

1. Consider the word unusual. How many unique subsets of 5 letters (of the 7) exist? How many different strings could be made from 5 of those 7 letters?

U, n, S, a, l unique strings is : 5! ways

2. Using a standard deck of playing cards, how many ways are to form a 5-card hand with 2 pairs (i.e. pair of one value, a pair of a different value, and a fifth card of some other value)?

2 pairs and one x card

$$\begin{array}{c} 1, 2, 3, 4 \\ (12), (13)(14) \\ (23)(34)(24) \end{array} \quad R = \begin{array}{c} 1 \\ 2 \\ A \\ J \\ K \end{array} \quad S = \begin{array}{c} \diamond \\ \heartsuit \\ \clubsuit \\ \spadesuit \\ \vdots \end{array}$$

How many pairs? 13 ranks w/ 4 suits

Choosing 2 unique ranks for the two pairs: $\binom{13}{2} = \frac{13!}{2!11!} = 78$

Choosing 2 suits for one pair of chosen rank: $\binom{4}{2} = \frac{4!}{2!2!} = 6$

Choosing 2 suits for other pair of chosen rank: $\binom{4}{2} = 6$

The fifth card must be of the 11 remaining rank as first pair

is a unique rank as is the second $13 - 1 - 1 = 11$

out of the 11 ranks it can be any of the 4 suits! = $\binom{11}{1} \binom{4}{1} = 44$

$$\# \text{ of ways} = (78)(6)(6)(44) = \boxed{123,552}$$

3. A violinist serenades couples at a romantic restaurant. She will play 16 songs in an hour and there are 7 couples. One couple is having a fight and will allow at most 1 song to be played to them before they ask the violist not to return to their table. If we care only about the number of songs each couple receives, how many ways can the songs be distributed amongst the couples.

Can think of this as 6 couples w/ 15 songs

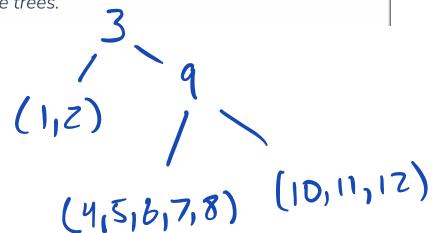
$$\binom{15}{6} = \frac{15!}{6! 9!} = 5005$$

But any of the 16 songs can be played to the fighting couple which is $\binom{16}{1} = 16$

$$\therefore \# \text{ways distributed} = (5005)(16) = \boxed{80,080}$$

4. There is a Binary Search Tree with 12 nodes. Each node has a distinct value between 1 and 12. The root has value 3, and its right child has value 9. How many possible Binary Search Trees could this be? Tip: Try to define how many ways there are to form a BST of 2 nodes. Then try to define how many ways there are to form a BST of 3 nodes (think about the possible insertion order based on rank: smallest, medium, largest) **in terms of 2 node trees** for certain cases. Continue to do this for 4 node trees (in terms of 3- and 2-node trees for various cases of insertion ordering based on rank) and 5 node trees.

12 nodes root = 3 rc = 9
 $[1, 12]$



$$\frac{\binom{2n}{n}}{n+1}$$

$$\frac{\binom{4}{2}}{3} = \frac{\frac{4!}{2!2!}}{3} = \frac{6}{3} = 2$$

$$\frac{\binom{10}{5}}{6} = \frac{\frac{10!}{5!5!}}{6} = \frac{252}{6} = 42$$

$$\frac{\binom{6}{3}}{4} = \frac{\frac{6!}{3!3!}}{4} = \frac{20}{4} = 5$$

$$(2)(42)(5) = \boxed{420 \text{ trees}}$$

5. 10 friends arrive to get their COVID vaccine during a particular time slot. During that time slot there are 4 identical nurses administering shots, but 1 of the nurses **may** (or **may not**) be scheduled for a break during the time slot in which the friends arrive. Also, how long it takes the nurses to administer a shot varies wildly, so the nurses working during the time slot are guaranteed to serve at least 1 person, but how many additional people they are able to serve is arbitrary. How many different combinations are there for the number of patients served by the nurses?

10 friends

4 nurses but 1 of them may be on break.
nurses guaranteed to serve at least one person.