Computational Mathematics

A bonus project worth an additional 10 points

Question	Points	Score
1	5	
2	5	
Total:	10	

1. (5 points) Apply the fourth order Runge-Kutta (RK4) method to model the spread of a viral outbreak in a population. The task involves solving a system of differential equations that describe the dynamics of the outbreak, analyzing the results, and proposing strategies to control the spread.

Problem Description

A new virus is spreading in a city with a population of 1 million people. The spread of the virus can be modeled using the **SIR model**, which divides the population into three groups:

- **Susceptible (S)**: People who can catch the virus.
- **Infected (I)**: People who are currently infected and can spread the virus.
- **Recovered (R)**: People who have recovered and are immune to the virus.

The dynamics of the outbreak are described by the following system of differential equations:

$$\frac{dS}{dt} = -\beta \cdot S \cdot I$$

$$\frac{dI}{dt} = \beta \cdot S \cdot I - \gamma \cdot I$$

$$\frac{dR}{dt} = \gamma \cdot I$$

Where:

- β is the infection rate (probability of transmission per contact).
- γ is the recovery rate (rate at which infected individuals recover).

Assume the following initial conditions:

- $S(0) = 999,000$ (initial susceptible population).
- $I(0) = 1,000$ (initial infected population).
- $R(0) = 0$ (initial recovered population).

Parameters:

- $\beta = 0.0003$ (per day).
- $\gamma = 0.1$ (per day).

Task Requirements

1. Write a program (in Python or any programming language of your choice) to solve the system of differential equations using the RK4 method.
2. Use a step size of $h = 0.1$ days and simulate the outbreak for 100 days.
3. Plot the curves for $S(t)$, $I(t)$, and $R(t)$ over time.
4. Determine the peak number of infected individuals and the day on which this peak occurs.
5. Calculate the total number of people who were infected at some point during the outbreak.
6. Simulate the effect of a vaccination campaign that reduces the initial susceptible population by 50%. Compare the results with the original scenario.
7. Simulate the effect of social distancing measures that reduce the infection rate β by 50%. Compare the results with the original scenario.
8. Write a detailed report explaining your results, and conclusions. Include graphs, tables, and any relevant calculations.

2. (5 points) Apply numerical integration methods (e.g., Trapezoidal rule, Simpson's rule, or other methods) to estimate the total energy consumption of a city over a 24-hour period. The task involves analyzing real-world data, implementing numerical integration techniques, and interpreting the results to make recommendations for energy efficiency.

Problem Description

A city's energy consumption varies throughout the day due to factors like industrial activity, residential usage, and commercial operations. The power consumption rate $P(t)$ (in megawatts, MW) as a function of time t (in hours) is given by the following data:

Time (hours)	**Power Consumption $P(t)$ (MW)**
0	500
2	480
4	450
6	600
8	800
10	950
12	1000
14	980
16	920
18	850
20	700
22	550
24	500

The total energy consumed E (in megawatt-hours, MWh) over a 24-hour period is given by the integral of $P(t)$ with respect to time:

$$E = \int_0^{24} P(t)\, dt$$

Since the data is discrete, numerical integration methods must be used to approximate the integral.

Task Requirements

1. Write a program (in Python or any programming language of your choice) to approximate the integral using at least two numerical integration methods (e.g., Trapezoidal Rule and Simpson's Rule).
2. Compare the results obtained from the different methods.

3. Calculate the total energy consumption E using each method.
4. Determine which method provides the most accurate result and justify your choice.
5. Write a detailed report explaining your methodology, results, and recommendations. Include graphs, tables, and any relevant calculations.

End of exam.