$$

Generally, for some γ-matrices $A, B, C, \ldots,$

$$\begin{aligned}\mathrm{Tr}(ABCDEF\cdots) &= \eta^{AB}\,\mathrm{Tr}(CDEF\cdots) - \eta^{AC}\,\mathrm{Tr}(BDEF\cdots)\\ &\quad + \eta^{AD}\,\mathrm{Tr}(BCEF\cdots) - \eta^{AE}\,\mathrm{Tr}(BCDF\cdots) + \cdots,\end{aligned} \tag{1.17}$$

$$\mathrm{Tr}(ABCDEF\cdots\gamma_5) = \textbf{Not Established.} \tag{1.18}$$

To prove the second equation, we use following technique:

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\cdots) = \mathrm{Tr}(\cdots\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\mu); \qquad \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\cdots\gamma_5) = \mathrm{Tr}(\gamma_5\cdots\gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\mu). \tag{1.19}$$

◆1.6.2 Contractions

$$\gamma^\mu\gamma_\mu = 4 \tag{1.20}$$
$$\gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \tag{1.21}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4\eta^{\nu\rho} \tag{1.22}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu \tag{1.23}$$

Generally, for some γ-matrices $A, B, C, \ldots,$

$$\text{ODD } \# : \quad \gamma^\mu ABC\cdots\gamma_\mu = -2(\cdots CBA), \tag{1.24}$$
$$\text{EVEN } \# : \quad \gamma^\mu ABC\cdots\gamma_\mu = \mathrm{Tr}(ABC\cdots) - \mathrm{Tr}(ABC\cdots\gamma_5)\cdot\gamma_5. \tag{1.25}$$

<u>Contractions in d-dimension</u>

$$\gamma^\mu\gamma_\mu = d \tag{1.26}$$
$$\gamma^\mu\gamma^\nu\gamma_\mu = -(d-2)\gamma^\nu \tag{1.27}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4\eta^{\nu\rho} - (4-d)\gamma^\nu\gamma^\rho \tag{1.28}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu + (4-d)\gamma^\nu\gamma^\rho\gamma^\sigma \tag{1.29}$$

<u>Contractions of ϵ's</u>

$$\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\alpha\beta\gamma\delta} = -24; \quad \epsilon^{\alpha\beta\gamma\mu}\epsilon_{\alpha\beta\gamma\nu} = -6\delta^\mu_\nu; \quad \epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\rho\sigma} = -2(\delta^\mu_\rho\delta^\nu_\sigma - \delta^\mu_\sigma\delta^\nu_\rho) \tag{1.30}$$
$$\epsilon^{\mu\alpha\beta\gamma}\epsilon_{\mu\alpha'\beta'\gamma'} = -\left(\delta^\alpha_{\alpha'}\delta^\beta_{\beta'}\delta^\gamma_{\gamma'} + \delta^\alpha_{\beta'}\delta^\beta_{\gamma'}\delta^\gamma_{\alpha'} + \delta^\alpha_{\gamma'}\delta^\beta_{\alpha'}\delta^\gamma_{\beta'} - \delta^\alpha_{\alpha'}\delta^\beta_{\gamma'}\delta^\gamma_{\beta'} - \delta^\alpha_{\beta'}\delta^\beta_{\alpha'}\delta^\gamma_{\gamma'} - \delta^\alpha_{\gamma'}\delta^\beta_{\beta'}\delta^\gamma_{\alpha'}\right) \tag{1.31}$$

1.7 Loop Integrals and Dimensional Regularization

◆1.7.1 Feynman Parameters

$$\frac{1}{A_1 A_2 \cdots A_n} = \int_0^1 \mathrm{d}x_1 \cdots x_n \; \delta\left(\sum x_i - 1\right) \frac{(n-1)!}{[x_1 A_1 + x_2 A_2 + \cdots + x_n A_n]^n} \tag{1.32}$$

$$\frac{1}{A_1 A_2} = \int_0^1 \mathrm{d}x \frac{1}{[xA_1 + (1-x)A_2]^2} \tag{1.33}$$

◆1.7.2 d-dimensional integrals in Minkowski space

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \tag{1.34}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^2}{(l^2-\Delta)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \tag{1.35}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2-\Delta)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{d/2}} \frac{\eta^{\mu\nu}}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \tag{1.36}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{(l^2)^2}{(l^2-\Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{d(d+2)}{4} \frac{\Gamma(n-\frac{d}{2}-2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-2} \tag{1.37}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^\mu l^\nu l^\rho l^\sigma}{(l^2-\Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2}-2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-2} \frac{\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}}{4} \tag{1.38}$$

Here we can use following expansions: $(\gamma \simeq 0.5772)$

$$\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} = 1 - (d-4)\frac{\log\Delta}{2} + \mathrm{O}\left((d-4)^2\right) \quad \text{around } d=4, \tag{1.39}$$

$$\Gamma(x) = \frac{1}{x} - \gamma + \mathrm{O}(x) \quad \text{around } x = 0, \tag{1.40}$$

$$\Gamma(x) = \frac{(-1)^n}{n!}\left[\frac{1}{x+n} - \gamma + \sum_{k=1}^{n}\frac{1}{k} + \mathrm{O}(x+n)\right] \quad \text{around } x = -n. \tag{1.41}$$

and we get following expansion:

$$\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} = \frac{1}{(4\pi)^2}\left[\left(\frac{2}{4-d} - \gamma + \log 4\pi\right) - \log\Delta + \mathrm{O}(4-d)\right]. \tag{1.42}$$

Usually this Δ is positive, but when Δ contains some timelike momenta, it becomes negative. Then these integrals acquire imaginary parts, which give the discontinuities of S-matrix elements. To compute the S-matrix in a physical region choose the correct branch

$$\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \to \left(\frac{1}{\Delta - \mathrm{i}\epsilon}\right)^{n-\frac{d}{2}}. \tag{1.43}$$

1.8 Cross Sections and Decay Rates

■General expression (The mass dimension of $\mathcal{M}$ is $2 - N_{\mathrm{f}}$ for $\mathrm{d}\sigma$ and $3 - N_{\mathrm{f}}$ for $\mathrm{d}\Gamma$.)

$$\mathrm{d}\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|}\left[\prod_f \frac{\mathrm{d}^3 p_f}{(2\pi)^3}\frac{1}{2E_f}\right]\left|\mathcal{M}(p_A, p_B \to \{p_f\})\right|^2 (2\pi)^4 \delta^{(4)}\left(p_A + p_B - \{p_f\}\right) \tag{1.44}$$

$$\mathrm{d}\Gamma = \frac{1}{2m_A}\left[\prod_f \frac{\mathrm{d}^3 p_f}{(2\pi)^3}\frac{1}{2E_f}\right]\left|\mathcal{M}(m_A \to \{p_f\})\right|^2 (2\pi)^4 \delta^{(4)}\left(m_A - \{p_f\}\right) \quad \text{(in A-rest frame.)} \tag{1.45}$$

■2-body phase space in center-of-mass frame

$$\int \Pi_2 := \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3}\int \frac{\mathrm{d}^3 p_2}{(2\pi)^3}\frac{1}{2E_1}\frac{1}{2E_2}(2\pi)^4\delta^{(4)}\left(E_{\mathrm{cm}} - (p_1 + p_2)\right) \qquad \text{(in center-of-mass frame)} \tag{1.46}$$

$$= \int \frac{\mathrm{d}\Omega}{4\pi}\frac{1}{8\pi}\frac{2\left\|\boldsymbol{p_1}\right\|}{E_{\mathrm{cm}}} \tag{1.47}$$

$$= \frac{1}{8\pi}\sqrt{1 - \frac{2({m_1}^2 + {m_2}^2)}{E_{\mathrm{cm}}^2} + \frac{({m_1}^2 - {m_2}^2)^2}{E_{\mathrm{cm}}^4}} \quad \xrightarrow{m_2 = 0} \quad \frac{1}{8\pi}\left(1 - \frac{{m_1}^2}{{E_{\mathrm{cm}}}^2}\right) \tag{1.48}$$

■Kinematics of Decay

$$K \to p_1 + p_2 \qquad \text{or} \qquad \begin{pmatrix} M \\ \boldsymbol{0} \end{pmatrix} \to \begin{pmatrix} \sqrt{p^2 + {m_1}^2} \\ \boldsymbol{p} \end{pmatrix} + \begin{pmatrix} \sqrt{p^2 + {m_2}^2} \\ -\boldsymbol{p} \end{pmatrix}; \tag{1.49}$$

$$\left\|\boldsymbol{p}\right\|^2 = \frac{1}{4}\left[M^2 - 2\left({m_1}^2 + {m_2}^2\right) + \frac{\left({m_1}^2 - {m_2}^2\right)^2}{M^2}\right] \approx \left(\frac{M^2 - {m_1}^2}{2M}\right)^2$$

$$E_1 = \frac{M^2 + {m_1}^2 - {m_2}^2}{2M}, \qquad E_2 = \frac{M^2 + {m_2}^2 - {m_1}^2}{2M};$$

$$K \cdot p_1 = \frac{M^2 + {m_1}^2 - {m_2}^2}{2}, \qquad p_1 \cdot p_2 = \frac{M^2 - \left({m_1}^2 + {m_2}^2\right)}{2}.$$

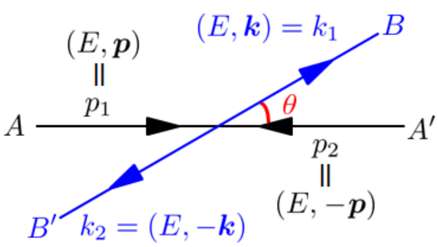
■Mandelstam Variables

For $p_1 + p_2 \to k_1 + k_2$ collision,

$$\begin{aligned} s &= (p_1 + p_2)^2 = (k_1 + k_2)^2, \\ t &= (p_1 - k_1)^2 = (p_2 - k_2)^2, \\ u &= (p_1 - k_2)^2 = (p_2 - k_1)^2, \end{aligned}$$

and $$s + t + u = p_1^2 + p_2^2 + k_1^2 + k_2^2 = \sum m^2.$$

■Kinematics of Collision (Same Mass)



$$\left\|\boldsymbol{p}\right\|^2 = E^2 - m_A^2 \qquad \boldsymbol{p}\cdot\boldsymbol{k} = \left\|\boldsymbol{p}\right\|\left\|\boldsymbol{k}\right\|\cos\theta$$

$$\left\|\boldsymbol{k}\right\|^2 = E^2 - m_B^2$$

$$p_1 \cdot p_2 = s/2 - m_A^2 \quad p_1 \cdot k_1 = p_2 \cdot k_2 = \tfrac{1}{2}(m_A^2 + m_B^2 - t);$$

$$k_1 \cdot k_2 = s/2 - m_B^2 \quad p_1 \cdot k_2 = p_1 \cdot k_2 = \tfrac{1}{2}(m_A^2 + m_B^2 - u);$$

$$\begin{aligned} &(p_1 - p_2)^2 = -4(E^2 - m_A^2) & s &= 4E^2, \\ & & t &= -(2E^2 - m_A^2 - m_B^2) + 2\boldsymbol{p}\cdot\boldsymbol{k} \\ &(k_1 - k_2)^2 = -4(E^2 - m_B^2) & u &= -(2E^2 - m_A^2 - m_B^2) - 2\boldsymbol{p}\cdot\boldsymbol{k} \end{aligned}$$

1.9 楊-Mills Theory (See App. ?? for verbose notes.)

◆1.9.1 Non-Abelian gauge theory

$$[T^a, T^b] = \mathrm{i} f^{ab}{}_c T^c, \qquad 0 = f^D{}_{ab} f^E{}_{Dc} + f^D{}_{ca} f^E{}_{Db} + f^D{}_{bc} f^E{}_{Da}, \qquad \mathrm{D}_\mu = \partial_\mu - \mathrm{i} g A_\mu$$

$$\operatorname{Tr} T^a T^b = \frac{1}{2}\delta^{ab}, \qquad [\widetilde{T}^a]_i{}^j := T^{\mathrm{ad}\ a}{}_i{}^j := -\mathrm{i} f^{aij} \qquad [\widetilde{\mathrm{D}}_\mu]_i{}^j := \delta_i^j \partial_\mu + g f^{iaj} A^a_\mu.$$

$$\begin{aligned} F_{\mu\nu} &= \frac{\mathrm{i}}{g}\left[\mathrm{D}_\mu, \mathrm{D}_\nu\right] \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{g}{\mathrm{i}}\left[A_\mu, A_\nu\right] \\ &= \Big[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu\Big] T^a \end{aligned} \qquad \begin{aligned} &\mathrm{D}_\mu \phi = \partial_\mu \phi - \mathrm{i} g A^a_\mu (T^a_\phi \phi) \\ &\mathrm{D}_\mu F_{\mu\nu}{}^a = \partial_\mu F^a_{\mu\nu} + g f^{abc} A^b_\mu F^c_{\mu\nu}, \\ &\Big(\mathrm{D}_\mu F_{\nu\rho} = \partial_\mu \lambda - \mathrm{i} g [A_\mu, F_{\nu\rho}]\Big).^{*1} \end{aligned}$$

$$\phi \mapsto V\phi := \mathrm{e}^{\mathrm{i} g\theta}\phi \qquad A_\mu \mapsto V\left(A_\mu + \frac{\mathrm{i}}{g}\partial_\mu\right)V^{-1} \qquad F_{\mu\nu} \mapsto V F_{\mu\nu} V^{-1}$$

$$\phi^{a\prime} \simeq \phi + \mathrm{i} g\theta^a T^a \phi \qquad A^{a\prime}_\mu \simeq A^a_\mu + \partial_\mu\theta^a + g f^{abc} A^b_\mu \theta^c \qquad F^a_{\mu\nu}{}' \simeq F^a_{\mu\nu} + g f^{abc} F^b_{\mu\nu}\theta^c$$

$$\epsilon^{\mu\nu\rho\sigma}\left[\mathrm{D}_\nu, [\mathrm{D}_\rho, \mathrm{D}_\sigma]\right] = \epsilon^{\mu\nu\rho\sigma} \mathrm{D}_\nu F_{\rho\sigma} = 0.$$

■Killing and Casimir Here we have two constants which **depend on representation** r.

$$\operatorname{Tr}(T^a T^b) =: C(r)\delta^{ab} \quad \text{(Killing form)}, \qquad T^a T^a =: C_2(r)\cdot \mathbf{1} \quad \text{(quadratic Casimir operator)}, \tag{1.50}$$

which satisfy

$$C(r) = \frac{d(r)}{d(\mathrm{ad})} C_2(r), \qquad T^a T^b T^a = \left[C_2(r) - \frac{1}{2} C_2(\mathrm{ad})\right] T^b, \tag{1.51}$$

$$f^{acd} f^{bcd} = C_2(\mathrm{ad})\delta^{ab}, \qquad f^{abc} T^b T^c = \frac{\mathrm{i}}{2} C_2(\mathrm{ad}) T^a. \tag{1.52}$$

<u>For $\mathrm{SU}(N)$</u> For its fundamental representation N with definition $C(N) := \frac{1}{2}$, we have

$$C(N) := \frac{1}{2}, \quad C_2(N) = \frac{N^2-1}{2N}, \quad C(\mathrm{ad}) = C_2(\mathrm{ad}) = N; \quad (T^a)_{ij}(T^a)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{\delta_{ij}\delta_{kl}}{N}\right).$$

◆1.9.2 Abelian gauge theory

In Abelian gauge theory, V and fields are always commutative, and thus we have charge freedom (Q).

$$\mathrm{D}_\mu\phi = (\partial_\mu - \mathrm{i} g A_\mu Q)\,\phi \qquad \phi \mapsto \mathrm{e}^{igQ\theta}\phi \qquad F_{\mu\nu} = \frac{\mathrm{i}}{g}\left[\mathrm{D}_\mu, \mathrm{D}_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathrm{D}_\mu \lambda^a = \partial_\mu \lambda^a \qquad A_\mu \mapsto A_\mu + \partial_\mu\theta \qquad F_{\mu\nu} \mapsto F_{\mu\nu}$$

◆1.9.3 Lagrangian Block

$$\mathcal{L} \ni |\mathrm{D}_\mu\phi|^2 - m^2|\phi|^2, \quad \overline{\psi}(\mathrm{i}\not{D} - m)\psi, \quad -\frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}\ \Big(= -\frac{1}{2}\operatorname{Tr} F^{\mu\nu}F_{\mu\nu}\Big), \quad \theta\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} \tag{1.53}$$

$$-\frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu} = -\frac{1}{2}\left[(\partial_\mu A^a_\nu)^2 + A^a_\mu\partial^\mu\partial^\nu A^a_\nu\right] - g f^{abc} A^a_\mu A^b_\nu \partial^\mu A^{c\nu} - \frac{g^2}{4} f^{abc} f^{ade} A^b_\mu A^c_\nu A^{d\mu} A^{e\nu} \tag{1.54}$$

---

*1 Note that we can use any representation T^a but must the same ones for $A^a_\mu T^a$ and $\lambda^a T^a$.

2 Standard Model

Any representations assumed to be *normalized Hermitian*. Note that the $SU(2)$ **2** representation is

$$T^a = \frac{1}{2}\sigma^a; \qquad \left[T^a, T^b\right] = \mathrm{i}\epsilon^{abc}T^c; \qquad T^{\pm} := T^1 \pm \mathrm{i}T^2. \tag{2.1}$$

We use the following abridged notations:

$$(\partial A)_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad F^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu. \tag{2.2}$$

2.1 Symmetries and Fields

	$\mathrm{SU}(3)_{\mathrm{strong}}$	$\mathrm{SU}(2)_{\mathrm{weak}}$	$\mathrm{U}(1)_Y$
Matter Fields (Fermionic / Lorentz Spinor)			
$P_{\mathrm{L}}Q_i$: Left-handed quarks	**3**	**2**	1/6
$P_{\mathrm{L}}U_i$: Right-handed up-type quarks	**3**	**1**	2/3
$P_{\mathrm{R}}D_i$: Right-handed down-type quarks	**3**	**1**	−1/3
$P_{\mathrm{R}}L_i$: Left-handed leptons	**1**	**2**	−1/2
$P_{\mathrm{R}}E_i$: Right-handed leptons	**1**	**1**	−1
Higgs Field (Bosonic / Lorentz Scalar)			
H : Higgs	**1**	**2**	1/2
Gauge Fields (Bosonic / Lorentz Vector)			
G : Gluons	**8**	**1**	0
W : Weak bosons	**1**	**3**	0
B : B boson	**1**	**1**	0

■Full Lagrangian $\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{湯川}}$

where
$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} \tag{2.3}$$

$$\mathcal{L}_{\text{Higgs}} = \left|\left(\partial_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{2}\mathrm{i}g_1 B_\mu\right)H\right|^2 - V(H), \tag{2.4}$$

$$\begin{aligned}\mathcal{L}_{\text{matter}} = {}& \overline{Q}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_3 G_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{6}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{L}} Q_i \\ &+ \overline{U}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_3 G_\mu - \frac{2}{3}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{R}} U_i \\ &+ \overline{D}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_3 G_\mu + \frac{1}{3}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{R}} D_i \\ &+ \overline{L}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_2 W_\mu + \frac{1}{2}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{L}} L_i \\ &+ \overline{E}_i \mathrm{i}\gamma^\mu \left(\partial_\mu + \mathrm{i}g_1 B_\mu\right) P_{\mathrm{R}} E_i,\end{aligned} \tag{2.5}$$

$$\mathcal{L}_{\text{湯川}} = \overline{U}_i (y_u)_{ij} H P_{\mathrm{L}} Q_j - \overline{D}_i (y_d)_{ij} H^\dagger P_{\mathrm{L}} Q_j - \overline{E}_i (y_e)_{ij} H^\dagger P_{\mathrm{L}} L_j + \mathrm{H.c.} \tag{2.6}$$

We have no freedom to add other terms into this Lagrangian of the gauge theory. See Appendix **??**.

■Gauge Kinetic Terms

the gauge kinetic terms can be expanded as

$$\begin{aligned}\mathcal{L}_{\text{gauge}} =& -\frac{1}{4}(\partial B)(\partial B)\\ &-\frac{1}{4}(\partial W^a)(\partial W^a) - g_2\epsilon^{abc}(\partial_\mu W^a_\nu)W^{\mu b}W^{\nu c} - \frac{{g_2}^2}{4}\left(\epsilon^{eab}W^a_\mu W^b_\nu\right)\left(\epsilon^{ecd}W^{c\mu}W^{d\nu}\right)\\ &-\frac{1}{4}(\partial G^a)(\partial G^a) - g_3 f^{abc}(\partial_\mu G^a_\nu)G^{\mu b}G^{\nu c} - \frac{{g_3}^2}{4}\left(f^{eab}G^a_\mu G^b_\nu\right)\left(f^{ecd}G^{c\mu}G^{d\nu}\right).\end{aligned} \tag{2.7}$$

2.2 Higgs Mechanism

■Higgs Potential

The (renormalizable) Higgs potential must be

$$V(H) = -\mu^2(H^\dagger H) + \lambda\left(H^\dagger H\right)^2. \tag{2.8}$$

for the SU(2), and $\lambda > 0$ in order not to run away the VEVs, while μ^2 is positive for the EWSB.

To discuss this clearly, let us *redefine* the Higgs field as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + \mathrm{i}\phi_2\\ v + (h + \mathrm{i}\phi_3)\end{pmatrix}, \qquad \text{where} \quad v = \sqrt{\frac{\mu^2}{\lambda}}. \tag{2.9}$$

Here h is the "Higgs boson," and ϕ_i are 南部–Goldstone bosons.

The Higgs potential becomes

$$V(h) = \frac{\mu^2}{4v^2}h^4 + \frac{\mu^2}{v}h^3 + \mu^2 h^2, \tag{2.10}$$

and now we know the Higgs boson has acquired mass $m_h = \sqrt{2}\mu$. Also

$$\mathcal{L}_{\text{Higgs}} = \left|\left(\partial_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{2}\mathrm{i}g_1 B_\mu\right)H\right|^2 \tag{2.11}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{8}\Big[{g_2}^2{W_1}^2 + {g_2}^2{W_2}^2 + (g_1 B - g_2 W_3)^2\Big]. \tag{2.12}$$

Redefining the gauge fields (with concerning the norms) as

$$W^\pm_\mu := \frac{1}{\sqrt{2}}(W^1_\mu \mp \mathrm{i}W^2_\mu), \qquad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} := \begin{pmatrix}\cos\theta_{\mathrm{w}} & -\sin\theta_{\mathrm{w}}\\ \sin\theta_{\mathrm{w}} & \cos\theta_{\mathrm{w}}\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix}, \tag{2.13}$$

where

$$\tan\theta_{\mathrm{w}} := \frac{g_1}{g_2}, \qquad e := -\frac{g_1 g_2}{\sqrt{{g_1}^2 + {g_2}^2}}; \qquad g_Z := \sqrt{{g_1}^2 + {g_2}^2}; \tag{2.14}$$

$$g_1 = \frac{|e|}{\cos\theta_{\mathrm{w}}} = g_Z\sin\theta_{\mathrm{w}}, \qquad g_2 = \frac{|e|}{\sin\theta_{\mathrm{w}}} = g_Z\cos\theta_{\mathrm{w}}. \tag{2.15}$$

We obtain the following terms in $\mathcal{L}_{\text{Higgs}}$:

$$\mathcal{L}_{\text{Higgs}} \supset \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{4}\Big[{g_2}^2 W^{+\mu}W^-_\mu + \frac{{g_Z}^2}{2}Z^\mu Z_\mu\Big]. \tag{2.16}$$

Here we have omitted the 南部–Goldstone bosons.

Here we present another form:

$$g_1 B_\mu = |e| A_\mu - \tan\theta_{\mathrm{w}} Z_\mu, \tag{2.17}$$

$$g_2 W_\mu = \frac{g_2}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) + \left(\frac{|e|}{\tan\theta_{\mathrm{w}}} Z_\mu + |e| A_\mu\right) T^3, \tag{2.18}$$

$$Z_\mu^0 := \frac{1}{\sqrt{{g_1}^2 + {g_2}^2}}(g_2 W_\mu^3 - g_1 B_\mu), \quad A_\mu := \frac{1}{\sqrt{{g_1}^2 + {g_2}^2}}(g_1 W_\mu^3 + g_2 B_\mu) \tag{2.19}$$

You can see the gauge bosons have acquired the masses

$$m_A = 0, \quad m_W := \frac{g_2}{2} v, \quad m_Z := \frac{g_Z}{2} v. \tag{2.20}$$

■Gauge Term The SU(2) gauge term is converted into

$$\begin{aligned} W^{a\mu\nu} W^a_{\mu\nu} = &(\partial W^3)(\partial W^3) + 2(\partial W^+)(\partial W^-) \\ &- 4\mathrm{i}g\left[(\partial W^3)^{\mu\nu} W_\mu^+ W_\nu^- + (\partial W^+)^{\mu\nu} W_\mu^- W_\nu^3 + (\partial W^-)^{\mu\nu} W_\mu^3 W_\nu^+\right] \\ &- 2g^2(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma})\left(W_\mu^+ W_\nu^+ W_\rho^- W_\sigma^- - 2W_\mu^3 W_\nu^3 W_\rho^+ W_\sigma^-\right), \end{aligned}$$

and therefore the final expression is

$$\begin{aligned} \mathcal{L}_{\text{gauge}} := &-\frac{1}{4}\left[G^{a\mu\nu} G^a_{\mu\nu} + (\partial Z)^{\mu\nu}(\partial Z)_{\mu\nu} + (\partial A)^{\mu\nu}(\partial A)_{\mu\nu} + 2(\partial W^+)^{\mu\nu}(\partial W^-)_{\mu\nu}\right] \\ &+ \frac{\mathrm{i}|e|}{\tan\theta_{\mathrm{w}}}\Big[(\partial W^+)^{\mu\nu} W_\mu^- Z_\nu + (\partial W^-)^{\mu\nu} Z_\mu W_\nu^+ + (\partial Z)^{\mu\nu} W_\mu^+ W_\nu^-\Big] \\ &+ \mathrm{i}|e|\Big[(\partial W^+)^{\mu\nu} W_\mu^- A_\nu + (\partial W^-)^{\mu\nu} A_\mu W_\nu^+ + (\partial A)^{\mu\nu} W_\mu^+ W_\nu^-\Big] \\ &+ (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma})\left[\frac{|e|^2}{2\sin^2\theta_{\mathrm{w}}} W_\mu^+ W_\nu^+ W_\rho^- W_\sigma^- + \frac{|e|^2}{\tan^2\theta_{\mathrm{w}}} W_\mu^+ Z_\nu W_\rho^- Z_\sigma\right. \\ &\qquad \left. + \frac{|e|^2}{\tan\theta_{\mathrm{w}}}\left(W_\mu^+ Z_\nu W_\rho^- A_\sigma + W_\mu^+ A_\nu W_\rho^- Z_\sigma\right) + |e|^2 W_\mu^+ A_\nu W_\rho^- A_\sigma\right]. \end{aligned} \tag{2.21}$$

■湯川 Term

$$\begin{aligned} \mathcal{L}_{\text{湯川}} = &\overline{U} y_u H P_{\mathrm{L}} Q - \overline{D} y_d H^\dagger P_{\mathrm{L}} Q - \overline{E} y_e H^\dagger P_{\mathrm{L}} L + \text{H.c.} \\ = &\overline{U} y_u \epsilon^{\alpha\beta} H^\alpha P_{\mathrm{L}} Q^\beta - \overline{D} y_d {H^\dagger}^\alpha P_{\mathrm{L}} Q^\alpha - \overline{E} y_e {H^\dagger}^\alpha P_{\mathrm{L}} L^\alpha + \text{H.c.} \\ = &-\frac{v+h}{\sqrt{2}}\left(\overline{U} y_u P_{\mathrm{L}} Q^1 + \overline{D} y_d P_{\mathrm{L}} Q^2 + \overline{E} y_e P_{\mathrm{L}} L^2\right) + \text{H.c.} \end{aligned} \tag{2.22}$$

2.3 Full Lagrangian After Higgs Mechanism

Now we have the following Lagrangian (with omitting P_L etc.):

$$
\begin{aligned}
\mathcal{L} = &\mathcal{L}_{\text{gauge}} + {m_W}^2 W^+W^- + \frac{{m_Z}^2}{2}Z^2 \\
\text{【Higgs】}\quad & + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}{m_h}^2h^2 - \sqrt{\frac{\lambda}{2}}m_h h^3 - \frac{1}{4}\lambda h^4 \\
& + \frac{v{g_2}^2}{4}W^+W^-h + \frac{v({g_1}^2+{g_2}^2)}{8}Z^2h \\
& + \frac{{g_2}^2}{4}W^+W^-h^2 + \frac{{g_1}^2+{g_2}^2}{8}Z^2h^2 \\
& - \left(\frac{1}{\sqrt{2}}h\bar{U}y_uQ^1 + \frac{1}{\sqrt{2}}h\bar{D}y_dQ^2 + \frac{1}{\sqrt{2}}h\bar{E}y_eL^2 + \text{H.c.}\right) \\
\text{【SU(3)】}\quad & + \bar{Q}\left(\mathrm{i}\partial\!\!\!/ + g_3 G\!\!\!/\right)Q + \bar{U}\left(\mathrm{i}\partial\!\!\!/ + g_3 G\!\!\!/\right)U + \bar{D}\left(\mathrm{i}\partial\!\!\!/ + g_3 G\!\!\!/\right)D + \bar{L}\left(\mathrm{i}\partial\!\!\!/\right)L + \bar{E}\left(\mathrm{i}\partial\!\!\!/\right)E \\
\text{【}W\text{】}\quad & + \bar{Q}\frac{g_2}{\sqrt{2}}\left(W\!\!\!\!/^{\,+}T^+ + W\!\!\!\!/^{\,-}T^-\right)Q + \bar{L}\frac{g_2}{\sqrt{2}}\left(W\!\!\!\!/^{\,+}T^+ + W\!\!\!\!/^{\,-}T^-\right)L \\
\text{【}A\&Z^0\text{】}\quad & + \bar{Q}\left[\left(T^3+\frac{1}{6}\right)|e|A\!\!\!/ + \left(\frac{|e|c}{s}T^3 - \frac{|e|s}{6c}\right)Z\!\!\!/^0\right]Q \\
& + \bar{U}\left(\frac{2}{3}|e|A\!\!\!/ - \frac{2|e|s}{3c}Z\!\!\!/\right)U \\
& + \bar{D}\left(-\frac{1}{3}|e|A\!\!\!/ + \frac{|e|s}{3c}Z\!\!\!/\right)D \\
& + \bar{L}\left[\left(T^3-\frac{1}{2}\right)|e|A\!\!\!/ + \left(\frac{|e|c}{s}T^3 + \frac{|e|s}{2c}\right)Z\!\!\!/^0\right]L \\
& + \bar{E}\left(-|e|A\!\!\!/ + \frac{|e|s}{c}Z\!\!\!/\right)E \\
\text{【湯川項】}\quad & - \left(\frac{1}{\sqrt{2}}v\bar{U}y_uQ^1 + \frac{1}{\sqrt{2}}v\bar{D}y_dQ^2 + \frac{1}{\sqrt{2}}v\bar{E}y_eL^2 + \text{H.c.}\right)
\end{aligned}
\tag{2.23}
$$

2.4 Mass Eigenstates

Here we will obtain the mass eigenstates of the fermions, by diagonalizing the 湯川 matrices.

We use the singular value decomposition method to mass matrices $Y_\bullet := vy_\bullet/\sqrt{2}$. Generally, any matrices can be transformed with two unitary matrices Ψ and Φ as

$$
Y = \Phi^\dagger \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \Psi =: \Phi^\dagger M \Psi \qquad (m_i \geq 0). \tag{2.24}
$$

Using this Ψ and Φ, we can rotate the basis as

$$
Q^1 \mapsto \Psi_u^\dagger Q^1, \quad Q^2 \mapsto \Psi_d^\dagger Q^2, \quad L \mapsto \Psi_e^\dagger L, \qquad U \mapsto \Phi_u^\dagger U, \quad D \mapsto \Phi_d^\dagger D, \quad E \mapsto \Phi_e^\dagger E, \tag{2.25}
$$

and now we have the 湯川 terms in mass eigenstates as

$$
\mathcal{L}_{湯川} = -\left(1+\frac{1}{v}h\right)\left[(m_u)_i\overline{U}_iP_\mathrm{L}Q_i^1 + (m_d)_i\overline{D}_iP_\mathrm{L}Q_i^2 + (m_e)_i\overline{E}_iP_\mathrm{L}L_i^2 + \text{H.c.}\right]. \tag{2.26}
$$

In the transformation from the gauge eigenstates to the mass eigenstates, almost all the terms in the Lagrangian are not modified. However, only the terms of quark–quark–W interactions do change drastically, as

$$\mathcal{L} \supset \overline{Q}\mathrm{i}\gamma^\mu \left(-\mathrm{i}g_2 W_\mu - \frac{1}{6}\mathrm{i}g_1 B_\mu\right) P_\mathrm{L} Q \tag{2.27}$$

$$= \overline{Q}\frac{g_2}{\sqrt{2}}\left(\not{W}^+ T^+ + \not{W}^- T^-\right) P_\mathrm{L} Q \quad + \quad \text{(interaction terms with Z and A)} \tag{2.28}$$

$$\mapsto \frac{g_2}{\sqrt{2}}\begin{pmatrix}\overline{Q}^1\Psi_u & \overline{Q}^2\Psi_d\end{pmatrix}\begin{pmatrix}0 & \not{W}^+ \\ \not{W}^- & 0\end{pmatrix} P_\mathrm{L}\begin{pmatrix}\Psi_u^\dagger Q^1 \\ \Psi_d^\dagger Q^2\end{pmatrix} + (\quad\cdots\quad) \tag{2.29}$$

$$= \frac{g_2}{\sqrt{2}}\left[\overline{Q}^2\not{W}^- X P_\mathrm{L} Q^1 + \overline{Q}^1\not{W}^+ X^\dagger P_\mathrm{L} Q^2\right] + (\quad\cdots\quad), \tag{2.30}$$

where $X := \Psi_d\Psi_u^\dagger$ is a matrix, so-called the Cabbibo–小林–益川 (CKM) matrix, which is *not* diagonal, and *not* real, generally. These terms violate the flavor symmetry of quarks, and even the CP-symmetry.

> In our notation, CP-transformation of a spinor is described as
>
> $$\mathscr{C}\mathscr{P}(\psi) = -\mathrm{i}\eta^*(\overline{\psi}\gamma^2)^\mathrm{T}, \quad \mathscr{C}\mathscr{P}\left(\overline{\psi}\right) = \mathrm{i}\eta(\gamma^2\psi)^\mathrm{T}, \tag{2.31}$$
>
> where η is a complex phase ($|\eta| = 1$). Under this transformation, those terms are transformed as, e.g.,
>
> $$\begin{aligned}\mathscr{C}\mathscr{P}\left(\overline{Q}^2\not{W}^- X P_\mathrm{L} Q^1\right) &= (\gamma^2 Q^2)^\mathrm{T}\mathscr{P}(-\not{W}^+) X P_\mathrm{L}(\overline{Q}^1\gamma^2)^\mathrm{T} \\ &= -W_\mu^{+P}(\gamma^2 Q^2)^\mathrm{T}(\overline{Q}^1 X^\mathrm{T}\gamma^2 P_\mathrm{L}\gamma^{\mu\mathrm{T}})^\mathrm{T} \\ &= (\overline{Q}^1\not{W}^+ X^\mathrm{T} P_\mathrm{L} Q^2).\end{aligned} \tag{2.32}$$
>
> Therefore, we can see that the CP-symmetry is maintained if and only if $X^\mathrm{T} = X^\dagger$, that is, if and only if X is a real matrix.

以上より，標準模型の Lagrangian は

$$
\begin{aligned}
\mathcal{L} =& \mathcal{L}_{\text{gauge}} \\
\text{【質量項】} \quad &+ \quad {m_W}^2 W^+ W^- \quad + \quad \frac{{m_Z}^2}{2} Z^2 \\
&- \quad \left(\bar{U} M_u P_{\mathrm{L}} Q^1 \ + \ \bar{D} M_d P_{\mathrm{L}} Q^2 \ + \ \bar{E} M_e P_{\mathrm{L}} L^2 \ + \ \text{H.c.} \right) \\
\text{【Higgs Field】} \quad &+ \quad \frac{1}{2} (\partial_\mu h)^2 \quad - \frac{1}{2} {m_h}^2 h^2 - \sqrt{\frac{\lambda}{2}} m_h h^3 - \frac{1}{4} \lambda h^4 \\
\text{【Higgs との結合】} \quad &+ \quad \frac{v {g_2}^2}{4} W^+ W^- h \quad + \quad \frac{v({g_1}^2 + {g_2}^2)}{8} Z^2 h \\
&+ \quad \frac{{g_2}^2}{4} W^+ W^- h^2 \quad + \quad \frac{{g_1}^2 + {g_2}^2}{8} Z^2 h^2 \\
&- \quad \left(\frac{1}{v} \bar{U} M_u P_{\mathrm{L}} Q^1 h + \frac{1}{v} \bar{D} M_d P_{\mathrm{L}} Q^2 h + \frac{1}{v} \bar{E} M_e P_{\mathrm{L}} L^2 h \ + \ \text{H.c.} \right) \\
\text{【SU(3) および微分項】} \quad &+ \quad \bar{Q} \left(\mathrm{i} \not{\partial} + g_3 \not{G} \right) P_{\mathrm{L}} Q \quad + \quad \bar{U} \left(\mathrm{i} \not{\partial} + g_3 \not{G} \right) P_{\mathrm{R}} U \quad + \quad \bar{D} \left(\mathrm{i} \not{\partial} + g_3 \not{G} \right) P_{\mathrm{R}} D \\
&+ \quad \bar{L} \left(\mathrm{i} \not{\partial} \right) P_{\mathrm{L}} L \quad + \quad \bar{E} \left(\mathrm{i} \not{\partial} \right) P_{\mathrm{R}} E \\
\text{【W boson】} \quad &+ \quad \frac{g_2}{\sqrt{2}} \left[\bar{Q}^2 \not{W}^- X P_{\mathrm{L}} Q^1 + \bar{Q}^1 \not{W}^+ X^\dagger P_{\mathrm{L}} Q^2 \right] \qquad \text{【$\leftarrow$ CP and flavor violating!】} \\
&+ \quad \bar{L} \frac{g_2}{\sqrt{2}} \left(\not{W}^+ T^+ + \not{W}^- T^- \right) P_{\mathrm{L}} L \\
\text{【A\&Z^0 boson】} \quad &+ \quad \bar{Q} \left[\left(T^3 + \frac{1}{6} \right) |e| \not{A} + \left(\frac{|e| c}{s} T^3 - \frac{|e| s}{6c} \right) \not{Z}^0 \right] P_{\mathrm{L}} Q \\
&+ \quad \bar{U} \left(\frac{2}{3} |e| \not{A} - \frac{2|e| s}{3c} \not{Z} \right) P_{\mathrm{R}} U \\
&+ \quad \bar{D} \left(-\frac{1}{3} |e| \not{A} + \frac{|e| s}{3c} \not{Z} \right) P_{\mathrm{R}} D \\
&+ \quad \bar{L} \left[\left(T^3 - \frac{1}{2} \right) |e| \not{A} + \left(\frac{|e| c}{s} T^3 + \frac{|e| s}{2c} \right) \not{Z}^0 \right] P_{\mathrm{L}} L \\
&+ \quad \bar{E} \left(-|e| \not{A} + \frac{|e| s}{c} \not{Z} \right) P_{\mathrm{R}} E
\end{aligned}
\tag{2.33}
$$

となる。

2.5 Chiral Notation

以上の Lagrangian を chiral 表示で表すと，まず最初は

$$
\begin{aligned}
\mathcal{L} =&(\text{Higgs terms}) + (\text{Gauge fields strength})\\
&+ Q_{\mathrm{L}}^{\dagger}\mathrm{i}\bar{\sigma}^{\mu}\left(\partial_\mu - \mathrm{i}g_3 G_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{6}\mathrm{i}g_1 B_\mu\right)Q_{\mathrm{L}}\\
&+ U_{\mathrm{R}}^{\dagger}\mathrm{i}\sigma^{\mu}\left(\partial_\mu - \mathrm{i}g_3 G_\mu - \frac{2}{3}\mathrm{i}g_1 B_\mu\right)U_{\mathrm{R}}\\
&+ D_{\mathrm{R}}^{\dagger}\mathrm{i}\sigma^{\mu}\left(\partial_\mu - \mathrm{i}g_3 G_\mu + \frac{1}{3}\mathrm{i}g_1 B_\mu\right)D_{\mathrm{R}}\\
&+ L_{\mathrm{L}}^{\dagger}\mathrm{i}\bar{\sigma}^{\mu}\left(\partial_\mu - \mathrm{i}g_2 W_\mu + \frac{1}{2}\mathrm{i}g_1 B_\mu\right)L_{\mathrm{L}}\\
&+ E_{\mathrm{R}}^{\dagger}\mathrm{i}\sigma^{\mu}\left(\partial_\mu + \mathrm{i}g_1 B_\mu\right)E_{\mathrm{R}}\\
&- \left(U_{\mathrm{R}}^{\dagger}y_u H Q_{\mathrm{L}} + D_{\mathrm{R}}^{\dagger}y_d H^{\dagger}Q_{\mathrm{L}} + E_{\mathrm{R}}^{\dagger}y_e H^{\dagger}L_{\mathrm{L}} + \text{H.c.}\right)\\
=&(\text{Higgs terms}) + (\text{Gauge fields strength})\\
&+ \mathrm{i}Q_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}\partial_\mu Q_{\mathrm{L}} + \mathrm{i}U_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_\mu U_{\mathrm{R}}^{\dagger} + \mathrm{i}D_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_\mu D_{\mathrm{R}}^{\dagger} + \mathrm{i}L_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}\partial_\mu L_{\mathrm{L}} + \mathrm{i}E_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_\mu E_{\mathrm{R}}^{\dagger}\\
&+ g_3\left(Q_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}G_\mu Q_{\mathrm{L}} + U_{\mathrm{R}}^{\dagger}\bar{\sigma}^{\mu}G_\mu U_{\mathrm{R}} + D_{\mathrm{R}}^{\dagger}\bar{\sigma}^{\mu}G_\mu D_{\mathrm{R}}\right)\\
&+ g_2\left(Q_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}W_\mu Q_{\mathrm{L}} + L_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}W_\mu L_{\mathrm{L}}\right)\\
&+ g_1\left(\frac{1}{6}Q_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}B_\mu Q_{\mathrm{L}} + \frac{2}{3}U_{\mathrm{R}}^{\dagger}\bar{\sigma}^{\mu}B_\mu U_{\mathrm{R}} - \frac{1}{3}D_{\mathrm{R}}^{\dagger}\bar{\sigma}^{\mu}B_\mu D_{\mathrm{R}} - \frac{1}{2}L_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}B_\mu L_{\mathrm{L}} - E_{\mathrm{R}}^{\dagger}\bar{\sigma}^{\mu}B_\mu E_{\mathrm{R}}\right)\\
&- \left(U_{\mathrm{R}}^{\dagger}y_u H Q_{\mathrm{L}} + D_{\mathrm{R}}^{\dagger}y_d H^{\dagger}Q_{\mathrm{L}} + E_{\mathrm{R}}^{\dagger}y_e H^{\dagger}L_{\mathrm{L}} + \text{H.c.}\right)
\end{aligned}
\tag{2.34}
$$

であり，そして最終的には

$$
\begin{aligned}
\mathcal{L} =&(\text{Gauge bosons and Higgs})\\
&+ \mathrm{i}Q_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}\partial_\mu Q_{\mathrm{L}} + \mathrm{i}U_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_\mu U_{\mathrm{R}}^{\dagger} + \mathrm{i}D_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_\mu D_{\mathrm{R}}^{\dagger} + \mathrm{i}L_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}\partial_\mu L_{\mathrm{L}} + \mathrm{i}E_{\mathrm{R}}\bar{\sigma}^{\mu}\partial_\mu E_{\mathrm{R}}^{\dagger}\\
&+ g_3\left(Q_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}G_\mu Q_{\mathrm{L}} + U_{\mathrm{R}}^{\dagger}\bar{\sigma}^{\mu}G_\mu U_{\mathrm{R}} + D_{\mathrm{R}}^{\dagger}\bar{\sigma}^{\mu}G_\mu D_{\mathrm{R}}\right)\\
&- m_u(u_{\mathrm{R}}^{\dagger}u_{\mathrm{L}} + u_{\mathrm{L}}^{\dagger}u_{\mathrm{R}}) - (\text{quarks}) - m_e(e_{\mathrm{R}}^{\dagger}e_{\mathrm{L}} + e_{\mathrm{L}}^{\dagger}e_{\mathrm{R}}) - (\text{leptons})\\
&- \frac{m_u}{v}(u_{\mathrm{R}}^{\dagger}u_{\mathrm{L}} + u_{\mathrm{L}}^{\dagger}u_{\mathrm{R}})h - (\text{quarks}) - \frac{m_e}{v}(e_{\mathrm{R}}^{\dagger}e_{\mathrm{L}} + e_{\mathrm{L}}^{\dagger}e_{\mathrm{R}})h - (\text{leptons})\\
&+ \frac{g_2}{\sqrt{2}}\left[\begin{pmatrix}d_{\mathrm{L}}^{\dagger} & s_{\mathrm{L}}^{\dagger} & b_{\mathrm{L}}^{\dagger}\end{pmatrix}\bar{\sigma}^{\mu}W_\mu^{-}X\begin{pmatrix}u_{\mathrm{L}}\\ c_{\mathrm{L}}\\ t_{\mathrm{L}}\end{pmatrix} + \begin{pmatrix}u_{\mathrm{L}}^{\dagger} & c_{\mathrm{L}}^{\dagger} & t_{\mathrm{L}}^{\dagger}\end{pmatrix}\bar{\sigma}^{\mu}W_\mu^{+}X^{\dagger}\begin{pmatrix}d_{\mathrm{L}}\\ s_{\mathrm{L}}\\ b_{\mathrm{L}}\end{pmatrix}\right]\\
&+ \frac{g_2}{\sqrt{2}}\left[\nu_e^{\dagger}\bar{\sigma}^{\mu}W_\mu^{+}e_{\mathrm{L}} + e_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}W_\mu^{-}\nu_e\right]\\
&+ |e|\left[\frac{2}{3}u_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}A_\mu u_{\mathrm{L}} - \frac{1}{3}d_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}A_\mu d_{\mathrm{L}} + \frac{2}{3}u_{\mathrm{R}}^{\dagger}\sigma^{\mu}A_\mu u_{\mathrm{R}} - \frac{1}{3}d_{\mathrm{R}}^{\dagger}\sigma^{\mu}A_\mu d_{\mathrm{R}} + (\text{quarks})\right.\\
&\qquad\left. - e_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}A_\mu e_{\mathrm{L}} - e_{\mathrm{R}}^{\dagger}\sigma^{\mu}A_\mu e_{\mathrm{R}} + (\text{leptons})\right]\\
&+ \frac{|e|s}{c}\left[\left(\frac{c^2}{2s^2} - \frac{1}{6}\right)u_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}Z_\mu u_{\mathrm{L}} - \left(\frac{c^2}{2s^2} + \frac{1}{6}\right)d_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}Z_\mu d_{\mathrm{L}} - \frac{2}{3}u_{\mathrm{R}}^{\dagger}\sigma^{\mu}Z_\mu u_{\mathrm{R}} + \frac{1}{3}d_{\mathrm{R}}^{\dagger}\sigma^{\mu}Z_\mu d_{\mathrm{R}}\right.\\
&\qquad\left. + \left(\frac{c^2}{2s^2} + \frac{1}{2}\right)\nu_e^{\dagger}\bar{\sigma}^{\mu}Z_\mu \nu_e - \left(\frac{c^2}{2s^2} - \frac{1}{2}\right)e_{\mathrm{L}}^{\dagger}\bar{\sigma}^{\mu}Z_\mu e_{\mathrm{L}} + e_{\mathrm{R}}^{\dagger}\sigma^{\mu}Z_\mu e_{\mathrm{R}} + (\text{others})\right]
\end{aligned}
\tag{2.35}
$$

となる。

2.6 Values of SM Parameters (Extracted from PDG 2010)

◆2.6.1 Experimental Values

Low energy values

$$\alpha_{\rm EM} = 1/137.035999679(94) \qquad G_{\rm F} = \frac{{g_2}^2}{4\sqrt{2}{m_W}^2} = \frac{1}{\sqrt{2}v^2} = 1.16637(1)\times 10^{-5}\,\mathrm{GeV}^{-2}$$

Electroweak scale 【These values are all in $\overline{\rm MS}$ scheme.】

$$\begin{aligned} \alpha_{\rm EM}^{-1}(m_Z) &= 127.925(16) & m_W(m_W) &= 80.399(23)\,\mathrm{GeV}\\ \alpha_{\rm EM}^{-1}(m_\tau) &= 133.452(16) & m_Z(m_Z) &= 91.1876(21)\,\mathrm{GeV}\\ \alpha_{\rm s}(m_Z) &= 0.1183(15) & \sin^2\theta_{\rm W}(m_Z) &= 0.23116(13)\\ \Gamma_{l^+l^-} &= 83.984(86)\,\mathrm{MeV} & \sin^2\theta_{\rm eff} &= 0.23146(12) \end{aligned}$$

Fundamental masses

$$\begin{array}{lll} e: 0.510998910(13)\,\mathrm{MeV} & u: 1.7 \text{ to } 3.3\,\mathrm{MeV} & d: 4.1 \text{ to } 5.8\,\mathrm{MeV}\\ \mu: 105.658367(4)\,\mathrm{MeV} & c: 1.27^{+0.07}_{-0.09}\,\mathrm{GeV} & s: 101^{+29}_{-21}\,\mathrm{MeV}\\ \tau: 1.77682(16)\,\mathrm{GeV} & t: 172.0_{\pm 2.2}\,\mathrm{GeV} & b: 4.19^{+0.18}_{-0.06}\,\mathrm{GeV} \end{array}$$

$$\begin{array}{lll} \pi^\pm: 139.57018(35)\,\mathrm{MeV} & K^\pm: 493.677(16)\,\mathrm{MeV} & p: 938.27203(8)\,\mathrm{MeV}\\ \pi^0: 134.9766(6)\,\mathrm{MeV} & K^0: 497.614(24)\,\mathrm{MeV} & n: 939.565346(23)\,\mathrm{MeV} \end{array}$$

Fundamental Lifetime (also $c\tau$ for some particles)

$$\begin{array}{lll} \mu: 2.197034(21)\,\mu\mathrm{s}\quad (659\,\mathrm{m}) & \pi^\pm: 2.6033(5)\times 10^{-8}\,\mathrm{s} & K^\pm: 1.2380(21)\times 10^{-8}\,\mathrm{s}\\ \tau: 2.906(10)\times 10^{-13}\,\mathrm{s}\quad (87\,\mu\mathrm{m}) & \pi^0: 8.4(5)\times 10^{-17}\,\mathrm{s} & K^0_{\rm S}: 8.953(5)\times 10^{-11}\,\mathrm{s}\\ & & K^0_{\rm L}: 5.116(20)\times 10^{-8}\,\mathrm{s} \end{array}$$

CKM matrix

$$V_{\rm CKM} = \begin{pmatrix} 0.97428(15) & 0.2253(7) & 0.00347(16)\\ 0.2252(7) & 0.97345(16) & 0.0410(11)\\ 0.00862(26) & 0.0403(11) & 0.999152(45) \end{pmatrix} \sim \begin{pmatrix} 1-\epsilon^2 & \epsilon & \epsilon^4\\ \epsilon & 1-\epsilon^2 & \epsilon^2\\ \epsilon^3 & \epsilon^2 & 1-\epsilon^4 \end{pmatrix} \quad \text{for } \epsilon\sim 0.23 \tag{2.36}$$

◆2.6.2 Estimation of SM Parameters

For EW scale, we can estimate the values as

$$e\sim 0.313,\qquad g_1\sim 0.358,\qquad g_2\sim 0.651,\qquad g_Z\sim 0.743;\qquad v=\sqrt{\frac{\mu^2}{\lambda}}\sim 246\,\mathrm{GeV} \tag{2.37}$$

Therefore 湯川 matrices are (after diagonalization), since $vy/\sqrt{2}=M$,

$$y_u\sim\begin{pmatrix} 10^{-5} & 0 & 0\\ 0 & 0.007 & 0\\ 0 & 0 & 0.98 \end{pmatrix},\quad y_d\sim\begin{pmatrix} 3\times 10^{-5} & 0 & 0\\ 0 & 0.0006 & 0\\ 0 & 0 & 0.02 \end{pmatrix},\quad y_e\sim\begin{pmatrix} 3\times 10^{-6} & 0 & 0\\ 0 & 0.0006 & 0\\ 0 & 0 & 0.01 \end{pmatrix}. \tag{2.38}$$

Also, for $m_h\sim 120\,\mathrm{GeV}$, we can estimate the Higgs potential as $\mu\sim 85\,\mathrm{GeV}$ and $\lambda\sim 0.12$.

3 Supersymmetry for $\eta = \mathrm{diag}(+,-,-,-)$

3.1 Spinor Convention

(See App. ?? for a verbose explanation.)

ϵ tensor : $\epsilon^{12} = \epsilon^{\dot{1}\dot{2}} = \epsilon_{21} = \epsilon_{\dot{2}\dot{1}} = 1$ (definition)

Sum Rule : ${}^{\alpha}{}_{\alpha}$ and ${}_{\dot\alpha}{}^{\dot\alpha}$, except for $\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta,\quad \xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta,\quad \xi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\xi^{\dot\beta},\quad \xi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\xi_{\dot\beta}$.

Lorentz 変換 : $\psi'_\alpha = \Lambda_\alpha{}^\beta\psi_\beta,\quad \bar\psi'_{\dot\alpha} = \bar\psi_{\dot\beta}\Lambda^{\dagger\dot\beta}{}_{\dot\alpha},\quad \psi'^\alpha = \psi^\beta\Lambda^{-1}{}_\beta{}^\alpha,\quad \bar\psi'^{\dot\alpha} = (\Lambda^{-1})^{\dagger\dot\alpha}{}_{\dot\beta}\bar\psi'^{\dot\beta}$.

σ matrices : $(\sigma^\mu)_{\alpha\dot\beta} := (1,\boldsymbol{\sigma})_{\alpha\dot\beta},\quad (\bar\sigma^\mu)^{\dot\alpha\alpha} := \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}(\sigma^\mu)_{\beta\dot\beta} = (1,-\boldsymbol{\sigma})^{\dot\alpha\beta}$.

3.2 Spinor Calculation Cheatsheet

$$\eta = (+,-,-,-),\qquad \epsilon^{0123} = -\epsilon_{0123} = 1;\qquad \textbf{Left Differential};$$
$$\epsilon^{12} = \epsilon_{21} = \epsilon^{\dot1\dot2} = \epsilon_{\dot2\dot1} = 1,\quad \xi^\alpha := \epsilon^{\alpha\beta}\xi_\beta,\quad \xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta,\quad \bar\xi^{\dot\alpha} := \epsilon^{\dot\alpha\dot\beta}\bar\xi_{\dot\beta},\quad \bar\xi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\xi^{\dot\beta}$$
$$\bar\sigma^{\mu\dot\alpha\alpha} := \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\sigma^\mu_{\beta\dot\beta}\qquad \sigma^\mu_{\alpha\dot\alpha} = \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\bar\sigma^{\mu\dot\beta\beta},\qquad \sigma^\mu := (1,\boldsymbol\sigma),\quad \bar\sigma^\mu := (1,-\boldsymbol\sigma)$$
$$(\sigma^{\mu\nu})_\alpha{}^\beta := \tfrac14\left(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu\right)_\alpha{}^\beta,\quad (\bar\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} := \tfrac14\left(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu\right)^{\dot\alpha}{}_{\dot\beta} = (\sigma^{\nu\mu})^{\dagger\dot\alpha}{}_{\dot\beta}.$$

$$\begin{array}{llll}
\theta^\alpha\theta^\beta = -\frac12\epsilon^{\alpha\beta}\theta\theta & \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \frac12\epsilon^{\dot\alpha\dot\beta}\bar\theta\bar\theta & (\theta\phi)(\theta\psi) = -\frac12(\psi\phi)(\theta\theta) & (\theta\sigma^\nu\bar\theta)\theta^\alpha = \frac12\theta\theta(\bar\theta\bar\sigma^\nu)^\alpha \\
\theta_\alpha\theta_\beta = \frac12\epsilon_{\alpha\beta}\theta\theta & \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\frac12\epsilon_{\dot\alpha\dot\beta}\bar\theta\bar\theta & (\bar\theta\bar\phi)(\bar\theta\bar\psi) = -\frac12(\bar\psi\bar\phi)(\bar\theta\bar\theta) & (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\frac12\bar\theta\bar\theta(\theta\sigma^\mu)_{\dot\alpha} \\
\theta^\alpha\theta_\beta = \frac12\delta^\alpha_\beta\theta\theta & \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \frac12\delta^{\dot\beta}_{\dot\alpha}\bar\theta\bar\theta & (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \frac12\theta\theta\bar\theta\bar\theta\eta^{\mu\nu} & (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \frac12(\sigma^\mu\bar\sigma^\nu\theta)_\alpha\bar\theta\bar\theta \\
\theta\sigma^\mu\bar\sigma^\nu\theta = \eta^{\mu\nu}\theta\theta & \bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta = \eta^{\mu\nu}\bar\theta\bar\theta & &
\end{array}$$

$$\begin{array}{ll}
\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} + 2\sigma^{\mu\nu} & \sigma^\mu\bar\sigma^\rho\sigma^\nu + \sigma^\nu\bar\sigma^\rho\sigma^\mu = 2\left(\eta^{\mu\rho}\sigma^\nu + \eta^{\nu\rho}\sigma^\mu - \eta^{\mu\nu}\sigma^\rho\right) \\
\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} + 2\bar\sigma^{\mu\nu} & \bar\sigma^\mu\sigma^\rho\bar\sigma^\nu + \bar\sigma^\nu\sigma^\rho\bar\sigma^\mu = 2\left(\eta^{\mu\rho}\bar\sigma^\nu + \eta^{\nu\rho}\bar\sigma^\mu - \eta^{\mu\nu}\bar\sigma^\rho\right) \\
\sigma^{\mu\nu} = -\sigma^{\nu\mu} & \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\mathrm{i}\epsilon^{\mu\nu\rho\sigma}\sigma_\sigma \\
\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu} & \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2\mathrm{i}\epsilon^{\mu\nu\rho\sigma}\bar\sigma_\sigma \\
\mathrm{Tr}\,\bar\sigma^\mu\sigma^\nu = \mathrm{Tr}\,\sigma^\mu\bar\sigma^\nu = 2\eta^{\mu\nu} & \mathrm{Tr}\,\sigma^{\mu\nu}\sigma^{\rho\sigma} = -\frac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) + \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho\sigma} \\
\mathrm{Tr}\,\sigma^{\mu\nu} = \mathrm{Tr}\,\bar\sigma^{\mu\nu} = 0 & \mathrm{Tr}\,\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma} = -\frac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho\sigma} \\
\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu^{\dot\beta\beta} = 2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta} & \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\left[(\sigma^{\mu\nu}\epsilon)_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} + (\epsilon\bar\sigma^{\mu\nu})_{\dot\alpha\dot\beta}\epsilon_{\alpha\beta}\right] \\
\sigma^\mu_{\alpha\dot\alpha}\sigma_{\mu\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} & \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = \eta^{\mu\nu}\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} - 4\eta_{\rho\sigma}(\sigma^{\rho\mu}\epsilon)_{\alpha\beta}(\epsilon\bar\sigma^{\sigma\nu})_{\dot\alpha\dot\beta}
\end{array}$$

$$\begin{array}{lll}
\bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma_\mu^{\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} & \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot\beta} & \epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma} = 2\mathrm{i}\sigma^{\mu\nu} \\
\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha} & \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon^{\dot\alpha\dot\beta}\sigma^\mu_{\beta\dot\beta} & \epsilon^{\mu\nu\rho\sigma}\bar\sigma_{\rho\sigma} = -2\mathrm{i}\bar\sigma^{\mu\nu}
\end{array}$$

$$\begin{array}{ll}
\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi = (\bar\chi\bar\sigma^\mu\xi)^* = -(\xi\sigma^\mu\bar\chi)^* & (\psi\phi)\chi_\alpha = -(\phi\chi)\psi_\alpha - (\chi\psi)\phi_\alpha \\
\xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi = (\bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi)^* = (\bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi)^* & (\psi\phi)\bar\chi_{\dot\alpha} = \frac12(\phi\sigma^\mu\bar\chi)(\psi\sigma_\mu)_{\dot\alpha}
\end{array}$$

$$\begin{array}{lll}
\epsilon^{\alpha\beta}\dfrac{\partial}{\partial\theta^\beta} = -\dfrac{\partial}{\partial\theta_\alpha} & \dfrac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha & \epsilon^{\alpha\beta}\dfrac{\partial}{\partial\theta^\alpha}\dfrac{\partial}{\partial\theta^\beta}\theta\theta = \dfrac{\partial}{\partial\theta_\alpha}\dfrac{\partial}{\partial\theta^\alpha}\theta\theta = 4 \\
\epsilon_{\alpha\beta}\dfrac{\partial}{\partial\theta_\beta} = -\dfrac{\partial}{\partial\theta^\alpha} & \dfrac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha & \epsilon_{\alpha\beta}\dfrac{\partial}{\partial\theta_\alpha}\dfrac{\partial}{\partial\theta_\beta}\theta\theta = \dfrac{\partial}{\partial\theta^\alpha}\dfrac{\partial}{\partial\theta_\alpha}\theta\theta = -4 \\
\epsilon^{\dot\alpha\dot\beta}\dfrac{\partial}{\partial\bar\theta^{\dot\beta}} = -\dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} & \dfrac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} & \epsilon_{\dot\alpha\dot\beta}\dfrac{\partial}{\partial\bar\theta_{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = \dfrac{\partial}{\partial\bar\theta^{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 4 \\
\epsilon_{\dot\alpha\dot\beta}\dfrac{\partial}{\partial\bar\theta_{\dot\beta}} = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} & \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} & \epsilon^{\dot\alpha\dot\beta}\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}}\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -4
\end{array}$$

3.3 General Relations (Note: $P_\mu = \mathrm{i}\partial_\mu$ in our convention.)

$$Q_\alpha := \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \qquad D_\alpha := \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \qquad y := x - \mathrm{i}\theta\sigma\bar\theta,$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \qquad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \qquad y^\dagger = x + \mathrm{i}\theta\sigma\bar\theta$$

$$\{Q_\alpha, \bar Q_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \{D_\alpha, \bar D_{\dot\alpha}\} = 2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad (\text{others}) = 0.$$

$$\begin{array}{llll}
 & < x\text{-basis} > & < y\text{-basis} > & < y^\dagger\text{-basis} > \\
D_\alpha & = \dfrac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu & = \dfrac{\partial}{\partial\theta^\alpha} - 2\mathrm{i}(\sigma^\mu\bar\theta)_\alpha\dfrac{\partial}{\partial y^\mu} & = \dfrac{\partial}{\partial\theta^\alpha} \qquad (3.1) \\
\bar D_{\dot\alpha} & = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu & = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} & = -\dfrac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2\mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\dfrac{\partial}{\partial(y^\dagger)^\mu} \qquad (3.2) \\
D^\alpha & = -\dfrac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu & = -\dfrac{\partial}{\partial\theta_\alpha} + 2\mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\dfrac{\partial}{\partial y^\mu} & = -\dfrac{\partial}{\partial\theta_\alpha} \qquad (3.3) \\
\bar D^{\dot\alpha} & = \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu & = \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} & = \dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} - 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\dfrac{\partial}{\partial(y^\dagger)^\mu} \qquad (3.4)
\end{array}$$

$$\phi(y) = \phi(x) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(x) = \phi(y^\dagger) - 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y^\dagger) - \theta\theta\bar\theta\bar\theta\partial^2\phi(y^\dagger) \tag{3.5}$$

$$\phi(y^\dagger) = \phi(x) + \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(x) = \phi(y) + 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y) - \theta\theta\bar\theta\bar\theta\partial^2\phi(y) \tag{3.6}$$

$$\phi(x) = \phi(y) + \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(y) = \phi(y^\dagger) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y^\dagger) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(y^\dagger) \tag{3.7}$$

3.4 Chiral Superfields : $\bar{D}_{\dot\alpha}\Phi = 0$

Explicit Expression

$$\begin{aligned}
\Phi &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) && (3.8)\\
&= \phi(x) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi(x) + \sqrt{2}\theta\psi(x) - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x) + \theta\theta F(x) && (3.9)\\
&= \phi(y^\dagger) - 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi(y^\dagger) - \theta\theta\bar\theta\bar\theta\partial^2\phi(y^\dagger) + \sqrt{2}\theta\psi(y^\dagger) - \sqrt{2}\mathrm{i}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(y^\dagger) + \theta\theta F(y^\dagger) && (3.10)\\
\Phi^\dagger &= \phi^*(y^\dagger) + \sqrt{2}\bar\theta\bar\psi(y^\dagger) + \bar\theta\bar\theta F^*(y^\dagger) && (3.11)\\
&= \phi^*(x) + \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi^*(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi^*(x) + \sqrt{2}\bar\theta\bar\psi(x) - \frac{\mathrm{i}}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\bar\psi(x) + \bar\theta\bar\theta F^*(x) && (3.12)\\
&= \phi^*(y) + 2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi^*(y) - \theta\theta\bar\theta\bar\theta\partial^2\phi^*(y) + \sqrt{2}\bar\theta\bar\psi(y) - \sqrt{2}\mathrm{i}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\bar\psi(y) + \bar\theta\bar\theta F^*(y) && (3.13)
\end{aligned}$$

Product of Chiral Superfields

$$\begin{aligned}
\Phi_i^\dagger\Phi_j(x,\theta,\bar\theta) &= \phi_i^*\phi_j + \sqrt{2}\phi_i^*\theta\psi_j + \sqrt{2}\bar\theta\bar\psi_i\phi_j + \theta\theta\phi_i^*F_j + \bar\theta\bar\theta F_i^*\phi_j\\
&\quad - \mathrm{i}\theta\sigma^\mu\bar\theta\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right) + 2\bar\theta\bar\psi_i\theta\psi_j\\
&\quad + \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\left(\phi_i^*\partial_\mu\psi_j - \partial_\mu\phi_i^*\psi_j\right)\sigma^\mu\bar\theta + \sqrt{2}\theta\theta\bar\theta\bar\psi_i F_j\\
&\quad - \frac{\mathrm{i}}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\left(\partial_\mu\bar\psi_i\phi_j - \bar\psi_i\partial_\mu\phi_j\right) + \sqrt{2}\bar\theta\bar\theta F_i^*\theta\psi_j && (3.14)\\
&\quad + \theta\theta\bar\theta\bar\theta\left[F_i^*F_j - \frac{1}{4}\phi_i^*\partial^2\phi_j - \frac{1}{4}\partial^2\phi_i^*\phi_j + \frac{1}{2}\partial_\mu\phi_i^*\partial_\mu\phi_j - \frac{\mathrm{i}}{2}\partial_\mu\bar\psi_i\bar\sigma^\mu\psi_j + \frac{\mathrm{i}}{2}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right]\\
&\rightsquigarrow \phi_i^*\phi_j + \sqrt{2}\phi_i^*\theta\psi_j + \sqrt{2}\bar\theta\bar\psi_i\phi_j + \theta\theta\phi_i^*F_j + \bar\theta\bar\theta F_i^*\phi_j\\
&\quad - 2\mathrm{i}(\theta\sigma^\mu\bar\theta)(\phi_i^*\partial_\mu\phi_j) + \sqrt{2}\mathrm{i}\theta\theta(\partial_\mu\phi_i^*)\bar\theta\bar\sigma^\mu\psi_j + \sqrt{2}\mathrm{i}\bar\theta\bar\theta\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j && (3.15)\\
&\quad + 2\bar\theta\bar\psi_i\theta\psi_j + \sqrt{2}\theta\theta\bar\theta\bar\psi_i F_j + \sqrt{2}\bar\theta\bar\theta F_i^*\theta\psi_j\\
&\quad + \theta\theta\bar\theta\bar\theta\left[F_i^*F_j + \partial^\mu\phi_i^*\partial_\mu\phi_j + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right]
\end{aligned}$$

$$\begin{aligned}
\Phi_i\Phi_j(\text{in } y\text{-basis}) &= \phi_i\phi_j + \sqrt{2}\theta\left[\psi_i\phi_j + \phi_i\psi_j\right] + \theta\theta\left[\phi_iF_j + F_i\phi_j - \psi_i\psi_j\right] && (3.16)\\
\Phi_i\Phi_j\Phi_k(\text{in } y\text{-basis}) &= \phi_i\phi_j\phi_k + \sqrt{2}\theta\left[\psi_i\phi_j\phi_k + \phi_i\psi_j\phi_k + \phi_i\phi_j\psi_k\right]\\
&\quad + \theta\theta\left[F_i\phi_j\phi_k + \phi_iF_j\phi_k + \phi_i\phi_jF_k - \psi_i\psi_j\phi_k - \psi_i\phi_j\psi_k - \phi_i\psi_j\psi_k\right] && (3.17)
\end{aligned}$$

(Products of chiral superfields are still chiral superfields.)

$$e^{\mathrm{i}k\Phi} = e^{\mathrm{i}k\phi(y)}\left[1 + \mathrm{i}k\left(\sqrt{2}\theta\psi(y) + \theta\theta F(y)\right) + \frac{k^2}{2}\theta\theta\psi(y)\psi(y)\right] \qquad (3.18)$$

Lagrangian Blocks

$$\mathcal{L}_{\text{kin.}} = \Phi_i^\dagger\Phi_j\Big|_{\theta\theta\bar\theta\bar\theta} \rightsquigarrow F_i^*F_j + \partial^\mu\phi_i^*\partial_\mu\phi_j + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j \qquad (3.19)$$

$$\begin{aligned}
\mathcal{L}_{\text{super}} &= W\Big|_{\theta\theta} + W^*\Big|_{\bar\theta\bar\theta} = \int \mathrm{d}^2\theta\left[\lambda_i\Phi_i + m_{ij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k\right] + \text{H.c.}\\
&= \lambda_iF_i + m_{ij}\left(\phi_iF_j + F_i\phi_j - \psi_i\psi_j\right) + y_{ijk}\Big[(F_i\phi_j\phi_k - \psi_i\psi_j\phi_k) + (jki \text{ and } kij \text{ terms})\Big] && (3.20)
\end{aligned}$$

3.5 Vector Superfields and Gauge Theory : $V = V^\dagger$

◆3.5.1 Abelian Case — Field Construction

Explicit Expression

$$\begin{aligned} V &= C(x) + \mathrm{i}\theta\chi(x) - \mathrm{i}\bar\theta\bar\chi(x) \\ &\quad + \frac{\mathrm{i}}{2}\theta\theta\left[M(x) + \mathrm{i}N(x)\right] - \frac{\mathrm{i}}{2}\bar\theta\bar\theta\left[M(x) - \mathrm{i}N(x)\right] - \theta\sigma^\mu\bar\theta A_\mu(x) \\ &\quad + \theta\theta\bar\theta\left[\bar\lambda(x) + \frac{1}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right] + \bar\theta\bar\theta\theta\left[\lambda(x) - \frac{1}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right] + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(x) - \frac{1}{2}\partial^2 C(x)\right] \end{aligned} \tag{3.21}$$

$$\begin{aligned} &= C(y) + \mathrm{i}\theta\chi(y) - \mathrm{i}\bar\theta\bar\chi(y) \\ &\quad + \frac{\mathrm{i}}{2}\theta\theta\left[M(y) + \mathrm{i}N(y)\right] - \frac{\mathrm{i}}{2}\bar\theta\bar\theta\left[M(y) - \mathrm{i}N(y)\right] - \theta\sigma^\mu\bar\theta\left[A_\mu(y) - \mathrm{i}\partial_\mu C(y)\right] \\ &\quad + \theta\theta\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\theta\left[\lambda(y) - \sigma^\mu\partial_\mu\bar\chi(y)\right] + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y) - \partial^2 C(y) - \mathrm{i}\partial_\mu A^\mu(y)\right] \end{aligned} \tag{3.22}$$

$$\begin{aligned} &= C(y^\dagger) + \mathrm{i}\theta\chi(y^\dagger) - \mathrm{i}\bar\theta\bar\chi(y^\dagger) \\ &\quad + \frac{\mathrm{i}}{2}\theta\theta\left[M(y^\dagger) + \mathrm{i}N(y^\dagger)\right] - \frac{\mathrm{i}}{2}\bar\theta\bar\theta\left[M(y^\dagger) - \mathrm{i}N(y^\dagger)\right] - \theta\sigma^\mu\bar\theta\left[A_\mu(y^\dagger) + \mathrm{i}\partial_\mu C(y^\dagger)\right] \\ &\quad + \theta\theta\bar\theta\left[\bar\lambda(y^\dagger) + \bar\sigma^\mu\partial_\mu\chi(y^\dagger)\right] + \bar\theta\bar\theta\theta\lambda(y^\dagger) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y^\dagger) - \partial^2 C(y^\dagger) + \mathrm{i}\partial_\mu A^\mu(y^\dagger)\right] \end{aligned} \tag{3.23}$$

Supersymmetric Gauge Transformation : $V \to V + \Phi + \Phi^\dagger$

$$\begin{aligned} C &\mapsto C + (\phi + \phi^*) & A_\mu &\mapsto A_\mu + \mathrm{i}\partial_\mu(\phi - \phi^*) \\ \chi &\mapsto \chi - \mathrm{i}\sqrt{2}\psi & \lambda &\mapsto \lambda \\ M + \mathrm{i}N &\mapsto M + \mathrm{i}N - 2\mathrm{i}F & D &\mapsto D \end{aligned} \tag{3.24}$$

Wess-Zumino Gauge $\quad C = \chi = M = N = 0$

Fixing this gauge breaks SUSY, but still allows the usual gauge transformation

$$A_\mu \mapsto A_\mu + \partial_\mu\alpha, \quad \lambda \mapsto \lambda, \quad D \mapsto D. \tag{3.25}$$

$$\begin{aligned} V &= -\theta\sigma^\mu\bar\theta A_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x) \\ &= -\theta\sigma^\mu\bar\theta A_\mu(y) + \theta\theta\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\theta\lambda(y) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y) - \mathrm{i}\partial_\mu A^\mu(y)\right] \\ &= -\theta\sigma^\mu\bar\theta A_\mu(y^\dagger) + \theta\theta\bar\theta\bar\lambda(y^\dagger) + \bar\theta\bar\theta\theta\lambda(y^\dagger) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(y^\dagger) + \mathrm{i}\partial_\mu A^\mu(y^\dagger)\right] \\ \mathrm{e}^{kV} &= 1 - k\theta\sigma^\mu\bar\theta A_\mu(x) + k\theta\theta\bar\theta\bar\lambda + k\bar\theta\bar\theta\theta\lambda + \theta\theta\bar\theta\bar\theta\left[\frac{k}{2}D + \frac{k^2}{4}A_\mu A^\mu\right] \end{aligned}$$

$$V^2 = \frac{1}{2}\theta\theta\bar\theta\bar\theta A_\mu A^\mu \qquad V^3 = 0$$

Field Strength

$$W_\alpha = -\frac{1}{4}\bar D\bar D D_\alpha V, \quad \bar W_{\dot\alpha} = -\frac{1}{4}DD\bar D_{\dot\alpha}V; \qquad W_\alpha \mapsto W_\alpha \quad \text{(gauge invariant)} \tag{3.26}$$

$$\bar D_{\dot\beta}W_\alpha = D_\beta\bar W_{\dot\alpha} = 0; \qquad D^\alpha W_\alpha = \bar D_{\dot\alpha}\bar W^{\dot\alpha} \tag{3.27}$$

$$W_\alpha = \lambda_\alpha(y) + \theta_\alpha D(y) + \mathrm{i}(\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}(y) + \mathrm{i}\theta\theta\left[\sigma^\mu\partial_\mu\bar\lambda(y)\right]_\alpha \tag{3.28}$$

$$\bar W_{\dot\alpha} = \bar\lambda_{\dot\alpha}(y^\dagger) + \bar\theta_{\dot\alpha}D(y^\dagger) + \mathrm{i}(\bar\theta\bar\sigma^{\mu\nu})_{\dot\alpha}F_{\mu\nu}(y^\dagger) - \mathrm{i}\bar\theta\bar\theta\left[\partial_\mu\lambda(y^\dagger)\sigma^\mu\right]_{\dot\alpha} \tag{3.29}$$

$$W^\alpha W_\alpha\big|_{\theta\theta} = -\frac{1}{4}\bar D\bar D W^\alpha D_\alpha V \rightsquigarrow -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{\mathrm{i}}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + 2\mathrm{i}\lambda\sigma^\mu\partial_\mu\bar\lambda + D^2 \tag{3.30}$$

Lagrangian Blocks ($\mathcal{L}_{\text{inv}}$ is SUSY- and gauge-invariant, while $\mathcal{L}_{\text{mass}}$ is not gauge-invariant.)

$$\begin{aligned}
\mathcal{L}_{\text{inv}} &= \frac{\tau}{4}W^\alpha W_\alpha\Big|_{\theta\theta} + \frac{\tau^*}{4}\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\Big|_{\bar\theta\bar\theta} \qquad \left(\text{with} \quad \tau := 1 + \frac{\mathrm{i}\theta}{8\pi^2}\right) \\
&\rightsquigarrow -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\lambda\sigma^\mu\partial_\mu\bar\lambda + \frac{1}{2}D^2 - \frac{\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \qquad (3.31) \\
\mathcal{L}_{\text{mass}} &= m^2V^2\Big|_{\theta\theta\bar\theta\bar\theta} \\
&= m^2\left(\frac{1}{2}A^\mu A_\mu + \mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - (\lambda\chi + \bar\lambda\bar\chi) + \frac{1}{2}(M^2+N^2) + CD + \frac{1}{2}\partial_\mu C\partial^\mu C\right) \\
&= \frac{1}{2}\partial_\mu C'\partial^\mu C' + \mathrm{i}\bar\chi'\bar\sigma^\mu\partial_\mu\chi' + \frac{m^2}{2}A^\mu A_\mu - m(\lambda\chi' + \bar\lambda\bar\chi') + mC'D + \frac{m^2}{2}(M^2+N^2) \qquad (3.32)
\end{aligned}$$

◆3.5.2 Abelian Case — Gauge Theory

Here we turn on the coupling constant g. When $\Lambda = \mathrm{i}\lambda(y) + \sqrt{2}\theta\xi(y) + \theta\theta K(y)$,

$$\mathcal{L} \ni \Phi^\dagger \mathrm{e}^{2gqV}\Phi; \qquad \Phi \mapsto \mathrm{e}^{\mathrm{i}qg\Lambda}\Phi, \quad \Phi^\dagger \mapsto \Phi^\dagger\mathrm{e}^{-\mathrm{i}qg\Lambda^\dagger}; \qquad 2V \mapsto 2V - \mathrm{i}(\Lambda - \Lambda^\dagger) \qquad (3.33)$$

$$\begin{aligned}
&\phi \mapsto \mathrm{e}^{-qg\lambda}\phi && C \mapsto C + \operatorname{Re}\lambda && M + \mathrm{i}N \mapsto M + \mathrm{i}N - K \\
&\psi \mapsto \mathrm{e}^{-qg\lambda}(\psi + \mathrm{i}qg\phi\cdot\xi) && \chi \mapsto \chi - \tfrac{1}{\sqrt2}\xi && A_\mu \mapsto A_\mu - \partial_\mu(\operatorname{Im}\lambda) \\
&F \mapsto \mathrm{e}^{-qg\lambda}\left(F + \mathrm{i}qg\phi K - \mathrm{i}qg\xi\psi + \frac{(qg)^2}{2}\xi\xi\phi\right) && \lambda \mapsto \lambda && D \mapsto D
\end{aligned}$$

(Very similar to the gauge transformations in Sec. 1.9.2.)

Lagrangian block

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} &= \frac{1}{4}W^\alpha W_\alpha\Big|_{\theta\theta} + \text{H.c.} && \rightsquigarrow -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathrm{i}\lambda\sigma^\mu\partial_\mu\bar\lambda + \frac{1}{2}D^2 \\
\mathcal{L}_{\text{chiral}} &= \Phi^\dagger\mathrm{e}^{2gqV}\Phi\Big|_{\theta\theta\bar\theta\bar\theta} && \rightsquigarrow F^*F + \mathrm{D}_\mu\phi^*\mathrm{D}^\mu\phi + \mathrm{i}\bar\psi\bar\sigma^\mu\mathrm{D}_\mu\psi + qgD\phi^*\phi - \sqrt{2}gq\left(\phi^*\lambda\psi + \phi\bar\lambda\bar\psi\right) \\
\mathcal{L}_{\cancel{CP}} &= \frac{\mathrm{i}\theta}{32\pi^2}W^\alpha W_\alpha\Big|_{\theta\theta} + \text{H.c.} && \rightsquigarrow -\frac{\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\
& && \text{with} \quad \mathrm{D}_\mu[\phi,\psi] = (\partial_\mu - \mathrm{i}gqA_\mu)[\phi,\psi], \qquad \mathrm{D}_\mu\lambda = \partial_\mu\lambda,
\end{aligned}$$

◆3.5.3 Non-Abelian Case

T^a : generators (Hermitian);
$\operatorname{Tr}T^aT^b = K\delta^{ab}$ $(K>0)$, $\qquad [T^a, T^b] = \mathrm{i}f^{abc}T^c$ (f is anti-symmetric)

Explicit Expression Same as the Abelian case.

Supersymmetric Gauge Transformation

$$\mathcal{L} \ni \Phi^\dagger\mathrm{e}^{2g\widetilde V}\Phi; \qquad \Phi \mapsto \mathrm{e}^{\mathrm{i}g\widetilde\Lambda}\Phi, \quad \Phi^\dagger \mapsto \Phi^\dagger\mathrm{e}^{-\mathrm{i}g\widetilde\Lambda^\dagger}; \qquad \mathrm{e}^{2g\widetilde V} \mapsto \mathrm{e}^{\mathrm{i}g\widetilde\Lambda^\dagger}\mathrm{e}^{2g\widetilde V}\mathrm{e}^{-\mathrm{i}g\widetilde\Lambda} \qquad (3.34)$$

$$\text{with } \widetilde V := V^aT^a, \quad \widetilde\Lambda := \Lambda^aT^a, \quad \widetilde\Lambda^\dagger := (\Lambda^a)^\dagger T^a$$

$$\begin{aligned}
2\widetilde V \mapsto 2\widetilde V - \mathrm{i}(\widetilde\Lambda - \widetilde\Lambda^\dagger) - \frac{g}{2}\left([\widetilde\Lambda, \widetilde\Lambda^\dagger] + \mathrm{i}[2\widetilde V, \widetilde\Lambda + \widetilde\Lambda^\dagger]\right) + \cdots \qquad (3.35) \\
= \left[2V^a - \mathrm{i}(\Lambda^a - \Lambda^{\dagger a}) + \frac{g}{2}\left(-\mathrm{i}\Lambda^b\Lambda^{\dagger c} + 2V^b(\Lambda^c + \Lambda^{\dagger c})\right)f^{abc} + \cdots\right]T^a \qquad (3.36)
\end{aligned}$$

We do not present the transformations of the components; note that λ and D are transformed in non-Abelian theories.

Wess-Zumino Gauge

$$V^a = -\theta\sigma^\mu\bar\theta A^a_\mu(x) + \theta\theta\bar\theta\bar\lambda^a(x) + \bar\theta\bar\theta\theta\lambda^a(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D^a(x) \tag{3.37}$$

$$\mathrm{e}^{kV^aT^a} = 1 + kV^aT^a + \frac{k^2}{4}\theta\theta\bar\theta\bar\theta A^a_\mu A^{b\mu}T^aT^b \tag{3.38}$$

Note that the lowest order term of the gauge transformation is independent of V, which guarantees that we can still take the Wess-Zumino gauge. The gauge transformation is restricted as $\mathrm{e}^{\mathrm{i}g\widetilde\Lambda}$, where $\Lambda^a = \xi^a(y) = \xi^a(x) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\xi^a(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\xi^a(x) : \xi\in\mathbb{R}$.

$$\begin{aligned}2V^e \mapsto{}& 2\left(V^e - gf^{eab}\xi^aV^b + 6g^2f^{abc}f^{ade}V^b\xi^c\xi^d\right)\\ &+ \theta\sigma^\mu\bar\theta\left(-2\partial_\mu\xi^e + gf^{eab}\xi^a\partial_\mu\xi^b + 4g^2f^{acd}f^{abe}\xi^b\xi^c\partial_\mu\xi^d\right) + \cdots\end{aligned} \tag{3.39}$$

$$\begin{aligned}A^e_\mu \mapsto{}& A^e_\mu + gf^{eab}A^a_\mu\xi^b + 6g^2f^{abc}f^{ade}A^b_\mu\xi^c\xi^d\\ &+ \left(\partial_\mu\xi^e - \frac{g}{2}f^{eab}\xi^a\partial_\mu\xi^b - 2g^2f^{acd}f^{abe}\xi^b\xi^c\partial_\mu\xi^d\right) + \cdots\end{aligned} \tag{3.40}$$

$$\lambda^e \mapsto \lambda^e + gf^{eab}\lambda^a\xi^b + 6g^2f^{abc}f^{ade}\lambda^b\xi^c\xi^d + \cdots \tag{3.41}$$

$$D^e \mapsto D^e + gf^{eab}D^a\xi^b + 6g^2f^{abc}f^{ade}D^b\xi^c\xi^d + \cdots \tag{3.42}$$

Note that C, χ, M and N are kept invariant automatically, for now we are under Wess-Zumino gauge.

Field Strength [*2]

$$\widetilde W_\alpha = -\frac{1}{8g}\bar D\bar D\mathrm{e}^{-2g\widetilde V}D_\alpha\mathrm{e}^{2g\widetilde V} \qquad \bar D_{\dot\beta}W_\alpha = 0 \qquad W_\alpha \mapsto \mathrm{e}^{\mathrm{i}g\widetilde\Lambda}W_\alpha\mathrm{e}^{-\mathrm{i}g\Lambda^\dagger} \tag{3.43}$$

$$W^a_\alpha = \lambda^a_\alpha(y) + \theta_\alpha D^a(y) + \mathrm{i}(\sigma^{\mu\nu}\theta)_\alpha F^a_{\mu\nu}(y) + \mathrm{i}\theta\theta(\sigma^\mu\mathrm{D}_\mu\bar\lambda^a(y))_\alpha \tag{3.44}$$

$$\mathrm{Tr}\,\widetilde W^\alpha\widetilde W_\alpha\Big|_{\theta\theta} = \mathrm{Tr}\left[DD + \mathrm{i}\lambda\sigma^\mu\mathrm{D}_\mu\bar\lambda + \mathrm{i}\mathrm{D}_\mu\bar\lambda\bar\sigma^\mu\lambda - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{\mathrm{i}}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right] \tag{3.45}$$

$$= K\left[D^aD^a + \mathrm{i}\lambda^a\sigma^\mu\mathrm{D}_\mu\bar\lambda^a + \mathrm{i}\mathrm{D}_\mu\bar\lambda^a\bar\sigma^\mu\lambda^a - \frac{1}{2}F^{\mu\nu\,a}F^a_{\mu\nu} + \frac{\mathrm{i}}{4}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\right] \tag{3.46}$$

Lagrangian Block

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4K}\mathrm{Tr}\,\widetilde W^\alpha\widetilde W_\alpha\Big|_{\theta\theta} + \text{H.c.} \qquad \rightsquigarrow -\frac{1}{4}F^{\mu\nu\,a}F^a_{\mu\nu} + \mathrm{i}\bar\lambda^a\bar\sigma^\mu\mathrm{D}_\mu\lambda^a + \frac{1}{2}D^aD^a \tag{3.47}$$

$$\mathcal{L}_{\cancel{CP}} = \frac{\mathrm{i}}{32K\pi^2}\mathrm{Tr}\,\widetilde W^\alpha\widetilde W_\alpha\Big|_{\theta\theta} + \text{H.c.} \qquad \rightsquigarrow -\frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} \tag{3.48}$$

$$\begin{aligned}\mathcal{L}_{\text{matter}} = \Phi^\dagger_i[\mathrm{e}^{2gV^aT^a}]_{ij}\Phi_j\Big|_{\theta\theta\bar\theta\bar\theta} \qquad &\rightsquigarrow \mathrm{D}^\mu\phi^*_i\mathrm{D}_\mu\phi_i + \mathrm{i}\bar\psi_i\bar\sigma^\mu\mathrm{D}_\mu\psi_i + F^*_iF_i\\ &\quad + gD^a(\phi^*T^a\phi) - \sqrt{2}g\left(\phi^*T^a\psi\lambda + \bar\psi\bar\lambda T^a\phi\right)\end{aligned} \tag{3.49}$$

$$\begin{aligned}\mathrm{D}_\mu\phi_i &= \partial_\mu\phi - \mathrm{i}gA^a_\mu(T^a\phi)_i & F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g[A_\mu, A_\nu],\\ \mathrm{D}_\mu\phi^*_i &= \partial_\mu\phi + \mathrm{i}gA^a_\mu(\phi^*T^a)_i & \mathrm{D}_\mu\lambda^a &= \partial_\mu\lambda^a + gf^{abc}A^b_\mu\lambda^c,\\ \mathrm{D}_\mu\psi_i &= \partial_\mu\psi - \mathrm{i}gA^a_\mu(T^a\psi)_i & \mathrm{D}_\mu\bar\lambda^a &= \partial_\mu\bar\lambda^a + gf^{abc}A^b_\mu\bar\lambda^c,\end{aligned}$$

[*2] Note the signs. $\bar D\bar D\mathrm{e}^{2gV}D_\alpha\mathrm{e}^{-2gV}$ is not gauge invariant! Also the curvature tensor and the covariant derivative is well-known ones: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g[A_\mu, A_\nu]$, and $\mathrm{D}_\mu\bar\lambda = \partial_\mu\bar\lambda - \mathrm{i}g[A_\mu, \bar\lambda] = \partial_\mu\bar\lambda + gf^{abc}A^b_\mu\bar\lambda^cT^a$.

3.6 Minimal Supersymmetric Standard Model

◆3.6.1 Definitions

Gauge Group and Superfields [*3]

$$\mathrm{SU}(3)_{\text{color}} \times \mathrm{SU}(2)_{\text{weak}} \times \mathrm{U}(1)_{\mathrm{Y}} \quad (\times \mathrm{Z}_{2R} : R\text{-parity}); \tag{3.50}$$

Field	SU(3)	SU(2)	U(1)	B	L
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$	
L_i		$\mathbf{2}$	$-1/2$		1
$\bar U_i$	$\bar{\mathbf{3}}$		$-2/3$	$-1/3$	
$\bar D_i$	$\bar{\mathbf{3}}$		$1/3$	$-1/3$	
$\bar E_i$			1		-1
H_u		$\mathbf{2}$	$1/2$		
H_d		$\mathbf{2}$	$-1/2$		

Field	SU(3)	SU(2)	U(1)
g	$\mathbf{8}$		
W		$\mathbf{3}$	
B			

Superpotential and ~~SUSY~~-terms

$$W_{\mathrm{RPC}} = \mu H_\mathrm{u} H_\mathrm{d} - y_{\mathrm{u}ij}\bar U_i H_\mathrm{u} Q_j + y_{\mathrm{d}ij}\bar D_i H_\mathrm{d} Q_j + y_{\mathrm{e}ij}\bar E_i H_\mathrm{d} L_j \tag{3.51}$$

$$W_{\mathrm{RPV}} = \mu_i H_\mathrm{u} L_i + \frac{1}{2}\lambda_{ijk} L_i L_j \bar E_k + \lambda'_{ijk} L_i Q_j \bar D_k + \frac{1}{2}\lambda''_{ijk}\bar U_i \bar D_j \bar D_k \tag{3.52}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3 \tilde g \tilde g + M_2 \tilde w \tilde w + M_1 \tilde b \tilde b + \text{H.c.}\right) - V_{\cancel{\mathrm{SUSY}}}; \tag{3.53}$$

$$\begin{aligned} V^{\mathrm{RPC}}_{\cancel{\mathrm{SUSY}}} = &\left(\tilde q^* m_Q^2 \tilde q + \tilde l^* m_L^2 \tilde l + \tilde u_\mathrm{R} m_{\bar U}^2 \tilde u_\mathrm{R}^* + \tilde d_\mathrm{R} m_{\bar D}^2 \tilde d_\mathrm{R}^* + \tilde e_\mathrm{R} m_{\bar E}^2 \tilde e_\mathrm{R}^* + m_{H_\mathrm{u}}^2 |H_\mathrm{u}|^2 + m_{H_\mathrm{d}}^2 |H_\mathrm{d}|^2\right) \\ &+ \left(-\tilde u_\mathrm{R}^* H_\mathrm{u} A^u \tilde q + \tilde d_\mathrm{R}^* H_\mathrm{d} A^d \tilde q + \tilde e_\mathrm{R}^* H_\mathrm{d} A^e \tilde l + b H_\mathrm{u} H_\mathrm{d} + \text{H.c.}\right) m \\ &+ \left(-\tilde u_\mathrm{R}^* H_\mathrm{d}^* C^u \tilde q + \tilde d_\mathrm{R}^* H_\mathrm{u}^* C^d \tilde q + \tilde e_\mathrm{R}^* H_\mathrm{u}^* C^e \tilde l + \text{H.c.}\right) \end{aligned} \tag{3.54}$$

$$V^{\mathrm{RPV}}_{\cancel{\mathrm{SUSY}}} = \left(b_i H_\mathrm{u} \tilde l_i + \frac{1}{2} A_{ijk} \tilde l_i \tilde l_j \tilde e^*_{\mathrm{R}k} + A'_{ijk} \tilde l_i \tilde q_j \tilde d^*_{\mathrm{R}k} + \frac{1}{2} A''_{ijk} \tilde u^*_{\mathrm{R}i} \tilde d^*_{\mathrm{R}j} \tilde d^*_{\mathrm{R}k} + M^2_{Li} H_\mathrm{d}^* \tilde l_i + \text{H.c.}\right) \tag{3.55}$$

$$+ \left(C^1_{ijk} \tilde l_i^* \tilde q_j \tilde u^*_{\mathrm{R}k} + C^2_i H_\mathrm{u}^* H_\mathrm{d} \tilde e^*_{\mathrm{R}i} + C^3_{ijk} \tilde d_{\mathrm{R}i} \tilde u^*_{\mathrm{R}j} \tilde e^*_{\mathrm{R}k} + \frac{1}{2} C^4_{ijk} \tilde d_{\mathrm{R}i} \tilde q_j \tilde q_k + \text{H.c.}\right), \tag{3.56}$$

where we define $\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C^4_{ijk} = C^4_{ikj}$

[*3] For left-handed fermions, the superfield will be written as, e.g., Q, and it contains a scalar component $\tilde q$ and a chiral fermion Q_α. Their complex conjugates will be shown as $\tilde q^*$ and $\bar Q_{\dot\alpha}$ as is done in the previous section.

For right-handed fermions such as $\bar U$, the superfield will be written as $\bar U$. Its scalar component is written as (or, in other words, equivalent to) $\tilde u^*_\mathrm{R}$, and its fermionic one is U^c_α. *c is just a label;* does not mean charge or complex conjugation. Their complex conjugates are $\tilde u_\mathrm{R}$ and $\bar U^\mathrm{c}_{\dot\alpha}$.

They form a Dirac fermion as $U = \begin{pmatrix} u_\alpha \\ \bar U^{\mathrm{c}\dot\alpha} \end{pmatrix} =: \begin{pmatrix} U_\mathrm{L} \\ U_\mathrm{R} \end{pmatrix}$; its *charge* conjugate is $U^\mathrm{C} = \begin{pmatrix} U^\mathrm{C}_\alpha \\ \bar u^{\dot\alpha} \end{pmatrix}$.

Majorana fermions are written as, e.g., $\tilde b = \tilde b^\mathrm{C} = \begin{pmatrix} \tilde b_\alpha \\ \bar{\tilde b}^{\dot\alpha} \end{pmatrix}$. Here $\bar{\tilde b}_{\dot\alpha}$ is the complex conjugate of $\tilde b_\alpha$.

scalars : $\tilde q\,(\tilde u_\mathrm{L}, \tilde d_\mathrm{L}), \tilde u^*_\mathrm{R}, \tilde d^*_\mathrm{R}, \tilde l\,(\tilde e_\mathrm{L}, \tilde\nu), \tilde e^*_\mathrm{R}, H_\mathrm{u}\,(H^+_\mathrm{u}, H^0_\mathrm{u}), H_\mathrm{d}\,(H^0_\mathrm{d}, H^-_\mathrm{d})$
$\tilde q^*\,(\tilde u^*_\mathrm{L}, \tilde d^*_\mathrm{L}), \tilde u_\mathrm{R}, \tilde d_\mathrm{R}, \tilde l^*\,(\tilde e^*_\mathrm{L}, \tilde\nu^*), \tilde e_\mathrm{R}, H^*_\mathrm{u}, H^*_\mathrm{d}$

Diracs : $U\,(U_\mathrm{L}, U_\mathrm{R}), D\,(D_\mathrm{L}, D_\mathrm{R}), E\,(E_\mathrm{L}, E_\mathrm{R}), \nu$

Weyls : $Q\,(u, d), U^\mathrm{c}, D^\mathrm{c}, L\,(\nu, e), E^\mathrm{c}$
$\bar Q\,(\bar u, \bar d), \bar U^\mathrm{c}, \bar D^\mathrm{c}, \bar L\,(\bar\nu, \bar e), \bar E^\mathrm{c}$
$\tilde h_\mathrm{u}\,(\tilde h^+_\mathrm{u}, \tilde h^0_\mathrm{u}), \tilde h_\mathrm{d}\,(\tilde h^0_\mathrm{d}, \tilde h^-_\mathrm{d}), \tilde b, \tilde w, \tilde g$
$\bar{\tilde h}_\mathrm{u}\,(\bar{\tilde h}^+_\mathrm{u}, \bar{\tilde h}^0_\mathrm{u}), \bar{\tilde h}_\mathrm{d}\,(\bar{\tilde h}^0_\mathrm{d}, \bar{\tilde h}^-_\mathrm{d}), \bar{\tilde b}, \bar{\tilde w}, \bar{\tilde g}$

◆3.6.2 Lagrangian Build Block

$$\mathcal{L}_{\mathrm{K;\,CP}} = -\frac{1}{4}F^{\mu\nu\,a}F^a_{\mu\nu} + \mathrm{D}^\mu\phi_i^*\mathrm{D}_\mu\phi_i + \mathrm{i}\bar{\psi}_i\bar{\sigma}^\mu\mathrm{D}_\mu\psi_i + \mathrm{i}\bar{\lambda}^a\bar{\sigma}^\mu\mathrm{D}_\mu\lambda^a - \sqrt{2}g\left(\phi^*T^a\psi\lambda + \bar{\psi}\bar{\lambda}T^a\phi\right) \tag{3.57}$$

$$\mathcal{L}^{\cancel{\mathrm{SUSY}}}_{\mathrm{gaugino}} = -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{w}\widetilde{w} + M_1\widetilde{b}\widetilde{b} + \mathrm{H.c.}\right) \tag{3.58}$$

$$\mathcal{L}_{\mathrm{scalar}} = -\left(\sum V^F + \sum V^D + \sum V_{\cancel{SUSY}}\right) \tag{3.59}$$

$$\mathcal{L}^{\mathrm{RPC}}_{\mathrm{S;\,fermi}} = -\left(\mu\widetilde{h}_\mathrm{u}\widetilde{h}_\mathrm{d} - y_{\mathrm{u}\,ij}U^\mathrm{c}_iH_\mathrm{u}Q_j + y_{\mathrm{d}\,ij}D^\mathrm{c}_iH_\mathrm{d}Q_j + y_{\mathrm{e}\,ij}E^\mathrm{c}_iH_\mathrm{d}L_j + \cdots + \mathrm{H.c.}\right) \tag{3.60}$$

$$\mathcal{L}^{\mathrm{RPV}}_{\mathrm{S;\,fermi}} = -\left(\mu_i\widetilde{h}_\mathrm{u}L_i + \frac{1}{2}\lambda_{ijk}L_iL_jE^\mathrm{c}_k + \lambda'_{ijk}L_iQ_jD^\mathrm{c}_k + \frac{1}{2}\lambda''_{ijk}U^\mathrm{c}_iD^\mathrm{c}_jD^\mathrm{c}_k + \cdots + \mathrm{H.c.}\right) \tag{3.61}$$

$$\mathcal{L}_{\mathrm{K;\,\cancel{CP}}} = -\frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} \tag{3.62}$$

The scalar potential is decomposed as

$$-F_{H_\mathrm{u}}{}^{a*} = \epsilon^{ab}\left(\mu H^b_\mathrm{d} - y_{\mathrm{u}\,ij}\bar{U}^x_iQ^{bx}_j + \mu_iL^b_i\right) \tag{3.63}$$

$$-F_{H_\mathrm{d}}{}^{a*} = \epsilon^{ab}\left(-\mu H^b_\mathrm{u} + y_{\mathrm{d}\,ij}\bar{D}^x_iQ^{bx}_j + y_{\mathrm{e}\,ij}\bar{E}_iL^b_j\right) \tag{3.64}$$

$$-F_{Q_i}{}^{ax*} = \epsilon^{ab}\left(y_{\mathrm{u}\,ji}H^b_\mathrm{u}\bar{U}^x_j - y_{\mathrm{d}\,ji}H^b_\mathrm{d}\bar{D}^x_j - \lambda'_{jik}L^b_i\bar{D}^x_j\right) \tag{3.65}$$

$$-F_{L_i}{}^{a*} = \epsilon^{ab}\left(-\mu_iH^b_\mathrm{u} - y_{\mathrm{e}\,ji}H^b_\mathrm{d}\bar{E}_j + \lambda_{ijk}L^b_j\bar{E}_k + \lambda'_{ijk}Q^{bx}_j\bar{D}^x_k\right) \tag{3.66}$$

$$-F_{\bar{U}_i}{}^{x*} = \left(-\epsilon^{ab}y_{\mathrm{u}\,ij}H^a_\mathrm{u}Q^{bx}_j + \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk}\bar{D}^y_j\bar{D}^z_k\right) \tag{3.67}$$

$$-F_{\bar{D}_i}{}^{x*} = \left(\epsilon^{ab}y_{\mathrm{d}\,ij}H^a_\mathrm{d}Q^{bx}_j + \epsilon^{ab}\lambda'_{jki}L^a_jQ^{bx}_k + \epsilon^{yzx}\lambda''_{jki}\bar{U}^y_j\bar{D}^z_k\right) \tag{3.68}$$

$$-F_{\bar{E}_i}{}^{*} = \left(\epsilon^{ab}y_{\mathrm{e}\,ij}H^a_\mathrm{d}L^b_j + \frac{1}{2}\epsilon^{ab}\lambda_{jki}L^a_jL^b_k\right) \tag{3.69}$$

and

$$D_g{}^\alpha = -g_3\sum_{i=1}^{3}\left[\sum_{a=1,2}Q^{ax*}_i(T^\alpha)_{xy}Q^{ay}_i - \bar{U}^{x*}_i(T^\alpha)_{xy}\bar{U}^y_i - \bar{D}^{x*}_i(T^\alpha)_{xy}\bar{D}^y_i\right] \tag{3.70}$$

$$D_W{}^\alpha = -g_2\left[\sum_{i=1}^{3}\sum_{x=1}^{3}Q^{ax*}_i(T^\alpha)_{ab}Q^{by}_i + \sum_{i=1}^{3}L^{a*}_i(T^\alpha)_{ab}L^b_i + H^{a*}_\mathrm{u}(T^\alpha)_{ab}H^b_\mathrm{u} + H^{a*}_\mathrm{d}(T^\alpha)_{ab}H^b_\mathrm{d}\right] \tag{3.71}$$

$$D_B = -g_1\left[\frac{1}{6}|Q^{ax}_i|^2 - \frac{1}{2}|L^a_i|^2 - \frac{2}{3}|\bar{U}^x_i|^2 + \frac{1}{3}|\bar{D}^x_i|^2 + |\bar{E}_i|^2 + \frac{1}{2}|H^a_\mathrm{u}|^2 - \frac{1}{2}|H^a_\mathrm{d}|^2\right]. \tag{3.72}$$

Here we use the superfield notation for simple appearance.

◆3.6.3 Scalar Potential (Verbose)

(with the superfield notation)

$$V^F_{H_\mathrm{u}} = |\mu|^2|H_\mathrm{d}|^2 + \sum_a\Big(|\bar{U}y^uQ^a|^2 + |\mu_i L_i^a|^2\Big) + \Big[\mu^*\mu_i H_\mathrm{d}^* L_i - \mu^* H_\mathrm{d}^*\bar{U}y^uQ - \mu_i^* L_i^*\bar{U}y^uQ + \text{H.c.}\Big] \tag{3.73}$$

$$V^F_{H_\mathrm{d}} = |\mu|^2|H_\mathrm{u}|^2 + \sum_a\Big(|\bar{D}y^dQ^a|^2 + |\bar{E}y^eL^a|^2\Big) + \Big[-\mu^* H_\mathrm{u}^*\bar{D}y^dQ - \mu^* H_\mathrm{u}^*\bar{E}y^eL + (\bar{D}y^dQ)^*(\bar{E}y^eL) + \text{H.c.}\Big] \tag{3.74}$$

$$V^F_Q = |H_\mathrm{u}|^2|\bar{U}_i y^u_{ij}|^2 + |H_\mathrm{d}|^2|\bar{D}_i y^d_{ij}|^2 + \lambda'^*_{jik}\lambda'_{lim}L^*_jL_l\bar{D}^*_k\bar{D}_m + \Big[-y^{u*}_{ji}y^d_{ki}H^*_\mathrm{u}H_\mathrm{d}\bar{U}^*_j\bar{D}_k - y^{u*}_{ji}\lambda'_{lim}H^*_\mathrm{u}L_l\bar{U}^*_j\bar{D}_m + y^{d*}_{ji}\lambda'_{lim}H^*_\mathrm{d}L_l\bar{D}^*_j\bar{D}_m + \text{H.c.}\Big] \tag{3.75}$$

$$\begin{aligned} V^F_L = {} & |\mu_i|^2|H_\mathrm{u}|^2 + |H_\mathrm{d}|^2(\bar{E}y^ey^{e\dagger}\bar{E}^*) + \lambda'_{ijk}\lambda'_{ilm}(Q^*_j\bar{D}^*_k)Q_l\bar{D}_m + \lambda^*_{ijk}\lambda_{ilm}L^*_jL_l\bar{E}^*_k\bar{E}_m \\ & + \Big[\mu^*_i y^e_{ji}\bar{E}_jH^*_\mathrm{u}H_\mathrm{d} - \mu^*_i\lambda'_{ijk}H^*_\mathrm{u}Q_j\bar{D}_k - \mu^*_i\lambda_{ijk}H^*_\mathrm{u}L_j\bar{E}_k \\ & \quad - y^{e*}_{ji}\lambda'_{ilm}\bar{E}^*_jH^*_\mathrm{d}Q_l\bar{D}_m - y^{e*}_{ji}\lambda_{ilm}\bar{E}^*_jH^*_\mathrm{d}L_l\bar{E}_m + \lambda'^*_{ijk}\lambda_{ilm}\bar{D}^*_kQ^*_jL_l\bar{E}_m + \text{H.c.}\Big] \end{aligned} \tag{3.76}$$

$$V^F_{\bar{U}} = y^{u*}_{ij}y^u_{ik}\epsilon^{ab}\epsilon^{cd}H^{a*}_\mathrm{u}H^c_\mathrm{u}Q^{b*}_jQ^d_k + \frac{1}{2}\lambda''^*_{ijk}\lambda''_{ilm}(\bar{D}^*_j\bar{D}_l)(\bar{D}^*_k\bar{D}_m) - \Big[y^{u*}_{il}\lambda''_{ijk}H^*_\mathrm{u}Q^*_l\bar{D}_j\bar{D}_k + \text{H.c.}\Big] \tag{3.77}$$

$$\begin{aligned} V^F_{\bar{D}} = {} & \epsilon^{ab}\epsilon^{cd}\left(y^d_{ij}H^a_\mathrm{d} + \lambda'_{kji}L^a_k\right)^*\left(y^d_{il}H^c_\mathrm{d} + \lambda'_{mli}L^c_m\right)Q^{b*}_jQ^d_l + \lambda''^*_{jki}\lambda''_{lmi}\left(\bar{U}^*_j\bar{U}_l\ \bar{D}^*_k\bar{D}_m - \bar{U}^*_j\bar{D}_m\ \bar{D}^*_k\bar{U}_l\right) \\ & + \Big[\lambda''_{lmi}(y^{d*}_{ij}H^*_\mathrm{d} + \lambda'^*_{kji}L^*_k)Q^*_j\bar{U}_l\bar{D}_m + \text{H.c.}\Big] \end{aligned} \tag{3.78}$$

$$V^F_{\bar{E}} = \epsilon^{ab}\epsilon^{cd}\left(y^e_{ij}H^a_\mathrm{d} + \frac{1}{2}\lambda_{kji}L^a_k\right)^* L^{*b}_j\left(y^e_{il}H^c_\mathrm{d} + \frac{1}{2}\lambda_{mli}L^c_m\right)L^d_l \tag{3.79}$$

$$V^D_g = \frac{g_3^2}{2}\left\{\sum_{\alpha=1}^{8}\sum_{i=1}^{3}\left[\sum_{a=1,2}Q^{a*}_i(t^\alpha)Q^a_i - \bar{U}^*_i(t^\alpha)\bar{U}_i - \bar{D}^*_i(t^\alpha)\bar{D}_i\right]\right\}^2 \tag{3.80}$$

$$V^D_W = \frac{g_2^2}{2}\left[\sum_{i=1}^{3}\sum_{x=1}^{3}Q^{x*}_i(T^\alpha)Q^x_i + \sum_{i=1}^{3}L^*_i(T^\alpha)L_i + H^*_\mathrm{u}(T^\alpha)H_\mathrm{u} + H^*_\mathrm{d}(T^\alpha)H_\mathrm{d}\right]^2 \tag{3.81}$$

$$V^D_B = \frac{g_1^2}{2}\left[\sum_i\left(\frac{1}{6}|Q_i|^2 - \frac{1}{2}|L_i|^2 - \frac{2}{3}|\bar{U}_i|^2 + \frac{1}{3}|\bar{D}_i|^2 + |\bar{E}_i|^2\right) + \frac{1}{2}|H_\mathrm{u}|^2 - \frac{1}{2}|H_\mathrm{d}|^2\right]^2 \tag{3.82}$$

$$\begin{aligned} V^\text{RPC}_{\cancel{\text{SUSY}}} = {} & \left(Q^*m^2_QQ + L^*m^2_LL + \bar{U}^*m^2_{\bar{U}}\bar{U} + \bar{D}^*m^2_{\bar{D}}\bar{D} + \bar{E}^*m^2_{\bar{E}}\bar{E} + m^2_{H_\mathrm{u}}|H_\mathrm{u}|^2 + m^2_{H_\mathrm{d}}|H_\mathrm{d}|^2\right) \\ & + \left(-\bar{U}H_\mathrm{u}A^uQ + \bar{D}H_\mathrm{d}A^dQ + \bar{E}H_\mathrm{d}A^eL + BH_\mathrm{u}H_\mathrm{d} + \text{H.c.}\right) \\ & + \left(-\bar{U}H^*_\mathrm{d}C^uQ + \bar{D}H^*_\mathrm{u}C^dQ + \bar{E}H^*_\mathrm{u}C^eL + \text{H.c.}\right) \end{aligned} \tag{3.83}$$

$$\begin{aligned} V^\text{RPV}_{\cancel{\text{SUSY}}} = {} & \left(B_iH_\mathrm{u}L_i + \frac{1}{2}A_{ijk}L_iL_j\bar{E}_k + A'_{ijk}L_iQ_j\bar{D}_k + \frac{1}{2}A''_{ijk}\bar{U}_i\bar{D}_j\bar{D}_k + \text{H.c.}\right) \\ & + \left(C^1_{ijk}L^*_iQ_j\bar{U}_k + C^2_iH^*_\mathrm{u}H_\mathrm{d}\bar{E}_i + C^3_{ijk}\bar{D}^*_i\bar{U}_j\bar{E}_k + \frac{1}{2}C^4_{ijk}\bar{D}^*_iQ_jQ_k + \text{H.c.}\right) \\ & + \left(M^2_{Li}H^*_\mathrm{d}L_i + \text{H.c.}\right) \end{aligned} \tag{3.84}$$

With R-parity

$$\begin{aligned}
V_{\text{full}}^{\text{RPC}} &= \left(Q^* m_Q^2 Q + L^* m_L^2 L + \bar{U}^* m_{\bar{U}}^2 \bar{U} + \bar{D}^* m_{\bar{D}}^2 \bar{D} + \bar{E}^* m_{\bar{E}}^2 \bar{E}\right) \\
&+ \left(|\mu|^2 + m_{H_\text{u}}^2\right)|H_\text{u}|^2 + \left(|\mu|^2 + m_{H_\text{d}}^2\right)|H_\text{d}|^2 + (B H_\text{u} H_\text{d} + \text{H.c.}) \\
&+ \Big[(-\mu^* y^u H_\text{d}^* - A^u H_\text{u} - C^u H_\text{d}^*)_{ij}\, \bar{U}_i Q_j + \text{H.c.}\Big] \\
&+ \Big[\left(-\mu^* y^d H_\text{u}^* + A^d H_\text{d} + C^d H_\text{u}^*\right)_{ij} \bar{D}_i Q_j + \text{H.c.}\Big] \\
&+ \Big[(-\mu^* y^e H_\text{u}^* + A^e H_\text{d} + C^e H_\text{u}^*)_{ij}\, \bar{E}_i L_j + \text{H.c.}\Big] \\
&+ |H_\text{u}|^2|\bar{U}y^u|^2 + |H_\text{d}|^2|\bar{D}y^d|^2 + |H_\text{d}|^2|\bar{E}y^e|^2 + |H_\text{u}|^2|y^u Q|^2 + |H_\text{d}|^2|y^d Q|^2 + |H_\text{d}|^2|y^e L|^2 \\
&+ \sum_a \Big(|\bar{U}y^u Q^a|^2 + |\bar{D}y^d Q^a|^2 + |\bar{E}y^e L^a|^2\Big) \\
&+ \Big[(\bar{D}y^d Q)^*(\bar{E}y^e L) - y_{ji}^{u*} y_{ki}^d H_\text{u}^* H_\text{d} \bar{U}_j^* \bar{D}_k + \text{H.c.}\Big] \\
&- \left[y_{ki}^{u*} y_{kj}^u (H_\text{u}^* Q_j)(Q_i^* H_\text{u}) + y_{ki}^{d*} y_{kj}^d (H_\text{d}^* Q_j)(Q_i^* H_\text{d}) + y_{ki}^{e*} y_{kj}^e (H_\text{d}^* L_j)(L_i^* H_\text{d})\right] \\
&+ \frac{g_3^2}{2}\left\{\sum_{\alpha=1}^{8}\sum_{i=1}^{3}\left[\sum_{a=1,2} Q_i^{a*}(t^\alpha)Q_i^a - \bar{U}_i^*(t^\alpha)\bar{U}_i - \bar{D}_i^*(t^\alpha)\bar{D}_i\right]\right\}^2 \\
&+ \frac{g_2^2}{2}\left[\sum_{i=1}^{3}\sum_{x=1}^{3} Q_i^{x*}(T^\alpha)Q_i^x + \sum_{i=1}^{3} L_i^*(T^\alpha)L_i + H_\text{u}^*(T^\alpha)H_\text{u} + H_\text{d}^*(T^\alpha)H_\text{d}\right]^2 \\
&+ \frac{g_1^2}{2}\left[\sum_i\left(\frac{1}{6}|Q_i|^2 - \frac{1}{2}|L_i|^2 - \frac{2}{3}|\bar{U}_i|^2 + \frac{1}{3}|\bar{D}_i|^2 + |\bar{E}_i|^2\right) + \frac{1}{2}|H_\text{u}|^2 - \frac{1}{2}|H_\text{d}|^2\right]^2
\end{aligned} \tag{3.85}$$

With Bilinear R-parity Violation

$$V_{H_\text{u}}^F \mathrel{+}= \sum_a |\mu_i L_i^a|^2 + \Big[\mu^* \mu_i H_\text{d}^* L_i - \mu_i^* L_i^* \bar{U} y^u Q + \text{H.c.}\Big] \tag{3.86}$$

$$V_L^F \mathrel{+}= |\mu_i|^2 |H_\text{u}|^2 + \Big[\mu_i^* y_{ji}^e \bar{E}_j H_\text{u}^* H_\text{d} + \text{H.c.}\Big] \tag{3.87}$$

$$V_{\cancel{\text{SUSY}}}^{\text{RPV}} = \left(B_i H_\text{u} L_i + M_{Li}^2 H_\text{d}^* L_i + \text{H.c.}\right) \tag{3.88}$$

With Trilinear leptonic R-parity Violation

$$V_Q^F \mathrel{+}= |\lambda'_{jik} L_j \bar{D}_k|^2 + \Big[-y_{ji}^{u*} \lambda'_{lim} H_\text{u}^* L_l \bar{U}_j^* \bar{D}_m + y_{ji}^{d*} \lambda'_{lim} H_\text{d}^* L_l \bar{D}_j^* \bar{D}_m + \text{H.c.}\Big] \tag{3.89}$$

$$\begin{aligned}
V_L^F \mathrel{+}= &|\lambda'_{ijk} Q_j \bar{D}_k|^2 + |\lambda_{ijk} L_j \bar{E}_k|^2 \\
&+ \Big[-y_{mi}^{e*} \lambda'_{ijk} \bar{E}_m^* H_\text{d}^* Q_j \bar{D}_k - y_{mi}^{e*} \lambda_{ijk} \bar{E}_m^* H_\text{d}^* L_j \bar{E}_k + \lambda'^*_{ijk} \lambda_{ilm} \bar{D}_k^* Q_j^* L_l \bar{E}_m + \text{H.c.}\Big]
\end{aligned} \tag{3.90}$$

$$\begin{aligned}
V_{\bar{D}}^F \mathrel{+}= &\lambda'^*_{kji} \lambda'_{mli} \Big[(L_k^* L_m)(Q_j^* Q_l) - (L_k^* Q_l)(Q_j^* L_m)\Big] \\
&+ \Big\{y_{ij}^{d\,*} \lambda'_{mli} \left[(H_\text{d}^* L_m)(Q_j Q_l) - (H_\text{d}^* Q_l)(Q_j^* L_m)\right] + \text{H.c.}\Big\}
\end{aligned} \tag{3.91}$$

$$V_{\bar{E}}^F = \frac{1}{2}\lambda_{kji}^* \lambda_{mli} (L_k^* L_m)(L_j^* L_l) + \lambda_{kji}^* y_{ilp}^e \Big[(L_k^* H_\text{d})(L_j^* L_l) + \text{H.c.}\Big] \tag{3.92}$$

◆3.6.4 Full Lagrangian (in Gauge eigenstates)

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{\mu\nu\,a}W^a_{\mu\nu} - \frac{1}{4}G^{\mu\nu\,a}G^a_{\mu\nu} \quad {}^{*4} \tag{3.93}$$

$$\begin{aligned}\mathcal{L}_{\text{gaugino}} = {}& -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{w}\widetilde{w} + M_1\widetilde{b}\widetilde{b} + \text{H.c.}\right)\\ & + \mathrm{i}\bar{\widetilde{b}}^a\bar{\sigma}^\mu\partial_\mu\widetilde{b}^a + \mathrm{i}\bar{\widetilde{w}}^a\bar{\sigma}^\mu\partial_\mu\widetilde{w}^a + \mathrm{i}\bar{\widetilde{g}}^a\bar{\sigma}^\mu\partial_\mu\widetilde{g}^a + \mathrm{i}g_2\epsilon^{abc}\bar{\widetilde{w}}^a\bar{\sigma}^\mu W^b_\mu\widetilde{w}^c + \mathrm{i}g_3 f^{abc}\bar{\widetilde{g}}^a\bar{\sigma}^\mu G^b_\mu\widetilde{g}^c\end{aligned} \tag{3.94}$$

$$\mathcal{L}_{\cancel{CP}} = -\frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}B_{\rho\sigma} - \frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^a_{\rho\sigma} - \frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} \tag{3.95}$$

$$\begin{aligned}\mathcal{L}_{\text{scalar}} = {}& \Big[\big(\partial^\mu + \mathrm{i}g_3G^\mu + \mathrm{i}g_2W^\mu + \tfrac{1}{6}\mathrm{i}g_1B^\mu\big)\widetilde{q}_i^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_3G_\mu - \mathrm{i}g_2W_\mu - \tfrac{1}{6}\mathrm{i}g_1B_\mu\big)\widetilde{q}_i\Big]\\ & + \Big[\big(\partial^\mu + \mathrm{i}g_3G^\mu + \tfrac{2}{3}\mathrm{i}g_1B^\mu\big)\widetilde{u}_{\mathrm{R}i}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_3G_\mu - \tfrac{2}{3}\mathrm{i}g_1B_\mu\big)\widetilde{u}_{\mathrm{R}i}\Big]\\ & + \Big[\big(\partial^\mu + \mathrm{i}g_3G^\mu - \tfrac{1}{3}\mathrm{i}g_1B^\mu\big)\widetilde{d}_{\mathrm{R}i}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_3G_\mu + \tfrac{1}{3}\mathrm{i}g_1B_\mu\big)\widetilde{d}_{\mathrm{R}i}\Big]\\ & + \Big[\big(\partial^\mu + \mathrm{i}g_2W^\mu - \tfrac{1}{2}\mathrm{i}g_1B^\mu\big)\widetilde{l}_i^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_2W_\mu + \tfrac{1}{2}\mathrm{i}g_1B_\mu\big)\widetilde{l}_i\Big]\\ & + \Big[\big(\partial^\mu - \mathrm{i}g_1B^\mu\big)\widetilde{e}_{\mathrm{R}i}^*\Big]\Big[\big(\partial_\mu + \mathrm{i}g_1B_\mu\big)\widetilde{e}_{\mathrm{R}i}\Big]\\ & + \Big[\big(\partial^\mu + \mathrm{i}g_2W^\mu + \tfrac{1}{2}\mathrm{i}g_1B^\mu\big)H_{\mathrm{u}}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_2W_\mu - \tfrac{1}{2}\mathrm{i}g_1B_\mu\big)H_{\mathrm{u}}\Big]\\ & + \Big[\big(\partial^\mu + \mathrm{i}g_2W^\mu - \tfrac{1}{2}\mathrm{i}g_1B^\mu\big)H_{\mathrm{d}}^*\Big]\Big[\big(\partial_\mu - \mathrm{i}g_2W_\mu + \tfrac{1}{2}\mathrm{i}g_1B_\mu\big)H_{\mathrm{d}}\Big],\end{aligned} \tag{3.96}$$

$$\begin{aligned}\mathcal{L}_{\text{fermion}} = {}& \mathrm{i}\bar{Q}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_3G_\mu - \mathrm{i}g_2W_\mu - \frac{1}{6}\mathrm{i}g_1B_\mu\right)Q_i\\ & + \mathrm{i}\bar{U}^{\mathrm{c}}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_3[-G^{\mathrm{T}}_\mu] + \frac{2}{3}\mathrm{i}g_1B_\mu\right)U^{\mathrm{c}}_i + \mathrm{i}\bar{D}^{\mathrm{c}}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_3[-G^{\mathrm{T}}_\mu] - \frac{1}{3}\mathrm{i}g_1B_\mu\right)D^{\mathrm{c}}_i\\ & + \mathrm{i}\bar{L}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)L_i + \mathrm{i}\bar{E}^{\mathrm{c}}_i\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_1B_\mu\right)E^{\mathrm{c}}_i\\ & + \mathrm{i}\bar{\widetilde{h}}_{\mathrm{u}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}_{\mathrm{u}} + \mathrm{i}\bar{\widetilde{h}}_{\mathrm{d}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\widetilde{h}_{\mathrm{d}},\end{aligned} \tag{3.97}$$

$$\begin{aligned}\mathcal{L}_{\text{SFG}} = {}& -\sqrt{2}g_3\Big[\big(\widetilde{q}_i^*\tau^aQ_i + \widetilde{u}_{\mathrm{R}i}[-\tau^{a\,\mathrm{T}}]U^{\mathrm{c}}_i + \widetilde{d}_{\mathrm{R}i}[-\tau^{a\,\mathrm{T}}]D^{\mathrm{c}}_i\big)\widetilde{g}^a\\ & \qquad\qquad + \bar{\widetilde{g}}^a\big(\bar{Q}_i\tau^a\widetilde{q}_i + \bar{U}^{\mathrm{c}}_i[-\tau^{a\,\mathrm{T}}]\widetilde{u}^*_{\mathrm{R}i} + \bar{D}^{\mathrm{c}}_i[-\tau^{a\,\mathrm{T}}]\widetilde{d}^*_{\mathrm{R}i}\big)\Big]\\ & - \sqrt{2}g_2\Big[\big(\widetilde{q}_i^*T^aQ_i + \widetilde{l}_i^*T^aL_i + H_{\mathrm{u}}^*T^a\widetilde{h}_{\mathrm{u}} + H_{\mathrm{d}}^*T^a\widetilde{h}_{\mathrm{d}}\big)\widetilde{w}^a\\ & \qquad\qquad + \bar{\widetilde{w}}^a\big(\bar{Q}_iT^a\widetilde{q}_i + \bar{L}_iT^a\widetilde{l}_i + \bar{\widetilde{h}}_{\mathrm{u}}T^aH_{\mathrm{u}} + \bar{\widetilde{h}}_{\mathrm{d}}T^aH_{\mathrm{d}}\big)\Big]\\ & - \sqrt{2}g_1\Big[\big(\tfrac{1}{6}\widetilde{q}_i^*Q_i - \tfrac{2}{3}\widetilde{u}_{\mathrm{R}i}U^{\mathrm{c}}_i + \tfrac{1}{3}\widetilde{d}_{\mathrm{R}i}D^{\mathrm{c}}_i - \tfrac{1}{2}\widetilde{l}_i^*L_i + \widetilde{e}_{\mathrm{R}i}E^{\mathrm{c}}_i + \tfrac{1}{2}H_{\mathrm{u}}^*\widetilde{h}_{\mathrm{u}} - \tfrac{1}{2}H_{\mathrm{d}}^*\widetilde{h}_{\mathrm{d}}\big)\widetilde{b}\\ & \qquad\qquad + \bar{\widetilde{b}}\big(\tfrac{1}{6}\bar{Q}_i\widetilde{q}_i - \tfrac{2}{3}\bar{U}^{\mathrm{c}}_i\widetilde{u}^*_{\mathrm{R}i} + \tfrac{1}{3}\bar{D}^{\mathrm{c}}_i\widetilde{d}^*_{\mathrm{R}i} - \tfrac{1}{2}\bar{L}_i\widetilde{l}_i + \bar{E}^{\mathrm{c}}_i\widetilde{e}^*_{\mathrm{R}i} + \tfrac{1}{2}\bar{\widetilde{h}}_{\mathrm{u}}H_{\mathrm{u}} - \tfrac{1}{2}\bar{\widetilde{h}}_{\mathrm{d}}H_{\mathrm{d}}\big)\Big]\end{aligned} \tag{3.98}$$

$$\begin{aligned}\mathcal{L}^{\text{RPC}}_{\text{super}} = {}& -\mu\widetilde{h}_{\mathrm{u}}\widetilde{h}_{\mathrm{d}} + y_{\mathrm{u}ij}U^{\mathrm{c}}_iH_{\mathrm{u}}Q_j - y_{\mathrm{d}ij}D^{\mathrm{c}}_iH_{\mathrm{d}}Q_j - y_{\mathrm{e}ij}E^{\mathrm{c}}_iH_{\mathrm{d}}L_j\\ & + y_{\mathrm{u}ij}U^{\mathrm{c}}_i\widetilde{h}_{\mathrm{u}}\widetilde{q}_j + y_{\mathrm{u}ij}\widetilde{u}^*_{\mathrm{R}i}\widetilde{h}_{\mathrm{u}}Q_j - y_{\mathrm{d}ij}D^{\mathrm{c}}_i\widetilde{h}_{\mathrm{d}}\widetilde{q}_j - y_{\mathrm{d}ij}\widetilde{d}^*_{\mathrm{R}i}\widetilde{h}_{\mathrm{d}}Q_j\\ & - y_{\mathrm{e}ij}E^{\mathrm{c}}_i\widetilde{h}_{\mathrm{d}}\widetilde{l}_j - y_{\mathrm{e}ij}\widetilde{e}^*_{\mathrm{R}i}\widetilde{h}_{\mathrm{d}}L_j + \text{H.c.}\end{aligned} \tag{3.99}$$

$$\mathcal{L}^{\text{RPC}}_{\text{pot.}} = -(3.85)\Big[V^{\text{RPC}}_{\text{full}}\Big] \tag{3.100}$$

[*4] Further decomposed results are shown in *Standard Model* section, Eqs. (2.7) and (2.21).

Fermion Composition

$$
\mathcal{L}_{\text{gaugino}} = \frac{1}{2}\bar{\tilde{b}}\left(\mathrm{i}\partial\!\!\!/ - M_1\right)\tilde{b} + \frac{1}{2}\bar{\tilde{w}}\left(\mathrm{i}D\!\!\!\!/ - M_2\right)\tilde{w} + \frac{1}{2}\bar{\tilde{g}}\left(\mathrm{i}D\!\!\!\!/ - M_3\right)\tilde{g} \tag{3.101}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{fermion}} &= (2.5)\left[\mathcal{L}^{\text{SM}}_{\text{matter}}\right] \\
&\quad - \left(\mu\tilde{h}_{\mathrm{u}}\tilde{h}_{\mathrm{d}} + \text{H.c.}\right) + \mathrm{i}\bar{\tilde{h}}_{\mathrm{u}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\tilde{h}_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}_{\mathrm{d}}\bar{\sigma}^\mu\left(\partial_\mu - \mathrm{i}g_2W_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\tilde{h}_{\mathrm{d}} \\
&= \mathcal{L}^{\text{SM}}_{\text{matter}} - \left[\mu\left(\tilde{h}^+_{\mathrm{u}}\tilde{h}^-_{\mathrm{d}} - \tilde{h}^0_{\mathrm{u}}\tilde{h}^0_{\mathrm{d}}\right) + \text{H.c.}\right] \\
&\quad + \mathrm{i}\bar{\tilde{h}}{}^+_{\mathrm{u}}\bar{\sigma}^\mu\left(\partial_\mu - \frac{1}{2}\mathrm{i}g_2W^3_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\tilde{h}^+_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}{}^-_{\mathrm{d}}\bar{\sigma}^\mu\left(\partial_\mu + \frac{1}{2}\mathrm{i}g_2W^3_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\tilde{h}^-_{\mathrm{d}} \\
&\quad + \mathrm{i}\bar{\tilde{h}}{}^0_{\mathrm{u}}\bar{\sigma}^\mu\left(\partial_\mu + \frac{1}{2}\mathrm{i}g_2W^3_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)\tilde{h}^0_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}{}^0_{\mathrm{d}}\bar{\sigma}^\mu\left(\partial_\mu - \frac{1}{2}\mathrm{i}g_2W^3_\mu + \frac{1}{2}\mathrm{i}g_1B_\mu\right)\tilde{h}^0_{\mathrm{d}} \\
&\quad + \frac{g_2}{\sqrt{2}}\left(\bar{\tilde{h}}{}^0_{\mathrm{u}}\bar{\sigma}^\mu W^-_\mu\tilde{h}^+_{\mathrm{u}} + \bar{\tilde{h}}{}^0_{\mathrm{d}}\bar{\sigma}^\mu W^+_\mu\tilde{h}^-_{\mathrm{d}} + \bar{\tilde{h}}{}^+_{\mathrm{u}}\bar{\sigma}^\mu W^+_\mu\tilde{h}^0_{\mathrm{u}} + \bar{\tilde{h}}{}^-_{\mathrm{d}}\bar{\sigma}^\mu W^-_\mu\tilde{h}^0_{\mathrm{d}}\right)
\end{aligned} \tag{3.102}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{SFG}} &= -\sqrt{2}g_3\Big[\tilde{u}^*_{\mathrm{L}i}\tau^a\left(\bar{\tilde{g}}{}^aP_{\mathrm{L}}U_i\right) + \tilde{d}^*_{\mathrm{L}i}\tau^a\left(\bar{\tilde{g}}{}^aP_{\mathrm{L}}D_i\right) - \left(\overline{U}_iP_{\mathrm{L}}\tilde{g}^a\right)\tau^a\tilde{u}_{\mathrm{R}i} - \left(\overline{D}_iP_{\mathrm{L}}\tilde{g}^a\right)\tau^a\tilde{d}_{\mathrm{R}i} \\
&\qquad + \left(\overline{U}_iP_{\mathrm{R}}\tilde{g}^a\right)\tau^a\tilde{u}_{\mathrm{L}i} + \left(\overline{D}_iP_{\mathrm{R}}\tilde{g}^a\right)\tau^a\tilde{d}_{\mathrm{L}i} - \tilde{u}^*_{\mathrm{R}i}\tau^a\left(\bar{\tilde{g}}{}^aP_{\mathrm{R}}U\right) - \tilde{d}^*_{\mathrm{R}i}\tau^a\left(\bar{\tilde{g}}{}^aP_{\mathrm{R}}D\right)\Big] \\
&\quad - g_2\Big[\tilde{w}^+\left(d_i\tilde{u}_{\mathrm{L}i}^* + e_i\tilde{\nu}^*_i + \tilde{h}^0_{\mathrm{u}}H^{+*}_{\mathrm{u}} + \tilde{h}^-_{\mathrm{d}}H^{0*}_{\mathrm{d}}\right) + \left(\tilde{d}_{\mathrm{L}i}\bar{U}_i + \tilde{e}_{\mathrm{L}i}\bar{\nu}_i + H^0_{\mathrm{u}}\bar{\tilde{h}}{}^+_{\mathrm{u}} + H^-_{\mathrm{d}}\bar{\tilde{h}}{}^0_{\mathrm{d}}\right)\bar{\tilde{w}}{}^- \\
&\qquad + \tilde{w}^-\left(u_i\tilde{d}_{\mathrm{L}i}^* + \nu_i\tilde{e}_{\mathrm{L}i}^* + \tilde{h}^+_{\mathrm{u}}H^{0*}_{\mathrm{u}} + \tilde{h}^0_{\mathrm{d}}H^{-*}_{\mathrm{d}}\right) + \left(\tilde{u}_{\mathrm{L}i}\bar{D}_i + \tilde{\nu}_i\bar{E}_i + H^+_{\mathrm{u}}\bar{\tilde{h}}{}^0_{\mathrm{u}} + H^0_{\mathrm{d}}\bar{\tilde{h}}{}^-_{\mathrm{d}}\right)\bar{\tilde{w}}{}^+\Big] \\
&\quad - \sqrt{2}g_2\Big[\tfrac{1}{2}\left(\tilde{u}_{\mathrm{L}i}^*u_i + \tilde{\nu}^*_i\nu_i + H^{+*}_{\mathrm{u}}\tilde{h}^+_{\mathrm{u}} + H^{0*}_{\mathrm{d}}\tilde{h}^0_{\mathrm{d}} - \tilde{d}_{\mathrm{L}i}^*d_i - \tilde{e}_{\mathrm{L}i}^*e_i - H^{0*}_{\mathrm{u}}\tilde{h}^0_{\mathrm{u}} - H^{-*}_{\mathrm{d}}\tilde{h}^-_{\mathrm{d}}\right)\tilde{w}^3 \\
&\qquad + \tfrac{1}{2}\bar{\tilde{w}}{}^a\left(\bar{U}_i\tilde{u}_{\mathrm{L}i} + \bar{\nu}_i\tilde{\nu}_i + \bar{\tilde{h}}{}^+_{\mathrm{u}}H^+_{\mathrm{u}} + \bar{\tilde{h}}{}^0_{\mathrm{d}}H^0_{\mathrm{d}} - \bar{D}_i\tilde{d}_{\mathrm{L}i} - \bar{E}_i\tilde{e}_{\mathrm{L}i} - \bar{\tilde{h}}{}^0_{\mathrm{u}}H^0_{\mathrm{u}} - \bar{\tilde{h}}{}^-_{\mathrm{d}}H^-_{\mathrm{d}}\right)\Big] \\
&\quad - \sqrt{2}g_1\Big[\tfrac{1}{6}\tilde{q}^*_i\bar{\tilde{b}}P_{\mathrm{L}}Q_i - \tfrac{2}{3}\tilde{u}_{\mathrm{R}i}\overline{U}_iP_{\mathrm{L}}\tilde{b} + \tfrac{1}{3}\tilde{d}_{\mathrm{R}i}\overline{D}_iP_{\mathrm{L}}\tilde{b} - \tfrac{1}{2}\tilde{l}^*_i\bar{\tilde{b}}P_{\mathrm{L}}L_i + \tilde{e}_{\mathrm{R}i}\overline{E}_iP_{\mathrm{L}}\tilde{b} \\
&\qquad + \tfrac{1}{6}\tilde{q}_i\overline{Q}_iP_{\mathrm{R}}\tilde{b} - \tfrac{2}{3}\tilde{u}^*_{\mathrm{R}i}\bar{\tilde{b}}P_{\mathrm{R}}U_i + \tfrac{1}{3}\tilde{d}^*_{\mathrm{R}i}\bar{\tilde{b}}P_{\mathrm{R}}D_i - \tfrac{1}{2}\tilde{l}_i\overline{L}_iP_{\mathrm{R}}\tilde{b} + \tilde{e}^*_{\mathrm{R}i}\bar{\tilde{b}}P_{\mathrm{R}}E_i \\
&\qquad + \tfrac{1}{2}H^*_{\mathrm{u}}\tilde{h}_{\mathrm{u}}\tilde{b} - \tfrac{1}{2}H^*_{\mathrm{d}}\tilde{h}_{\mathrm{d}}\tilde{b} + \tfrac{1}{2}H_{\mathrm{u}}\bar{\tilde{h}}_{\mathrm{u}}\bar{\tilde{b}} - \tfrac{1}{2}H_{\mathrm{d}}\bar{\tilde{h}}_{\mathrm{d}}\bar{\tilde{b}}\Big]
\end{aligned} \tag{3.103}
$$

<u>TODO</u>: SFG と湯川，compose 可能？ $\tilde{w}$ を Dirac にするときの C は Weyl 位相に非依存？

$$
\begin{aligned}
\mathcal{L}_{\text{湯川}} &= \Big(-H^0_{\mathrm{u}}\overline{U}y_{\mathrm{u}}P_{\mathrm{L}}U - H^0_{\mathrm{d}}\overline{D}y_{\mathrm{d}}P_{\mathrm{L}}D - H^0_{\mathrm{d}}\overline{E}y_{\mathrm{e}}P_{\mathrm{L}}E \\
&\qquad + H^+_{\mathrm{u}}\overline{U}y_{\mathrm{u}}P_{\mathrm{L}}D + H^-_{\mathrm{d}}\overline{D}y_{\mathrm{d}}P_{\mathrm{L}}U + H^-_{\mathrm{d}}\overline{E}y_{\mathrm{e}}P_{\mathrm{L}}\nu + \text{H.c.}\Big) \\
&\quad + y_{\mathrm{u}ij}U^{\mathrm{c}}_i\left(\tilde{h}^+_{\mathrm{u}}\tilde{u}_{\mathrm{L}j} - \tilde{h}^0_{\mathrm{u}}\tilde{d}_{\mathrm{L}j}\right) + y_{\mathrm{d}ij}D^{\mathrm{c}}_i\left(\tilde{h}^-_{\mathrm{d}}\tilde{d}_{\mathrm{L}j} - \tilde{h}^0_{\mathrm{d}}\tilde{u}_{\mathrm{L}j}\right) + y_{\mathrm{e}ij}E^{\mathrm{c}}_i\left(\tilde{h}^-_{\mathrm{d}}\tilde{e}_{\mathrm{L}j} - \tilde{h}^0_{\mathrm{d}}\tilde{\nu}_j\right) \\
&\quad + y_{\mathrm{u}ij}\tilde{u}^*_{\mathrm{R}i}\left(\tilde{h}^+_{\mathrm{u}}u_j - \tilde{h}^0_{\mathrm{u}}U_j\right) + y_{\mathrm{d}ij}\tilde{d}^*_{\mathrm{R}i}\left(\tilde{h}^-_{\mathrm{d}}d_j - \tilde{h}^0_{\mathrm{d}}D_j\right) + y_{\mathrm{e}ij}\tilde{e}^*_{\mathrm{R}i}\left(\tilde{h}^-_{\mathrm{d}}e_j - \tilde{h}^0_{\mathrm{d}}e_j\right) \\
&\quad + y^*_{\mathrm{u}ij}\bar{U}^{\mathrm{c}}_i\left(\bar{\tilde{h}}{}^+_{\mathrm{u}}\tilde{d}^*_{\mathrm{L}j} - \bar{\tilde{h}}{}^0_{\mathrm{u}}\tilde{u}^*_{\mathrm{L}j}\right) + y^*_{\mathrm{d}ij}\bar{D}^{\mathrm{c}}_i\left(\bar{\tilde{h}}{}^-_{\mathrm{d}}\tilde{u}^*_{\mathrm{L}j} - \bar{\tilde{h}}{}^0_{\mathrm{d}}\tilde{d}^*_{\mathrm{L}j}\right) + y^*_{\mathrm{e}ij}\bar{E}^{\mathrm{c}}_i\left(\bar{\tilde{h}}{}^-_{\mathrm{d}}\tilde{\nu}^*{}_j - \bar{\tilde{h}}{}^0_{\mathrm{d}}\tilde{e}^*_{\mathrm{L}j}\right) \\
&\quad + y^*_{\mathrm{u}ij}\tilde{u}_{\mathrm{R}i}\left(\bar{\tilde{h}}{}^+_{\mathrm{u}}\bar{D}_j - \bar{\tilde{h}}{}^0_{\mathrm{u}}\bar{U}_j\right) + y^*_{\mathrm{d}ij}\tilde{d}_{\mathrm{R}i}\left(\bar{\tilde{h}}{}^-_{\mathrm{d}}\bar{U}_j - \bar{\tilde{h}}{}^0_{\mathrm{d}}\bar{D}_j\right) + y^*_{\mathrm{e}ij}\tilde{e}_{\mathrm{R}i}\left(\bar{\tilde{h}}{}^-_{\mathrm{d}}\bar{\nu}_j - \bar{\tilde{h}}{}^0_{\mathrm{d}}\bar{E}_j\right)
\end{aligned} \tag{3.104}
$$

and the rest part is $\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}^{\text{RPC}}_{\text{pot.}}$.