Bayesian Statistics and Bayesian Cognitive Modeling

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Recap

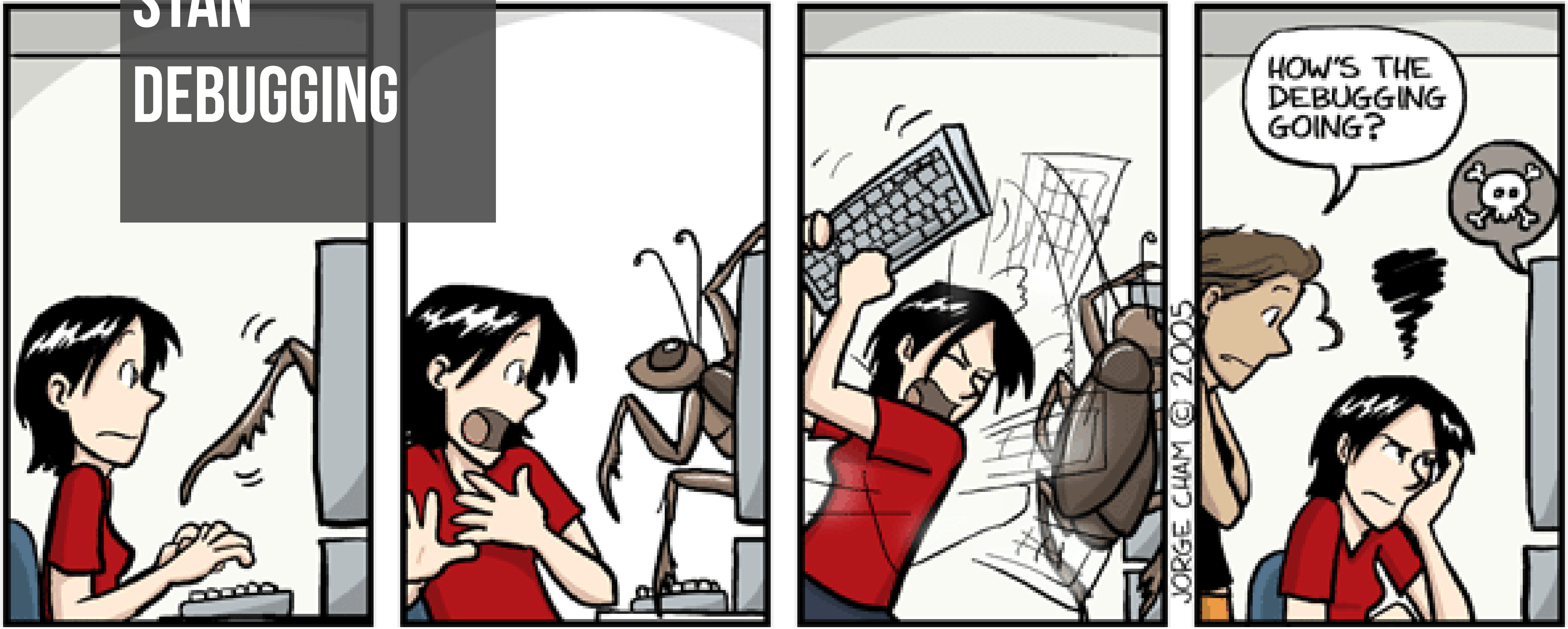
What we've learned…

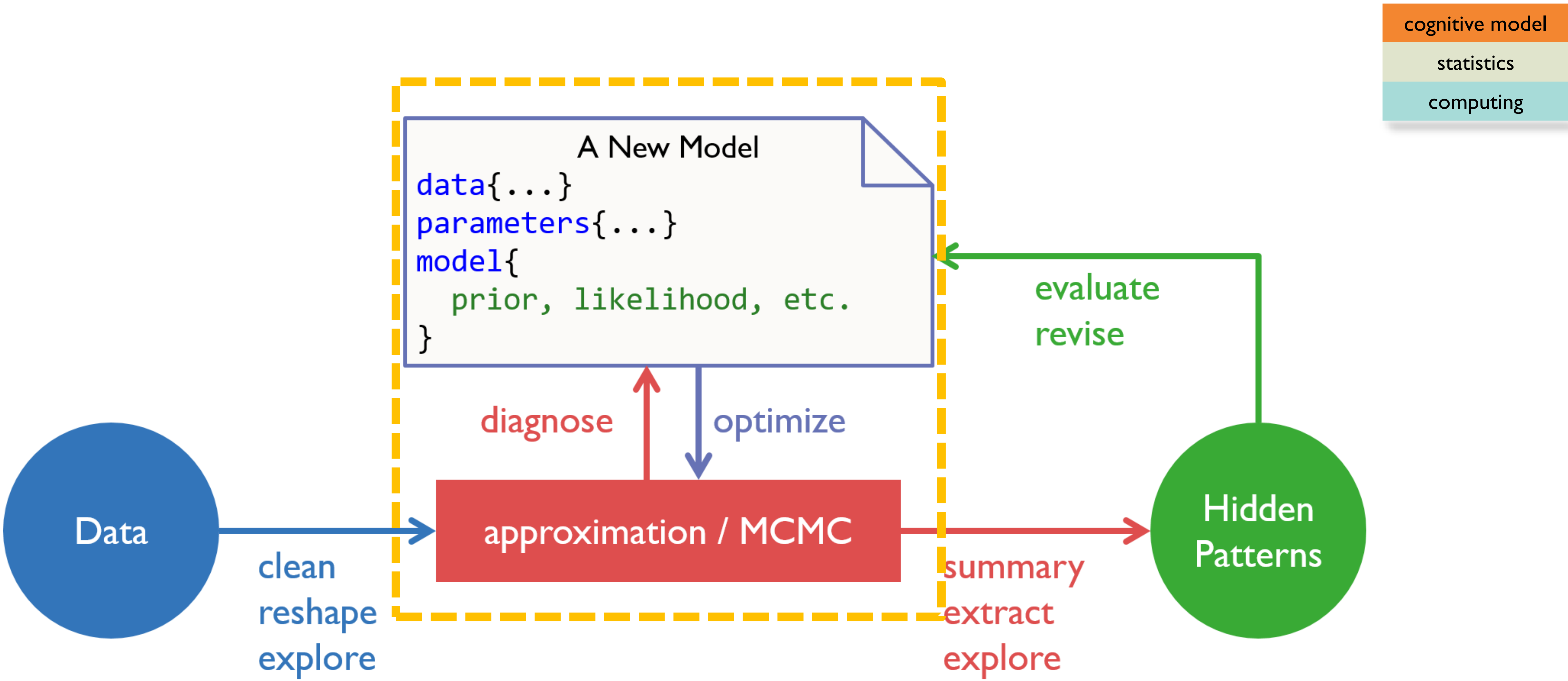
- Linear Regression Model
- Posterior Predictive Check
- Cognitive Modeling
- Reinforcement Learning Model
- Hierarchical Modeling
- Optimizing Stan Codes
- Model Comparison

DAY3

09:00 – 09:30	Implementing Model Comparison
09:30 – 10:15	Stan Style Tips and Debugging
10:15 – 11:00	Introduction to Model-based fMRI
11:00 – 11:15	Coffee break
11:15 – 12:15	Capstone Project: Delay Discounting
12:15 – 13:00	Summary, Misc., Q&A

STAN DEBUGGING




cognitive model
statistics
computing
A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
evaluate
revise
diagnose
optimize
Data
approximation / MCMC
Hidden Patterns
clean
reshape
explore
summary
extract
explore

Stan Style Tips

Make it Reproducible

- Scripts are good documentation!
- Save your seed (not cross platform*)

Make it Readable

- Choose a consistent style
- Give meaningful variable names

Start with Simulated Data

Design Top-Down, Code Bottom-Up

Write Comments

- Code never lies!

* Stan seed depends on hardware etc.

The Editor of your Choice

```
data {
  int<lower=0> w;
  int<lower=0> N;
}

parameters {
  real<lower=0,upper=1> p;
}

model {
  p ~ uniform(0,1);
  w ~ binomial(N, p);
}
```


```
data {
  int<lower=0> w;
  int<lower=0> N;
}

parameters {
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}
```


* Click on each logo to visit their homepage.
** Comparison

Common Error / Warning Types

ERRORS

- forget “ ; ”
- mis-indexing: mismatch or constant
- support mismatch
- improper constrain
- improper dimension declaration
- vectorizing when not supported
- wrong data type
- wrong distribution names
- forget priors
- miss spelling

WARNINGS

- forget last blank line
- use earlier version of Stan
- numerical problems
- divergent transitions
- hit max_treedepth
- BFMI too low
- improper prior

see also: Brief Guide to Stan’s Warnings

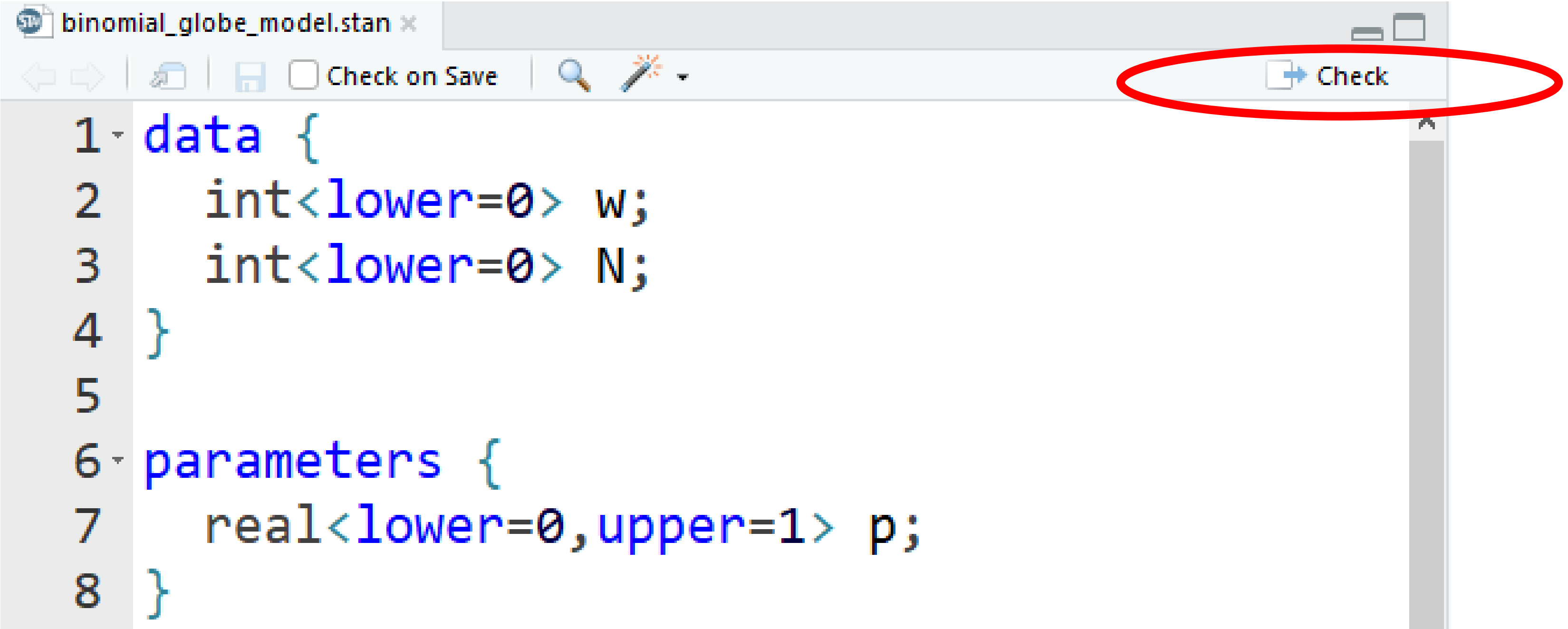
Debugging in Stan

- always use a *.stan file
- press Check in RStudio
- use lookup()
- start with simulated data
- be careful with copy/paste
- run 1 chain, 1 sample
- debugging by printing

```
for (s in 1:1) {
  vector[2] v;
  real pe;
  v <- initV;

  for (t in 1:nTrials) {
    choice[s,t] ~ categorical_logit( tau[s] * v );

    print("s = ", s, ", t = ", t, ", v = ", v);

    pe <- reward[s,t] - v[choice[s,t]];
    v[choice[s,t]] <- v[choice[s,t]] + lr[s] * pe;
  }
}
```

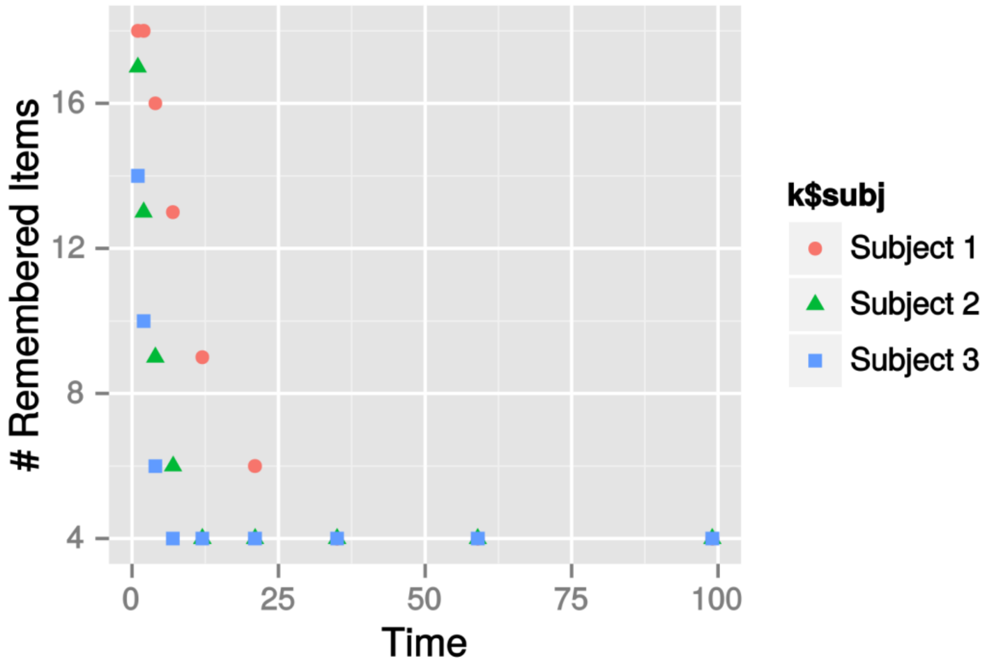
```
> lookup(dnorm)
    StanFunction                           Arguments ReturnType Page SamplingStatement
344       normal             (reals mu, reals sigma)       real  369              TRUE
348   normal_log (reals y, reals mu, reals sigma)       real  369             FALSE
```

Debugging Stan in RStudio




```
rstan::rstudio_stanc("_scripts/binomial_globe_model.stan")
```

Example: Memory Retention



	Time Interval								
Subject	1	2	4	7	12	21	35	59	99
1	18	18	16	13	9	6	4	4	4
2	17	13	9	6	4	4	4	4	4
3	14	10	6	4	4	4	4	4	4

Lee & Wagenmakers (2013)

Simple Exponential Decay Model

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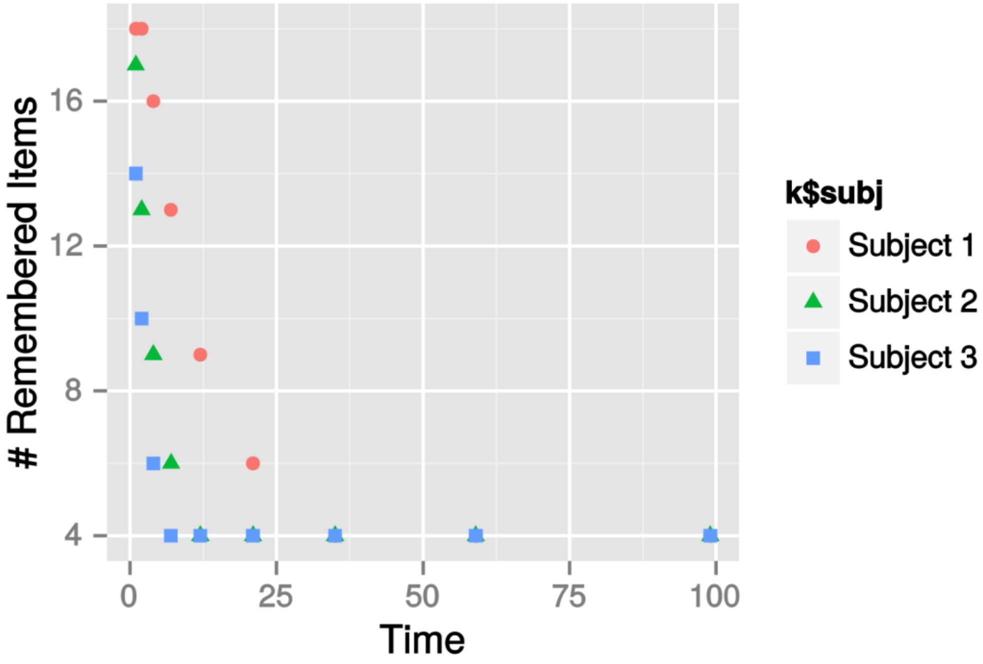
computing



$$\theta_t = \min(1.0, \exp(-\alpha t) + \beta)$$

p(remember)   decay rate   baseline

Lee & Wagenmakers (2013)

Exercise I


```
.../BayesCog/09.debugging/_scripts/exp_decay_main.R
```

TASK: Debugging the Memory retention model

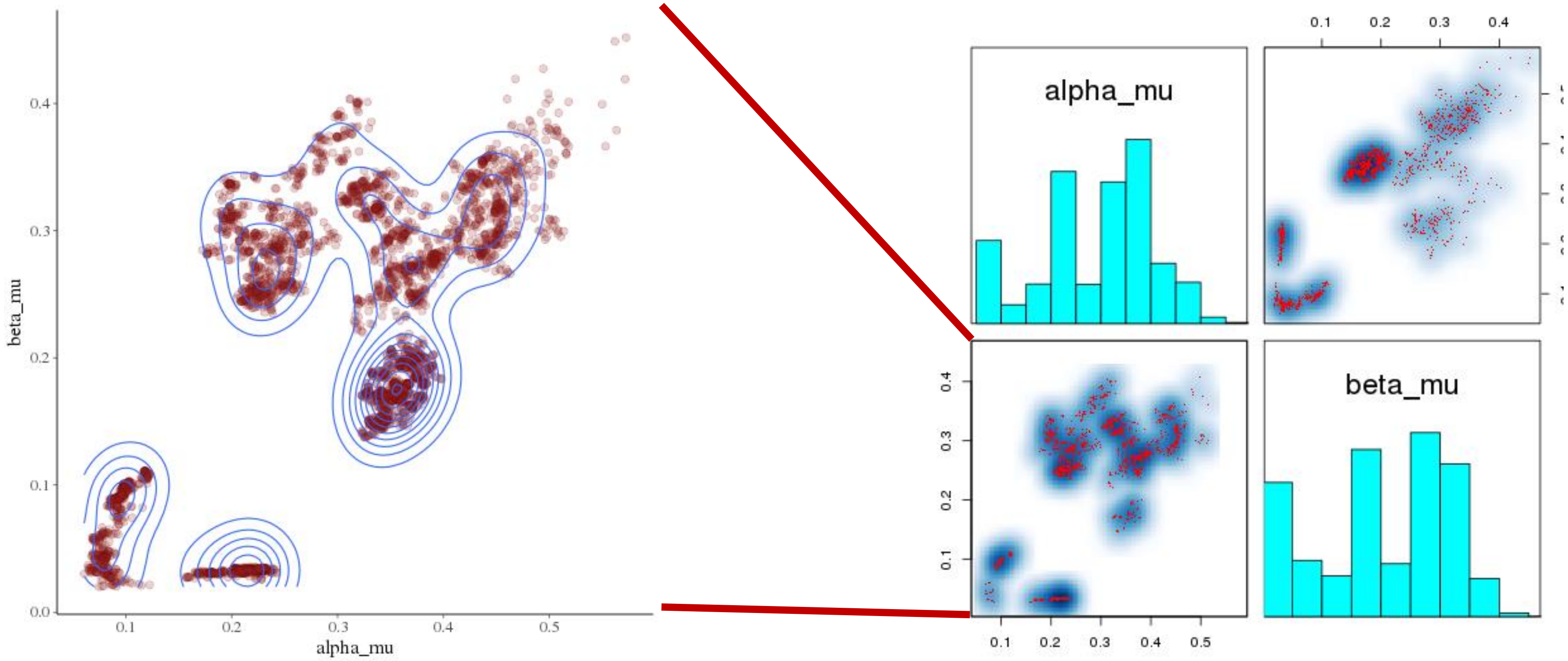
```
> dataList
$`k`
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,]   18   18   16   13    9    6    4    4    4
[2,]   17   13    9    6    4    4    4    4    4
[3,]   14   10    6    4    4    4    4    4    4

$nItem
[1] 18

$intervals
[1]  1  2  4  7 12 21 35 59 99

$ns
[1] 3

$nt
[1] 9
```

>= 9 errors!

Viel Spaß!

Satisfied with the results?

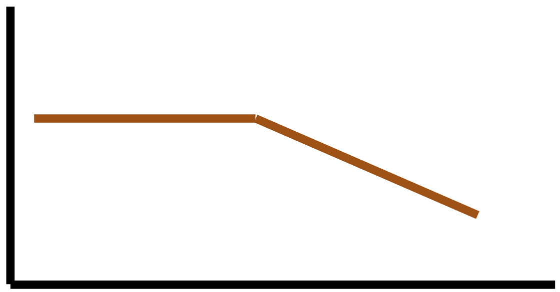
```
Warning messages:
1: There were 3998 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help. See
http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
2: Examine the pairs() plot to diagnose sampling problems
```



Why Stan Fails?

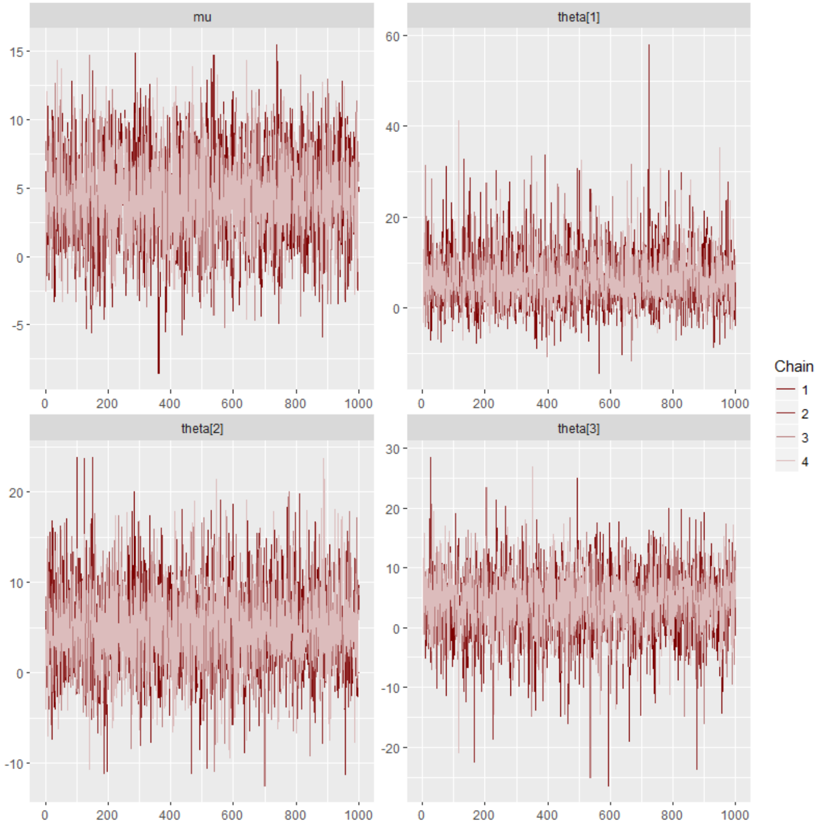

```
for (s in 1:ns) {
  for (t in 1:nt) {
    theta[s,t] = fmin(1.0, exp(-alpha[s] * intervals[t]) + beta[s]);
    k[s,t] ~ binomial(nItem, theta[s,t]);
  }
}
```

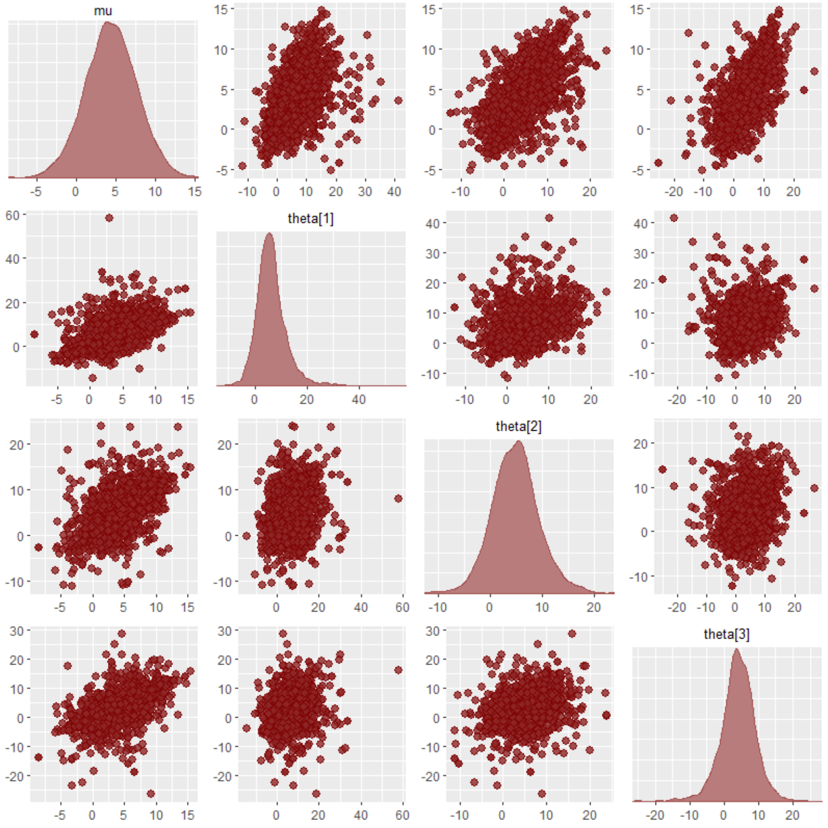
Non-differentiable link (likelihood) functions are bad news, particularly in Stan, which relies on derivatives.



What to look for?

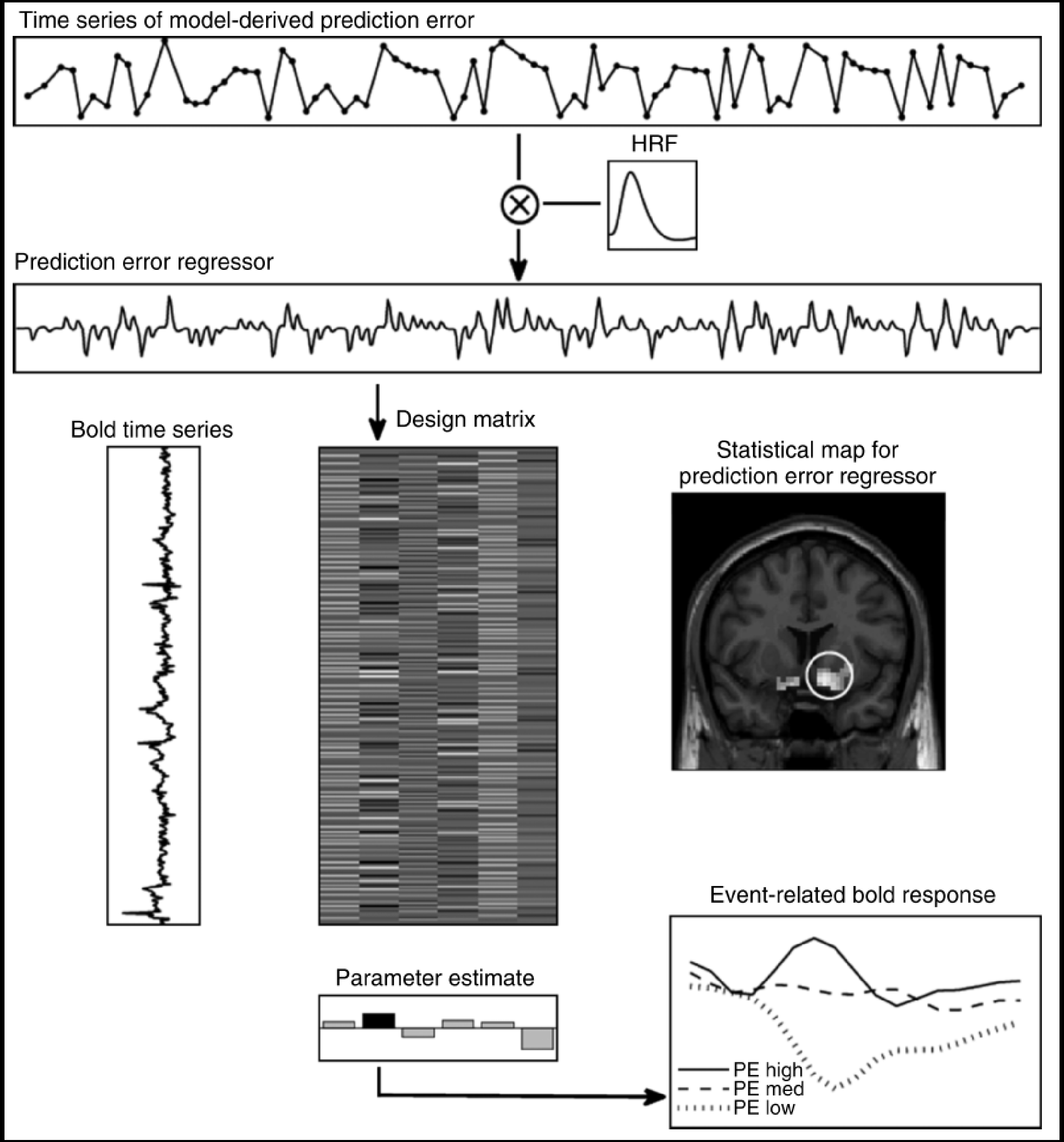
```
> source('stan_utility.R')
> check_all_diagnostics(fit)
[1] "n_eff / iter looks reasonable for all parameters"
[1] "Rhat looks reasonable for all parameters"
[1] "0 of 4000 iterations ended with a divergence (0%)"
[1] "0 of 4000 iterations saturated the maximum tree depth of 10 (0%)"
[1] "E-BFMI indicated no pathological behavior"
```





Credit: [Michael Betancourt]

INTRODUCTION TO MODEL-BASED FMRI



Model-based Analysis

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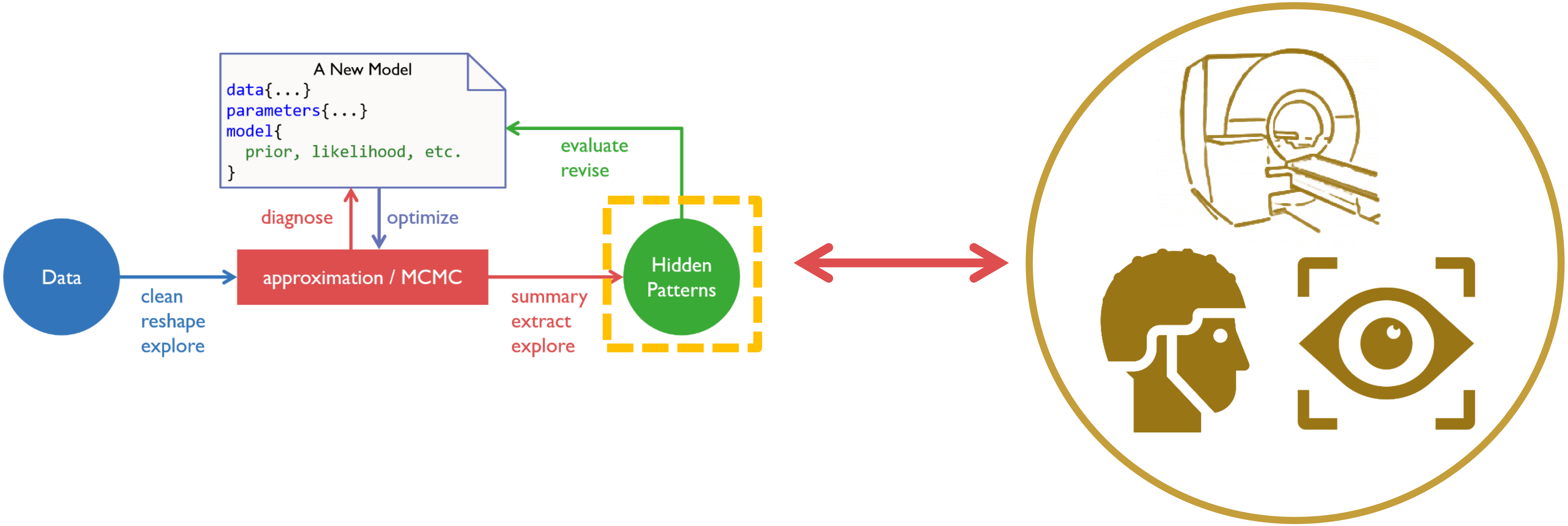
computing



Explore Hidden Patterns in Stan

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statistics
computing

```
generated quantities {
  ...

  vector[2] v[nSubjects, nTrials+1];
  real vc[nSubjects,nTrials]; //chosen value
  real pe[nSubjects,nTrials];

  {
    for (s in 1:nSubjects) {
      log_lik[s] <- 0;
      v[s,1] <- initV;

      for (t in 1:nTrials) {
        log_lik[s] = log_lik[s] + categorical_logit_log(choice[s,t], tau[s] * v[s,t] );

        vc[s,t] = v[s,t,choice[s,t]];

        pe[s,t] = reward[s,t] - v[s,t,choice[s,t]];

        v[s,t+1] = v[s,t];
        v[s,t+1,choice[s,t]] = v[s,t,choice[s,t]] + lr[s] * pe[s,t];
```

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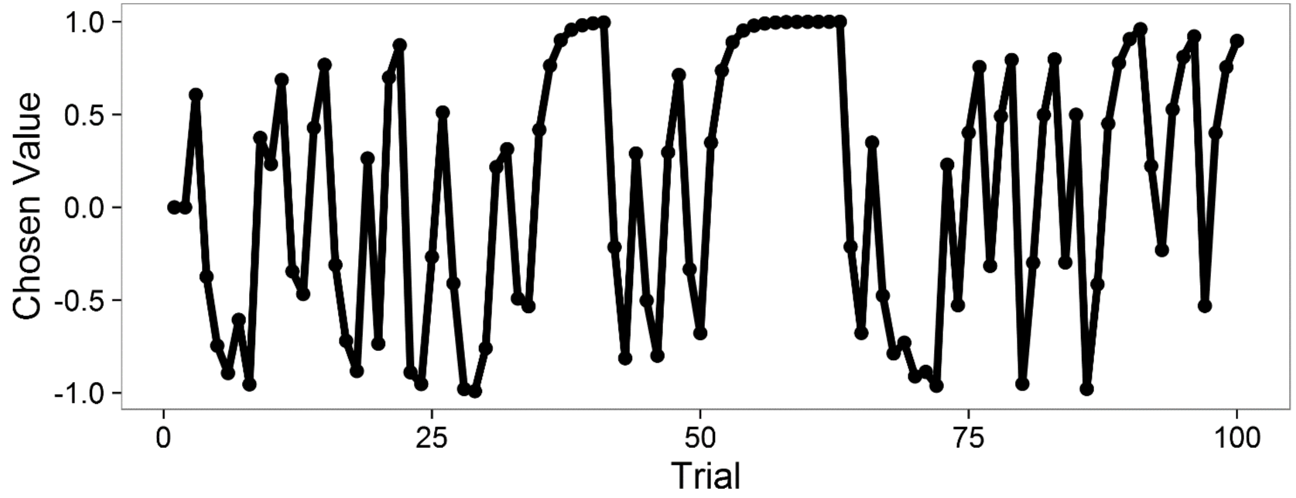
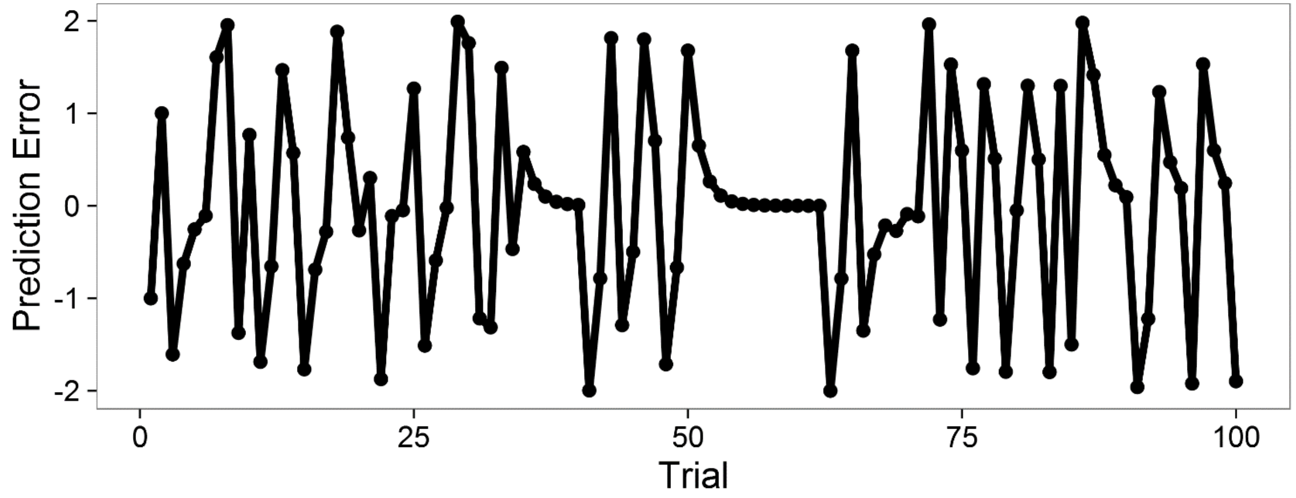
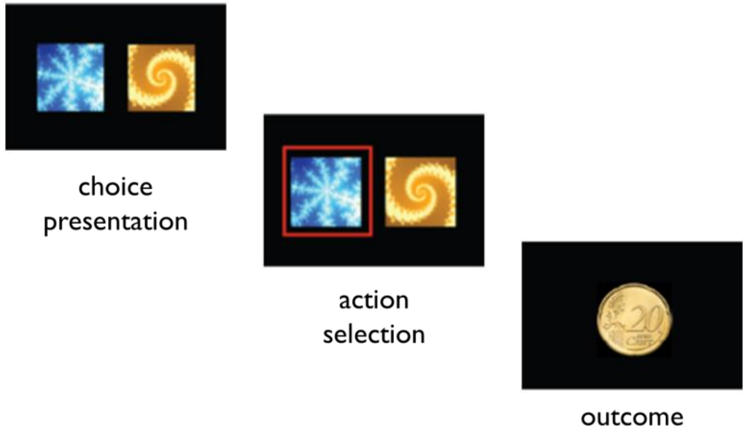
Obtain Decision Variables

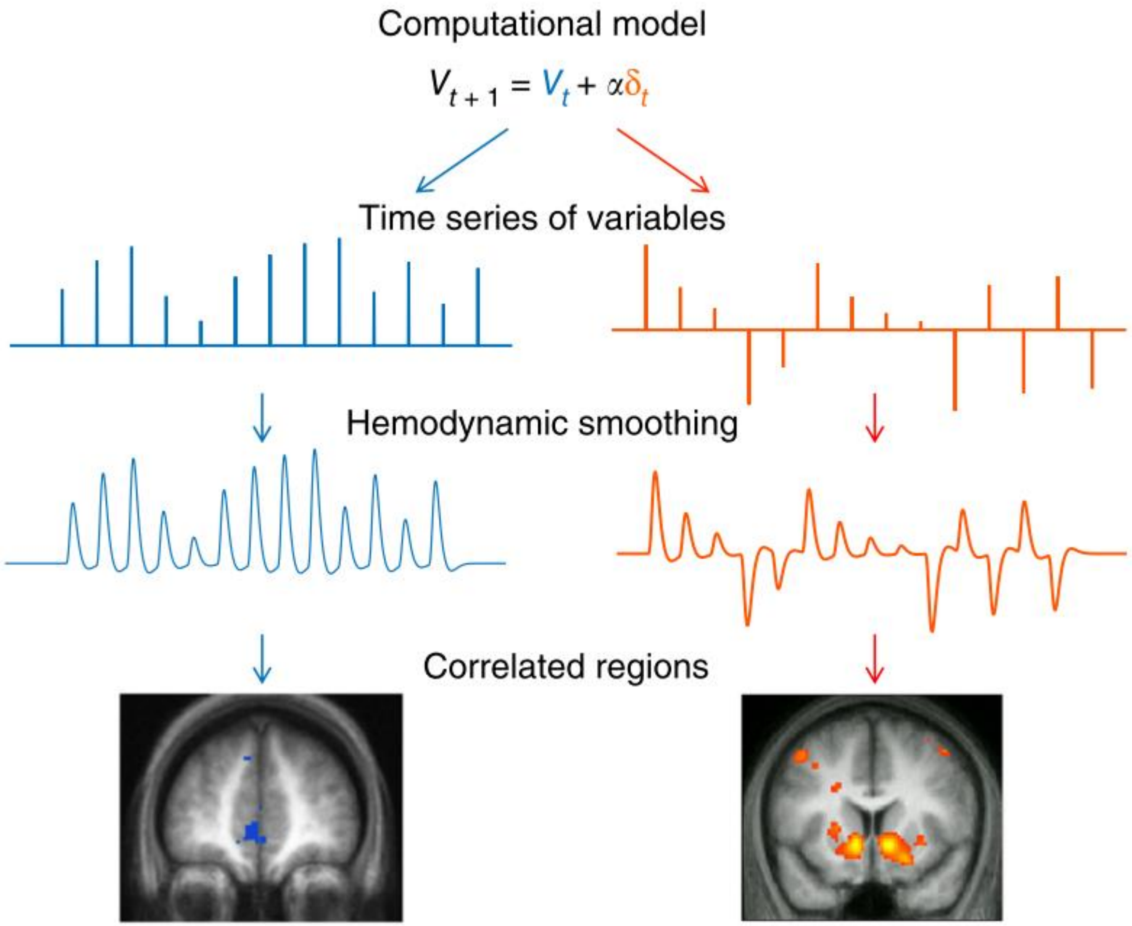
subject01





Perform Model-based fMRI





O’Doherty et al. (2007); Gläscher & O’Doherty (2010); Forstmann & Wagenmarkers (2015); Cohen et al. (2017)

Perform Model-based fMRI (cont.)

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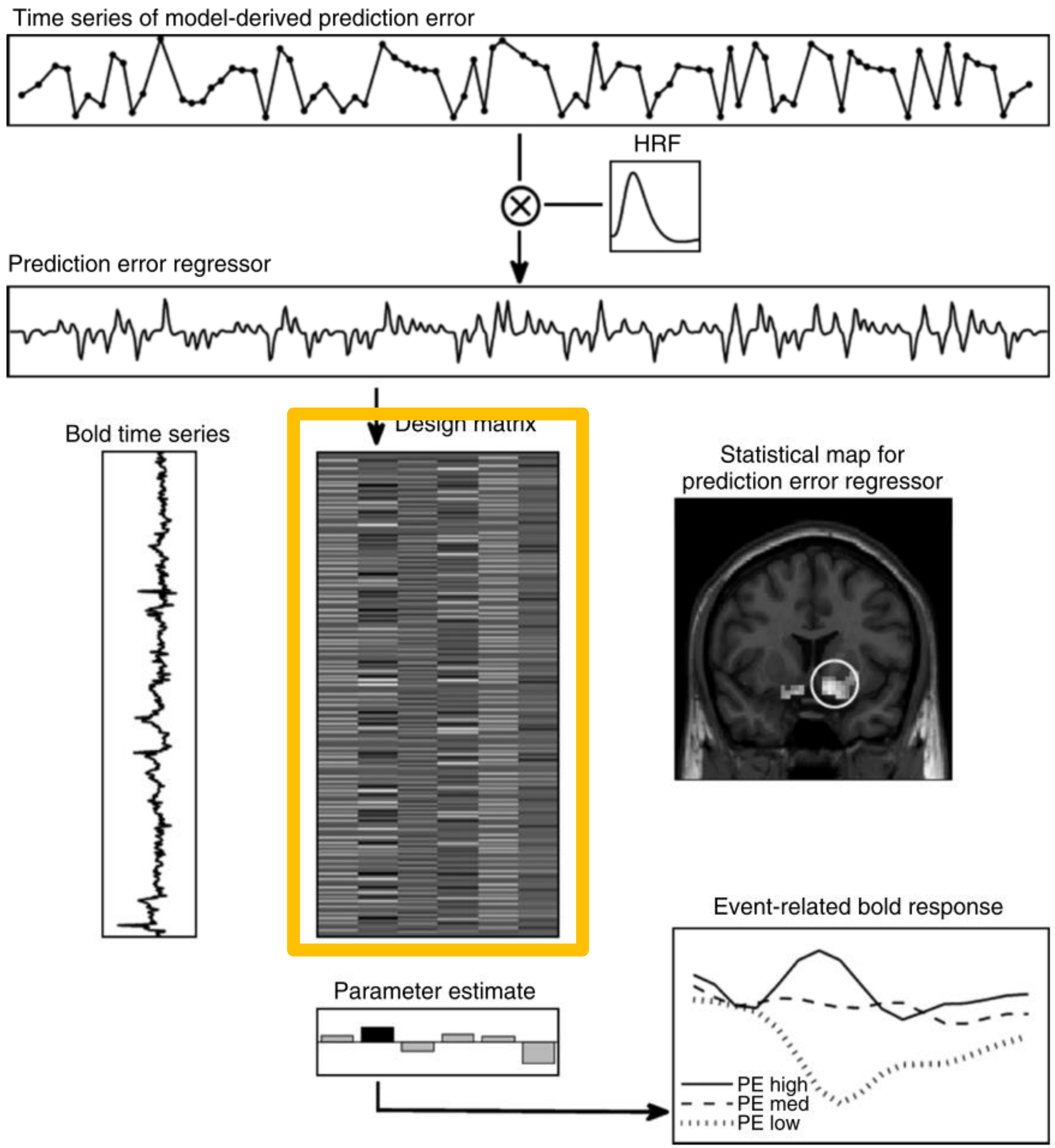
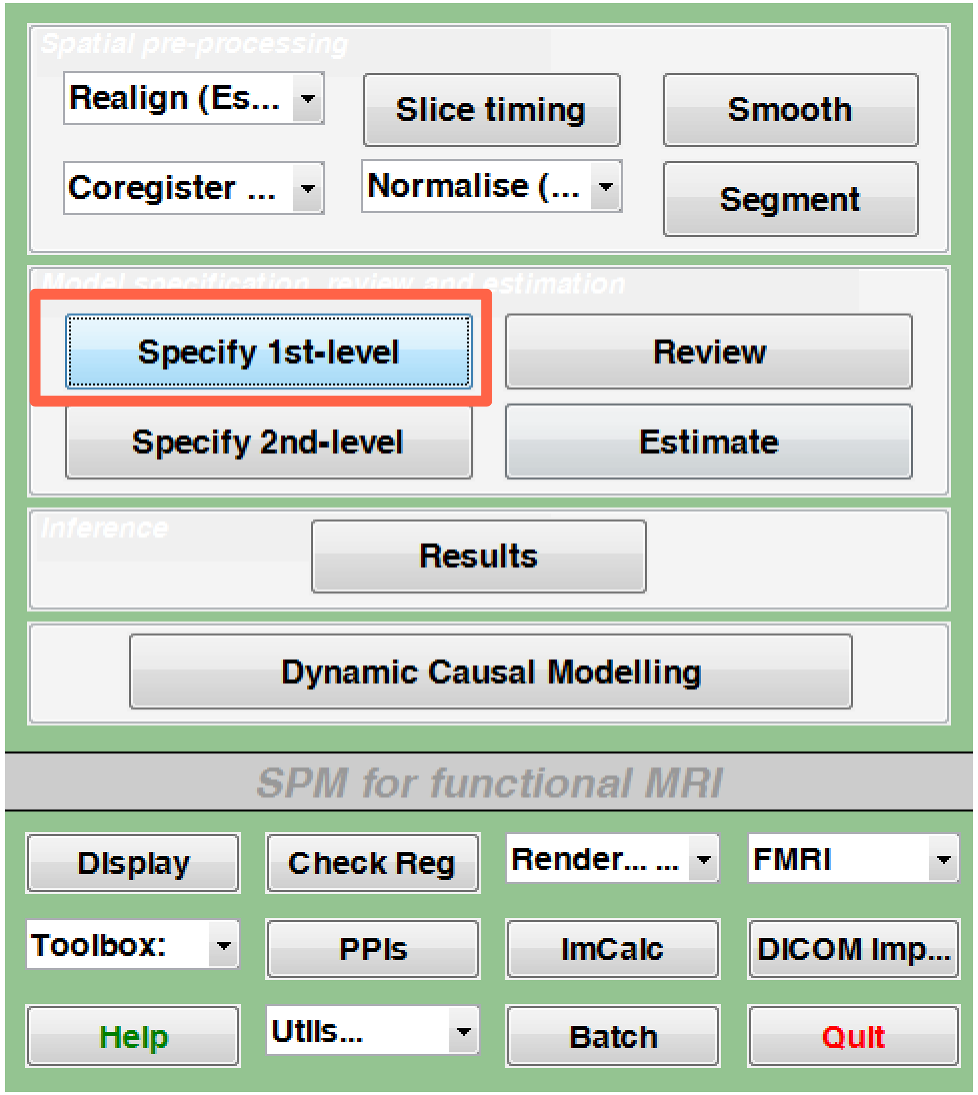
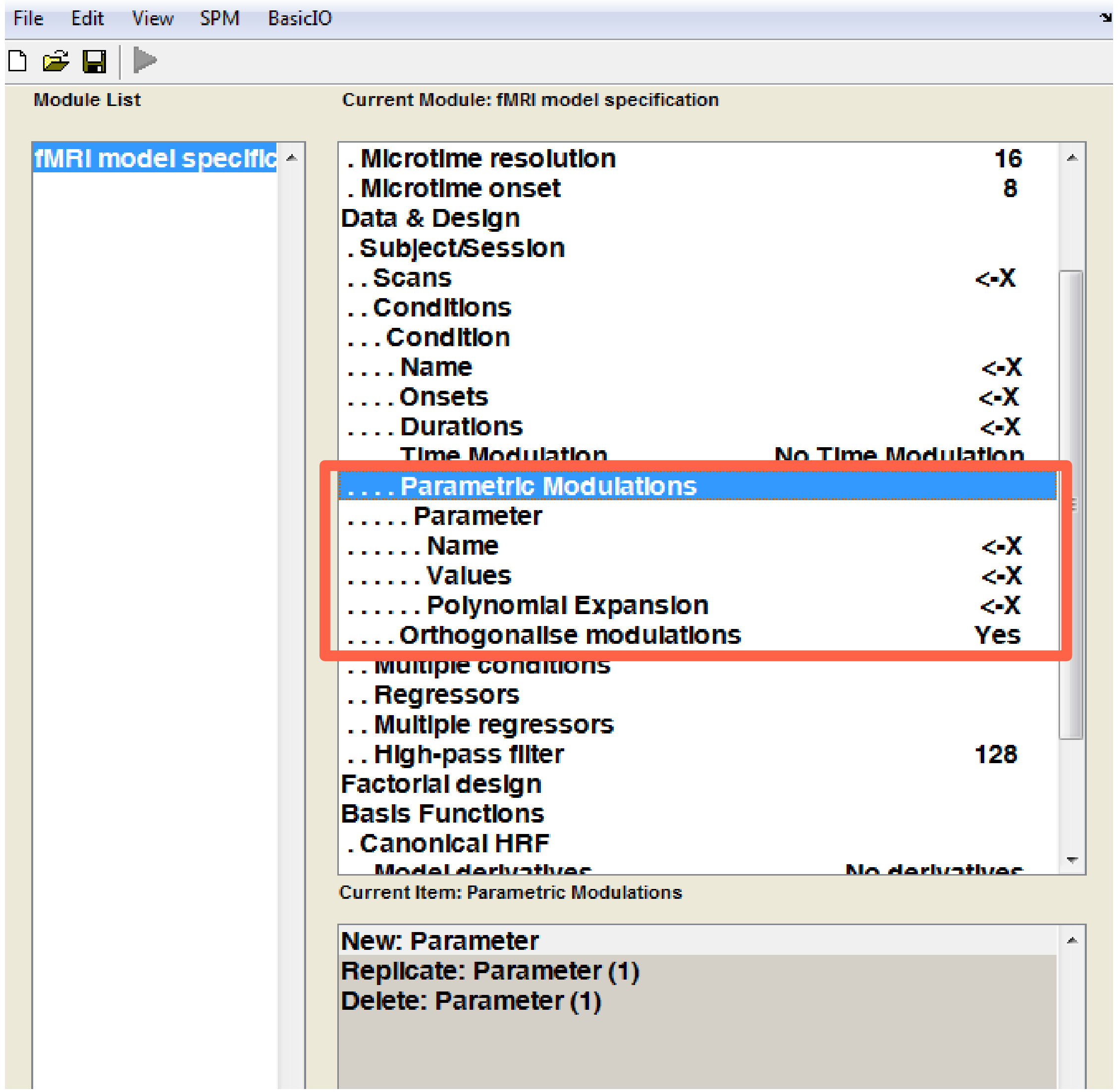
statistics
computing



Gläscher & O'Doherty (2010)

Implementing in SPM12





SPM12 – batch scripting


```
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).name = 'onsetPE';
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).onset = onscat.sub(i_sub).cueoutcome;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).duration = 0;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).tmod = 0;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).pmod.name = 'PE';
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).pmod.param = pe(i_sub);
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).pmod.poly = 1;
matlabbatch{1}.spm.stats.fmri_spec.sess.cond(cnt).orth = 0;
```

make sure: length(onset) == length(PE)

A closer look at PE

Prediction error:

$$PE = R_t - V_t$$

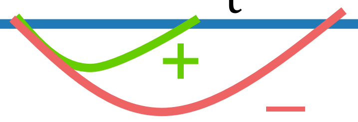
outcome

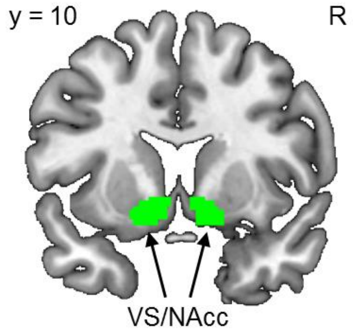
Q: how to justify the striatal activity is indeed associated with PE, rather than reward?

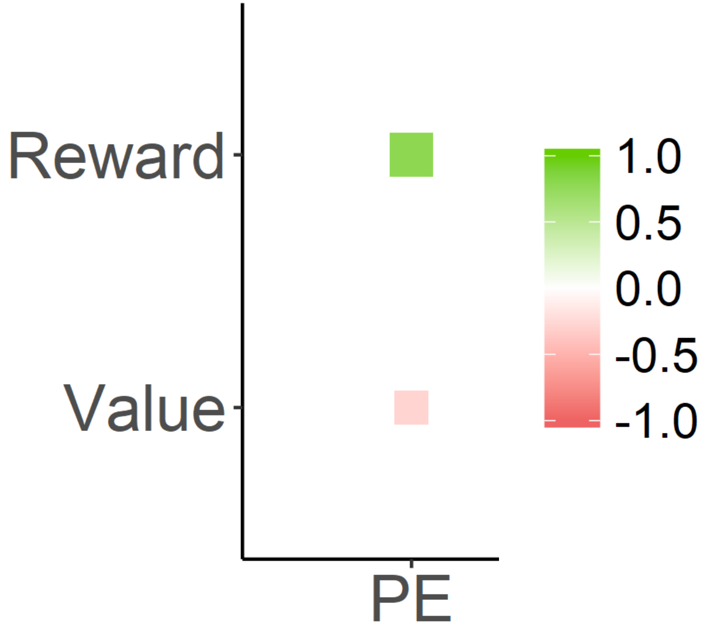
A closer look at PE

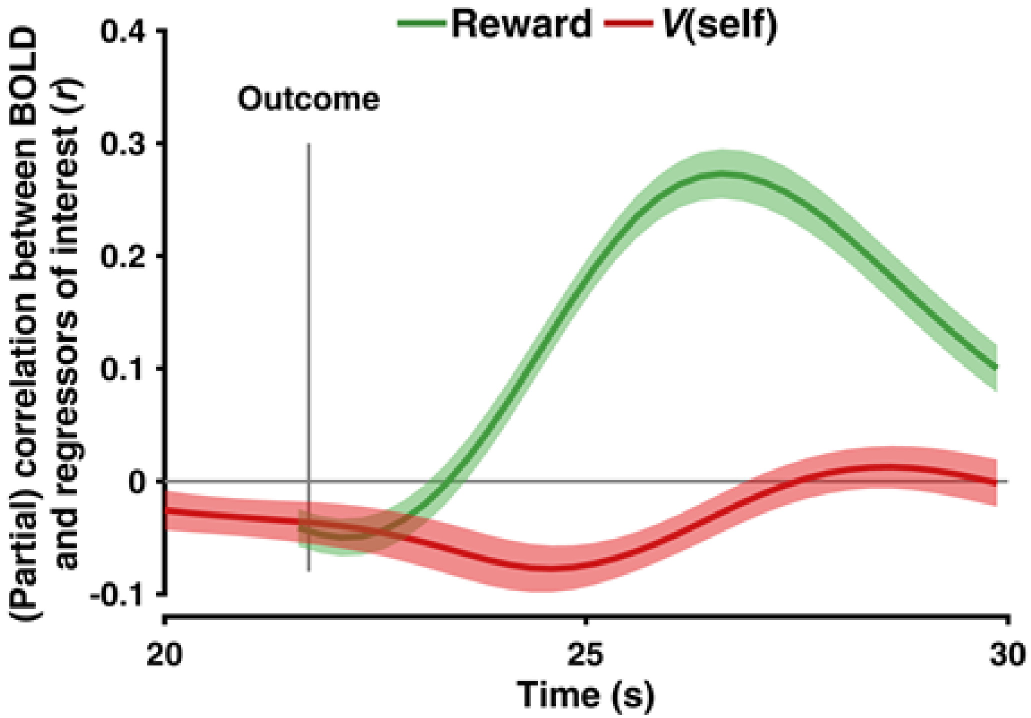
Prediction error:

$$PE = R_t - V_t$$









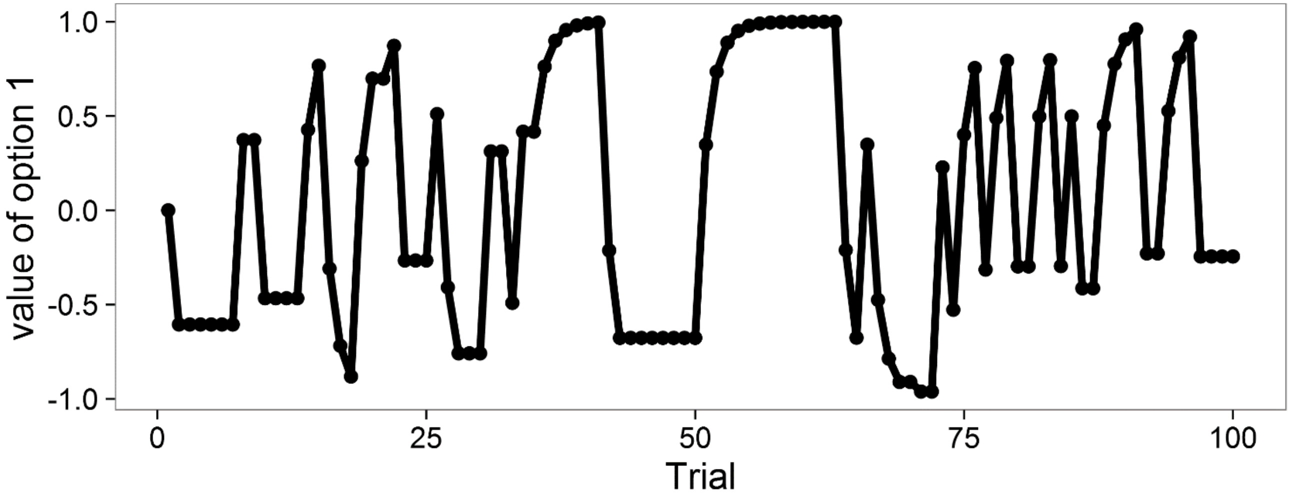
Zhang & Gläscher (in prep); Behrens et al. (2008)

Exercise II


```
…/BayesCog/10.model_based/_scripts/reinforcement_learning_model_based_main.R
```

TASK: extract and plot V(option=1), for subject01 (from L65/L105)

TIP: `fit_rl <- readRDS('_outputs/fit_rl.RData')`



*Value of the first option, instead of the chosen value

DELAY DISCOUNTING

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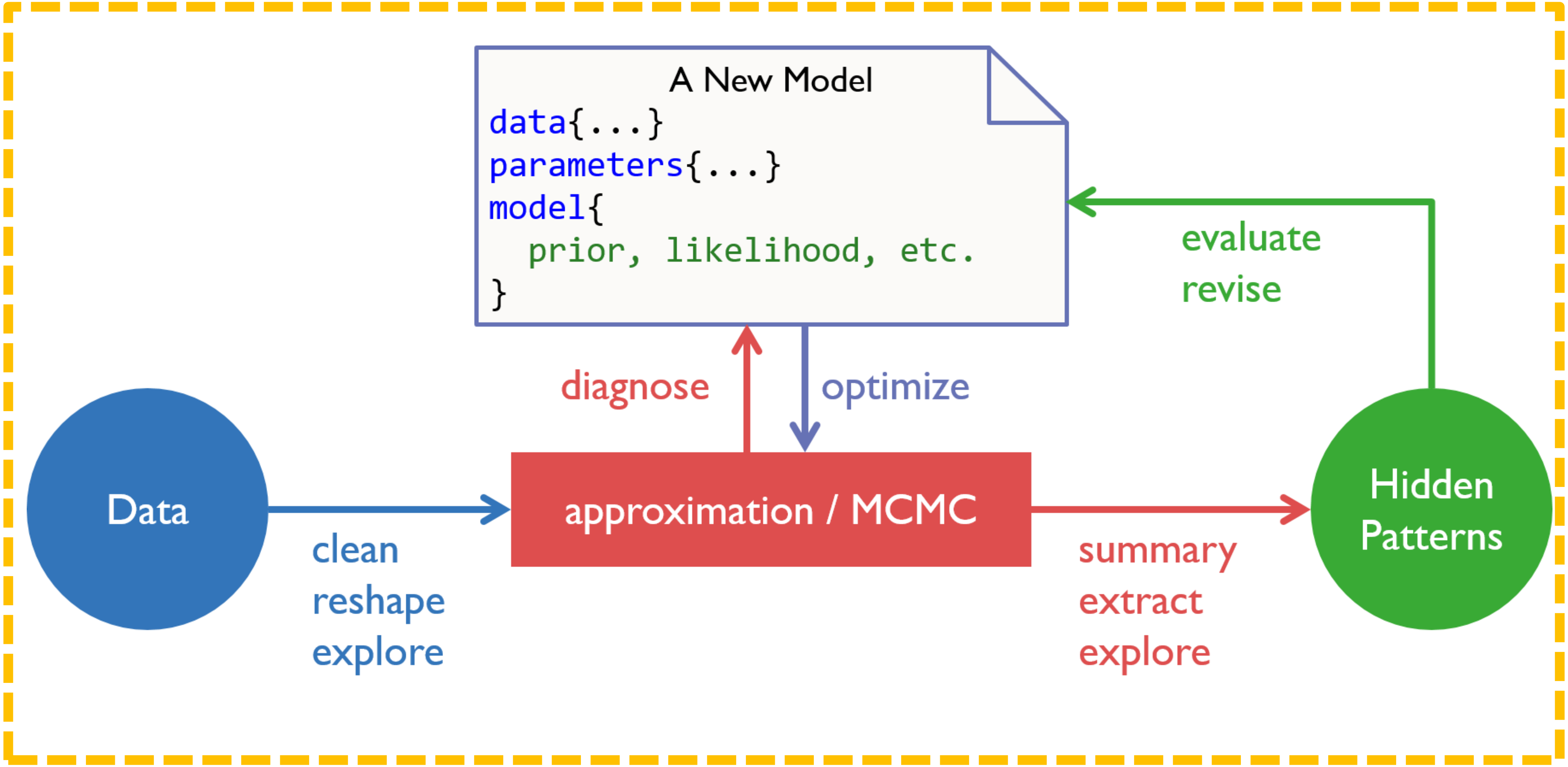
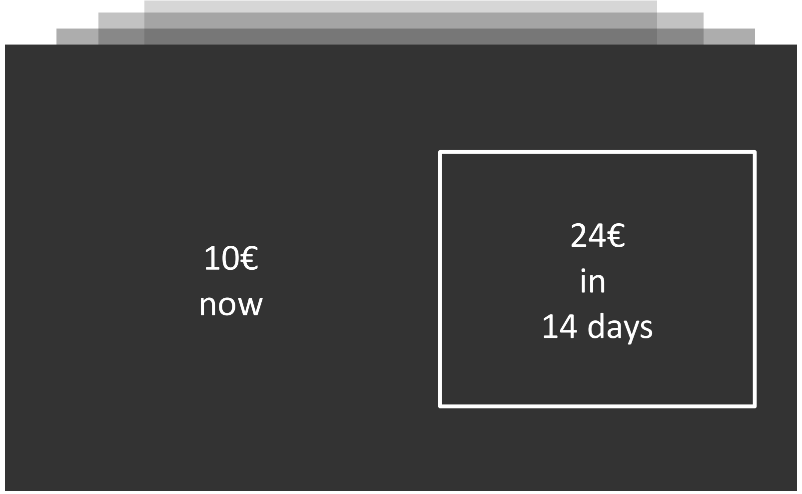
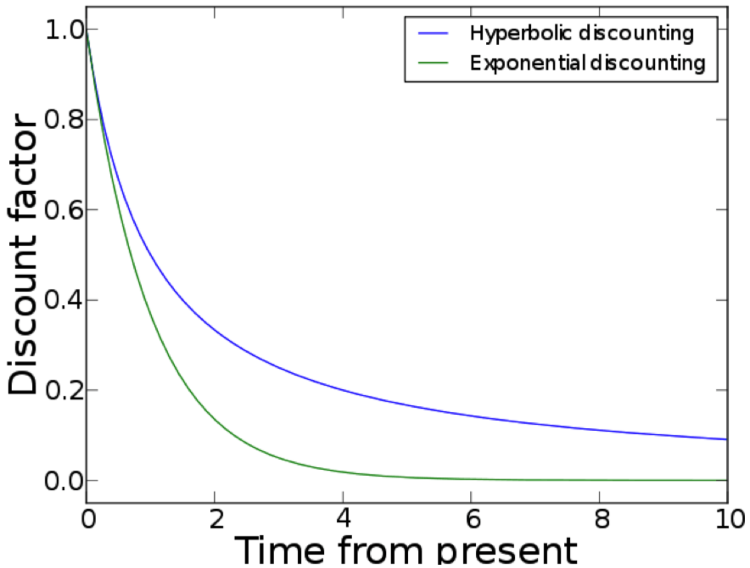
statistics
computing



Delay Discounting Task and Models

cognitive model
statistics
computing





Hyperbolic Discounting Model

$$SV = \frac{A}{1 + k * delay}$$

Exponential Discounting Model

$$SV = A * \exp(-r * delay)$$

$$p(LL) = \frac{1}{1 + \exp^{temp(v(SS) - v(LL))}}$$

```
LL – late large option
SS – soon small option
```

Exercise III

cognitive model

statistics

computing

```
.../BayesCog/11.delay_discounting/_scripts/delay_discounting_main.R
```

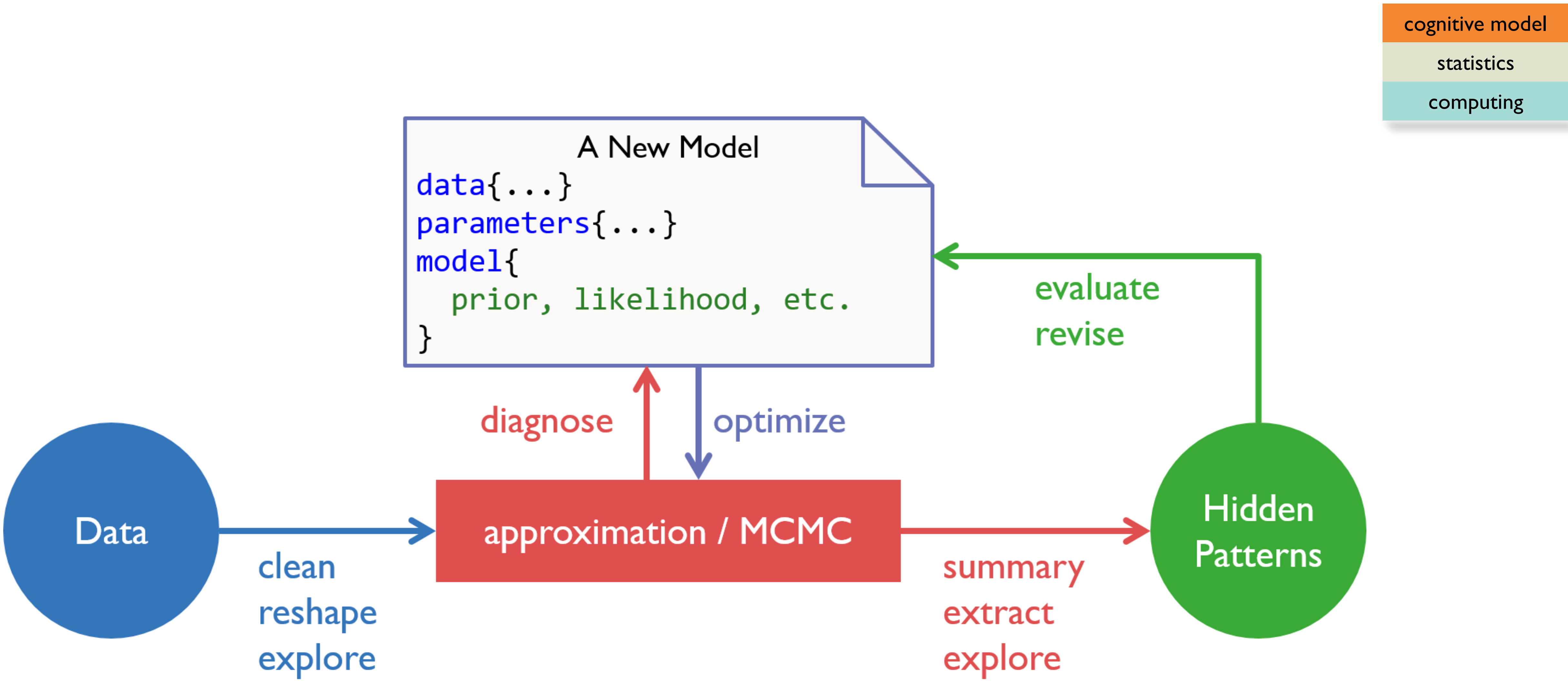
TASK:

(1) understand how to deal with missing trials

(2) complete and fit both models

(3) complete the main script for comparing the two models


Summary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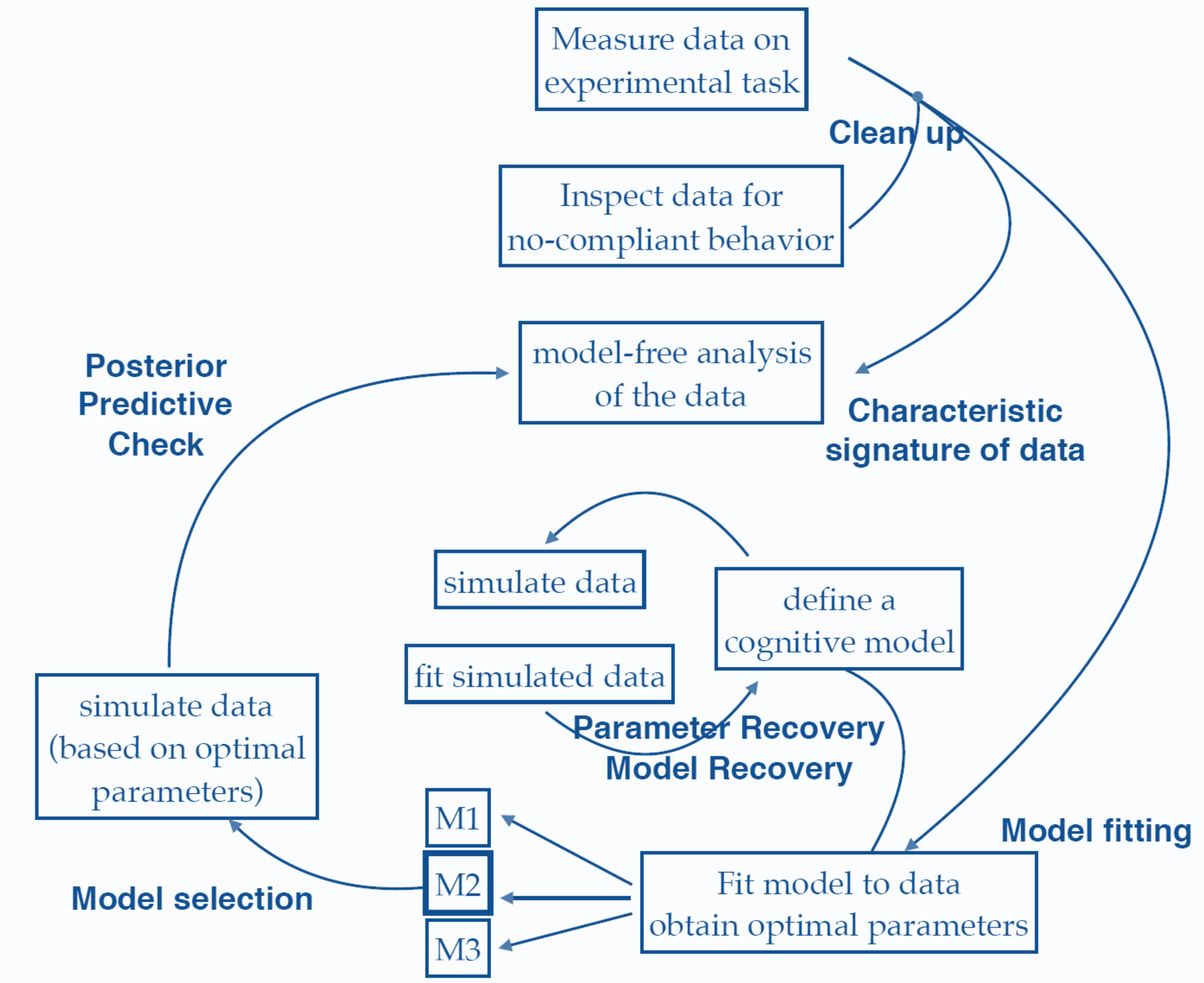
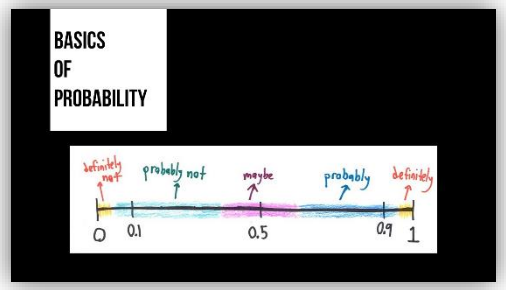
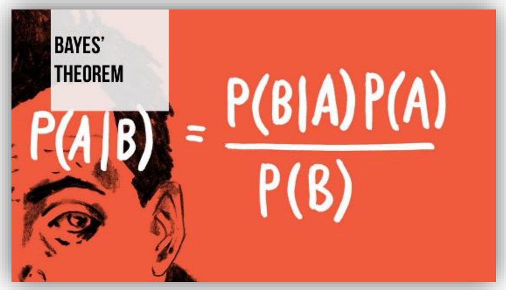


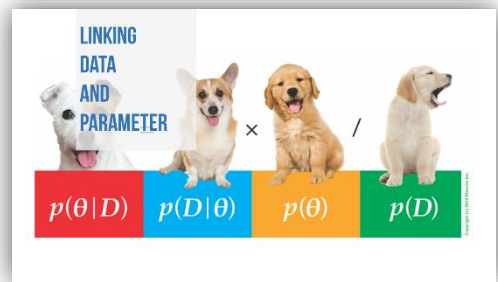
cognitive model
statistics
computing

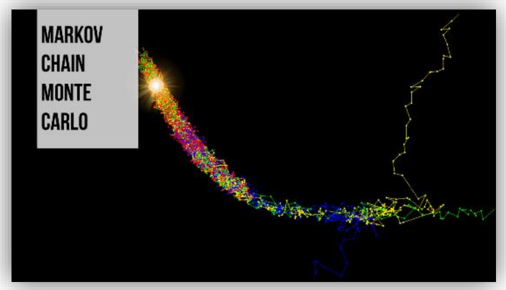
Adapted from Jan Gläscher's workshop

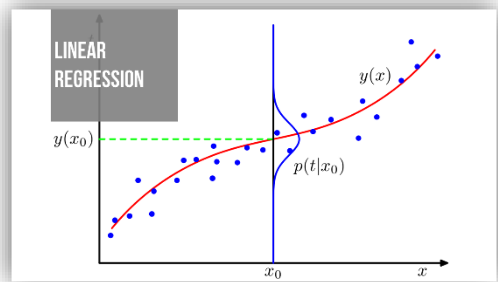
Summary of Topics

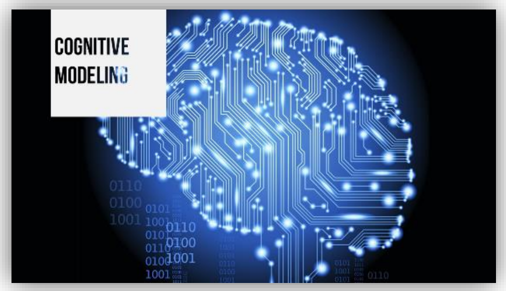


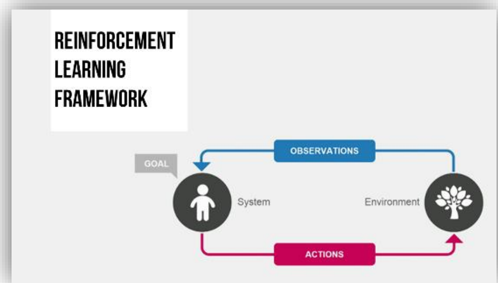


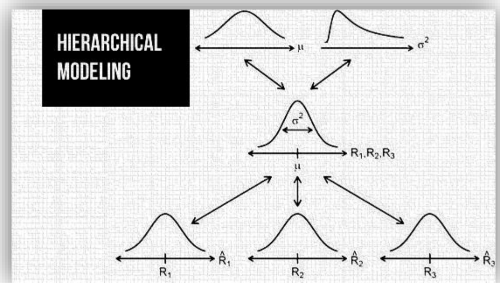


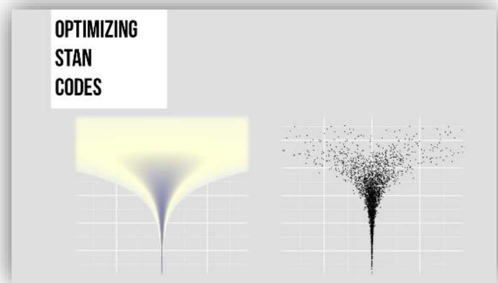


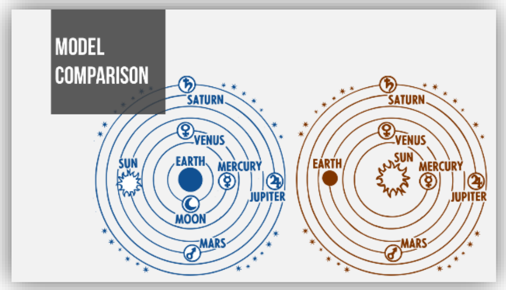


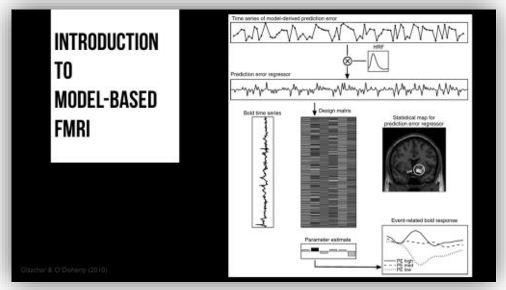


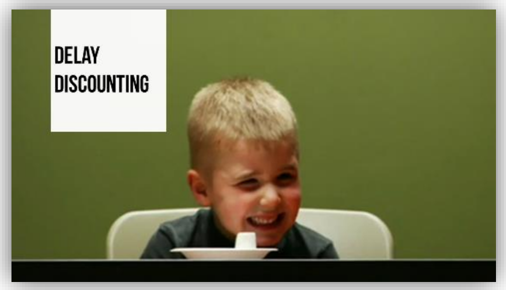












Summary of Examples/Exercises

FOLDER	TASK	MODEL
01.R_basics	NA	NA
02.binomial_globe	Globe toss	Binomial Model
03.bernoulli_coin	Coin flip	Bernoulli Model
04.regression_height 05.regression_height_poly	Observed weight and height	Linear regression model
06.reinforcement_learning 07.optm_rl	2-armed bandit task	Simple reinforcement learning (RL) model
08.compare_models	Probabilistic reversal learning task	Simple and fictitious RL models
09.debugging	Memory Retention	Exponential decay model
10.model_based	2-armed bandit task	Simple RL model
11.delay_discounting	Delay discounting task	Hyperbolic and exponential discounting model

After the Workshop, you...

…are able to implement your own model

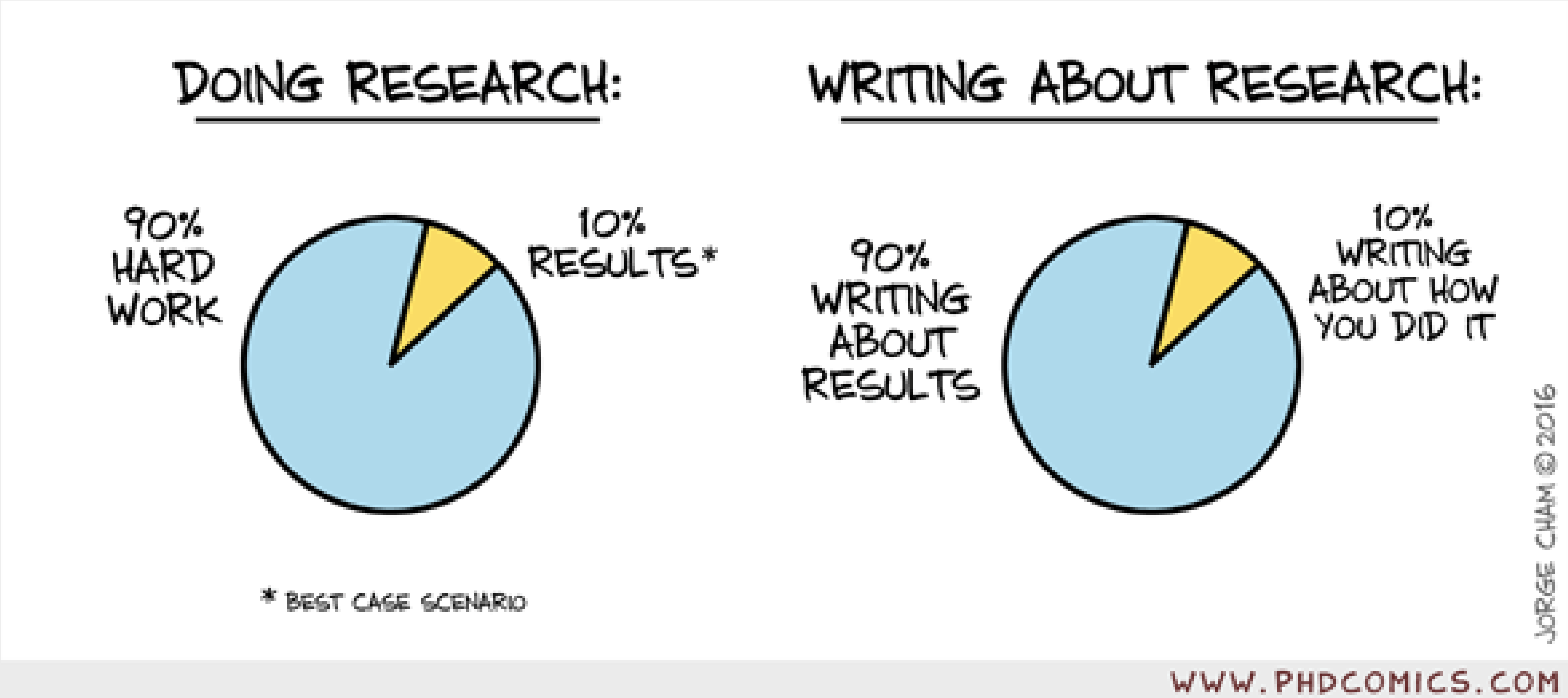
…feel comfortable with reading mathematical equations

…consider the implementation of the "computational modeling" section

…gain insightful understanding of Bayesian stats and modeling

…take it as a good start and work on it later

Remember: practice makes perfect!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90% HARD WORK
10% RESULTS*
* BEST CASE SCENARIO
WRITING ABOUT RESEARCH:
90% WRITING ABOUT RESULTS
10% WRITING ABOUT HOW YOU DID IT
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RESEARCH

Revealing Neurocomputational Mechanisms of Reinforcement Learning and Decision-Making With the hBayesDM Package

Woo-Young Ahn[1], Nathaniel Haines[1], and Lei Zhang[2]

[1]Department of Psychology, The Ohio State University, Columbus, OH

[2]Institute for Systems Neuroscience, University Medical Center Hamburg-Eppendorf, Hamburg, Germany

Keywords: Reinforcement learning; Decision-making, Hierarchical Bayesian modeling, Model-based fMRI

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RESEARCH

Revealing Neurocomputational Mechanisms of Reinforcement Learning and Decision-Making With the hBayesDM Package

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Task (alphabetical order)	Model name	hBayesDM function	References (see below for full citations)
Balloon Analogue Risk Task	4 parameter model	bart_4par	Wallsten et al. (2005)
Choice reaction time Task	Drift diffusion model Linear Ballistic Accumulator model	choiceRT_ddm choiceRT_lba	Ratcliff (1978) S. Brown & Heathcote (2008) Annis et al. (2017)
Choice under Risk and Ambiguity (CRA) Task	Linear model Exponential model	cra_linear cra_exp	Levy et al. (2009)
Delay Discounting Task	Constant-Sensitivity (CS) model Exponential model Hyperbolic model	dd_cs dd_exp dd_hyp	Ebert & Prelec (2007) Samuelson (1937) Mazur (1987)
Iowa Gambling Task (IGT)	Prospect Valence Learning-DecayRI Prospect Valence Learning-Delta Value-Plus-Perseverance (VPP) Outcome-Represent. Learning (ORL)	igt_pvl_decay igt_pvl_delta igt_vpp igt_orl	Ahn et al. (2011; 2014) Ahn et al. (2008) Worthy et al. (2013) Haines et al. (in press)
Orthogonalized Go/Nogo Task	RW+noise RW+noise+go bias RW+noise+go bias+Pav. bias M5 (see Table 1 of the reference)	gng_m1 gng_m2 gng_m3 gng_m4	Guitart-Masip et al. (2012) Guitart-Masip et al. (2012) Guitart-Masip et al. (2012) Cavanagh et al. (2013)
Peer influence task	Other-conferred utility (OCU)	peer_ocu	Chung et al. (2015)
Probabilistic Reversal Learning (PRL) Task	Experience-Weighted Attraction Fictitious update Reward-Punishment (Rew.-Pun.) Fictitious + Rew.-Pun. Fictitious + Rew.-Pun. w/o alpha Fictitious w/o alpha	prl_ewa prl_fictitious prl_rp prl_fictitious_rp prl_fictitious_rp_woa prl_fictitious_woa	Ouden et al. (2013) Gläscher et al. (2009) Ouden et al. (2013)
Probabilistic Selection Task	Q-learning with two learning rates	pst_gainloss_Q	M. J. Frank et al. (2007)
Risk-Aversion Task	Prospect Theory (PT) PT without loss aversion (LA) PT without risk aversion (RA)	ra_prospect ra_noLA ra_noRA	Sokol-Hessner et al. (2009) Tom et al. (2007)
Risky Decision Task	Happiness model	rdt_happiness	Rutledge et al. (2014)
Two-Armed Bandit (Experience-based) Task	Rescorla-Wagner (delta) model	bandit2arm_delta	Erev et al. (2010) Hertwig et al. (2004)
Two Step (TS) Task	7 parameter model 6 parameter model 4 parameter model	ts_7par ts_6par ts_4par	Daw et al. (2011) Wunderlich et al. (2012)
Four-Armed Bandit (Experience-based) Task	Fictive upd.+rew/pun sens. Fictive upd.+rew/pun sens.+lapse	bandit4arm_4par bandit4arm_lapse	Seymour et al. (2012) Seymour et al. (2012)
Ultimatum Game	Ideal Bayesian observer model Rescorla-Wagner (delta) model	ug_bayes ug_delta	Xiang et al. (2013) Gu et al. (2015)
Wisconsin Card Sorting Task	Sequential learning model	wcs_sql	A. J. Bishara et al. (2010)

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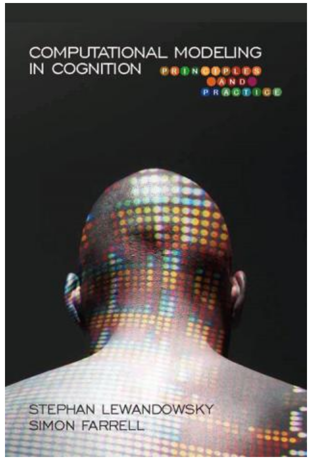
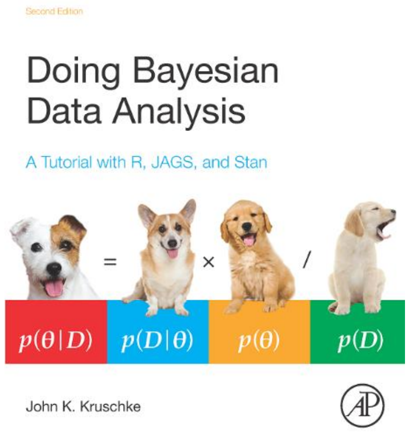
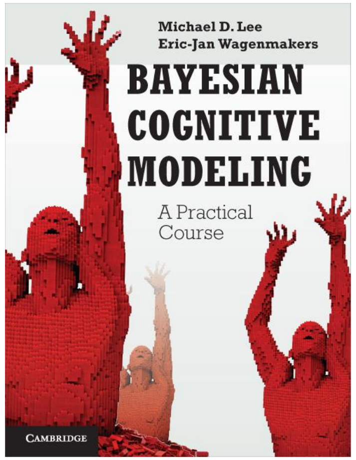
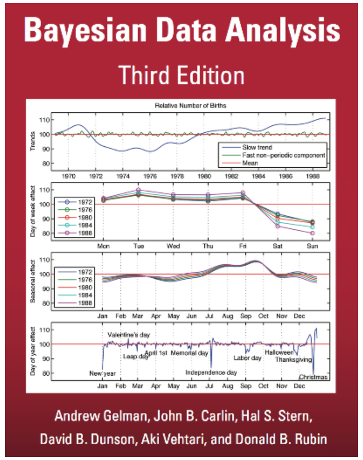
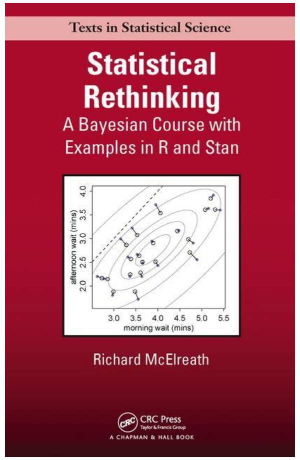
Resources

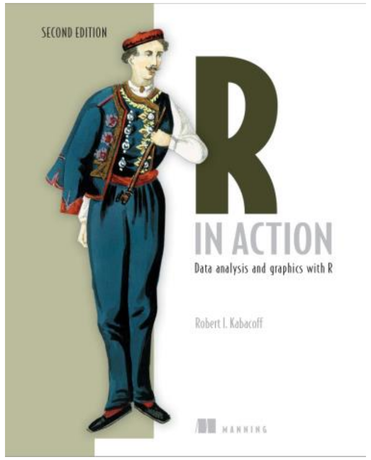












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- JAGS and WinBUGS Workshop @ Amsterdam, NL (annual)
- Model-based Neuroscience Summer School @ Amsterdam, NL (annual)
- European Summer School on Computational and Mathematical Modeling of Cognition @ multiple EU sites (biannual)
- Computational Psychiatry Course @ Zürich, CH (annual)
- London Computational Psychiatry Course @ London, UK (annual?)
- Methods In Neuroscience At Dartmouth Computational Summer School @ Dartmouth, US (annual)
- Brains, Minds & Machines Summer Course @ MIT, US (annual)

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ANY
QUESTIONS
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Happy Computing!