控制系统计算

1. 概述

根据电机的矢量控制原理图，对系统算法进行数值计算。采用的定点数计算控制电机，存在溢出等问题。那么对函数进行幅值限制就很有必要，同时也为测试用例的设计提供相应的理论依据。边界的界定是鲁棒性控制的重要一环。下面将以离散系统为例进行计算。

基本假设：

1. 所有的电流信号都符合正弦和直流分量叠加的信号
2. 所有的信号流都是符合原理计算的结果
3. 电流、电压信号的相位与理论的相位偏差很小。

2. 系统计算

考虑三相电的电流波形，噪声比较小的条件下，采样得到的电流符合以下表达式

$Iu \ = \ 8 * Isin(wt) = Asin(\omega t)$ (2-1)

$Iv \ = \ 8 * Isin(wt - \varphi) = Asin(\omega t - \varphi)$ (2-2)

其中 $A > 0$ 是幅值，φ是在 120°波动的相位。

定点数的范围是$[-32767,32767]$， 那么 $A \in [0,32767]$。

采用等幅值的 Clark 变换

$i_\alpha = i_u$ (2-3)

$i_\beta = \frac{(i_u+2i_v)}{\sqrt{3}}$ (2-4)

$i_\alpha = i_u = Asin(\omega t)$ (2-5)

$i_\beta = \frac{(i_u+2i_v)}{\sqrt{3}} = \frac{A\sqrt{m^2+n^2}}{\sqrt{3}} sin\,(\omega t - \varphi_1)$ (2-6)

Clark 变换

$$i_\alpha = i_u$$

$$i_\beta = \frac{(i_u + 2i_v)}{\sqrt{3}} = \frac{A}{\sqrt{3}}(sin(\omega t) + 2sin(\omega t - \varphi))$$

$$= \frac{A}{\sqrt{3}}((1 + 2\,cos\,(\varphi)\,)\,sin\,(\omega t) - 2\,sin\,(\varphi)\,cos\,(\omega t))$$

$$= \frac{A\sqrt{m^2 + n^2}}{\sqrt{3}} sin\,(\omega t - \varphi_1)$$

$$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \begin{bmatrix} A_1 sin(\omega t) \\ A_1\,sin\,(\omega t - \varphi_1) \end{bmatrix}$$

$$\varphi_1 = arctan\left(\frac{n}{m}\right)$$

$$m \ = \ 1 + 2cos(\varphi)$$

$$n \ = \ 2sin(\varphi)$$

$$0 \leq \frac{\mathrm{A}\sqrt{\mathrm{m}^2+\mathrm{n}^2}}{\sqrt{3}} \leq 32767$$

将静止坐标投射到同步旋转坐标

$i_d = i_\alpha \cos\theta + i_\beta \sin\theta$ (2-7)

$i_q = -i_\alpha \sin\theta + i_\beta \cos\theta$ (2-8)

假设估计出来的位置$\theta = \omega t$ 是准确的，那么

$i_d = A_1\sqrt{m_1^2+n_1^2} sin(2\omega t - \varphi_2) + P$ (2-9)

$i_q = A_1\sqrt{m_1^2+n_1^2} sin(2\omega t + \varphi_3) + Q$ (2-10)

如果估计出来的位置$\theta = \omega t - \Delta\theta$，那么

$i_d = A_1\sqrt{m_2^2+n_2^2} sin(2\omega t + \varphi_4) + P_1$ (2-11)

$i_q = A_1\sqrt{m_2^2+n_2^2} sin(2\omega t + \varphi_5) + Q_1$ (2-12)

假设估计出来的位置$\theta = \omega t$ 是准确的，那么

Park 变换

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} cos\,(\omega t) & sin\,(\omega t) \\ -\,sin\,(\omega t) & cos\,(\omega t) \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix}$$

$$= \begin{bmatrix} cos\,(\omega t) & sin\,(\omega t) \\ -\,sin\,(\omega t) & cos\,(\omega t) \end{bmatrix} \begin{bmatrix} A_1 sin(\omega t) \\ A_1\, sin\,(\omega t - \varphi_1) \end{bmatrix}$$

$$= \begin{bmatrix} A_1 sin(\omega t)\cos(\omega t) + A_1\, sin\,(\omega t - \varphi_1)\, sin\,(\omega t) \\ -\,A_1\, sin\,(\omega t)\, sin(\omega t) +\ A_1\, sin\,(\omega t - \varphi_1)\, cos\,(\omega t) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{A_1}{2}(1 - sin\varphi_1) sin(2\omega t) - \frac{A_1}{2} cos\varphi_1\, cos\,(2\omega t) + \frac{A_1}{2} cos\varphi_1 \\ \frac{A_1}{2} cos\varphi_1\, sin\,(2\omega t) + \frac{A_1}{2}(1 - sin\varphi_1)\, cos\,(2\omega t) - \frac{A_1}{2}(1 + sin\varphi_1) \end{bmatrix}$$

$$= A_1 \begin{bmatrix} \sqrt{m_1^2+n_1^2} sin(2\omega t - \varphi_2) + P \\ \sqrt{m_1^2+n_1^2} sin(2\omega t + \varphi_3) + Q \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}(1 - sin\varphi_1) & -\frac{1}{2} cos\varphi_1 \\ \frac{1}{2} cos\varphi_1 & \frac{1}{2}(1 - sin\varphi_1) \end{bmatrix} \begin{bmatrix} A_1 sin(2\omega t) \\ A_1\, cos\,(2\omega t) \end{bmatrix} + A_1 \begin{bmatrix} \frac{1}{2} cos\varphi_1 \\ -\frac{1}{2}(1 + sin\varphi_1) \end{bmatrix}$$

$$= \begin{bmatrix} m_1 & -\,n_1 \\ n_1 & m_1 \end{bmatrix} \begin{bmatrix} A_1 sin(2\omega t) \\ A_1\, cos\,(2\omega t) \end{bmatrix} + A_1 \begin{bmatrix} P \\ Q \end{bmatrix}$$

$$\le A_1\begin{bmatrix}\sqrt{m_1^2+n_1^2}+|P|\\ \sqrt{m_1^2+n_1^2}+|Q|\end{bmatrix}$$

$$0\le A_1\left(\sqrt{m_1^2+n_1^2}+|P|\right)\le 32767$$

$$0\le A_1\left(\sqrt{m_1^2+n_1^2}+|Q|\right)\le 32767$$

如果估计出来的位置$\theta=\omega t-\Delta\theta$，那么

$$\begin{bmatrix}I_d\\ I_q\end{bmatrix}_{\Delta\theta}=\begin{bmatrix}cos\,(\omega t-\Delta\theta) & sin\,(\omega t-\Delta\theta)\\ -sin\,(\omega t-\Delta\theta) & cos\,(\omega t-\Delta\theta)\end{bmatrix}\begin{bmatrix}A_1 sin(\omega t)\\ A_1\,sin\,(\omega t-\varphi_1)\end{bmatrix}$$

$$=A_1\begin{bmatrix}sin(\omega t)\,cos\,(\omega t-\Delta\theta)+sin\,(\omega t-\varphi_1)\,sin\,(\omega t-\Delta\theta)\\ -sin\,(\omega t)\,sin(\omega t-\Delta\theta)+sin\,(\omega t-\varphi_1)\,cos\,(\omega t-\Delta\theta)\end{bmatrix}$$

$$=A_1\begin{bmatrix}\frac{1}{2}(1-sin\varphi_1-\Delta\theta cos\varphi_1)sin(2\omega t)-\frac{1}{2}(cos\varphi_1+\Delta\theta-\Delta\theta sin\varphi_1)\,cos\,(2\omega t)+\frac{1}{2}(cos\varphi_1+\Delta\theta+\Delta\theta sin\varphi_1)\\ \frac{1}{2}(cos\varphi_1+\Delta\theta-\Delta\theta sin\varphi_1)\,sin\,(2\omega t)+\frac{1}{2}(1-sin\varphi_1-\Delta\theta cos\varphi_1)\,cos\,(2\omega t)-\frac{1}{2}(1+sin\varphi_1-\Delta\theta cos\varphi_1)\end{bmatrix}$$

$$=A_1\begin{bmatrix}\sqrt{m_2^2+n_2^2}sin(2\omega t-\varphi_4)+P_1\\ \sqrt{m_2^2+n_2^2}sin(2\omega t+\varphi_5)+Q_1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2}(1-sin\varphi_1-\Delta\theta cos\varphi_1) & -\frac{1}{2}(cos\varphi_1+\Delta\theta-\Delta\theta sin\varphi_1)\\ \frac{1}{2}(cos\varphi_1+\Delta\theta-\Delta\theta sin\varphi_1) & \frac{1}{2}(1-sin\varphi_1-\Delta\theta cos\varphi_1)\end{bmatrix}\begin{bmatrix}A_1 sin(2\omega t)\\ A_1\,cos\,(2\omega t)\end{bmatrix}$$

$$+A_1\begin{bmatrix}\frac{1}{2}(cos\varphi_1+\Delta\theta+\Delta\theta sin\varphi_1)\\ -\frac{1}{2}(1+sin\varphi_1-\Delta\theta cos\varphi_1)\end{bmatrix}$$

$$=\begin{bmatrix}m_2 & -n_2\\ n_2 & m_2\end{bmatrix}\begin{bmatrix}A_1 sin(2\omega t)\\ A_1\,cos\,(2\omega t)\end{bmatrix}+A_1\begin{bmatrix}P_1\\ Q_1\end{bmatrix}$$

$$\le A_1\begin{bmatrix}\sqrt{m_2^2+n_2^2}+|P_1|\\ \sqrt{m_2^2+n_2^2}+|Q_1|\end{bmatrix}$$

$$0\le A_1\left(\sqrt{m_2^2+n_2^2}+|P_1|\right)\le 32767$$

$$0\le A_1\left(\sqrt{m_2^2+n_2^2}+|Q_1|\right)\le 32767$$

与位置估计无偏差时比较，得到误差表达式

$$\begin{bmatrix}\Delta I_d\\ \Delta I_q\end{bmatrix} = \begin{bmatrix}I_d\\ I_q\end{bmatrix}_{\Delta\theta} - \begin{bmatrix}I_d\\ I_q\end{bmatrix}$$

$$= A_1\begin{bmatrix}-\frac{1}{2}\Delta\theta cos\varphi_1 sin(2\omega t) - \frac{1}{2}(\Delta\theta - \Delta\theta sin\varphi_1)\cos(2\omega t) + \frac{1}{2}(\Delta\theta + \Delta\theta sin\varphi_1)\\ \frac{1}{2}(\Delta\theta - \Delta\theta sin\varphi_1)\sin(2\omega t) - \frac{1}{2}\Delta\theta cos\varphi_1\cos(2\omega t) + \frac{1}{2}\Delta\theta cos\varphi_1\end{bmatrix}$$

$$= A_1\begin{bmatrix}-\frac{1}{2}\Delta\theta cos\varphi_1 & -\frac{1}{2}(\Delta\theta - \Delta\theta sin\varphi_1)\\ \frac{1}{2}(\Delta\theta - \Delta\theta sin\varphi_1) & -\frac{1}{2}\Delta\theta cos\varphi_1\end{bmatrix}\begin{bmatrix}sin(2\omega t)\\ cos(2\omega t)\end{bmatrix} + A_1\begin{bmatrix}\frac{1}{2}(\Delta\theta + \Delta\theta sin\varphi_1)\\ \frac{1}{2}\Delta\theta cos\varphi_1\end{bmatrix}$$

磁链估计连续域的表达式为

$$d\psi = U - RI \tag{2-13}$$

采用积分器和高通滤波器进行磁链的求解

$$d\psi\frac{1}{s}\frac{\tau s}{1+\tau s} = y \tag{2-14}$$

整理上式得到

$$d\psi\frac{\tau}{1+\tau s} = y \tag{2-15}$$

结合前向差分公式，将公式离散化

$$s = \frac{z-1}{T} \tag{2-16}$$

代入式（2-15），得到

$$y(k+1) = \frac{U(k)-RI(k)}{f} - \frac{\left(\frac{1}{\tau}-\frac{1}{T}\right)}{f}y(k) \tag{2-17}$$

$$y(k) = y(k+1) \tag{2-18}$$

磁链估计

$$d\psi(k)\frac{\tau}{1+\tau\frac{z-1}{T}} = y(k)$$

$$d\psi(k)\frac{\tau}{1+\tau\frac{z-1}{T}} = y(k)$$

$$(U(k) - RI(k))\tau = y\left(1 + \tau\frac{z-1}{T}\right)$$

$$= \left(y(k) + \frac{\tau}{T}y(k+1) - \frac{\tau}{T}y(k)\right)$$

$$\frac{\tau}{T}y(k+1) = (U(k) - RI(k))\tau + \left(1 - \frac{\tau}{T}\right)y(k)$$

$$y(k+1)=\frac{U(k)-RI(k)}{f}-\frac{\left(\frac{1}{\tau}-\frac{1}{T}\right)}{f}y(k)$$

$$y(k)=y(k+1)$$

将式（2-17）和式（2-18）应用到α和β轴上磁链的估计，考虑到式（2-17）和式（2-18）中的迭代项 $y(k)$，防止累计误差偏大，应进行相应的误差补偿， 得到如下表达式。

$$\psi_{\alpha}^{temp}(k+1)=\psi_{\alpha}^{temp}(k)+\frac{u_{\alpha}(k)*Vmax-\frac{R_{S}i_{\alpha}(k)}{64}-K_{gain}\psi_{\alpha}(k)}{f_{pwm}} \quad (2\text{-}19)$$

$$\psi_{\alpha}(k+1)=\psi_{\alpha}^{temp}(k+1)-\frac{i_{\alpha}(k)\left(L_{D}+L_{Q}\right)}{2^{13}} \quad (2\text{-}20)$$

$$\psi_{\beta}^{temp}(k+1)=\psi_{\beta}^{temp}(k)+\frac{u_{\beta}(k)*Vmax-\frac{R_{S}i_{\beta}(k)}{64}-K_{gain}\psi_{\beta}(k)}{f_{pwm}} \quad (2\text{-}21)$$

$$\psi_{\beta}(k+1)=\psi_{\beta}^{temp}(k+1)-\frac{i_{\beta}(k)\left(L_{D}+L_{Q}\right)}{2^{13}} \quad (2\text{-}22)$$

为了分析简单，将下标略去，得到如下表达式

$$\psi^{temp}(k+1)=\psi^{temp}(k)+\frac{u(k)*V_{max}-\frac{R_{S}i(k)}{64}-K_{gain}\psi(k)}{f_{pwm}} \quad (2\text{-}23)$$

$$\psi(k+1)=\psi^{temp}(k+1)-\frac{i(k)L}{8192} \quad (2\text{-}24)$$

$$\psi^{temp}(k)=\frac{1}{k_g}\left(V_{max}u(k)+\left(-\frac{R}{64}+\frac{k_gL}{8192}\right)i(k)\right) \quad (2\text{-}25)$$

$$\psi(k)=\frac{1}{k_g}\left(V_{max}u(k)-\frac{R}{64}i(k)\right) \quad (2\text{-}26)$$

在稳态时可以近似求解出式（2-23）和式（2-24）中$\psi(k)$和$\psi^{temp}(k)$的表达式。

磁链估计

$$\psi^{temp}(k+1)=\psi^{temp}(k)+\frac{u(k)*V_{max}-\frac{R_{s}i(k)}{64}-K_{gain}\psi(k)}{f_{pwm}}$$

$$\psi(k+1)=\psi^{temp}(k+1)-\frac{iL}{8192}$$

$$\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}\begin{bmatrix}\psi^{temp}(k+1)\\ \psi(k+1)\end{bmatrix}=\begin{bmatrix}1 & -\frac{k_g}{f}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\psi^{temp}(k)\\ \psi(k)\end{bmatrix}+\begin{bmatrix}\frac{V_{max}}{f} & -\frac{R}{64f}\\ 0 & -\frac{L}{8192}\end{bmatrix}\begin{bmatrix}u(k)\\ i(k)\end{bmatrix}$$

假设稳态时，$\begin{bmatrix}\psi^{temp}(k+1)\\ \psi(k+1)\end{bmatrix}=\begin{bmatrix}\psi^{temp}(k)\\ \psi(k)\end{bmatrix}=\psi$，那么

$$\left(\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}-\begin{bmatrix}1 & -\frac{k_g}{f}\\ 0 & 0\end{bmatrix}\right)\psi=\begin{bmatrix}\frac{V_{max}}{f} & -\frac{R}{64f}\\ 0 & -\frac{L}{8192}\end{bmatrix}\begin{bmatrix}u(k)\\ i(k)\end{bmatrix}$$

$$\psi = \frac{1}{k_g}\begin{bmatrix} V_{max} & -\frac{R}{64}+\frac{k_g L}{8192} \\ V_{max} & -\frac{R}{64} \end{bmatrix}\begin{bmatrix} u(k) \\ i(k) \end{bmatrix}$$

$$\begin{bmatrix} \psi^{temp}(k) \\ \psi(k) \end{bmatrix} = \begin{bmatrix} \frac{1}{k_g}\left(V_{max}u(k)+\left(-\frac{R}{64}+\frac{k_g L}{8192}\right)i(k)\right) \\ \frac{1}{k_g}\left(V_{max}u(k)-\frac{R}{64}i(k)\right) \end{bmatrix}$$

结合 Clark 变换得到的α和β轴上的电流

$$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \begin{bmatrix} A_1 sin(\omega k) \\ A_1 sin\,(\omega k-\varphi_1) \end{bmatrix}$$

因为这里的$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix}$是正弦信号，假设电压信号也是正弦信号，那么可以得到

$$\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} B_1 sin(\omega k-\varphi_6) \\ B_1 sin\,(\omega k-\varphi_7) \end{bmatrix}$$

$$\psi_\alpha^{temp}(k) = \frac{1}{k_g}\left(V_{max}u(k)+\left(-\frac{R}{64}+\frac{k_g L}{8192}\right)i(k)\right)$$

$$= \frac{V_{max}}{k_g}u(k)+\left(-\frac{R}{64k_g}+\frac{L}{8192}\right)i(k)$$

$$= \frac{V_{max}}{k_g}B_1 sin(\omega k-\varphi_6)+\left(-\frac{R}{64k_g}+\frac{L}{8192}\right)A_1 sin(\omega k)$$

$$= M_1 sin(\omega k-\varphi_6)+N_1 sin(\omega k)$$

$$= (M_1 cos(\varphi_6)+N_1)sin(\omega k)-M_1 sin(\varphi_6)cos(\omega k)$$

$$= P_1\,sin\,(\omega k)-Q_1 cos(\omega k)$$

$$= \sqrt{P_1^2+Q_1^2}\,sin\,(\omega k-\varphi_7)$$

$$\varphi_7 = arctan\left(\frac{Q_1}{P_1}\right)$$

$$\psi_\beta^{temp}(k) = \frac{V_{max}}{k_g}u(k)+\left(-\frac{R}{64k_g}+\frac{L}{8192}\right)i(k)$$

$$= \frac{V_{max}}{k_g}B_1\,sin\,(\omega k-\varphi_7)+\left(-\frac{R}{64k_g}+\frac{L}{8192}\right)A_1\,sin\,(\omega k-\varphi_1)$$

$$= M_2\,sin\,(\omega k-\varphi_7)+N_2\,sin\,(\omega k-\varphi_1)$$

$$= (M_2 cos(\varphi_7)+N_2 cos(\varphi_1))sin(\omega k)-(M_2 sin(\varphi_7)+N_2 sin(\varphi_1))cos(\omega k)$$

$$= P_2\,sin\,(\omega k)-Q_2 cos(\omega k)$$

$$= \sqrt{P_2^2+Q_2^2}\,sin\,(\omega k-\varphi_8)$$

$$\varphi_8 = arctan\left(\frac{Q_2}{P_2}\right)$$

$$\psi_\alpha(k) = \frac{V_{max}}{k_g}B_1 sin(\omega k-\varphi_6)-\frac{R}{64k_g}A_1 sin(\omega k)$$

$$= M_3 sin(\omega k-\varphi_6)-N_3 sin(\omega k)$$

$$= (M_3 cos(\varphi_6) - N_3) sin(\omega k) - M_3 sin(\varphi_6) cos(\omega k)$$
$$= P_3 \sin(\omega k) - Q_3 cos(\omega k)$$
$$= \sqrt{P_3^2 + Q_3^2} \sin(\omega k - \varphi_9)$$
$$\varphi_9 = arctan\left(\frac{Q_3}{P_3}\right)$$

$$\psi_\beta(k) = \frac{V_{max}}{k_g} u(k) - \frac{R}{64k_g} i(k)$$
$$= \frac{V_{max}}{k_g} B_1 \sin(\omega k - \varphi_7) - \frac{R}{64k_g} A_1 \sin(\omega k - \varphi_1)$$
$$= M_4 \sin(\omega k - \varphi_7) - N_4 \sin(\omega k - \varphi_1)$$
$$= (M_4 cos(\varphi_7) - N_4 cos(\varphi_1)) sin(\omega k) - (M_4 sin(\varphi_7) - N_4 sin(\varphi_1)) cos(\omega k)$$
$$= P_4 \sin(\omega k) - Q_4 cos(\omega k)$$
$$= \sqrt{P_4^2 + Q_4^2} \sin(\omega k - \varphi_{10})$$
$$\varphi_{10} = arctan\left(\frac{Q_4}{P_4}\right)$$

根据式（2-25）和式（2-26），可以求得相应的边界

速度位置估计

$\omega = \Delta\theta\left(k_p + \frac{k_i}{s}\right)$ (2-27)

$\theta = \theta + \frac{\omega}{f_{pwm}}$ (2-28)

采用磁链速度和位置进行估计

锁相环估计速度和位置

$$\psi_\alpha = \psi cos\theta_r$$
$$\psi_\beta = \psi sin\theta_r$$
$$\Delta\theta = \psi_\beta cos\theta - \psi_\alpha sin\theta$$
$$= \psi(sin\theta_r cos\theta - cos\theta_r sin\theta)$$
$$= \psi \sin(\theta_r - \theta) \approx K(\theta_r - \theta)$$

采用 PI 求得速度

$$\omega = \Delta\theta\left(k_p + \frac{k_i}{s}\right)$$

前向差分公式 $s = \frac{(z-1)}{T}$

$$\omega(k) = \Delta\theta(k)\left(k_p + \frac{k_i}{\frac{(z-1)}{T}}\right)$$

$$= \Delta\theta(k)\left(k_p + \frac{k_i T}{z-1}\right)$$
$$= \omega_1(k) + \omega_2(k)$$
$$\omega_1(k) = \Delta\theta(k) k_p$$
$$\omega_2(k) = \Delta\theta(k)\frac{k_i T}{z-1}$$
$$\omega_2(k) = \omega_2(k-1) + \Delta\theta(k) k_i T$$
$$\theta(k+1) = \theta(k) + \omega(k)T$$
$$= \theta(k) + \frac{\omega(k)}{f_{pwm}}$$

观察到式（2-25）和式（2-26）估计得到的磁链，幅值和相位都存在偏差，那么可以假设磁链的表达式符合以下表达式。

$$\psi_\alpha = \psi_1 \cos(\theta_r + \phi_\alpha) \tag{2-29}$$
$$\psi_\beta = \psi_2 \sin(\theta_r + \phi_\beta) \tag{2-30}$$

$$\tilde{\theta} = \begin{cases} \frac{\psi_1\phi_\alpha + \psi_2\phi_\beta}{2} + \frac{\psi_2 - \psi_1}{2}sin(2\theta) + \left(\frac{\psi_2\phi_\beta}{2} - \frac{\psi_1\phi_\alpha}{2}\right)cos(2\theta),\ \psi_1 \le \psi_2 \\ \frac{\psi_1 - \psi_2}{2}(1 + cos(2\theta)) + \frac{\psi_2\phi_\beta}{2}(1 + cos(2\theta)) + \frac{\psi_1\phi_\alpha}{2}(1 - cos(2\theta)),\ \psi_1 > \psi_2 \end{cases} \tag{2-31}$$

公式推导内在假设

$$cos\phi \approx 1, sin\phi \approx \phi \tag{2-32}$$

锁相环偏差分析

$$\Delta\theta_1 = \psi_\beta cos\theta - \psi_\alpha sin\theta = \psi_2 \sin(\theta_r + \phi_\beta) cos\theta - \psi_1 \cos(\theta_r + \phi_\alpha) sin\theta$$
$$= \psi_2(\sin\theta_r\, cos\phi_\beta + \cos\theta_r\, sin\phi_\beta)cos\theta - \psi_1(\cos\theta_r\, cos\phi_\alpha - \sin\theta_r\, sin\phi_\alpha)sin\theta$$
$$= \psi_2(\sin\theta_r\, cos\theta + \phi_\beta \cos\theta_r\, cos\theta) - \psi_1(\cos\theta_r\, sin\theta - \phi_\alpha \sin\theta_r\, sin\theta)$$

稳态下，假设$\theta_r = \theta$。

当 $\psi_1 \le \psi_2$

$$\Delta\theta_1 = \psi_1(\sin\theta_r\, cos\theta - \cos\theta_r\, sin\theta) + (\psi_2 - \psi_1)\sin\theta_r\, cos\theta + \psi_2\phi_\beta \cos\theta_r\, cos\theta + \psi_1\phi_\alpha \sin\theta_r\, sin\theta$$
$$\tilde{\theta} = \Delta\theta_1 - \Delta\theta = (\psi_2 - \psi_1)\sin\theta_r\, cos\theta + \psi_2\phi_\beta \cos\theta_r\, cos\theta + \psi_1\phi_\alpha \sin\theta_r\, sin\theta$$
$$= \frac{\psi_2 - \psi_1}{2}sin(2\theta) + \frac{\psi_2\phi_\beta}{2}(1 + cos(2\theta)) + \frac{\psi_1\phi_\alpha}{2}(1 - cos(2\theta))$$
$$= \frac{\psi_1\phi_\alpha + \psi_2\phi_\beta}{2} + \frac{\psi_2 - \psi_1}{2}sin(2\theta) + \left(\frac{\psi_2\phi_\beta}{2} - \frac{\psi_1\phi_\alpha}{2}\right)cos(2\theta)$$

当 $\psi_1 > \psi_2$

$$\Delta\theta_1 = \psi_2(\sin\theta_r\, cos\theta - \cos\theta_r\, sin\theta) + (\psi_1 - \psi_2)\cos\theta_r\, sin\theta + \psi_2\phi_\beta \cos\theta_r\, cos\theta + \psi_1\phi_\alpha \sin\theta_r\, sin\theta$$
$$\tilde{\theta} = \Delta\theta_1 - \Delta\theta = (\psi_1 - \psi_2)\cos\theta_r\, sin\theta + \psi_2\phi_\beta \cos\theta_r\, cos\theta + \psi_1\phi_\alpha \sin\theta_r\, sin\theta$$
$$= \frac{\psi_1 - \psi_2}{2}(1 + cos(2\theta)) + \frac{\psi_2\phi_\beta}{2}(1 + cos(2\theta)) + \frac{\psi_1\phi_\alpha}{2}(1 - cos(2\theta))$$
$$= \frac{\psi_1\phi_\alpha + \psi_2\phi_\beta + \psi_2 - \psi_1}{2} + \left(\frac{\psi_1 - \psi_2}{2} + \frac{\psi_2\phi_\beta}{2} - \frac{\psi_1\phi_\alpha}{2}\right)cos(2\theta)$$

信号本身有误差，采样也存在误差，那么滤波器就显得很有必要了。接下来设计一个低通滤波器。
采用一阶滤波器进行设计，其表达式为

$$G(s)=\frac{1}{\tau s+1} \tag{2-33}$$

采用前向差分 $s=\frac{z-1}{T}$，得到

$$y(k+1)=\frac{\omega_c}{f}u(k)+\left(1-\frac{\omega_c}{f}\right)y(k) \tag{2-34}$$

低通滤波器

$$G(s)=\frac{1}{\tau s+1}$$

前向差分 $\mathrm{s}=\frac{\mathrm{z}-1}{\mathrm{T}}$

$$\frac{y(k)}{u(k)}=\frac{1}{\tau s+1}=\frac{1}{\tau\frac{z-1}{T}+1}$$

$$u(k)=y(k)\left(\tau\frac{z-1}{T}+1\right)$$

$$\frac{\tau}{T}y(k+1)-\left(\frac{\tau}{T}-1\right)y(k)=u(k)$$

$$\begin{aligned}y(k+1)&=\frac{\omega_c}{f}u(k)+\left(1-\frac{\omega_c}{f}\right)y(k)\\&=\alpha u(k)+(1-\alpha)y(k)\end{aligned}$$

归一化项$\alpha=\frac{\omega_c}{\mathrm{f}}\in[0,1]$。

速度环

$$\tilde{\omega}(k)=\omega^*(k)-\omega(k)$$

$$\begin{aligned}i_q(k)&=\tilde{\omega}(k)\left(k_p+\frac{k_iT}{z-1}\right)\\&=i_{q1}(k)+i_{q2}(k)\end{aligned}$$

$$i_{q1}(k)=\tilde{\omega}(k)k_p$$

$$i_{q2}(k)=\tilde{\omega}(k)\frac{k_iT}{z-1}$$

$$i_{q2}(k)=i_{q2}(k-1)+\tilde{\omega}(k)k_iT$$

电流环

$$\tilde{i}_q(k)=i_q^*(k)-i_q(k)$$

$$U_q(k)=\tilde{i}_q(k)\left(k_p+\frac{k_iT}{z-1}\right)$$

$$= U_{q1}(k) + U_{q2}(k)$$
$$U_{q1}(k) = \tilde{i}_q(k)k_p$$
$$U_{q2}(k) = \tilde{i}_q(k)\frac{k_iT}{z-1}$$
$$U_{q2}(k) = U_{q2}(k-1) + \tilde{i}_q(k)k_iT$$

$$\tilde{i}_d(k) = i_d^*(k) - i_d(k)$$
$$U_d(k) = \tilde{i}_d(k)\left(k_p + \frac{k_iT}{z-1}\right)$$
$$= U_{d1}(k) + U_{d2}(k)$$
$$U_{d1}(k) = \tilde{i}_d(k)k_p$$
$$U_{d2}(k) = \tilde{i}_d(k)\frac{k_iT}{z-1}$$
$$U_{d2}(k) = U_{d2}(k-1) + \tilde{i}_d(k)k_iT$$

将得到的 U_d 和 U_q 变换到变换到α和β轴上的电压。

$$\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} cos\,(\omega t) & -sin\,(\omega t) \\ sin\,(\omega t) & cos\,(\omega t) \end{bmatrix} \begin{bmatrix} U_d \\ U_q \end{bmatrix}$$

理论情况下，$\begin{bmatrix} U_d \\ U_q \end{bmatrix}$应该是直流分量，但是在实际计算中会有谐波叠加在$\begin{bmatrix} U_d \\ U_q \end{bmatrix}$中。假设$\begin{bmatrix} U_d \\ U_q \end{bmatrix}$符合以下表达式

$$\begin{bmatrix} U_d \\ U_q \end{bmatrix} = \begin{bmatrix} M + A_d\,sin\,(\omega t + \varphi_d) \\ N + A_q\,sin\left(\omega t + \varphi_q\right) \end{bmatrix}$$
$$= \begin{bmatrix} M \\ N \end{bmatrix} + \begin{bmatrix} A_d cos\varphi_d & A_d sin\varphi_d \\ A_q cos\varphi_q & A_q cos\varphi_q \end{bmatrix} \begin{bmatrix} sin\,(\omega t) \\ cos\,(\omega t) \end{bmatrix}$$

$$\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} cos\,(\omega t) & -sin\,(\omega t) \\ sin\,(\omega t) & cos\,(\omega t) \end{bmatrix} \left(\begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} A_d cos\varphi_d & A_d sin\varphi_d \\ A_q cos\varphi_q & A_q sin\varphi_q \end{bmatrix} \begin{bmatrix} sin\,(\omega t) \\ cos\,(\omega t) \end{bmatrix}\right)$$
$$= \begin{bmatrix} X\,cos\,(\omega t) - Y\,sin\,(\omega t) \\ X\,sin\,(\omega t) + Y\,cos\,(\omega t) \end{bmatrix}$$
$$+ \begin{bmatrix} \frac{A_d cos\varphi_d - A_q sin\varphi_q}{2} sin(2\omega t) + \frac{A_d sin\varphi_d + A_q cos\varphi_q}{2} cos(2\omega t) + \frac{A_d sin\varphi_d - A_q cos\varphi_q}{2} \\ \frac{A_d sin\varphi_d + A_q cos\varphi_q}{2} sin(2\omega t) + \frac{A_q sin\varphi_q - A_d cos\varphi_d}{2} cos(2\omega t) + \frac{A_d cos\varphi_d + A_q sin\varphi_q}{2} \end{bmatrix}$$

$$\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} Z_1\,sin\left(\omega t + \varphi_{dq1}\right) + A_{dq1} sin\left(2\omega t + \varphi_{dq2}\right) + C_1 \\ Z_2\,sin\left(\omega t + \varphi_{dq3}\right) + A_{dq2} sin\left(2\omega t + \varphi_{dq4}\right) + C_2 \end{bmatrix}$$
$$\leq \begin{bmatrix} Z_1 + A_{dq1} + C_1 \\ Z_2 + A_{dq2} + C_2 \end{bmatrix}$$

将由反 Park 变换得到的α和β轴上的电压通过反 Clark 变换转换到 U、V、W 三相电压。

$$\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix} = \begin{bmatrix} Z_1 \, sin\left(\omega t + \varphi_{dq1}\right) + A_{dq1} sin\left(2\omega t + \varphi_{dq2}\right) + C_1 \\ Z_2 \, sin\left(\omega t + \varphi_{dq3}\right) + A_{dq2} sin\left(2\omega t + \varphi_{dq4}\right) + C_2 \end{bmatrix}$$

$$\begin{bmatrix} U_u \\ U_v \\ U_w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix}$$

$$= \begin{bmatrix} U_\alpha \\ \frac{\sqrt{3}U_\alpha - U_\beta}{2} \\ -\frac{\sqrt{3}U_\alpha + U_\beta}{2} \end{bmatrix}$$

$$\begin{bmatrix} U_u \\ U_v \\ U_w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} Z_1 \, sin\left(\omega t + \varphi_{dq1}\right) + A_{dq1} sin\left(2\omega t + \varphi_{dq2}\right) + C_1 \\ Z_2 \, sin\left(\omega t + \varphi_{dq3}\right) + A_{dq2} sin\left(2\omega t + \varphi_{dq4}\right) + C_2 \end{bmatrix}$$

矢量控制计算

反 Clark 变换计算得到的三相电压

计算三相电压输出的时间

$$T_a = \frac{U_u}{Vdc} T_{pwm}$$

$$T_b = \frac{U_v}{Vdc} T_{pwm}$$

$$T_c = \frac{U_w}{Vdc} T_{pwm}$$

加入死区补偿

$$T_a = T_a + T_{ud}$$
$$T_b = T_b + T_{vd}$$
$$T_c = T_c + T_{wd}$$

扇区判断

$$T_a \geq T_b \geq T_c, Sector\ 1$$
$$T_a \geq T_c \geq T_b, Sector\ 2$$
$$T_b \geq T_c \geq T_a, Sector\ 3$$
$$T_b \geq T_a \geq T_c, Sector\ 4$$
$$T_c \geq T_a \geq T_b, Sector\ 5$$
$$T_c \geq T_b \geq T_a, Sector\ 6$$

计算中间时间

$$Sector\ 1, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_c}{2}$$

$$Sector\ 2, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_b}{2}$$

$$Sector\ 3, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_b}{2}$$

$$Sector\ 4, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_b + T_c}{2}$$

$$Sector\ 5, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_b + T_c}{2}$$

$$Sector\ 6, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_c}{2}$$

注入三次谐波

$$T_a = T_a + T_{mid}$$
$$T_b = T_b + T_{mid}$$
$$T_c = T_c + T_{mid}$$

计算 T1 和 T2

$$Sector\ 1, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_c}{2}$$

$$Sector\ 2, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_b}{2}$$

$$Sector\ 3, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_b}{2}$$

$$Sector\ 4, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_b + T_c}{2}$$

$$Sector\ 5, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_b + T_c}{2}$$

$$Sector\ 6, T_{mid} = T_{\frac{pwm}{2}} - \frac{T_a + T_c}{2}$$