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Assignment 1
Robot Estimation

1 Kalman Filter

Let the motion model of a simplified 2D robot as follows:

$$\dot{x} = \frac{r}{2}(u_r + u_l) + w_x, \qquad \dot{y} = \frac{r}{2}(u_r + u_l) + w_y \tag{1}$$

where the radius of the wheel is $r = 0.1m$, u_r and u_l are the control signal employed to the right and left wheels, respectively. $w_x = N(0, 0.1)$ and $w_y = N(0, 0.15)$. The speed of each wheel is assumed as $0.1m/s$. In addition, the initial values considered as:

$$x_0 = 0, \quad y_0 = 0, \quad P_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2}$$

It is assumed that the motion model is computed 8 times a second, and every second a measurement is provided as:

$$z = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} r_x & 0 \\ 0 & r_y \end{bmatrix} \tag{3}$$

Here $r_x = N(0, 0.05)$ and $r_y = N(0, 0.75)$. The system need to discretize as:$\dot{x} = \frac{x_k - x_{k-1}}{T}$, where T is 1/8 sec. According to equation 1, the process noise covariance is provided by:

$$Q_k = \begin{bmatrix} w_x & 0 \\ 0 & w_y \end{bmatrix} \tag{4}$$

and the measurement noise covariance is given by:

$$R_k = \begin{bmatrix} r_x & 0 \\ 0 & r_y \end{bmatrix} \tag{5}$$

The Kalman filter estimates a process by using a form of feedback control. The equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of predictor-corrector algorithm for solving numerical problems. The Kalman filter algorithm is depicted in Figure 1. Please noted that $\hat{x_k}^-$ (super minus) is denoted to be a prioir state estimate at step k, $\hat{x_k}$ is a posteriori state estimate at step k.

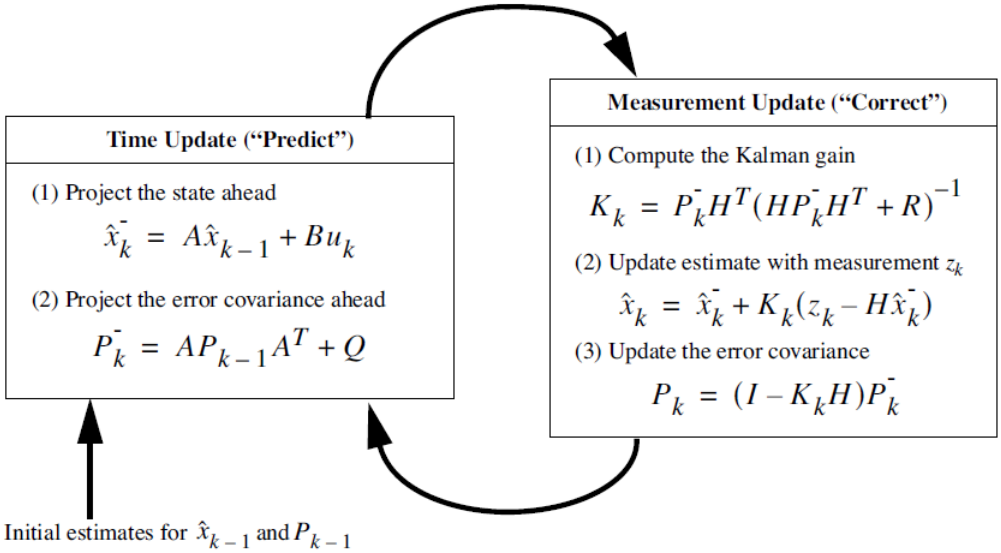

Figure 1: Kalman filter algorithm

The simplified system of 2D robot is simulated by using pyGame to estimate the state employing the Kalman filter algorithm. The outcomes of this method is illustrated in Figure 2. The motion update, the measurement correction and covariance ellipse are shown by pink dot, the red line, and green ellipse, respectively. Since, the process noise covariance and the measurement noise covariance in the direction of y axis is more than x axis, then the system is reluctant to the y axis.

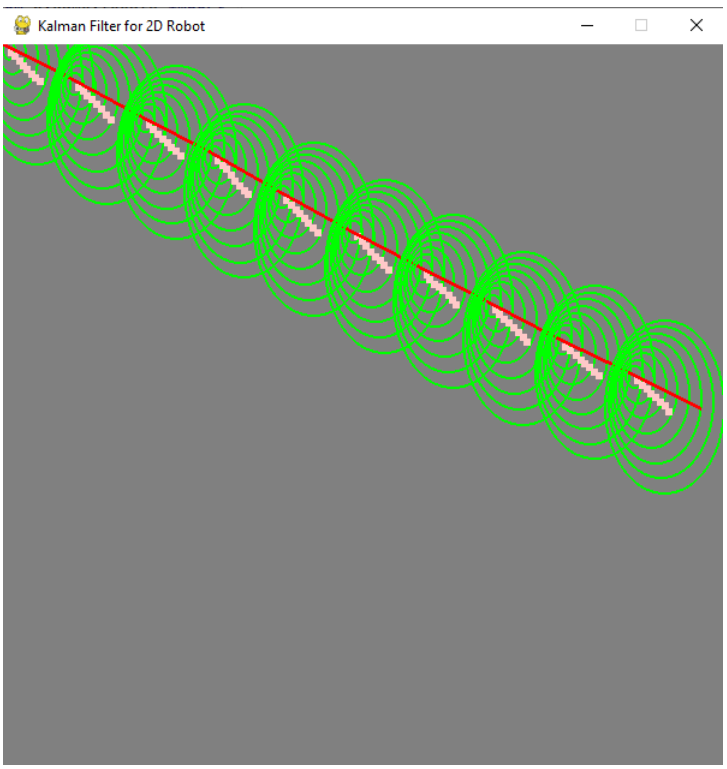

Figure 2: The outcome of state estimation applying Kalman filter algorithm