LAB1: STATE FEEDBACK ADAPTIVE CONTROL

Zein Alabedeen Barhoum – Rahaf Alshaowa

Introduction:

In the laboratory work, we discussed the use of adaptive control in systems with unknown parameters. First, we built a conventional controller assuming knowledge of the system parameters, and then changed the system until the conventional controller couldn't stabilize and control the system. Then repeated the same experiments but using an adaptive controller and observed the results. We saw how the adaptive controller was able to achieve the required response with very low tracking error, moreover, it was able to calculate the system parameters with very good parametric error.

In this report, we will state the main theory used, and then go over the steps of the work with the plotted graphs of each step.

Theory:

Let's consider a system of the form:

$$\dot{x} = Ax + bu$$

where A is the system's matrix and b is the input's matrix with u (one element) being the input and x the state (vector of shape n). We assume that the matrix b is known and of the shape

$$b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_0 \end{bmatrix}$$

The systems parameters are impeded in the matrix A which has the form:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & 0 & \cdots & -a_{n-1} \end{bmatrix}$$

The parameters of the system are $a_0, a_1, \ldots, a_{n-1}$

And we want this state vector to follow a reference state vector x_M that has the following reference model :

$$\dot{x_M} = A_M x_M + b_M g$$

where g is a reference signal that's piecewise continuous. The reference model's matrices are known and of the shape:

$$A_M = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_{M0} & -a_{M1} & 0 & \cdots & -a_{M,n-1} \end{bmatrix}, b_M = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ a_{M0} \end{bmatrix}$$

The reference model is selected to achieved certain properties like overshoot and settling time.

If we know the systems parameters, then a non-adaptive controller can be designed as follows,

$$u = \theta^T x + \frac{1}{\kappa} g$$

where $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T ; \theta_i = \frac{-a_{M(i-1)} + a_{(i-1)}}{b_0}$ and $\kappa = \frac{b_0}{a_{M0}}$

However, if the system parameters a_i are not known, then we can design a controller using the estimation of θ noted as $\hat{\theta}$ and the adaptive controller is written as:

$$u = \hat{\theta}^T x + \frac{1}{\kappa} g$$

In order to estimate the values of $\hat{\theta}$, we need to use an adaptation algorithm. We can use the following adaptation algorithm:

$$\dot{\hat{\theta}} = \gamma x b^T P e ; \hat{\theta}(0) = 0$$

Where γ is the adaptation gain an P is a matrix that satisfies the equation $A_M^T P + P A_M = -Q$ where Q is a positive defined matrix.

This choice of adaptation algorithm guarantees (1) the boundedness of all signals of the system , (2) the convergence (asymptotic) of the control error $e = x_M - x$ to zero, and (3) the convergence (exponential) of the parametric error $\tilde{\theta} = \theta - \hat{\theta}$ to zero if x satisfies the persistent excitation condition. Moreover, there is a adaptation gain γ that's optimal in terms of rate of convergence of the parametric error to zero .

The order of the work:

In the work, we discuss a second order system, the initial values of matrices A and b are:

$$A=\begin{bmatrix}0 & 1\\ 10 & 1\end{bmatrix}, b=\begin{bmatrix}0\\ 1\end{bmatrix}$$

1. Reference Model:

We can see that to define the reference model , we need to define the values of a_{M0} and a_{M1} . The denominator of the transfer function of the reference model can be written as:

$$det(sI-A_M)=det\left(\begin{bmatrix}s & -1\\ a_{M0} & s+a_{M1}\end{bmatrix}\right)=s^2+a_{M1}s+a_{M0}$$

And since we have a desired overshoot of $\bar{\sigma}=0\%$ we can achieve the desired transient response by choosing a design depending on the binomial polynomial of the second order: $s^2+2\omega_{0s}+\omega_0^2$ we can see that $a_{M1}=2\omega_0, a_{M0}=\omega_0^2$.

The value of ω_0 can be calculated using the formula $\omega_0=\frac{t_T{}'}{t_T}$ where t_T is the desired transient time (or settling time). And $t_T{}'$ is the normalized transient time which is the transient time of the system: $h(s)=\frac{1}{s^2+2s+1}$ under a unit step input. Figure 1A shows the system h with step input. Figure 1 shows the normalized transient time (using 2% margins) which is **5.83 s** Therefore, $t_T{}'=5.83$ and since $t_T=0.2$ we have $\omega_0=\frac{5.83}{0.2}=29.15$ which leads to

$$a_{M1}=58.3, a_{M0}=849.7225$$

So, the reference model is the following:

$$\dot{x_M}=\begin{bmatrix}0 & 1\\ -849.7225 & -58.3\end{bmatrix}x_M+\begin{bmatrix}0\\ 849.7225\end{bmatrix}g$$
$$y_M=[1\quad 0]x_M$$

The step response of the reference model is shown in Figure 2 , we can see that the settling time is $t_T=0.2\,s$ and the overshoot is $\bar{\sigma}=0\%$

Note: Figure1 and Figure2 were produced using MATLAB and the ‘step’ function in order to automatically calculate the transient response (settling time and overshoot). Figures after these two are produced using Simulink.

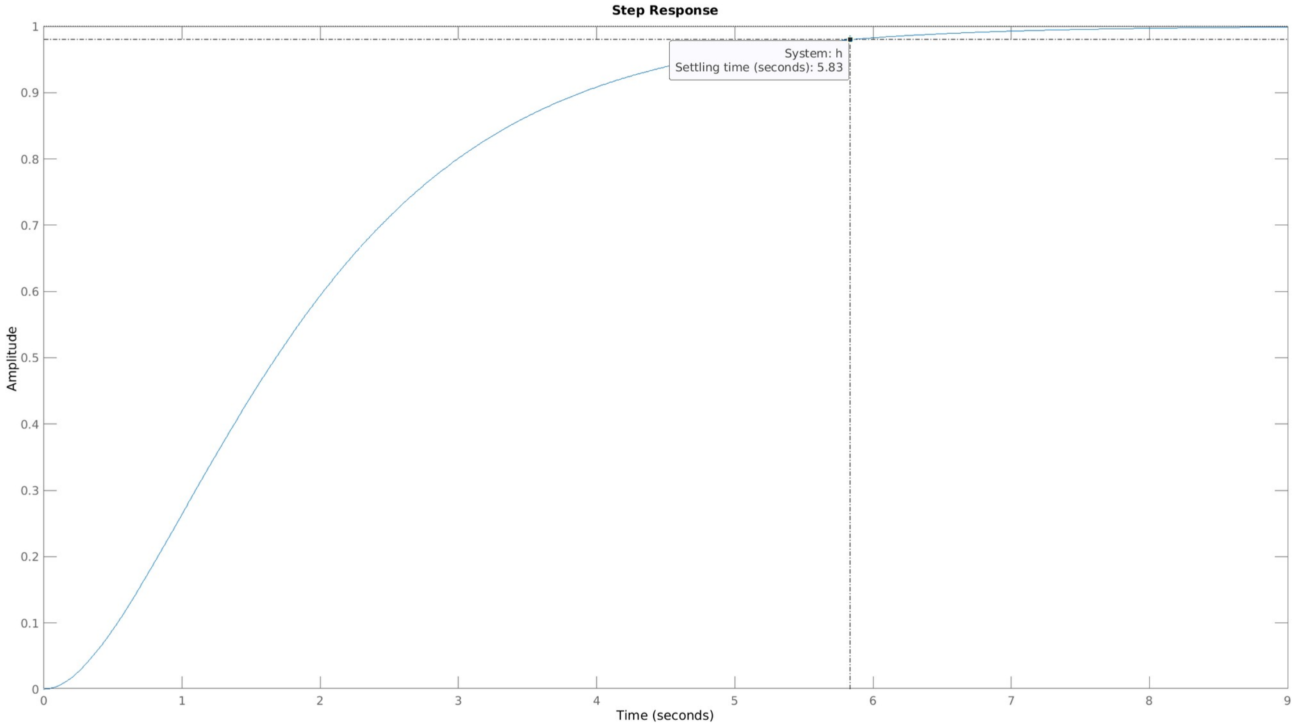


Figure 1: Normalized transient time

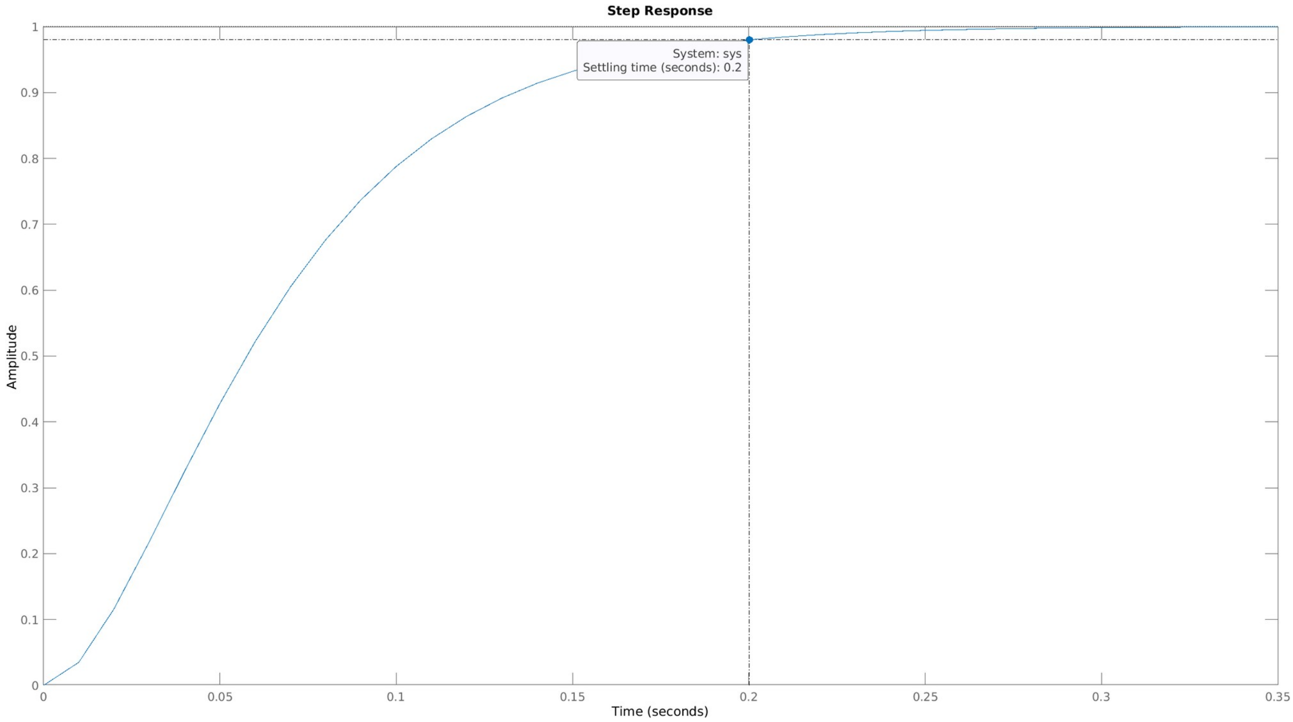


Figure 2: Reference model step response

2. Non_Adaptive Control:

First, assuming that the system parameters are $a_0 = -10, a_1 = -1$ and they are known , from the theory, we can use a non-adaptive controller of the form:

$$u = \theta^T x + \frac{1}{\kappa} g = [\theta_1 \quad \theta_2] x + \frac{1}{\kappa} g$$

Where,

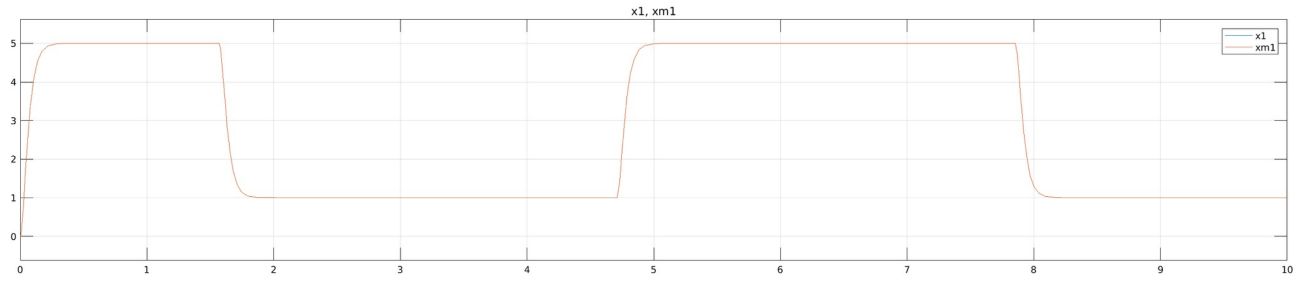
$$\theta_1 = \frac{-a_{M0} + a_0}{b_0} = \frac{-849.7225 - 10}{1} = -859.7225$$
$$\theta_2 = \frac{-a_{M1} + a_1}{b_0} = \frac{-58.3 - 1}{1} = -59.3$$

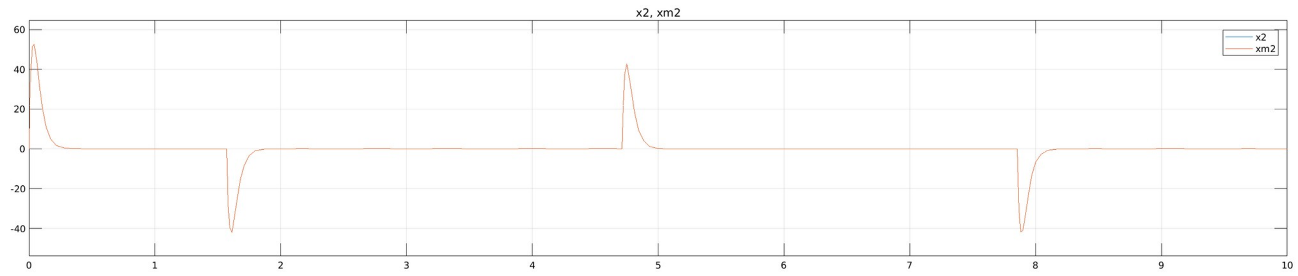
And $\kappa = \frac{b_0}{a_{M0}} = \frac{1}{849.7225} = 0.001176855$

Therefore, the non-adaptive controller is:

$$u(t) = [-859.7225 \quad -59.3] x + 849.7225 g(t)$$

Experiment 1: the actual plant parameters are $a_0 = -10, a_1 = -1$, the same as the parameters used to design the controller. We can see in Figure 3 that the controller follows the reference model exactly with zero error (error of order 10^{-11} is neglected as simulation error).





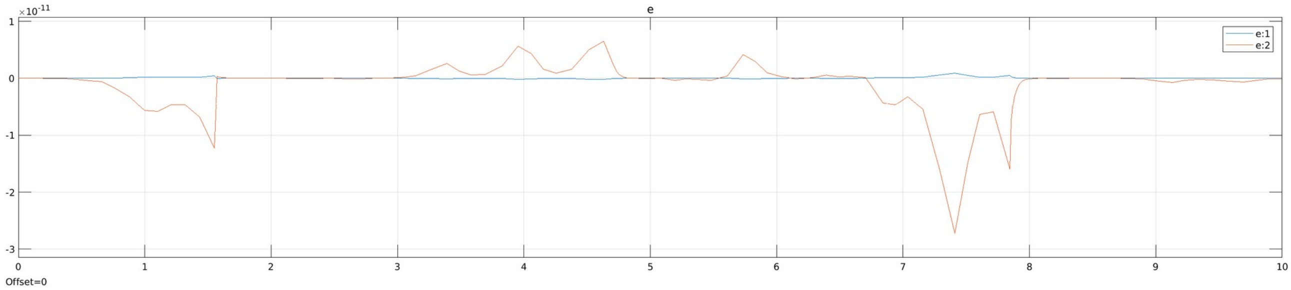


Figure 3: Non-adaptive controller, original plant parameters

Experiment 2: In this experiment, we changed the actual plant parameters to become $a_0 = -20, a_1 = -10$ but the controller formula is the same as the first experiment. We can see from Figure 4 that the control error is now apparent ; however, the controller is capable of stabilizing the plant.

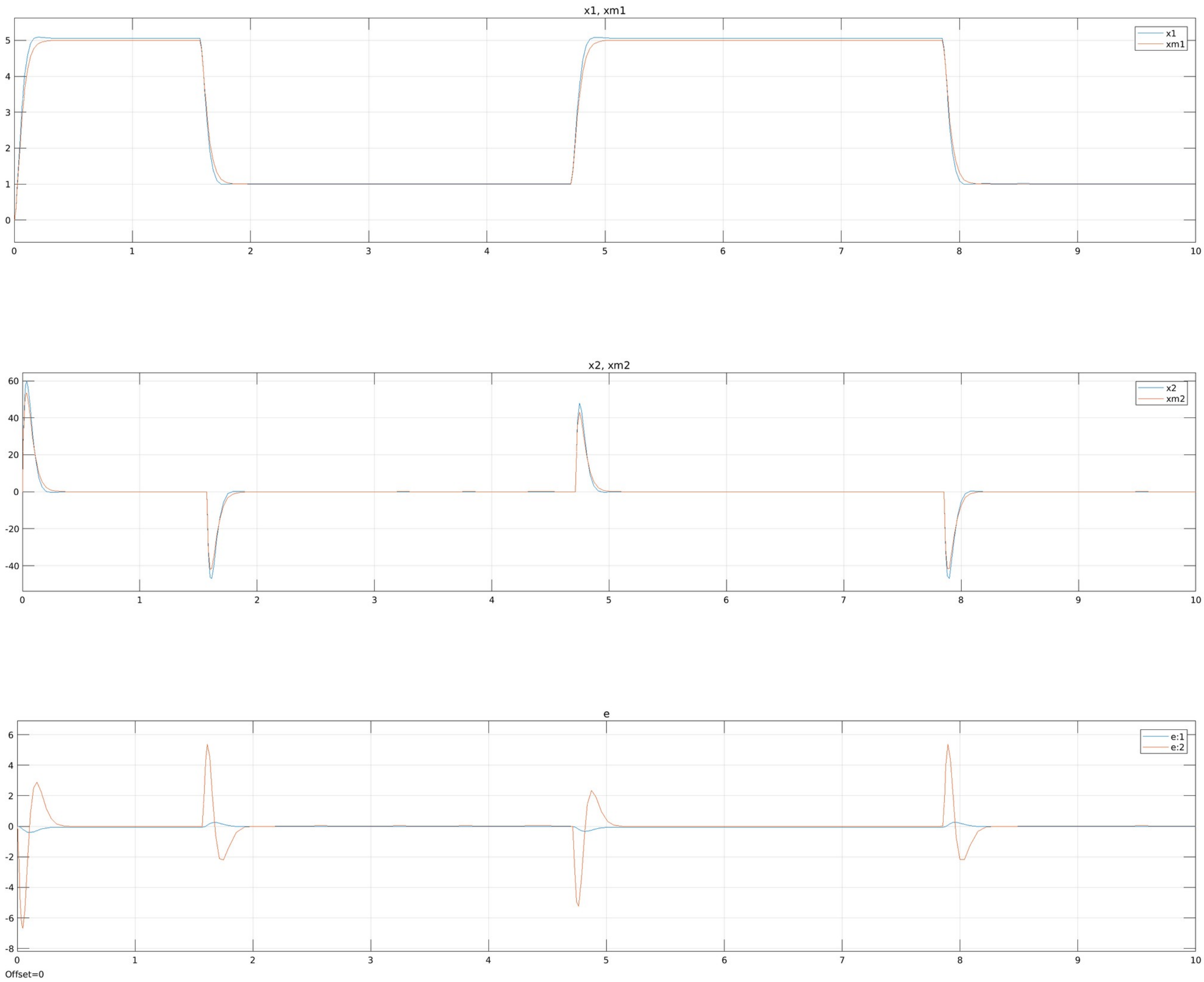


Figure 4: Non-adaptive controller, slightly changed plant parameters

Experiment 3: In this experiment, we changed the actual plant parameters to become $a_0 = -30, a_1 = -60$ but the controller formula is the same as the first two experiments. We can see from Figure 5 that the system is not stable and the control error is exponentially increasing.

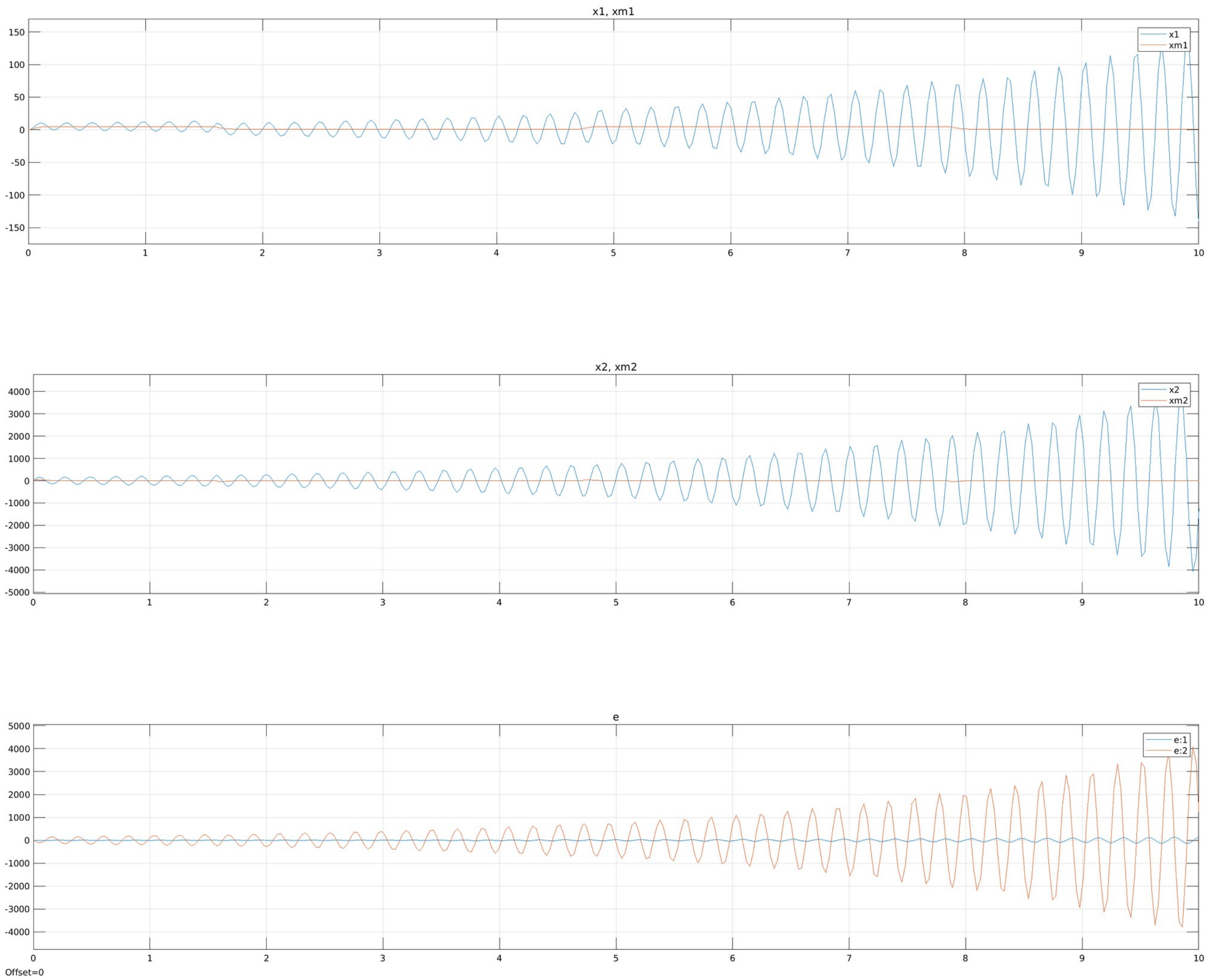


Figure 5: Non-adaptive controller, significantly changed plant parameters

3. Adaptive Control:

We saw from the non-adaptive controller experiments that if the parameters of the plant changed from our original expectations, then the system might loose stability. Moreover, sometimes even original expectations of the parameters are not available. In this case, an adaptive controller can be applied which estimates the parameters $\hat{\theta}$ using an adaptation algorithm $\dot{\hat{\theta}}=\gamma x b^T P e$ and then apply an adapted version of the controller using the estimated parameters , $u=\hat{\theta}^T x+\frac{1}{\kappa}g$.

The matrix P should be selected so it satisfies the equation $A_M^T P+PA_M=-Q$ where Q is a positive defined matrix. A positive definite matrix Q is a matrix that's for every non-zero vector $z=\begin{bmatrix}a\\b\end{bmatrix}$ the following inequality is satisfied $z^T Q z>0$. If we represented Q as $Q=\begin{bmatrix}q_{11} & q_{12}\\q_{21} & q_{22}\end{bmatrix}$ we see that the inequality becomes: $[a \quad b]\times\begin{bmatrix}q_{11} & q_{12}\\q_{21} & q_{22}\end{bmatrix}\times\begin{bmatrix}a\\b\end{bmatrix}>0$, therefore; $q_{11}a^2+(q_{12}+q_{21})ab+q_{22}b^2>0$, if we choose $q_{11}=1600, q_{12}=q_{21}=800, q_{22}=500$, this becomes $1600a^2+1600ab+500b^2>0 \rightarrow (40a+20b)^2+100b^2>0$ which is correct for any non-zero vector z , therefore, $Q=\begin{bmatrix}1600 & 800\\800 & 500\end{bmatrix}$ is positive defined and was chosen for this task.

To calculate the matrix P we use the equation $A_M^T P+PA_M=-Q$. we have the matrix $A_M=\begin{bmatrix}0 & 1\\-849.7225 & -58.3\end{bmatrix}$ and representing the matrix $P=\begin{bmatrix}p_{11} & p_{12}\\p_{21} & p_{22}\end{bmatrix}$ we see that the equation becomes:

$$\begin{bmatrix}0 & -849.7225\\1 & -58.3\end{bmatrix}\times\begin{bmatrix}p_{11} & p_{12}\\p_{21} & p_{22}\end{bmatrix}+\begin{bmatrix}p_{11} & p_{12}\\p_{21} & p_{22}\end{bmatrix}\times\begin{bmatrix}0 & 1\\-849.7225 & -58.3\end{bmatrix}=-\begin{bmatrix}16 & 8\\8 & 5\end{bmatrix}$$

Solving this equation we /see that

$$P=\begin{bmatrix}2912.36 & 0.94\\0.94 & 4.3\end{bmatrix}$$

Therefore, the adaptation algorithm is written as

$$\dot{\hat{\theta}}=\gamma x[0 \quad 1]\begin{bmatrix}2912.36 & 0.94\\0.94 & 4.3\end{bmatrix}(x_M-x);\hat{\theta}(0)=\begin{bmatrix}0\\0\end{bmatrix}$$

Experiment 1: the actual plant parameters are $a_0 = -10, a_1 = -1$, the adaptation gain was set to $\gamma = 50$. We can see in Figure 8 how the error between the system state and the reference state gets smaller with time converging to zero. Moreover, we can see that the parametric error $\bar{\theta} = \theta - \hat{\theta}$ also converges to zero.

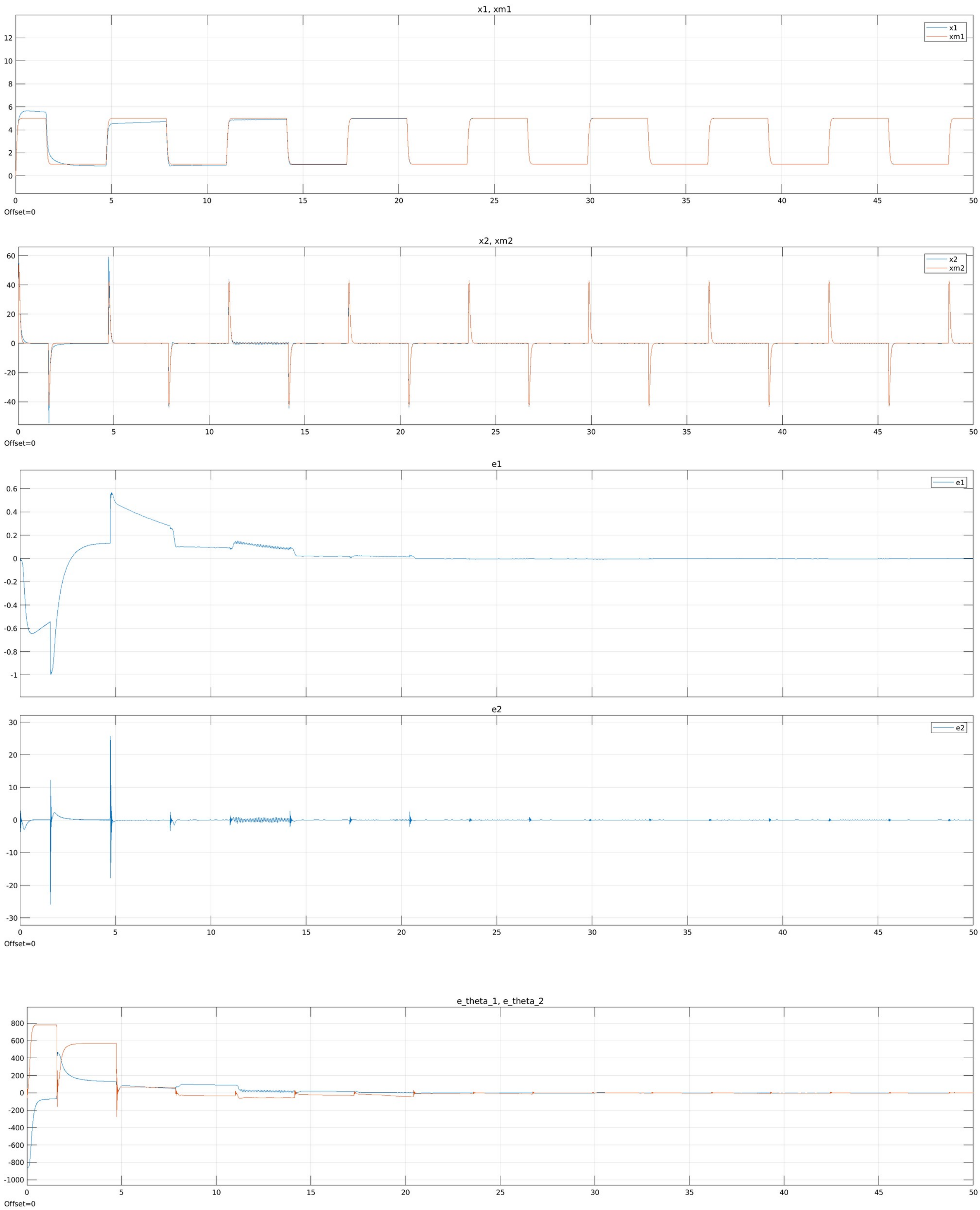


Figure 6: Adaptive controller, original plant parameters

Experiment 2: the actual plant parameters are $a_0 = -20, a_1 = -10$, we can see from Figure 7 that the adaptive controller is able to stabilize and control the system.

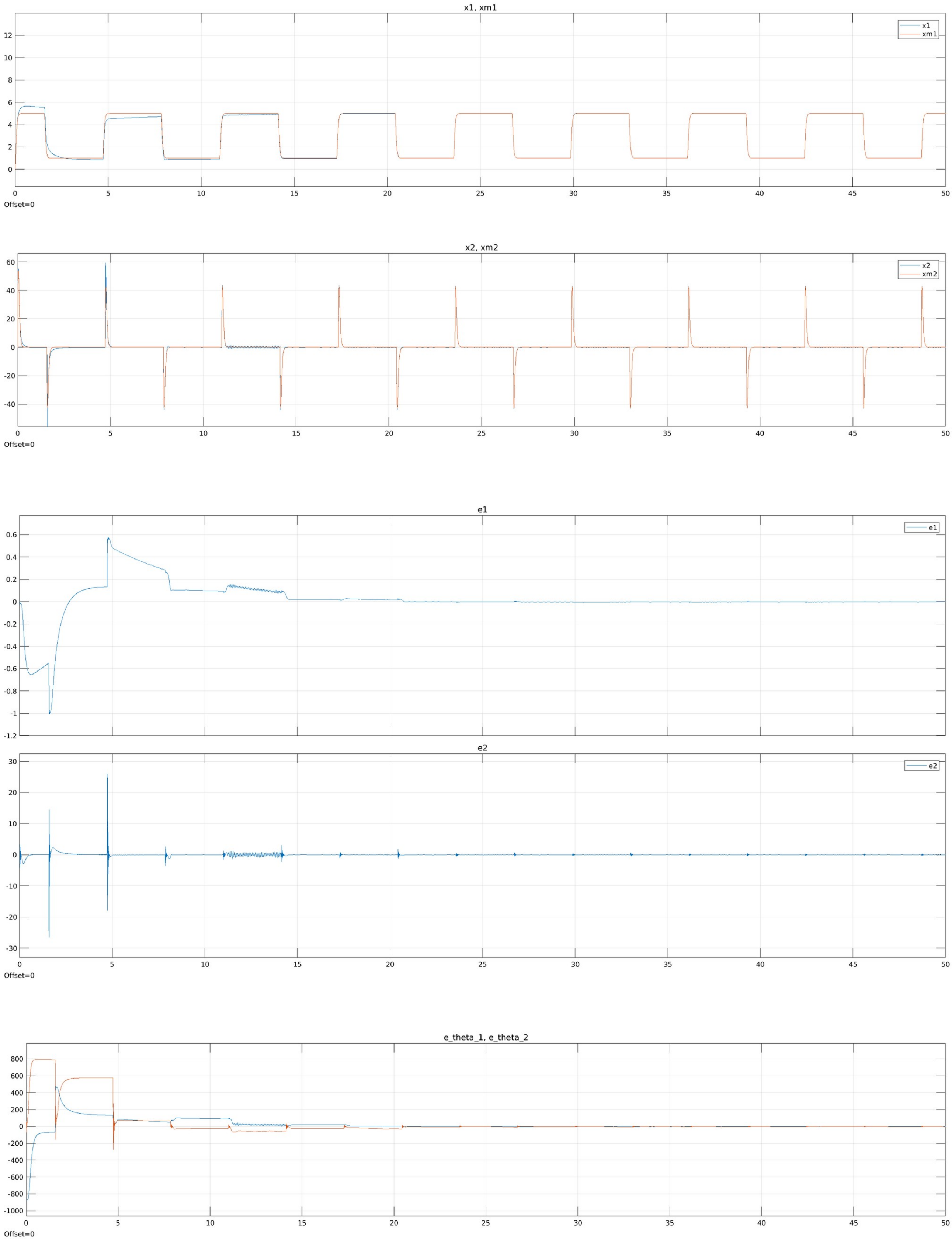


Figure 7: Adaptive controller, slightly changed plant parameters

Experiment 3: the actual plant parameters are, $a_0 = -30, a_1 = -60$ we can see from Figure 8 that the adaptive controller is able to stabilize and control the system which the non-adaptive controller wasn't able to control.

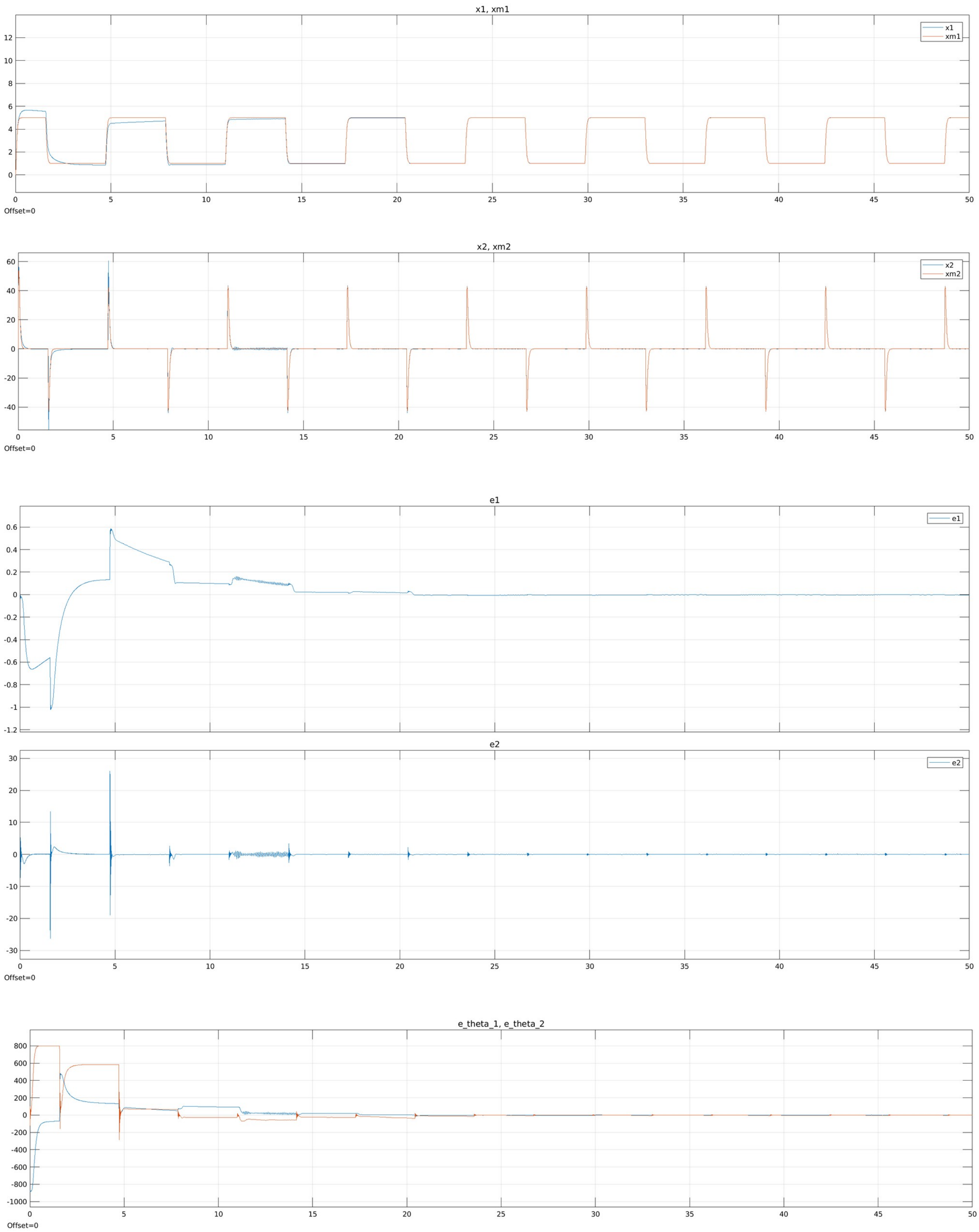


Figure 8: Adaptive controller, significantly changed plant parameters

We can observe from the previous experiments the following:

1. All signals are bounded
2. The control error converges to zero
3. The parametric error converges to zero

This is due to the fact that the signal x has more than two harmonics (the signal g is oscillating in a square signal which has infinite number of frequencies).

Experiment 4: In this experiment, three different values of the adaptation gain will be tested which are $\gamma=10,100,1000$ with the original system parameters $a_0=-10, a_1=-1$ The results are shown in Figures 9, 10 and 11.

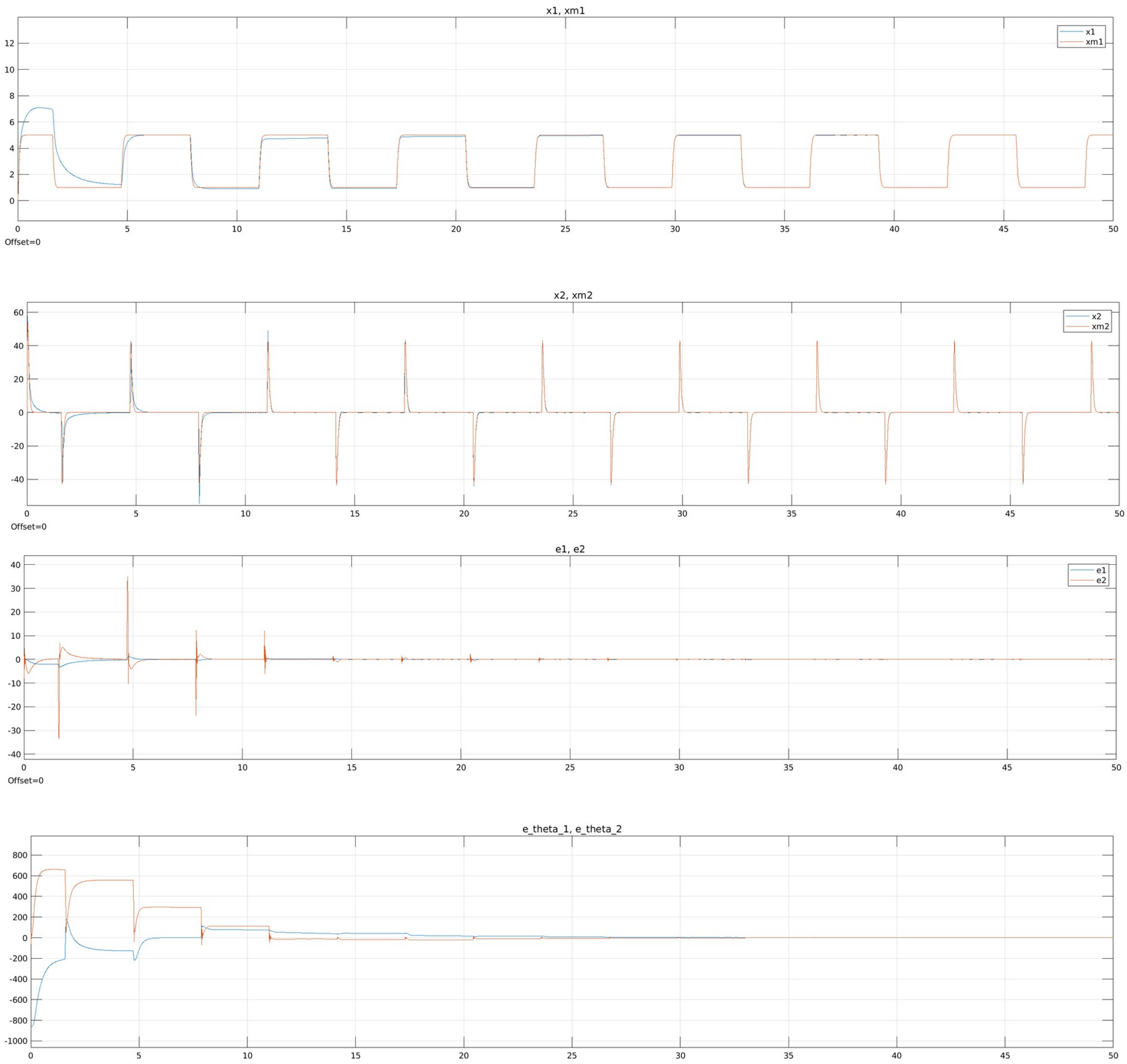


Figure 9: Adaptive controller, original plant parameters, gain= 10

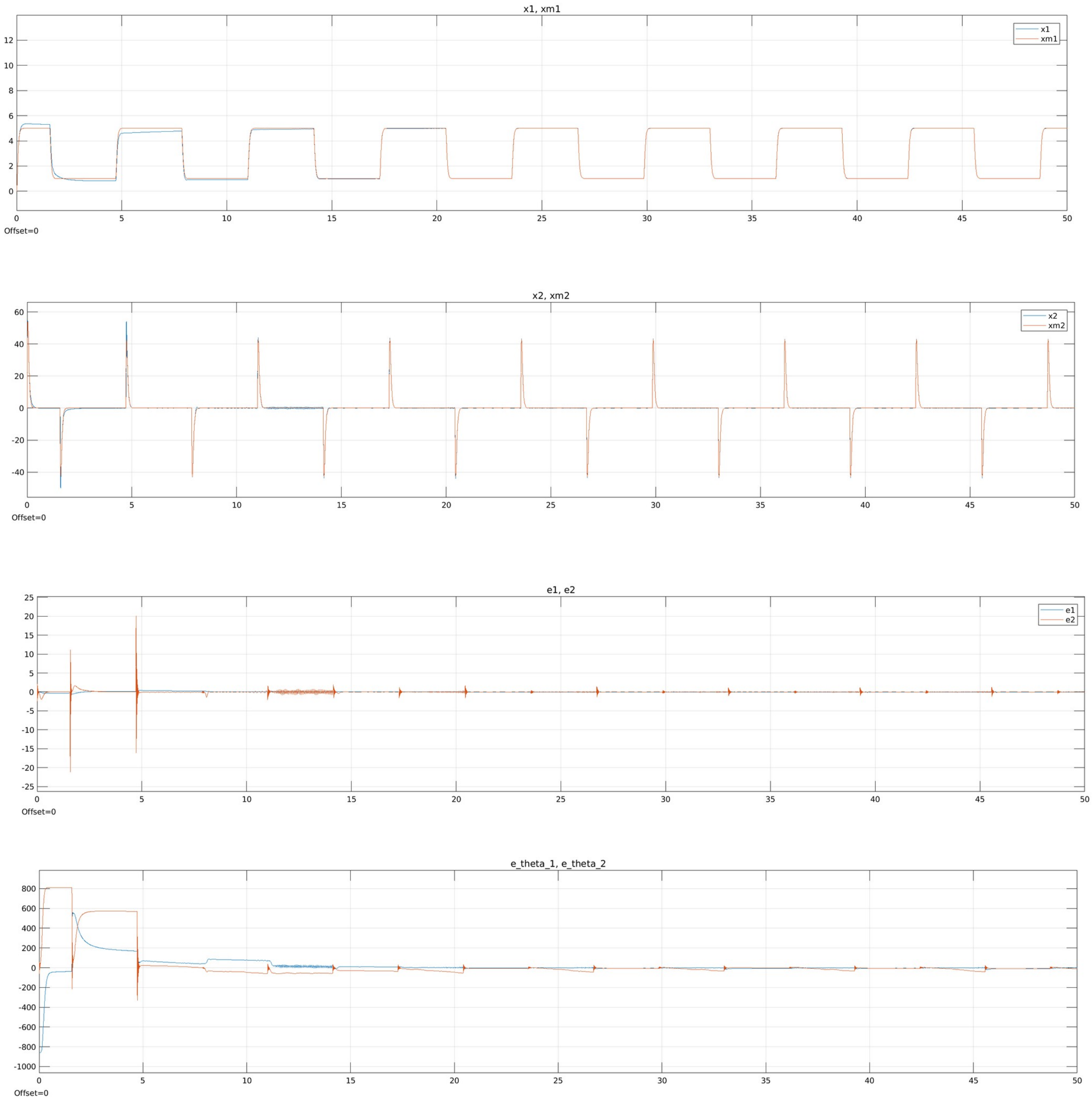


Figure 10: Adaptive controller, original plant parameters, gain= 100

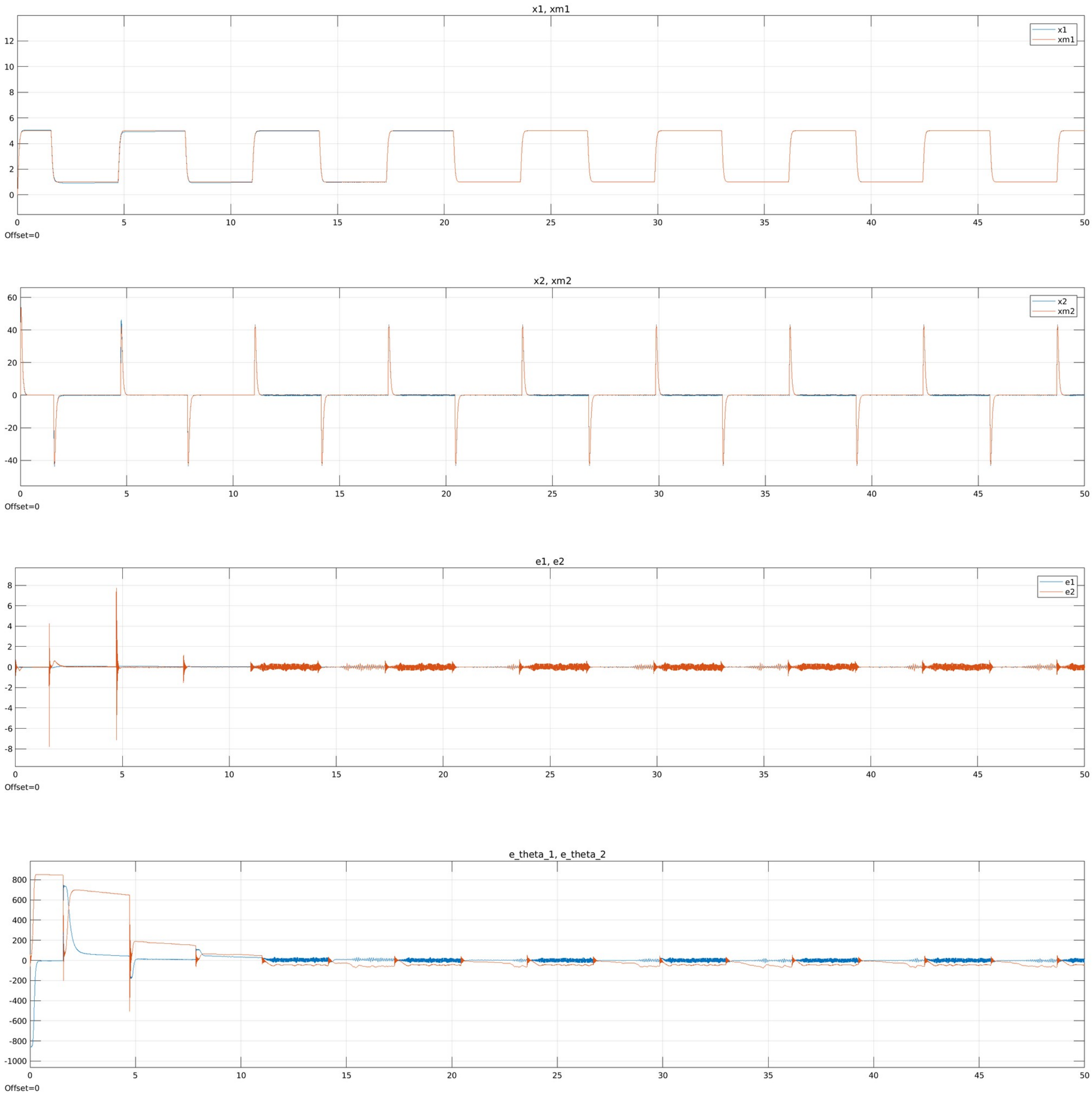


Figure 11: Adaptive controller, original plant parameters, gain= 1000

Experiment 5: In this experiment, we repeated experiment 4 with the same gains, but with different reference signal which is $g(t)=1$ (doesn't have enough harmonics)

The results are shown in Figures 12, 13, 14.

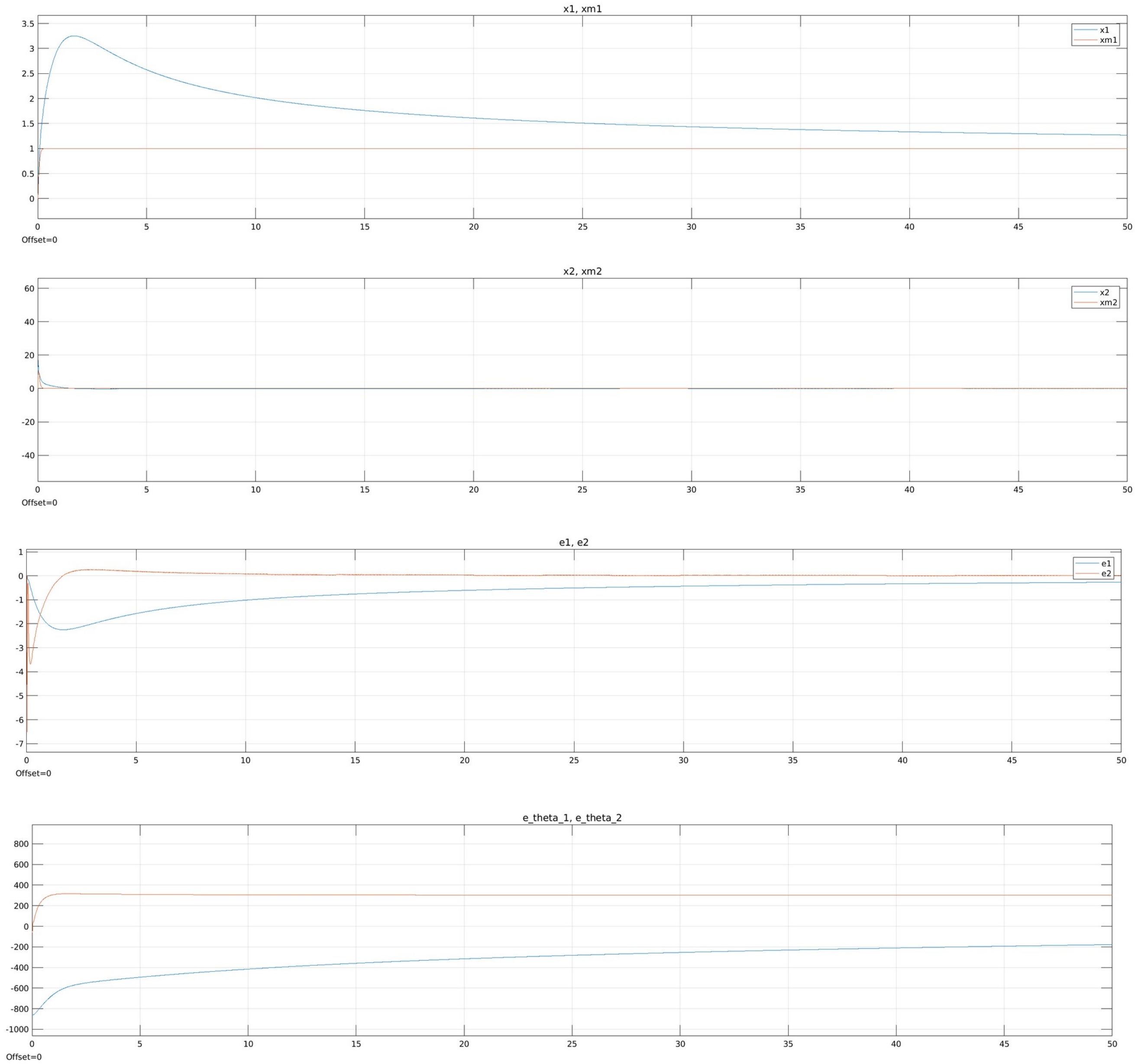


Figure 12: Adaptive controller, constant reference, gain= 10

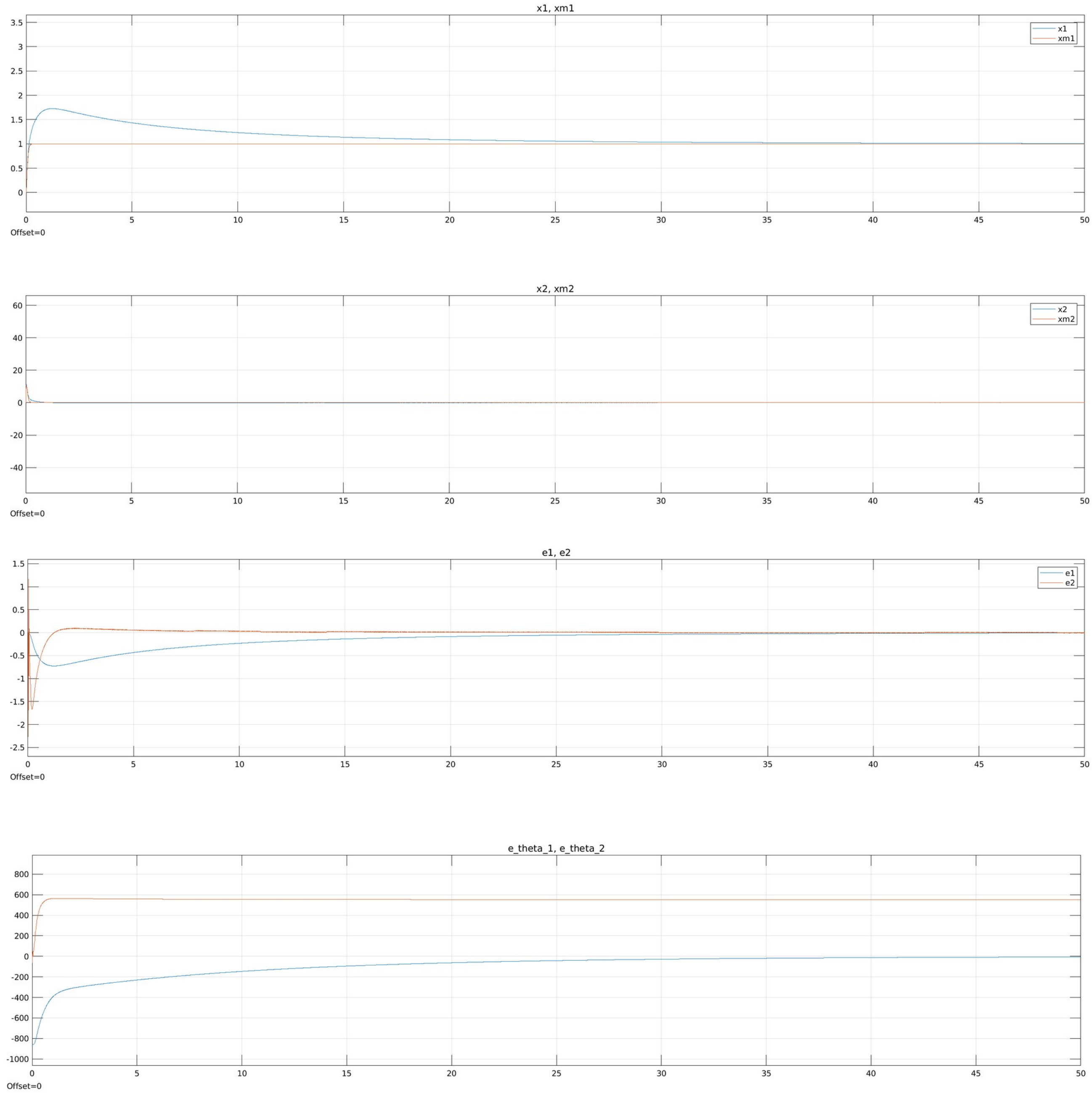


Figure 13: Adaptive controller, constant reference, gain= 100

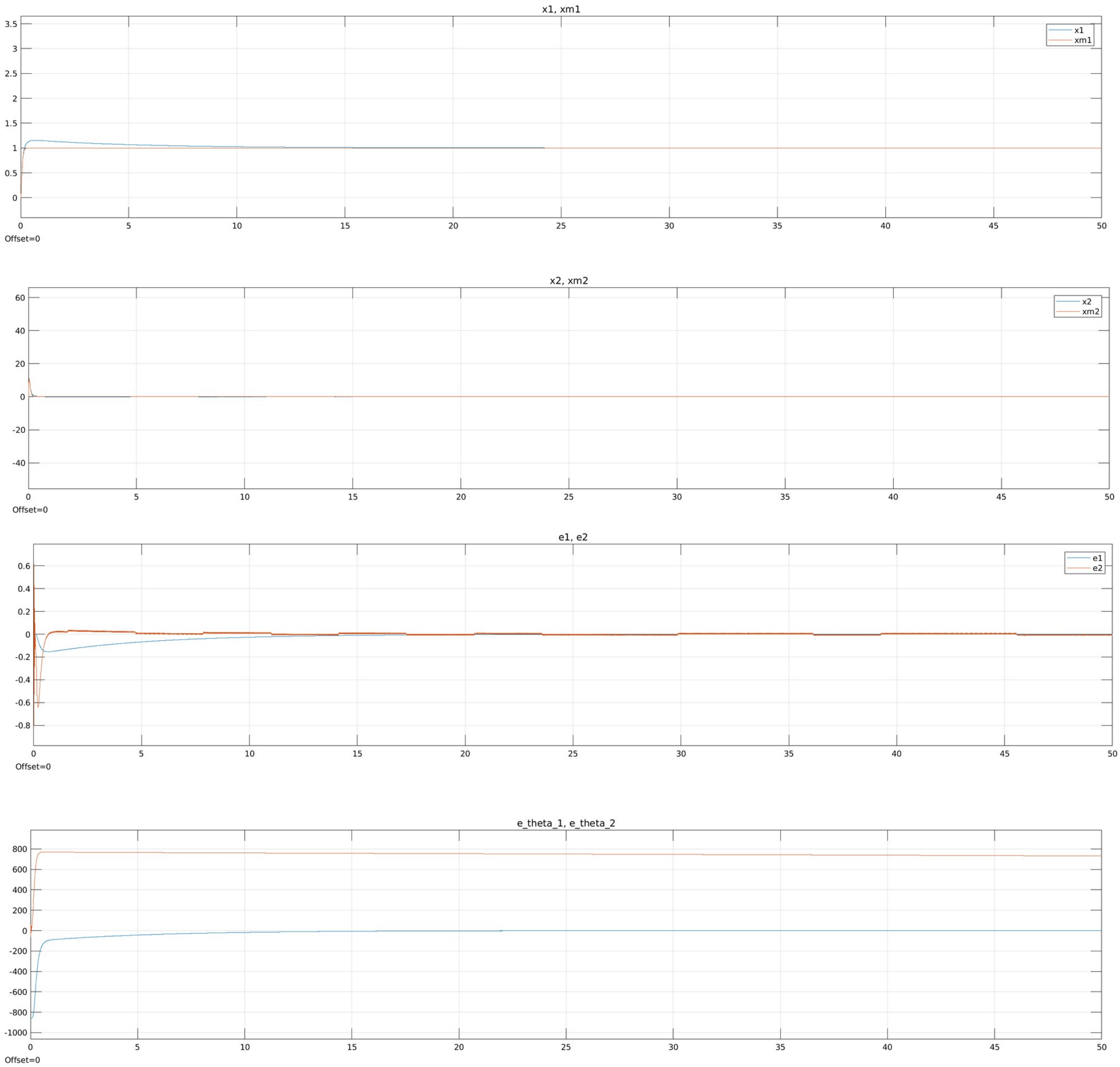


Figure 14: Adaptive controller, constant reference, gain= 1000

Conclusions:

We see that non-adaptive controllers assume knowledge of the system model. This assumption can be relaxed by observing that the non-adaptive controller can control and stabilize a system with slightly different parameters. However, we also observed that the plant-controller system can become unstable if the parameters are changed significantly.

To solve this problem, adaptive controllers are used which simultaneously predict the plant's parameters using an adaptation algorithm and using these parameters to control the plant. The selected adaptation algorithm ensures the boundedness of the signals, convergence of the control error to zero and if the persistent excitation condition is met, then the adaptation algorithm ensures the convergence of the estimated parameters to the actual parameters (parametric error to zero).

From experiments, we saw that when a reference signal with more than 1.5 harmonics (satisfies the persistent excitation), both the control error and parametric error goes to zero for any adaptation gain. However, when we set the reference signal to be constant which violate the presistent excitation condition, only the control error converged to zero while only one of the two elements of the parametric error converged to zero because there are enough excitation for one parametric identification.

We also observed from the experiments that increasing the adaptation gain doesn't mean better rate of convergence. We can see that for gain (100), the parametric convergence rate is better than for gains (10, 1000).

Appendix: Simulink Model:

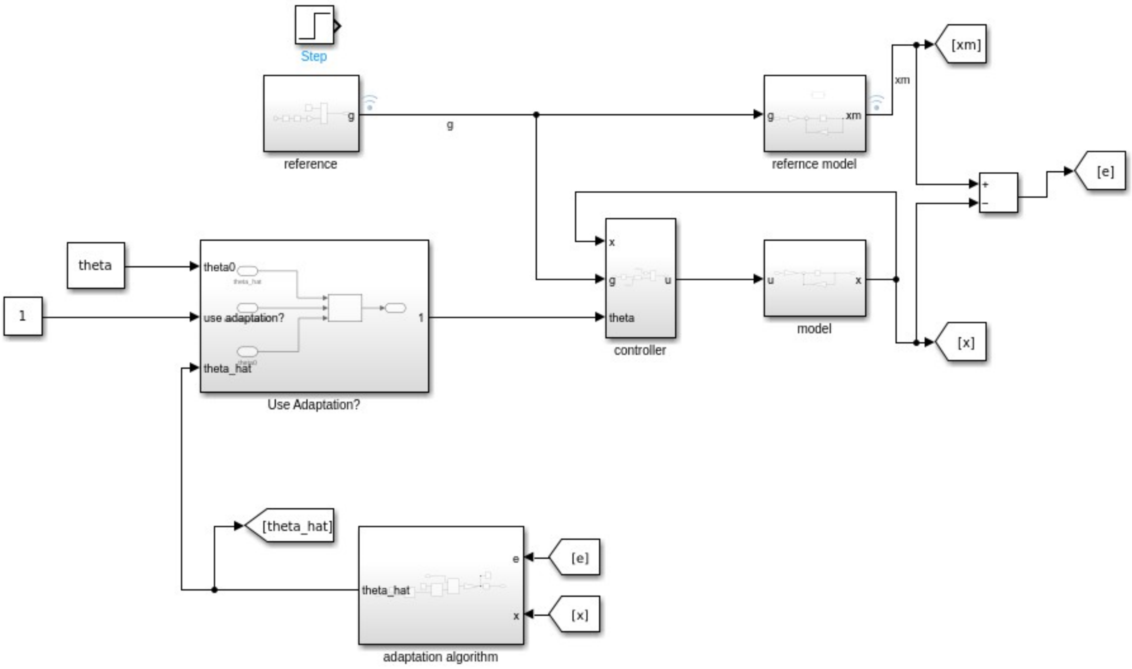


Figure 15: Full Model

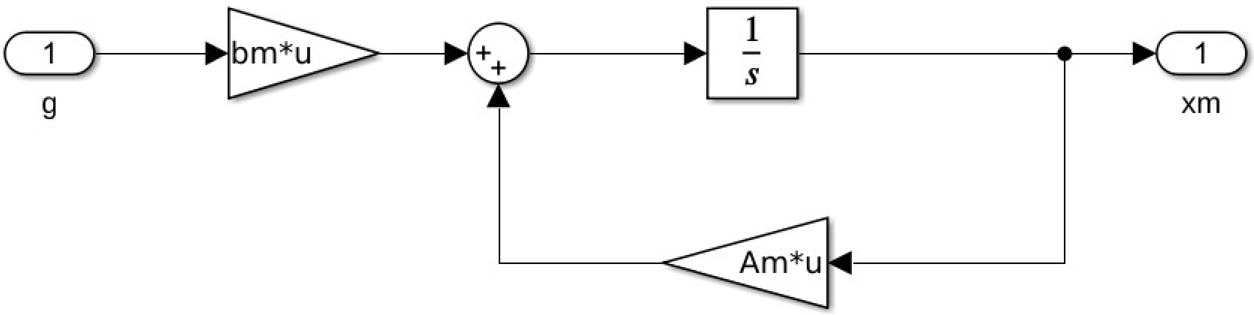


Figure 16: Reference model

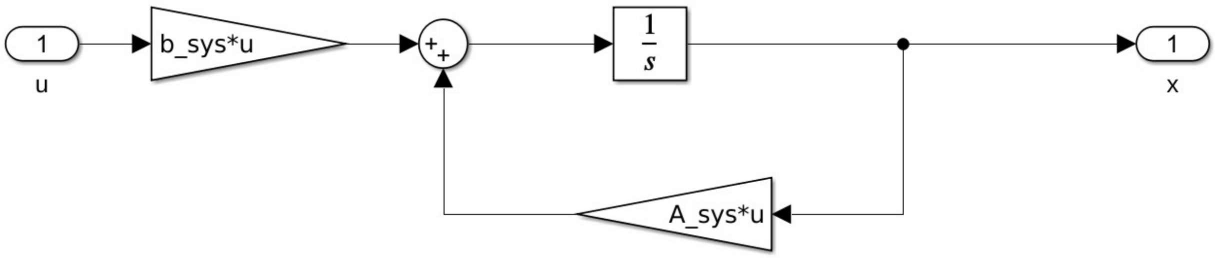


Figure 17: plant

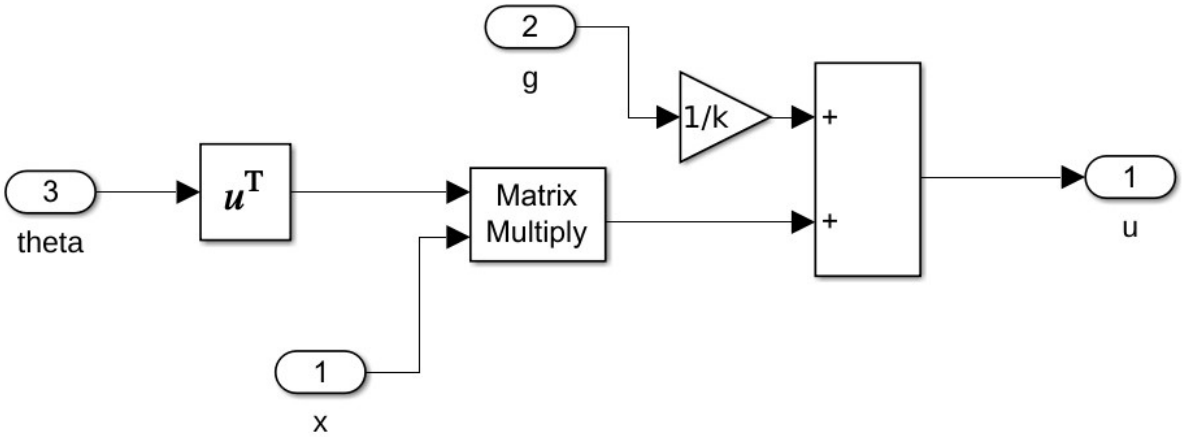


Figure 18: Control

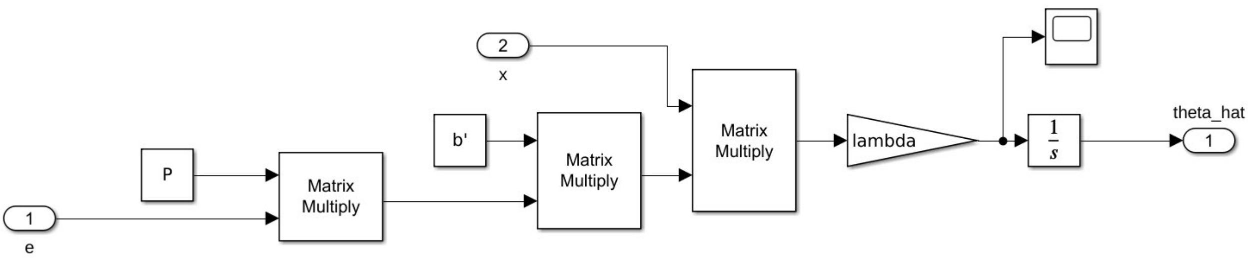


Figure 19: Adaptation algorithm