1. Kinematics

Decay rate and cross section (Note: $\mathcal{M}$ has a mass dimension of $4 - N_\mathrm{i} - N_\mathrm{f}$.)

$$\text{decay rate (rest frame; } \sqrt{s} = M_0) : \quad \mathrm{d}\Gamma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2M_0} \Big| \mathcal{M}(M_0 \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\}) \Big|^2, \tag{1.1}$$

$$\text{cross section (Lorentz invariant)} : \quad \mathrm{d}\sigma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2E_A 2E_B\, v_{\text{Møl}}} \Big| \mathcal{M}(p_A, p_B \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\}) \Big|^2, \tag{1.2}$$

where $\overline{\mathrm{d}\Pi^n}$ is n-particle Lorentz-invariant phase space with momentum conservation

$$\overline{\mathrm{d}\Pi^n} := \mathrm{d}\Pi_1\, \mathrm{d}\Pi_2 \cdots \mathrm{d}\Pi_n\, (2\pi)^4\, \delta^{(4)} \Big(P_0 - \sum p_n\Big); \quad \mathrm{d}\Pi := \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{2E_{\boldsymbol{p}}}. \tag{1.3}$$

At the CM frame, two-body phase-space are characterized by the final momentum $\|\boldsymbol{p}\|$ and given by

$$\overline{\mathrm{d}\Pi^2} = \frac{\|\boldsymbol{p}\|}{4\pi\sqrt{s}} \frac{\mathrm{d}\Omega}{4\pi} = \frac{\|\boldsymbol{p}\|}{8\pi\sqrt{s}}\, \mathrm{d}\cos\theta = \frac{1}{16\pi} \sqrt{1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}}\, \mathrm{d}\cos\theta \tag{1.4}$$

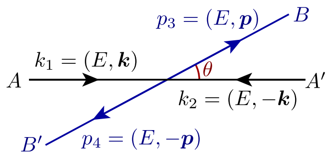
with $\sqrt{s} = M_0$ or E_CM, θ is the angle between initial and final motion, and

$$\|\boldsymbol{p}\| = \frac{\sqrt{s}}{2}\, \lambda^{1/2} \left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right), \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad p_1 \cdot p_2 = \frac{s - (m_1^2 + m_2^2)}{2}.$$

Mandelstam variables For $(k_1, k_2) \to (p_3, p_4)$ collision,

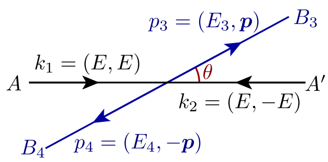
$$s = (k_1 + k_2)^2 = (p_3 + p_4)^2, \qquad t = (p_3 - k_1)^2 = (p_4 - k_2)^2, \qquad u = (p_3 - k_2)^2 = (p_4 - k_1)^2;$$
$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

If the collision is with the "same mass" $(m_A, m_A) \to (m_B, m_B)$,



$$t = m_A^2 + m_B^2 - s/2 + 2kp\cos\theta, \qquad (k_1 - k_2)^2 = 4m_A^2 - s,$$
$$u = m_A^2 + m_B^2 - s/2 - 2kp\cos\theta, \qquad (p_3 - p_4)^2 = 4m_B^2 - s,$$
$$k = \frac{\sqrt{s - 4m_A^2}}{2}, \qquad k_1 \cdot k_2 = \frac{s}{2} - m_A^2, \qquad k_1 \cdot p_3 = k_2 \cdot p_4 = \frac{m_A^2 + m_B^2 - t}{2},$$
$$p = \frac{\sqrt{s - 4m_B^2}}{2}, \qquad p_3 \cdot p_4 = \frac{s}{2} - m_B^2, \qquad k_1 \cdot p_4 = k_2 \cdot p_3 = \frac{m_A^2 + m_B^2 - u}{2}.$$

Instead, if the collision is "initially massless" $(0, 0) \to (m_3, m_4)$,



$$t = (m_3^2 + m_4^2 - s)/2 + p\sqrt{s}\cos\theta,$$
$$u = (m_3^2 + m_4^2 - s)/2 - p\sqrt{s}\cos\theta,$$
$$p = (\sqrt{s}/2)\, \lambda^{1/2} \left(1; m_3^2/s, m_4^2/s\right).$$

1.1. Fundamentals

Lorentz-invariant phase space:

$$\int \mathrm{d}\Pi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\sqrt{m^2+\|\boldsymbol{p}\|^2}} = \int \frac{\mathrm{d}p_0\,\mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}(2\pi)\,\delta\left(p_0^2-\|\boldsymbol{p}\|^2-m^2\right)\Theta(p_0)$$

Källén function:

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zx = (x-y-z)^2-4yz;$$

$$\lambda(1;\alpha_1^2,\alpha_2^2) = (1-(\alpha_1+\alpha_2)^2)(1-(\alpha_1-\alpha_2)^2) = (1+\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)(1-\alpha_1+\alpha_2).$$

$$\lambda^{1/2}\left(s;m_1^2,m_2^2\right) = s\,\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right); \qquad \lambda^{1/2}\left(1;\frac{m^2}{s},\frac{m^2}{s}\right) = \sqrt{1-\frac{4m^2}{s}},$$

$$\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = \sqrt{1-\frac{2(m_1^2+m_2^2)}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}}, \qquad \lambda^{1/2}\left(1;\frac{m_1^2}{s},0\right) = \frac{s-m_1^2}{s}.$$

Two-body phase space If $f(p_1^\mu,p_2^\mu)$ is Lorentz invariant, $f\equiv f(p_1^2,p_2^2,p_1^\mu p_{2\mu})\equiv f(p_1,p_2,\cos\theta_{12})$. Meanwhile,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \int \frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3}\frac{1}{2E_12E_2} = \int \frac{(4\pi)\,\mathrm{d}p_1\,p_1^2}{(2\pi)^3}\frac{(2\pi)\,\mathrm{d}p_2\,p_2^2\,\mathrm{d}\cos\theta_{12}}{(2\pi)^3}\frac{1}{2E_12E_2} = \int \frac{\mathrm{d}E_+\,\mathrm{d}E_-\,\mathrm{d}s}{128\pi^4}, \tag{1.5}$$

with the replacement of the variables

$$E_\pm = E_1\pm E_2, \qquad s=(p_1+p_2)^2 = m_1^2+m_2^2+2E_1E_2-2\|\boldsymbol{p}_1\|\|\boldsymbol{p}_2\|\cos\theta_{12};$$

$$\left|\frac{\mathrm{d}(E_+,E_-,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{4p_1^2p_2^2}{E_1E_2}, \qquad \left|\frac{\mathrm{d}(E_1,E_2,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{2p_1^2p_2^2}{E_1E_2}.$$

Therefore,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \frac{1}{128\pi^4}\int_{(m_1+m_2)^2}^{\infty}\mathrm{d}s\int_{\sqrt{s}}^{\infty}\mathrm{d}E_+\int_{\mathrm{min}}^{\mathrm{max}}\mathrm{d}E_-, \tag{1.6}$$

where the boundary of E_- is given by

$$\cos\theta_{12} = \frac{E_+^2-E_-^2+2\left(m_1^2+m_2^2-s\right)}{\sqrt{(E_++E_-)^2-4m_1^2}\sqrt{(E_+-E_-)^2-4m_2^2}} \in [-1,1]$$

$$\therefore \quad \left|E_- - \frac{m_1^2-m_2^2}{s}E_+\right| \le \sqrt{E_+^2-s}\cdot\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = 2p\sqrt{\frac{E_+^2-s}{s}}.$$

Two-body phase space with momentum conservation As a general representation in any frame,

$$\overline{\mathrm{d}\Pi^2} = \frac{\mathrm{d}p_1\,\mathrm{d}\Omega\,p_1^2}{16\pi^2}\frac{\delta(E_0-\sqrt{m_1^2+p_1^2}-\sqrt{m_2^2+\|\boldsymbol{P}_0-\boldsymbol{p}_1\|^2})}{E_1E_2} = \frac{1}{8\pi}\,\mathrm{d}\cos\theta_1\frac{p_1^2}{E_0p_1-P_0E_1\cos\theta_1}, \tag{1.7}$$

where the momentum p_1 is given by

$$p_1 = \frac{(E_0^2+m_1^2-m_2^2-P_0^2)P_0\cos\theta_1+E_0\sqrt{\lambda(E_0^2,m_1^2,m_2^2)+P_0^4-2P_0^2(E_0^2+m_1^2-2m_1^2\cos^2\theta_1-m_2^2)}}{2(E_0^2-P_0^2\cos^2\theta_1)}. \tag{1.8}$$

CM frame result is recovered by setting $E_0=\sqrt{s}$ and $P_0=0$.

1.2. Decay rate and Cross section

As $\langle\mathrm{out}|\mathrm{in}\rangle = (2\pi)^4\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f})\mathrm{i}\mathcal{M}$ (for in $\neq$ out) and $\langle\boldsymbol{p}|\boldsymbol{p}\rangle = 2E_{\boldsymbol{p}}(2\pi)^3\,\delta^{(3)}(\boldsymbol{0}) = 2E_{\boldsymbol{p}}V$ for one-particle state,

$$\frac{N_\mathrm{ev}}{\prod_\mathrm{in}N_\mathrm{particle}} = \int \mathrm{d}\Pi^\mathrm{out}\frac{|\langle\mathrm{out}|\mathrm{in}\rangle|^2}{\langle\mathrm{in}|\mathrm{in}\rangle} = \int \mathrm{d}\Pi^\mathrm{out}\frac{(2\pi)^8|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}\frac{VT}{(2\pi^4)}\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f}) = VT\int\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}. \tag{1.9}$$

Therefore, decay rate (at the rest frame) is given by

$$\mathrm{d}\Gamma := \frac{1}{T}\frac{\mathrm{d}N_\mathrm{ev}}{N_\mathrm{particle}} = \frac{1}{T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.10}$$

We also define Lorentz-invariant cross section σ by $N_\mathrm{ev} =: (\rho_Av_\mathrm{Møl}T\sigma)N_B = (\rho_Av_\mathrm{Møl}T\sigma)(\rho_BV)$, or

$$\mathrm{d}\sigma := \frac{\mathrm{d}N_\mathrm{ev}}{\rho_Av_\mathrm{Møl}TN_B} = \frac{V}{v_\mathrm{Møl}T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{2E_A2E_BV^2} = \frac{1}{2E_A2E_B\,v_\mathrm{Møl}}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.11}$$

where the Møller parameter $v_\mathrm{Møl}$ is equal to $v_\mathrm{rel}^\mathrm{NR} = \|\boldsymbol{v}_A-\boldsymbol{v}_B\|$ if $\boldsymbol{v}_A \mathbin{/\!/} \boldsymbol{v}_B$ (cf. Ref. [?]). Generally,

$$v_\mathrm{Møl} := \frac{\sqrt{(p_A\cdot p_B)^2-m_A^2m_B^2}}{E_AE_B} = \frac{p_A\cdot p_B}{E_AE_B}v_\mathrm{rel} = (1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B)v_\mathrm{rel}, \tag{1.12}$$

where v_rel is the actual relative velocity

$$v_\mathrm{rel} = \sqrt{1-\frac{(1-v_A^2)(1-v_B^2)}{1-(\boldsymbol{v}_A\cdot\boldsymbol{v}_B)^2}} = \frac{\sqrt{\|\boldsymbol{v}_A-\boldsymbol{v}_B\|^2-\|\boldsymbol{v}_A\times\boldsymbol{v}_B\|^2}}{1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B} = \frac{\lambda^{1/2}(s,m_A^2,m_B^2)}{s-(m_A^2+m_B^2)} \neq v_\mathrm{rel}^\mathrm{NR}. \tag{1.13}$$

(Note that $p_A\cdot p_B/E_AE_B = 1$ if $\boldsymbol{p}_A=0$ or $\boldsymbol{p}_B=0$. Also, Each of v_rel, VT, and $E_AE_Bv_\mathrm{Møl}$ is Lorentz invariant.)

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation adj. $= (\epsilon^a)^{bc}$.[*1]

$$T_a = \frac{1}{2}\sigma_a, \qquad \mathrm{Tr}(T_aT_b) = \frac{1}{2}\delta_{ab}, \qquad [T_a, T_b] = \mathrm{i}\epsilon^{abc}T^c, \qquad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\overline{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a\epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\overline{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \to \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp(-\mathrm{i}g\theta^\alpha T^{\alpha*})]^{ab}(\epsilon^{bc}\mathbf{2}^c). \tag{2.1}$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\overline{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation adj. $= \mathbf{8} = (f^a)^{bc}$.
Gell-Mann matrices:

$$\lambda_{1\text{–}8} = \begin{pmatrix} 0&1&0\\ 1&0&0\\ 0&0&0 \end{pmatrix}, \begin{pmatrix} 0&-\mathrm{i}&0\\ \mathrm{i}&0&0\\ 0&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0\\ 0&-1&0\\ 0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&1\\ 0&0&0\\ 1&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&-\mathrm{i}\\ 0&0&0\\ \mathrm{i}&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\ 0&0&1\\ 0&1&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\ 0&0&-\mathrm{i}\\ 0&\mathrm{i}&0 \end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix} 1&0&0\\ 0&1&0\\ 0&0&-2 \end{pmatrix}. \tag{2.2}$$

$$\tau_a = \frac{1}{2}\lambda_a, \qquad \mathrm{Tr}(\tau_a\tau_b) = \frac{1}{2}\delta_{ab}, \qquad [\tau_a, \tau_b] = \mathrm{i}f^{abc}\tau^c, \qquad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\begin{aligned} \mathbf{3}: \quad \phi_a &\to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi_b \simeq \phi_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi_b \\ \phi^*_a &\to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi^*_b \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi^*_b \\ &= \phi^*_b[\exp(-\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ba} \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\phi^*_b\tau^\alpha_{ba} \end{aligned}$$

$$\begin{aligned} \overline{\mathbf{3}}: \quad \phi_a &\to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi_b \\ \phi^*_a &\to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi^*_b \simeq \phi^*_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi^*_b \end{aligned}$$

[*1] We do not distinguish sub- and superscripts for gauge indices.

3. Spinors

$$(\overline{\psi_1}\psi_2)^* = (\psi_2)^\dagger(\overline{\psi}_1)^\dagger = \overline{\psi_2}\psi_1. \tag{3.1}$$

4. Supersymmetry with $\eta = \mathrm{diag}(+,-,-,-)$

Convention Our convention follows DHM (except for D_μ):

$$\eta = \mathrm{diag}(1,-1,-1,-1);\quad \epsilon^{0123} = -\epsilon_{0123} = 1,\quad \epsilon^{12} = \epsilon_{21} = \epsilon^{\dot 1\dot 2} = \epsilon_{\dot 2\dot 1} = 1 \quad \left(\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma\right),$$

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta,\quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta,\quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta},\quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta};$$

$$\sigma^\mu_{\alpha\dot\alpha} := (\mathbf{1},\boldsymbol{\sigma})_{\alpha\dot\alpha},\qquad \sigma^{\mu\nu}{}_\alpha{}^\beta := \frac{\mathrm{i}}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)_\alpha{}^\beta,$$ [*2] $$\left(\sigma^\mu_{\alpha\dot\beta} = \epsilon_{\alpha\delta}\epsilon_{\dot\beta\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta},\quad \bar\sigma^{\mu\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\delta}\epsilon^{\beta\gamma}\sigma^\mu_{\gamma\dot\delta}\right)$$

$$\bar\sigma^{\mu\dot\alpha\alpha} := (\mathbf{1},-\boldsymbol{\sigma})^{\dot\alpha\alpha},\quad \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta} := \frac{\mathrm{i}}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta},$$ [*2]

$$(\psi\xi) := \psi^\alpha\xi_\alpha,\quad (\bar\psi\bar\chi) := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha};\qquad \frac{\mathrm{d}}{\mathrm{d}\theta^\alpha}(\theta\theta) := \theta_\alpha \quad \text{[left derivative]}.$$

Especially, spinor-index contraction is done as ${}^\alpha{}_\alpha$ and ${}_{\dot\alpha}{}^{\dot\alpha}$ except for ϵ_{ab} (which always comes from left). Noting that complex conjugate reverses spinor order: $(\psi^\alpha\xi^\beta)^* := (\xi^\beta)^*(\psi^\alpha)^*$,

$$\bar\psi^{\dot\alpha} := (\psi^\alpha)^*,\quad \epsilon^{\dot a\dot b} := (\epsilon^{ab})^*,\qquad \left(\psi\chi\right)^* = (\bar\psi\bar\chi),$$

$$\left(\sigma^\mu_{\alpha\dot\beta}\right)^* = \bar\sigma^\mu{}_{\dot\alpha\beta} = \epsilon_{\beta\delta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta},\qquad \left(\sigma^{\mu\nu}\right)^{\dagger\alpha}{}_\beta = \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta},\qquad \left(\sigma^{\mu\nu}{}_\alpha{}^\beta\right)^* = \bar\sigma^{\mu\nu\dot\beta}{}_{\dot\alpha} = \bar\sigma^{\mu\nu}{}_{\dot\alpha}{}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\delta},$$

$$\left(\bar\sigma^{\mu\dot\alpha\beta}\right)^* = \sigma^{\mu\alpha\dot\beta} = \epsilon^{\dot\beta\dot\delta}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\delta},\qquad \left(\bar\sigma^{\mu\nu}\right)^\dagger{}_{\dot\alpha}{}^{\dot\beta} = \sigma^{\mu\nu}{}_\alpha{}^\beta,\qquad \left(\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\right)^* = \sigma^{\mu\nu}{}_\beta{}^\alpha = \bar\sigma^{\mu\nu\alpha}{}_\beta = \epsilon_{\beta\delta}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\delta.$$

Contraction formulae

$$\theta^\alpha\theta^\beta = -\tfrac12(\theta\theta)\epsilon^{\alpha\beta}\qquad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\epsilon^{\dot\alpha\dot\beta}\qquad (\theta\xi)(\theta\chi) = -\tfrac12(\theta\theta)(\xi\chi)\qquad (\theta\sigma^\nu\bar\theta)\theta^\alpha = \tfrac12(\theta\theta)(\bar\theta\bar\sigma^\nu)^\alpha$$

$$\theta_\alpha\theta_\beta = \tfrac12(\theta\theta)\epsilon_{\alpha\beta}\qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac12(\bar\theta\bar\theta)\epsilon_{\dot\alpha\dot\beta}\qquad (\bar\theta\bar\xi)(\bar\theta\bar\chi) = -\tfrac12(\bar\theta\bar\theta)(\bar\xi\bar\chi)\qquad (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\tfrac12(\theta\sigma^\nu)_{\dot\alpha}(\bar\theta\bar\theta)$$

$$\theta^\alpha\theta_\beta = \tfrac12(\theta\theta)\delta^\alpha_\beta\qquad \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\delta^{\dot\alpha}_{\dot\beta}\qquad (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \tfrac12(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu}$$

$$(\theta\sigma^\mu\bar\sigma^\nu\theta) = (\theta\theta)\eta^{\mu\nu}\qquad (\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta) = (\bar\theta\bar\theta)\eta^{\mu\nu}\qquad (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \tfrac12(\bar\theta\bar\theta)(\sigma^\mu\bar\sigma^\nu\theta)_\alpha$$

$$\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\sigma^{\mu\nu}\qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho + \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\left(\sigma^\mu\eta^{\rho\nu} - \sigma^\nu\eta^{\mu\rho} + \sigma^\rho\eta^{\mu\nu}\right)$$

$$\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\bar\sigma^{\mu\nu}\qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\mathrm{i}\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}\left(\sigma^\mu\bar\sigma^\nu\right) = \mathrm{Tr}\left(\bar\sigma^\mu\sigma^\nu\right) = 2\eta^{\mu\nu}\qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho + \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = 2\left(\bar\sigma^\mu\eta^{\rho\nu} - \bar\sigma^\nu\eta^{\mu\rho} + \bar\sigma^\rho\eta^{\mu\nu}\right)$$

$$\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu{}^{\dot\beta\beta} = 2\delta^{\dot\beta}_{\dot\alpha}\delta^\alpha_\beta\qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2\mathrm{i}\bar\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$

$$\sigma_{\mu\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\qquad \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\dot\beta\dot\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\beta}$$

$$\bar\sigma_\mu{}^{\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\qquad \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\beta\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\dot\alpha\dot\gamma}\sigma^\mu_{\beta\dot\gamma}$$

$$\mathrm{Tr}\left(\sigma^{\mu\nu}\right) = \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\right) = 0\qquad \mathrm{Tr}\left(\sigma^{\mu\nu}\sigma^{\rho\sigma}\right) = \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma}$$

$$\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu}\qquad \sigma^{\mu\nu} = -\sigma^{\nu\mu}\qquad \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma}\right) = \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma} + \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$

$$\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha}\qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = -2\mathrm{i}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon_{\alpha\beta} - 2\mathrm{i}\sigma^{\mu\nu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}$$

$$\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon^{\dot\beta\dot\alpha}\qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 4\sigma^{\rho\mu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\sigma\nu\dot\gamma}{}_{\dot\beta}\eta_{\rho\sigma} + \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\eta^{\mu\nu}$$

$$\bar\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = -2\mathrm{i}\bar\sigma^{\mu\nu}\qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} - \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = -2\mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\epsilon^{\alpha\beta} - 2\mathrm{i}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon^{\dot\alpha\dot\beta}$$

$$\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 2\mathrm{i}\sigma^{\mu\nu}\qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} + \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 4\epsilon^{\alpha\gamma}\sigma^{\sigma\nu}{}_\gamma{}^\beta\bar\sigma^{\rho\mu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\eta_{\rho\sigma} + \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\eta^{\mu\nu}$$

$$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi\qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi = \bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi\qquad \xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi\qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho\chi = -\chi\sigma^\rho\bar\sigma^\nu\sigma^\mu\bar\xi$$

$$\left(\xi\sigma^\mu\bar\chi\right)^* = \chi\sigma^\mu\bar\xi\qquad \left(\bar\xi\bar\sigma^\mu\chi\right)^* = \bar\chi\bar\sigma^\mu\xi\qquad \left(\bar\chi\bar\sigma^\mu\sigma^\nu\bar\xi\right)^* = \xi\sigma^\nu\bar\sigma^\mu\chi\qquad (\xi[\sigma\mathrm{s}]\chi)^* = \bar\chi[\sigma\mathrm{s}_{\mathrm{reversed}}]\bar\xi$$

$$(\xi\chi)\psi^\alpha = -(\psi\xi)\chi^\alpha - (\psi\chi)\xi^\alpha\qquad (\xi\chi)\bar\psi_{\dot\alpha} = \tfrac12(\xi\sigma^\mu\bar\psi)(\chi\sigma_\mu)_{\dot\alpha}$$

Superfields

[*2] As the definition of $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are not unified in literature, they are not used in this CheatSheet except for this page.

4.1. Lorentz symmetry as SU(2)×SU(2)

4.2. Supersymmetry algebra

We define the generators as

$$P_\mu := \mathrm{i}\partial_\mu, \quad \{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \quad \{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{Q}}_{\dot\beta}\} = 0, \tag{4.1}$$

which is realized by

$$\mathcal{Q}_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{Q}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{Q}^\alpha = -\frac{\partial}{\partial\theta_\alpha} - \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{Q}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu,$$

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{D}^\alpha = -\frac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{D}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu;$$

$\mathcal{D}_\alpha$ etc. works as covariant derivatives because of the commutation relations

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = +2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \{\mathcal{Q}_\alpha, \mathcal{D}_\beta\} = \{\mathcal{Q}_\alpha, \bar{\mathcal{D}}_{\dot\beta}\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \mathcal{D}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 0.$$

Derivative formulae

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = -2\delta^\beta_\alpha \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = 2\delta^{\dot\beta}_{\dot\alpha}$$

$$\epsilon_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} = -\frac{\partial}{\partial\theta^\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = 2\epsilon^{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = -2\epsilon^{\dot\alpha\dot\beta}$$

$$\epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\beta}} = -\frac{\partial}{\partial\bar\theta_{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = 2\delta^\alpha_\beta \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = -2\delta^{\dot\alpha}_{\dot\beta}$$

$$\epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\beta}} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = -2\epsilon_{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = 2\epsilon_{\dot\alpha\dot\beta}$$

In addition, we define

$$(y, \theta', \bar\theta') := (x - \mathrm{i}\theta\sigma^\mu\bar\theta, \theta, \bar\theta) : \tag{4.2}$$

$$\bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta'^{\dot\alpha}}; \qquad \begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix} = \begin{pmatrix} \delta^\nu_\mu & 0 & 0 \\ -\mathrm{i}(\sigma^\nu\bar\theta)_\alpha & \delta^\beta_\alpha & 0 \\ \mathrm{i}(\theta\sigma^\nu)_{\dot\alpha} & 0 & \delta^{\dot\beta}_{\dot\alpha} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\beta} \\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}} \end{pmatrix}, \qquad \begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\beta} \\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}} \end{pmatrix} = \begin{pmatrix} \delta^\mu_\nu & 0 & 0 \\ \mathrm{i}(\sigma^\mu\bar\theta)_\beta & \delta^\alpha_\beta & 0 \\ -\mathrm{i}(\theta\sigma^\mu)_{\dot\beta} & 0 & \delta^{\dot\alpha}_{\dot\beta} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix}, \tag{4.3}$$

and a function $f : \mathbb{C}^4 \to \mathbb{C}$ (independent of θ' and $\bar\theta'$) is expanded as

$$f(y) = f(x - \mathrm{i}\theta\sigma\bar\theta) = f(x) - \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f(x) - \frac{1}{4}\theta^4\partial^2 f(x). \tag{4.4}$$

Note that we differentiate $[f(y)]^*$ and $f^*(y)$:

$$[f(y)]^* = f(x) + \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f^*(x) - \frac{1}{4}\theta^4\partial^2 f^*(x) = f^*(y + \mathrm{i}\theta\sigma\bar\theta) = f^*(y^*). \tag{4.5}$$

4.3. Superfields

SUSY-invariant Lagrangian SUSY transformation is induced by $\xi\mathcal{Q} + \bar\xi\bar{\mathcal{Q}} = \xi^\alpha\partial_\alpha + \bar\xi_{\dot\alpha}\partial^{\dot\alpha} + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu$. Therefore, for an object Ψ in the superspace,

$$\bigl[\Psi\bigr]_{\theta^4} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \xi^\alpha\partial_\alpha\Psi + \bar\xi_{\dot\alpha}\partial^{\dot\alpha}\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\Bigr]_{\theta^4} = \bigl[\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\bigr]_{\theta^4}, \tag{4.6}$$

which means $\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant up to total derivative, i.e., $\int \mathrm{d}^4x\bigl[\Psi\bigr]_{\theta^4}$ is SUSY-invariant action. Also,

$$\bigl[\Psi\bigr]_{\theta^2} \xrightarrow{\text{SUSY}} \Bigl[\Psi + \bar\xi_{\dot\alpha}\Bigl(\partial^{\dot\alpha} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Bigr)\Psi\Bigr]_{\theta^2} = \Bigl[\Psi + \bar\xi_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\Psi + 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Psi\Bigr]_{\theta^2} \tag{4.7}$$

will be SUSY-invariant if $\bar{\mathcal{D}}_{\dot\alpha}\Psi = 0$, i.e., Ψ is a chiral superfield. Therefore, SUSY-invariant Lagrangian is given by

$$\mathcal{L} = \Bigl[(\text{any real superfield})\Bigr]_{\theta^4} + \Bigl[(\text{any chiral superfield})\Bigr]_{\theta^2} + \Bigl[(\text{any chiral superfield})^*\Bigr]_{\bar\theta^2}. \tag{4.8}$$

Chiral superfield A chiral superfield is a superfield that satisfies $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$, i.e., we find

$$\Phi = \phi(y) + \sqrt{2}\theta'\psi(y) + \theta'^2 F(y) \tag{4.9}$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{1}{4}\partial^2\phi(x)\theta^4 \tag{4.10}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{1}{4}\partial^2\phi^*(x)\theta^4; \tag{4.11}$$

their product is expanded as

$$\begin{aligned}\Phi_i^*\Phi_j &= \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - \mathrm{i}\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2\\
&\quad + \left[\sqrt{2}\bar\psi_i\bar\theta F_j - \frac{\mathrm{i}\left(\partial_\mu\phi_i^*\cdot\psi_j\sigma^\mu\bar\theta - \phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)}{\sqrt{2}}\right]\theta^2 + \left[\sqrt{2}F_i^*\theta\psi_j + \frac{\mathrm{i}\left(\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j - \theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)}{\sqrt{2}}\right]\bar\theta^2\\
&\quad + \frac{1}{4}\left(4F_i^*F_j - \phi_i^*\partial^2\phi_j - (\partial^2\phi_i^*)\phi_j + 2(\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + 2\mathrm{i}(\psi_j\sigma^\mu\partial_\mu\bar\psi_i) - 2\mathrm{i}(\partial_\mu\psi_j\sigma^\mu\bar\psi_i)\right)\theta^4
\end{aligned}\tag{4.12}$$

$$\begin{aligned}&\equiv \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - 2\mathrm{i}\left(\phi_i^*\partial_\mu\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2\\
&\quad + \sqrt{2}\left(\bar\psi_i\bar\theta F_j + \mathrm{i}\phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)\theta^2 + \sqrt{2}\left(F_i^*\theta\psi_j - \mathrm{i}\theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)\bar\theta^2\\
&\quad + \left(F_i^*F_j + (\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + \mathrm{i}\bar\psi_i\sigma^\mu\partial_\mu\psi_j\right)\theta^4
\end{aligned}\tag{4.13}$$

$$\Phi_i\Phi_j\Big|_{\theta^2} = -\psi_i\psi_j + F_i\phi_j + \phi_iF_j \tag{4.14}$$

$$\Phi_i\Phi_j\Phi_k\Big|_{\theta^2} = -(\psi_i\psi_j)\phi_k - (\psi_k\psi_i)\phi_j - (\psi_j\psi_k)\phi_i + \phi_i\phi_jF_k + \phi_k\phi_iF_j + \phi_j\phi_kF_i \tag{4.15}$$

$$\mathrm{e}^{k\Phi} = \mathrm{e}^{k\phi}\left[1 + \sqrt{2}k\theta\psi + \left(kF - \frac{k^2}{2}\psi\psi\right)\theta^2 - \mathrm{i}k\partial_\mu\phi(\theta\sigma^\mu\bar\theta) + \frac{\mathrm{i}k\left(\partial_\mu\psi + k\psi\partial_\mu\phi\right)\sigma^\mu\bar\theta\theta^2}{\sqrt{2}} - \frac{k}{4}\left(\partial^2\phi + k\partial_\mu\phi\partial^\mu\phi\right)\theta^4\right]; \tag{4.16}$$

note that $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$, and $\mathrm{e}^{k\Phi}$ are all chiral superfields.

Vector superfield A vector superfield is a superfield V that satisfies $V = V^*$. It is given by real fields $\{C, M, N, D, A_\mu\}$ and Grassmann fields $\{\chi, \lambda\}$ as[*3]

$$\begin{aligned}V(x,\theta,\bar\theta) &= C(x) + \mathrm{i}\theta\chi(x) - \mathrm{i}\bar\theta\bar\chi(x) + \frac{1}{2}\left(M(x) + \mathrm{i}N(x)\right)\theta^2 + \frac{1}{2}\left(M(x) - \mathrm{i}N(x)\right)\bar\theta^2 + (\bar\theta\bar\sigma^\mu\theta)A_\mu(x)\\
&\quad \left(\lambda(x) + \frac{1}{2}\partial_\mu\bar\chi(x)\bar\sigma^\mu\right)\theta\bar\theta^2 + \theta^2\bar\theta\left(\bar\lambda(x) + \frac{1}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right) + \frac{1}{2}\left(D(x) - \frac{1}{2}\partial^2C(x)\right)\theta^4.
\end{aligned}\tag{4.17}$$

With this convention,

$$V \to V - \mathrm{i}\Phi + \mathrm{i}\Phi^* \iff \begin{cases} C \to C - \mathrm{i}\phi + \mathrm{i}\phi^*, & \chi \to \chi - \sqrt{2}\psi, & \lambda \to \lambda,\\ M + \mathrm{i}N \to M + \mathrm{i}N - 2\mathrm{i}F, & A_\mu \to A_\mu + \partial_\mu(\phi + \phi^*), & D \to D.\end{cases} \tag{4.18}$$

The exponential of a vector superfield is also a vector superfield:

$$\begin{aligned}\mathrm{e}^{kV} = \mathrm{e}^{kC}\Bigg\{&1 + \mathrm{i}k(\theta\chi - \bar\theta\bar\chi) + \left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\theta^2 + \left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\bar\theta^2 + (k^2\theta\chi\bar\theta\bar\chi - k\theta\sigma^\mu\bar\theta A_\mu)\\
&+ \left[k\bar\theta\bar\lambda - \mathrm{i}k\bar\theta\bar\chi\left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right) + \frac{1}{2}k\bar\theta\bar\sigma^\mu\left(\partial_\mu\chi - \mathrm{i}k\chi A_\mu\right)\right]\theta^2\\
&+ \left[k\theta\lambda + \mathrm{i}k\theta\chi\left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right) - \frac{1}{2}k\theta\sigma^\mu\left(\partial_\mu\bar\chi + \mathrm{i}k\bar\chi A_\mu\right)\right]\bar\theta^2\\
&+ \left[\frac{k}{2}\left(D - \frac{1}{2}\partial^2C\right) - \frac{1}{2}\mathrm{i}k^2(\lambda\chi - \bar\lambda\bar\chi) + \left(\frac{M+\mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\left(\frac{M-\mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\right.\\
&\quad\left. + \frac{k^3}{4}\bar\chi\bar\sigma^\mu\chi A_\mu + \frac{k^2}{4}\left(\mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - \mathrm{i}\partial_\mu\bar\chi\bar\sigma^\mu\chi + A^\mu A_\mu\right)\right]\theta^4\Bigg\}.
\end{aligned}\tag{4.19}$$

Supergauge symmetry The gauge transformation $\phi(x) \to \mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\phi(x)$ is not closed in the chiral superfield; i.e., $\mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\Phi(x)$ is not a chiral superfield if the parameter $\theta(x)$ has x^μ-dependence. Hence, in supersymmetric theories, it is extended to *supergauge symmetry* parameterized by a chiral superfield $\Omega(x)$, which is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Omega^a(x)t^a}\Phi, \qquad \Phi^* \to \Phi^*\,\mathrm{e}^{-2\mathrm{i}g\Omega^{*a}(x)t^a} \tag{4.20}$$

for a chiral superfield Φ and an anti-chiral superfield Φ^*. The supergauge-invariant Lagrangian should be

$$\mathcal{L} \sim \Phi^*\cdot(\text{real superfield})\cdot\Phi; \tag{4.21}$$

we parameterize the "real superfield" as $\mathrm{e}^{2gV^a(x)t^a}$:

$$\mathcal{L} = \left[\Phi^*\,\mathrm{e}^{2gV^a(x)t^a}\Phi\right]_{\theta^4}; \qquad \mathrm{e}^{2gV^a(x)t^a} \to \mathrm{e}^{2\mathrm{i}g\Omega^{*a}(x)t^a}\,\mathrm{e}^{2gV^a(x)t^a}\,\mathrm{e}^{-2\mathrm{i}g\Omega^a(x)t^a}. \tag{4.22}$$

[*3]Different coordination of "i"s are found in literature. Take care, especially, λ(ours) = iλ(Wess-Bagger) = iλ(SLHA).

In Abelian case, t^a is replaced by the charge Q of Φ and

$$\mathcal{L} = \left[\Phi^* \, \mathrm{e}^{2gQV(x)}\Phi\right]_{\theta^4}; \qquad \Phi \to \mathrm{e}^{2\mathrm{i}gQ\Omega(x)}\Phi, \quad \Phi^* \to \Phi^* \, \mathrm{e}^{-2\mathrm{i}gQ\Omega^*(x)}, \tag{4.23}$$

$$\mathrm{e}^{2gQV(x)} \to \mathrm{e}^{2\mathrm{i}gQ\Omega^*(x)} \, \mathrm{e}^{2gQV(x)} \, \mathrm{e}^{-2\mathrm{i}gQ\Omega(x)} = \mathrm{e}^{2gQ\left(V - \mathrm{i}\Omega + \mathrm{i}\Omega^*\right)}. \tag{4.24}$$

The usual gauge transformation corresponds to the real part of the lowest component of Ω, i.e., $\theta \equiv 2\,\mathrm{Re}\,\phi = \phi + \phi^*$, and we use the other components to fix the supergauge so that C, M, N and χ are eliminated:

$$\text{supergauge fixing:} \quad V(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{1}{2}D(x) \qquad \text{(Wess-Zumino gauge);} \tag{4.25}$$

$$\mathrm{e}^{2gQV} \longrightarrow 1 + gQ\left(-2\theta\sigma^\mu\bar\theta A_\mu + 2\theta^2\bar\theta\bar\lambda + 2\bar\theta^2\theta\lambda + D\theta^4\right) + g^2Q^2A^\mu A_\mu\theta^4. \tag{4.26}$$

The gauge transformation is the remnant freedom: $\Theta = \phi(y) = \phi - \mathrm{i}\partial_\mu\phi(\theta\sigma^\mu\bar\theta) - \partial^2\phi\theta^4/4$ with ϕ being real;

$$\Phi_i \to \mathrm{e}^{2\mathrm{i}gQ\Theta}\Phi_i, \qquad \mathrm{e}^{2gQV} \to \mathrm{e}^{2gQ(V - \mathrm{i}\Theta + \mathrm{i}\Theta^*)}. \tag{4.27}$$

Rules for each component is obvious in $(y, \theta, \bar\theta)$-basis and given by

$$\{\phi, \psi, F\} \to \mathrm{e}^{\mathrm{i}gQ\theta}\{\phi, \psi, F\}, \qquad A_\mu \to A_\mu + \partial_\mu\theta, \qquad \lambda \to \lambda, \qquad D \to D. \tag{4.28}$$

For non-Abelian gauges, the supergauge transformation for the real field is evaluated as

$$\mathrm{e}^{2gV} \to \mathrm{e}^{2\mathrm{i}g\Omega^*} \, \mathrm{e}^{2gV} \, \mathrm{e}^{-2\mathrm{i}g\Omega} \tag{4.29}$$

$$= \left(\mathrm{e}^{2\mathrm{i}g\Omega^*} \, \mathrm{e}^{2gV} \, \mathrm{e}^{-2\mathrm{i}g\Omega^*}\right) \left(\mathrm{e}^{2\mathrm{i}g\Omega^*} \, \mathrm{e}^{-2\mathrm{i}g\Omega}\right) \tag{4.30}$$

$$= \exp\left(\mathrm{e}^{[2\mathrm{i}g\Omega^*,}2gV\right) \mathrm{e}^{2\mathrm{i}g(\Omega^* - \Omega)} + \mathcal{O}(\Omega^2) \tag{4.31}$$

$$= \exp\left(2gV + [2\mathrm{i}g\Omega^*, 2gV]\right) \mathrm{e}^{2\mathrm{i}g(\Omega^* - \Omega)} + \mathcal{O}(\Omega^2); \tag{4.32}$$

$$= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[2gV,})2\mathrm{i}g(\Omega^* - \Omega)] + \mathcal{O}(\Omega^2)\right] \tag{4.33}$$

$$= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \sum_{n=0}^{\infty} \frac{B_n\left([2gV,)^n\right.}{n!} 2\mathrm{i}g(\Omega^* - \Omega)]\right] + \mathcal{O}(\Omega^2) \tag{4.34}$$

$$= \exp\left[2g\left(V + \mathrm{i}(\Omega^* - \Omega) - [V, \mathrm{i}g(\Omega^* + \Omega)] + \sum_{n=2}^{\infty} \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Omega^* - \Omega)]\right) + \mathcal{O}(\Omega^2)\right]. \tag{4.35}$$

Here, again we can use the "non-gauge" component of Ω to eliminate the C-term etc., i.e., we fix $\mathrm{i}(\Omega^* - \Omega)$, the second term of the expansion, to remove those terms:

$$V - [V, \mathrm{i}g(\Omega^* + \Omega)] + \left(\mathrm{i} + \sum_{n=2}^{\infty} \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}\right)(\Omega^* - \Omega)] + \mathcal{O}(\Omega^2) = (\bar\theta\bar\sigma^\mu\theta)A_\mu + \bar\theta^2\theta\lambda + \theta^2\bar\theta\bar\lambda + \frac{1}{2}D; \tag{4.36}$$

this defines the Wess-Zumino gauge:

$$\text{supergauge fixing:} \quad V^a(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A^a_\mu(x) + \bar\theta^2\theta\lambda^a(x) + \theta^2\bar\theta\bar\lambda^a(x) + \frac{1}{2}D^a(x), \tag{4.37}$$

$$\mathrm{e}^{2gV^at^a} \longrightarrow 1 + g\left(-2\theta\sigma^\mu\bar\theta A^a_\mu + 2\theta^2\bar\theta\bar\lambda^a + 2\bar\theta^2\theta\lambda^a + D^a\theta^4\right)t^a + g^2Q^2A^{a\mu}A^b_\mu\theta^4t^at^b. \tag{4.38}$$

The gauge transformation is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Theta^at^a}\Phi, \quad \mathrm{e}^{2gV^at^a} \to \mathrm{e}^{2\mathrm{i}g\Theta^bt^b} \, \mathrm{e}^{2gV^at^a} \, \mathrm{e}^{-2\mathrm{i}g\Theta^ct^c}. \tag{4.39}$$

For components in chiral superfields,

$$\{\phi, \psi, F\} \to \mathrm{e}^{\mathrm{i}g\theta^at^a}\{\phi, \psi, F\}, \tag{4.40}$$

while for vector superfield we can express as infinitesimal transformation:

$$V \to V' \simeq V + \mathrm{i}(\Theta^* - \Theta) - [V, \mathrm{i}g(\Theta^* + \Theta)] + \sum_{n=2}^{\infty} \frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Theta^* - \Theta)] \tag{4.41}$$

$$= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi - \left[V, \mathrm{i}g\left(2\phi - \frac{\theta^4}{2}\partial^2\phi\right)\right] + 2\sum_{n=2}^{\infty} \frac{B_n\left([2gV,)^n\right.}{n!}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi] \tag{4.42}$$

$$= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi + 2gf^{abc}V^b\phi^ct^a \qquad \text{(Wess-Zumino gauge)} \tag{4.43}$$

$$\begin{aligned} \therefore A^a_\mu &\to A^a_\mu + \partial_\mu\theta^a + gf^{abc}A^b_\mu\theta^c + \mathcal{O}(\theta^2), & \lambda^a &\to \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2), \\ D^a &\to D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), & \bar\lambda^a &\to \bar\lambda^a + gf^{abc}\bar\lambda^b\theta^c + \mathcal{O}(\theta^2). \end{aligned} \tag{4.44}$$

Gauge-field strength The real superfield e^V is gauge-invariant in Abelian case and a candidate in Lagrangian term, but this is not case in non-Abelian case. We thus define a chiral superfield from e^V:

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right); \qquad \mathcal{W}_\alpha \xrightarrow{\text{gauge}} \mathrm{e}^{2\mathrm{i}g\Omega}\mathcal{W}_\alpha\,\mathrm{e}^{-2\mathrm{i}g\Omega}; \tag{4.45}$$

it is not supergauge- or Lorentz-invariatn, but $\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha) = \mathrm{Tr}(\epsilon^{\alpha\beta}\mathcal{W}_\beta\mathcal{W}_\alpha)$ is supergauge- and Lorentz-invariant, and its θ^2-term is SUSY-invariant, which becomes a candidate in SUSY Lagrangian with its Hermitian conjugate.

In Wess-Zumino gauge, it is given by

$$\mathcal{W}_\alpha = \left\{\lambda^a_\alpha(y) + \theta_\alpha D^a(y) + \frac{[\mathrm{i}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)\theta]_\alpha}{4}F^a_{\mu\nu}(y) + \theta^2\left[\mathrm{i}\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a(y^*)\right]_\alpha\right\}T^a \tag{4.46}$$

$$\begin{aligned}&= \Bigg[\lambda^a_\alpha + \theta_\alpha D^a + \frac{\mathrm{i}}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F^a_{\mu\nu} + \mathrm{i}\theta^2(\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a)_\alpha + \mathrm{i}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\lambda^a_\alpha - \frac{\theta^4}{4}\partial^2\lambda^a_\alpha \\ &\qquad + \frac{\mathrm{i}\theta^2(\sigma^\mu\bar\theta)_\alpha}{2}\left(\partial_\mu D^a + \mathrm{i}\partial^\nu F^a_{\mu\nu} - gf^{abc}\epsilon_{\mu\nu\rho\sigma}A^{\nu b}\partial^\rho A^{\sigma c}\right)\Bigg]T^a,\end{aligned} \tag{4.47}$$

where, as usual,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gA^b_\mu A^c_\nu f^{abc}, \qquad \mathrm{D}_\mu\lambda^a_\alpha = \partial_\mu\lambda^a_\alpha + gf^{abc}A^b_\mu\lambda^c_\alpha. \tag{4.48}$$

Also,

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^2} = \left[\mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^b + \mathrm{i}\lambda^b\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + D^aD^b - \frac{1}{4}\left(\mathrm{i}\epsilon^{\sigma\mu\nu\rho} + 2\eta^{\mu\rho}\eta^{\nu\sigma}\right)F^a_{\mu\nu}F^b_{\rho\sigma}\right]\mathrm{Tr}(T^aT^b) \tag{4.49}$$

$$= \mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + \frac{1}{2}D^aD^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}. \tag{4.50}$$

For Abelian theory,

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right) = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}_\alpha(2gV), \tag{4.51}$$

$$\frac{1}{2}\left[\mathcal{W}^\alpha\mathcal{W}_\alpha\right]_{\theta^2} = \mathrm{i}\lambda\sigma^\mu\,\mathrm{D}_\mu\bar\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}. \tag{4.52}$$

5. Minimal Supersymmetric Standard Model

Gauge symmetry: $\mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_{\mathrm{weak}} \times \mathrm{U}(1)_Y$

Particle content:

(a) Chiral superfields

	SU(3)	SU(2)	U(1)	B	L	scalar/spinor
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$		$\tilde{q}_{\mathrm{L}}, q_{\mathrm{L}}$ $[\to (u_{\mathrm{L}}, d_{\mathrm{L}})]$
L_i		$\mathbf{2}$	$-1/2$		1	$\tilde{l}_{\mathrm{L}}, l_{\mathrm{L}}$ $[\to (\nu_{\mathrm{L}}, l_{\mathrm{L}})]$
U_i^{c}	$\overline{\mathbf{3}}$		$-2/3$	$-1/3$		$\tilde{u}_{\mathrm{R}}^{\mathrm{c}}, u_{\mathrm{R}}^{\mathrm{c}}$
D_i^{c}	$\overline{\mathbf{3}}$		$1/3$	$-1/3$		$\tilde{d}_{\mathrm{R}}^{\mathrm{c}}, d_{\mathrm{R}}^{\mathrm{c}}$
E_i^{c}			1		-1	$\tilde{e}_{\mathrm{R}}^{\mathrm{c}}, e_{\mathrm{R}}^{\mathrm{c}}$
H_{u}		$\mathbf{2}$	$1/2$			$h_{\mathrm{u}}, \tilde{h}_{\mathrm{u}}$ $[\to (h_{\mathrm{u}}^+, h_{\mathrm{u}}^0)]$
H_{d}		$\mathbf{2}$	$-1/2$			$h_{\mathrm{d}}, \tilde{h}_{\mathrm{d}}$ $[\to (h_{\mathrm{d}}^0, h_{\mathrm{d}}^-)]$

(b) Vector superfields

	SU(3)	SU(2)	U(1)	ino/boson
g	adj.			$\tilde{g}, g_\mu$
W		adj.		$\tilde{w}, W_\mu$
B				$\tilde{b}, B_\mu$

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and SU(2) decomposition).

"c"-notation For scalars, $\tilde{\phi}_{\mathrm{R}}^{\mathrm{c}} := \phi_{\mathrm{R}}^* = \mathsf{C}\phi_{\mathrm{R}}\mathsf{C}$ (because the intrinsic phase for C is $+1$ for quarks and leptons.)

For matter spinors, $\psi_{\mathrm{R}}^{\mathrm{c}} := \bar{\psi}_{\mathrm{R}}$ (and $\psi_{\mathrm{R}} = \bar{\psi}_{\mathrm{R}}^{\mathrm{c}}$); Dirac spinors are thus

$$\psi_{\mathrm{L}} = \begin{pmatrix}\psi_{\mathrm{L}}\\0\end{pmatrix}, \quad \overline{\psi_{\mathrm{L}}} = \begin{pmatrix}0 & \bar{\psi}_{\mathrm{L}}\end{pmatrix}, \quad \psi_{\mathrm{R}}^{\mathrm{c}} := \begin{pmatrix}\psi_{\mathrm{R}}^{\mathrm{c}}\\0\end{pmatrix} = C\begin{pmatrix}0\\\psi_{\mathrm{R}}\end{pmatrix} = C\psi_{\mathrm{R}}, \quad \overline{\psi_{\mathrm{R}}^{\mathrm{c}}} = \begin{pmatrix}0 & \psi_{\mathrm{R}}\end{pmatrix} = \begin{pmatrix}\bar{\psi}_{\mathrm{R}} & 0\end{pmatrix}C = \overline{\psi_{\mathrm{R}}}C.$$

Superpotential and $\cancel{\mathrm{SUSY}}$-terms

$$W_{\mathrm{RPC}} = \mu H_{\mathrm{u}}H_{\mathrm{d}} - y_{\mathrm{u}ij}U_i^{\mathrm{c}}H_{\mathrm{u}}Q_j + y_{\mathrm{d}ij}D_i^{\mathrm{c}}H_{\mathrm{d}}Q_j + y_{\mathrm{e}ij}E_i^{\mathrm{c}}H_{\mathrm{d}}L_j, \tag{5.1}$$

$$W_{\mathrm{RPV}} = -\kappa_i L_i H_{\mathrm{u}} + \frac{1}{2}\lambda_{ijk}L_iL_jE_k^{\mathrm{c}} + \lambda'_{ijk}L_iQ_jD_k^{\mathrm{c}} + \frac{1}{2}\lambda''_{ijk}U_i^{\mathrm{c}}D_j^{\mathrm{c}}D_k^{\mathrm{c}}, \tag{5.2}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\Big(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \mathrm{H.c.}\Big) - V_{\cancel{\mathrm{SUSY}}}; \tag{5.3}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^{\mathrm{RPC}} = &\Big(\tilde{q}_{\mathrm{L}}^* m_Q^2 \tilde{q}_{\mathrm{L}} + \tilde{l}_{\mathrm{L}}^* m_L^2 \tilde{l}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^* m_{U^{\mathrm{c}}}^2 \tilde{u}_{\mathrm{R}} + \tilde{d}_{\mathrm{R}}^* m_{D^{\mathrm{c}}}^2 \tilde{d}_{\mathrm{R}} + \tilde{e}_{\mathrm{R}}^* m_{E^{\mathrm{c}}}^2 \tilde{e}_{\mathrm{R}} + m_{H_{\mathrm{u}}}^2|h_{\mathrm{u}}|^2 + m_{H_{\mathrm{d}}}^2|h_{\mathrm{d}}|^2\Big) \\ &+ \Big(-\tilde{u}_{\mathrm{R}}^* h_{\mathrm{u}} a_{\mathrm{u}} \tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* h_{\mathrm{d}} a_{\mathrm{d}} \tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^* h_{\mathrm{d}} a_{\mathrm{e}} \tilde{l}_{\mathrm{L}} + bH_{\mathrm{u}}H_{\mathrm{d}} + \mathrm{H.c.}\Big) \\ &+ \Big(-\tilde{u}_{\mathrm{R}}^* h_{\mathrm{d}}^* c_{\mathrm{u}} \tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* h_{\mathrm{u}}^* c_{\mathrm{d}} \tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^* h_{\mathrm{u}}^* c_{\mathrm{e}} \tilde{l}_{\mathrm{L}} + \mathrm{H.c.}\Big), \end{aligned} \tag{5.4}$$

$$\begin{aligned} V_{\cancel{\mathrm{SUSY}}}^{\mathrm{RPV}} = &\Big(-b_i\tilde{l}_{\mathrm{L}i}H_{\mathrm{u}} + \frac{1}{2}T_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{l}_{\mathrm{L}j}\tilde{e}_{\mathrm{R}k}^* + T'_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{q}_{\mathrm{L}j}\tilde{d}_{\mathrm{R}k}^* + \frac{1}{2}T''_{ijk}\tilde{u}_{\mathrm{R}i}^*\tilde{d}_{\mathrm{R}j}^*\tilde{d}_{\mathrm{R}k}^* + \tilde{l}_{\mathrm{L}i}^* M_{Li}^2 H_{\mathrm{d}} + \mathrm{H.c.}\Big) \\ &+ \Big(C_{ijk}^1\tilde{l}_{\mathrm{L}i}^*\tilde{q}_{\mathrm{L}j}\tilde{u}_{\mathrm{R}k}^* + C_i^2 h_{\mathrm{u}}^* h_{\mathrm{d}}\tilde{e}_{\mathrm{R}i}^* + C_{ijk}^3\tilde{d}_{\mathrm{R}i}\tilde{u}_{\mathrm{R}j}^*\tilde{e}_{\mathrm{R}k}^* + \frac{1}{2}C_{ijk}^4\tilde{d}_{\mathrm{R}i}\tilde{q}_{\mathrm{L}j}\tilde{q}_{\mathrm{L}k} + \mathrm{H.c.}\Big), \end{aligned} \tag{5.5}$$

($\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C_{ijk}^4 = C_{ikj}^4$.)

We follow the notation of DHM [**?**, PhysRept] and Martin [**?**, v7] (but note that Martin uses $(-,+,+,+)$-metric) for RPC part and SLHA2 convention for RPV part.

5.1. Scalar potential

The MSSM scalar potential has contributions from F-terms and D-terms:

$$V_{\mathrm{SUSY}} = F_i^* F_i + \frac{1}{2} D^a D^a; \qquad F_i = -W_i^* = -\frac{\delta W^*}{\delta \phi_i^*}, \qquad D^a = -g(\phi^* T^a \phi), \tag{5.6}$$

where T_a corresponds to the gauge-symmetry generator relevant for each ϕ. They are given by

$$-F^*_{h_{\mathrm{u}}^a} = \epsilon^{ab}\left(-\tilde{u}_{\mathrm{R}}^{x*} y_{\mathrm{u}} \tilde{q}_{\mathrm{L}}^{bx} + \mu h_{\mathrm{d}}^b + \mu_i' \tilde{l}_{\mathrm{L}i}^b\right), \tag{5.7}$$

$$-F^*_{h_{\mathrm{d}}^a} = \epsilon^{ab}\left(\tilde{e}_{\mathrm{R}}^* y_{\mathrm{e}} \tilde{l}_{\mathrm{L}}^b + \tilde{d}_{\mathrm{R}}^{x*} y_{\mathrm{d}} \tilde{q}_{\mathrm{L}}^{bx} - \mu h_{\mathrm{u}}^b\right), \tag{5.8}$$

$$-F^*_{\tilde{q}_{\mathrm{L}i}^{ax}} = \epsilon^{ab}\left(-y_{\mathrm{d}ji} h_{\mathrm{d}}^b \tilde{d}_{\mathrm{R}j}^{x*} + y_{\mathrm{u}ji} h_{\mathrm{u}}^b \tilde{u}_{\mathrm{R}j}^{x*} - \lambda'_{kij} \tilde{d}_{\mathrm{R}j}^{x*} \tilde{l}_{\mathrm{L}k}^b\right), \tag{5.9}$$

$$-F^*_{\tilde{u}_{\mathrm{R}i}^{x*}} = -y_{\mathrm{u}ij} h_{\mathrm{u}} \tilde{q}_{\mathrm{L}j}^x + \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk} \tilde{d}_{\mathrm{R}j}^{y*} \tilde{d}_{\mathrm{R}k}^{z*}, \tag{5.10}$$

$$-F^*_{\tilde{d}_{\mathrm{R}i}^{x*}} = y_{\mathrm{d}ij} h_{\mathrm{d}} \tilde{q}_{\mathrm{L}j}^x + \lambda'_{jki} \tilde{l}_{\mathrm{L}j} \tilde{q}_{\mathrm{L}k}^x - \lambda''_{jik}\epsilon^{xyz} \tilde{u}_{\mathrm{R}j}^{y*} \tilde{d}_{\mathrm{R}k}^{z*}, \tag{5.11}$$

$$-F^*_{\tilde{l}_{\mathrm{L}i}^a} = \epsilon^{ab}\left(-y_{\mathrm{e}ji} \tilde{e}_{\mathrm{R}j}^* h_{\mathrm{d}}^b - \mu_i' h_{\mathrm{u}}^b + \lambda_{ijk} \tilde{l}_{\mathrm{L}j}^b \tilde{e}_{\mathrm{R}k}^* + \lambda'_{ijk} \tilde{q}_{\mathrm{L}j}^{bx} \tilde{d}_{\mathrm{R}k}^{x*}\right), \tag{5.12}$$

$$-F^*_{\tilde{e}_{\mathrm{R}i}^*} = y_{\mathrm{e}ij} h_{\mathrm{d}} \tilde{l}_{\mathrm{L}j} + \frac{1}{2}\lambda_{jki} \tilde{l}_{\mathrm{L}j} \tilde{l}_{\mathrm{L}k}. \tag{5.13}$$

$$D^\alpha_{\mathrm{SU}(3)} = -g_3 \sum_{i=1}^{3}\left(\sum_{a=1,2} \tilde{q}_{\mathrm{L}i}^{a*} \tau^\alpha \tilde{q}_{\mathrm{L}i}^a - \tilde{u}_{\mathrm{R}i}^* \tau^\alpha \tilde{u}_{\mathrm{R}i} - \tilde{d}_{\mathrm{R}i}^* \tau^\alpha \tilde{d}_{\mathrm{R}i}\right), \tag{5.14}$$

$$D^\alpha_{\mathrm{SU}(2)} = -g_2 \left[\sum_{i=1}^{3}\left(\sum_{x=1}^{3} \tilde{q}_{\mathrm{L}i}^{x*} T^\alpha \tilde{q}_{\mathrm{L}i}^x + \tilde{l}_{\mathrm{L}i}^* T^\alpha \tilde{l}_{\mathrm{L}i}\right) + h_{\mathrm{u}}^* T^\alpha h_{\mathrm{u}} + h_{\mathrm{d}}^* T^\alpha h_{\mathrm{d}}\right], \tag{5.15}$$

$$D_{\mathrm{U}(1)} = -g_1 \left(\frac{1}{6}|\tilde{q}_{\mathrm{L}}|^2 - \frac{1}{2}|\tilde{l}_{\mathrm{L}}|^2 - \frac{2}{3}|\tilde{u}_{\mathrm{R}}|^2 + \frac{1}{3}|\tilde{d}_{\mathrm{R}}|^2 + |\tilde{e}_{\mathrm{R}}|^2 + \frac{1}{2}|h_{\mathrm{u}}|^2 - \frac{1}{2}|h_{\mathrm{d}}|^2\right). \tag{5.16}$$

Combining these, the full SUSY scalar potential is given by

$$V = V_{\mathrm{SUSY}} + V_{\cancel{\mathrm{SUSY}}}; \tag{5.17}$$

$$\begin{aligned}
V_{\mathrm{SUSY}} = {}& |h_{\mathrm{u}}|^2 \left(|\mu|^2 + |\mu'|^2\right) + |\mu|^2|h_{\mathrm{d}}|^2 + \left(\mu_i'^{*} \mu \tilde{l}_{\mathrm{L}i}^{*} h_{\mathrm{d}} + \mathrm{H.c.}\right) + \mu_i'^{*}\mu_j' \tilde{l}_{\mathrm{L}i}^{*}\tilde{l}_{\mathrm{L}j} \\
&+ \Big[- y_{\mathrm{u}ij}\mu^{*} h_{\mathrm{d}}^{*}\tilde{u}_{\mathrm{R}i}^{*}\tilde{q}_{\mathrm{L}j} - y_{\mathrm{u}ij}\mu_k'^{*}\tilde{u}_{\mathrm{R}i}^{*}\tilde{q}_{\mathrm{L}j}\tilde{l}_{\mathrm{L}k}^{*} - \left(y_{\mathrm{d}ij}\mu^{*} + \lambda'_{kji}\mu_k'^{*}\right) h_{\mathrm{u}}^{*}\tilde{d}_{\mathrm{R}i}^{*}\tilde{q}_{\mathrm{L}j} \\
&\qquad + y_{\mathrm{e}ij}\mu_j'^{*}\tilde{e}_{\mathrm{R}i}^{*}h_{\mathrm{u}}^{*}h_{\mathrm{d}} + \left(\lambda_{jki}\mu_k'^{*} - y_{\mathrm{e}ij}\mu^{*}\right) h_{\mathrm{u}}^{*}\tilde{e}_{\mathrm{R}i}^{*}\tilde{l}_{\mathrm{L}j} + \mathrm{H.c.}\Big] \\
&+ \frac{1}{8}\left(g_2^2 + g_Y^2\right)|h_{\mathrm{d}}|^4 + \frac{1}{8}\left(g_2^2 + g_Y^2\right)|h_{\mathrm{u}}|^4 + \left(-\frac{g_2^2}{4}|h_{\mathrm{d}}|^2|h_{\mathrm{u}}|^2 - \frac{g_Y^2}{4}|h_{\mathrm{d}}|^2|h_{\mathrm{u}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{d}}^{*}h_{\mathrm{u}}|^2\right) \\
&+ \left(-\frac{g_2^2}{4}|h_{\mathrm{u}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_Y^2}{12}|h_{\mathrm{u}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{u}}^{b}\tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_{\mathrm{u}}^{\dagger}y_{\mathrm{u}})_{ji}h_{\mathrm{u}}^{a}h_{\mathrm{u}}^{b*}\tilde{q}_{\mathrm{L}i}^{c}\tilde{q}_{\mathrm{L}j}^{d*}\right) \\
&+ \left(-\frac{g_2^2}{4}|h_{\mathrm{d}}|^2|\tilde{q}_{\mathrm{L}}|^2 - \frac{g_Y^2}{12}|h_{\mathrm{d}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{d}}^{*}\tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_{\mathrm{d}}^{\dagger}y_{\mathrm{d}})_{ji}h_{\mathrm{d}}^{a}h_{\mathrm{d}}^{b*}\tilde{q}_{\mathrm{L}i}^{c}\tilde{q}_{\mathrm{L}j}^{d*}\right) \\
&+ \left(-\frac{g_2^2}{4}|h_{\mathrm{u}}|^2|\tilde{l}_{\mathrm{L}}|^2 - \frac{g_Y^2}{4}|h_{\mathrm{u}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{u}}^{*}\tilde{l}_{\mathrm{L}i}|^2\right) \\
&+ \left(-\frac{g_2^2}{4}|h_{\mathrm{d}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \frac{g_Y^2}{4}|h_{\mathrm{d}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}|h_{\mathrm{d}}^{*}\tilde{l}_{\mathrm{L}i}|^2 + \epsilon^{ac}\epsilon^{bd}(y_{\mathrm{e}}^{\dagger}y_{\mathrm{e}})_{ji}h_{\mathrm{d}}^{a}h_{\mathrm{d}}^{b*}\tilde{l}_{\mathrm{L}i}^{c}\tilde{l}_{\mathrm{L}j}^{d*}\right) \\
&+ |h_{\mathrm{u}}|^2\left(-\frac{g_Y^2}{3}|\tilde{u}_{\mathrm{R}}|^2 + (y_{\mathrm{u}}y_{\mathrm{u}}^{\dagger})_{ij}\tilde{u}_{\mathrm{R}i}^{*}\tilde{u}_{\mathrm{R}j}\right) + \frac{g_Y^2}{3}|h_{\mathrm{d}}|^2|\tilde{u}_{\mathrm{R}}|^2 \\
&+ \frac{g_Y^2}{6}|h_{\mathrm{u}}|^2|\tilde{d}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2\left(-\frac{g_Y^2}{6}|\tilde{d}_{\mathrm{R}}|^2 + (y_{\mathrm{d}}y_{\mathrm{d}}^{\dagger})_{ij}\tilde{d}_{\mathrm{R}i}^{*}\tilde{d}_{\mathrm{R}j}\right) - \left[(y_{\mathrm{u}}y_{\mathrm{d}}^{\dagger})_{ij}\tilde{u}_{\mathrm{R}i}^{*}\tilde{d}_{\mathrm{R}j}(h_{\mathrm{d}}^{*}h_{\mathrm{u}}) + \mathrm{H.c.}\right] \\
&+ \frac{g_Y^2}{2}|h_{\mathrm{u}}|^2|\tilde{e}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2\left(-\frac{g_Y^2}{2}|\tilde{e}_{\mathrm{R}}|^2 + (y_{\mathrm{e}}y_{\mathrm{e}}^{\dagger})_{ij}\tilde{e}_{\mathrm{R}i}^{*}\tilde{e}_{\mathrm{R}j}\right) \\
&+ \Big[-\frac{1}{2}\epsilon^{ab}\epsilon^{xyz}y_{\mathrm{u}lk}\lambda_{lij}''^{*}h_{\mathrm{u}}^{a}\tilde{d}_{\mathrm{R}i}^{x}\tilde{d}_{\mathrm{R}j}^{y}\tilde{q}_{\mathrm{L}k}^{bz} + \epsilon^{ab}\epsilon^{xyz}y_{\mathrm{d}lk}\lambda_{ijl}''^{*}h_{\mathrm{d}}^{a}\tilde{u}_{\mathrm{R}i}^{x}\tilde{d}_{\mathrm{R}j}^{y}\tilde{q}_{\mathrm{L}k}^{bz} - y_{\mathrm{u}il}\lambda_{klj}'^{*}h_{\mathrm{u}}^{a}\tilde{u}_{\mathrm{R}i}^{*}\tilde{d}_{\mathrm{R}j}\tilde{l}_{\mathrm{L}k}^{a*} \\
&\qquad + y_{\mathrm{d}il}\lambda_{klj}'^{*}h_{\mathrm{d}}^{a}\tilde{d}_{\mathrm{R}i}^{*}\tilde{d}_{\mathrm{R}j}\tilde{l}_{\mathrm{L}k}^{a*} - \epsilon^{ab}\epsilon^{cd}y_{\mathrm{d}li}\lambda_{kjl}'^{*}h_{\mathrm{d}}^{a}\tilde{q}_{\mathrm{L}i}^{b}\tilde{q}_{\mathrm{L}j}^{c*}\tilde{l}_{\mathrm{L}k}^{d*} + y_{\mathrm{e}il}\lambda_{klj}^{*}h_{\mathrm{d}}^{a}\tilde{e}_{\mathrm{R}i}^{*}\tilde{e}_{\mathrm{R}j}\tilde{l}_{\mathrm{L}k}^{a*} \\
&\qquad - y_{\mathrm{e}jl}\lambda_{lki}'^{*}h_{\mathrm{d}}^{a}\tilde{e}_{\mathrm{R}j}^{*}\tilde{d}_{\mathrm{R}i}\tilde{q}_{\mathrm{L}k}^{a*} + \frac{1}{2}\epsilon^{ab}\epsilon^{cd}y_{\mathrm{e}li}\lambda_{jkl}^{*}h_{\mathrm{d}}^{a}\tilde{l}_{\mathrm{L}i}^{b}\tilde{l}_{\mathrm{L}j}^{c*}\tilde{l}_{\mathrm{L}k}^{d*} + \mathrm{H.c.}\Big] \\
&+ \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8}\right)|\tilde{q}_{\mathrm{L}}|^4 + \frac{g_2^2}{4}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{q}_{\mathrm{L}j}^{by}\tilde{q}_{\mathrm{L}i}^{bx*}\tilde{q}_{\mathrm{L}j}^{ay*} + \frac{g_3^2}{4}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{q}_{\mathrm{L}j}^{by}\tilde{q}_{\mathrm{L}i}^{ay*}\tilde{q}_{\mathrm{L}j}^{bx*}\right] \\
&+ \left[\left(-\frac{g_3^2}{12} + \frac{2g_Y^2}{9}\right)|\tilde{u}_{\mathrm{R}}|^4 + \frac{g_3^2}{4}\tilde{u}_{\mathrm{R}i}^{x*}\tilde{u}_{\mathrm{R}j}^{y*}\tilde{u}_{\mathrm{R}i}^{y}\tilde{u}_{\mathrm{R}j}^{x}\right] \\
&+ \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18}\right)|\tilde{d}_{\mathrm{R}}|^4 + \frac{g_3^2}{4}\tilde{d}_{\mathrm{R}i}^{x*}\tilde{d}_{\mathrm{R}j}^{y*}\tilde{d}_{\mathrm{R}i}^{y}\tilde{d}_{\mathrm{R}j}^{x} + \frac{1}{2}\lambda''_{mij}\lambda''^{*}_{mkl}\tilde{d}_{\mathrm{R}i}^{x*}\tilde{d}_{\mathrm{R}j}^{y*}\tilde{d}_{\mathrm{R}k}^{x}\tilde{d}_{\mathrm{R}l}^{y}\right] \\
&+ \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9}\right)|\tilde{u}_{\mathrm{R}}|^2|\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2}\tilde{u}_{\mathrm{R}i}^{x*}\tilde{q}_{\mathrm{L}j}^{ax}\tilde{u}_{\mathrm{R}i}^{y}\tilde{q}_{\mathrm{L}j}^{ay*} + y_{\mathrm{u}ik}y_{\mathrm{u}jl}^{*}\tilde{u}_{\mathrm{R}i}^{x*}\tilde{u}_{\mathrm{R}j}^{y}\tilde{q}_{\mathrm{L}k}^{ax}\tilde{q}_{\mathrm{L}l}^{ay*}\right] \\
&+ \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18}\right)|\tilde{d}_{\mathrm{R}}|^2|\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2}\tilde{d}_{\mathrm{R}i}^{x*}\tilde{q}_{\mathrm{L}j}^{ax}\tilde{d}_{\mathrm{R}i}^{y}\tilde{q}_{\mathrm{L}j}^{ay*} + \left(y_{\mathrm{d}ik}y_{\mathrm{d}jl}^{*} + \lambda'_{mki}\lambda'^{*}_{mlj}\right)\tilde{d}_{\mathrm{R}i}^{x*}\tilde{q}_{\mathrm{L}k}^{ax}\tilde{d}_{\mathrm{R}j}^{y}\tilde{q}_{\mathrm{L}l}^{ay*}\right] \\
&+ \left[-\left(\frac{g_3^2}{6} + \frac{2g_Y^2}{9}\right)|\tilde{d}_{\mathrm{R}}|^2|\tilde{u}_{\mathrm{R}}|^2 + \left(\frac{g_3^2}{2} - \lambda''_{ikm}\lambda''^{*}_{ljm}\right)\tilde{u}_{\mathrm{R}i}^{x*}\tilde{d}_{\mathrm{R}j}^{x}\tilde{d}_{\mathrm{R}j}^{y*}\tilde{u}_{\mathrm{R}i}^{y} + \lambda''_{ikm}\lambda''^{*}_{jlm}\tilde{u}_{\mathrm{R}i}^{x*}\tilde{u}_{\mathrm{R}j}^{x}\tilde{d}_{\mathrm{R}k}^{y*}\tilde{d}_{\mathrm{R}l}^{y}\right] \\
&+ \left[-\left(\frac{g_2^2}{4} + \frac{g_Y^2}{12}\right)|\tilde{l}_{\mathrm{L}}|^2|\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{q}_{\mathrm{L}i}^{bx*}\tilde{l}_{\mathrm{L}j}^{b}\tilde{l}_{\mathrm{L}j}^{a*} + \epsilon^{ac}\epsilon^{bd}\lambda'_{kim}\lambda'^{*}_{ljm}\tilde{q}_{\mathrm{L}i}^{ax}\tilde{q}_{\mathrm{L}j}^{bx*}\tilde{l}_{\mathrm{L}k}^{c}\tilde{l}_{\mathrm{L}l}^{d*}\right] + \frac{g_Y^2}{6}|\tilde{e}_{\mathrm{R}}|^2|\tilde{q}_{\mathrm{L}}|^2 \\
&+ \left(-\frac{g_Y^2}{6}|\tilde{d}_{\mathrm{R}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \lambda'_{lmi}\lambda'^{*}_{kmj}\tilde{d}_{\mathrm{R}i}^{*}\tilde{d}_{\mathrm{R}j}\tilde{l}_{\mathrm{L}k}^{*}\tilde{l}_{\mathrm{L}l}\right) + \frac{g_Y^2}{3}|\tilde{u}_{\mathrm{R}}|^2|\tilde{l}_{\mathrm{L}}|^2 - \frac{2g_Y^2}{3}|\tilde{u}_{\mathrm{R}}|^2|\tilde{e}_{\mathrm{R}}|^2 + \frac{g_Y^2}{3}|\tilde{d}_{\mathrm{R}}|^2|\tilde{e}_{\mathrm{R}}|^2 \\
&+ \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8}\right)|\tilde{l}_{\mathrm{L}}|^4 + \frac{g_2^2}{4}\tilde{l}_{\mathrm{L}i}^{a}\tilde{l}_{\mathrm{L}j}^{b}\tilde{l}_{\mathrm{L}i}^{b*}\tilde{l}_{\mathrm{L}j}^{a*} + \frac{1}{4}\epsilon^{ab}\epsilon^{cd}\lambda_{ijm}\lambda_{klm}^{*}\tilde{l}_{\mathrm{L}i}^{a}\tilde{l}_{\mathrm{L}j}^{b}\tilde{l}_{\mathrm{L}k}^{c*}\tilde{l}_{\mathrm{L}l}^{d*}\right] \\
&+ \frac{g_Y^2}{2}|\tilde{e}_{\mathrm{R}}|^4 + \left[-\frac{g_Y^2}{2}|\tilde{e}_{\mathrm{R}}|^2|\tilde{l}_{\mathrm{L}}|^2 + \left(y_{\mathrm{e}ik}y_{\mathrm{e}jl}^{*} + \lambda_{kmi}\lambda_{lmj}^{*}\right)\tilde{e}_{\mathrm{R}i}^{*}\tilde{e}_{\mathrm{R}j}\tilde{l}_{\mathrm{L}k}\tilde{l}_{\mathrm{L}l}^{*}\right] \\
&+ \left[\left(y_{\mathrm{d}ik}y_{\mathrm{e}jl}^{*} - \lambda'_{mki}\lambda_{lmj}^{*}\right)\tilde{d}_{\mathrm{R}i}^{x*}\tilde{q}_{\mathrm{L}k}^{ax}\tilde{e}_{\mathrm{R}j}\tilde{l}_{\mathrm{L}l}^{a*} - \epsilon^{ab}\epsilon^{xyz}\lambda'_{lkm}\lambda''^{*}_{ijm}\tilde{l}_{\mathrm{L}l}^{b}\tilde{q}_{\mathrm{L}k}^{az}\tilde{d}_{\mathrm{R}j}^{y}\tilde{u}_{\mathrm{R}i}^{x} + \mathrm{H.c.}\right].
\end{aligned} \tag{5.18}$$

5.2. SLHA convention

The SLHA convention [?] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table (**magenta color** for objects in other conventions), while

$\mu, b, m^2_{Q,L,H_\mathrm{u},H_\mathrm{d}}$, RPV-trilinears ($\lambda$s and Ts)

are in common.

SLHA		our notation	Martin/DHM
(H_1, H_2)	$=$	$(H_\mathrm{d}, H_\mathrm{u})$	
$Y_{\mathrm{u,d,e}}$	$=$	$(y_{\mathrm{u,d,e}})^\mathrm{T}$	
$T_{\mathrm{u,d,e}}$	$=$	$(a_{\mathrm{u,d,e}})^\mathrm{T}$	
$A_{\mathrm{u,d,e}}$	$=$	$(A_{\mathrm{u,d,e}})^\mathrm{T}$	
$m^2_{U^\mathrm{c},D^\mathrm{c},E\mathrm{c}}$	$=$	$(m^2_{U^\mathrm{c},D^\mathrm{c},E^\mathrm{c}})^\dagger$	
$M_{1,2,3}$	$=$	$-M_{1,2,3}$	
m_3^2	$=$	b	
m_A^2	$=$	$m^2_{A_0}$ (tree)	
		κ_i	$=-\mu'_i$ (rarely used)
D_i	$=$	b_i	
$m^2_{\tilde{L}_i H_1}$	$=$	M^2_{Li}	

In particular, the chargino/neutralino mass terms in RPC case are given by

$$\mathcal{L} \supset \left[\frac{1}{2}M_1\tilde{b}\tilde{b} + \frac{1}{2}M_2\tilde{w}\tilde{w} - \mu\tilde{h}_{\mathrm{u}}\tilde{h}_{\mathrm{d}} - \frac{g_Y}{2\sqrt{2}}\left(h_{\mathrm{u}}^*\tilde{h}_{\mathrm{u}} - h_{\mathrm{d}}^*\tilde{h}_{\mathrm{d}}\right)\tilde{b} - \sqrt{2}g_2\left(h_{\mathrm{u}}^*T^a\tilde{h}_{\mathrm{u}} + h_{\mathrm{d}}^*T_a\tilde{h}_{\mathrm{d}}\right)\tilde{w}\right] + \mathrm{H.c.} \tag{5.19}$$

$$\to \frac{1}{2}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_{\mathrm{u}}^0\\ h_{\mathrm{d}}^0\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}-M_1 & 0 & -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{s}_\beta\mathrm{s}_w\\ 0 & -M_2 & m_Z\mathrm{c}_\beta\mathrm{c}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w\\ -m_Z\mathrm{c}_\beta\mathrm{s}_w & m_Z\mathrm{c}_\beta\mathrm{c}_w & 0 & -\mu\\ m_Z\mathrm{s}_\beta\mathrm{s}_w & -m_Z\mathrm{s}_\beta\mathrm{c}_w & -\mu & 0\end{pmatrix}\begin{pmatrix}\tilde{b}\\ \tilde{w}\\ h_{\mathrm{u}}^0\\ h_{\mathrm{d}}^0\end{pmatrix} \tag{5.20}$$

A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [?]:

$$\mathrm{e}^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \text{ (converges for any } X), \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(A-1)^m}{m} \text{ (conv. if } \|A-I\| < 1). \tag{A.1}$$

$$\mathrm{e}^{\log A} = A \text{ (if } \|A-I\| < 1), \quad \log \mathrm{e}^X = X \text{ and } \|\mathrm{e}^X - 1\| < 1 \text{ (if } \|X\| < \log 2). \tag{A.2}$$

$$\text{Hilbert-Schmidt norm : } \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \operatorname{Tr} X^\dagger X. \tag{A.3}$$

Properties:

$$\mathrm{e}^{(X^\mathrm{T})} = (\mathrm{e}^X)^\mathrm{T}, \qquad \mathrm{e}^{(X^*)} = (\mathrm{e}^X)^*, \qquad (\mathrm{e}^X)^{-1} = \mathrm{e}^{-X}, \qquad \mathrm{e}^{YXY^{-1}} = Y\,\mathrm{e}^X Y^{-1},$$

$$\det\exp X = \exp\operatorname{Tr} X, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{tX} = X\,\mathrm{e}^{tX} = \mathrm{e}^{tX} X \qquad \mathrm{e}^{(\alpha+\beta)X} = \mathrm{e}^{\alpha X}\,\mathrm{e}^{\beta X} \text{ for } \alpha,\beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$\mathrm{e}^X Y\,\mathrm{e}^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots = \mathrm{e}^{[X,]}Y; \tag{A.4}$$

$$\mathrm{e}^X\,\mathrm{e}^Y\,\mathrm{e}^{-X} = \sum_{n=0}^{\infty} \frac{1}{n!}(\mathrm{e}^X Y\,\mathrm{e}^{-X})^n = \exp\left(\mathrm{e}^{[X,]}Y\right); \tag{A.5}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = X + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[X,}\,\mathrm{e}^{t[Y,})Y \qquad \left[g(z) = \frac{\log z}{1-z^{-1}} = 1 - \sum_{n=1}^{\infty}\frac{(1-z)^n}{n(n+1)}; \quad g(\mathrm{e}^y) = \sum_{n=0}^{\infty}\frac{B_n y^n}{n!}\right] \tag{A.6}$$

$$= X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots \quad \text{(Baker-Campbell-Hausdorff).} \tag{A.7}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\sum_{m,n=0}^{\infty}\frac{X^m Y^n}{m!n!} - 1\right)^k = \sum_{k=1}^{\infty}\sum_{m_1+n_1>0}\cdots\sum_{m_k+n_k>0}\frac{(-1)^{k-1}}{k}\frac{X^{m_1}Y^{n_1}\cdots X^{m_k}Y^{n_k}}{m_1!n_1!\cdots m_k!n_k!} \tag{A.8}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = \sum_{k=1}^{\infty}\sum_{m_1+n_1>0}\cdots\sum_{m_k+n_k>0}\frac{(-1)^{k-1}}{k\sum_{i=1}^{k}(m_i+n_i)}\frac{\left([X,\right)^{m_1}\left([Y,\right)^{n_1}\cdots\left([X,\right)^{m_k}\left([Y,\right)^{n_k}]\cdots]}{m_1!n_1!\cdots m_k!n_k!} \tag{A.9}$$

withe $[X] := X$ understood.