LAB3: OUTPUT FEEDBACK MODEL REFERENCE ADAPTIVE CONTROL WITH THE USE OF AUGMENTED ERROR APPROACH

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Introduction:

In this laboratory work, we discussed the use of output feedback adaptive control in systems with unknown parameters. In the previous laboratories, we attempted to control the system to achieve a certain reference **state vector** assuming the existence of state vector measurements. However, in this laboratory, we only care about the **output**. We need the output to follow a reference output assuming the existence of an output measurement to use it as feedback.

Moreover, an approach called augmented error approach will be used to estimate the system's parameters. This approach relaxed the need for strictly positive real (SPR) transfer function in the reference model, which is a requirement to use normal output feedback adaptive control.

In this report, we will state the main theory used, and then go over the steps of the work and list the results and conclusions.

Theory:

Let's consider a s system defined using the following differential equation:

$$y^{(n)}+a_{n-1}y^{(n-1)}+...+a_0 y=b_m u^{(m)}+b_{m-1}u^{(m-1)}+...+b_0 u$$

Where the constants a_i, b_i are unknowns (we only know that the sign of b_m is positive)

We want to define a control law $u(t)$ which drive the output $y(t)$ to follow a reference model output y_M. The reference model is given as:

$$y_M(t)=\frac{k_0}{K_M(s)}[g(t)]$$

Where $g(t)$ is a piecewise continuous reference signal . The polynomial $K_M(s)$ can be designed to achieve dynamic response with certain characteristics using plot-placement approach.

In addition to the plant, we will define two filters with state vectors v_1, v_2 defined as follows,

$$\dot{v}_1 = \Lambda v_1 + e_{n-1} u$$
$$\dot{v}_2 = \Lambda v_2 + e_{n-1} y$$

Where $\Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_0 & -k_1 & -k_2 & \cdots & -k_{n-2} \end{bmatrix}$ and $e_{n-1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$, both of them has $n-1$ rows.

It's proven that any system represented in the stated differential equations has an output:

$$y(t) = \frac{b_m}{K_M(s)}[\psi^T w(t) + u(t)] + \delta(t); w = \begin{bmatrix} v_1 \\ v_2 \\ y \end{bmatrix}$$

And if we define an error signal of the form $\epsilon(t) = y_M(t) - y(t)$ we can substitute $y_M(t), y(t)$ so;

$$\epsilon = \frac{k_0}{K_M(s)}[g(t)] - \frac{b_m}{K_M(s)}[\psi^T w(t) + u(t)] + \delta(t) = \frac{b_m}{K_M(s)}[\frac{k_0}{b_m} g(t) - \psi^T w(t) - u(t)]$$

And if we used the notion $\theta = \begin{bmatrix} -\psi^T \\ \frac{k_0}{b_m} \end{bmatrix}, \phi = \begin{bmatrix} w \\ g \end{bmatrix}$ we can rewrite the error as:

$$\epsilon = \frac{b_m}{K_M(s)}[\theta^T \phi - u]$$

We see that the vector ϕ is known since y, u, g are known and measurable and therefore v_1, v_2 are known. However, the vector of unknown parameters is θ . We can see that if we have the vector of parameters θ we can apply a conventional controllers $u = \theta^T \phi$ which guarantees a zero-error control. But without the knowledge of this vector, we can implement an adaptive controller using adaptation algorithm to estimate this vector as $\hat{\theta}$, we can then apply an adaptive control low $u = \hat{\theta}^T \phi$.

One possible adaptation algorithm is :

$$\dot{\hat{\theta}} = \gamma \phi \epsilon$$

However, this algorithm works only if the transfer function $H(s) = \frac{1}{K_M(s)}$ is strictly positive real (SPR) function which is very limiting requirement.

A more general adaptation algorithm can be obtained using the augmented error approach.

We start by writing the augmented error as:

$$\hat{\epsilon}=\epsilon-\hat{b}_m\zeta$$

Where $\zeta=\hat{\theta}^T\frac{1}{K_M(s)}[\phi]-\frac{1}{K_M(s)}[\hat{\theta}^T\phi]$

The adaptation algorithm is then written for two value $\hat{b}_m,\hat{\theta}$ as follows,

$$\dot{\hat{\theta}}=\gamma\frac{\bar{\phi}}{1+\bar{\phi}^T\bar{\phi}}\hat{\epsilon},\dot{\hat{b}}_m=\gamma\frac{\zeta}{1+\bar{\phi}^T\bar{\phi}}\hat{\epsilon}$$

Where $\bar{\phi}=\frac{1}{K_M(s)}[\phi]$ and γ is an adaptation gain.

This adaptation algorithm guarantees the following:

1. Boundedness of all closed-loop signals
2. The control error ϵ tends to zero asymptotically

the order of the work:

In this laboratory work, we consider the system of second order with parameters $a_0=a_1=0,b_0=1$. The differential equation of the system is

$$\ddot{y}=u$$

The transfer function of the plant is therefore, $G(s)=\frac{1}{s^2}$, we see that $n=2,m=0$ We want the system to follow a reference model represented using the function $K_M(s)=s^2+2s+1$ which is of order $n-m=2$ where $k_0=1$ as

$$y_M=\frac{1}{s^2+2s+1}[g(t)]$$

Where $g(t)$ is the reference signal. We will consider stabilization control where $g(t)=0$ in addition to tracking control where $g(t)=2sign(\cos(t))$. The filters used follow from $k_0=1$ as :

$$\dot{v}_1=[-k_0]v_1+[1]u=-v_1+u$$
$$\dot{v}_2=[-k_0]v_2+[1]y=-v_2+y$$

The vector of known signals is $\phi^T = [v1 \quad v2 \quad y \quad g]$

Experiment 1:

Three different adaptation gains $\gamma = 1, 10, 100$ were tested to stabilize the output $y(t)$ to 0 with initial conditions $y(0) = 5$. For each one, plots of $y, \hat{u}, \theta, \hat{b}_m$ will be included.

Gain: 1

We see in Figure 1 that the controller is able to stabilize the output

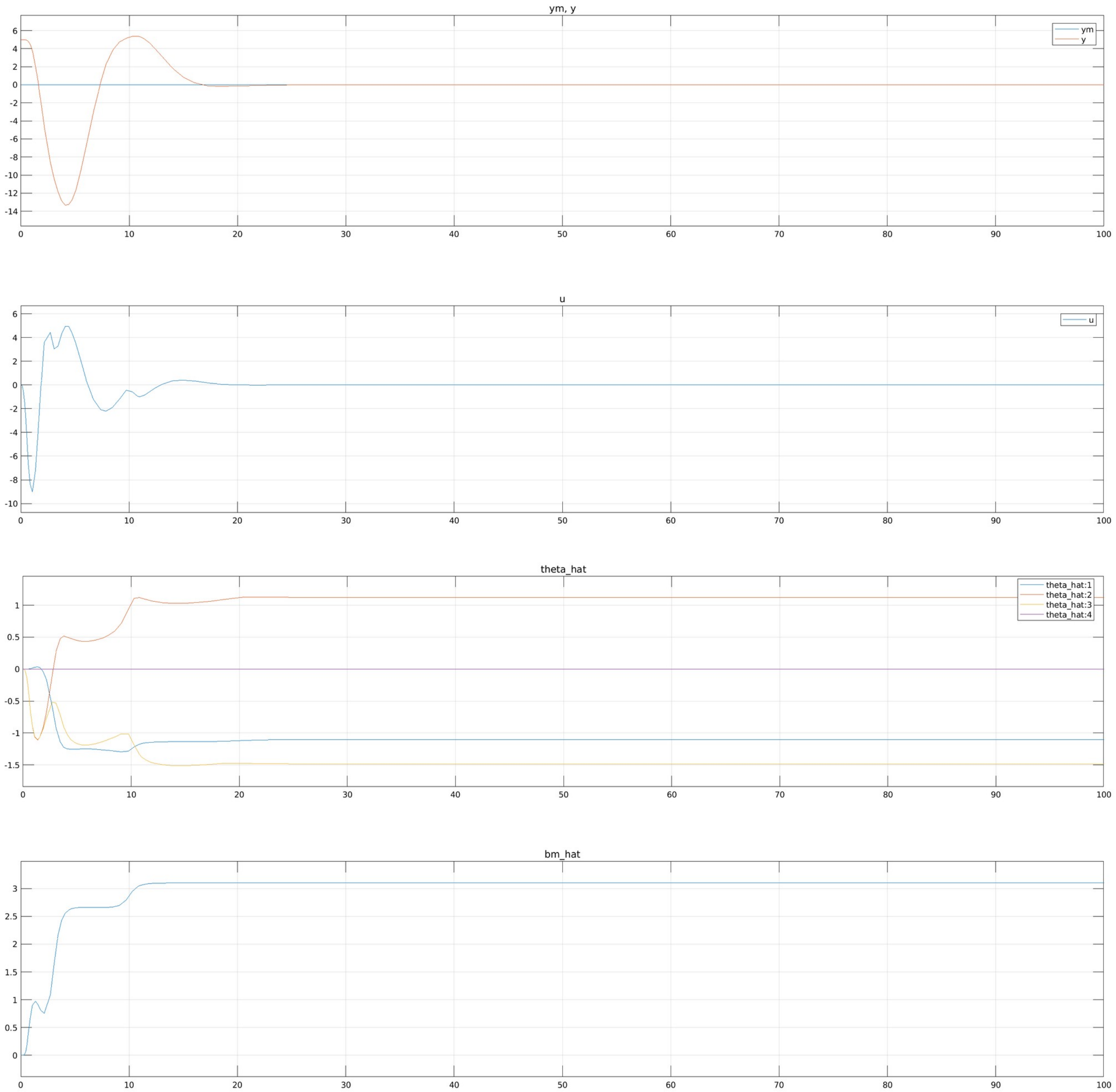

Figure 1: Results of stabilization control, gain = 1

Gain: 10

We see in Figure 2 that the controller is able to stabilize the output in around 11 seconds with maximum undershoot of -9

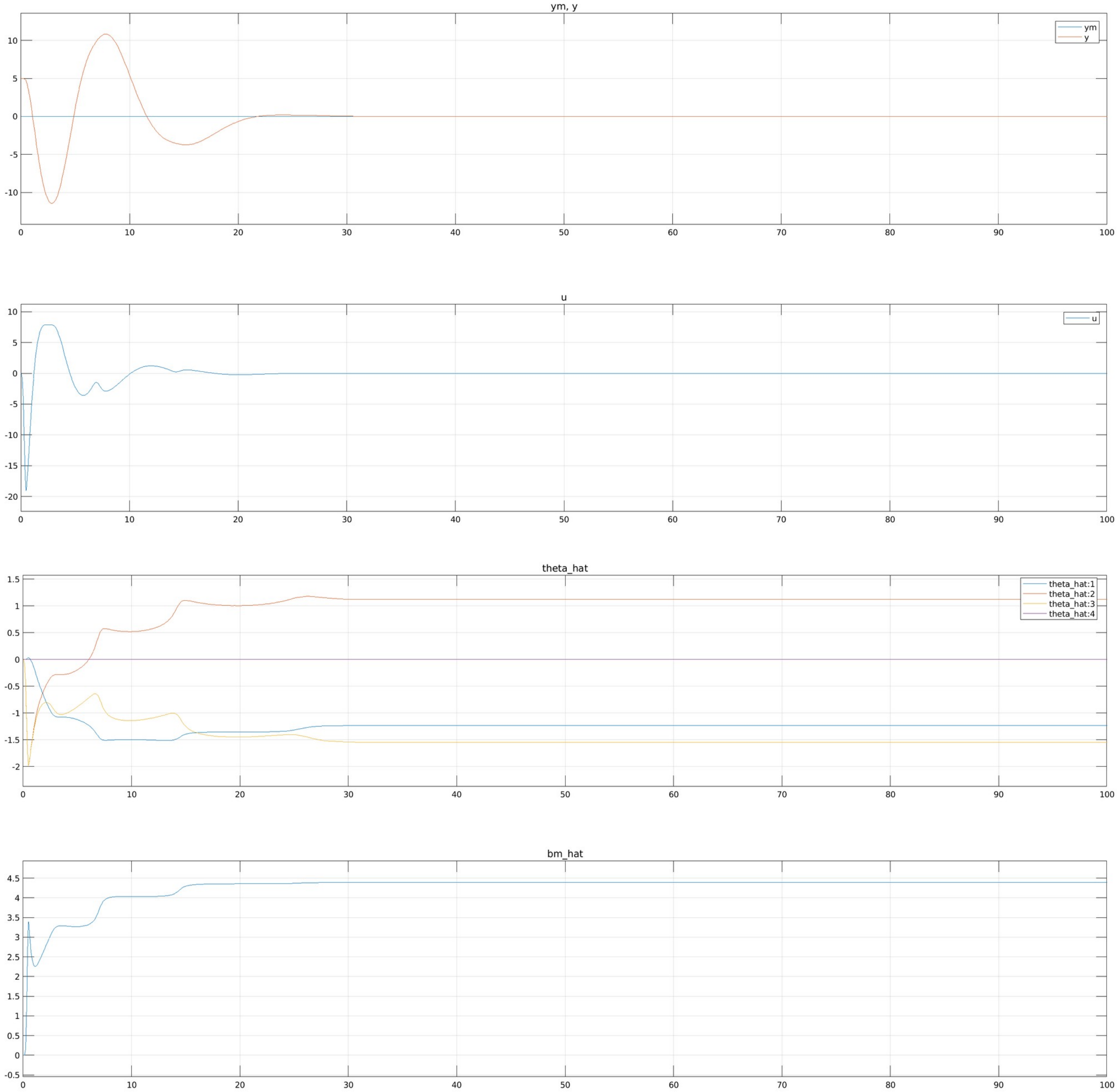

Figure 2: Results of stabilization control, gain = 10

Gain: 100

We see in Figure 3 that the controller is able to stabilize the output but takes longer than the other gain values

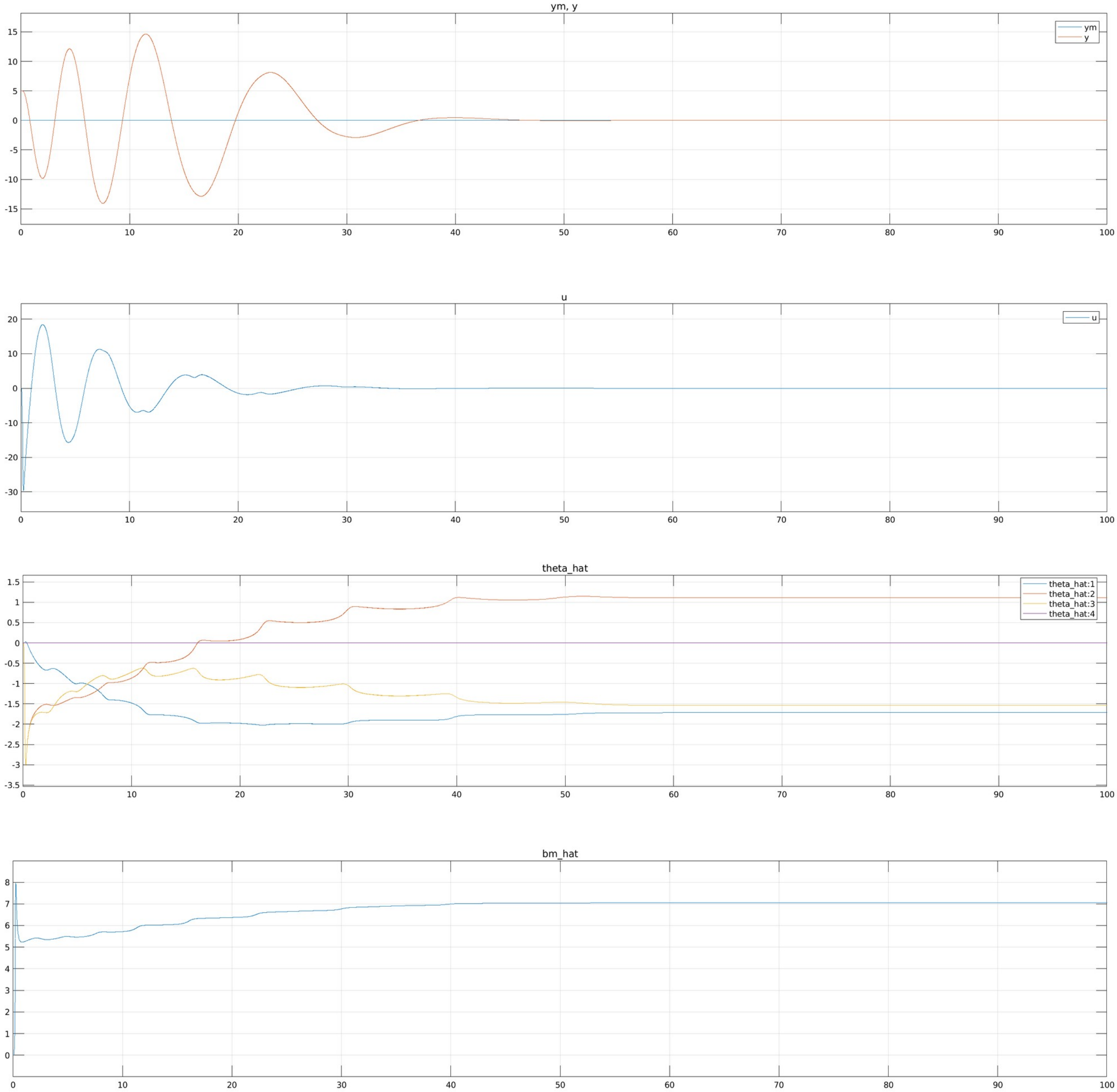

Figure 3: Results of stabilization control, gain = 100

Experiment 2:

Three different adaptation gains $\gamma=0.5, 1, 10$ were tested to drive the output $y(t)$ to $y_M(t)=\frac{1}{(s^2+2s+1)}[g(t)]; g(t)=2\,sign(\cos(t))$ with initial conditions $y(0)=0$. For each gain, plots of $y, \hat{u}, \theta, \hat{b}_m$ will be included.

Gain: 1

We see in Figure 4 that the controller is able to drive $y \rightarrow y_m$.

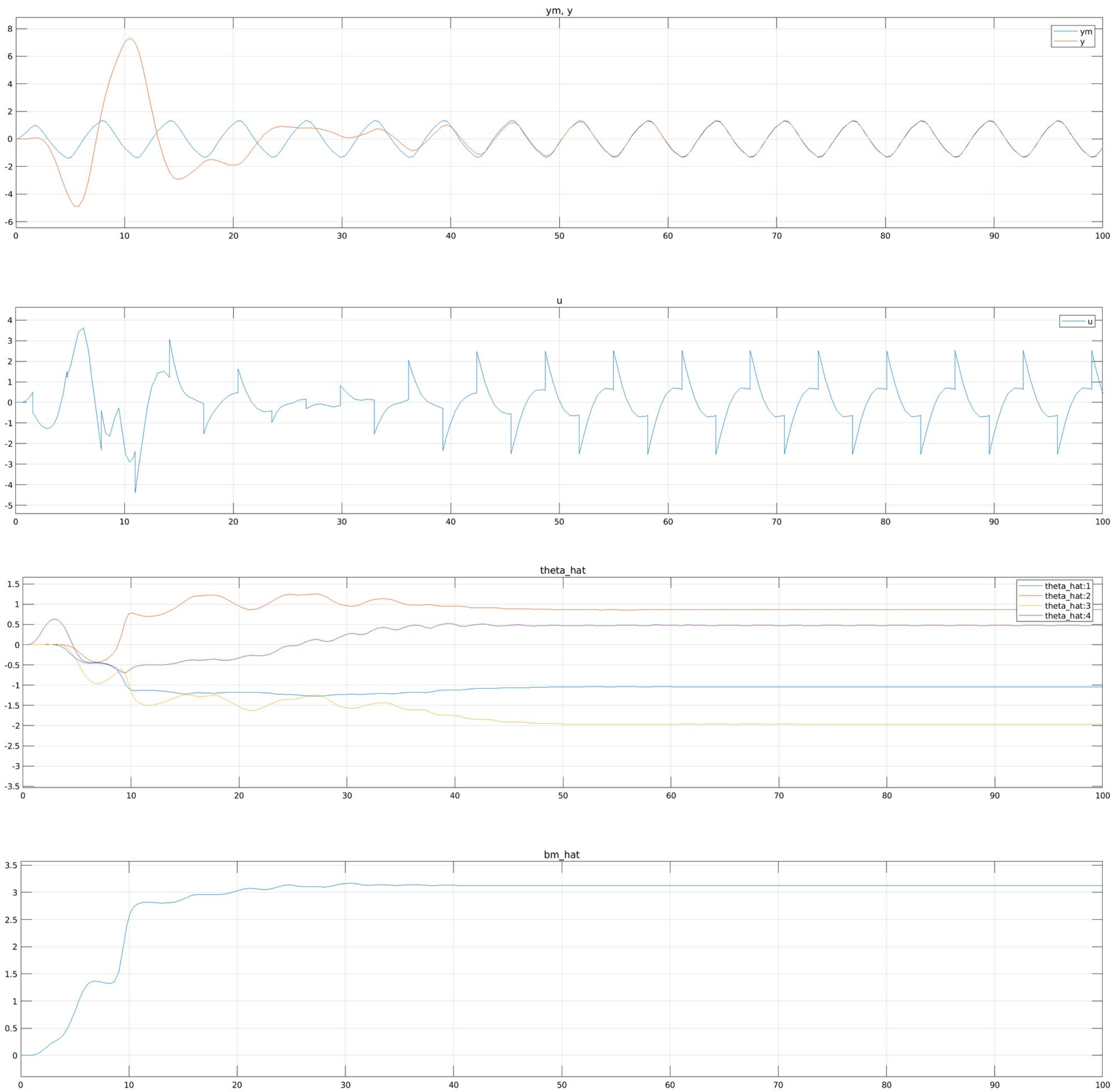

Figure 4: Results of tracking control, gain = 1

Gain: 10

We see in Figure 5that the controller is able to drive $y \rightarrow y_m$.

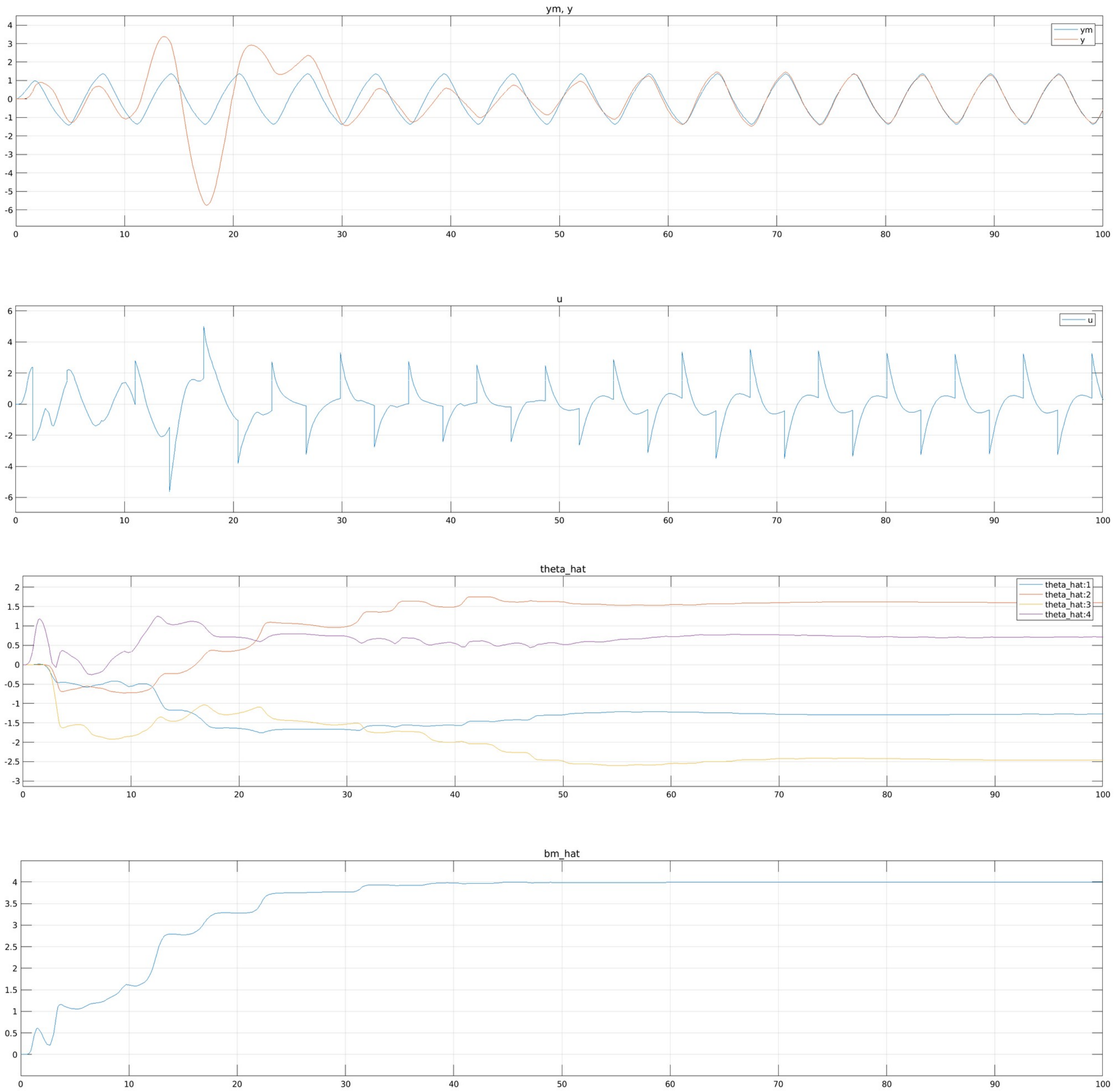

Figure 5: Results of tracking control, gain = 10

Gain: 0.5

We see in Figure 6 that the controller have very good good performance where the error tends to 0 fast.

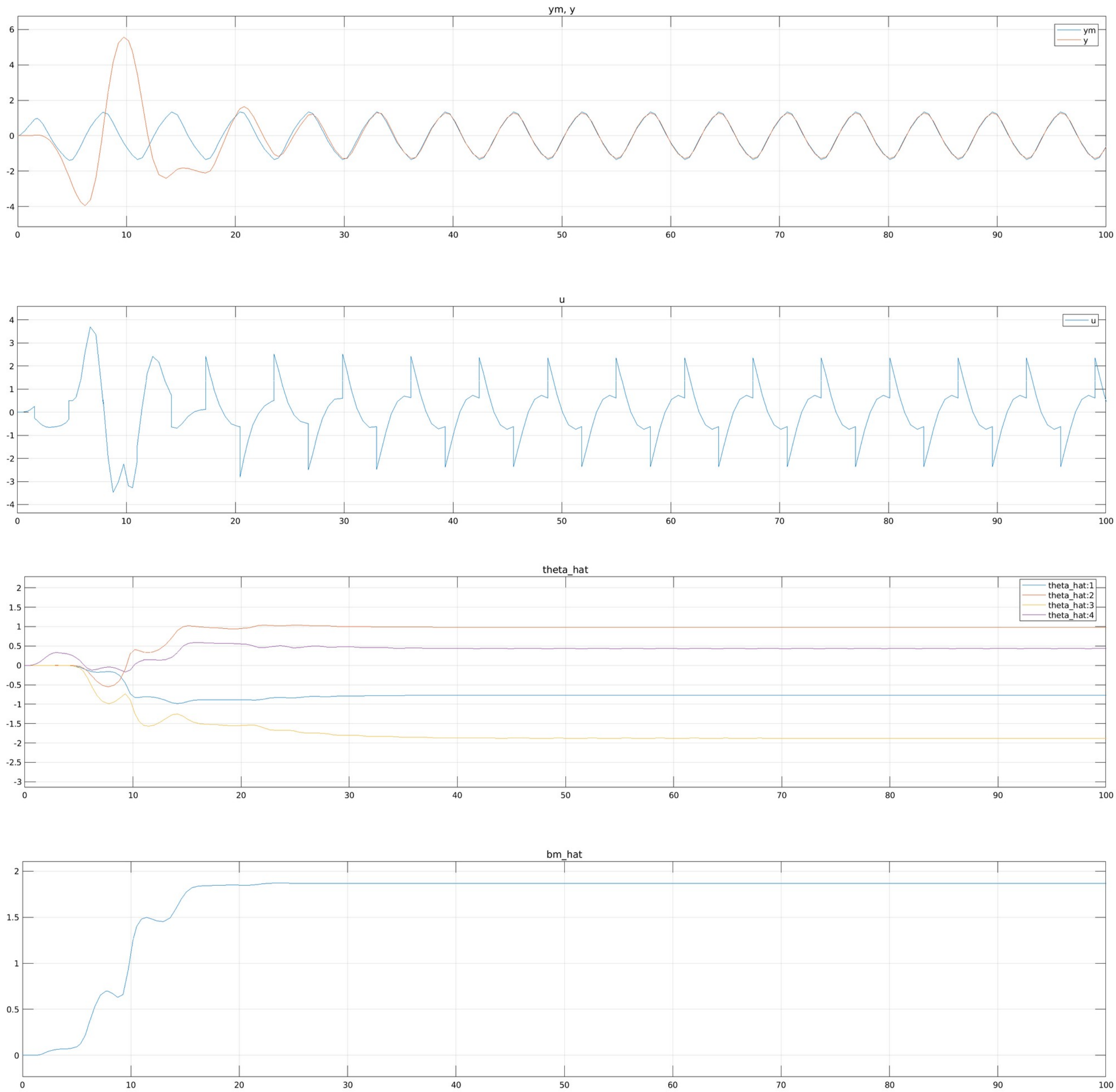

Figure 6: Results of stabilization control, gain = 0.5

Conclusions:

From the results shown before we can conclude the following:

1. The adaptive output feedback controller using augmented error approach is able to control the plant in two modes, stabilization control and tracking control.
2. There is no limitation on the reference model polynomial to be of the first order (SPR) as with the simple adaptive output-feedback control.
3. We can see that For any adaptation gain, the controller preserve the boundedness of all signals in the closed loop system.
4. We can see that the adaptation gain has a large effect on the performance of the controller.
5. Higher adaptation gain doesn’t mean better control as we saw in the tracking control, where a smaller gain gave better results. However, this isn’t the case in the stabilization control.
6. We can say that there is no linear (direct or inverse) of the control performance with the adaptation gain.

Appendix : Simulink Model

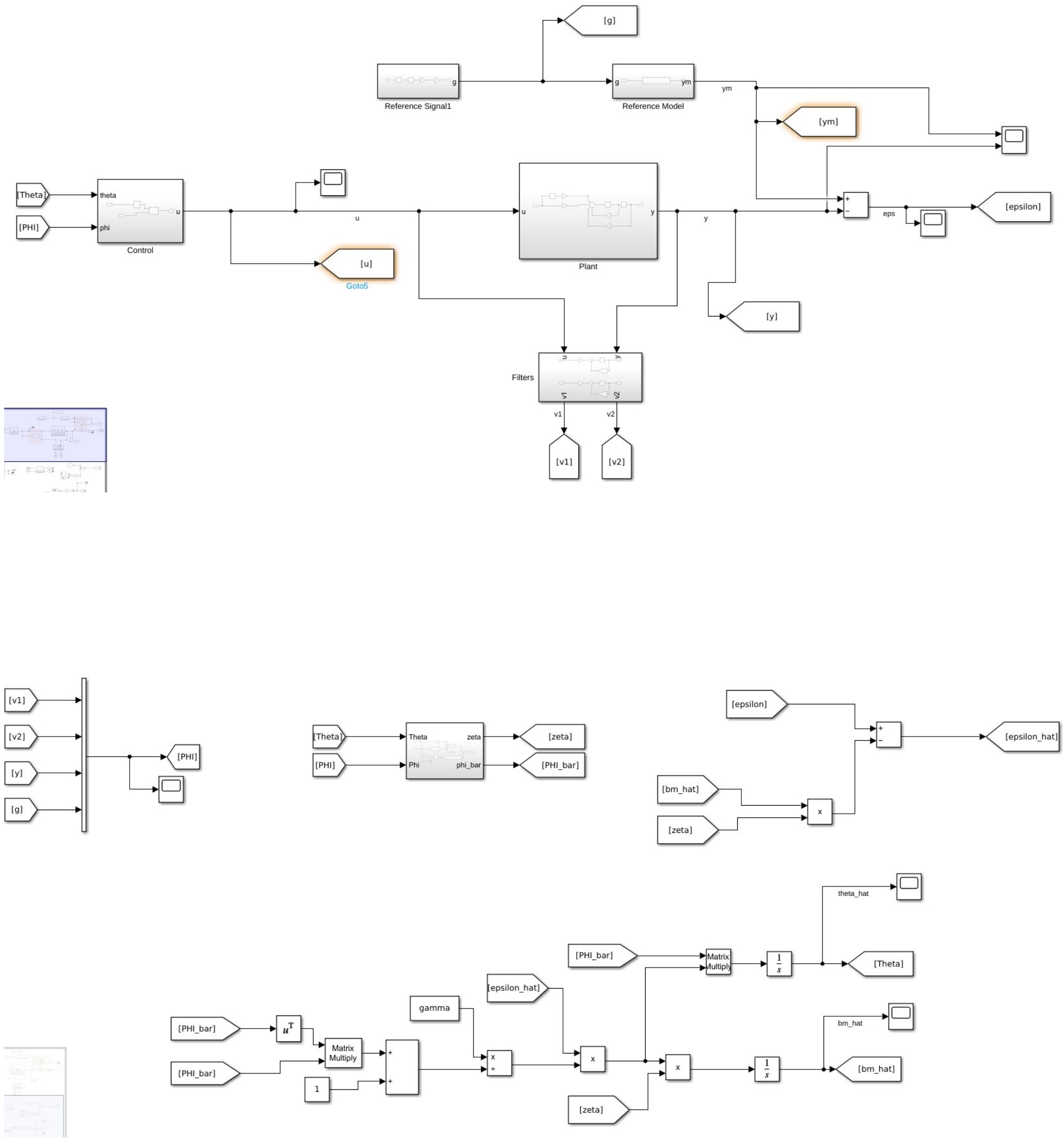

Figure 7: Full model