LAB2: STATE FEEDBACK ROBUST AND ROBUST ADAPTIVE CONTROL

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Introduction:

In the laboratory work, we discussed the use of robust and adaptive-robust controllers in systems with unknown parameters and disturbances. First, both types of controllers will be tested on a plant with unknown parameters and added disturbance, the difference in terms of control error and parametric error will be discussed between the two. Next, the effect of controllers' parameters on the errors will be investigated.

In this report, we will state the main theory used, and then go over the steps of the work with the plotted graphs of each step.

Theory:

Let's consider a system affected by disturbance written in the form:

$$\dot{x} = Ax + bu + \delta$$
$$y = c^T x$$

Where δ is the vector of disturbances and it satisfies $\|\delta\| < \bar{\delta}$. We need this plant to follow a reference model that satisfies a desired transient response (settling time and overshoot) given as:

$$\dot{x}_M = A_M x_M + b_M g$$

Where g is a reference signal that's piecewise continuous.

We need to implement a controller $u(t)$ without knowledge of the parameters of the plant (elements of A) or the disturbance signal. We need to ensure that $\|x_M(t) - x(t)\| \leq \Delta$ for any $t \geq T$ where T is the transient time and Δ is the steady state error. We see that in existence of disturbances, we cannot strictly condition the control error to converge to 0, however, we will see that we can decrease Δ sufficiently by adjusting the controllers parameters.

We saw in LAB1 that assuming a full knowledge of the plant, one can design a controller of the form $u=\theta^T x+\frac{1}{\kappa}g$ where $\theta=[\theta_1,\theta_2,\ldots,\theta_n]^T;\theta_i=\frac{-a_{M(i-1)}+a_{(i-1)}}{b_0}$ and $\kappa=\frac{b_0}{a_{M0}}$

However, if the system parameters are unknown, we can adjust this control law to use the estimated values of θ denoted as $\hat{\theta}$ to become:

$$u=\hat{\theta}^T x+\frac{1}{\kappa}g$$

Two types of methods that can be used to produce the values of $\hat{\theta}$ will be discussed and compared:

1. Nonlinear static feedback (Robust Control):

$$\hat{\theta}=\gamma x b^T P e$$

where P is the solution to the equation $A_M^T P+PA_M=-Q$ given that Q is a positive defined matrix. γ is a feedback gain, x is the state vector and e is the control error vector.

This choice ensures the following properties:

   1. Boundedness of all the signals in the closed-loop system
   2. Convergence of the value $||e||$ to residual set including zero origin.

We can see that there is no guarantee of the control error convergence to zero (even in the zero-disturbances case). However, the radius of the residual set can be decreased by increasing the feedback gain γ . Another important note is there's no guarantee of the parametric error $\bar{\theta}=\theta-\hat{\theta}$ to converge at all. This controller 'suppresses' the effect of the disturbance and parametric error.

2. Adaptive control with robust modification:

$$\dot{\hat{\theta}}=\gamma x b^T P e-\sigma\hat{\theta};\hat{\theta}(0)=0$$

Where γ is the adaptation gain and σ is the static feedback gain. Choosing this formula ensures all the properties of the first formula and add to it one property ; Convergence of the value $||\bar{\theta}||$ to residual set including zero origin. Both the parametric error and control error cannot be guaranteed to go to zero even with zero disturbances. However, the radius of the residual sets can be decreased by increasing the adaptation gain γ and decreasing the feedback gain σ .

The order of the work:

In the work, we discuss a second order system, the values of matrices A and b are:

$$A=\begin{bmatrix}0 & 1\\ 10 & 1\end{bmatrix}, b=\begin{bmatrix}0\\ 1\end{bmatrix}$$

The system's parameters aren't known for the controllers. The system is effected by disturbance vector of the form :

$$\delta(t)=[0.6\sin(10t)+0.1\sin(50t) \quad 0.5\cos(12t)+0.2\sin(30t)]^T$$

The reference model satisfies a transient time $t_T=0.2\,s$ and overshoot $\bar{\sigma}=0\%$. We showed in the first LAB that the reference model that satisfies this transient response is:

$$\dot{x}_M=\begin{bmatrix}0 & 1\\ -849.7225 & -58.3\end{bmatrix}x_M+\begin{bmatrix}0\\ 849.7225\end{bmatrix}g$$

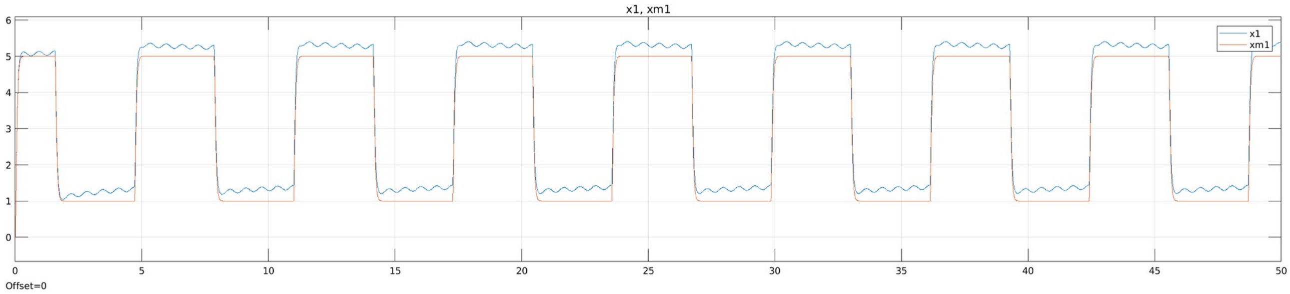
We considered a reference signal $g(t)=2\,sign(\cos(t))+3$.

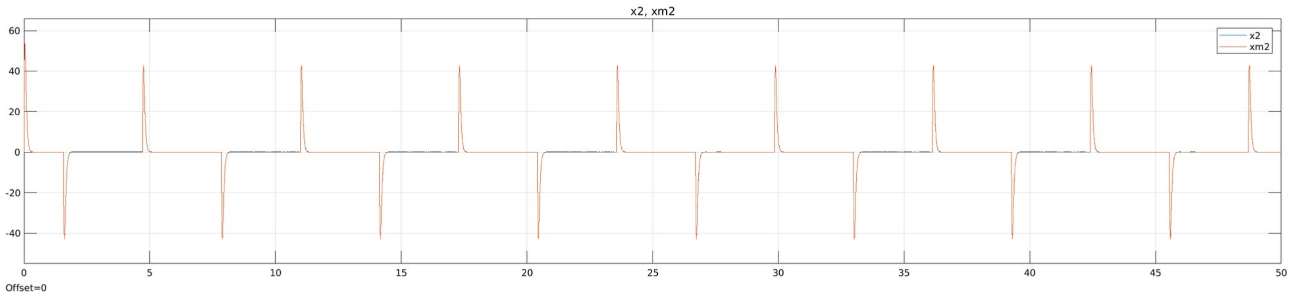
Experiment 1: In this experiment, the robust and adaptive-robust controllers will be compared on the model that include the disturbance vector stated before.

For the robust controller, the value of the feedback gain used is $\gamma=1000$.

For the adaptive-robust controller, the value of the adaptation gain used is $\gamma=1000$ and the value of the feedback gain is $\sigma=0.05$

The results of the adaptation algorithms are shown in Figure 1 and 2.





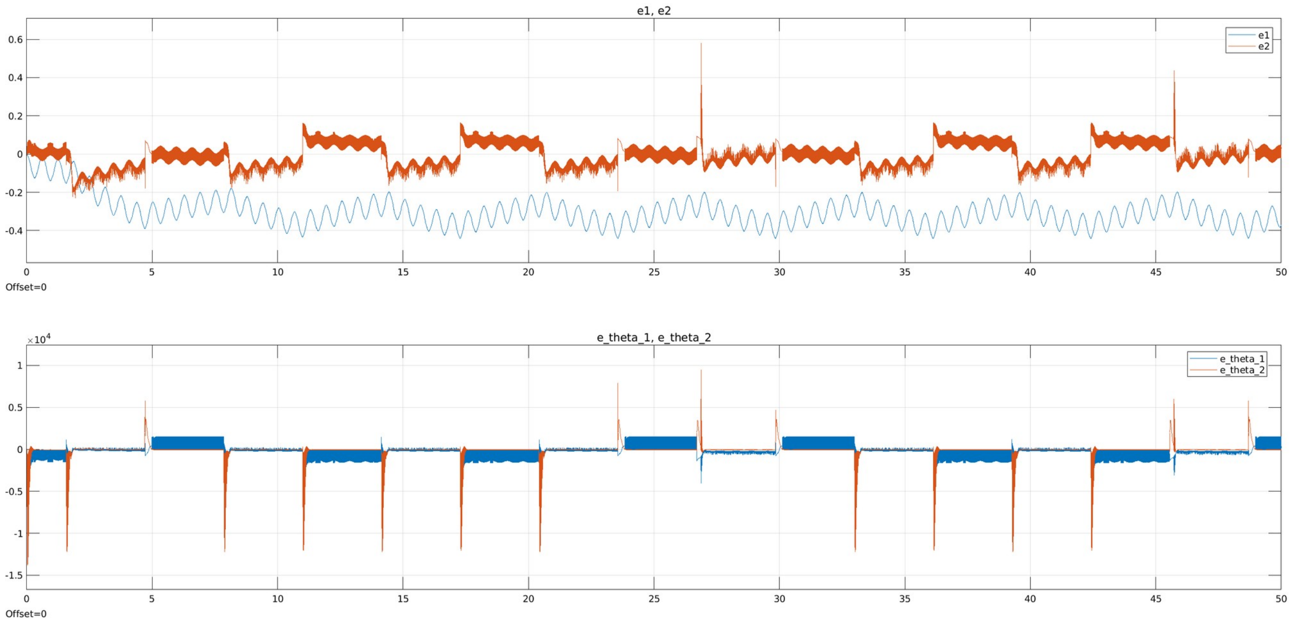
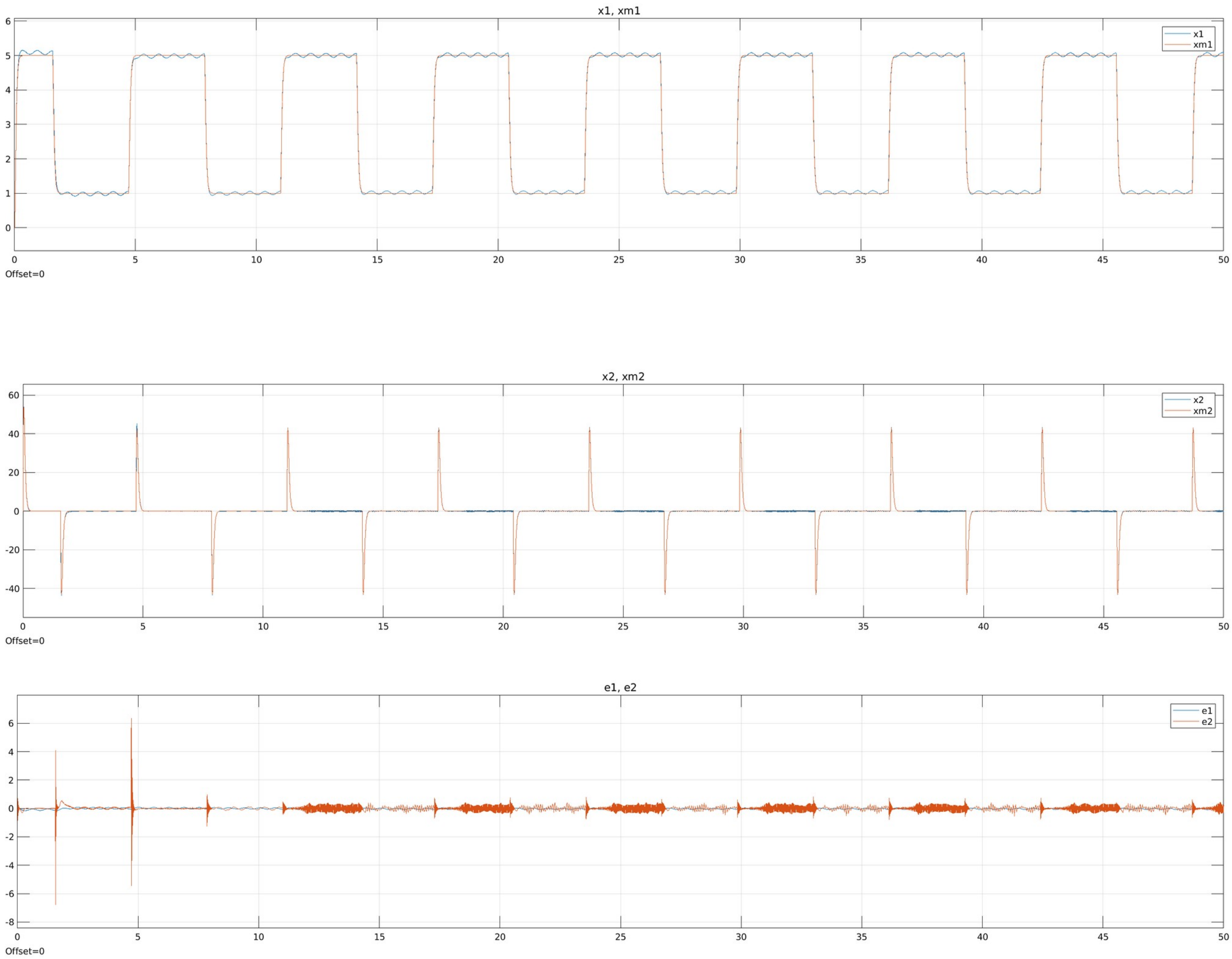


Figure 1: Results of the robust controller, feedback gain: 1000



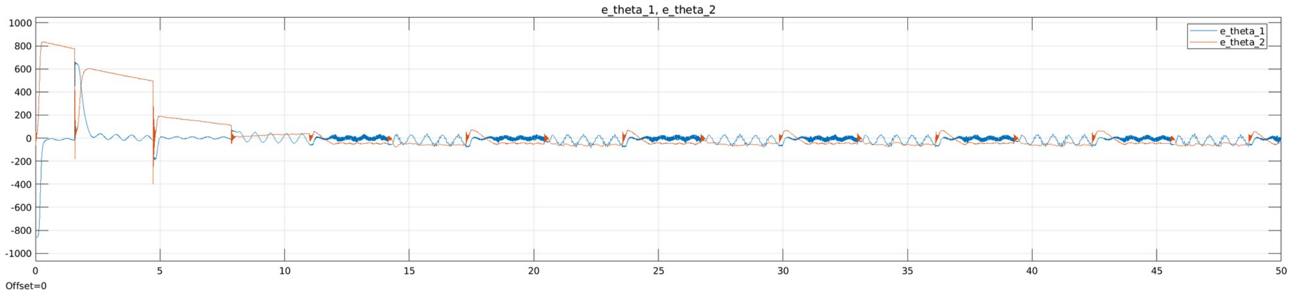
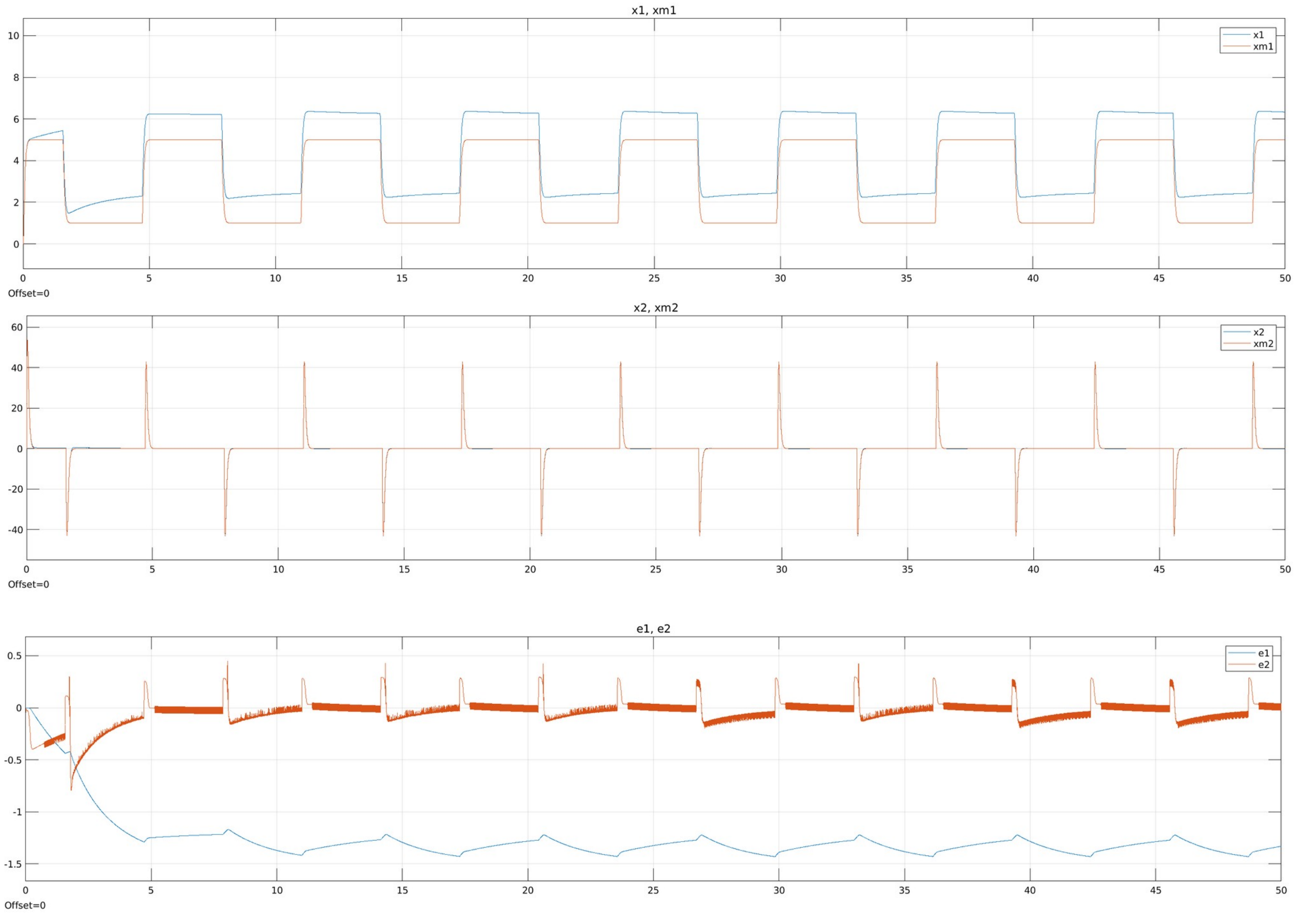


Figure 2: Results of the adaptive-robust controller, static feedback gain: 0.05, adaptation gain: 1000

We can see that both controllers were able to control the system and stabilize it. However, non of them were able to converge the control error to zero which is expected. Moreover, we can notice that only the adaptive-robust controller was able to produce good estimations of the plant parameters θ as expected.

Experiment 2: In this experiment we tested the effect of the feedback gain γ on the performance of the robust controller without the existence of disturbances. Three gain values were tested $\gamma = 100, 500, 1000$. The results are shown in Figures 3, 4, 5 .



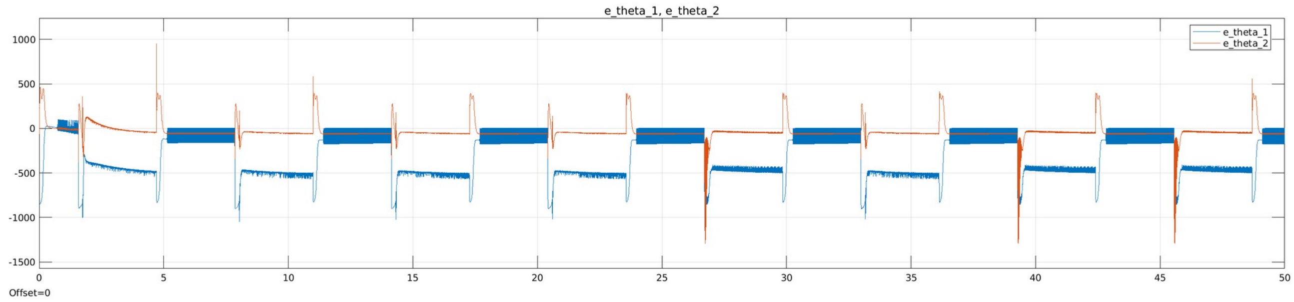


Figure 3: Results of the robust controller, feedback gain: 100 (no disturbances)

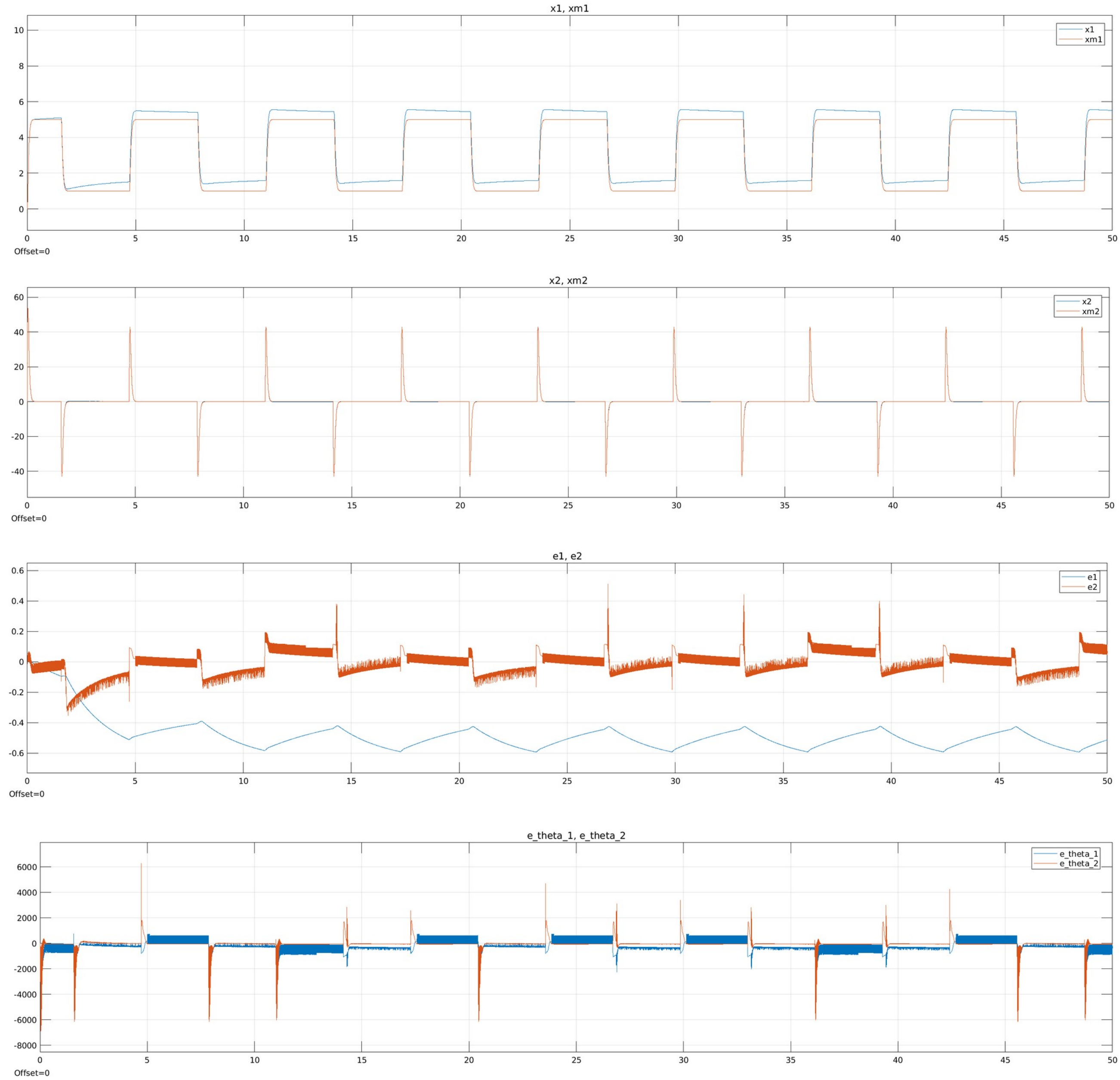


Figure 4: Results of the robust controller, feedback gain: 500 (no disturbances)

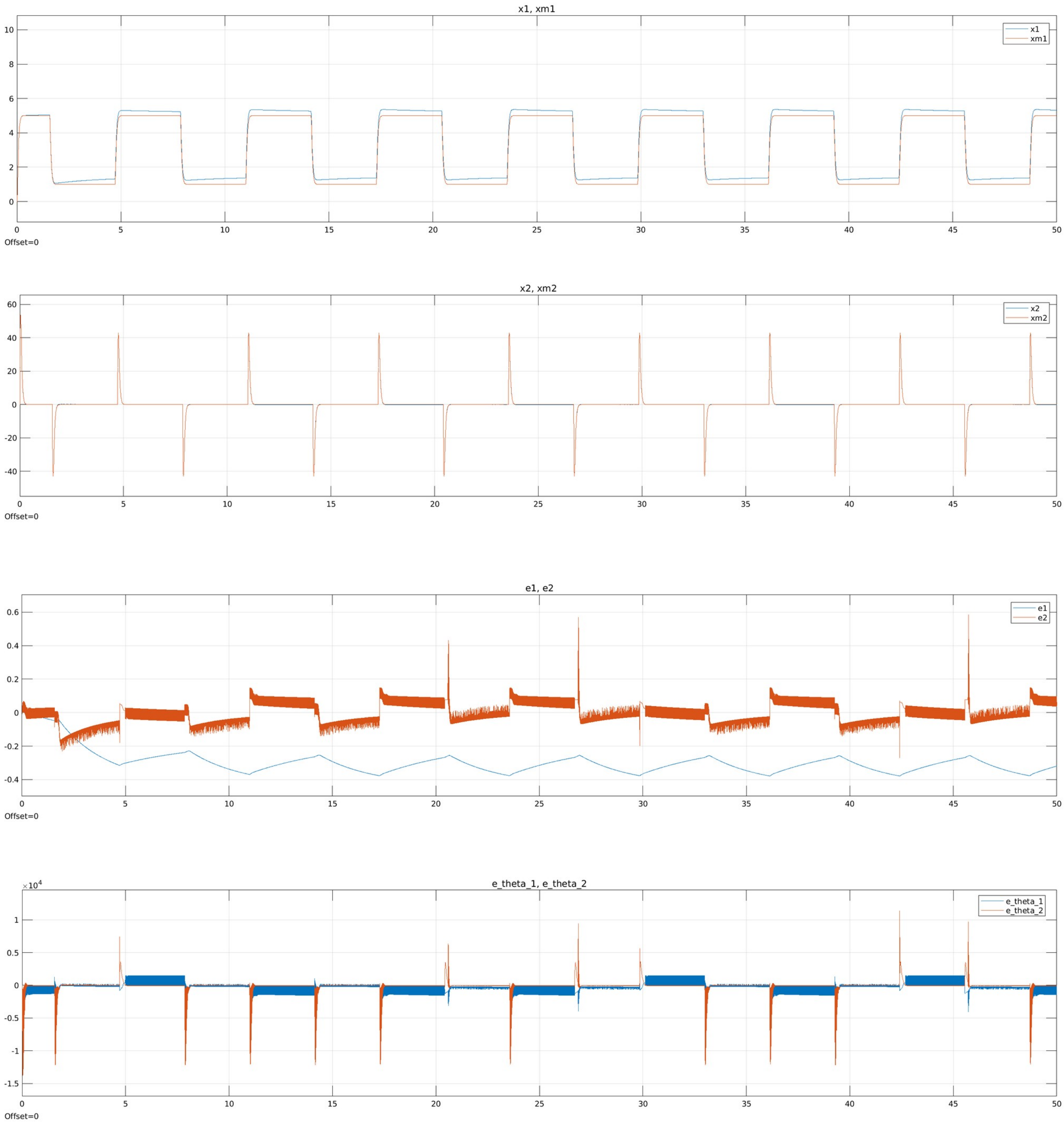


Figure 5: Results of the robust controller, feedback gain: 1000 (no disturbances)

We can clearly see the effect of the feedback gain on the radius of the residual set to which the control error converges. However, we can see that although there are no disturbances, this controller isn't able to converge the control error to zero. Moreover, it's apparent that this controller cannot produce meaningful estimation of the plant parameters.

Experiment 3: In this experiment , we will investigate the effect of the adaptation gain γ and the static feedback gain σ on the adaptive-robust controller. First, two different values of adaptation gain $\gamma=500,1500$ will be tested with $\sigma=0.1$ with and without disturbances. Then we we will decrease the value of σ to 0.01 with $\gamma=500$ and compare the results. The results are shown in Figures 6, 7, 8, 9, 10, 11.

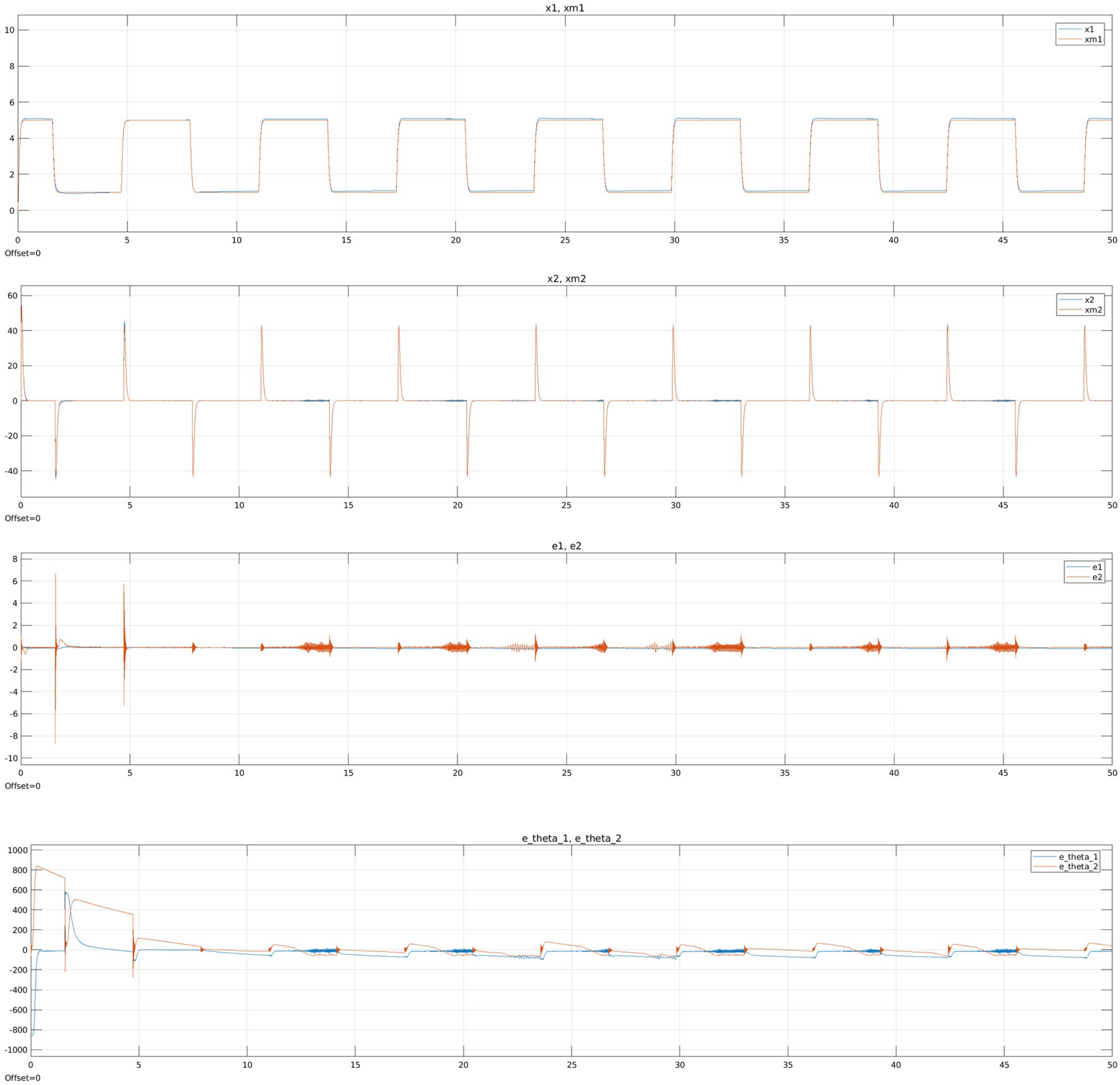


Figure 6: Results of the adaptive-robust controller, static feedback gain: 0.1, adaptation gain: 500 (no disturbances)

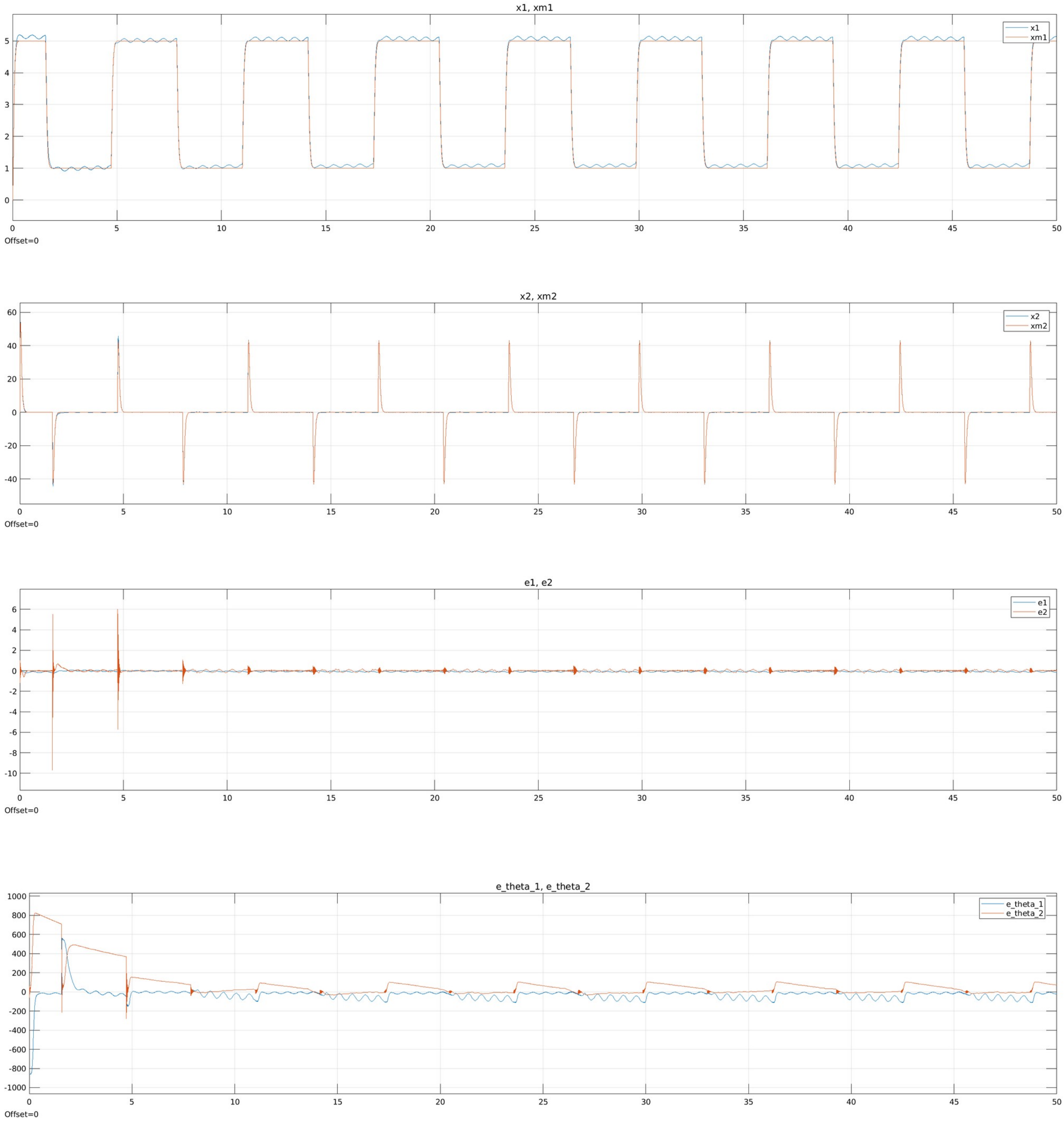


Figure 7: Results of the adaptive-robust controller, static feedback gain: 0.1, adaptation gain: 500 (with disturbances)

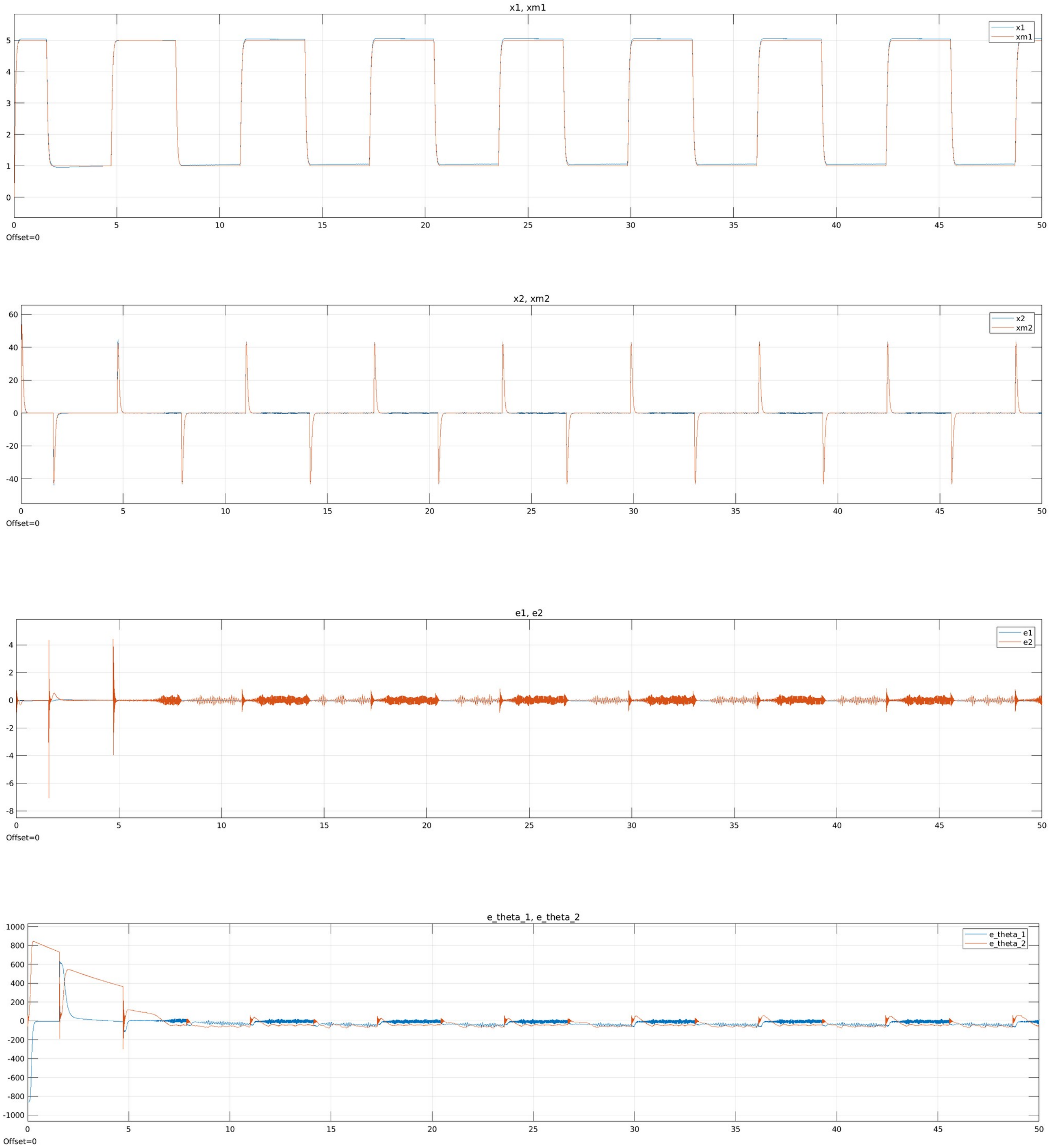


Figure 8: Results of the adaptive-robust controller, static feedback gain: 0.1, adaptation gain: 1500 (no disturbances)

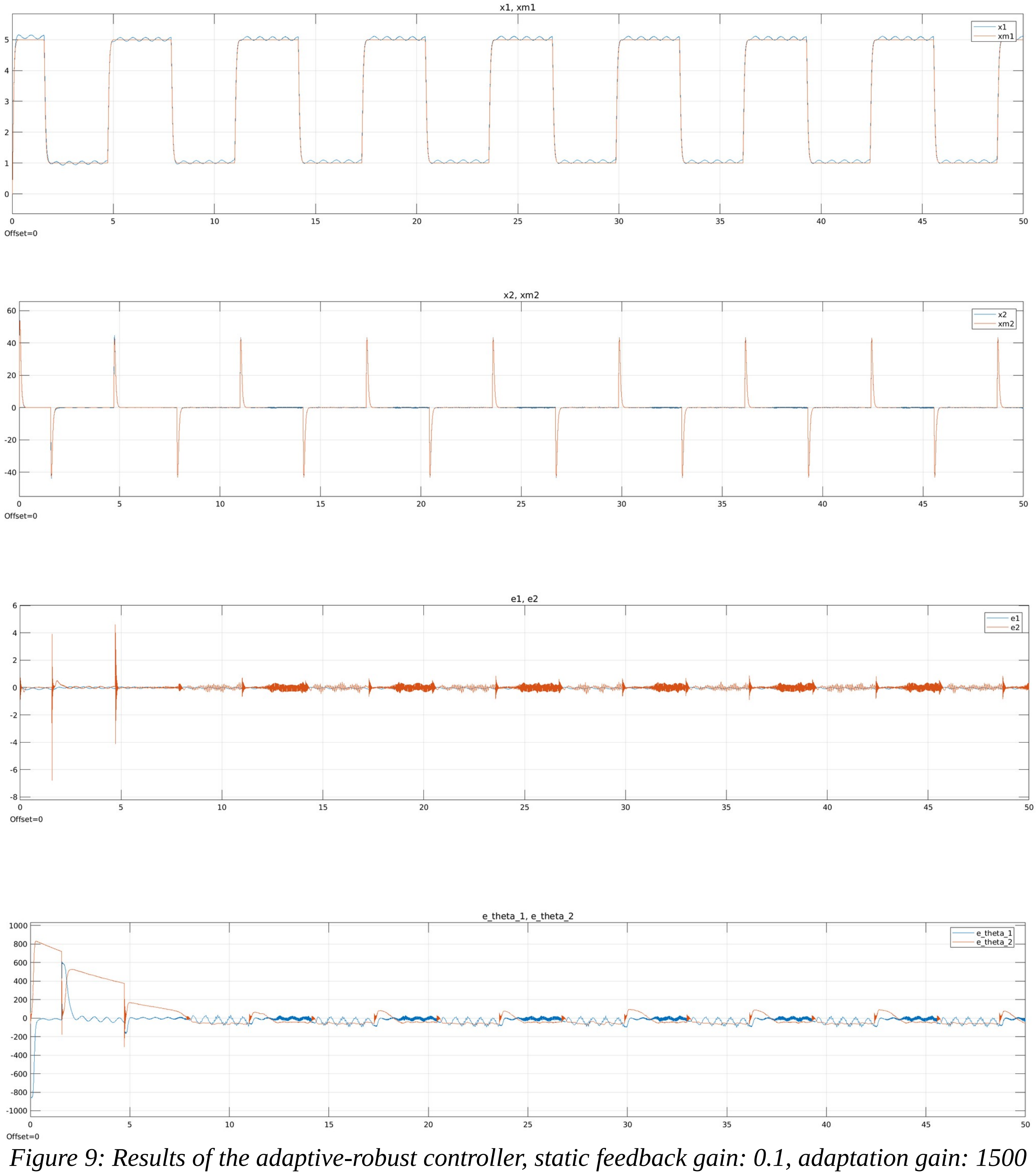


Figure 9: Results of the adaptive-robust controller, static feedback gain: 0.1, adaptation gain: 1500 (with disturbances)

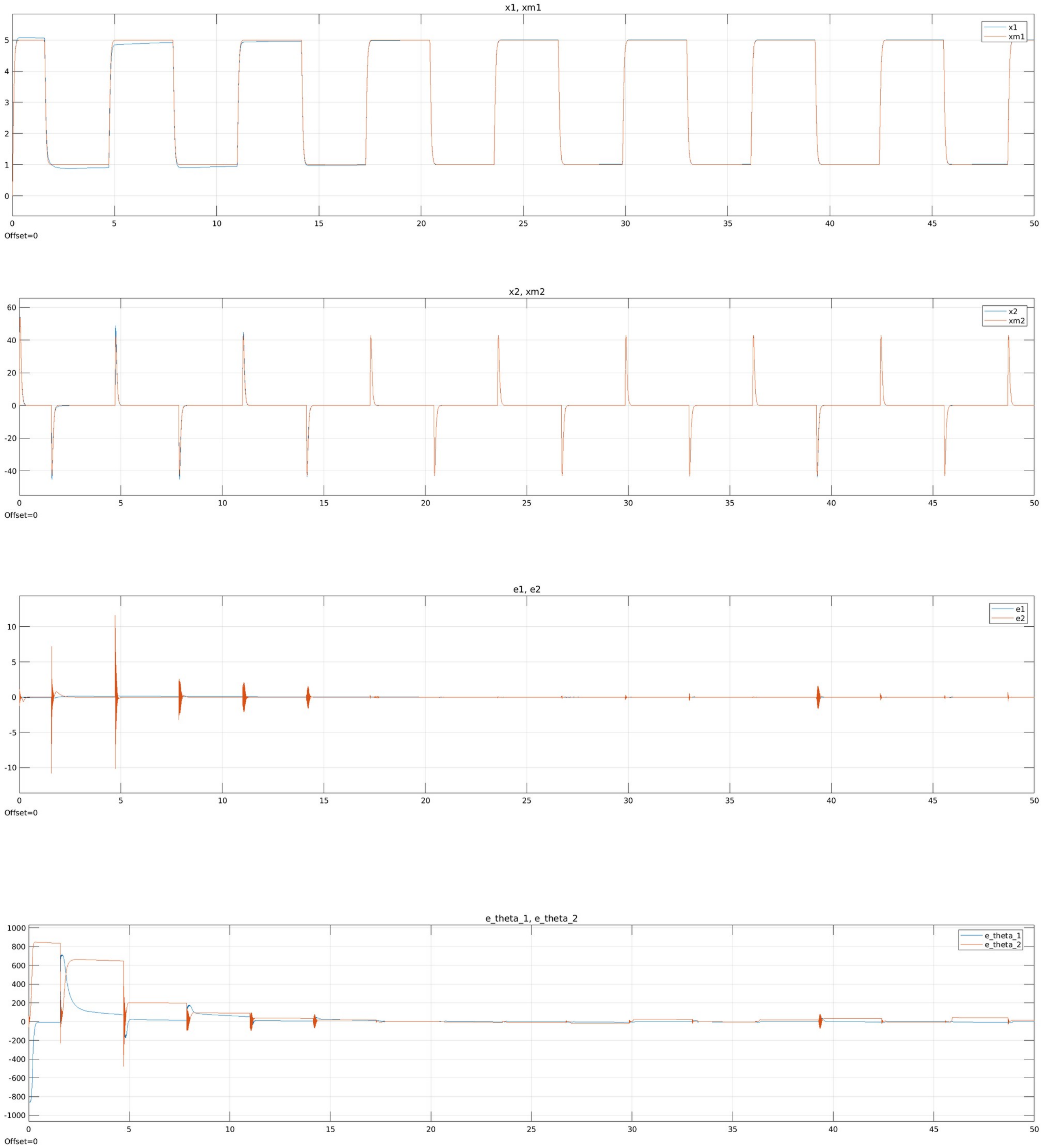


Figure 10: Results of the adaptive-robust controller, static feedback gain: 0.01, adaptation gain: 500 (no disturbances)

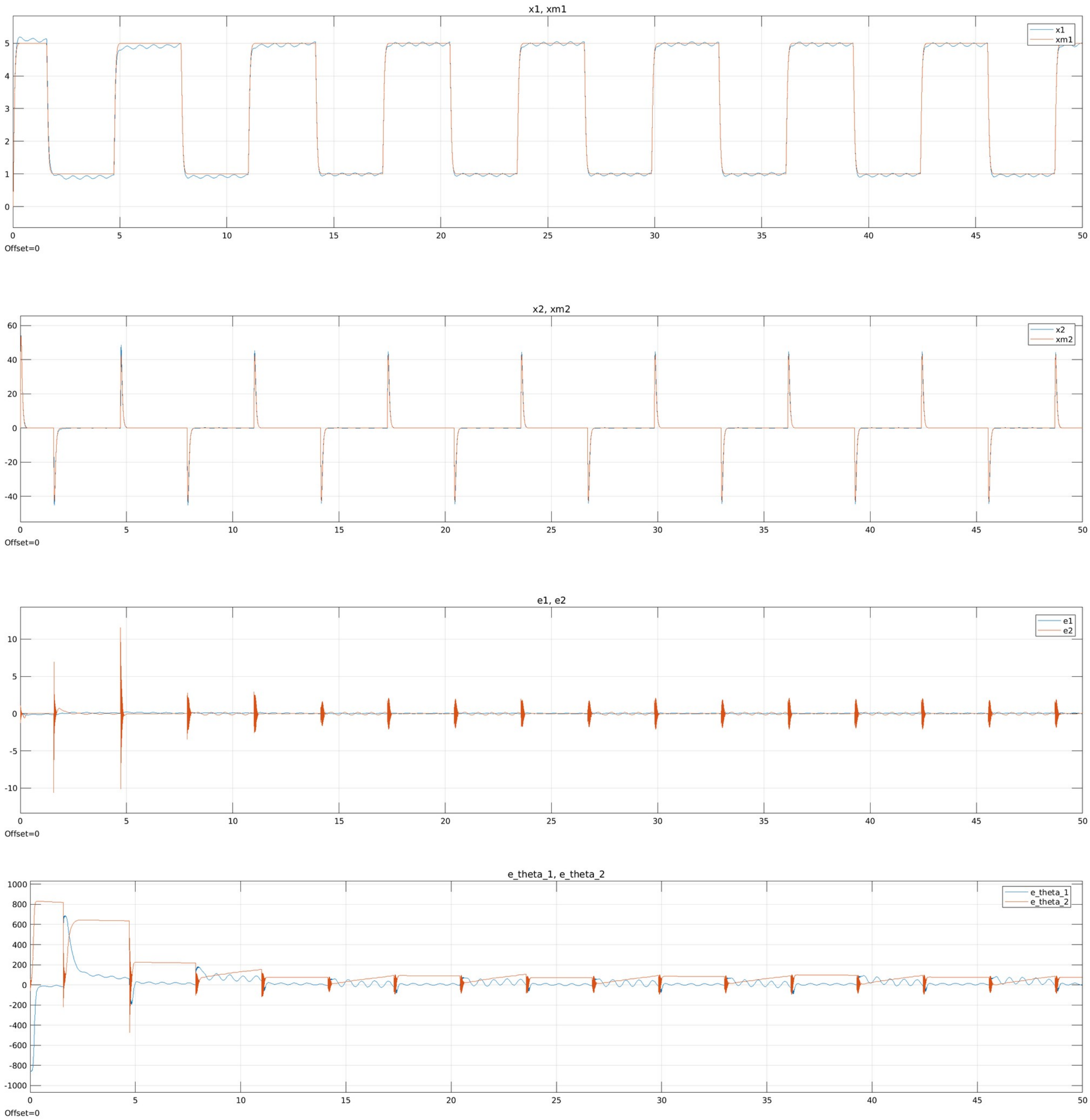


Figure 11: Results of the adaptive-robust controller, static feedback gain: 0.01, adaptation gain: 500 (with disturbances)

Conclusions:

From the results above we can conclude the following:

1. The robust controller is always able to converge the control error into a zero-centered range (residual set). However, the width of this range can be decreased (error closer to zero) by increasing the feedback margin γ. These properties are ensured with and without disturbances.
2. Increasing the feedback gain doesn't relate to the decrease in the range's width linearly. We saw that the proximity of the error to zero increased a lot by increasing γ from 100 to 500. And while additional enhancement was recorded when increasing the gain up to 1000, the difference wasn't of the same significance.
3. The robust controller cannot ensure control error convergence to zero even without any disturbances.
4. The robust controller produce an estimation of the plant parameters θ. However, these estimations are not meaningful since this controller doesn't achieve control by driving the parametric error to 0, but by suppressing the effect of the parametric error in addition to the disturbances.
5. The adaptive-robust controller can guarantee all the robust controller properties in terms of control error. However, it adds up the ability to converge the parametric error to a zero-centered range as well.
6. We saw from the experiments that both the feedback gain and adaptation gain have effects on the width (radius) of the ranges to which the parametric and control errors converge.
7. We noticed that with the absence of the disturbances, the adaptive-robust controller with high γ and low σ is able to converge the control error very nearly to zero. The robust controller wasn't able to match this performance.

Appendix: Simulink Models

The full model is the same as with LAB1, but the adaptation / robust algorithm that produce $\hat{\theta}$ are the following:

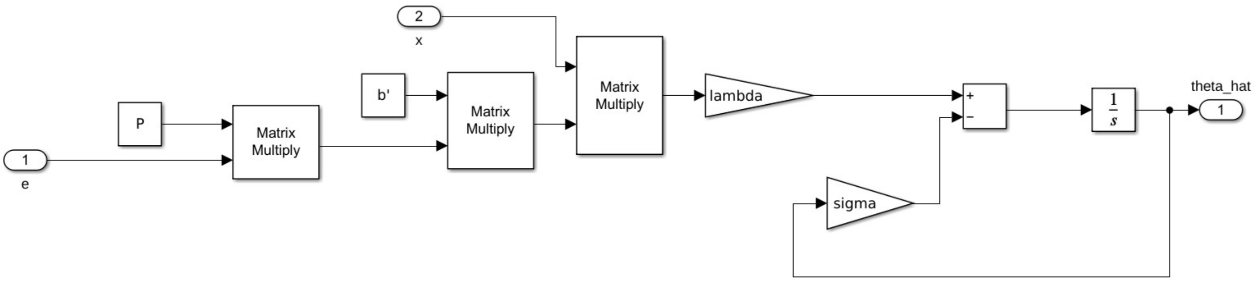


Figure 12: Adaptive-robust control

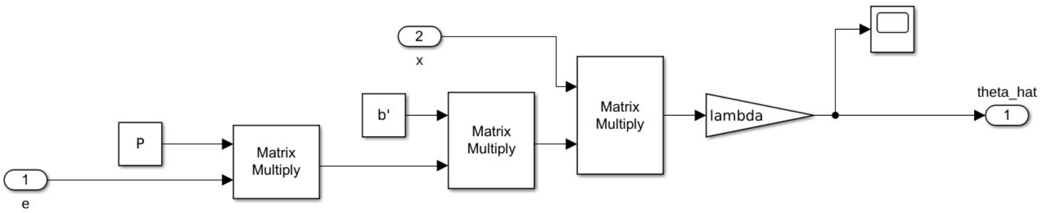


Figure 13: Robust control