1. Kinematics

Decay rate and cross section (Note: $\mathcal{M}$ has a mass dimension of $4-N_{\rm i}-N_{\rm f}$.)

$$\text{decay rate (rest frame; } \sqrt{s}=M_0\text{)}: \quad \mathrm{d}\Gamma = \frac{\overline{\mathrm{d}\Pi^{N_{\rm f}}}}{2M_0}\left|\mathcal{M}(M_0\to\{p_1,p_2,\cdots,p_{N_{\rm f}}\})\right|^2, \tag{1.1}$$

$$\text{cross section (Lorentz invariant)}: \quad \mathrm{d}\sigma = \frac{\overline{\mathrm{d}\Pi^{N_{\rm f}}}}{2E_A 2E_B\, v_{\rm Møl}}\left|\mathcal{M}(p_A,p_B\to\{p_1,p_2,\cdots,p_{N_{\rm f}}\})\right|^2, \tag{1.2}$$

where $\overline{\mathrm{d}\Pi^n}$ is n-particle Lorentz-invariant phase space with momentum conservation

$$\overline{\mathrm{d}\Pi^n} := \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2\cdots\mathrm{d}\Pi_n\,(2\pi)^4\,\delta^{(4)}\left(P_0-\sum p_n\right); \quad \mathrm{d}\Pi := \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}}. \tag{1.3}$$

At the CM frame, two-body phase-space are characterized by the final momentum $\|\boldsymbol{p}\|$ and given by

$$\overline{\mathrm{d}\Pi^2} = \frac{\|\boldsymbol{p}\|}{4\pi\sqrt{s}}\frac{\mathrm{d}\Omega}{4\pi} = \frac{\|\boldsymbol{p}\|}{8\pi\sqrt{s}}\,\mathrm{d}\cos\theta = \frac{1}{16\pi}\sqrt{1-\frac{2(m_1^2+m_2^2)}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}}\,\mathrm{d}\cos\theta \tag{1.4}$$

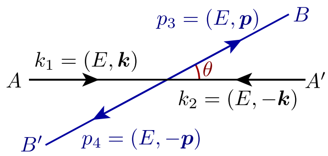
with $\sqrt{s}=M_0$ or $E_{\rm CM}$, θ is the angle between initial and final motion, and

$$\|\boldsymbol{p}\| = \frac{\sqrt{s}}{2}\,\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right), \quad E_1=\frac{s+m_1^2-m_2^2}{2\sqrt{s}}, \quad E_2=\frac{s-m_1^2+m_2^2}{2\sqrt{s}}, \quad p_1\cdot p_2=\frac{s-(m_1^2+m_2^2)}{2}.$$

Mandelstam variables For $(k_1,k_2)\to(p_3,p_4)$ collision,

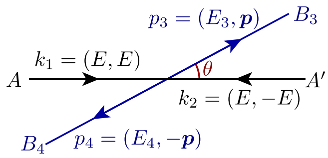
$$s=(k_1+k_2)^2=(p_3+p_4)^2, \qquad t=(p_3-k_1)^2=(p_4-k_2)^2, \qquad u=(p_3-k_2)^2=(p_4-k_1)^2;$$
$$s+t+u=m_1^2+m_2^2+m_3^2+m_4^2.$$

If the collision is with the "same mass" $(m_A,m_A)\to(m_B,m_B)$,



$$t=m_A^2+m_B^2-s/2+2kp\cos\theta, \qquad (k_1-k_2)^2=4m_A^2-s,$$
$$u=m_A^2+m_B^2-s/2-2kp\cos\theta, \qquad (p_3-p_4)^2=4m_B^2-s,$$
$$k=\frac{\sqrt{s-4m_A^2}}{2}, \qquad k_1\cdot k_2=\frac{s}{2}-m_A^2, \qquad k_1\cdot p_3=k_2\cdot p_4=\frac{m_A^2+m_B^2-t}{2},$$
$$p=\frac{\sqrt{s-4m_B^2}}{2}, \qquad p_3\cdot p_4=\frac{s}{2}-m_B^2, \qquad k_1\cdot p_4=k_2\cdot p_3=\frac{m_A^2+m_B^2-u}{2}.$$

Instead, if the collision is "initially massless" $(0,0)\to(m_3,m_4)$,



$$t=(m_3^2+m_4^2-s)/2+p\sqrt{s}\cos\theta,$$
$$u=(m_3^2+m_4^2-s)/2-p\sqrt{s}\cos\theta,$$
$$p=(\sqrt{s}/2)\,\lambda^{1/2}\left(1;m_3^2/s,m_4^2/s\right).$$

1.1. Fundamentals

Lorentz-invariant phase space:

$$\int \mathrm{d}\Pi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\sqrt{m^2+\|\boldsymbol{p}\|^2}} = \int \frac{\mathrm{d}p_0\,\mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}(2\pi)\,\delta\left(p_0^2-\|\boldsymbol{p}\|^2-m^2\right)\Theta(p_0)$$

Källén function:

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zx = (x-y-z)^2-4yz;$$
$$\lambda(1;\alpha_1^2,\alpha_2^2) = (1-(\alpha_1+\alpha_2)^2)(1-(\alpha_1-\alpha_2)^2) = (1+\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)(1-\alpha_1+\alpha_2).$$
$$\lambda^{1/2}\left(s;m_1^2,m_2^2\right) = s\,\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right); \qquad \lambda^{1/2}\left(1;\frac{m^2}{s},\frac{m^2}{s}\right) = \sqrt{1-\frac{4m^2}{s}},$$
$$\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = \sqrt{1-\frac{2(m_1^2+m_2^2)}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}}, \qquad \lambda^{1/2}\left(1;\frac{m_1^2}{s},0\right) = \frac{s-m_1^2}{s}.$$

Two-body phase space If $f(p_1^\mu,p_2^\mu)$ is Lorentz invariant, $f\equiv f(p_1^2,p_2^2,p_1^\mu p_{2\mu})\equiv f(p_1,p_2,\cos\theta_{12})$. Meanwhile,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \int \frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{(4\pi)\,\mathrm{d}p_1\,p_1^2}{(2\pi)^3}\frac{(2\pi)\,\mathrm{d}p_2\,p_2^2\,\mathrm{d}\cos\theta_{12}}{(2\pi)^3}\frac{1}{2E_1 2E_2} = \int \frac{\mathrm{d}E_+\,\mathrm{d}E_-\,\mathrm{d}s}{128\pi^4}, \tag{1.5}$$

with the replacement of the variables

$$E_\pm = E_1\pm E_2, \qquad s=(p_1+p_2)^2 = m_1^2+m_2^2+2E_1E_2-2\|\boldsymbol{p}_1\|\|\boldsymbol{p}_2\|\cos\theta_{12};$$
$$\left|\frac{\mathrm{d}(E_+,E_-,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{4p_1^2p_2^2}{E_1E_2}, \qquad \left|\frac{\mathrm{d}(E_1,E_2,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{2p_1^2p_2^2}{E_1E_2}.$$

Therefore,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \frac{1}{128\pi^4}\int_{(m_1+m_2)^2}^\infty \mathrm{d}s\int_{\sqrt{s}}^\infty \mathrm{d}E_+\int_{\mathrm{min}}^{\mathrm{max}}\mathrm{d}E_-, \tag{1.6}$$

where the boundary of E_- is given by

$$\cos\theta_{12} = \frac{E_+^2-E_-^2+2\left(m_1^2+m_2^2-s\right)}{\sqrt{(E_++E_-)^2-4m_1^2}\sqrt{(E_+-E_-)^2-4m_2^2}}\in[-1,1]$$
$$\therefore\quad \left|E_- - \frac{m_1^2-m_2^2}{s}E_+\right| \le \sqrt{E_+^2-s}\cdot\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = 2p\sqrt{\frac{E_+^2-s}{s}}.$$

Two-body phase space with momentum conservation As a general representation in any frame,

$$\overline{\mathrm{d}\Pi^2} = \frac{\mathrm{d}p_1\,\mathrm{d}\Omega\,p_1^2}{16\pi^2}\frac{\delta(E_0-\sqrt{m_1^2+p_1^2}-\sqrt{m_2^2+\|\boldsymbol{P}_0-\boldsymbol{p}_1\|^2})}{E_1E_2} = \frac{1}{8\pi}\,\mathrm{d}\cos\theta_1\frac{p_1^2}{E_0p_1-P_0E_1\cos\theta_1}, \tag{1.7}$$

where the momentum p_1 is given by

$$p_1 = \frac{(E_0^2+m_1^2-m_2^2-P_0^2)P_0\cos\theta_1+E_0\sqrt{\lambda(E_0^2,m_1^2,m_2^2)+P_0^4-2P_0^2(E_0^2+m_1^2-2m_1^2\cos^2\theta_1-m_2^2)}}{2(E_0^2-P_0^2\cos^2\theta_1)}. \tag{1.8}$$

CM frame result is recovered by setting $E_0=\sqrt{s}$ and $P_0=0$.

1.2. Decay rate and Cross section

As $\langle\mathrm{out}|\mathrm{in}\rangle = (2\pi)^4\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f})\mathrm{i}\mathcal{M}$ (for in $\neq$ out) and $\langle\boldsymbol{p}|\boldsymbol{p}\rangle = 2E_{\boldsymbol{p}}(2\pi)^3\,\delta^{(3)}(\boldsymbol{0}) = 2E_{\boldsymbol{p}}V$ for one-particle state,

$$\frac{N_\mathrm{ev}}{\prod_\mathrm{in}N_\mathrm{particle}} = \int \mathrm{d}\Pi^\mathrm{out}\frac{|\langle\mathrm{out}|\mathrm{in}\rangle|^2}{\langle\mathrm{in}|\mathrm{in}\rangle} = \int \mathrm{d}\Pi^\mathrm{out}\frac{(2\pi)^8|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}\frac{VT}{(2\pi^4)}\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f}) = VT\int\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}. \tag{1.9}$$

Therefore, decay rate (at the rest frame) is given by

$$\mathrm{d}\Gamma := \frac{1}{T}\frac{\mathrm{d}N_\mathrm{ev}}{N_\mathrm{particle}} = \frac{1}{T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.10}$$

We also define Lorentz-invariant cross section σ by $N_\mathrm{ev} =: (\rho_Av_\mathrm{Møl}T\sigma)N_B = (\rho_Av_\mathrm{Møl}T\sigma)(\rho_BV)$, or

$$\mathrm{d}\sigma := \frac{\mathrm{d}N_\mathrm{ev}}{\rho_Av_\mathrm{Møl}TN_B} = \frac{V}{v_\mathrm{Møl}T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{2E_A2E_BV^2} = \frac{1}{2E_A2E_B\,v_\mathrm{Møl}}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.11}$$

where the Møller parameter $v_\mathrm{Møl}$ is equal to $v_\mathrm{rel}^\mathrm{NR} = \|\boldsymbol{v}_A-\boldsymbol{v}_B\|$ if $\boldsymbol{v}_A\,/\!\!/\,\boldsymbol{v}_B$ (cf. Ref. [**?**]). Generally,

$$v_\mathrm{Møl} := \frac{\sqrt{(p_A\cdot p_B)^2-m_A^2m_B^2}}{E_AE_B} = \frac{p_A\cdot p_B}{E_AE_B}v_\mathrm{rel} = (1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B)v_\mathrm{rel}, \tag{1.12}$$

where v_rel is the actual relative velocity

$$v_\mathrm{rel} = \sqrt{1-\frac{(1-v_A^2)(1-v_B^2)}{1-(\boldsymbol{v}_A\cdot\boldsymbol{v}_B)^2}} = \frac{\sqrt{\|\boldsymbol{v}_A-\boldsymbol{v}_B\|^2-\|\boldsymbol{v}_A\times\boldsymbol{v}_B\|^2}}{1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B} = \frac{\lambda^{1/2}(s,m_A^2,m_B^2)}{s-(m_A^2+m_B^2)} \neq v_\mathrm{rel}^\mathrm{NR}. \tag{1.13}$$

(Note that $p_A\cdot p_B/E_AE_B = 1$ if $\boldsymbol{p}_A=0$ or $\boldsymbol{p}_B=0$. Also, Each of v_rel, VT, and $E_AE_Bv_\mathrm{Møl}$ is Lorentz invariant.)

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation adj. $= (\epsilon^a)^{bc}$.[*1]

$$T_a = \frac{1}{2}\sigma_a, \qquad \mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \qquad [T_a, T_b] = \mathrm{i}\epsilon^{abc}T^c, \qquad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\overline{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a \epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\overline{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \to \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp(-\mathrm{i}g\theta^\alpha T^{\alpha*})]^{ab}(\epsilon^{bc}\mathbf{2}^c). \tag{2.1}$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\overline{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation adj. $= \mathbf{8} = (f^a)^{bc}$.
Gell-Mann matrices:

$$\lambda_{1\text{–}8} = \begin{pmatrix} 0&1&0\\1&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&-\mathrm{i}&0\\ \mathrm{i}&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0\\0&-1&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&-\mathrm{i}\\0&0&0\\ \mathrm{i}&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&1\\0&1&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&-\mathrm{i}\\0&\mathrm{i}&0 \end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix} 1&0&0\\0&1&0\\0&0&-2 \end{pmatrix}. \tag{2.2}$$

$$\tau_a = \frac{1}{2}\lambda_a, \qquad \mathrm{Tr}(\tau_a\tau_b) = \frac{1}{2}\delta_{ab}, \qquad [\tau_a, \tau_b] = \mathrm{i}f^{abc}\tau^c, \qquad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\begin{aligned} \mathbf{3}: \quad & \phi_a \to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi_b \simeq \phi_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi_b \\ & \phi^*_a \to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi^*_b \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi^*_b \\ & \quad = \phi^*_b[\exp(-\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ba} \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\phi^*_b\tau^\alpha_{ba} \end{aligned}$$

$$\begin{aligned} \overline{\mathbf{3}}: \quad & \phi_a \to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi_b \\ & \phi^*_a \to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi^*_b \simeq \phi^*_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi^*_b \end{aligned}$$

[*1] We do not distinguish sub- and superscripts for gauge indices.

3. Spinors

$$(\overline{\psi_1}\psi_2)^* = (\psi_2)^\dagger(\overline{\psi}_1)^\dagger = \overline{\psi_2}\psi_1. \tag{3.1}$$

4. Supersymmetry with $\eta = \mathrm{diag}(+,-,-,-)$

Convention Our convention follows DHM (except for D_μ):

$$\eta = \mathrm{diag}(1,-1,-1,-1);\quad \epsilon^{0123} = -\epsilon_{0123} = 1,\quad \epsilon^{12} = \epsilon_{21} = \epsilon^{\dot 1\dot 2} = \epsilon_{\dot 2\dot 1} = 1\quad \left(\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma\right),$$
$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta,\quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta,\quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta},\quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta};$$
$$\sigma^\mu_{\alpha\dot\alpha} := (\mathbf{1},\boldsymbol{\sigma})_{\alpha\dot\alpha},\qquad \sigma^{\mu\nu}{}_\alpha{}^\beta := \frac{\mathrm{i}}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)_\alpha{}^\beta,^{*2}\qquad \left(\sigma^\mu_{\alpha\dot\beta} = \epsilon_{\alpha\delta}\epsilon_{\dot\beta\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta},\quad \bar\sigma^{\mu\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\delta}\epsilon^{\beta\gamma}\sigma^\mu_{\gamma\dot\delta}\right)$$
$$\bar\sigma^{\mu\dot\alpha\alpha} := (\mathbf{1},-\boldsymbol{\sigma})^{\dot\alpha\alpha},\quad \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta} := \frac{\mathrm{i}}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta},^{*2}$$
$$(\psi\xi) := \psi^\alpha\xi_\alpha,\quad (\bar\psi\bar\chi) := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha};\qquad \frac{\mathrm{d}}{\mathrm{d}\theta^\alpha}(\theta\theta) := \theta_\alpha\quad \text{[left derivative]}.$$

Especially, spinor-index contraction is done as ${}^\alpha{}_\alpha$ and ${}_{\dot\alpha}{}^{\dot\alpha}$ except for ϵ_{ab} (which always comes from left). Noting that complex conjugate reverses spinor order: $(\psi^\alpha\xi^\beta)^* := (\xi^\beta)^*(\psi^\alpha)^*$,

$$\bar\psi^{\dot\alpha} := (\psi^\alpha)^*,\quad \epsilon^{\dot a\dot b} := (\epsilon^{ab})^*,\qquad (\psi\chi)^* = (\bar\psi\bar\chi),$$
$$\left(\sigma^\mu_{\alpha\dot\beta}\right)^* = \bar\sigma^\mu{}_{\dot\alpha\beta} = \epsilon_{\beta\delta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta},\qquad (\sigma^{\mu\nu})^\dagger{}^\alpha{}_\beta = \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta},\qquad \left(\sigma^{\mu\nu}{}_\alpha{}^\beta\right)^* = \bar\sigma^{\mu\nu\dot\beta}{}_{\dot\alpha} = \bar\sigma^{\mu\nu}{}_{\dot\alpha}{}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\delta},$$
$$\left(\bar\sigma^{\mu\dot\alpha\beta}\right)^* = \sigma^{\mu\alpha\dot\beta} = \epsilon^{\dot\beta\dot\delta}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\delta},\qquad (\bar\sigma^{\mu\nu})^\dagger{}_{\dot\alpha}{}^{\dot\beta} = \sigma^{\mu\nu}{}_\alpha{}^\beta,\qquad \left(\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\right)^* = \sigma^{\mu\nu}{}_\beta{}^\alpha = \bar\sigma^{\mu\nu\alpha}{}_\beta = \epsilon_{\beta\delta}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\delta.$$

Contraction formulae

$$\theta^\alpha\theta^\beta = -\tfrac12(\theta\theta)\epsilon^{\alpha\beta}\qquad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\epsilon^{\dot\alpha\dot\beta}\qquad (\theta\xi)(\theta\chi) = -\tfrac12(\theta\theta)(\xi\chi)\qquad (\theta\sigma^\nu\bar\theta)\theta^\alpha = \tfrac12(\theta\theta)(\bar\theta\bar\sigma^\nu)^\alpha$$
$$\theta_\alpha\theta_\beta = \tfrac12(\theta\theta)\epsilon_{\alpha\beta}\qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac12(\bar\theta\bar\theta)\epsilon_{\dot\alpha\dot\beta}\qquad (\bar\theta\bar\xi)(\bar\theta\bar\chi) = -\tfrac12(\bar\theta\bar\theta)(\bar\xi\bar\chi)\qquad (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\tfrac12(\theta\sigma^\nu)_{\dot\alpha}(\bar\theta\bar\theta)$$
$$\theta^\alpha\theta_\beta = \tfrac12(\theta\theta)\delta^\alpha_\beta\qquad \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\delta^{\dot\alpha}_{\dot\beta}\qquad (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \tfrac12(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu}$$
$$(\theta\sigma^\mu\bar\sigma^\nu\theta) = (\theta\theta)\eta^{\mu\nu}\qquad (\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta) = (\bar\theta\bar\theta)\eta^{\mu\nu}\qquad (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \tfrac12(\bar\theta\bar\theta)(\sigma^\mu\bar\sigma^\nu\theta)_\alpha$$

$$\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\sigma^{\mu\nu}\qquad \sigma^\mu\bar\sigma^\rho\sigma^\nu + \sigma^\nu\bar\sigma^\rho\sigma^\mu = 2\left(\sigma^\mu\eta^{\nu\rho} + \sigma^\nu\eta^{\mu\rho} - \sigma^\rho\eta^{\mu\nu}\right)$$
$$\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\bar\sigma^{\mu\nu}\qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\mathrm{i}\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$\mathrm{Tr}\left(\sigma^\mu\bar\sigma^\nu\right) = \mathrm{Tr}\left(\bar\sigma^\mu\sigma^\nu\right) = 2\eta^{\mu\nu}\qquad \bar\sigma^\mu\sigma^\rho\bar\sigma^\nu + \bar\sigma^\nu\sigma^\rho\bar\sigma^\mu = 2\left(\bar\sigma^\mu\eta^{\nu\rho} + \bar\sigma^\nu\eta^{\mu\rho} - \bar\sigma^\rho\eta^{\mu\nu}\right)$$
$$\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu{}^{\dot\beta\beta} = 2\delta^{\dot\beta}_{\dot\alpha}\delta^\alpha_\beta\qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2\mathrm{i}\bar\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$\sigma_{\mu\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\qquad \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\dot\beta\dot\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\beta}$$
$$\bar\sigma_\mu{}^{\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\qquad \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\beta\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\dot\alpha\dot\gamma}\sigma^\mu_{\beta\dot\gamma}$$
$$\mathrm{Tr}\left(\sigma^{\mu\nu}\right) = \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\right) = 0\qquad \mathrm{Tr}\left(\sigma^{\mu\nu}\sigma^{\rho\sigma}\right) = \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma}$$
$$\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu}\qquad \sigma^{\mu\nu} = -\sigma^{\nu\mu}\qquad \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma}\right) = \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma} + \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$
$$\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha}\qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = -2\mathrm{i}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon_{\alpha\beta} - 2\mathrm{i}\sigma^{\mu\nu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}$$
$$\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon^{\dot\beta\dot\alpha}\qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 4\sigma^{\rho\mu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\sigma\nu\dot\gamma}{}_{\dot\beta}\eta_{\rho\sigma} + \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\eta^{\mu\nu}$$
$$\bar\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = -2\mathrm{i}\bar\sigma^{\mu\nu}\qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} - \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = -2\mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\epsilon^{\alpha\beta} - 2\mathrm{i}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon^{\dot\alpha\dot\beta}$$
$$\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 2\mathrm{i}\sigma^{\mu\nu}\qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} + \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 4\epsilon^{\alpha\gamma}\sigma^{\sigma\nu}{}_\gamma{}^\beta\bar\sigma^{\rho\mu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\eta_{\rho\sigma} + \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\eta^{\mu\nu}$$

$$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi\qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi = \bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi\qquad \xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi\qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho\chi = -\chi\sigma^\rho\bar\sigma^\nu\sigma^\mu\bar\xi$$
$$\left(\xi\sigma^\mu\bar\chi\right)^* = \chi\sigma^\mu\bar\xi\qquad \left(\bar\xi\bar\sigma^\mu\chi\right)^* = \bar\chi\bar\sigma^\mu\xi\qquad \left(\bar\chi\bar\sigma^\mu\sigma^\nu\bar\xi\right)^* = \xi\sigma^\nu\bar\sigma^\mu\chi\qquad (\xi[\sigma\mathrm{s}]\chi)^* = \bar\chi[\sigma\mathrm{s}_{\mathrm{reversed}}]\bar\xi$$
$$(\xi\chi)\psi^\alpha = -(\psi\xi)\chi^\alpha - (\psi\chi)\xi^\alpha\qquad (\xi\chi)\bar\psi_{\dot\alpha} = \tfrac12(\xi\sigma^\mu\bar\psi)(\chi\sigma_\mu)_{\dot\alpha}$$

Superfields

*2 As the definition of $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are not unified in literature, they are not used in this CheatSheet except for this page.

4.1. Lorentz symmetry as SU(2)×SU(2)

4.2. Supersymmetry algebra

We define the generators as

$$P_\mu := \mathrm{i}\partial_\mu, \quad \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \quad \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \tag{4.1}$$

which is realized by

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad Q^\alpha = -\frac{\partial}{\partial\theta_\alpha} - \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{Q}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu,$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad D^\alpha = -\frac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{D}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu;$$

D_α etc. works as covariant derivatives because of the commutation relations

$$\{D_\alpha, \bar{D}_{\dot\alpha}\} = +2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \{Q_\alpha, D_\beta\} = \{Q_\alpha, \bar{D}_{\dot\beta}\} = \{\bar{Q}_{\dot\alpha}, D_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = \{D_\alpha, D_\beta\} = \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0.$$

Derivative formulae

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = -2\delta^\beta_\alpha \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = 2\delta^{\dot\beta}_{\dot\alpha}$$

$$\epsilon_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} = -\frac{\partial}{\partial\theta^\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = 2\epsilon^{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = -2\epsilon^{\dot\alpha\dot\beta}$$

$$\epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\beta}} = -\frac{\partial}{\partial\bar\theta_{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = 2\delta^\alpha_\beta \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = -2\delta^{\dot\alpha}_{\dot\beta}$$

$$\epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\beta}} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = -2\epsilon_{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = 2\epsilon_{\dot\alpha\dot\beta}$$

4.3. Superfields

SUSY-invariant Lagrangian SUSY transformation is induced by $\xi Q + \bar\xi\bar Q = \xi^\alpha\partial_\alpha + \bar\xi_{\dot\alpha}\partial^{\dot\alpha} + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu$. Therefore, for an object Ψ in the superspace,

$$\bigl[\Psi\bigr]_{\theta^4} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \xi^\alpha\partial_\alpha\Psi + \bar\xi_{\dot\alpha}\partial^{\dot\alpha}\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\Bigr]_{\theta^4} = \bigl[\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\bigr]_{\theta^4}, \tag{4.2}$$

which means $\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant up to total derivative, i.e., $\int \mathrm{d}^4x\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant action. Also,

$$\bigl[\Psi\bigr]_{\theta^2} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \bar\xi_{\dot\alpha}\left(\partial^{\dot\alpha} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\right)\Psi\Bigr]_{\theta^2} = \Bigl[\Psi + \bar\xi_{\dot\alpha}\bar{D}^{\dot\alpha}\Psi + 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Psi\Bigr]_{\theta^2} \tag{4.3}$$

will be SUSY-invariant if $\bar{D}_{\dot\alpha}\Psi = 0$, i.e., Ψ is a chiral superfield. Therefore, SUSY-invariant Lagrangian is given by

$$\mathcal{L} = \bigl[(\text{any real superfield})\bigr]_{\theta^4} + \bigl[(\text{any chiral superfield})\bigr]_{\theta^2} + \bigl[(\text{any chiral superfield})^*\bigr]_{\bar\theta^2}. \tag{4.4}$$

Chiral superfield A chiral superfield is a superfield that satisfies $\bar{D}_{\dot\alpha}\Phi = 0$. Because of $\bar{D}_{\dot\alpha}y^\mu = \bar{D}_{\dot\alpha}\theta = 0$ holds for a variable $y^\mu := x^\mu - \mathrm{i}(\theta\sigma^\mu\bar\theta)$, this condition is equivalent to $\partial_{\dot\alpha}\Phi(y,\theta,\bar\theta) = 0$. Using the Taylor expansion formula

$$f(y,\theta,\bar\theta) = f(x,\theta,\bar\theta) - \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f(x,\theta,\bar\theta) - \frac{1}{4}\theta^4\partial^2 f(x,\theta,\bar\theta), \tag{4.5}$$

we find

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \tag{4.6}$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{1}{4}\partial^2\phi(x)\theta^4 \tag{4.7}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{1}{4}\partial^2\phi^*(x)\theta^4; \tag{4.8}$$

their product is expanded as

$$\begin{aligned}\Phi_i^*\Phi_j = {}& \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - \mathrm{i}\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2 \\ &+ \left[\sqrt{2}\bar\psi_i\bar\theta F_j - \frac{\mathrm{i}\left(\partial_\mu\phi_i^*\cdot\psi_j\sigma^\mu\bar\theta - \phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)}{\sqrt{2}}\right]\theta^2 + \left[\sqrt{2}F_i^*\theta\psi_j + \frac{\mathrm{i}\left(\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j - \theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)}{\sqrt{2}}\right]\bar\theta^2 \\ &+ \frac{1}{4}\left(4F_i^*F_j - \phi_i^*\partial^2\phi_j - (\partial^2\phi_i^*)\phi_j + 2(\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + 2\mathrm{i}(\psi_j\sigma^\mu\partial_\mu\bar\psi_i) - 2\mathrm{i}(\partial_\mu\psi_j\sigma^\mu\bar\psi_i)\right)\theta^4\end{aligned} \tag{4.9}$$

$$\begin{aligned}\equiv {}& \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - 2\mathrm{i}\left(\phi_i^*\partial_\mu\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2 \\ &+ \sqrt{2}\left(\bar\psi_i\bar\theta F_j + \mathrm{i}\phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)\theta^2 + \sqrt{2}\left(F_i^*\theta\psi_j - \mathrm{i}\theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)\bar\theta^2 \\ &+ \left(F_i^*F_j + (\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + \mathrm{i}\bar\psi_i\sigma^\mu\partial_\mu\psi_j\right)\theta^4\end{aligned} \tag{4.10}$$

$$\Phi_i \Phi_j \Big|_{\theta^2} = -\psi_i \psi_j + F_i \phi_j + \phi_i F_j \tag{4.11}$$

$$\Phi_i \Phi_j \Phi_k \Big|_{\theta^2} = -(\psi_i \psi_j)\phi_k - (\psi_k \psi_i)\phi_j - (\psi_j \psi_k)\phi_i + \phi_i \phi_j F_k + \phi_k \phi_i F_j + \phi_j \phi_k F_i \tag{4.12}$$

$$\mathrm{e}^{k\Phi} = \mathrm{e}^{k\phi}\left[1 + \sqrt{2}k\theta\psi + \left(kF - \frac{k^2}{2}\psi\psi\right)\theta^2 - \mathrm{i}k\partial_\mu\phi(\theta\sigma^\mu\bar{\theta}) + \frac{\mathrm{i}k\left(\partial_\mu\psi + k\psi\partial_\mu\phi\right)\sigma^\mu\bar{\theta}\theta^2}{\sqrt{2}} - \frac{k}{4}\left(\partial^2\phi + k\partial_\mu\phi\partial^\mu\phi\right)\theta^4\right]; \tag{4.13}$$

note that $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$, and $\mathrm{e}^{k\Phi}$ are all chiral superfields.

5. Minimal Supersymmetric Standard Model

Gauge symmetry: $\mathrm{SU(3)_{color}} \times \mathrm{SU(2)_{weak}} \times \mathrm{U(1)}_Y$

Particle content:

(a) Chiral superfields

	SU(3)	SU(2)	U(1)	B	L	scalar/spinor
Q_i	$\mathbf{3}$	$\mathbf{2}$	1/6	1/3		$\tilde{q}_\mathrm{L}\,, q_\mathrm{L}$ $[\to (u_\mathrm{L}, d_\mathrm{L})]$
L_i		$\mathbf{2}$	$-1/2$		1	$\tilde{l}_\mathrm{L}\,, l_\mathrm{L}$ $[\to (\nu_\mathrm{L}, l_\mathrm{L})]$
U_i^c	$\overline{\mathbf{3}}$		$-2/3$	$-1/3$		$\tilde{u}_\mathrm{R}^\mathrm{c}, u_\mathrm{R}^\mathrm{c}$
D_i^c	$\overline{\mathbf{3}}$		1/3	$-1/3$		$\tilde{d}_\mathrm{R}^\mathrm{c}, d_\mathrm{R}^\mathrm{c}$
E_i^c			1		-1	$\tilde{e}_\mathrm{R}^\mathrm{c}\,, e_\mathrm{R}^\mathrm{c}$
H_u		$\mathbf{2}$	1/2			$h_\mathrm{u}\,, \tilde{h}_\mathrm{u}$ $[\to (h_\mathrm{u}^+, h_\mathrm{u}^0)]$
H_d		$\mathbf{2}$	$-1/2$			$h_\mathrm{d}\,, \tilde{h}_\mathrm{d}$ $[\to (h_\mathrm{d}^0, h_\mathrm{d}^-)]$

(b) Vector superfields

	SU(3)	SU(2)	U(1)	ino/boson
g	adj.			$\tilde{g}, g_\mu$
W		adj.		$\tilde{w}, W_\mu$
B				$\tilde{b}, B_\mu$

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and SU(2) decomposition).

"c"-notation For scalars, $\tilde{\phi}_\mathrm{R}^\mathrm{c} := \phi_\mathrm{R}^* = \mathsf{C}\phi_\mathrm{R}\mathsf{C}$ (because the intrinsic phase for C is $+1$ for quarks and leptons.)

For matter spinors, $\psi_\mathrm{R}^\mathrm{c} := \bar{\psi}_\mathrm{R}$ (and $\psi_\mathrm{R} = \bar{\psi}_\mathrm{R}^\mathrm{c}$); Dirac spinors are thus

$$\psi_\mathrm{L} = \begin{pmatrix}\psi_\mathrm{L}\\ 0\end{pmatrix}, \quad \overline{\psi_\mathrm{L}} = \begin{pmatrix}0 & \bar{\psi}_\mathrm{L}\end{pmatrix}, \quad \psi_\mathrm{R}^\mathrm{c} := \begin{pmatrix}\psi_\mathrm{R}^\mathrm{c}\\ 0\end{pmatrix} = C\begin{pmatrix}0\\ \psi_\mathrm{R}\end{pmatrix} = C\psi_\mathrm{R}, \quad \overline{\psi_\mathrm{R}^\mathrm{c}} = \begin{pmatrix}0 & \psi_\mathrm{R}\end{pmatrix} = \begin{pmatrix}\bar{\psi}_\mathrm{R} & 0\end{pmatrix} C = \overline{\psi_\mathrm{R}}C.$$

Superpotential and ~~SUSY~~-terms

$$W_\mathrm{RPC} = \mu H_\mathrm{u} H_\mathrm{d} - y_{\mathrm{u}ij} U_i^\mathrm{c} H_\mathrm{u} Q_j + y_{\mathrm{d}ij} D_i^\mathrm{c} H_\mathrm{d} Q_j + y_{\mathrm{e}ij} E_i^\mathrm{c} H_\mathrm{d} L_j, \tag{5.1}$$

$$W_\mathrm{RPV} = -\kappa_i L_i H_\mathrm{u} + \frac{1}{2}\lambda_{ijk} L_i L_j E_k^\mathrm{c} + \lambda'_{ijk} L_i Q_j D_k^\mathrm{c} + \frac{1}{2}\lambda''_{ijk} U_i^\mathrm{c} D_j^\mathrm{c} D_k^\mathrm{c}, \tag{5.2}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\Big(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\Big) - V_{\cancel{\mathrm{SUSY}}}; \tag{5.3}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^\mathrm{RPC} = &\Big(\tilde{q}_\mathrm{L}^* m_Q^2 \tilde{q}_\mathrm{L} + \tilde{l}_\mathrm{L}^* m_L^2 \tilde{l}_\mathrm{L} + \tilde{u}_\mathrm{R}^* m_{U^\mathrm{c}}^2 \tilde{u}_\mathrm{R} + \tilde{d}_\mathrm{R}^* m_{D^\mathrm{c}}^2 \tilde{d}_\mathrm{R} + \tilde{e}_\mathrm{R}^* m_{E^\mathrm{c}}^2 \tilde{e}_\mathrm{R} + m_{H_\mathrm{u}}^2 |h_\mathrm{u}|^2 + m_{H_\mathrm{d}}^2 |h_\mathrm{d}|^2\Big) \\ &+ \Big(-\tilde{u}_\mathrm{R}^* h_\mathrm{u} a_\mathrm{u} \tilde{q}_\mathrm{L} + \tilde{d}_\mathrm{R}^* h_\mathrm{d} a_\mathrm{d} \tilde{q}_\mathrm{L} + \tilde{e}_\mathrm{R}^* h_\mathrm{d} a_\mathrm{e} \tilde{l}_\mathrm{L} + b H_\mathrm{u} H_\mathrm{d} + \text{H.c.}\Big) \\ &+ \Big(-\tilde{u}_\mathrm{R}^* h_\mathrm{d}^* c_\mathrm{u} \tilde{q}_\mathrm{L} + \tilde{d}_\mathrm{R}^* h_\mathrm{u}^* c_\mathrm{d} \tilde{q}_\mathrm{L} + \tilde{e}_\mathrm{R}^* h_\mathrm{u}^* c_\mathrm{e} \tilde{l}_\mathrm{L} + \text{H.c.}\Big), \end{aligned} \tag{5.4}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^\mathrm{RPV} = &\Big(-b_i \tilde{l}_{\mathrm{L}i} H_\mathrm{u} + \frac{1}{2} T_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{l}_{\mathrm{L}j} \tilde{e}_{\mathrm{R}k}^* + T'_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{q}_{\mathrm{L}j} \tilde{d}_{\mathrm{R}k}^* + \frac{1}{2} T''_{ijk} \tilde{u}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j}^* \tilde{d}_{\mathrm{R}k}^* + \tilde{l}_{\mathrm{L}i}^* M_{Li}^2 H_\mathrm{d} + \text{H.c.}\Big) \\ &+ \Big(C_{ijk}^1 \tilde{l}_{\mathrm{L}i}^* \tilde{q}_{\mathrm{L}j} \tilde{u}_{\mathrm{R}k}^* + C_i^2 h_\mathrm{u}^* h_\mathrm{d} \tilde{e}_{\mathrm{R}i}^* + C_{ijk}^3 \tilde{d}_{\mathrm{R}i} \tilde{u}_{\mathrm{R}j}^* \tilde{e}_{\mathrm{R}k}^* + \frac{1}{2} C_{ijk}^4 \tilde{d}_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} \tilde{q}_{\mathrm{L}k} + \text{H.c.}\Big), \end{aligned} \tag{5.5}$$

($\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C_{ijk}^4 = C_{ikj}^4$.)

We follow the notation of DHM [?, PhysRept] and Martin [?, v7] (but note that Martin uses $(-,+,+,+)$-metric) for RPC part and SLHA2 convention for RPV part.

5.1. Scalar potential

The MSSM scalar potential has contributions from F-terms and D-terms:

$$V_{\mathrm{SUSY}} = F_i^* F_i + \frac{1}{2} D^a D^a; \qquad F_i = -W_i^* = -\frac{\delta W^*}{\delta \phi_i^*}, \qquad D^a = -g(\phi^* T^a \phi), \tag{5.6}$$

where T_a corresponds to the gauge-symmetry generator relevant for each ϕ. They are given by

$$-F^*_{h_{\mathrm{u}}^a} = \epsilon^{ab}\left(-\tilde{u}_{\mathrm{R}}^{x*} y_{\mathrm{u}} \tilde{q}_{\mathrm{L}}^{bx} + \mu h_{\mathrm{d}}^b + \mu_i' \tilde{l}_{\mathrm{L}i}^b\right), \tag{5.7}$$

$$-F^*_{h_{\mathrm{d}}^a} = \epsilon^{ab}\left(\tilde{e}_{\mathrm{R}}^* y_{\mathrm{e}} \tilde{l}_{\mathrm{L}}^b + \tilde{d}_{\mathrm{R}}^{x*} y_{\mathrm{d}} \tilde{q}_{\mathrm{L}}^{bx} - \mu h_{\mathrm{u}}^b\right), \tag{5.8}$$

$$-F^*_{\tilde{q}_{\mathrm{L}i}^{ax}} = \epsilon^{ab}\left(-y_{\mathrm{d}ji} h_{\mathrm{d}}^b \tilde{d}_{\mathrm{R}j}^{x*} + y_{\mathrm{u}ji} h_{\mathrm{u}}^b \tilde{u}_{\mathrm{R}j}^{x*} - \lambda'_{kij} \tilde{d}_{\mathrm{R}j}^{x*} \tilde{l}_{\mathrm{L}k}^b\right), \tag{5.9}$$

$$-F^*_{\tilde{u}_{\mathrm{R}i}^{x*}} = -y_{\mathrm{u}ij} h_{\mathrm{u}} \tilde{q}_{\mathrm{L}j}^x + \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk} \tilde{d}_{\mathrm{R}j}^{y*} \tilde{d}_{\mathrm{R}k}^{z*}, \tag{5.10}$$

$$-F^*_{\tilde{d}_{\mathrm{R}i}^{x*}} = y_{\mathrm{d}ij} h_{\mathrm{d}} \tilde{q}_{\mathrm{L}j}^x + \lambda'_{jki} \tilde{l}_{\mathrm{L}j} \tilde{q}_{\mathrm{L}k}^x - \lambda''_{jik}\epsilon^{xyz} \tilde{u}_{\mathrm{R}j}^{y*} \tilde{d}_{\mathrm{R}k}^{z*}, \tag{5.11}$$

$$-F^*_{\tilde{l}_{\mathrm{L}i}^a} = \epsilon^{ab}\left(-y_{\mathrm{e}ji} \tilde{e}_{\mathrm{R}j}^* h_{\mathrm{d}}^b - \mu_i' h_{\mathrm{u}}^b + \lambda_{ijk} \tilde{l}_{\mathrm{L}j}^b \tilde{e}_{\mathrm{R}k}^* + \lambda'_{ijk} \tilde{q}_{\mathrm{L}j}^{bx} \tilde{d}_{\mathrm{R}k}^{x*}\right), \tag{5.12}$$

$$-F^*_{\tilde{e}_{\mathrm{R}i}^*} = y_{\mathrm{e}ij} h_{\mathrm{d}} \tilde{l}_{\mathrm{L}j} + \frac{1}{2}\lambda_{jki} \tilde{l}_{\mathrm{L}j} \tilde{l}_{\mathrm{L}k}. \tag{5.13}$$

$$D^\alpha_{\mathrm{SU}(3)} = -g_3 \sum_{i=1}^{3}\left(\sum_{a=1,2} \tilde{q}_{\mathrm{L}i}^{a*} \tau^\alpha \tilde{q}_{\mathrm{L}i}^a - \tilde{u}_{\mathrm{R}i}^* \tau^\alpha \tilde{u}_{\mathrm{R}i} - \tilde{d}_{\mathrm{R}i}^* \tau^\alpha \tilde{d}_{\mathrm{R}i}\right), \tag{5.14}$$

$$D^\alpha_{\mathrm{SU}(2)} = -g_2 \left[\sum_{i=1}^{3}\left(\sum_{x=1}^{3} \tilde{q}_{\mathrm{L}i}^{x*} T^\alpha \tilde{q}_{\mathrm{L}i}^x + \tilde{l}_{\mathrm{L}i}^* T^\alpha \tilde{l}_{\mathrm{L}i}\right) + h_{\mathrm{u}}^* T^\alpha h_{\mathrm{u}} + h_{\mathrm{d}}^* T^\alpha h_{\mathrm{d}}\right], \tag{5.15}$$

$$D_{\mathrm{U}(1)} = -g_1\left(\frac{1}{6}|\tilde{q}_{\mathrm{L}}|^2 - \frac{1}{2}|\tilde{l}_{\mathrm{L}}|^2 - \frac{2}{3}|\tilde{u}_{\mathrm{R}}|^2 + \frac{1}{3}|\tilde{d}_{\mathrm{R}}|^2 + |\tilde{e}_{\mathrm{R}}|^2 + \frac{1}{2}|h_{\mathrm{u}}|^2 - \frac{1}{2}|h_{\mathrm{d}}|^2\right). \tag{5.16}$$

Combining these, the full SUSY scalar potential is given by

$$V = V_{\mathrm{SUSY}} + V_{\cancel{\mathrm{SUSY}}}; \tag{5.17}$$

$$\begin{aligned}
V_{\mathrm{SUSY}} = {}& |h_{\mathrm{u}}|^2 \left(|\mu|^2 + |\mu'|^2\right) + |\mu|^2 |h_{\mathrm{d}}|^2 + \left(\mu_i'^* \mu \tilde{l}^*_{\mathrm{L}i} h_{\mathrm{d}} + \mathrm{H.c.}\right) + \mu_i'^* \mu_j' \tilde{l}^*_{\mathrm{L}i} \tilde{l}_{\mathrm{L}j} \\
& + \Big[- y_{\mathrm{u}ij} \mu^* h_{\mathrm{d}}^* \tilde{u}^*_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} - y_{\mathrm{u}ij} \mu_k'^* \tilde{u}^*_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} \tilde{l}^*_{\mathrm{L}k} - \left(y_{\mathrm{d}ij} \mu^* + \lambda'_{kji} \mu_k'^*\right) h_{\mathrm{u}}^* \tilde{d}^*_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} \\
& \qquad + y_{\mathrm{e}ij} \mu_j'^* \tilde{e}^*_{\mathrm{R}i} h_{\mathrm{u}}^* h_{\mathrm{d}} + \left(\lambda_{jki} \mu_k'^* - y_{\mathrm{e}ij} \mu^*\right) h_{\mathrm{u}}^* \tilde{e}^*_{\mathrm{R}i} \tilde{l}_{\mathrm{L}j} + \mathrm{H.c.} \Big] \\
& + \frac{1}{8} \left(g_2^2 + g_Y^2\right) |h_{\mathrm{d}}|^4 + \frac{1}{8} \left(g_2^2 + g_Y^2\right) |h_{\mathrm{u}}|^4 + \left(-\frac{g_2^2}{4} |h_{\mathrm{d}}|^2 |h_{\mathrm{u}}|^2 - \frac{g_Y^2}{4} |h_{\mathrm{d}}|^2 |h_{\mathrm{u}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{d}}^* h_{\mathrm{u}}|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{u}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_Y^2}{12} |h_{\mathrm{u}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{u}}^b \tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_{\mathrm{u}}^\dagger y_{\mathrm{u}})_{ji} h_{\mathrm{u}}^a h_{\mathrm{u}}^{b*} \tilde{q}^c_{\mathrm{L}i} \tilde{q}^{d*}_{\mathrm{L}j} \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{d}}|^2 |\tilde{q}_{\mathrm{L}}|^2 - \frac{g_Y^2}{12} |h_{\mathrm{d}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{d}}^* \tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_{\mathrm{d}}^\dagger y_{\mathrm{d}})_{ji} h_{\mathrm{d}}^a h_{\mathrm{d}}^{b*} \tilde{q}^c_{\mathrm{L}i} \tilde{q}^{d*}_{\mathrm{L}j} \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{u}}|^2 |\tilde{l}_{\mathrm{L}}|^2 - \frac{g_Y^2}{4} |h_{\mathrm{u}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{u}}^* \tilde{l}_{\mathrm{L}i}|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{d}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \frac{g_Y^2}{4} |h_{\mathrm{d}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{d}}^* \tilde{l}_{\mathrm{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_{\mathrm{e}}^\dagger y_{\mathrm{e}})_{ji} h_{\mathrm{d}}^a h_{\mathrm{d}}^{b*} \tilde{l}^c_{\mathrm{L}i} \tilde{l}^{d*}_{\mathrm{L}j} \right) \\
& + |h_{\mathrm{u}}|^2 \left(-\frac{g_Y^2}{3} |\tilde{u}_{\mathrm{R}}|^2 + (y_{\mathrm{u}} y_{\mathrm{u}}^\dagger)_{ij} \tilde{u}^*_{\mathrm{R}i} \tilde{u}_{\mathrm{R}j} \right) + \frac{g_Y^2}{3} |h_{\mathrm{d}}|^2 |\tilde{u}_{\mathrm{R}}|^2 \\
& + \frac{g_Y^2}{6} |h_{\mathrm{u}}|^2 |\tilde{d}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2 \left(-\frac{g_Y^2}{6} |\tilde{d}_{\mathrm{R}}|^2 + (y_{\mathrm{d}} y_{\mathrm{d}}^\dagger)_{ij} \tilde{d}^*_{\mathrm{R}i} \tilde{d}_{\mathrm{R}j} \right) - \left[(y_{\mathrm{u}} y_{\mathrm{d}}^\dagger)_{ij} \tilde{u}^*_{\mathrm{R}i} \tilde{d}_{\mathrm{R}j} (h_{\mathrm{d}}^* h_{\mathrm{u}}) + \mathrm{H.c.} \right] \\
& + \frac{g_Y^2}{2} |h_{\mathrm{u}}|^2 |\tilde{e}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2 \left(-\frac{g_Y^2}{2} |\tilde{e}_{\mathrm{R}}|^2 + (y_{\mathrm{e}} y_{\mathrm{e}}^\dagger)_{ij} \tilde{e}^*_{\mathrm{R}i} \tilde{e}_{\mathrm{R}j} \right) \\
& + \Big[-\frac{1}{2} \epsilon^{ab} \epsilon^{xyz} y_{\mathrm{u}lk} \lambda''^*_{lij} h_{\mathrm{u}}^a \tilde{d}^x_{\mathrm{R}i} \tilde{d}^y_{\mathrm{R}j} \tilde{q}^{bz}_{\mathrm{L}k} + \epsilon^{ab} \epsilon^{xyz} y_{\mathrm{d}lk} \lambda''^*_{ijl} h_{\mathrm{d}}^a \tilde{u}^x_{\mathrm{R}i} \tilde{d}^y_{\mathrm{R}j} \tilde{q}^{bz}_{\mathrm{L}k} - y_{\mathrm{u}il} \lambda'^*_{klj} h_{\mathrm{u}}^a \tilde{u}^*_{\mathrm{R}i} \tilde{d}_{\mathrm{R}j} \tilde{l}^{a*}_{\mathrm{L}k} \\
& \qquad + y_{\mathrm{d}il} \lambda'^*_{klj} h_{\mathrm{d}}^a \tilde{d}^*_{\mathrm{R}i} \tilde{d}_{\mathrm{R}j} \tilde{l}^{a*}_{\mathrm{L}k} - \epsilon^{ab} \epsilon^{cd} y_{\mathrm{d}li} \lambda'^*_{kjl} h_{\mathrm{d}}^a \tilde{q}^b_{\mathrm{L}i} \tilde{q}^{c*}_{\mathrm{L}j} \tilde{l}^{d*}_{\mathrm{L}k} + y_{\mathrm{e}il} \lambda^*_{klj} h_{\mathrm{d}}^a \tilde{e}^*_{\mathrm{R}i} \tilde{e}_{\mathrm{R}j} \tilde{l}^{a*}_{\mathrm{L}k} \\
& \qquad - y_{\mathrm{e}jl} \lambda'^*_{lki} h_{\mathrm{d}}^a \tilde{e}^*_{\mathrm{R}j} \tilde{d}_{\mathrm{R}i} \tilde{q}^{a*}_{\mathrm{L}k} + \frac{1}{2} \epsilon^{ab} \epsilon^{cd} y_{\mathrm{e}li} \lambda^*_{jkl} h_{\mathrm{d}}^a \tilde{l}^b_{\mathrm{L}i} \tilde{l}^{c*}_{\mathrm{L}j} \tilde{l}^{d*}_{\mathrm{L}k} + \mathrm{H.c.} \Big] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8} \right) |\tilde{q}_{\mathrm{L}}|^4 + \frac{g_2^2}{4} \tilde{q}^{ax}_{\mathrm{L}i} \tilde{q}^{by}_{\mathrm{L}j} \tilde{q}^{bx*}_{\mathrm{L}i} \tilde{q}^{ay*}_{\mathrm{L}j} + \frac{g_3^2}{4} \tilde{q}^{ax}_{\mathrm{L}i} \tilde{q}^{by}_{\mathrm{L}j} \tilde{q}^{ay*}_{\mathrm{L}i} \tilde{q}^{bx*}_{\mathrm{L}j} \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{2 g_Y^2}{9} \right) |\tilde{u}_{\mathrm{R}}|^4 + \frac{g_3^2}{4} \tilde{u}^{x*}_{\mathrm{R}i} \tilde{u}^{y*}_{\mathrm{R}j} \tilde{u}^y_{\mathrm{R}i} \tilde{u}^x_{\mathrm{R}j} \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18} \right) |\tilde{d}_{\mathrm{R}}|^4 + \frac{g_3^2}{4} \tilde{d}^{x*}_{\mathrm{R}i} \tilde{d}^{y*}_{\mathrm{R}j} \tilde{d}^y_{\mathrm{R}i} \tilde{d}^x_{\mathrm{R}j} + \frac{1}{2} \lambda''_{mij} \lambda''^*_{mkl} \tilde{d}^{x*}_{\mathrm{R}i} \tilde{d}^{y*}_{\mathrm{R}j} \tilde{d}^x_{\mathrm{R}k} \tilde{d}^y_{\mathrm{R}l} \right] \\
& + \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9} \right) |\tilde{u}_{\mathrm{R}}|^2 |\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2} \tilde{u}^{x*}_{\mathrm{R}i} \tilde{q}^{ax}_{\mathrm{L}j} \tilde{u}^y_{\mathrm{R}i} \tilde{q}^{ay*}_{\mathrm{L}j} + y_{\mathrm{u}ik} y^*_{\mathrm{u}jl} \tilde{u}^{x*}_{\mathrm{R}i} \tilde{u}^y_{\mathrm{R}j} \tilde{q}^{ax}_{\mathrm{L}k} \tilde{q}^{ay*}_{\mathrm{L}l} \right] \\
& + \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18} \right) |\tilde{d}_{\mathrm{R}}|^2 |\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2} \tilde{d}^{x*}_{\mathrm{R}i} \tilde{q}^{ax}_{\mathrm{L}j} \tilde{d}^y_{\mathrm{R}i} \tilde{q}^{ay*}_{\mathrm{L}j} + \left(y_{\mathrm{d}ik} y^*_{\mathrm{d}jl} + \lambda'_{mki} \lambda'^*_{mlj} \right) \tilde{d}^{x*}_{\mathrm{R}i} \tilde{q}^{ax}_{\mathrm{L}k} \tilde{d}^y_{\mathrm{R}j} \tilde{q}^{ay*}_{\mathrm{L}l} \right] \\
& + \left[-\left(\frac{g_3^2}{6} + \frac{2 g_Y^2}{9} \right) |\tilde{d}_{\mathrm{R}}|^2 |\tilde{u}_{\mathrm{R}}|^2 + \left(\frac{g_3^2}{2} - \lambda''_{ikm} \lambda''^*_{ljm} \right) \tilde{u}^{x*}_{\mathrm{R}i} \tilde{d}^x_{\mathrm{R}j} \tilde{d}^{y*}_{\mathrm{R}j} \tilde{u}^y_{\mathrm{R}i} + \lambda''_{ikm} \lambda''^*_{jlm} \tilde{u}^{x*}_{\mathrm{R}i} \tilde{u}^x_{\mathrm{R}j} \tilde{d}^{y*}_{\mathrm{R}k} \tilde{d}^y_{\mathrm{R}l} \right] \\
& + \left[-\left(\frac{g_2^2}{4} + \frac{g_Y^2}{12} \right) |\tilde{l}_{\mathrm{L}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} \tilde{q}^{ax}_{\mathrm{L}i} \tilde{q}^{bx*}_{\mathrm{L}i} \tilde{l}^b_{\mathrm{L}j} \tilde{l}^{a*}_{\mathrm{L}j} + \epsilon^{ac} \epsilon^{bd} \lambda'_{kim} \lambda'^*_{ljm} \tilde{q}^{ax}_{\mathrm{L}i} \tilde{q}^{bx*}_{\mathrm{L}j} \tilde{l}^c_{\mathrm{L}k} \tilde{l}^{d*}_{\mathrm{L}l} \right] + \frac{g_Y^2}{6} |\tilde{e}_{\mathrm{R}}|^2 |\tilde{q}_{\mathrm{L}}|^2 \\
& + \left(-\frac{g_Y^2}{6} |\tilde{d}_{\mathrm{R}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \lambda'_{lmi} \lambda'^*_{kmj} \tilde{d}^*_{\mathrm{R}i} \tilde{d}_{\mathrm{R}j} \tilde{l}^*_{\mathrm{L}k} \tilde{l}_{\mathrm{L}l} \right) + \frac{g_Y^2}{3} |\tilde{u}_{\mathrm{R}}|^2 |\tilde{l}_{\mathrm{L}}|^2 - \frac{2 g_Y^2}{3} |\tilde{u}_{\mathrm{R}}|^2 |\tilde{e}_{\mathrm{R}}|^2 + \frac{g_Y^2}{3} |\tilde{d}_{\mathrm{R}}|^2 |\tilde{e}_{\mathrm{R}}|^2 \\
& + \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8} \right) |\tilde{l}_{\mathrm{L}}|^4 + \frac{g_2^2}{4} \tilde{l}^a_{\mathrm{L}i} \tilde{l}^b_{\mathrm{L}j} \tilde{l}^{b*}_{\mathrm{L}i} \tilde{l}^{a*}_{\mathrm{L}j} + \frac{1}{4} \epsilon^{ab} \epsilon^{cd} \lambda_{ijm} \lambda^*_{klm} \tilde{l}^a_{\mathrm{L}i} \tilde{l}^b_{\mathrm{L}j} \tilde{l}^{c*}_{\mathrm{L}k} \tilde{l}^{d*}_{\mathrm{L}l} \right] \\
& + \frac{g_Y^2}{2} |\tilde{e}_{\mathrm{R}}|^4 + \left[-\frac{g_Y^2}{2} |\tilde{e}_{\mathrm{R}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \left(y_{\mathrm{e}ik} y^*_{\mathrm{e}jl} + \lambda_{kmi} \lambda^*_{lmj} \right) \tilde{e}^*_{\mathrm{R}i} \tilde{e}_{\mathrm{R}j} \tilde{l}_{\mathrm{L}k} \tilde{l}^*_{\mathrm{L}l} \right] \\
& + \left[\left(y_{\mathrm{d}ik} y^*_{\mathrm{e}jl} - \lambda'_{mki} \lambda^*_{lmj} \right) \tilde{d}^{x*}_{\mathrm{R}i} \tilde{q}^{ax}_{\mathrm{L}k} \tilde{e}_{\mathrm{R}j} \tilde{l}^{a*}_{\mathrm{L}l} - \epsilon^{ab} \epsilon^{xyz} \lambda'_{lkm} \lambda''^*_{ijm} \tilde{l}^b_{\mathrm{L}l} \tilde{q}^{az}_{\mathrm{L}k} \tilde{d}^y_{\mathrm{R}j} \tilde{u}^x_{\mathrm{R}i} + \mathrm{H.c.} \right].
\end{aligned} \tag{5.18}$$

5.2. SLHA convention

The SLHA convention [?] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table (**magenta color** for objects in other conventions), while

$$\mu, b, m^2_{Q,L,H_\mathrm{u},H_\mathrm{d}}, \text{RPV-trilinears } (\lambda\text{s and } T\text{s})$$

are in common.

SLHA		our notation	Martin/DHM
(H_1, H_2)	$=$	$(H_\mathrm{d}, H_\mathrm{u})$	
$Y_{\mathrm{u,d,e}}$	$=$	$(y_{\mathrm{u,d,e}})^\mathrm{T}$	
$T_{\mathrm{u,d,e}}$	$=$	$(a_{\mathrm{u,d,e}})^\mathrm{T}$	
$A_{\mathrm{u,d,e}}$	$=$	$(A_{\mathrm{u,d,e}})^\mathrm{T}$	
$m^2_{U^\mathrm{c},D^\mathrm{c},E\mathrm{c}}$	$=$	$(m^2_{U^\mathrm{c},D^\mathrm{c},E^\mathrm{c}})^\dagger$	
$M_{1,2,3}$	$=$	$-M_{1,2,3}$	
m_3^2	$=$	b	
m_A^2	$=$	$m^2_{A_0}$ (tree)	
		κ_i	$= -\mu'_i$ (rarely used)
D_i	$=$	b_i	
$m^2_{\tilde{L}_iH_1}$	$=$	M^2_{Li}	

In particular, the chargino/neutralino mass terms in RPC case are given by

$$\mathcal{L} \supset \left[\frac{1}{2}M_1\tilde{b}\tilde{b} + \frac{1}{2}M_2\tilde{w}\tilde{w} - \mu\tilde{h}_\mathrm{u}\tilde{h}_\mathrm{d} - \frac{g_Y}{2\sqrt{2}}\left(h_\mathrm{u}^*\tilde{h}_\mathrm{u} - h_\mathrm{d}^*\tilde{h}_\mathrm{d}\right)\tilde{b} - \sqrt{2}g_2\left(h_\mathrm{u}^*T^a\tilde{h}_\mathrm{u} + h_\mathrm{d}^*T_a\tilde{h}_\mathrm{d}\right)\tilde{w}\right] + \mathrm{H.c.} \tag{5.19}$$

$$\to \frac{1}{2}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_\mathrm{u}^0\\ h_\mathrm{d}^0\end{pmatrix}^\mathrm{T}\begin{pmatrix}-M_1 & 0 & -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{s}_\beta\mathrm{s}_w\\ 0 & -M_2 & m_Z\mathrm{c}_\beta\mathrm{c}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w\\ -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{c}_\beta\mathrm{c}_w & 0 & -\mu\\ m_Z\mathrm{s}_\beta\mathrm{s}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w & -\mu & 0\end{pmatrix}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_\mathrm{u}^0\\ h_\mathrm{d}^0\end{pmatrix} \tag{5.20}$$

A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [**?**]:

$$\mathrm{e}^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \quad \text{(converges for any } X\text{)}, \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(A-1)^m}{m} \quad \text{(conv. if } \|A - I\| < 1\text{)}. \tag{A.1}$$

$$\mathrm{e}^{\log A} = A \quad \text{(if } \|A-I\| < 1\text{)}, \quad \log \mathrm{e}^X = X \text{ and } \|\mathrm{e}^X - 1\| < 1 \quad \text{(if } \|X\| < \log 2\text{)}. \tag{A.2}$$

$$\text{Hilbert-Schmidt norm : } \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \operatorname{Tr} X^\dagger X. \tag{A.3}$$

Properties:

$$\mathrm{e}^{(X^{\mathrm{T}})} = (\mathrm{e}^X)^{\mathrm{T}}, \qquad \mathrm{e}^{(X^*)} = (\mathrm{e}^X)^*, \qquad (\mathrm{e}^X)^{-1} = \mathrm{e}^{-X}, \qquad \mathrm{e}^{YXY^{-1}} = Y \mathrm{e}^X Y^{-1},$$

$$\det \exp X = \exp \operatorname{Tr} X, \qquad \frac{\mathrm{d}}{\mathrm{d}t} \mathrm{e}^{tX} = X \mathrm{e}^{tX} = \mathrm{e}^{tX} X \qquad \mathrm{e}^{(\alpha+\beta)X} = \mathrm{e}^{\alpha X} \mathrm{e}^{\beta X} \text{ for } \alpha, \beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$\mathrm{e}^X Y \mathrm{e}^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots; \tag{A.4}$$

$$\log(\mathrm{e}^X \mathrm{e}^Y) = X + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[X,\cdot]} \mathrm{e}^{t[Y,\cdot]}) Y \qquad \left[g(z) = \frac{\log z}{1 - z^{-1}} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(z-1)^n}{n(n+1)} \right] \tag{A.5}$$

$$= X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots \quad \text{(Baker-Campbell-Hausdorff)}. \tag{A.6}$$