

# Formally verified algorithms and data structures in Coq: concepts and techniques

(Formalnie zweryfikowane algorytmy i struktury danych w Coqu: koncepty i  
techniki)

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## Abstract

We discuss how to design, implement, specify and verify functional algorithms and data structures, concentrating on formal proofs rather than asymptotic complexity or actual performance. We present concepts and techniques, both of which often rely on one key principle – the reification and representation, using Coq’s powerful type system, of something which in the classical-imperative approach is intangible, like the flow of information in a proof or the shape of a function’s recursion. We illustrate our approach using rich examples and case studies.

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Omawiamy sposoby projektowania, implementowania, specyfikowania i weryfikowania funkcyjnych algorytmów i struktur danych, skupiając się bardziej na dowodach formalnych niż na asymptotycznej złożoności czy faktycznym czasie działania. Prezentujemy koncepty i techniki, obie często opierające na jednej kluczowej zasadzie – reifikacji i reprezentacji, za pomocą potężnego systemu typów Coq, czegoś co w klasycznym, imperatywnym podejściu jest nieuchwytne, jak przepływ informacji w dowodzie czy kształt rekursji funkcji. Nasze podejście bogato ilustrujemy przykładami i studiami przypadku.



# Contents

<b>1</b>	<b>Introduction</b>	<b>7</b>
1.1	Imperative and functional algorithms . . . . .	7
1.2	Complexity, performance and correctness . . . . .	8
1.3	An overview of available literature . . . . .	8
<b>2</b>	<b>Binary search trees – an extended case study</b>	<b>9</b>
<b>3</b>	<b>A man, a plan, a canal – MSc thesis</b>	<b>11</b>
3.1	Things to write about . . . . .	11
	<b>Bibliography</b>	<b>13</b>



# Chapter 1

## Introduction

The topic of this thesis are functional algorithms and data structures <sup>1</sup>. The Free Dictionary says [1] that an algorithm is

A finite set of unambiguous instructions that, given some set of initial conditions, can be performed in a prescribed sequence to achieve a certain goal and that has a recognizable set of end conditions.

The purpose of this entry is to explain the concept to a lay person, but it likely sounds just about right to the imperative programmer’s ear too. To a functional ear, however, talking about sequences of instructions most certainly sounds as un-functional as it possibly could. It is no wonder then that some people wonder if it is even possible for algorithms to “exist” in a functional programming language, as exemplified by this StackOverflow question [2]. The poor soul asking this question had strongly associated algorithms with imperative languages in his head, even though functional languages are based on lambda calculus, which was invented precisely to formalize what an algorithm is.

### 1.1 Imperative and functional algorithms

This situation is not uncommon and rather easy to explain. Imperative algorithms and data structures form one of the oldest, biggest, most widespread and prestigious fields of computer science. They are taught to every student in every computer science programme at every university. There’s a huge amount of textbooks, with classics such as [3] [4] known to pretty much everybody, at least by title. There’s an even huger and still growing mass of research articles and conferences and I’m pretty sure there are at least some (imperative) algorithm researchers at every computer

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<sup>1</sup>In order not to be too repetitive, in this thesis the word “algorithms” means “algorithms and data structures”

science department in existence. Imperative algorithms and data structures and pretty much synonymous with “competitive programming”

## 1.2 Complexity, performance and correctness

Differences between performance-oriented design and formal-correctness-oriented design.

## 1.3 An overview of available literature

Literature review, Okasaki is old and bad for Coq, SF3 is shallow.



## Chapter 2

# Binary search trees – an extended case study

Binary search trees: a case study to show the basic workflow and that it's not that obvious how to get basic stuff right.



## Chapter 3

# A man, a plan, a canal – MSc thesis

### 3.1 Things to write about

- Design: we shouldn't require proofs in order to run programs. Ways of doing general recursion and, connected with it, functional induction as the way-to-go proof technique. Maybe something about the equations plugin. A word about classes, records and modules.
- Quicksort: in functional languages we have so powerful abstractions that we can actually implement algorithms and not just programs.
- Braun mergesort: in order not to waste resources, we sometimes have to reify abstract patterns, like the splitting in mergesort.
- Cool data structures: ternary search trees, finger trees.



# Bibliography

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