Deep Learning
Recurrent Networks : 1
Spring 2019

Instructor: Bhiksha Raj

Which open source project?

```
/*
 * Increment the size file of the new incorrect UI_FILTER group information
 * of the size generatively.
 */
static int indicate_policy(void)
{
  int error;
  if (fd == MARN_EPT) {
    /*
     * The kernel blank will coeld it to userspace.
     */
    if (ss->segment < mem_total)
      unblock_graph_and_set_blocked();
    else
      ret = 1;
    goto bail;
  }
  segaddr = in_SB(in.addr);
  selector = seg / 16;
  setup_works = true;
  for (i = 0; i < blocks; i++) {
    seq = buf[i++];
    bpf = bd->bd.next + i * search;
    if (fd) {
      current = blocked;
    }
  }
  rw->name = "Getjbbregs";
  bprm_self_clearl(&iv->version);
  regs->new = blocks[(BPF_STATS << info->historidac)] | PFMR_CLOBATHINC_SECON
  return segtable;
}
```

Related math. What is it talking about?

Proof. Omitted. ☐

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{\acute{e}tale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. ☐

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. ☐

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

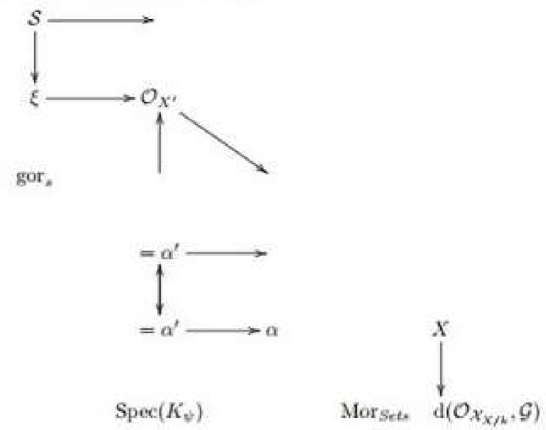
be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. ☐

This since $\mathcal{F} \in \mathcal{F}$ and $x \in \mathcal{G}$ the diagram



is a limit. Then $\mathcal{G}$ is a finite type and assume S is a flat and $\mathcal{F}$ and $\mathcal{G}$ is a finite type f_*. This is of finite type diagrams, and

- the composition of $\mathcal{G}$ is a regular sequence,
- $\mathcal{O}_{X'}$ is a sheaf of rings.

☐

Proof. We have see that $X = \mathrm{Spec}(R)$ and $\mathcal{F}$ is a finite type representable by algebraic space. The property $\mathcal{F}$ is a finite morphism of algebraic stacks. Then the cohomology of X is an open neighbourhood of U. ☐

Proof. This is clear that $\mathcal{G}$ is a finite presentation, see Lemmas ??.
A *reduced above* we conclude that U is an open covering of $\mathcal{C}$. The functor $\mathcal{F}$ is a "field

$$\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\overline{x}} \quad \text{-1}(\mathcal{O}_{X_{\acute{e}tale}}) \longrightarrow \mathcal{O}_{X_\ell}^{-1}\mathcal{O}_{X_\lambda}(\mathcal{O}_{X_\eta}^{\overline{v}})$$

is an isomorphism of covering of $\mathcal{O}_{X_i}$. If $\mathcal{F}$ is the unique element of $\mathcal{F}$ such that X is an isomorphism.
The property $\mathcal{F}$ is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme $\mathcal{O}_X$-algebra with $\mathcal{F}$ are opens of finite type over S.
If $\mathcal{F}$ is a scheme theoretic image points. ☐

If $\mathcal{F}$ is a finite direct sum $\mathcal{O}_{X_\lambda}$ is a closed immersion, see Lemma ??. This is a sequence of $\mathcal{F}$ is a similar morphism.

And a Wikipedia page explaining it all

```
Naturalism and decision for the majority of Arab countries' capitalide was grounded
by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated
with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal
in the [[Protestant Immineners]], which could be said to be directly in Cantonese
Communication, which followed a ceremony and set inspired prison, training. The
emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom
of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known
in western [[Scotland]], near Italy to the conquest of India with the conflict.
Copyright was the succession of independence in the slop of Syrian influence that
was a famous German movement based on a more popular servicious, non-doctrinal
and sexual power post. Many governments recognize the military housing of the
[[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]],
that is sympathetic to be to the [[Punjab Resolution]]
(PJS)[http://www.humah.yahoo.com/guardian.
cfm/7754800786d17551963s89.htm Official economics Adjoint for the Nazism, Montgomery
was swear to advance to the resources for those Socialism's rule,
was starting to signing a major tripad of aid exile.]]
```

The unreasonable effectiveness of recurrent neural networks..

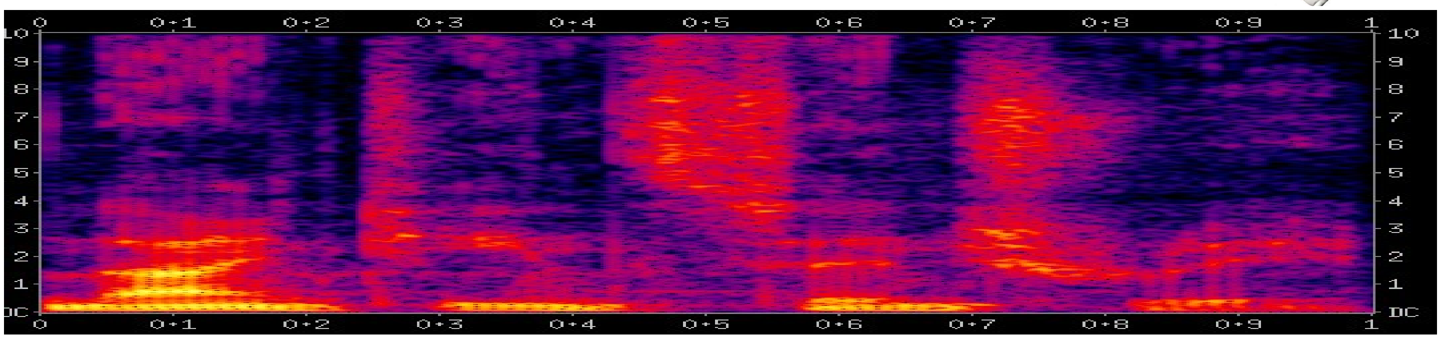
- All previous examples were *generated* blindly by a *recurrent* neural network..
- http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Modelling Series

- In many situations one must consider a *series* of inputs to produce an output
  - Outputs too may be a series
- Examples: ..

What did I say?

“To be” or not “to be”??



- Speech Recognition
  - Analyze a series of spectral vectors, determine what was said
- Note: Inputs are vectors. Output is a classification result

What is he talking about?

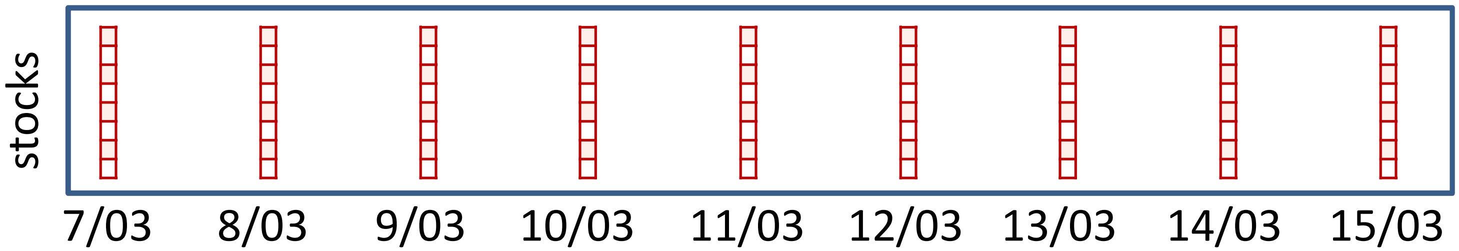
"Football" or "basketball"?

The Steelers, meanwhile, continue to struggle to make stops on defense. They've allowed, on average, 30 points a game, and have shown no signs of improving anytime soon.

- Text analysis
  - E.g. analyze document, identify topic
    - Input series of words, output classification output
  - E.g. read English, output French
    - Input series of words, output series of words

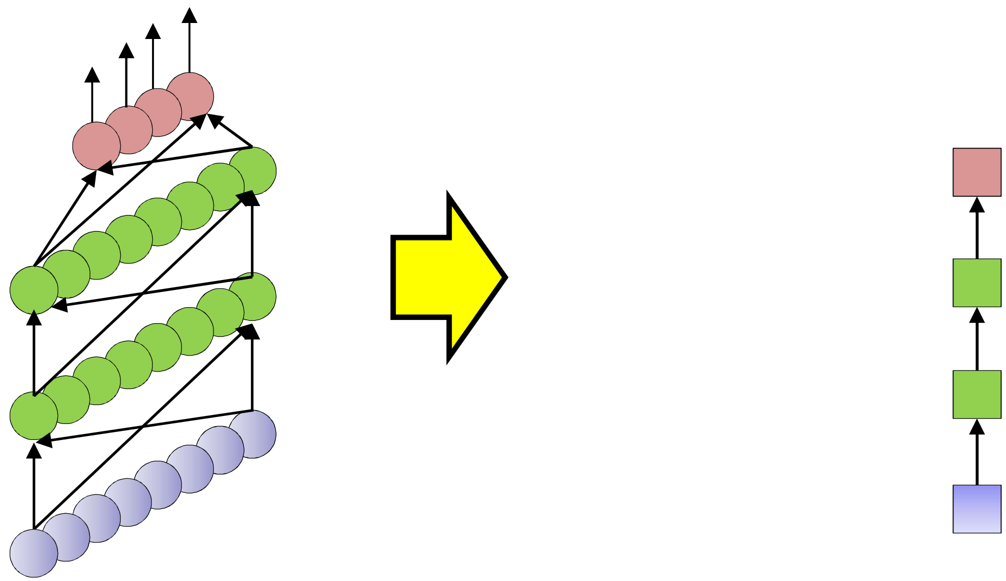
Should I invest..

To invest or not to invest?



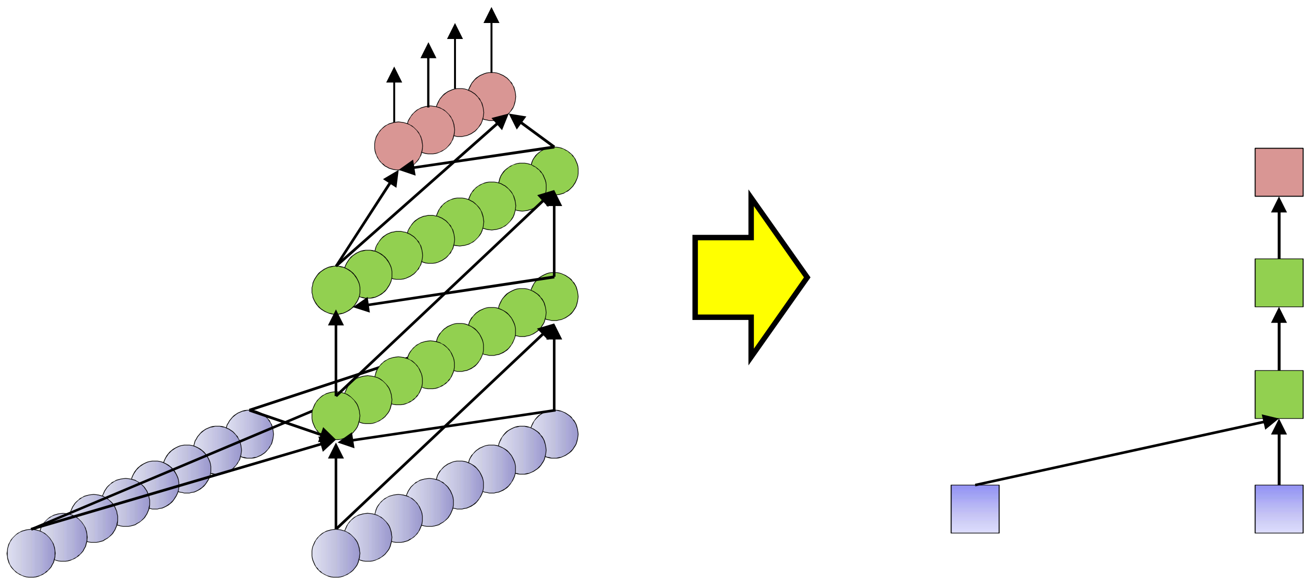
- Stock market
  - Must consider the series of stock values in the past several days to decide if it is wise to invest today
    - Ideally consider *all* of history
- Note: Inputs are vectors. Output may be scalar or vector
  - Should I invest, vs. should I invest in X

Representational shortcut



- Input at each time is a *vector*
- Each layer has many neurons
  - Output layer too may have many neurons
- But will represent everything by simple boxes
  - Each box actually represents an entire *layer with many units*

Representational shortcut



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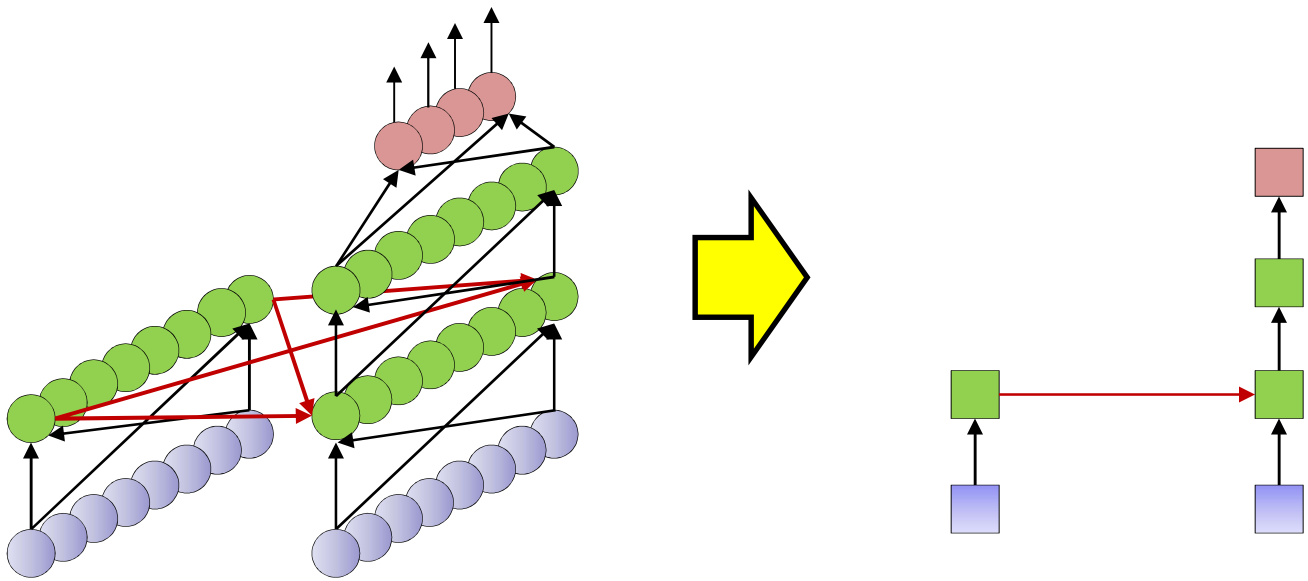

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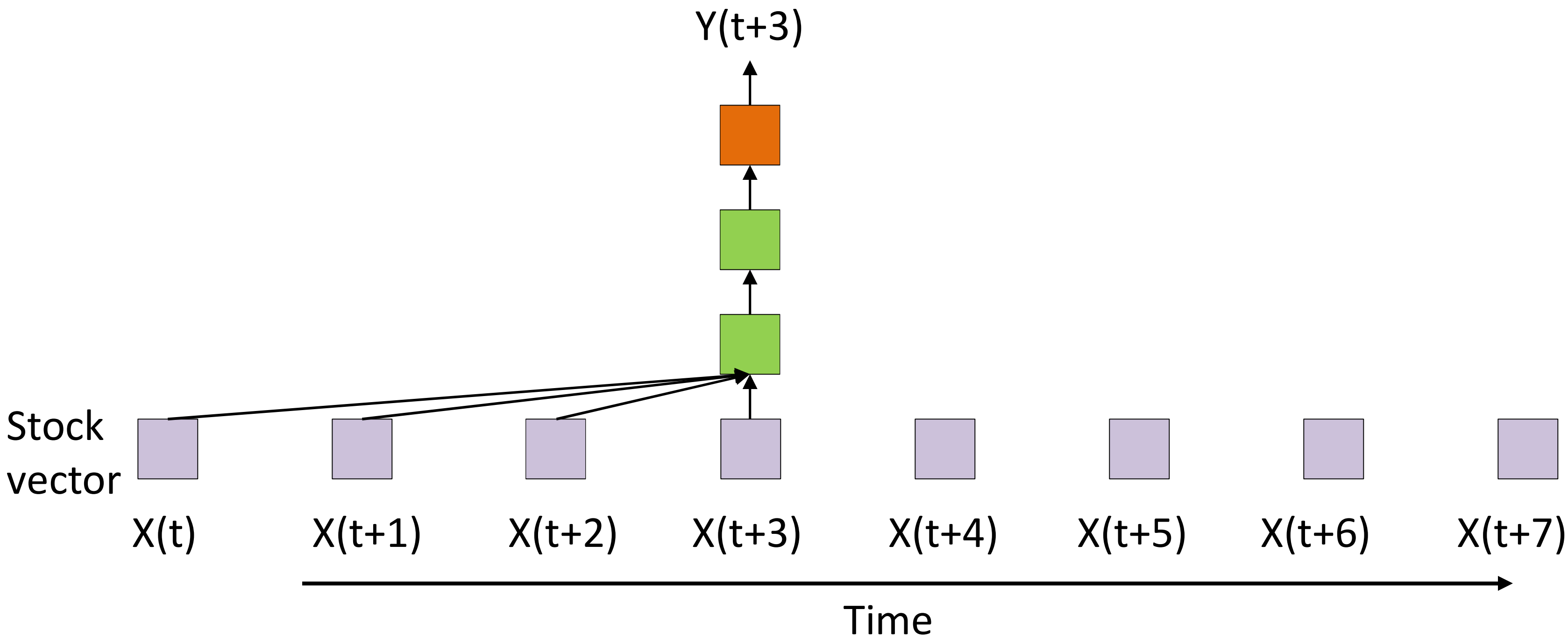
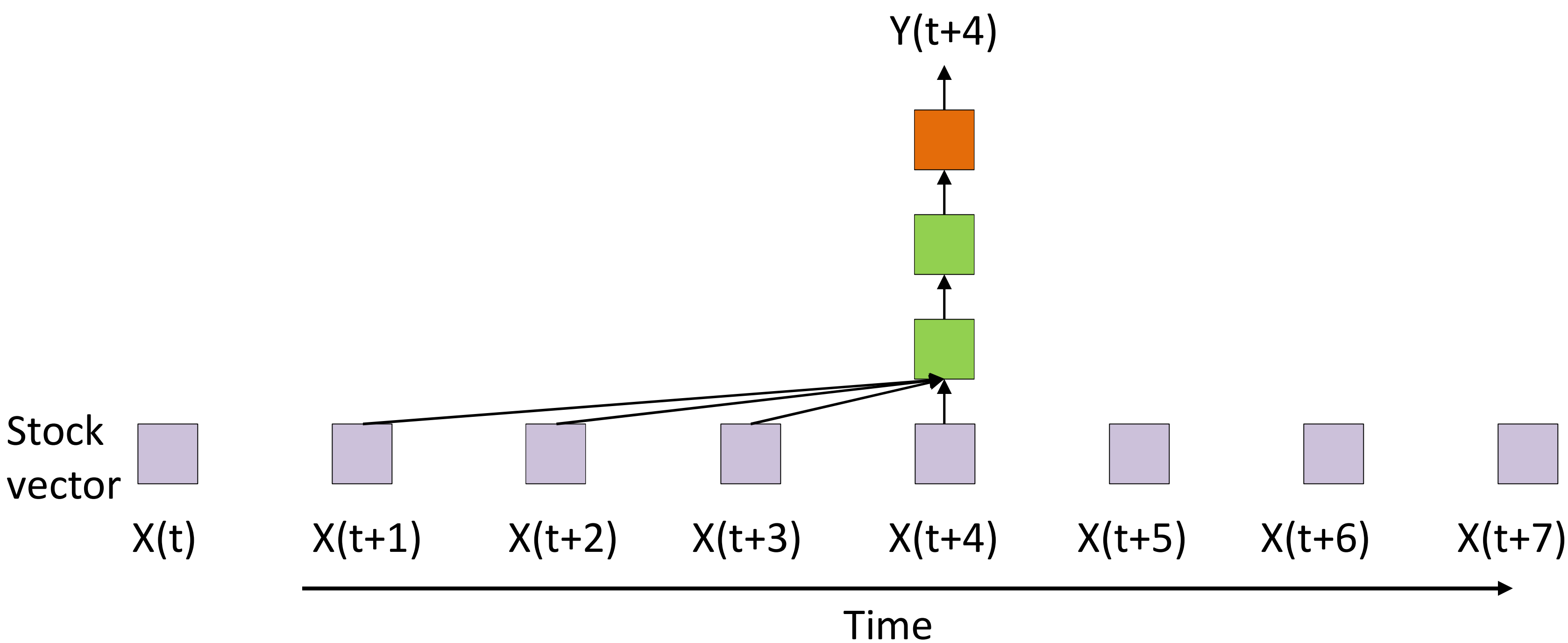
The stock predictor



- The sliding predictor
  - Look at the last few days
  - This is just a convolutional neural net applied to series data
    - Also called a *Time-Delay neural network*

The stock predictor



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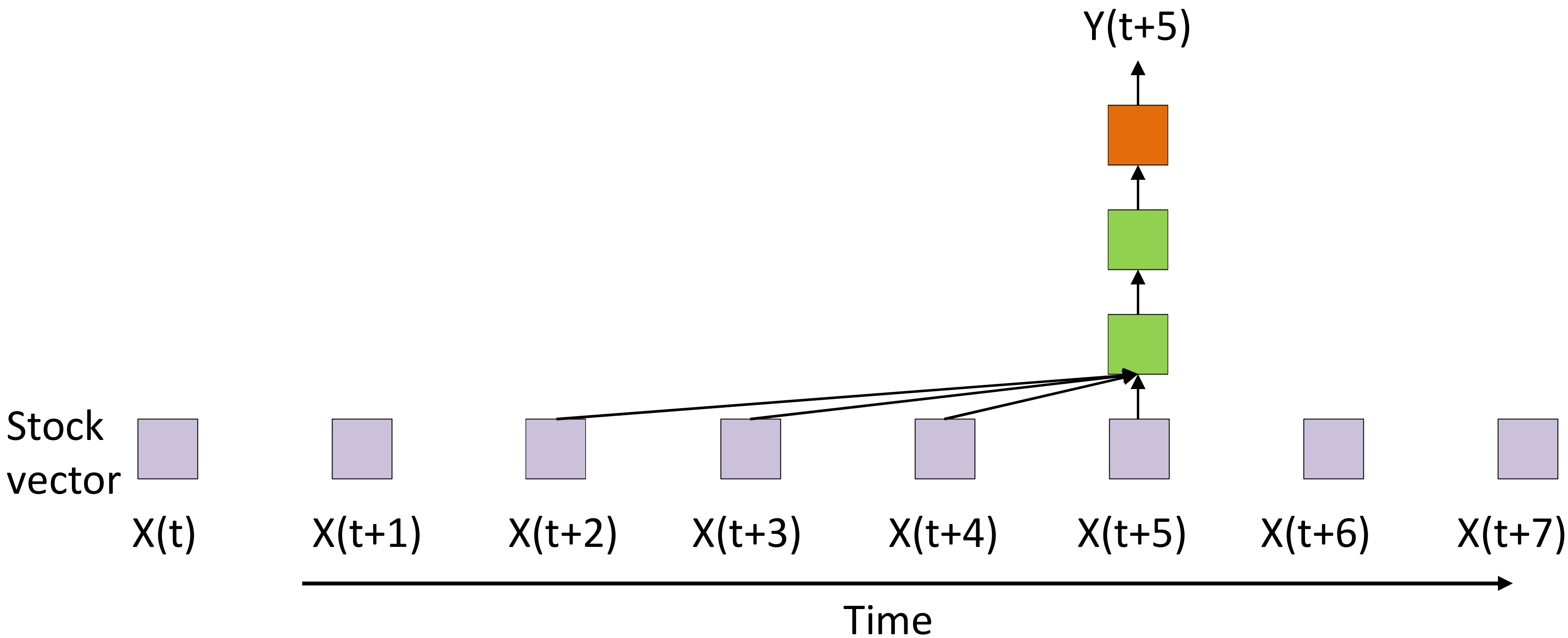

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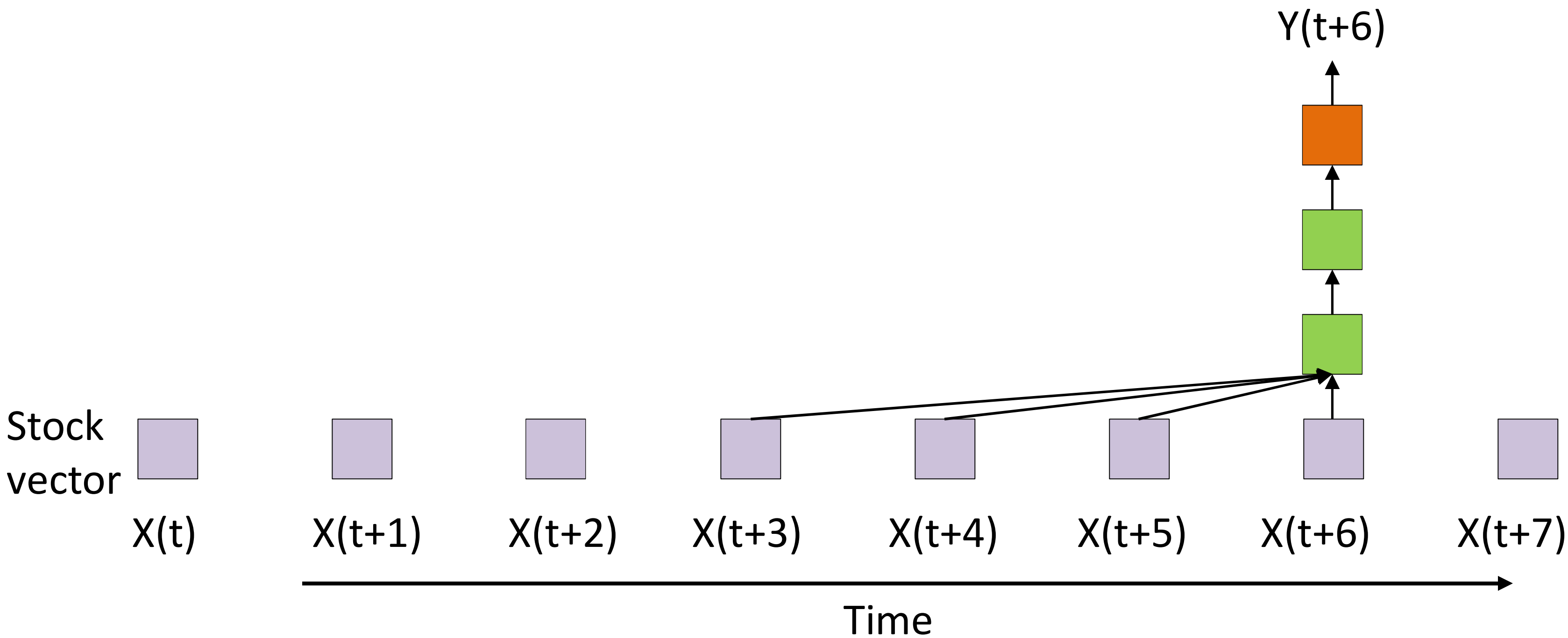

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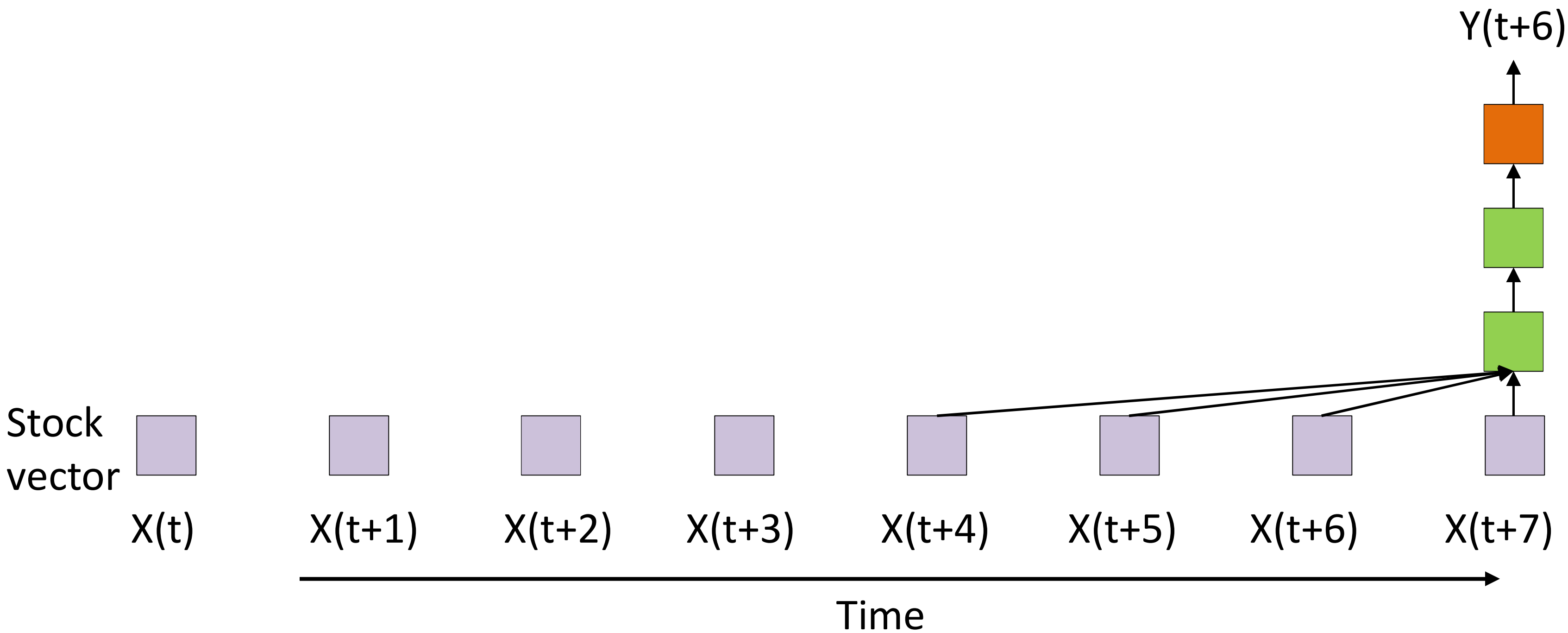

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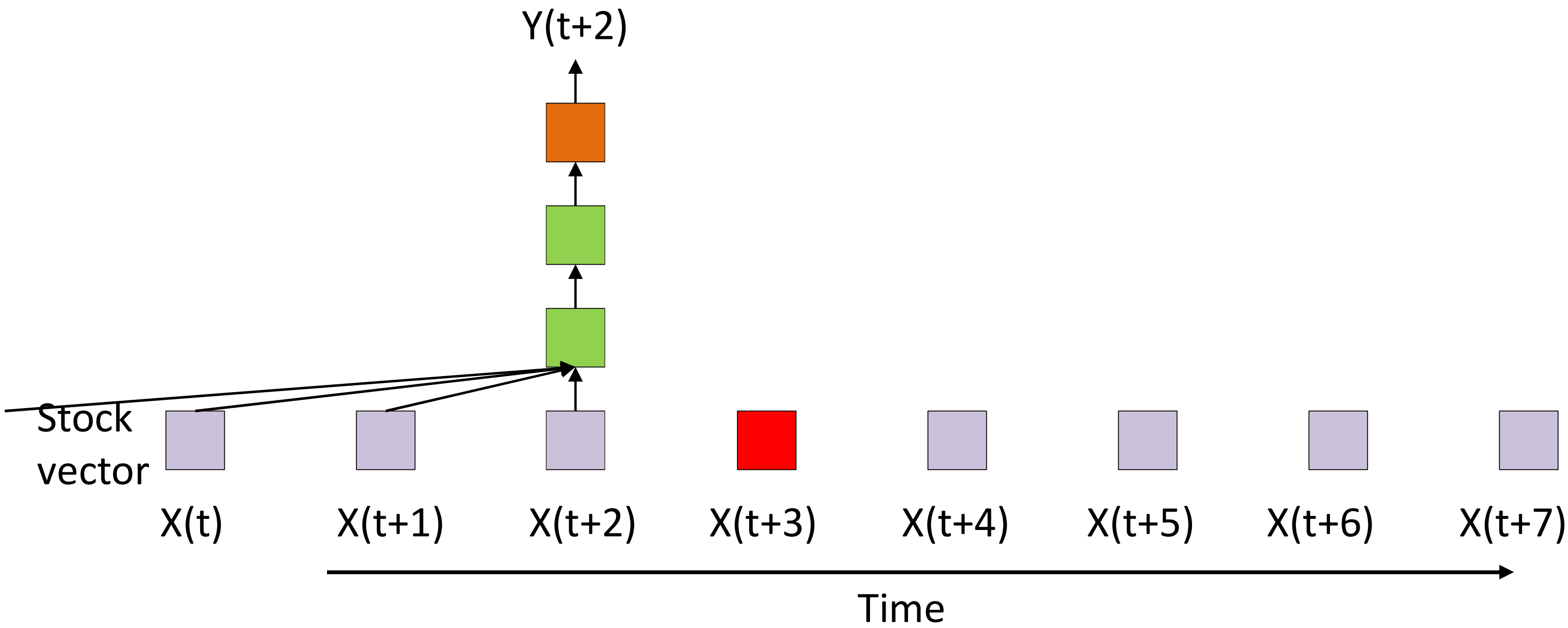
Finite-response model

- This is a *finite response* system
  - Something that happens *today* only affects the output of the system for N days into the future
    - N is the *width* of the system

$$Y_t = f(X_t, X_{t-1}, \ldots, X_{t-N})$$

The stock predictor



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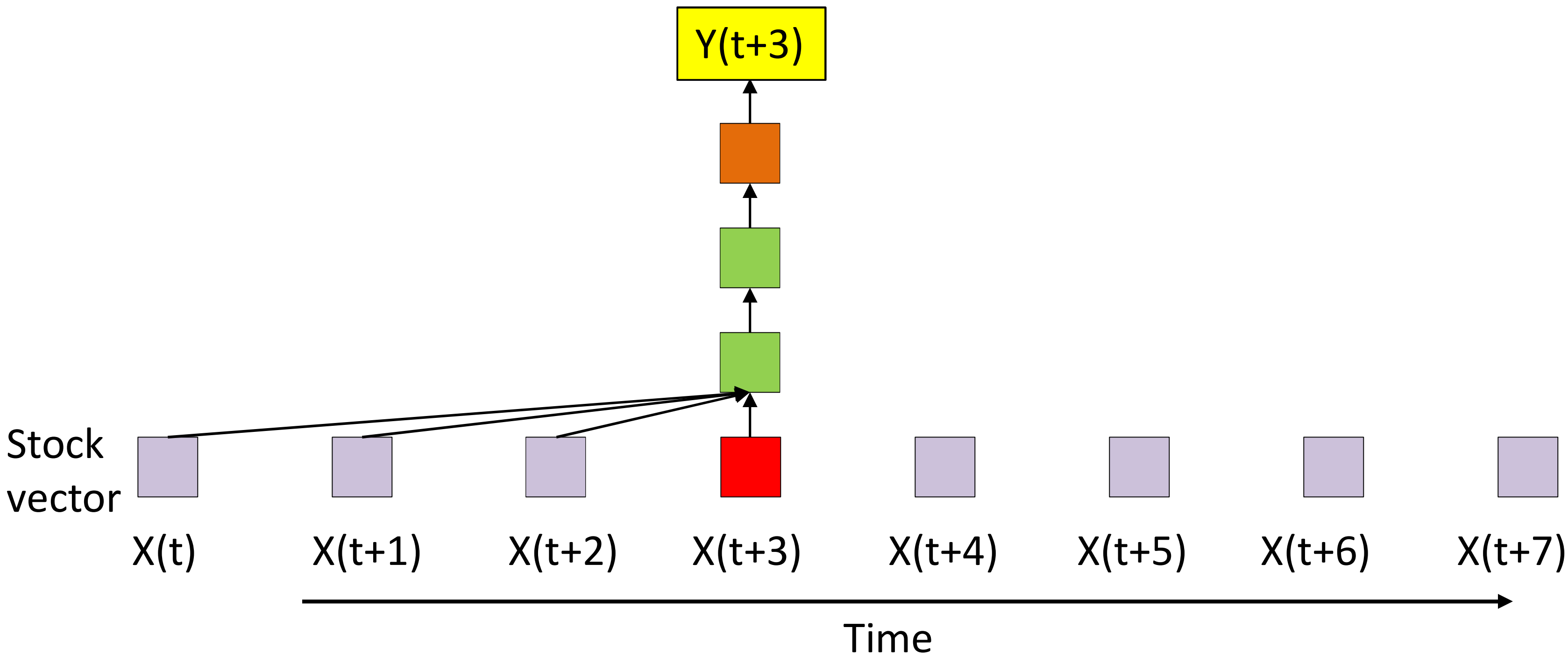

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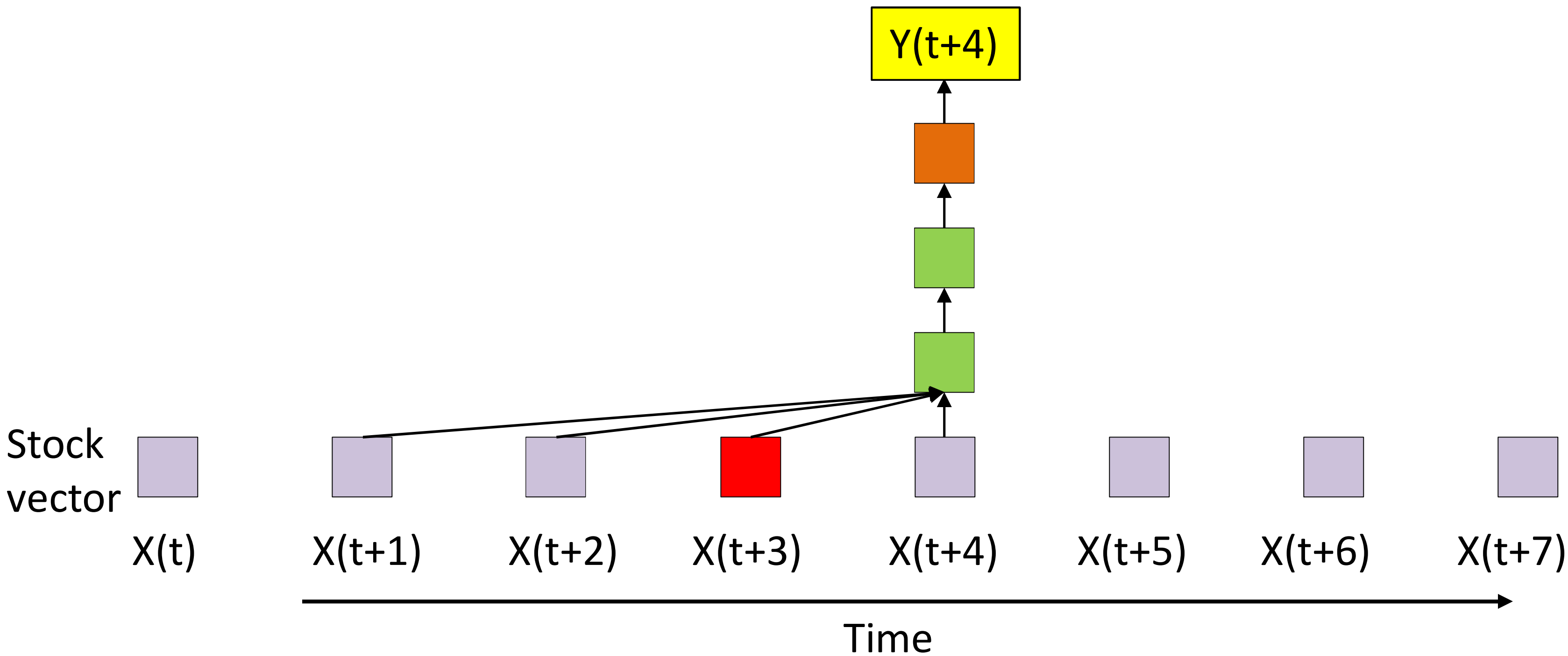

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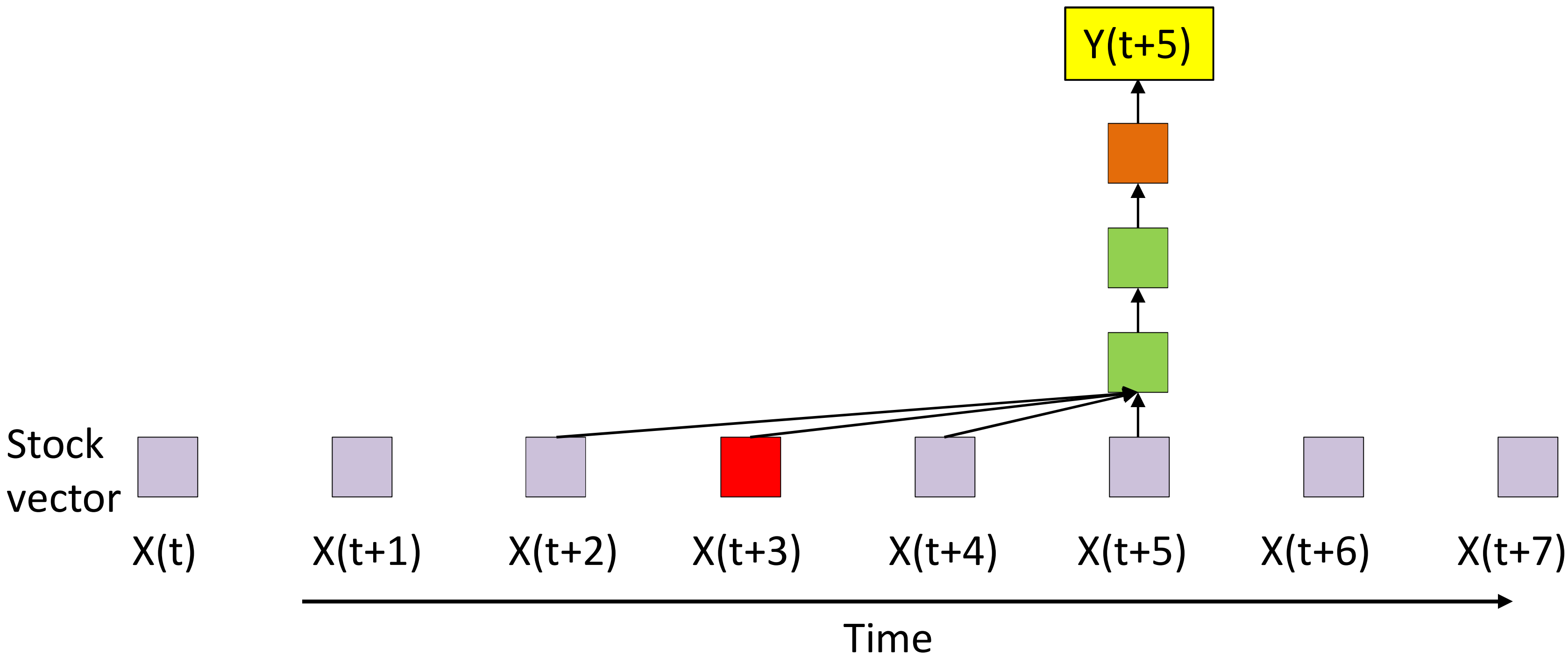

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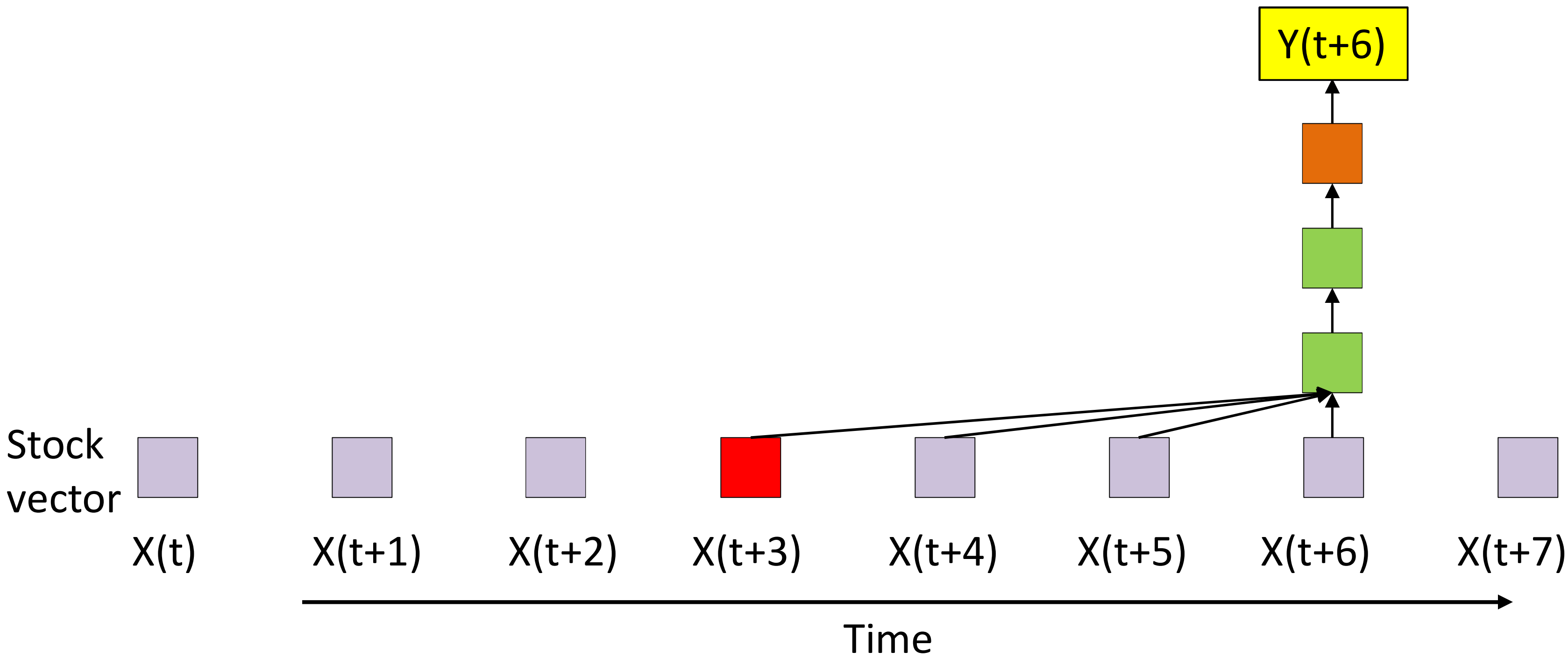

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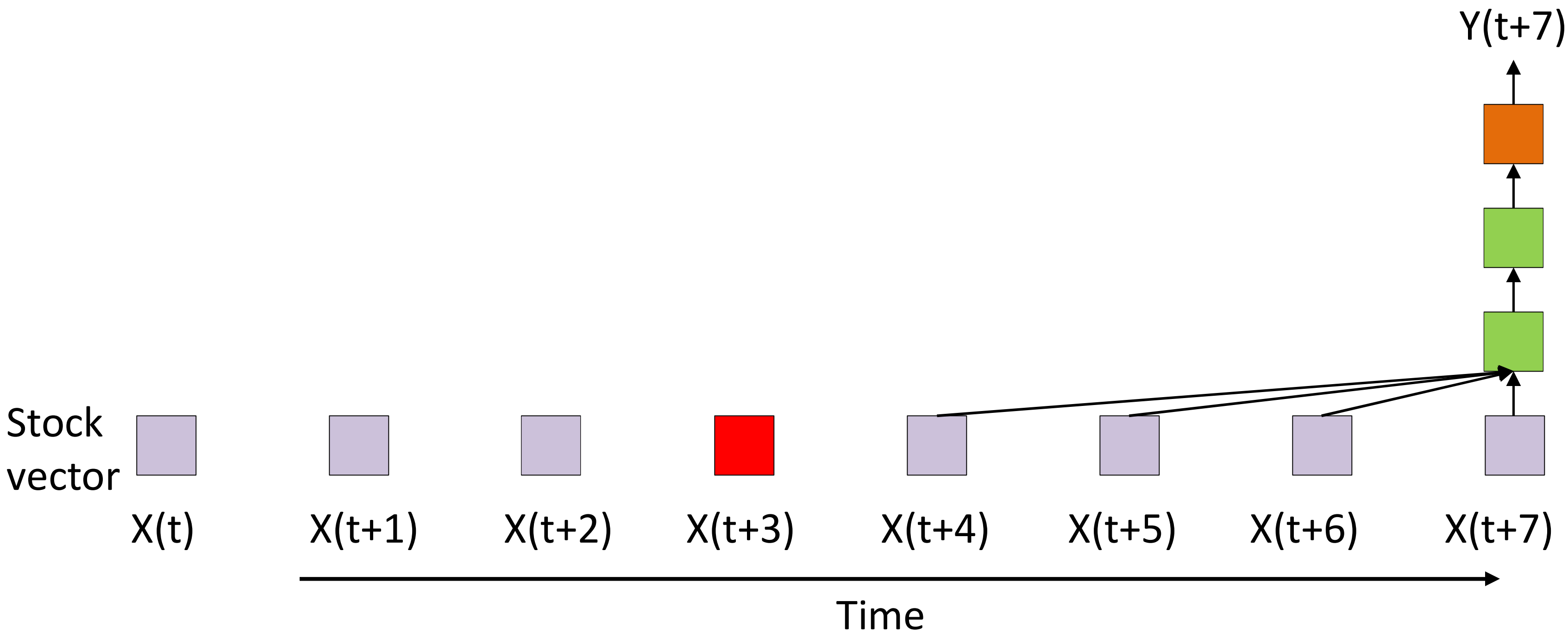

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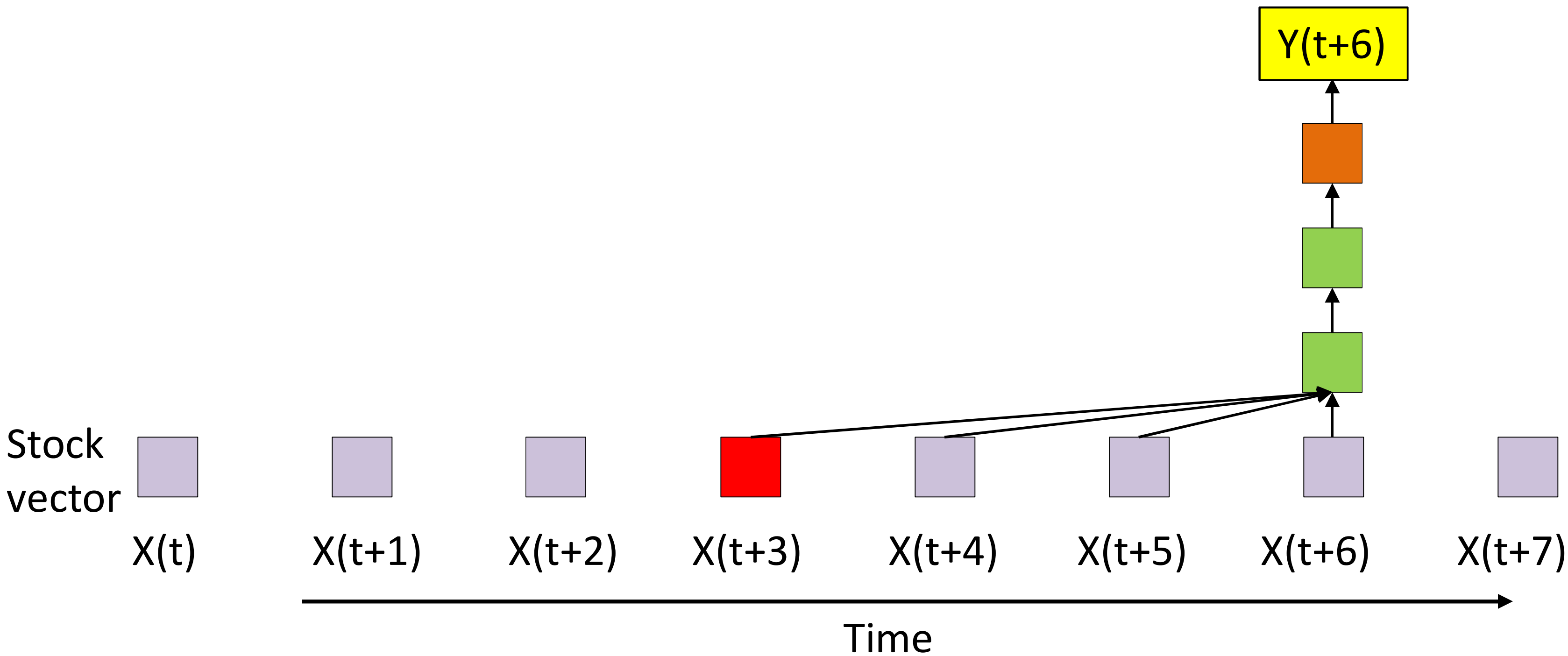

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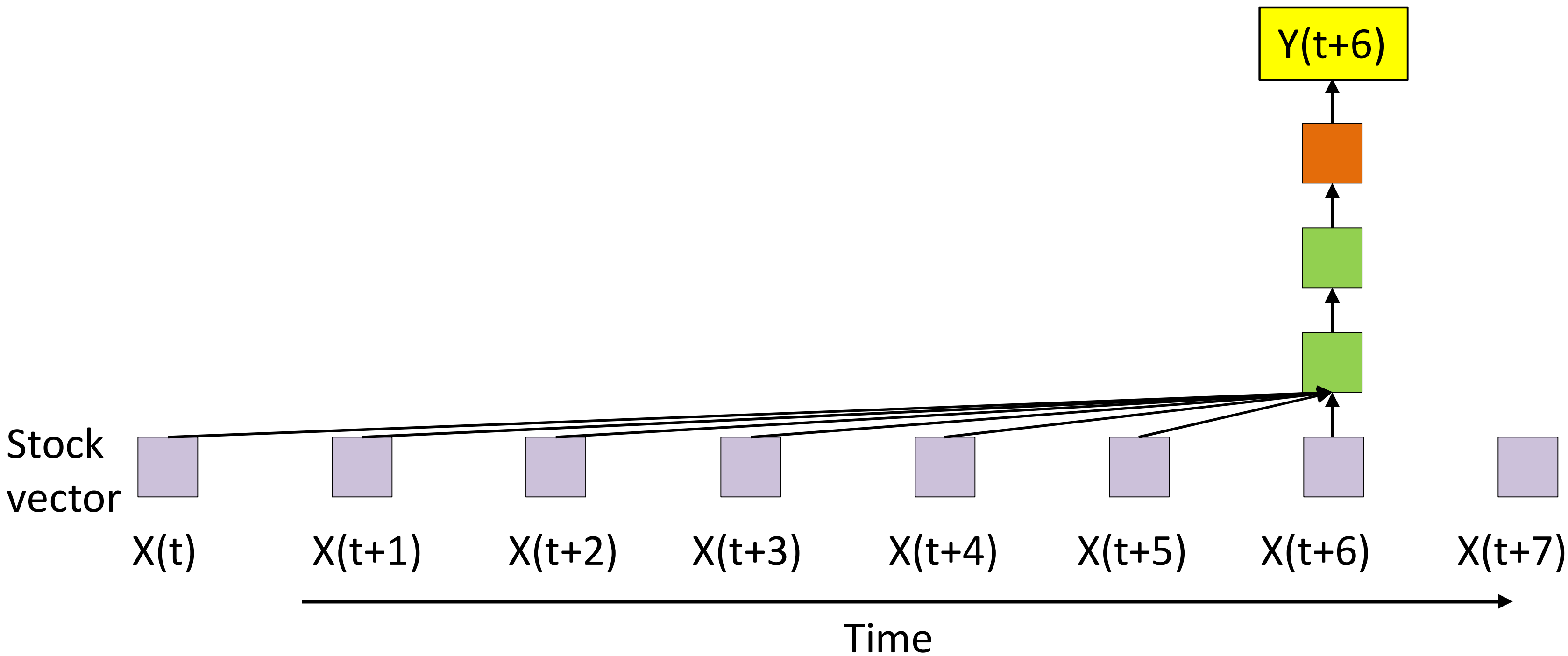
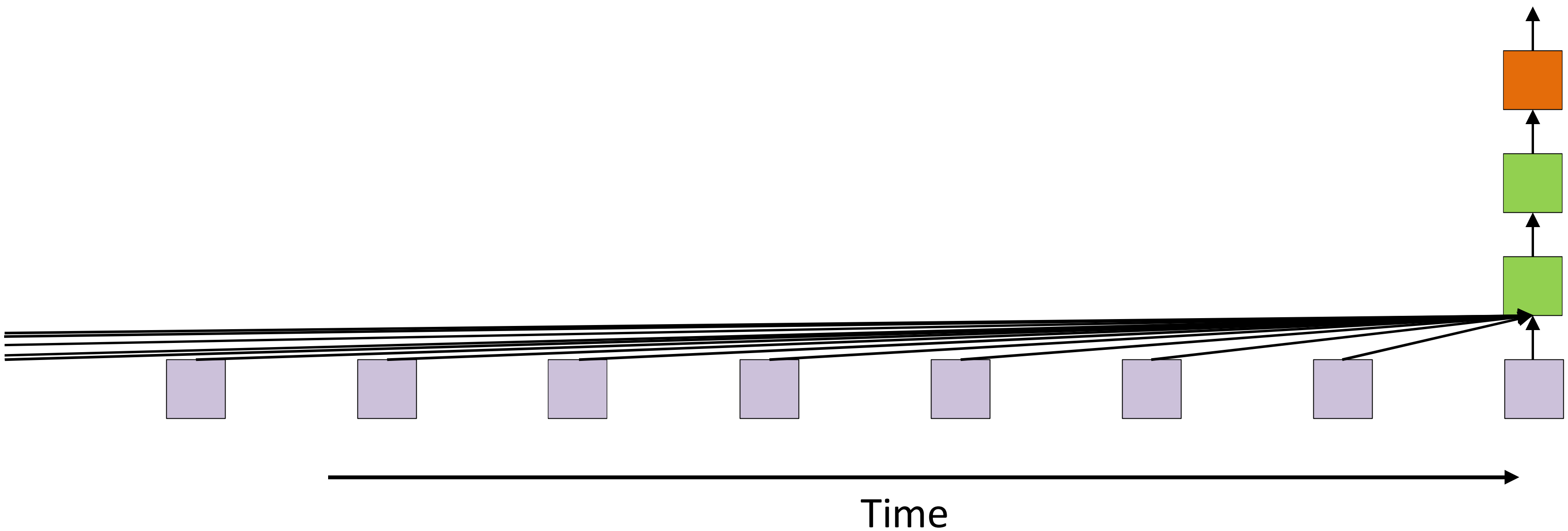


- Problem: Increasing the "history" makes the network more complex
  - No worries, we have the CPU and memory
    - Or do we?

Systems often have long-term dependencies

- Longer-term trends –
  - Weekly trends in the market
  - Monthly trends in the market
  - Annual trends
  - Though longer historic tends to affect us less than more recent events..

We want *infinite* memory



- Required: *Infinite* response systems
  - What happens today can continue to affect the output forever
    - Possibly with weaker and weaker influence

$$Y_t = f(X_t, X_{t-1}, \dots, X_{t-\infty})$$

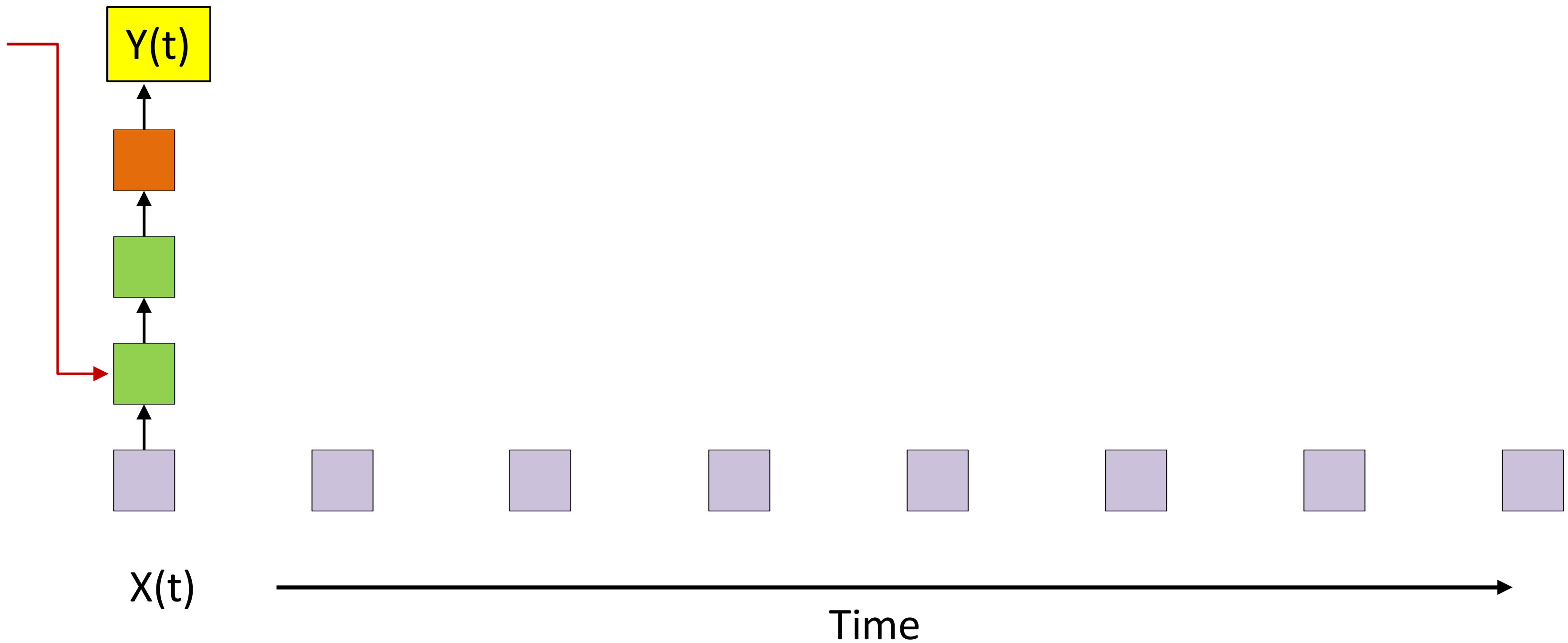
Examples of infinite response systems

$$Y_t = f(X_t, Y_{t-1})$$

– Required: Define initial state: Y_{-1} for $t = 0$
– An input at X_0 at $t = 0$ produces Y_0
– Y_0 produces Y_1 which produces Y_2 and so on until Y_∞ *even if* $X_1 \dots X_\infty$ *are 0*
  - i.e. even if there are no further inputs!

- This is an instance of a NARX network
  – "nonlinear autoregressive network with exogenous inputs"
  – $Y_t = f(X_{0:t}, Y_{0:t-1})$
- *Output* contains information about the entire past

A one-tap NARX network



- A NARX net with recursion from the output

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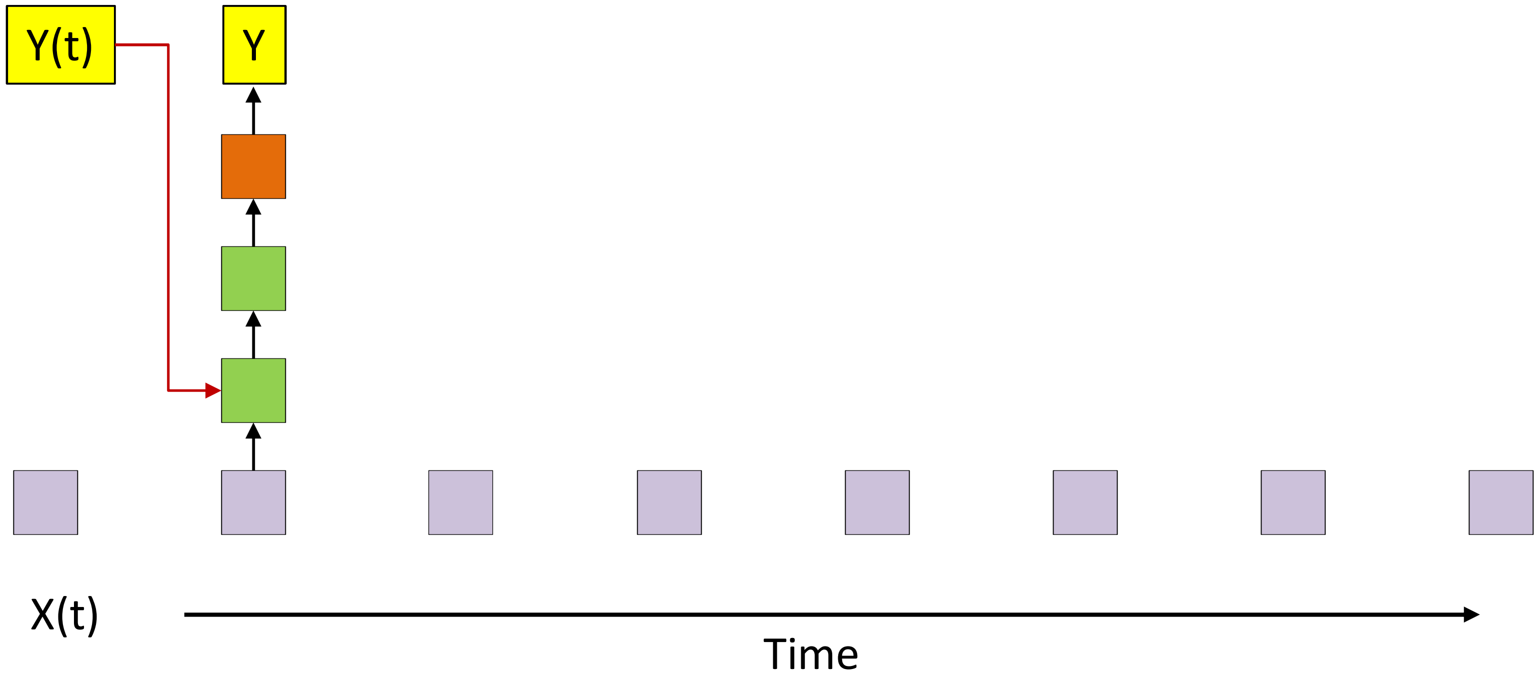

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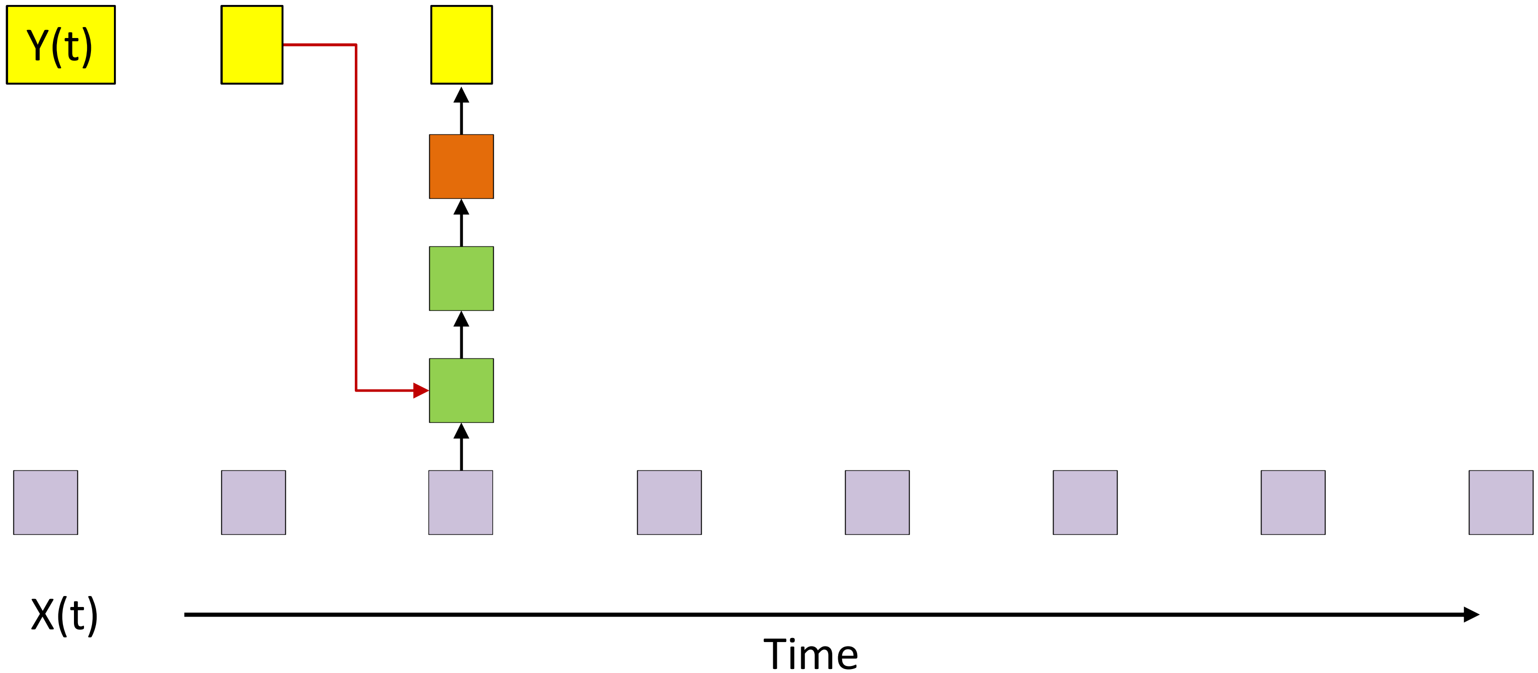

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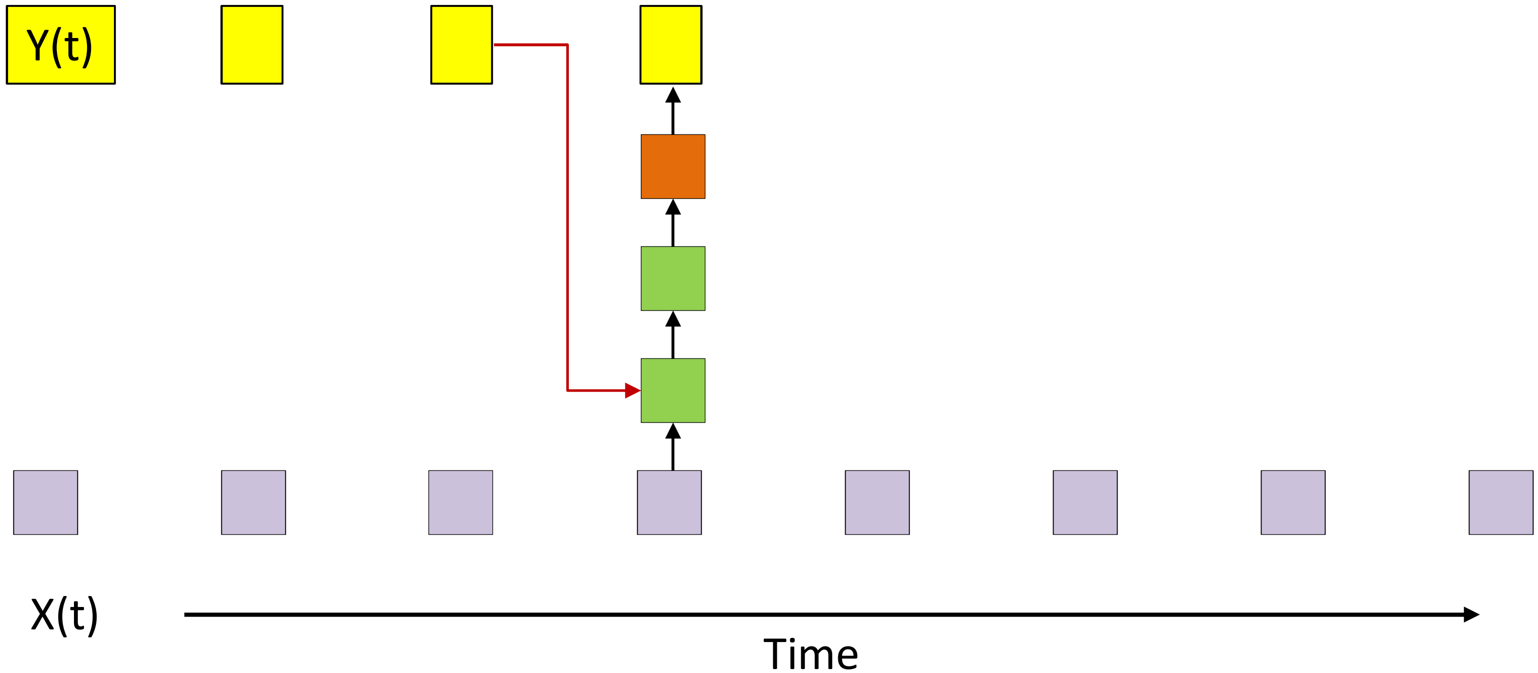

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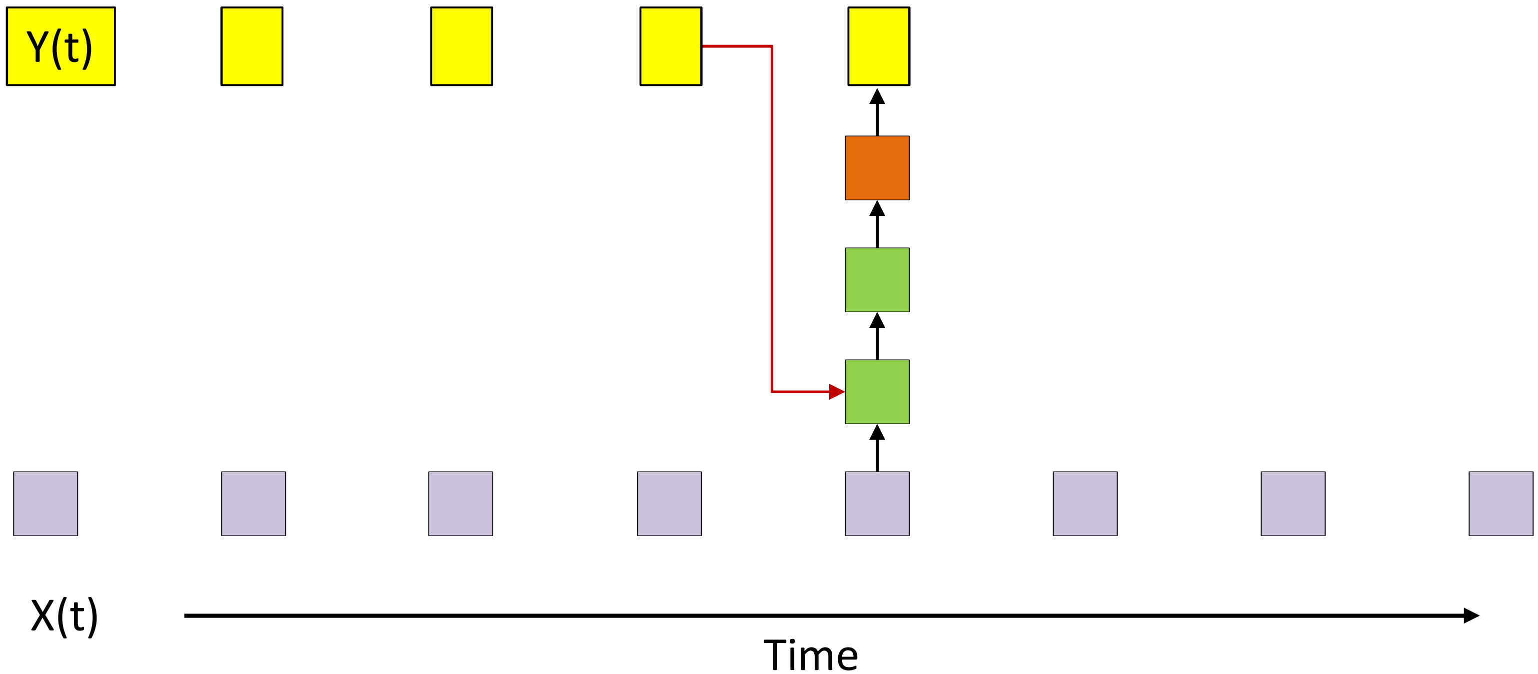

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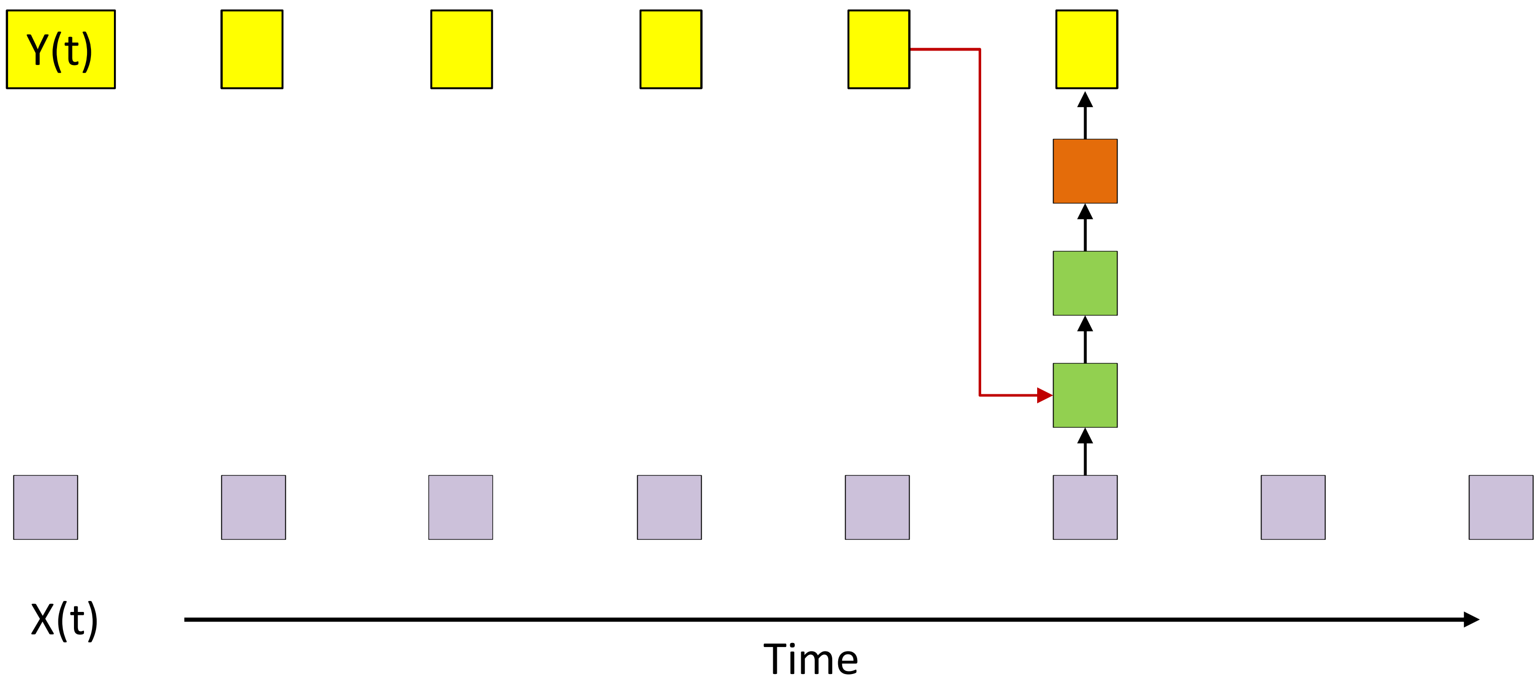

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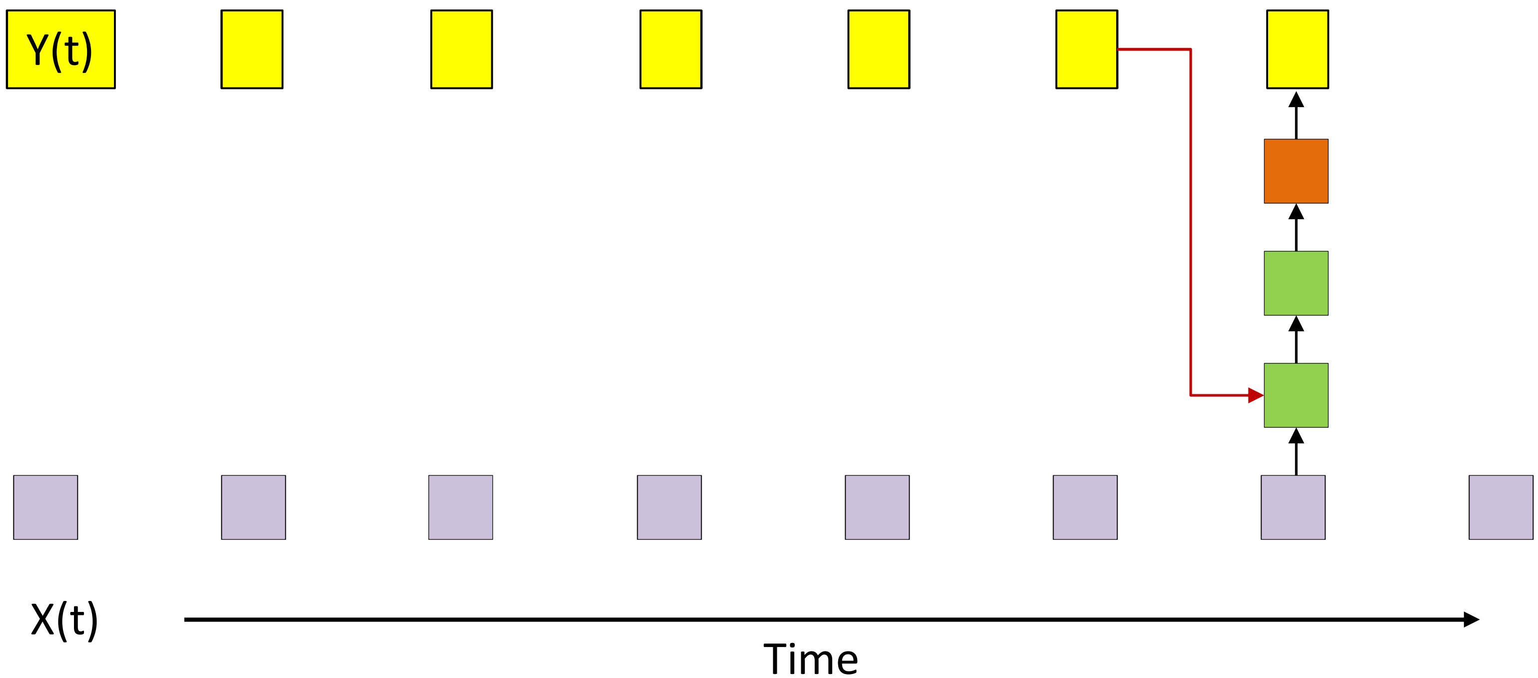

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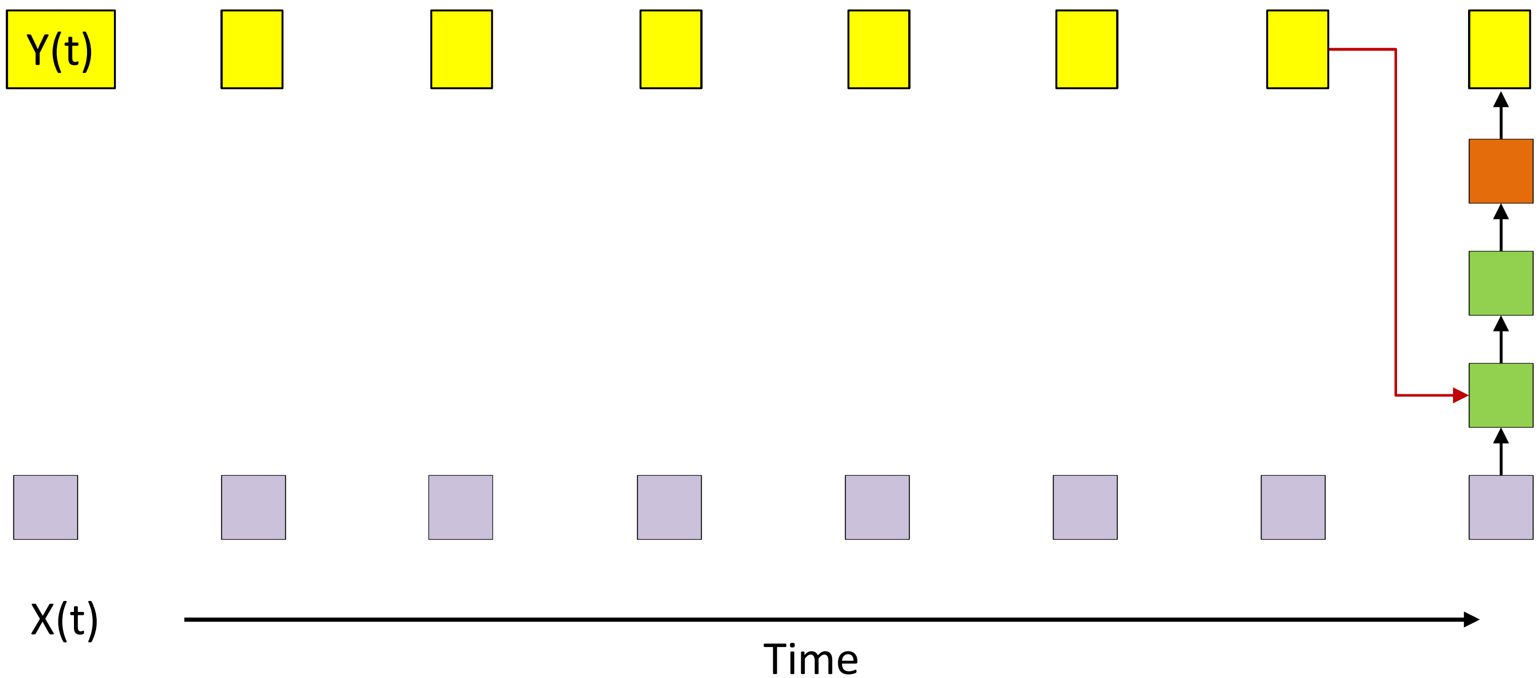
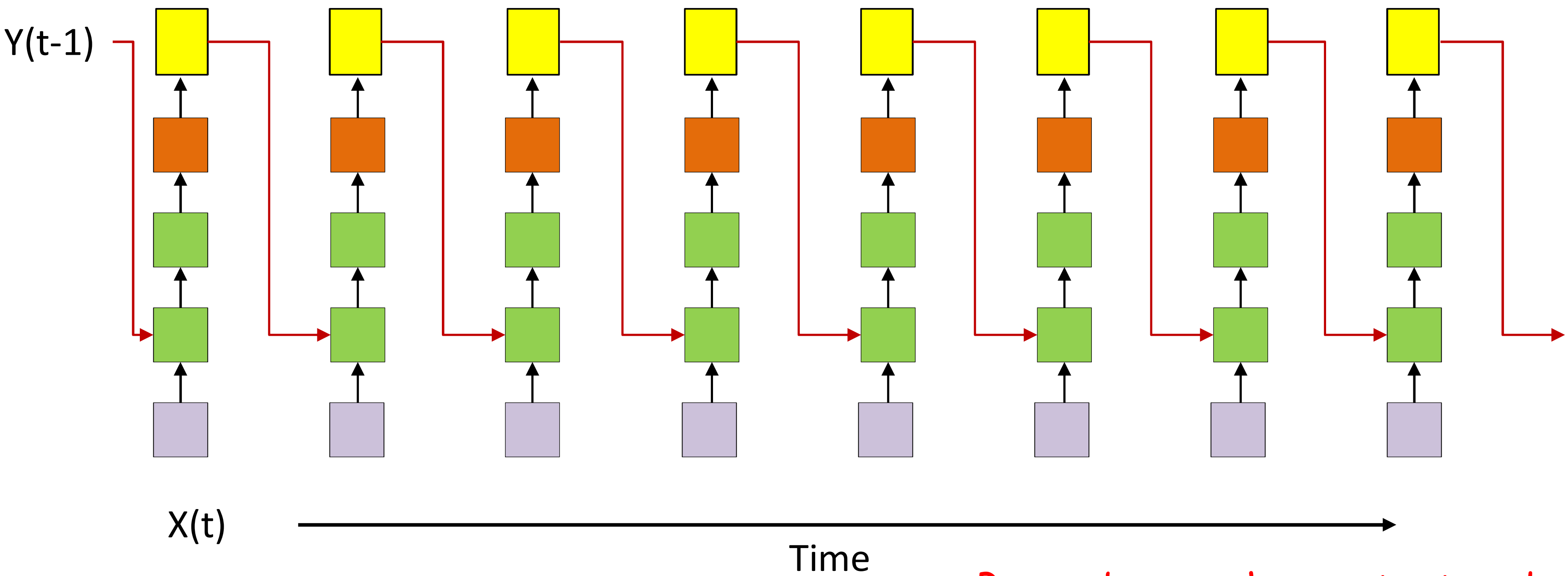


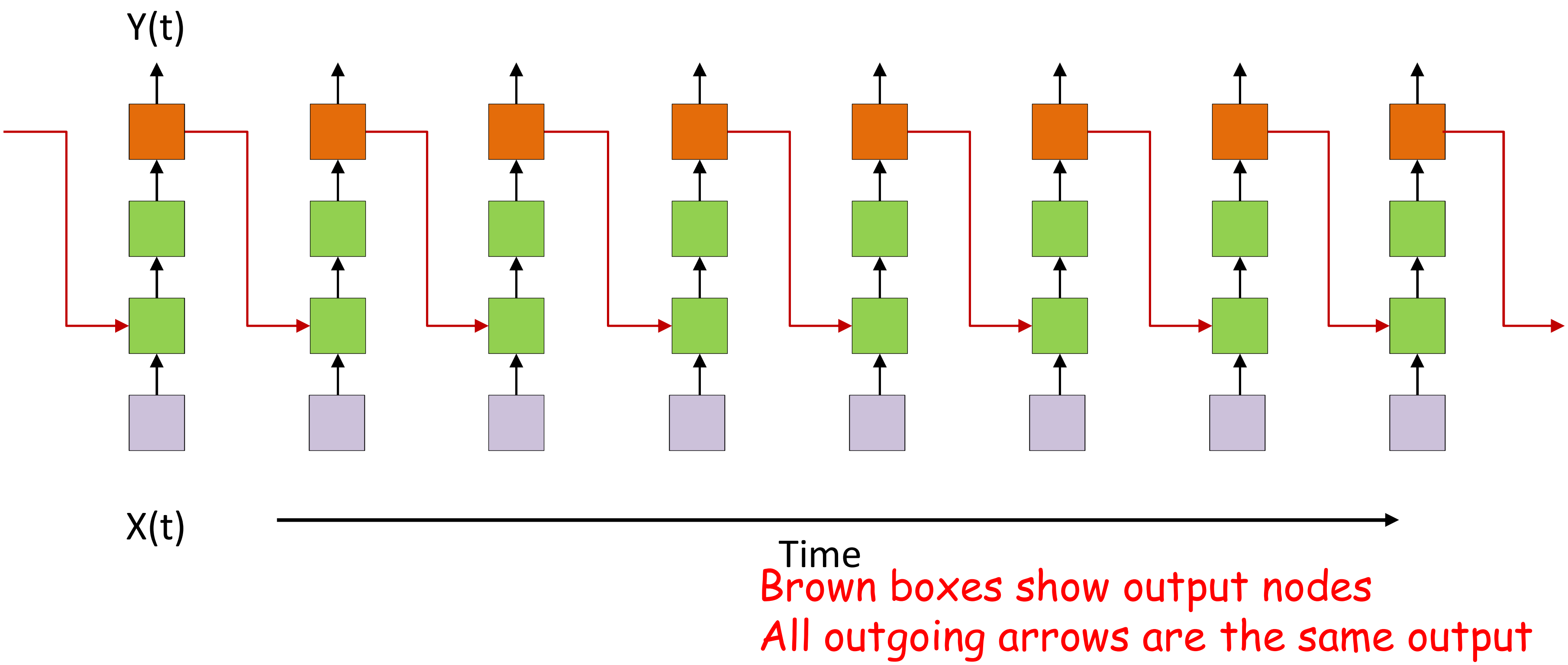
- A NARX net with recursion from the output

A more complete representation



- A NARX net with recursion from the output
- Showing all computations
- All columns are identical
- *An input at t=0 affects outputs forever*

Same figure redrawn



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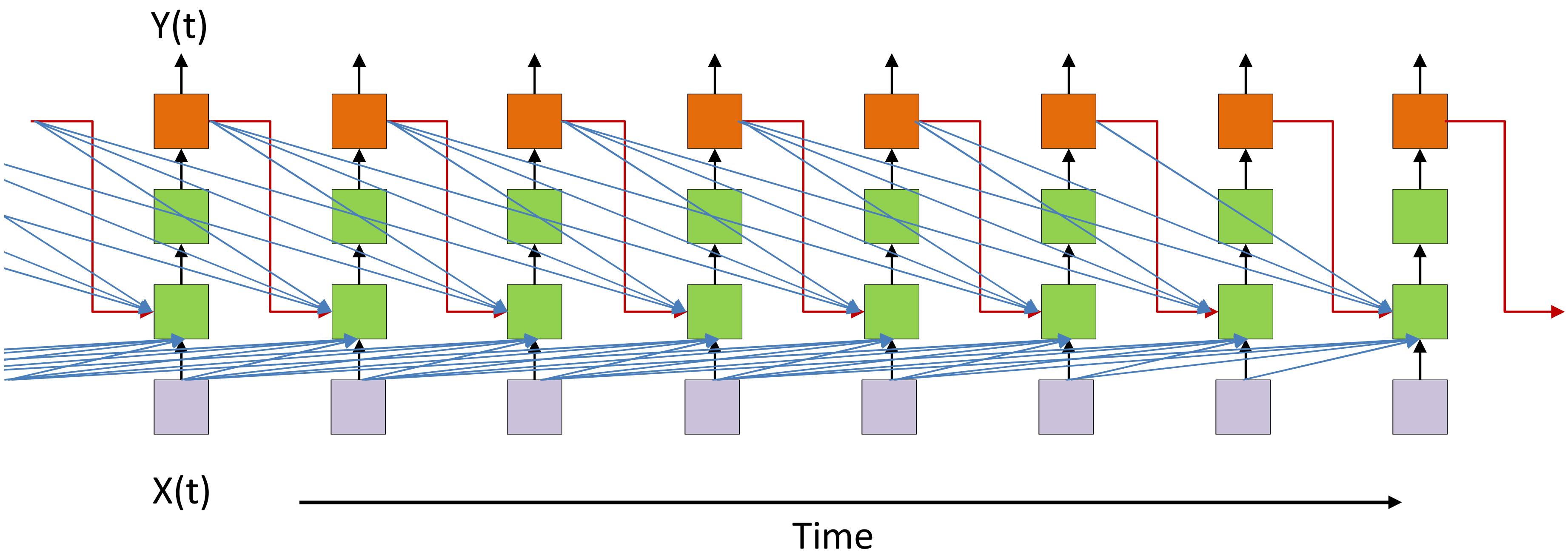
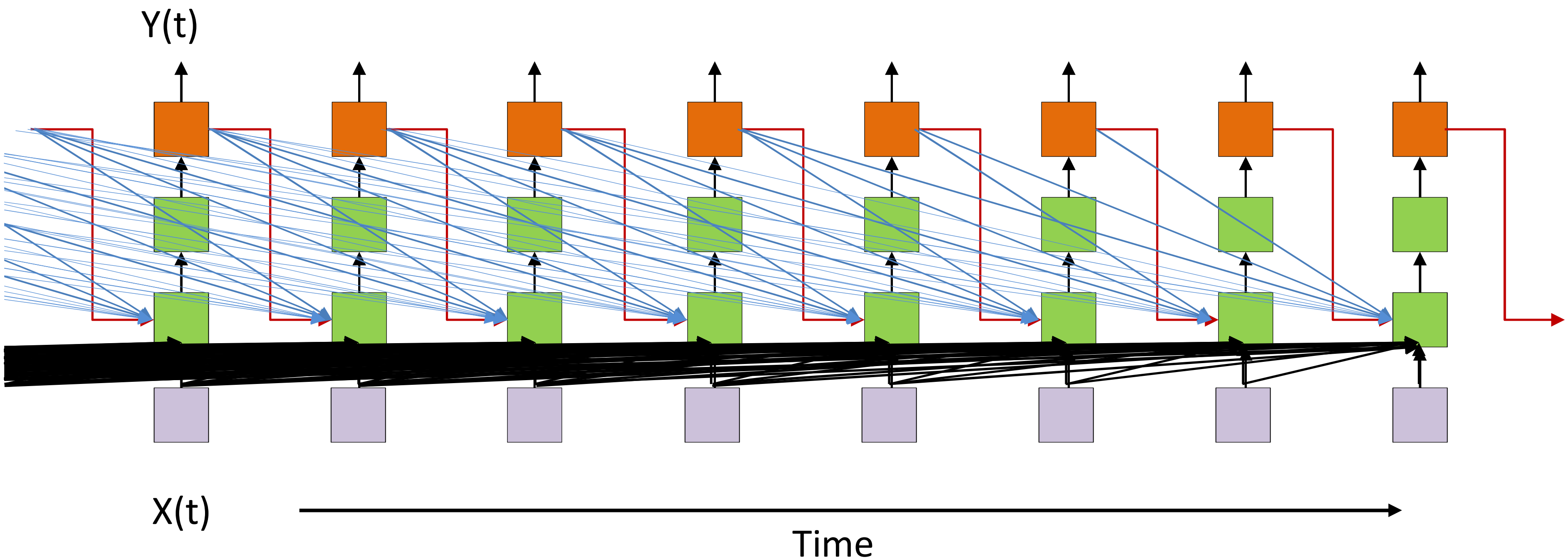
A more generic NARX network



- The output Y_t at time t is computed from the past K outputs $Y_{t-1}, \ldots, Y_{t-K}$ and the current and past L inputs $X_t, \ldots, X_{t-L}$

A "complete" NARX network



- The output Y_t at time t is computed from *all* past outputs and *all* inputs until time t
  - Not really a practical model

NARX Networks

- Very popular for time-series prediction
  - Weather
  - Stock markets
  - As alternate system models in tracking systems
- Any phenomena with distinct “innovations” that “drive” an output

- Note: here the “memory” of the past is in the output itself, and not in the network

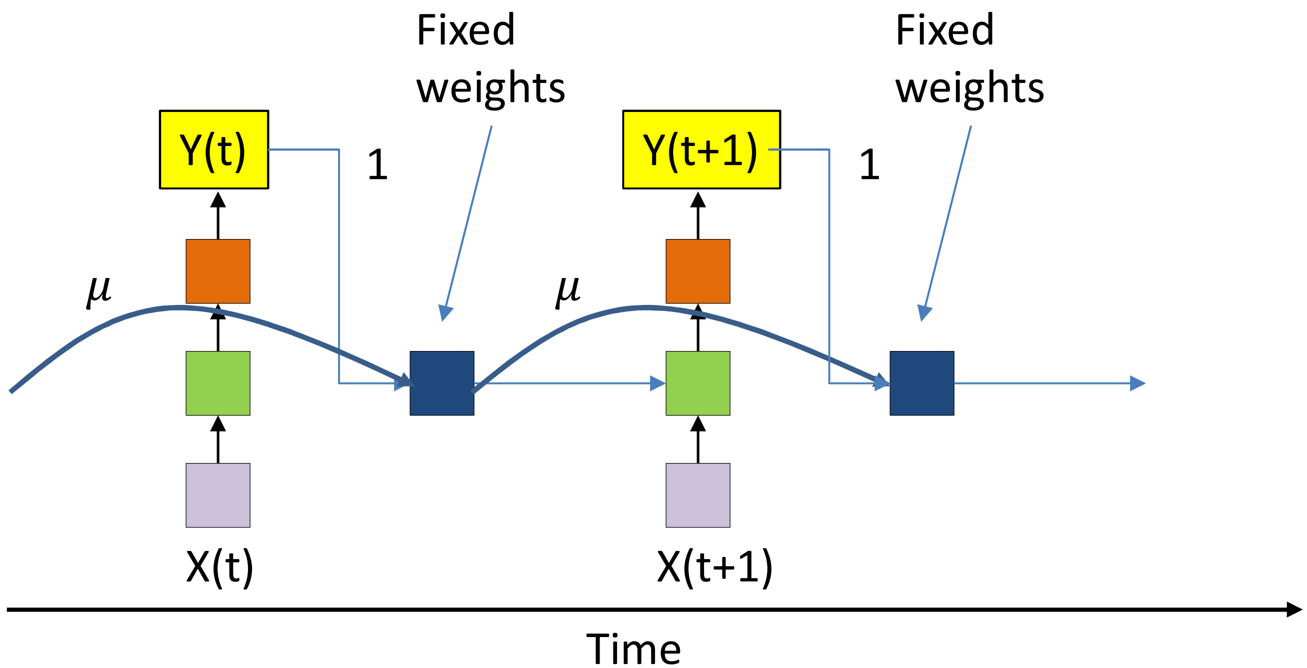
Lets make memory more explicit

- Task is to "remember" the past
- Introduce an explicit *memory* variable whose *job* it is to remember

$$m_t = r(y_{t-1}, h_{t-1}, m_{t-1})$$
$$h_t = f(x_t, m_t)$$
$$y_t = g(h_t\ )$$

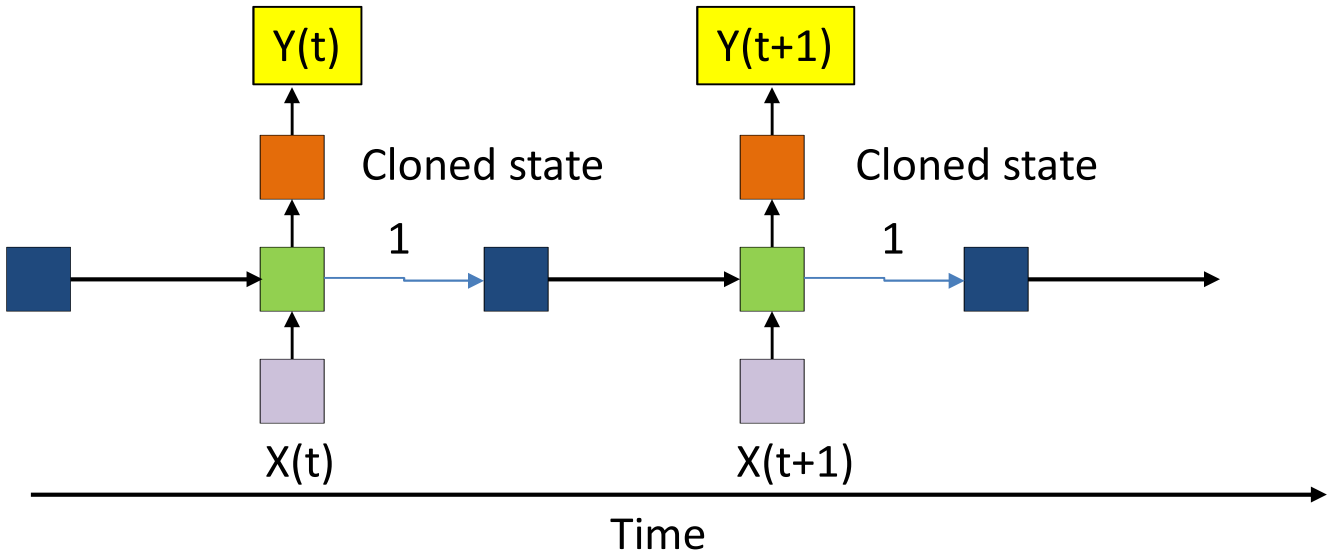
- m_t is a "memory" variable
  – Generally stored in a "memory" unit
  – Used to "remember" the past

Jordan Network



- Memory unit simply retains a running average of past outputs
  - "Serial order: A parallel distributed processing approach", M.I.Jordan, 1986
    - Input is constant (called a "plan")
    - Objective is to train net to produce a specific output, given an input plan
  - Memory has fixed structure; does not "learn" to remember
    - The running average of outputs considers entire past, rather than immediate past

Elman Networks



- Separate memory state from output
  - "Context" units that carry historical state
  - "Finding structure in time", Jeffrey Elman, Cognitive Science, 1990
    - For the purpose of training, this was approximated as a set of T independent 1-step history nets
- Only the weight *from* the memory unit to the hidden unit is learned
  - But during training no gradient is backpropagated over the "1" link

Story so far

- In time series analysis, models must look at past inputs along with current input
  - Looking at a finite horizon of past inputs gives us a convolutional network
- Looking into the infinite past requires recursion

- NARX networks recurse by feeding back the output to the input
  - May feed back a finite horizon of outputs

- "Simple" recurrent networks:
  - Jordon networks maintain a running average of outputs in a "memory" unit
  - Elman networks store *hidden* unit values for one time instant in a "context" unit
  - "Simple" (or partially recurrent) because during *learning* current error does not actually propagate to the past
    - "Blocked" at the memory units in Jordan networks
    - "Blocked" at the "context" unit in Elman networks

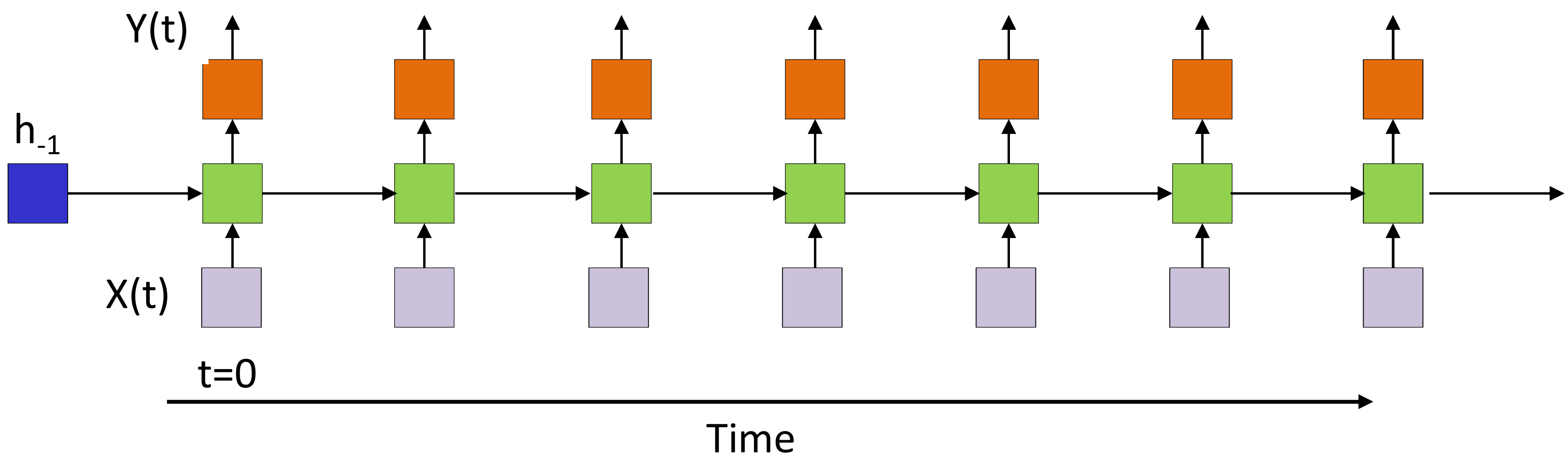
An alternate model for infinite response systems: the state-space model

$$h_t = f(x_t, h_{t-1})$$
$$y_t = g(h_t)$$

- h_t is the *state* of the network
  - Model directly embeds the memory in the state
- Need to define initial state h_{-1}

- This is a *fully recurrent* neural network
  - Or simply a *recurrent neural network*
- *State* summarizes information about the entire past

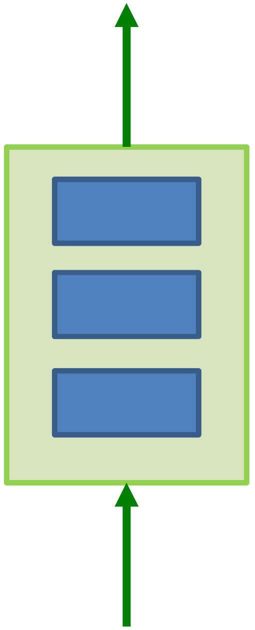
The simple state-space model



- The state (green) at any time is determined by the input at that time, and the state at the previous time
- *An input at t=0 affects outputs forever*
- Also known as a recurrent neural net

An alternate model for infinite response systems: the state-space model

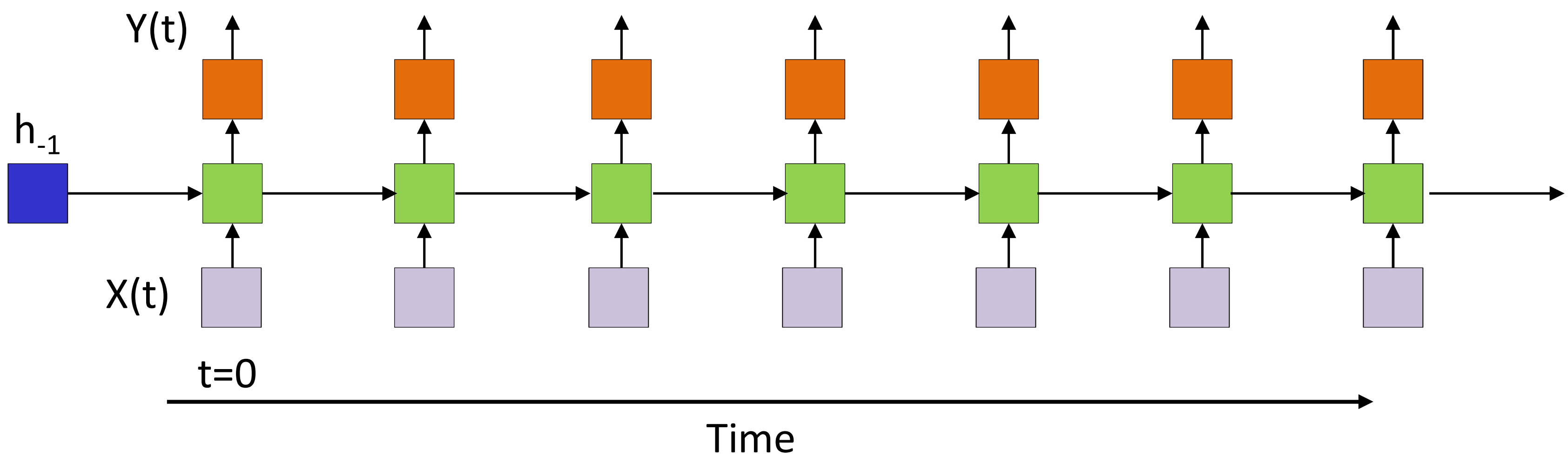
$$h_t = f(x_t, h_{t-1})$$
$$y_t = g(h_t)$$



- h_t is the *state* of the network
- Need to define initial state h_{-1}

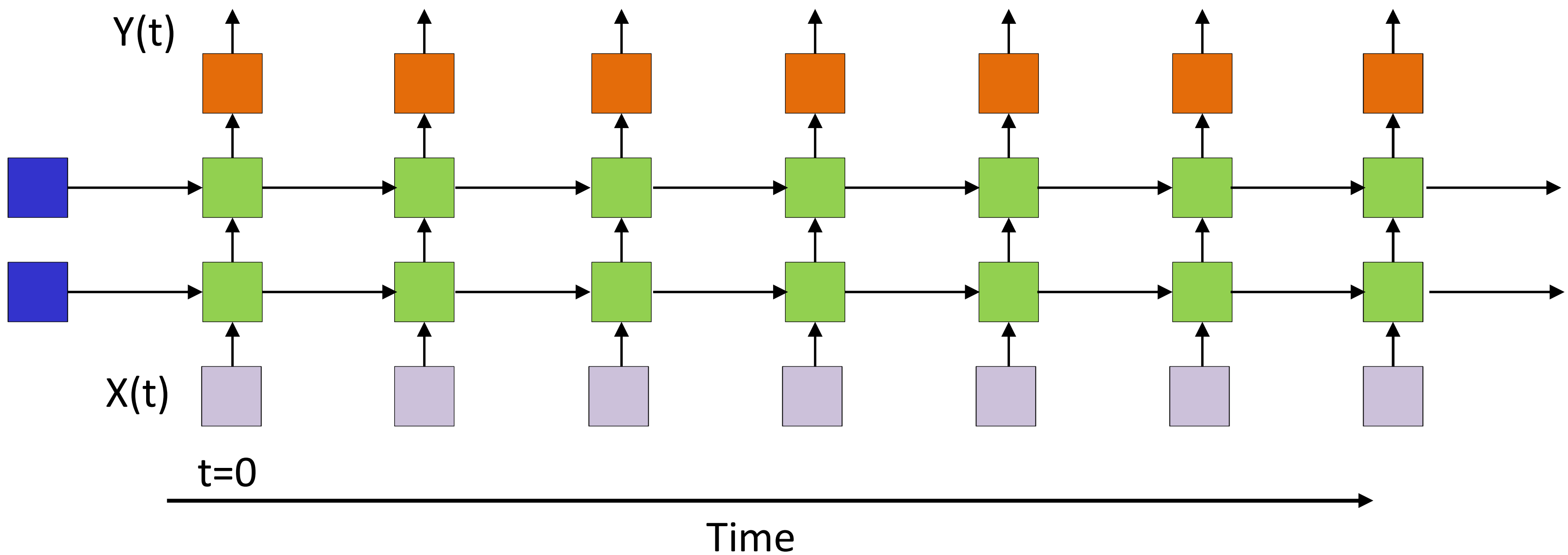
- The state an be arbitrarily complex

Single hidden layer RNN



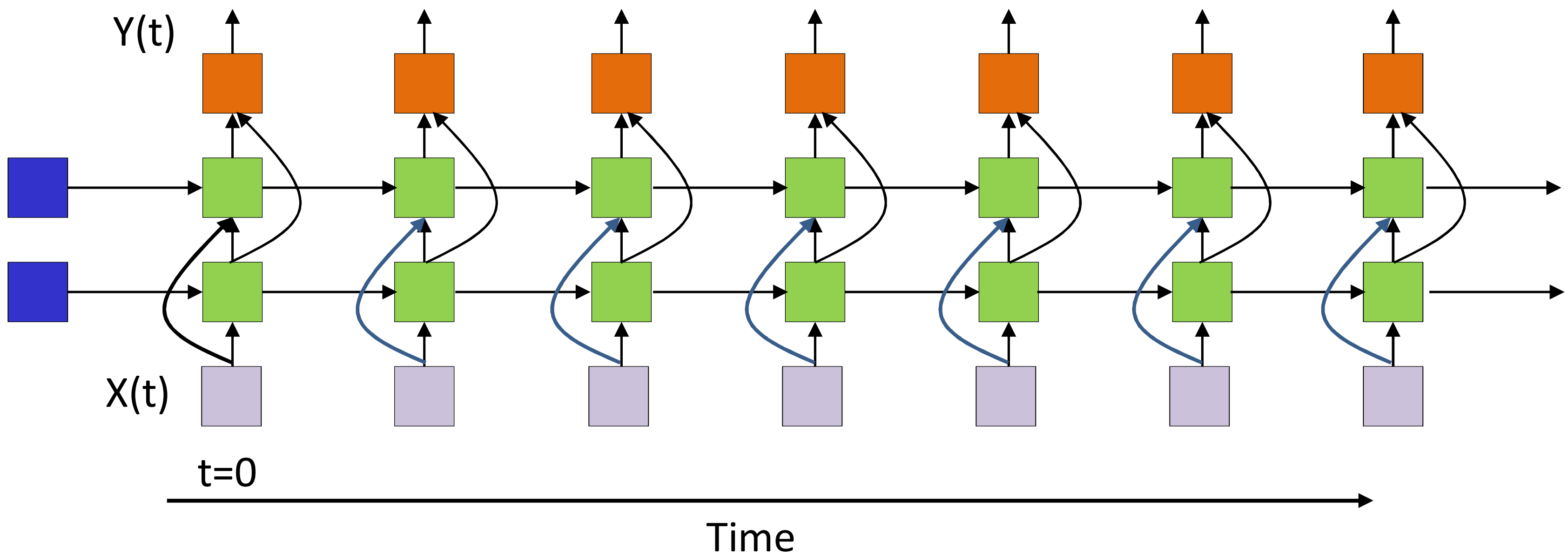
- Recurrent neural network
- All columns are identical
- *An input at t=0 affects outputs forever*

Multiple recurrent layer RNN



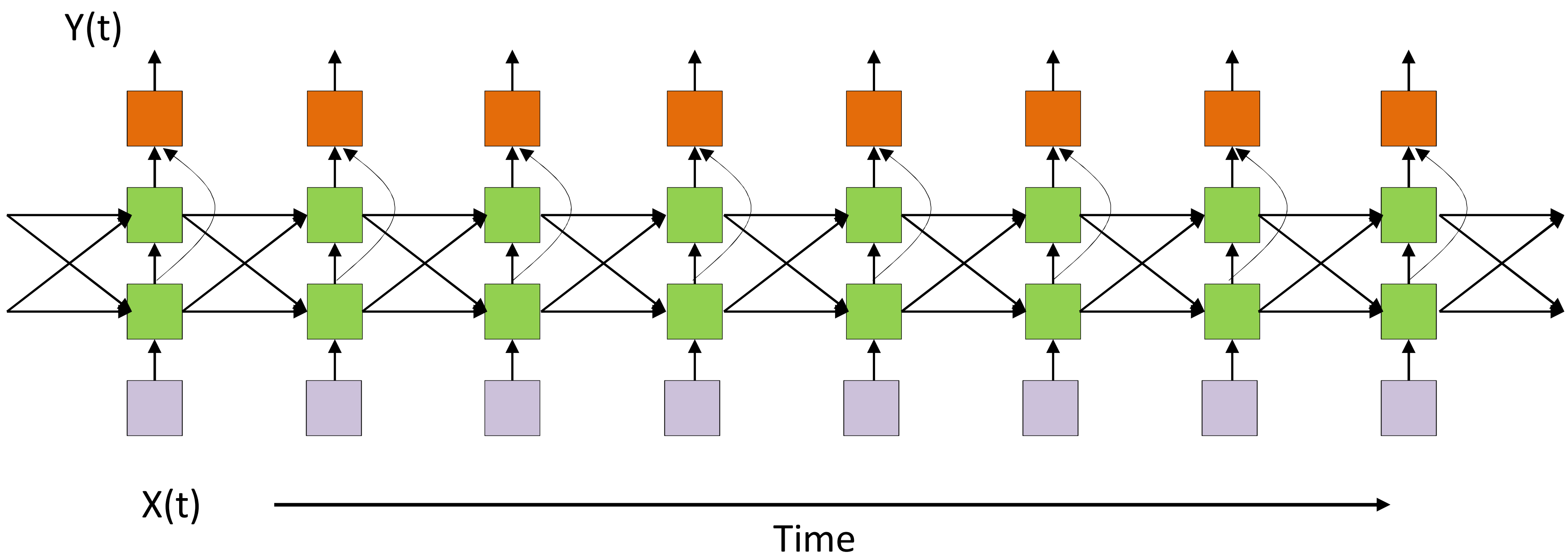
- Recurrent neural network
- All columns are identical
- *An input at t=0 affects outputs forever*

Multiple recurrent layer RNN



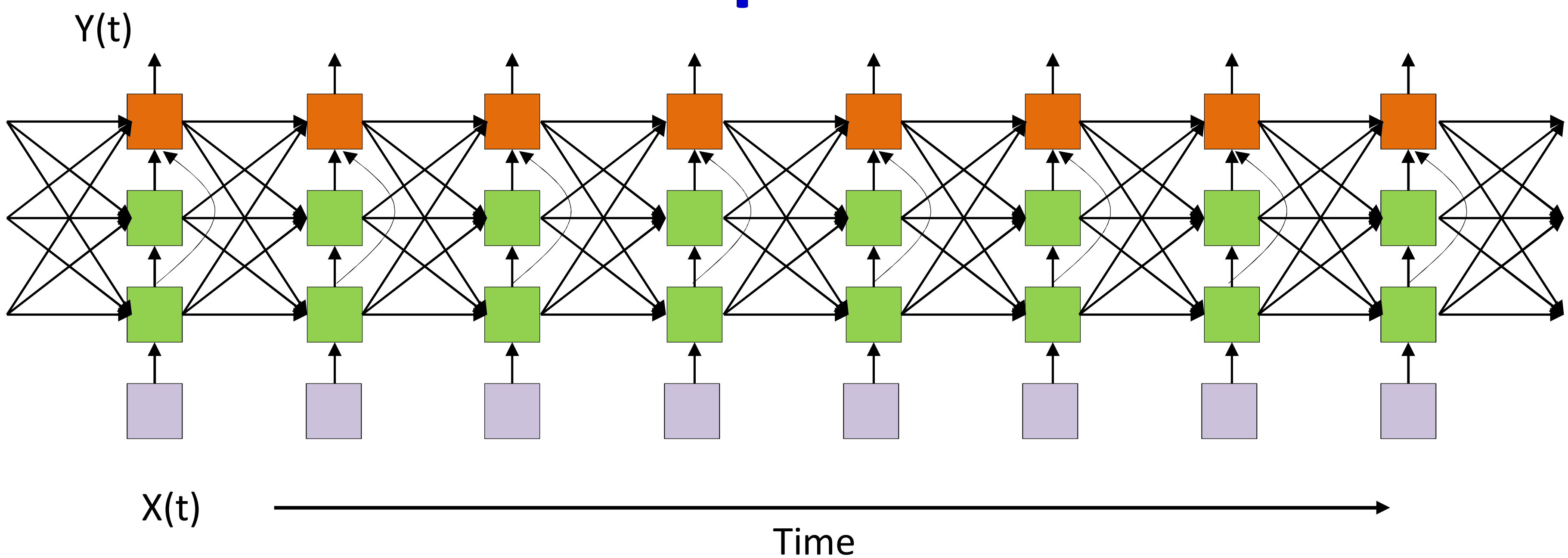
- We can also have skips..

A more complex state



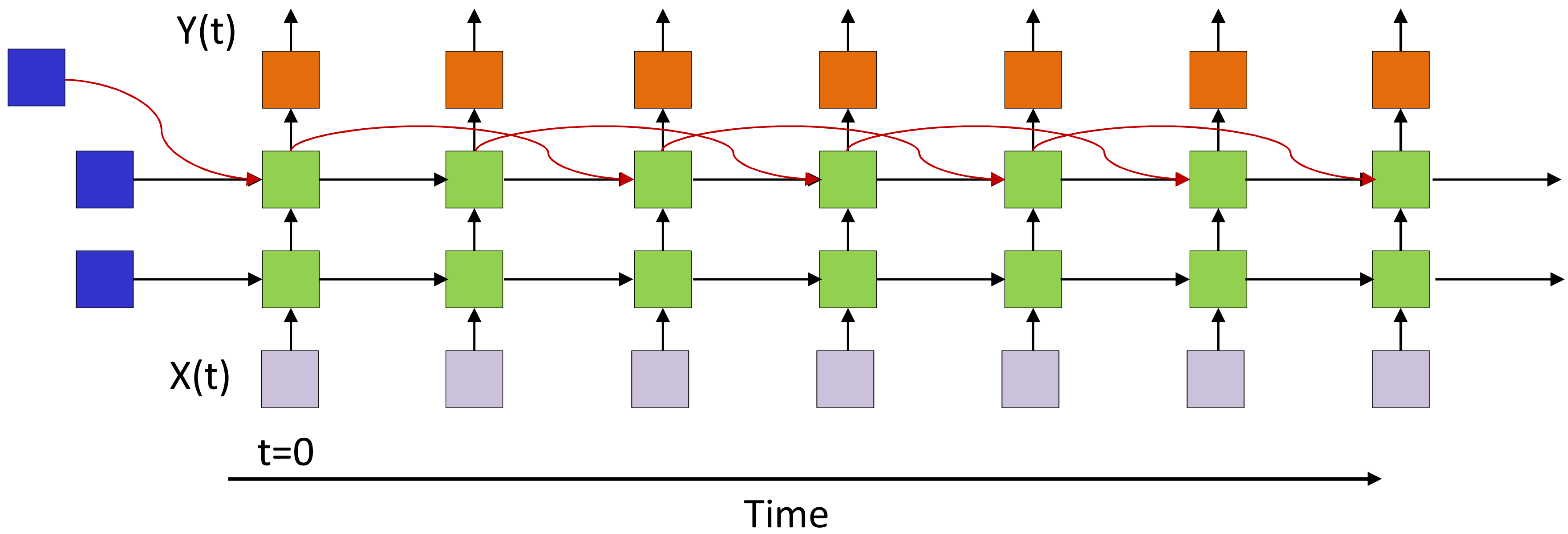
- All columns are identical
- *An input at t=0 affects outputs forever*

Or the network may be even more complicated



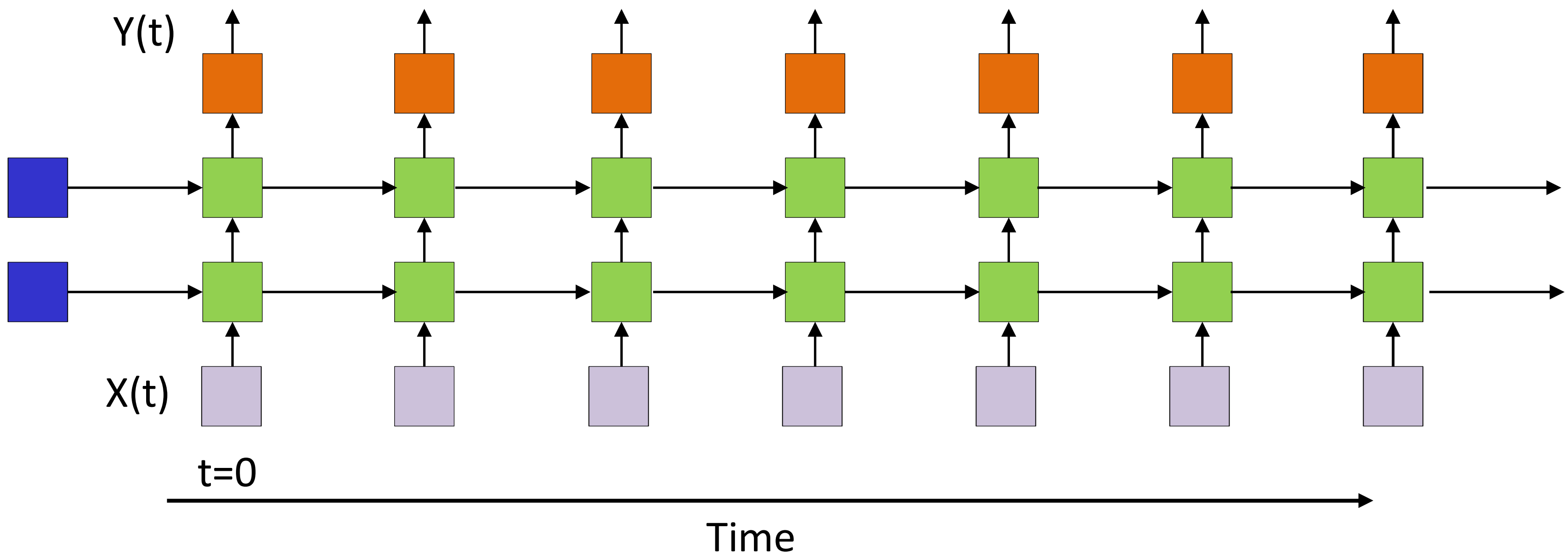
- Shades of NARX
- All columns are identical
- *An input at t=0 affects outputs forever*

Generalization with other recurrences



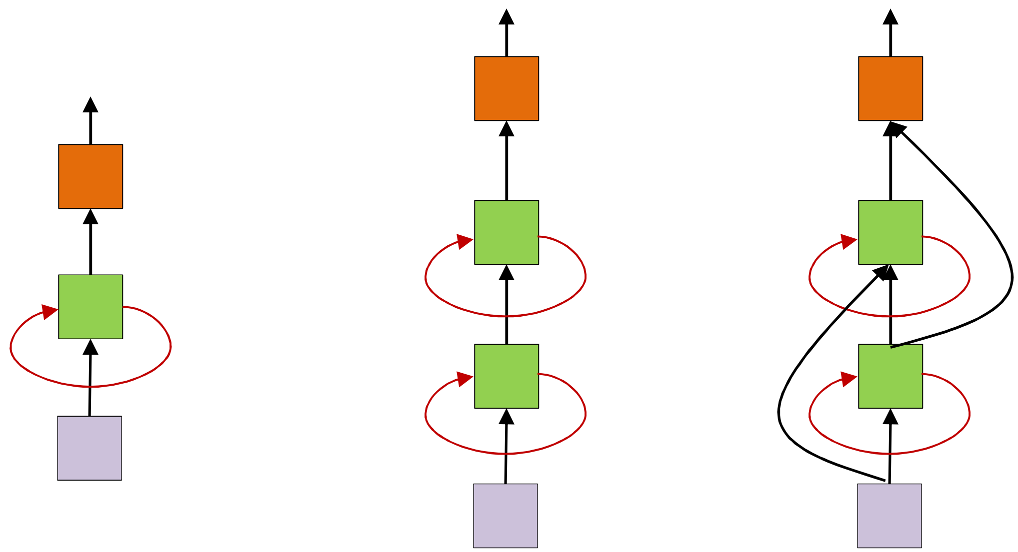
- All columns (including incoming edges) are identical

The simplest structures are most popular



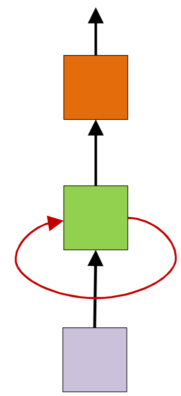
- Recurrent neural network
- All columns are identical
- *An input at t=0 affects outputs forever*

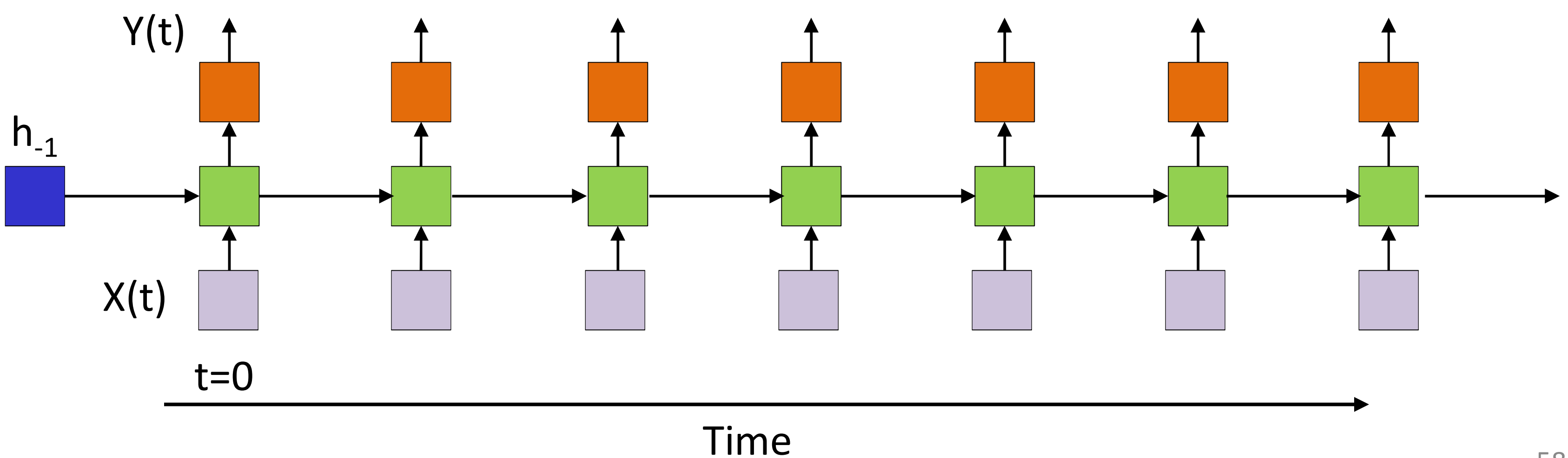
A Recurrent Neural Network



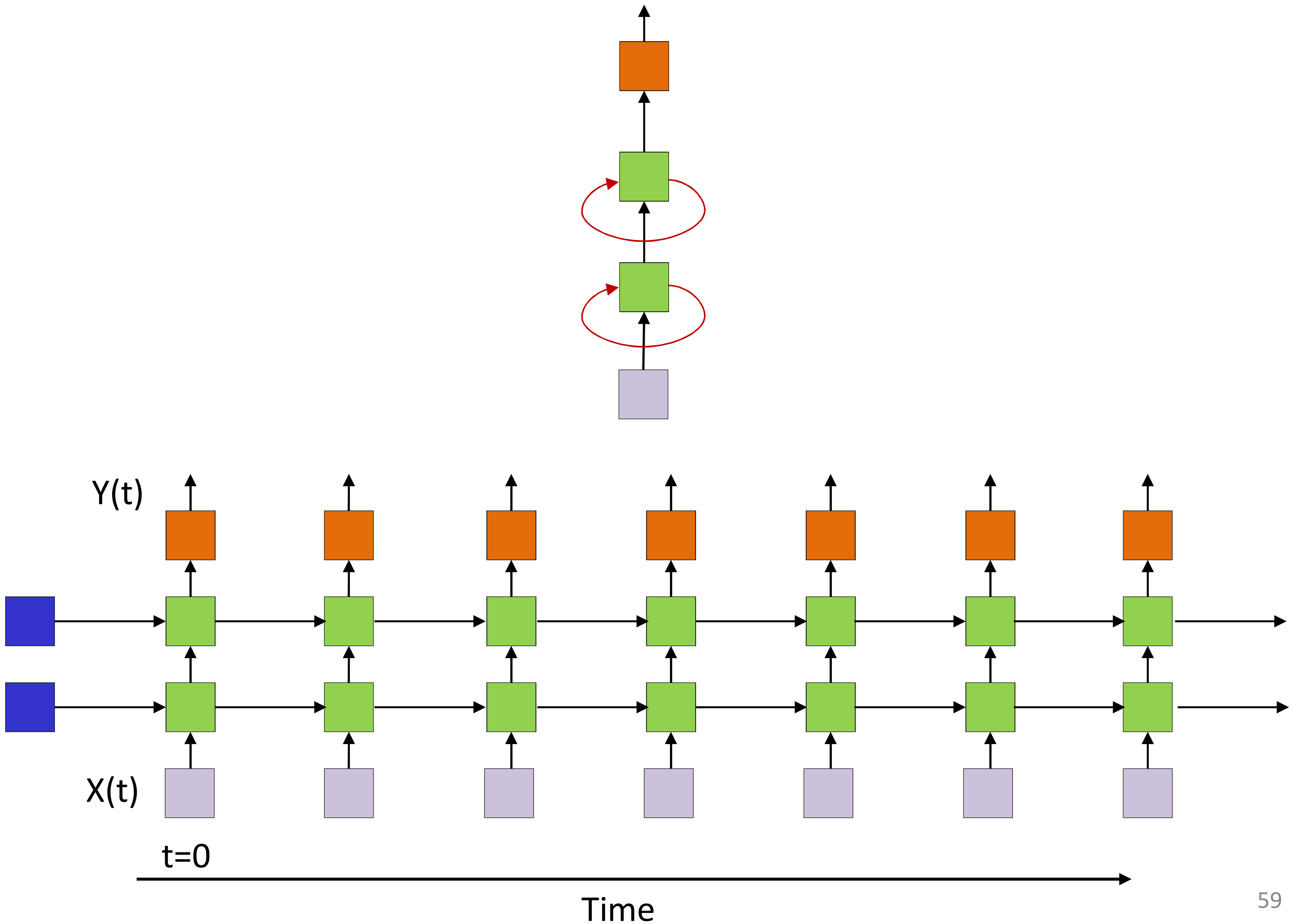
- Simplified models often drawn
- The loops imply recurrence

The detailed version of the simplified representation

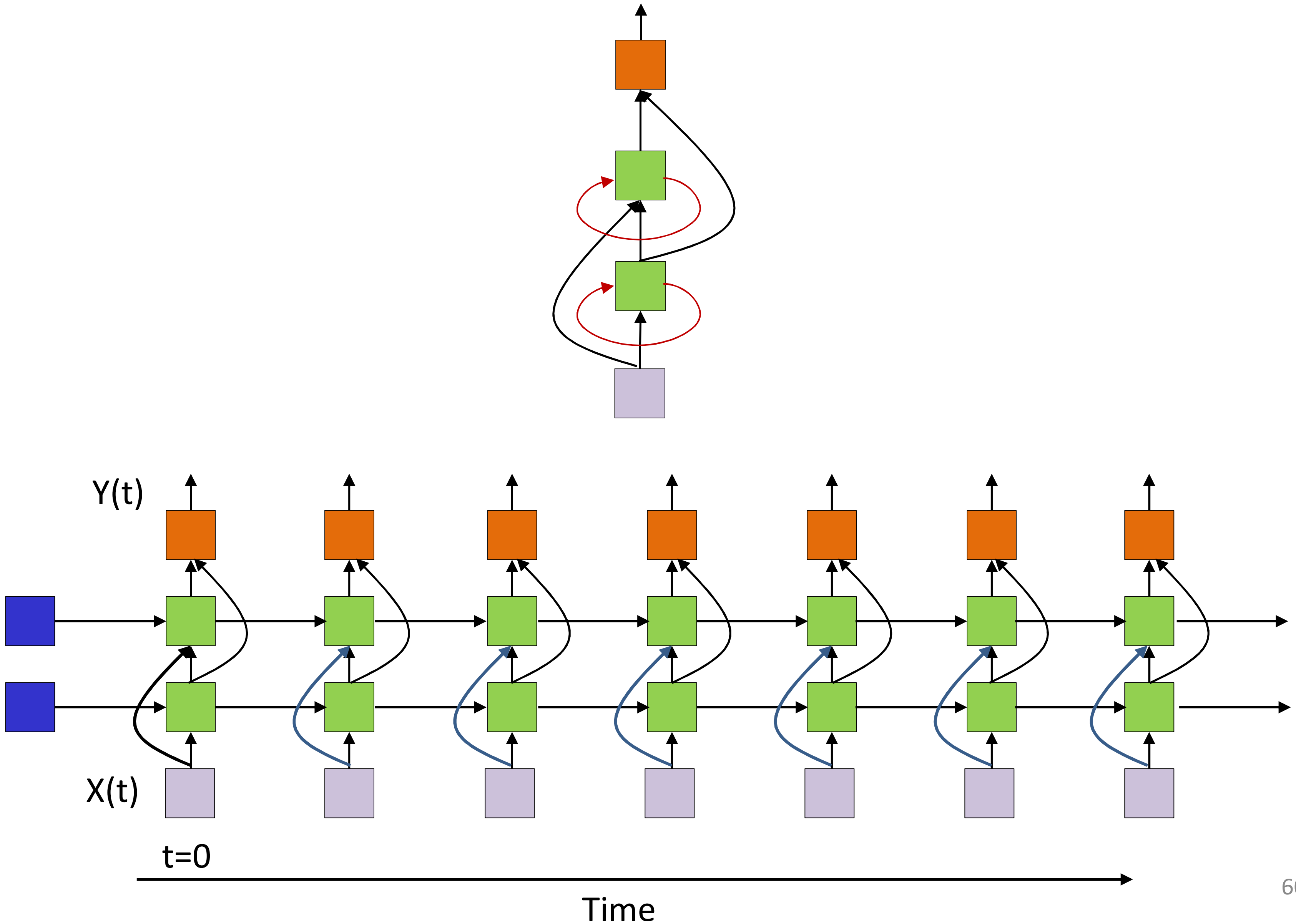




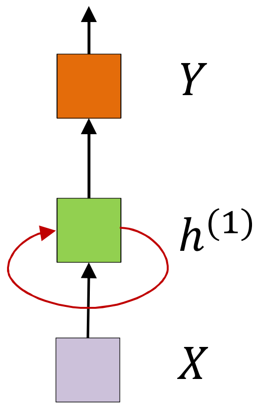
Multiple recurrent layer RNN



Multiple recurrent layer RNN



Equations



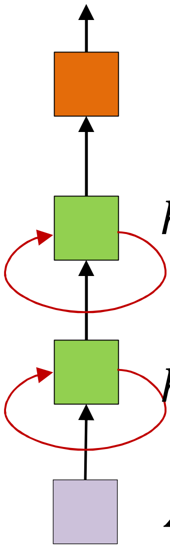
$$h_i^{(1)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(1)}(t) = f_1\left(\sum_j w_{ji}^{(0)} X_j(t) + \sum_j w_{ji}^{(11)} h_i^{(1)}(t-1) + b_i^{(1)}\right)$$

$$Y(t) = f_2\left(\sum_j w_{jk}^{(1)} h_j^{(1)}(t) + b_k^{(1)}, k = 1..M\right)$$

- Note superscript in indexing, which indicates layer of network from which inputs are obtained
- Assuming vector function at output, e.g. softmax
- The *state* node activation, $f_1()$ is typically $\tanh()$
- Every neuron also has a *bias* input

Equations



$$h_i^{(1)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(2)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(1)}(t) = f_1\left(\sum_j w_{ji}^{(0)} X_j(t) + \sum_j w_{ji}^{(11)} h_i^{(1)}(t-1) + b_i^{(1)}\right)$$

$$h_i^{(2)}(t) = f_2\left(\sum_j w_{ji}^{(1)} h_j^{(1)}(t) + \sum_j w_{ji}^{(22)} h_i^{(2)}(t-1) + b_i^{(2)}\right)$$

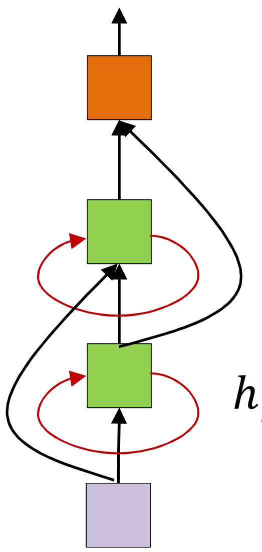
$$Y(t) = f_3\left(\sum_j w_{jk}^{(2)} h_j^{(2)}(t) + b_k^{(3)}, k = 1..M\right)$$

- Assuming vector function at output, e.g. softmax $f_3()$
- The *state* node activations, $f_k()$ are typically $\tanh()$
- Every neuron also has a *bias* input

Equations

$$h_i^{(1)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(2)}(-1) = part\ of\ network\ parameters$$

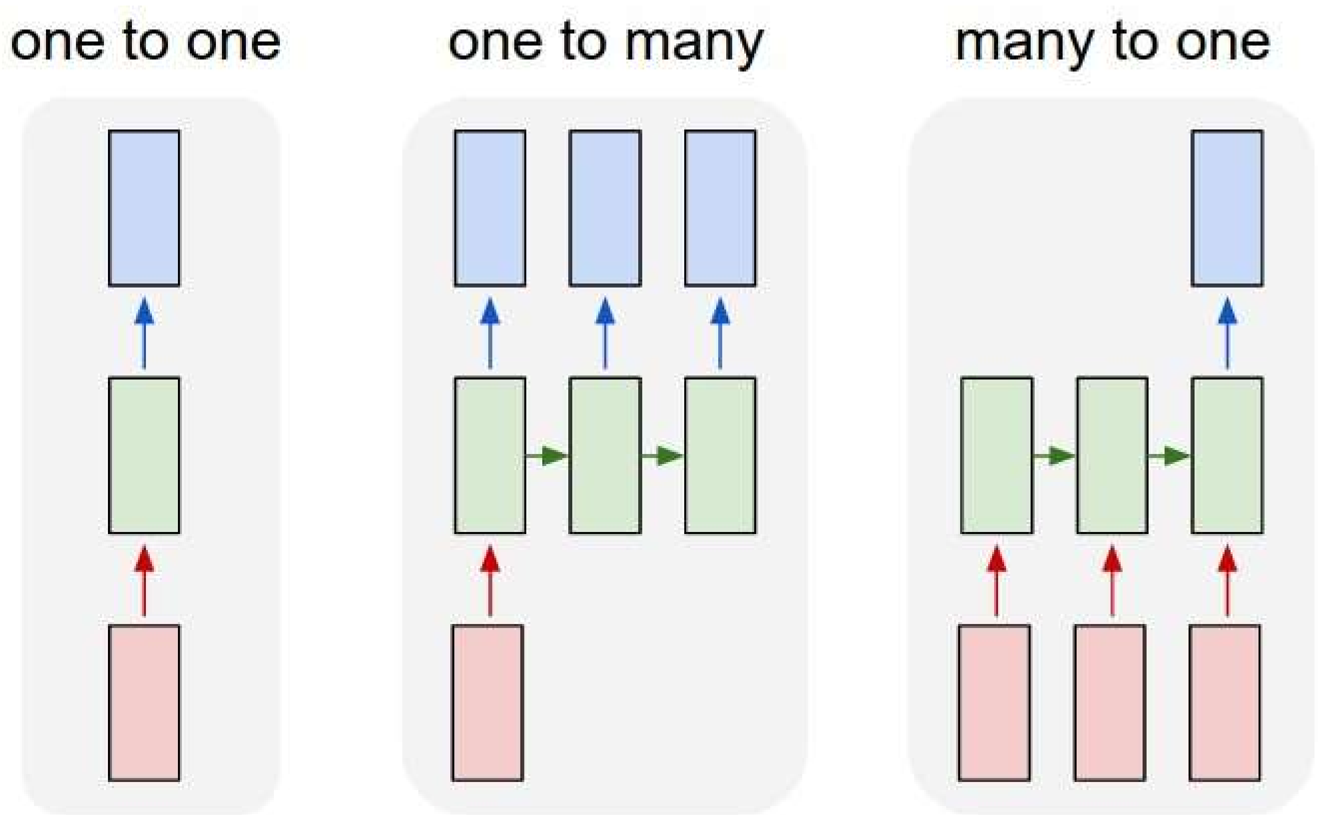


$$h_i^{(1)}(t) = f_1\left(\sum_j w_{ji}^{(0,1)} X_j(t) + \sum_i w_{ii}^{(1,1)} h_i^{(1)}(t-1) + b_i^{(1)}\right)$$

$$h_i^{(2)}(t) = f_2\left(\sum_j w_{ji}^{(1,2)} h_j^{(1)}(t) + \sum_j w_{ji}^{(0,2)} X_j(t) + \sum_i w_{ii}^{(2,2)} h_i^{(2)}(t-1) + b_i^{(2)}\right)$$

$$Y_i(t) = f_3\left(\sum_j w_{jk}^{(2)} h_j^{(2)}(t) + \sum_j w_{jk}^{(1,3)} h_j^{(1)}(t) + b_k^{(3)}, k = 1..M\right)$$

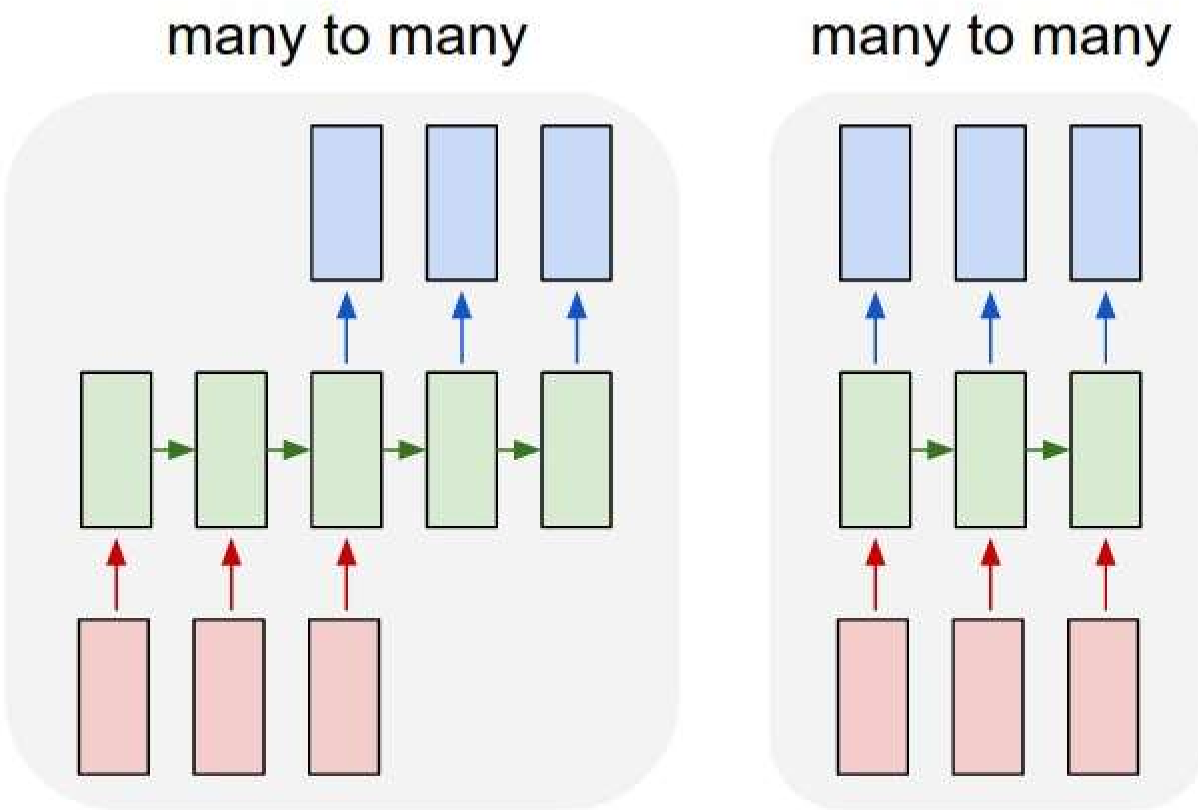
Variants on recurrent nets



Images from Karpathy

- 1: Conventional MLP
- 2: Sequence *generation*, e.g. image to caption
- 3: Sequence based *prediction or classification,* e.g. Speech recognition, text classification

Variants



Images from Karpathy

- 1: *Delayed* sequence to sequence
- 2: Sequence to sequence, e.g. stock problem, label prediction
- Etc…

Story so far

- Time series analysis must consider past inputs along with current input
- Looking into the infinite past requires recursion

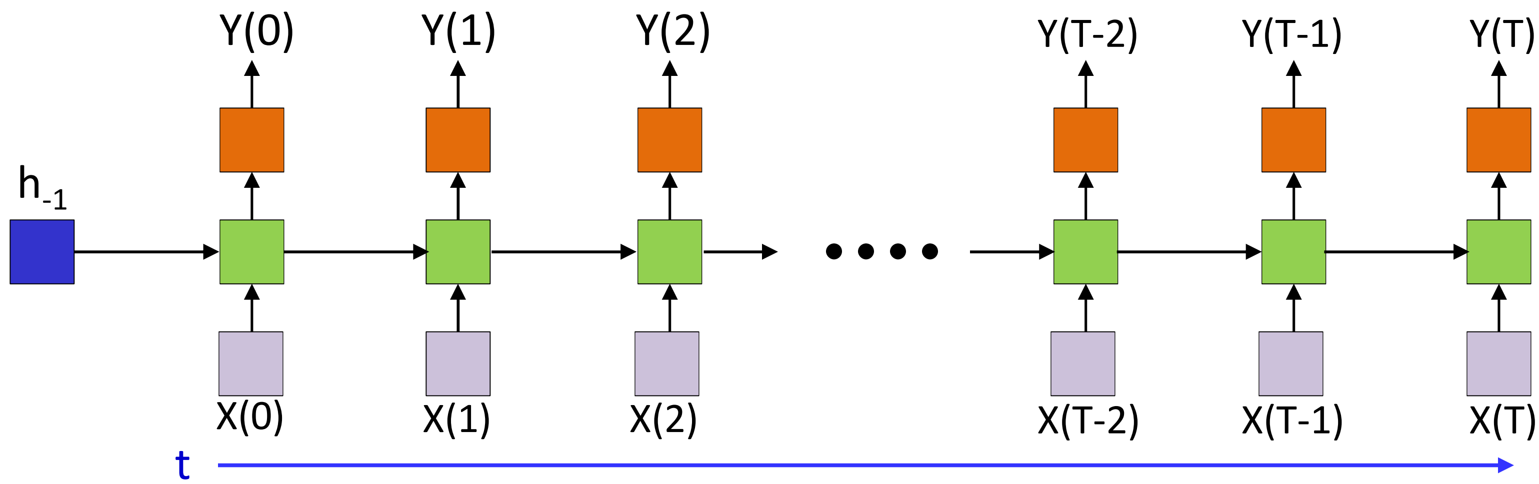
- NARX networks achieve this by feeding back the output to the input

- "Simple" recurrent networks maintain separate "memory" or "context" units to retain some information about the past
  - But during learning the current error does not influence the past

- State-space models retain information about the past through recurrent hidden states
  - These are "fully recurrent" networks
  - The initial values of the hidden states are generally learnable parameters as well

- State-space models enable current error to update parameters in the past

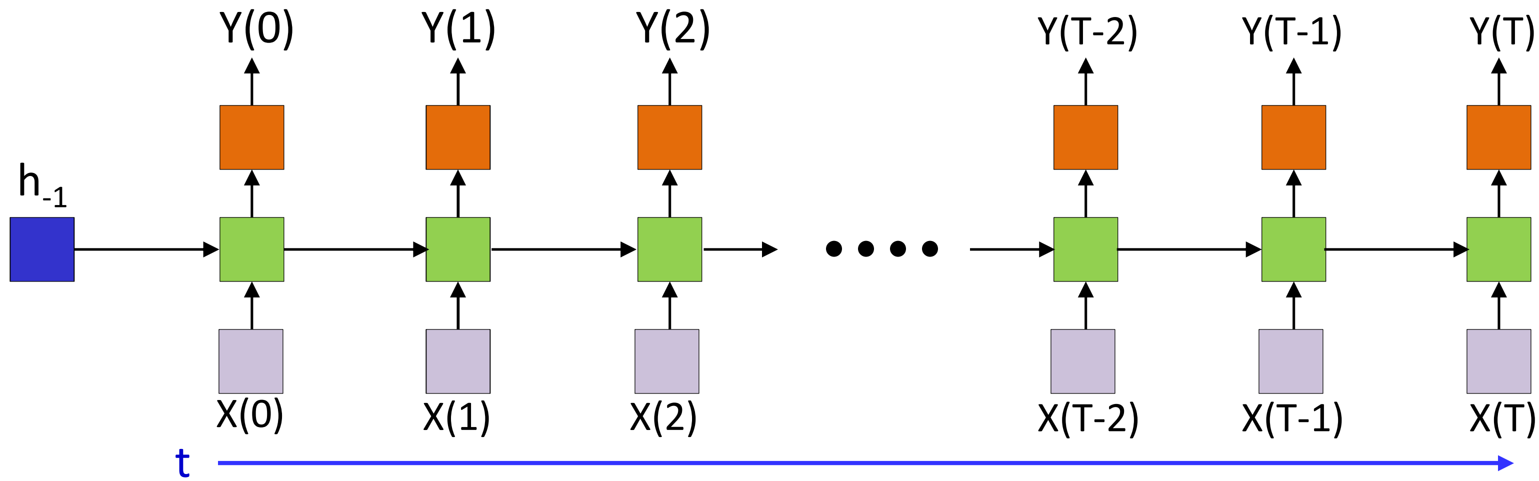
How do we *train* the network



- Back propagation through time (BPTT)

- Given a collection of *sequence* inputs
  - $(\mathbf{X}_i, \mathbf{D}_i)$, where
  - $\mathbf{X}_i = X_{i,0}, \dots, X_{i,T}$
  - $\mathbf{D}_i = D_{i,0}, \dots, D_{i,T}$
- Train network parameters to minimize the error between the output of the network $\mathbf{Y}_i = Y_{i,0}, \dots, Y_{i,T}$ and the desired outputs
  - This is the most generic setting. In other settings we just "remove" some of the input or output entries

Training: Forward pass



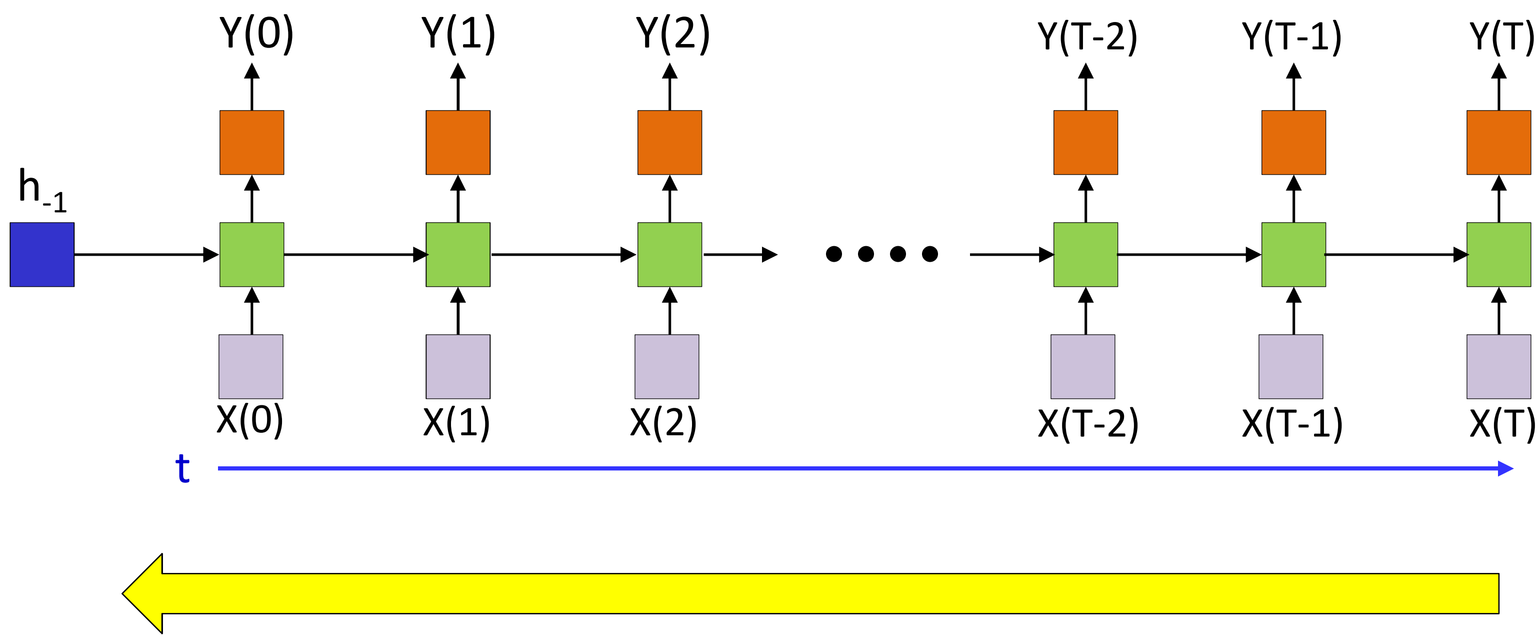
- For each training input:
- Forward pass: pass the entire data sequence through the network, generate outputs

Recurrent Neural Net
Assuming time-synchronous output

```
# Assuming h(-1,*) is known
# Assuming L hidden-state layers and an output layer
# Wc(*) and Wr(*) are matrics, b(*) are vectors
# Wc are weights for inputs from current time
# Wr is recurrent weight applied to the previous time
# Wo are output layre weights

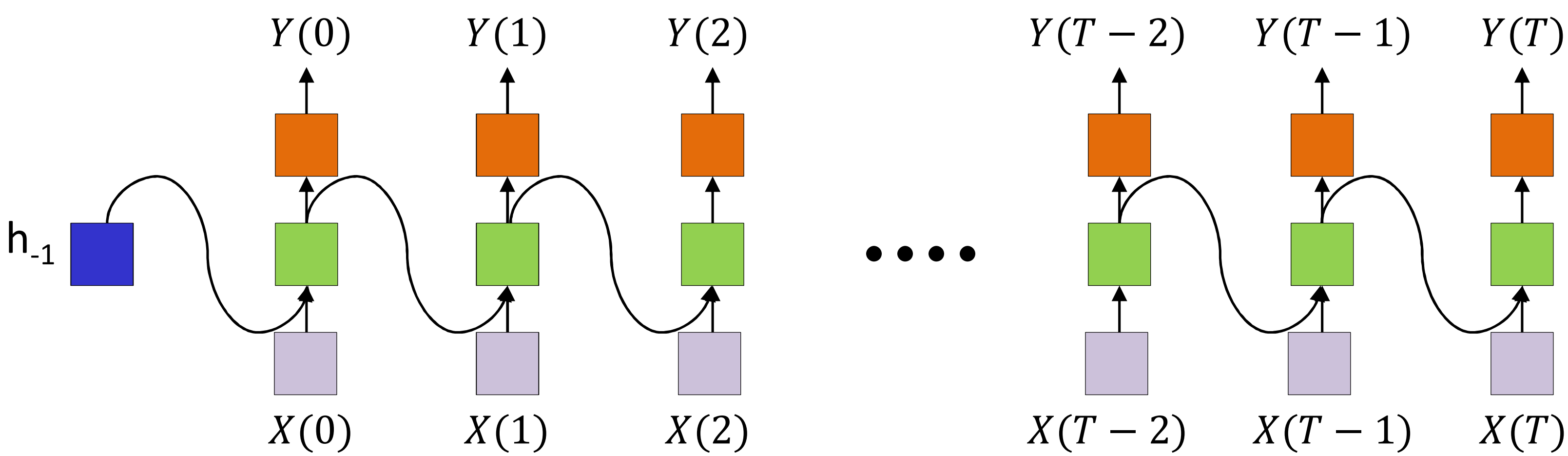
for t = 0:T-1  # Including both ends of the index
    h(t,0) = x(t) # Vectors. Initialize h(0) to input
    for l = 1:L  # hidden layers operate at time t
        z(t,l) = Wc(l)h(t,l-1) + Wr(l)h(t-1,l) + b(l)
        h(t,l) = tanh(z(t,l)) # Assuming tanh activ.
    zo(t) = Woh(t,L) + bo
    Y(t) = softmax( zo(t) )
```

Training: Computing gradients



- For each training input:
- Backward pass: Compute gradients via backpropagation
  - *Back Propagation Through Time*

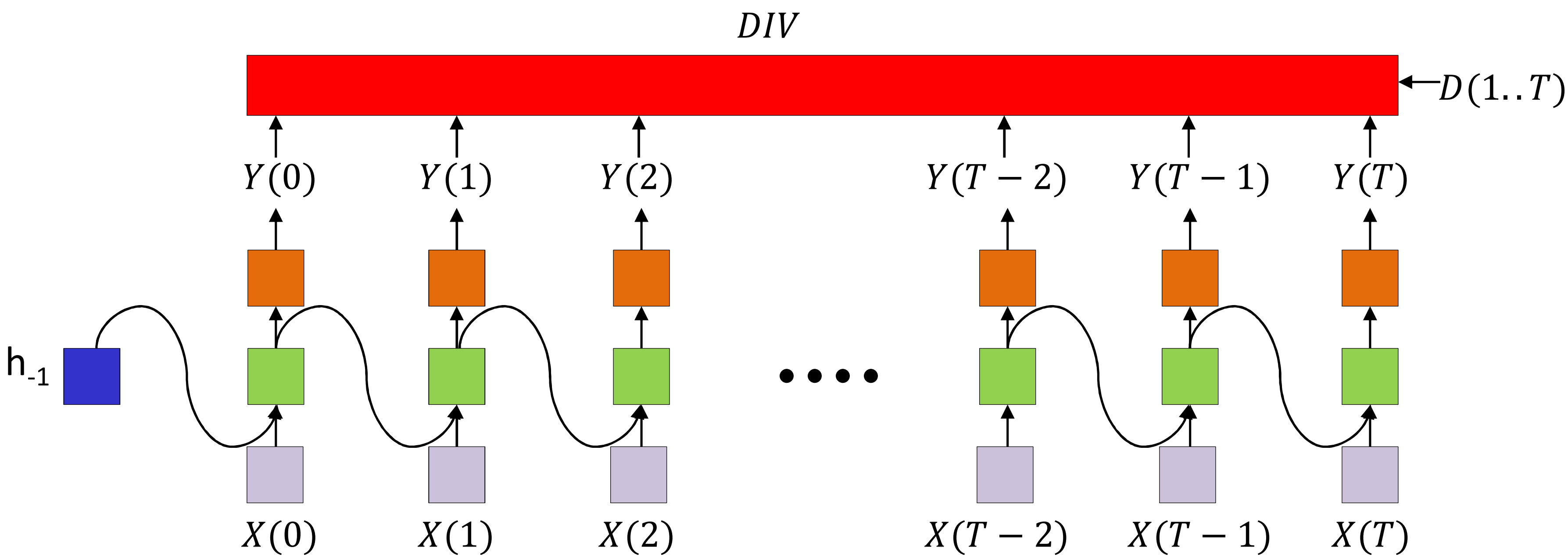
Back Propagation Through Time



Will only focus on *one* training instance

All subscripts represent *components* and not training instance index

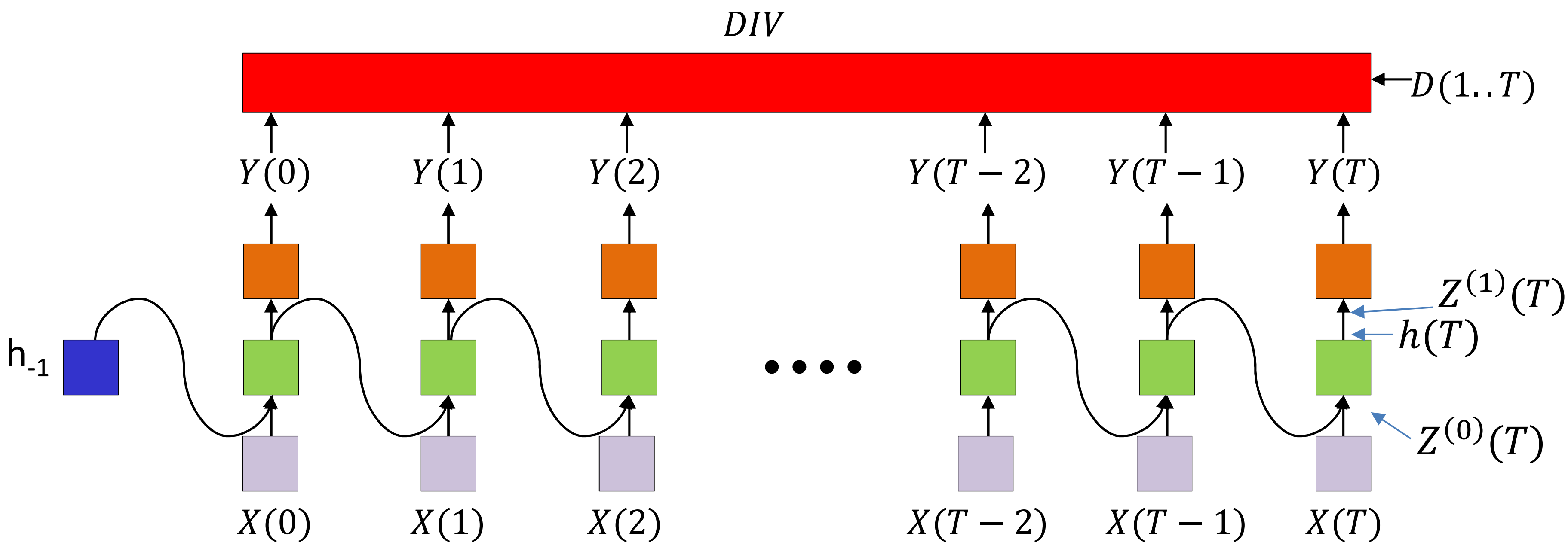
Back Propagation Through Time



- The divergence computed is between the *sequence of outputs* by the network and the *desired sequence of outputs*
  - DIV is a scalar function of a series of vectors!

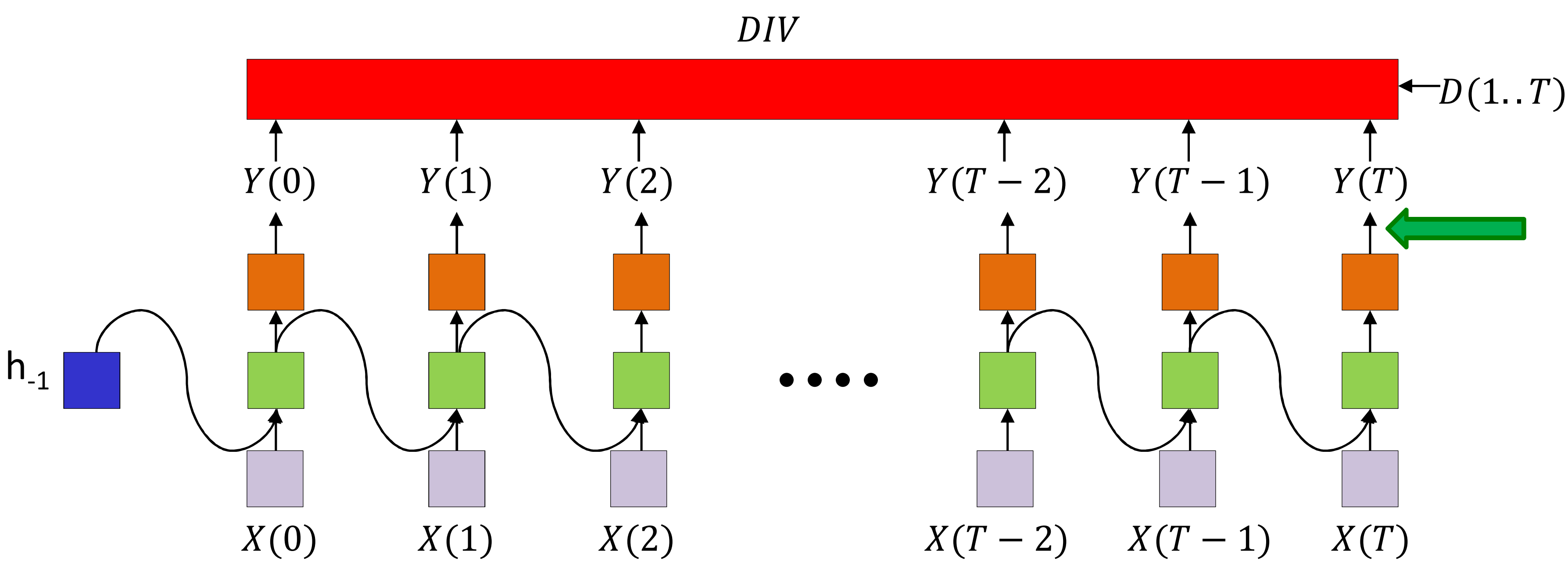
- This is *not* just the sum of the divergences at individual times
  - Unless we explicitly define it that way

Notation



- $Y(t)$ is the output at time t
  - $Y_i(t)$ is the ith output
- $Z^{(1)}(t)$ is the pre-activation value of the neurons at the output layer at time t
- $h(t)$ is the output of the hidden layer at time t
  - Assuming only one hidden layer in this example
- $Z^{(0)}(t)$ is the pre-activation value of the hidden layer at time t

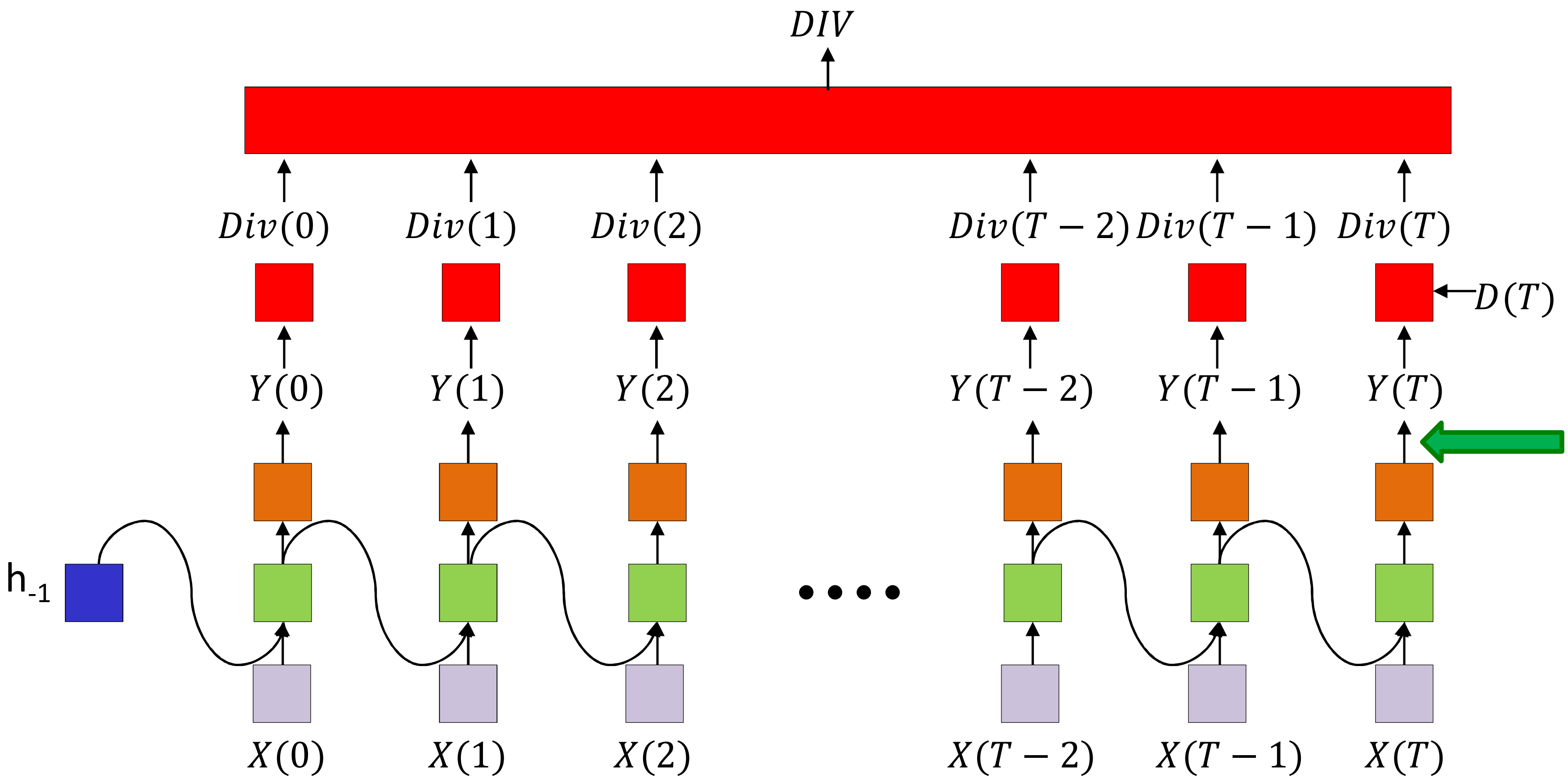
Back Propagation Through Time



First step of backprop: Compute $\frac{dDIV}{dY_i(T)}$ for all i

Note: DIV is a function of *all* outputs Y(0) … Y(T)

In general we will be required to compute $\frac{dDIV}{dY_i(t)}$ for all i and t as we will see. This can be a source of significant difficulty in many scenarios.



Special case, when the overall divergence is a simple combination of local divergences at each time:

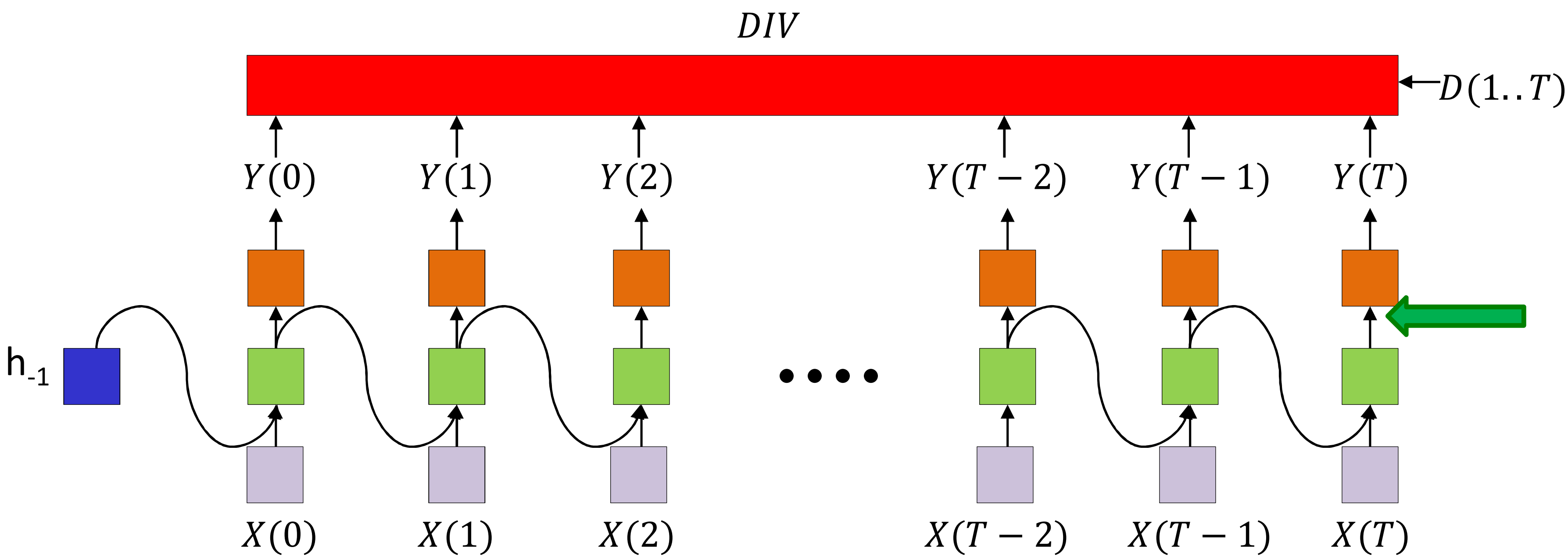
Must compute

$$\boxed{\frac{dDIV}{dY_i(t)} \text{ for all i for all T}}$$

Will usually get

$$\boxed{\frac{dDIV}{dY_i(t)} = \frac{dDiv(t)}{dY_i(t)}}$$

Back Propagation Through Time



First step of backprop: Compute $\frac{dDIV}{dY_i(T)}$ for all i

$$\nabla_{Z^{(1)}(T)}DIV = \nabla_{Y(T)}DIV \nabla_{Z^{(1)}(T)}Y(T)$$

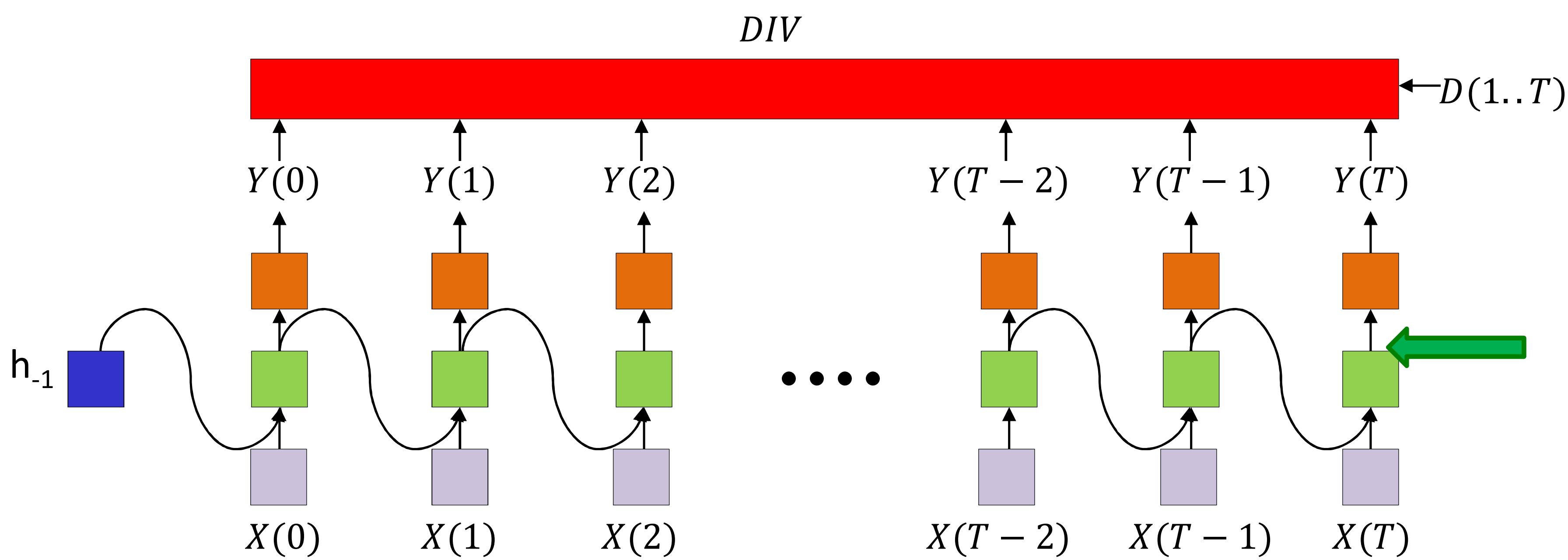
Vector output activation

$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDIV}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(1)}(T)}$$

OR

$$\frac{dDIV}{dZ_i(T)} = \sum_j \frac{dDIV}{dY_j(T)} \frac{dY_j(T)}{dZ_j^{(1)}(T)}$$

Back Propagation Through Time



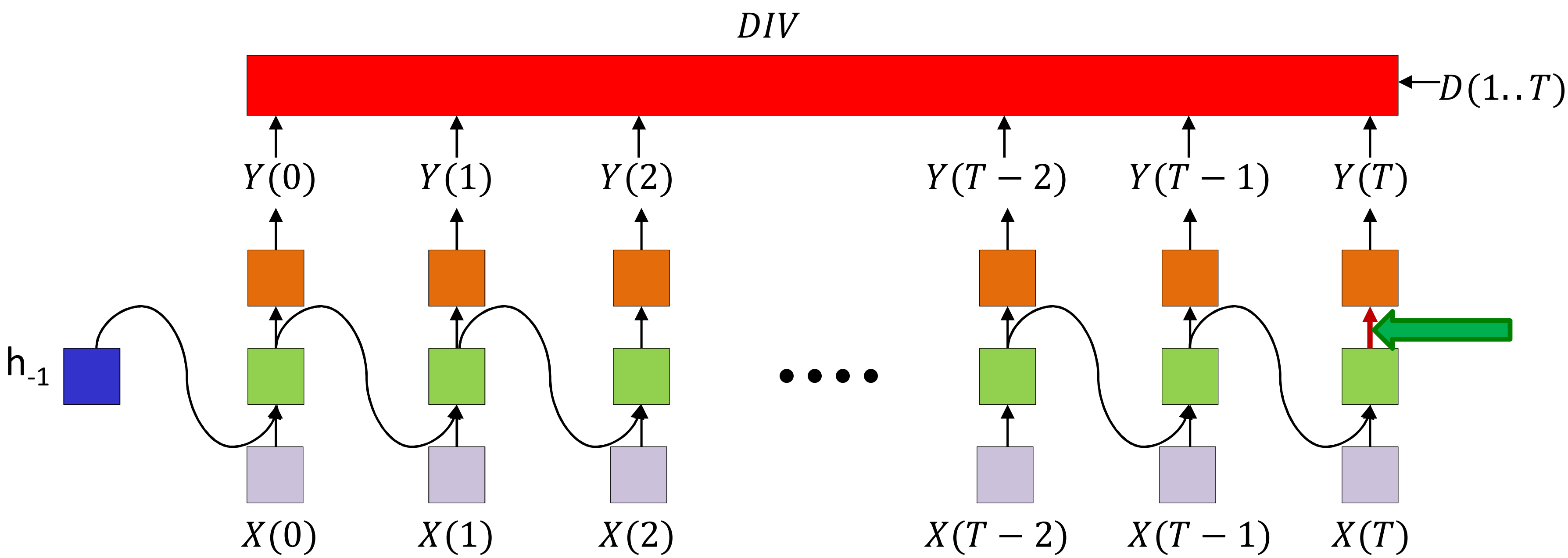
$$\frac{dDIV}{dY_i(T)} \text{ for all i}$$

$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDiv(T)}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(1)}(T)}$$

$$\frac{dDIV}{dh_i(T)} = \sum_j \frac{dDIV}{dZ_j^{(1)}(T)} \frac{dZ_j^{(1)}(T)}{dh_i(T)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T)}$$

$$\nabla_{h(T)} DIV = \nabla_{Z^{(1)}(T)} DIV \, W^{(1)}$$

Back Propagation Through Time

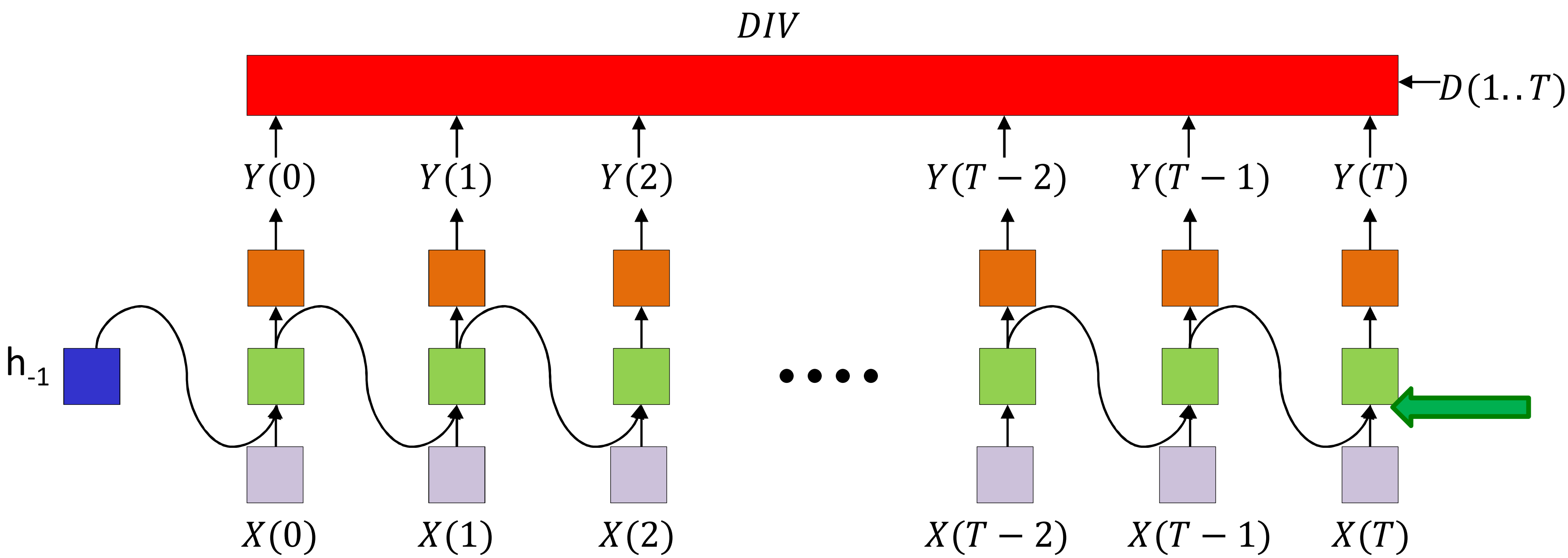


$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDiv(T)}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(1)}(T)} \qquad \frac{dDIV}{dh_i(T)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T)}$$

$$\boxed{\frac{dDIV}{dw_{ij}^{(1)}} = \frac{dDIV}{dZ_j^{(1)}(T)} h_i(T)}$$

$$\boxed{\nabla_{W^{(1)}} DIV = h(T) \nabla_{Z^{(1)}(T)} DIV}$$

Back Propagation Through Time



$$\nabla_{Z^{(0)}(T)} DIV = \nabla_{h(T)} DIV \, \nabla_{Z^{(0)}(T)} h(T)$$

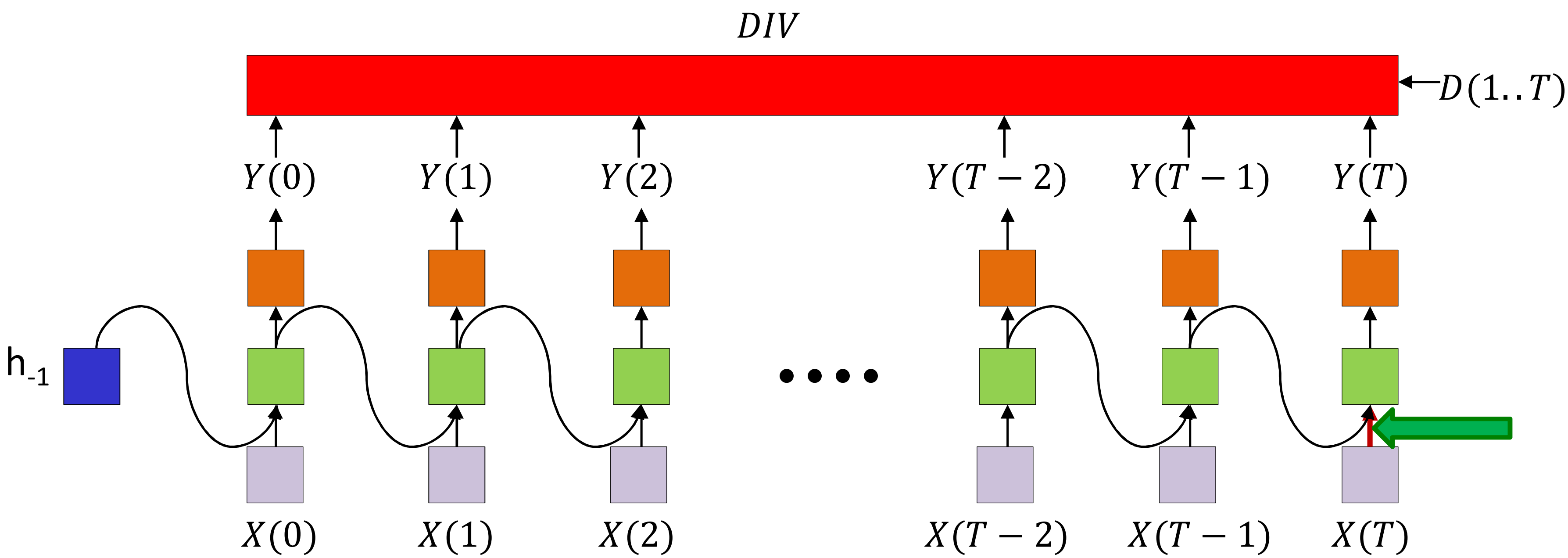
$$\frac{dDIV}{dZ_i^{(0)}(T)} = \frac{dDIV}{dh_i(T)} \frac{dh_i(T)}{dZ_i^{(0)}(T)}$$

$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDIV}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(1)}(T)}$$

$$\frac{dDIV}{dh_i(T)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T)}$$

$$\frac{dDIV}{dw_{ij}^{(1)}} = \frac{dDIV}{dZ_j^{(1)}(T)} h_i(T)$$

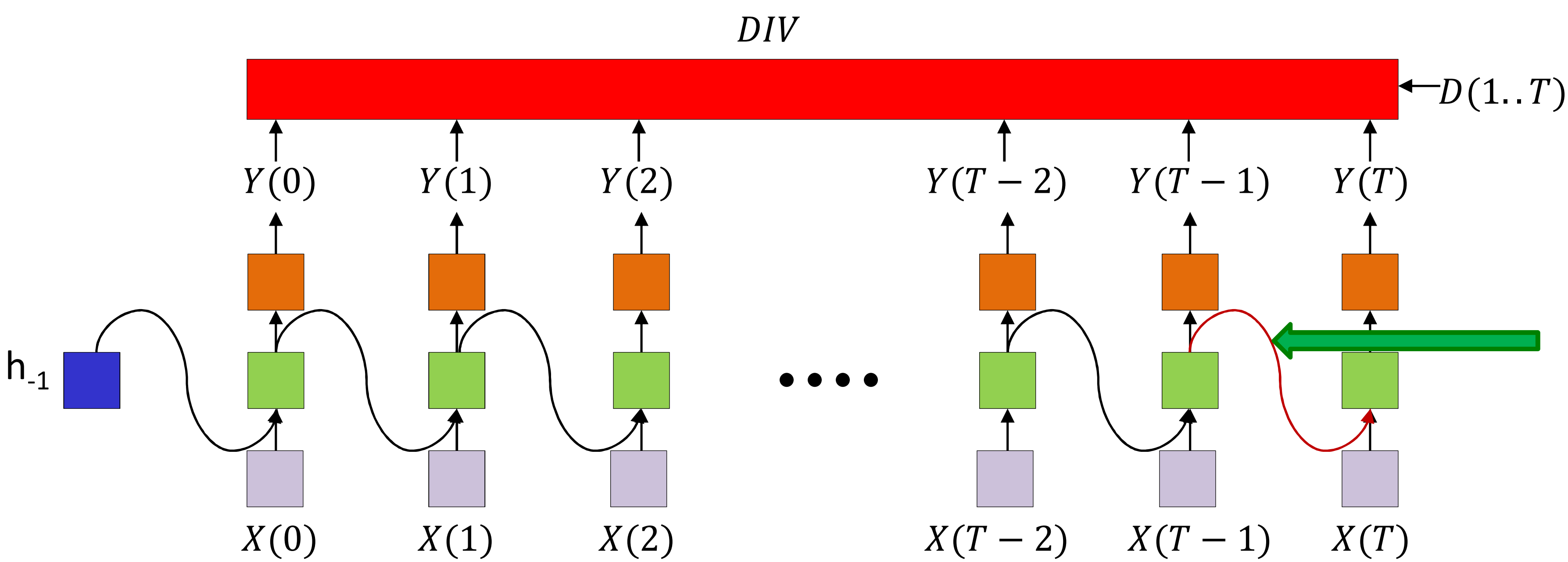
Back Propagation Through Time



$$\frac{dDIV}{dw_{ij}^{(0)}} = \frac{dDIV}{dZ_j^{(0)}(T)} X_i(T)$$

$$\nabla_{W^{(0)}} DIV = X(T) \nabla_{Z^{(0)}(T)} DIV$$

Back Propagation Through Time

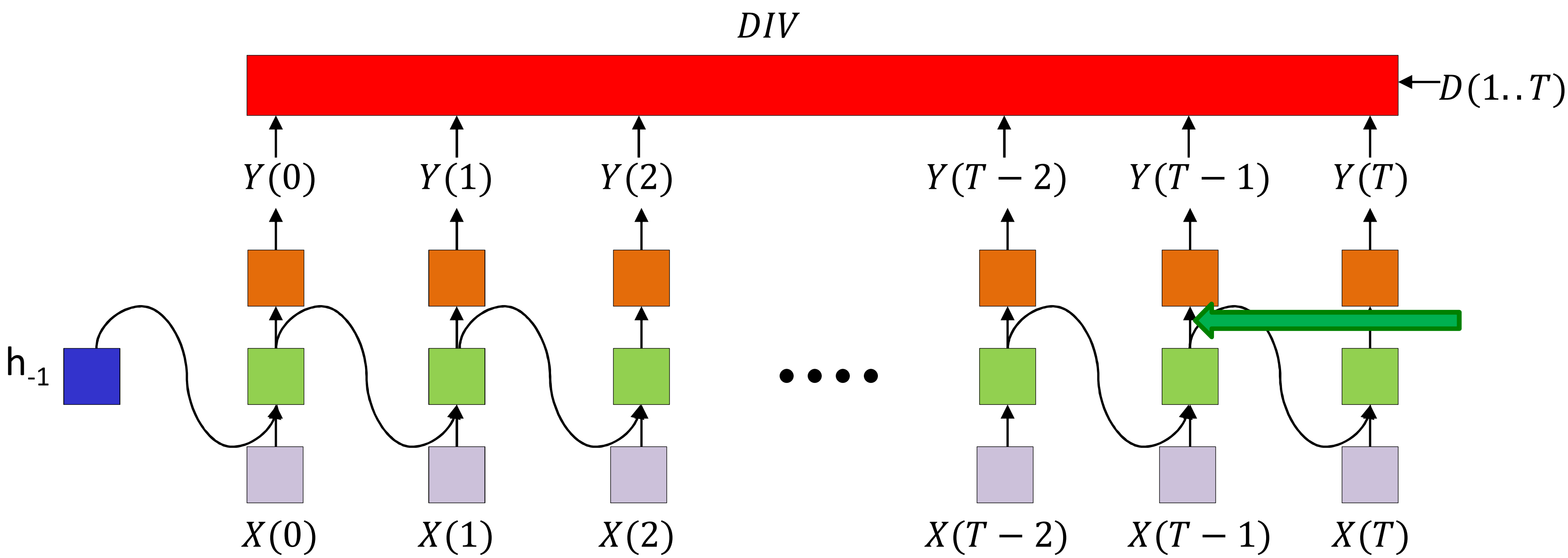


$$\nabla_{W^{(11)}} DIV = h(T-1) \nabla_{Z^{(0)}(T)} DIV$$

$$\frac{dDIV}{dw_{ij}^{(0)}} = \frac{dDIV}{dZ_j^{(0)}(T)} X_i(T)$$

$$\frac{dDIV}{dw_{ij}^{(11)}} = \frac{dDIV}{dZ_j^{(0)}(T)} h_i(T-1)$$

Back Propagation Through Time



$$\nabla_{Z^{(1)}(T-1)}DIV = \nabla_{Y(T-1)}DIV \; \nabla_{Z^{(1)}(T)}Y(T-1)$$

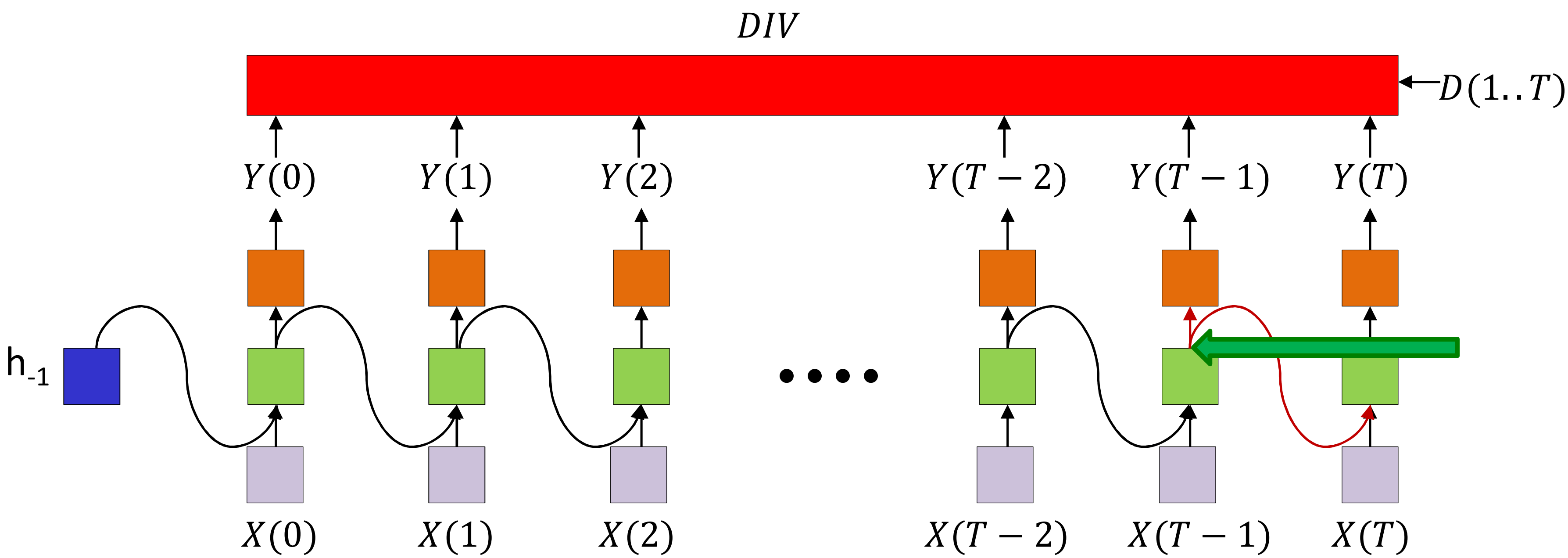
Vector output activation

$$\frac{dDIV}{dZ_i^{(1)}(T-1)} = \frac{dDIV}{dY_i(T-1)} \frac{dY_i(T-1)}{dZ_i^{(1)}(T-1)}$$

OR

$$\frac{dDIV}{dZ_i^{(1)}(T-1)} = \sum_j \frac{dDIV}{dY_j(T-1)} \frac{dY_j(T-1)}{dZ_i^{(1)}(T-1)}$$

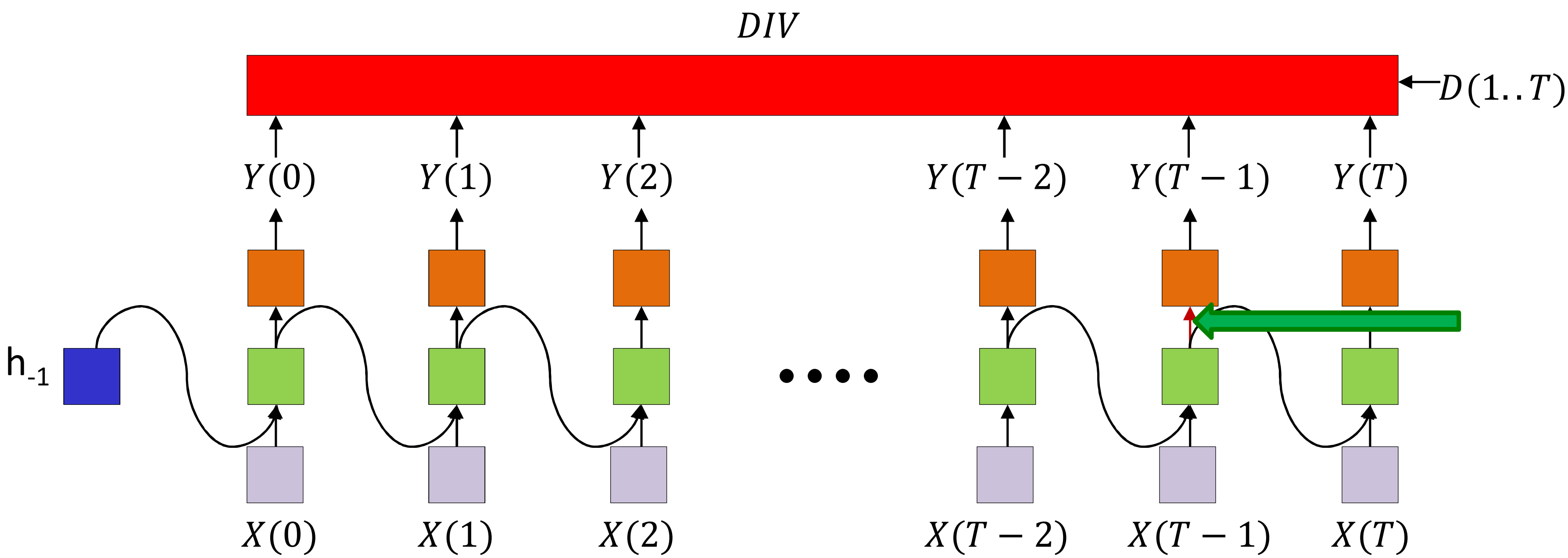
Back Propagation Through Time



$$\frac{dDIV}{dh_i(T-1)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T-1)} + \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(0)}(T)}$$

$$\nabla_{h(T-1)}DIV = \nabla_{Z^{(1)}(T-1)}DIV\ W^{(1)} + \nabla_{Z^{(0)}(T)}DIV\ W^{(11)}$$

Back Propagation Through Time



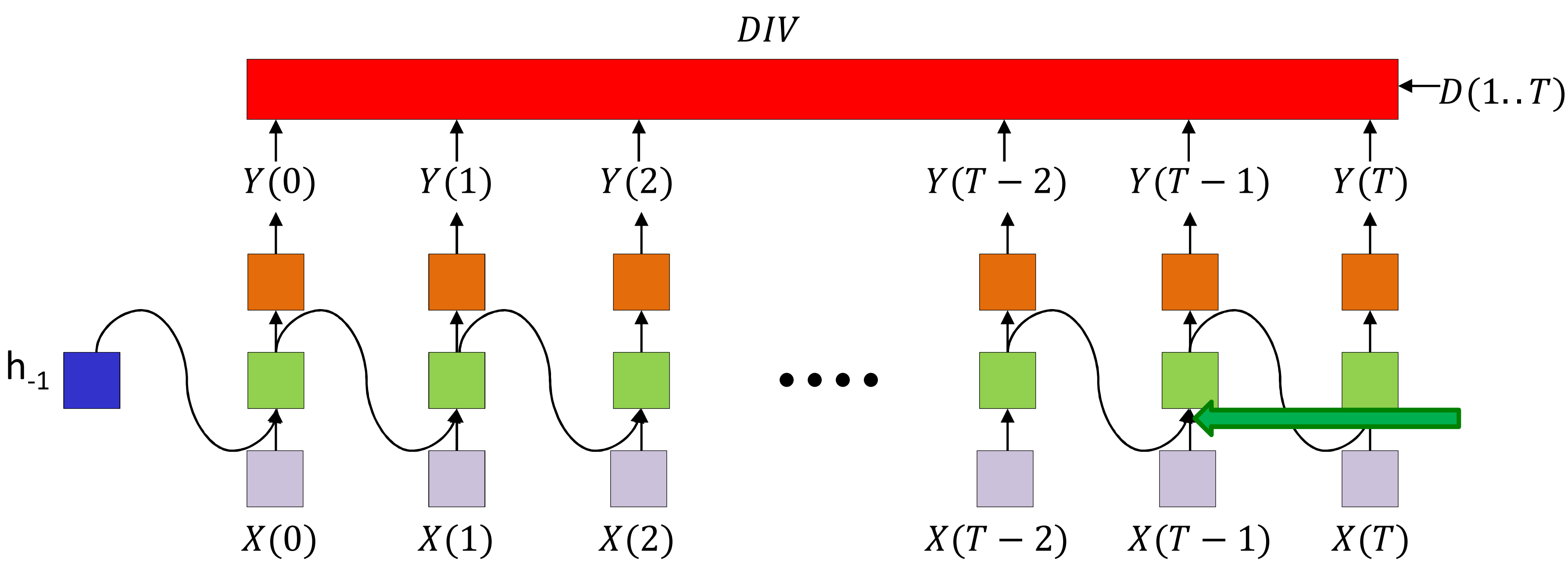
$$\frac{dDIV}{dh_i(T-1)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T-1)} + \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(0)}(T)}$$

Note the addition

$$\frac{dDIV}{dw_{ij}^{(1)}} += \frac{dDIV}{dZ_j^{(1)}(T-1)} h_i(T-1)$$

$$\nabla_{W^{(1)}} DIV += h(T-1) \nabla_{Z^{(1)}(T-1)} DIV$$

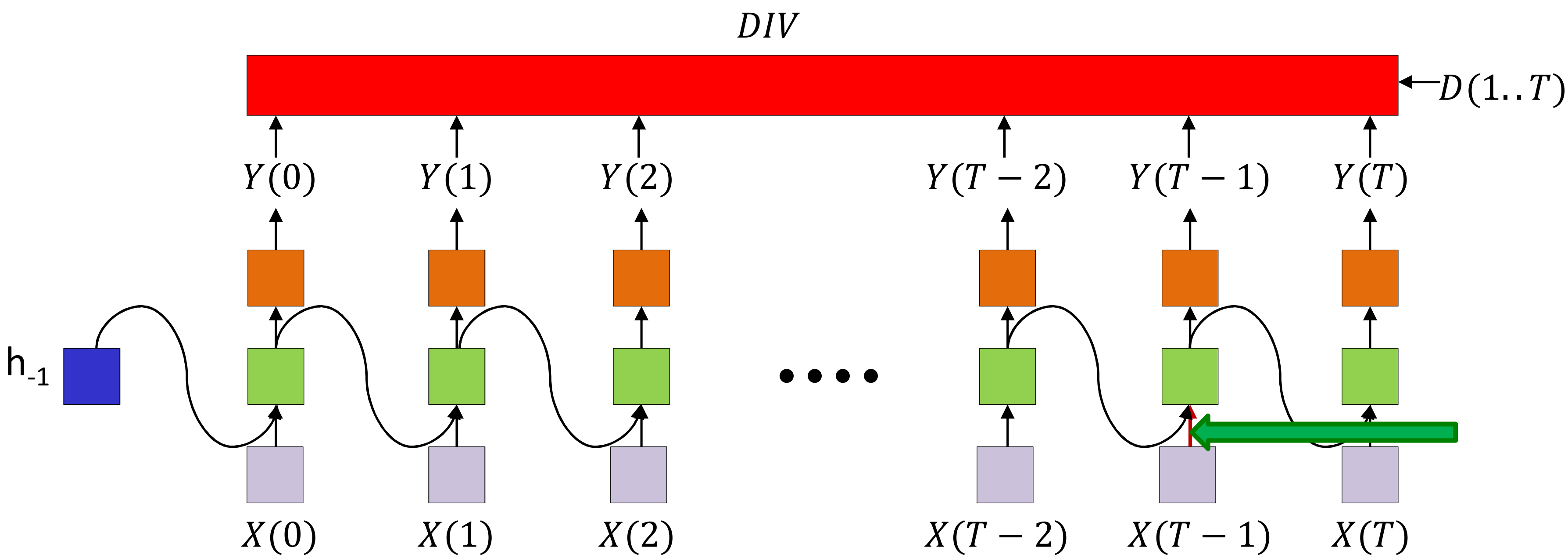
Back Propagation Through Time



$$\frac{dDIV}{dZ_i^{(0)}(T-1)} = \frac{dDIV}{dh_i(T-1)} \frac{dh_i(T-1)}{dZ_i^{(0)}(T-1)}$$

$$\nabla_{Z^{(0)}(T-1)} DIV = \nabla_{h(T-1)} DIV \; \nabla_{Z^{(0)}(T-1)} h(T-1)$$

Back Propagation Through Time



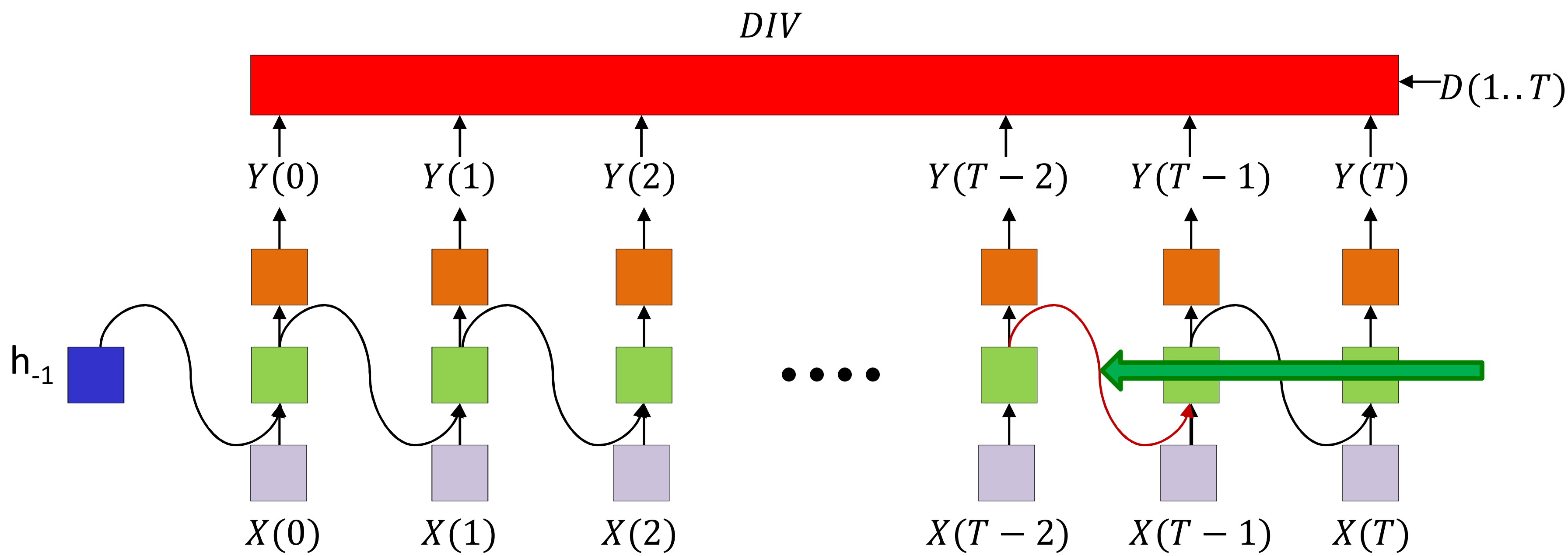
$$\frac{dDIV}{dZ_i^{(0)}(T-1)} = \frac{dDIV}{dh_i(T-1)} \frac{dh_i(T-1)}{dZ_i^{(0)}(T-1)}$$

$$\frac{dDIV}{dw_{ij}^{(0)}} += \frac{dDIV}{dZ_j^{(0)}(T-1)} X_i(T-1)$$

Note the addition

$$\nabla_{W^{(0)}} DIV += X(T-1) \nabla_{Z^{(0)}(T-1)} DIV$$

Back Propagation Through Time



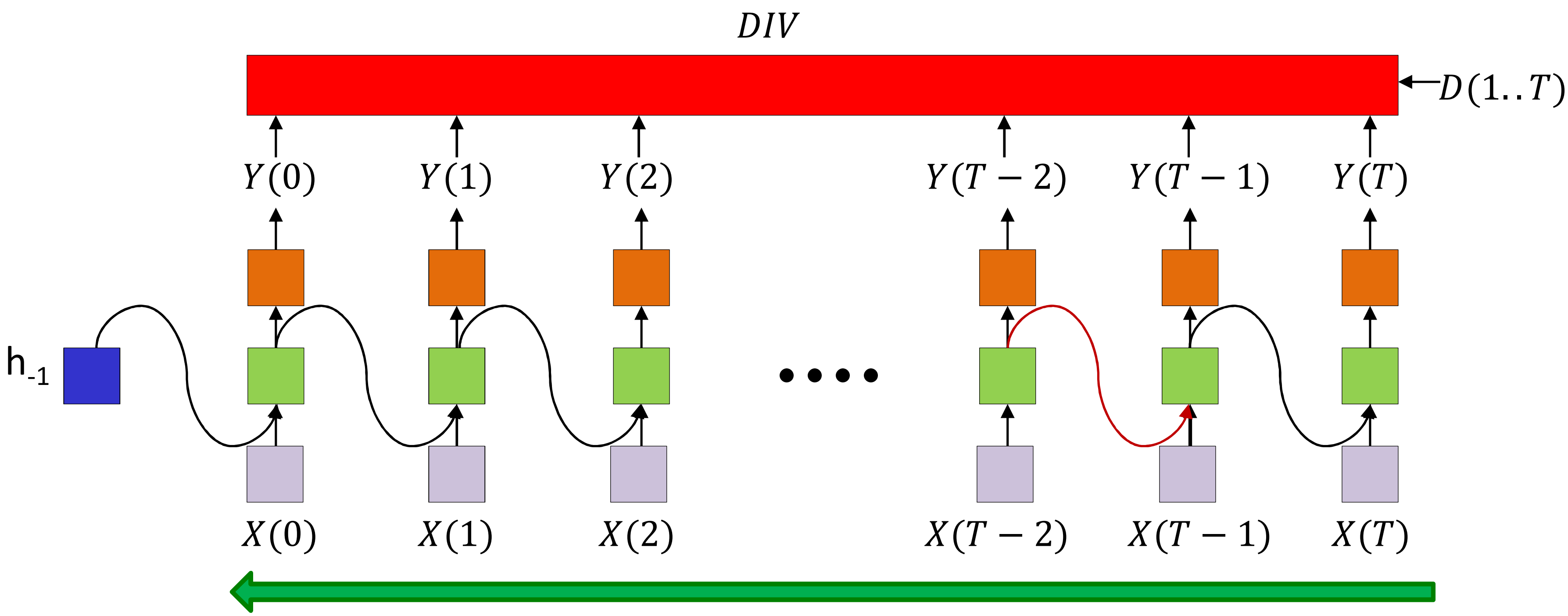
$$\frac{dDIV}{dw_{ij}^{(0)}} += \frac{dDIV}{dZ_j^{(0)}(T-1)} X_i(T-1)$$

Note the addition

$$\frac{dDIV}{dw_{ij}^{(11)}} += \frac{dDIV}{dZ_j^{(0)}(T-1)} h_i(T-2)$$

$$\nabla_{W^{(11)}} DIV += h(T-2) \nabla_{Z^{(0)}(T-1)} DIV$$

Back Propagation Through Time

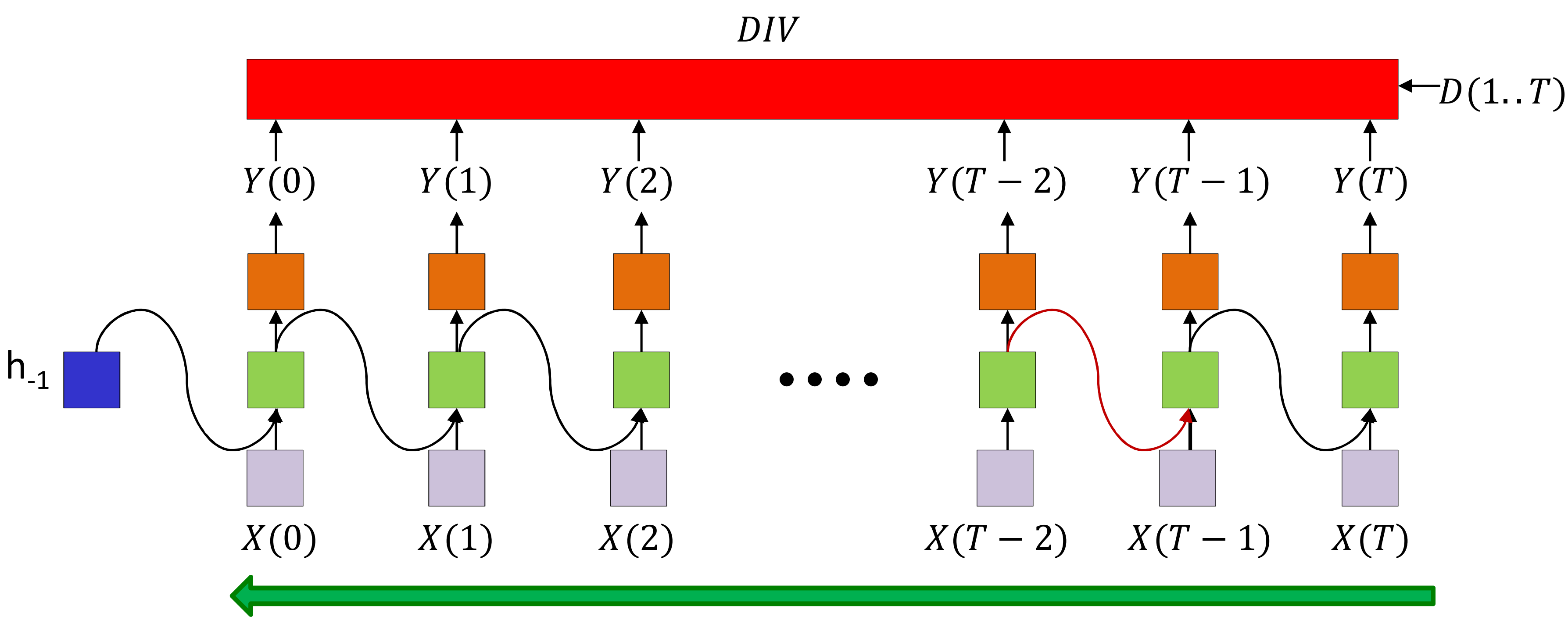


Continue computing derivatives going backward through time until..

$$\frac{dDIV}{dh_{-1}} = \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(1)}(0)}$$

$$\nabla_{h_{-1}} DIV = \nabla_{Z^{(1)}(0)} DIV\, W^{(11)}$$

Back Propagation Through Time

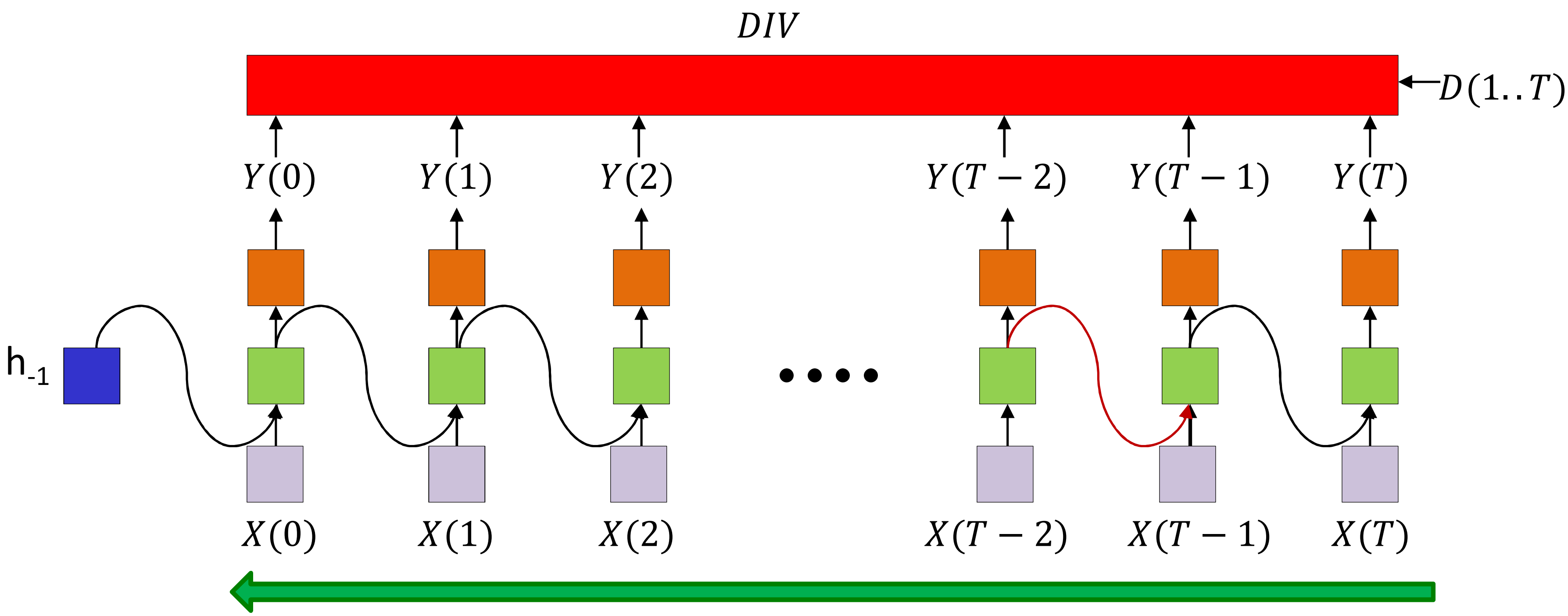


$$\frac{dDIV}{dh_i^{(k)}(t)} = \sum_j w_{i,j}^{(k)} \frac{dDIV}{dZ_j^{(k+1)}(t)} + \sum_j w_{i,j}^{(k,k)} \frac{dDIV}{dZ_j^{(k)}(t+1)}$$

Not showing derivatives at output neurons

$$\frac{dDIV}{dZ_i^{(k)}(t)} = \frac{dDIV}{dh_i^{(k)}(t)} f_k'\left(Z_i^{(k)}(t)\right)$$

Back Propagation Through Time



$$\frac{dDIV}{dh_{-1}} = \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(1)}(0)}$$

$$\frac{dDIV}{dw_{ij}^{(0)}} = \sum_t \frac{dDIV}{dZ_j^{(0)}(t)} X_i(t)$$

$$\frac{dDIV}{dw_{ij}^{(11)}} = \sum_t \frac{dDIV}{dZ_j^{(0)}(t)} h_i(t-1)$$

BPTT

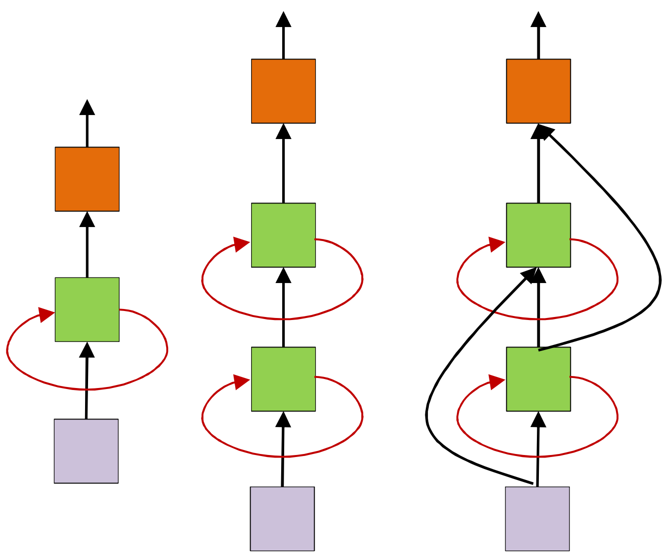
```
# Assuming forward pass has been completed
# Jacobian(x,y) is the jacobian of x w.r.t. y
# Assuming dY(t) = gradient(div,Y(t)) available for all t
# Assuming all dz, dh, dW and db are initialized to 0

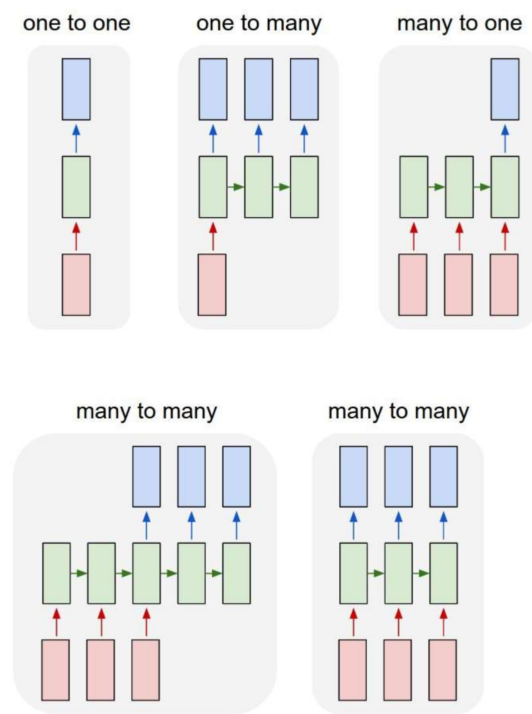
for t = T-1:downto:0  # Backward through time
    dz_o(t) = dY(t)Jacobian(Y(t),z_o(t))
    dW_o += h(t,L)dz_o(t)
    db(L) += dz_o(t)
    dh(t,L) += dz_o(t)W_o

    for l = L:1  # Reverse through layers
        dz(t,l) = dh(t,l)Jacobian(h(t,l),z(t,l))
        dh(t,l-1) += dz(t,l) W_c(l)
        dh(t-1,l) += dz(t,l) W_r(l)

        dW_c(l) += h(t,l-1)dz(t,l)
        dW_r(l) += h(t-1,l)dz(t,l)
        db(l) += dz(t,l)
```

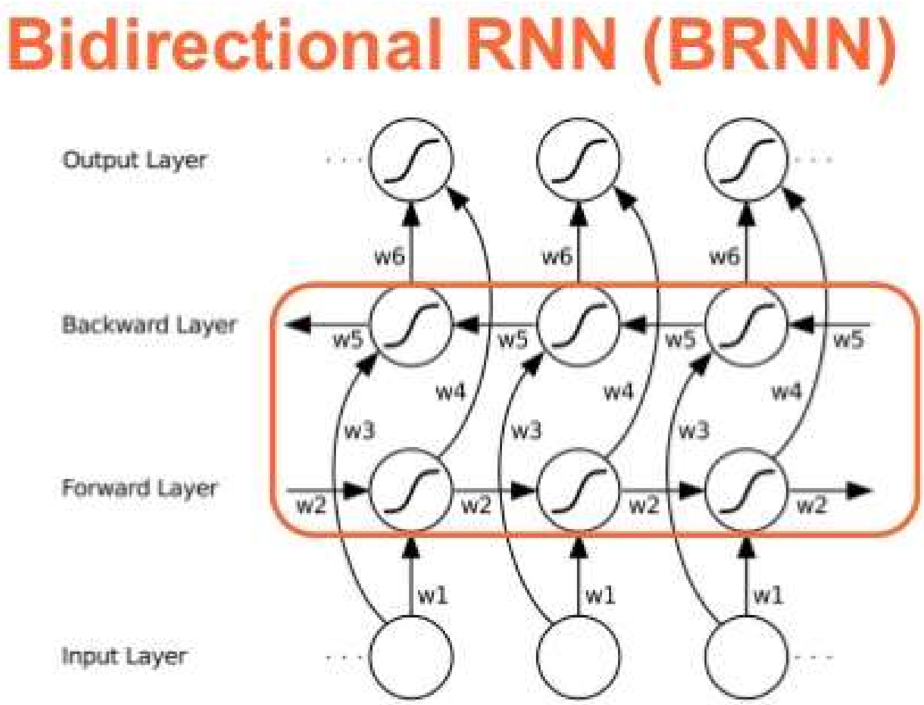
BPTT





- Can be generalized to any architecture

Extensions to the RNN: *Bidirectional RNN*



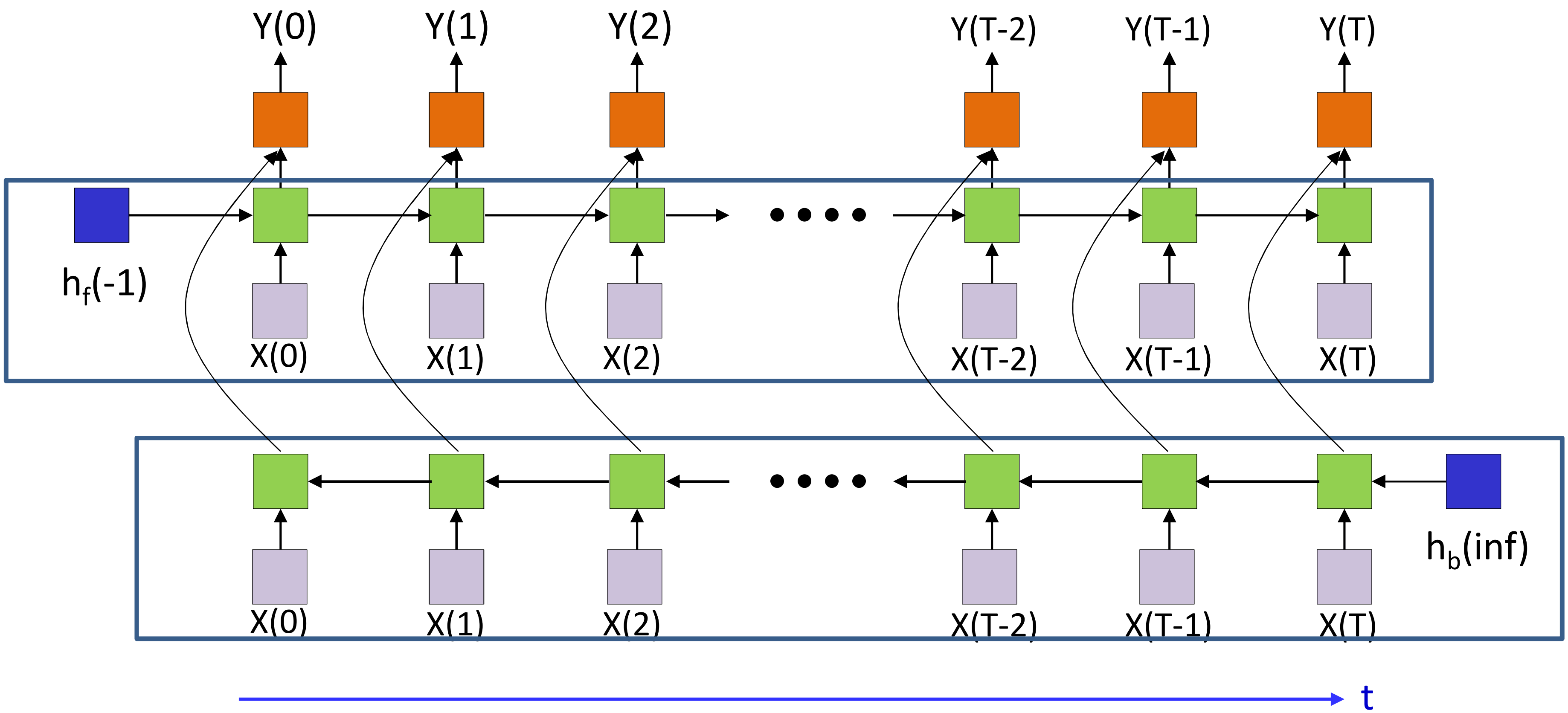
Must learn weights w2, w3, w4 & w5; in addition to w1 & w6.

Proposed by Schuster and Paliwal 1997

Alex Graves, "Supervised Sequence Labelling with Recurrent Neural Networks"

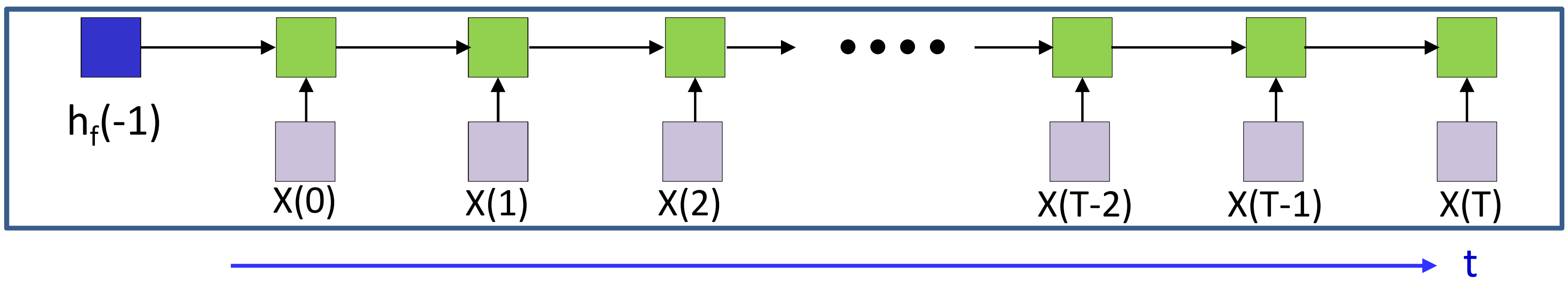
- RNN with both forward and backward recursion
  - Explicitly models the fact that just as the future can be predicted from the past, the past can be deduced from the future

Bidirectional RNN



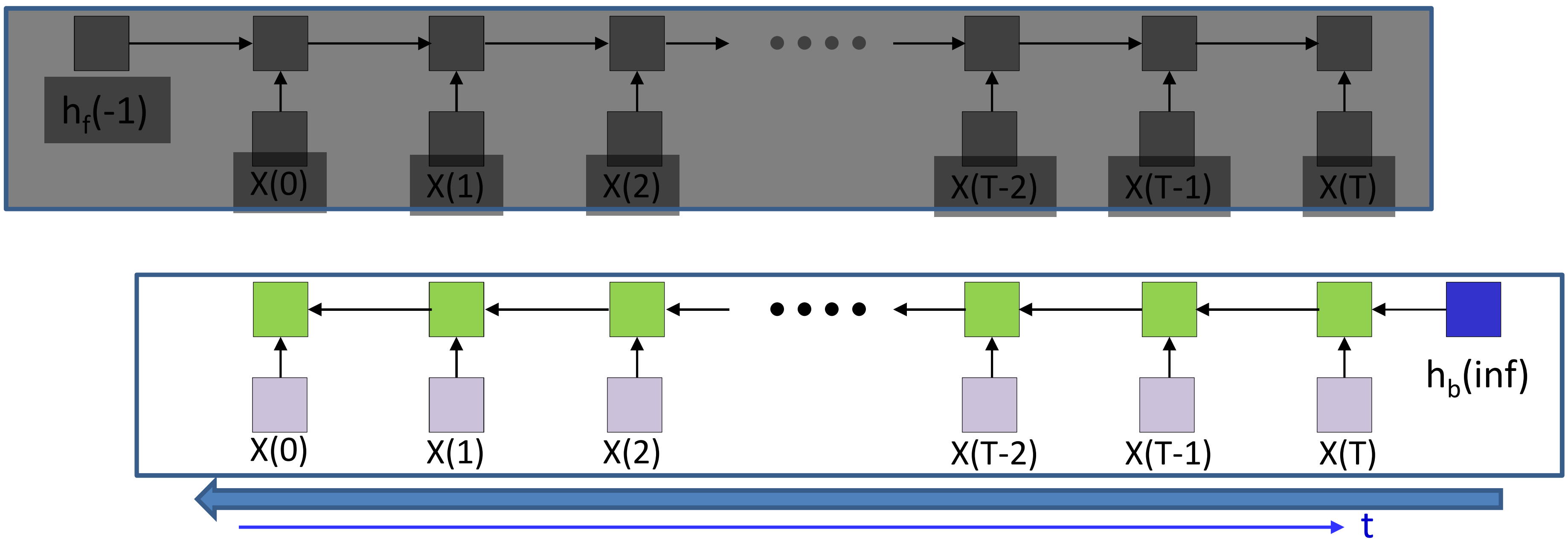
- A forward net process the data from t=0 to t=T
- A backward net processes it backward from t=T down to t=0

Bidirectional RNN: Processing an input string



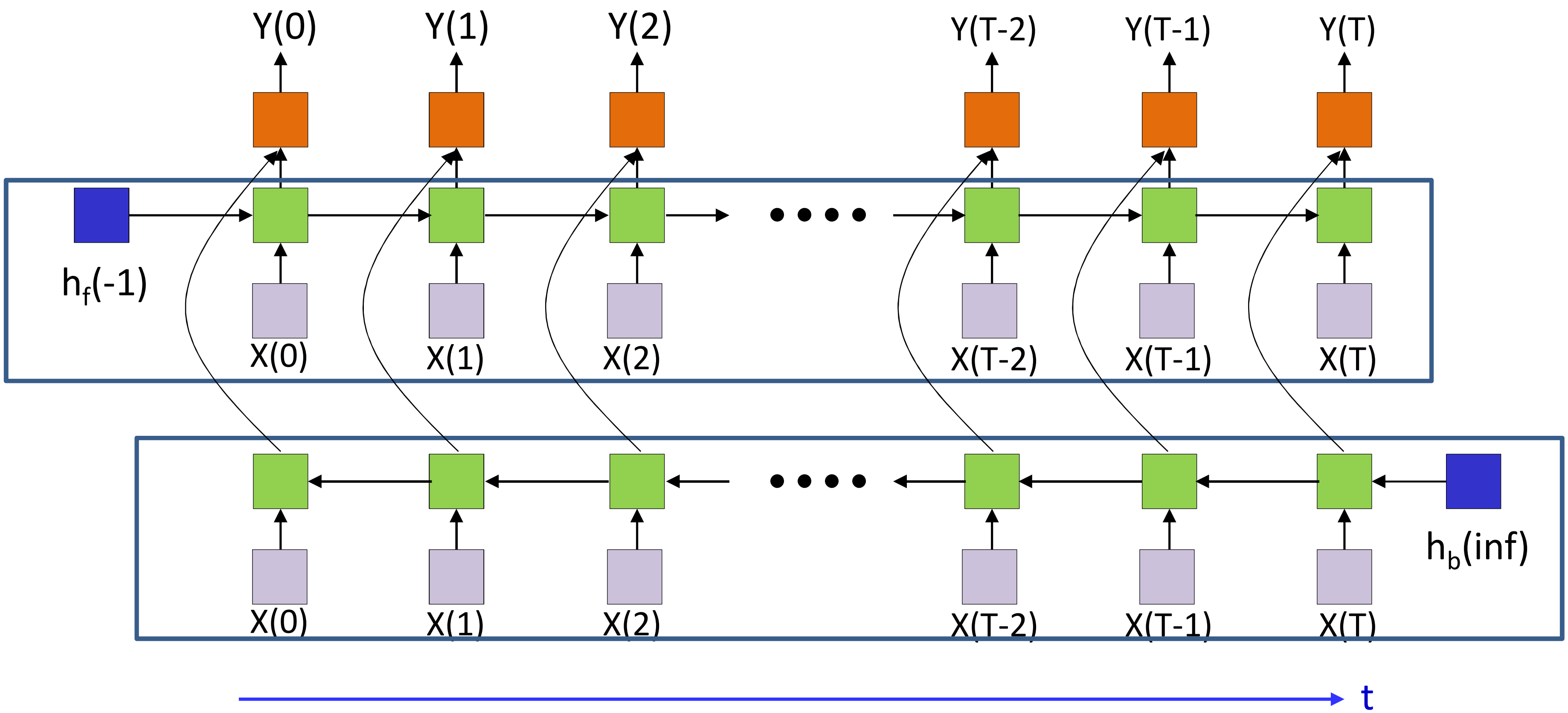
- The forward net process the data from t=0 to t=T
  - Only computing the hidden states, initially

Bidirectional RNN: Processing an input string



- The backward nets processes the input data in *reverse time,* end to beginning
  - Initially only the hidden state values are computed
    - Clearly, this is not an online process and requires the *entire* input data
  - Note: *This is not the backward pass of backprop.*

Bidirectional RNN: Processing an input string



- The computed states of both networks are used to compute the final output at each time

Bidirectional RNN
Assuming time-synchronous output

```
# Subscript f represents forward net, b is backward net
# Assuming hf(-1,*) and hb(inf,*) are known

#forward pass
for t = 0:T-1 # Going forward in time
    hf(t,0) = x(t) # Vectors. Initialize h(0) to input
    for l = 1:Lf  # Lf is depth of forward network hidden layers
        zf(t,l) = Wfc(l)hf(t,l-1) + Wfr(l)hf(t-1,l) + bf(l)
        hf(t,l) = tanh(zf(t,l)) # Assuming tanh activ.

#backward
h(T,:,:) = h(inf,:,:) # Just the initial value
for t = T-1:downto:0 # Going backward in time
    hb(t,0) = x(t) # Vectors. Initialize h(0) to input
    for l = 1:Lb # Lb is depth of backward network hidden layers
        zb(t,l) = Wbc(l)hb(t,l-1) + Wbr(l)h(t+1,l) + bb(l)
        hb(t,l) = tanh(zb(t,l)) # Assuming tanh activ.

for t = 0:T-1  # The output combines forward and backward
     zo(t) = Wfohf(t,Lf) +  Wbohb(t,Lb) + bo
    Y(t) = softmax( zo(t) )
```

Bidirectional RNN: Simplified code

- Code can be made modular and simplified for better interpretability…

First: Define basic RNN with only hidden units

```
# Inputs:
#    L : Number of hidden layers
#    Wc,Wr,b: current weights,  recurrent weights, biases
#    hinit:  initial value of h(representing h(-1,*))
#    x: input vector sequence
#    T: Length of input vector sequence
# Output:
#    h, z: sequence of pre-and post activation hidden
#          representations from all layers of the RNN

function [h,z] = RNN_forward(L, Wc, Wr, b, hinit, x, T)
    h(-1,:) = hinit  # hinit is the initial value for all layers
    for t = 0:T-1 # Going forward in time
        h(t,0) = x(t) # Vectors. Initialize h(0) to input
        for l = 1:L
            z(t,l) = Wc(l)h(t,l-1) + Wr(l)h(t-1,l) + b(l)
            h(t,l) = tanh(z(t,l)) # Assuming tanh activ.
    return h,z
```

Bidirectional RNN
Assuming time-synchronous output

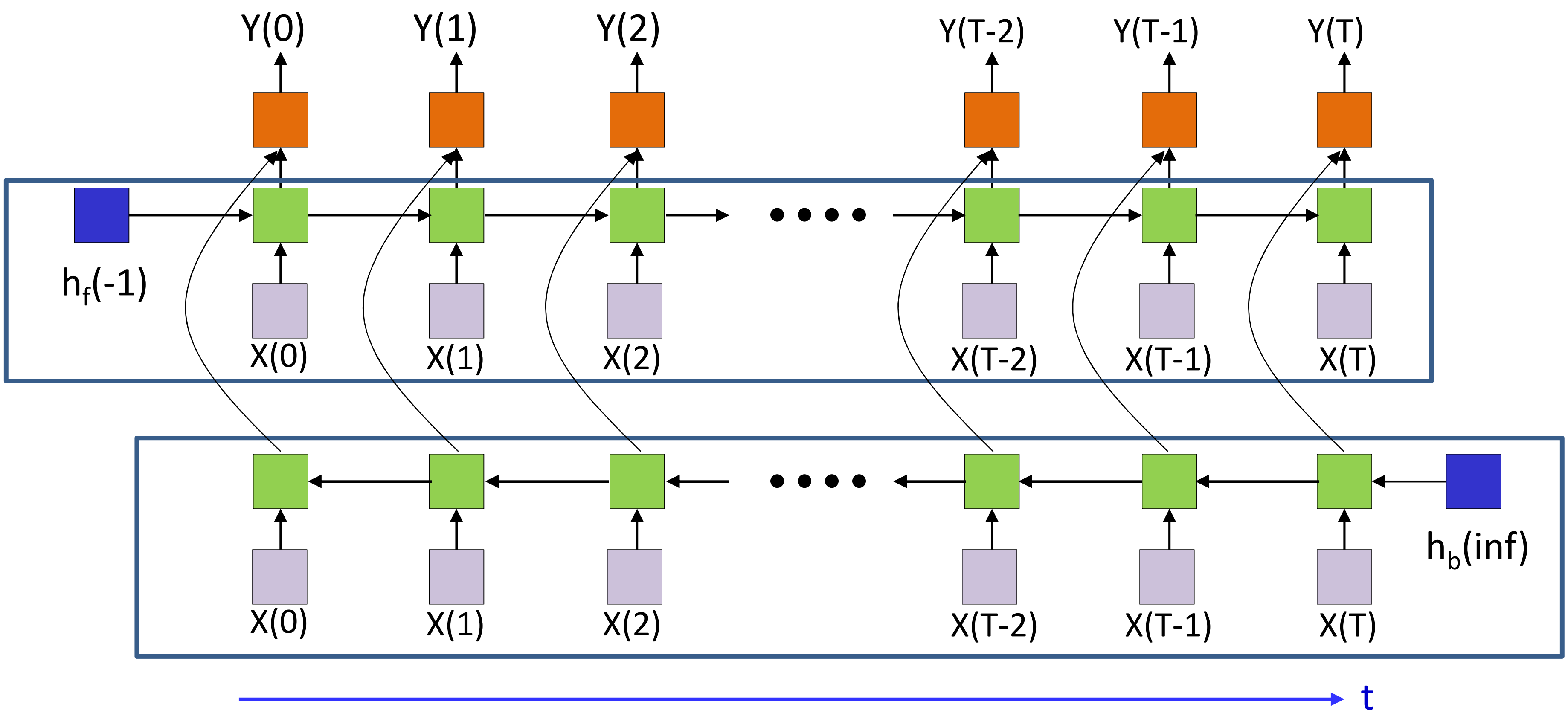
```
# Subscript f represents forward net, b is backward net
# Assuming h_f(-1,*) and h_b(inf,*) are known

#forward pass
[h_f, z_f] = RNN_forward(L_f, W_fc, W_fr, b_f, h(-1,:), x, T)

#backward pass
x_rev = fliplr(x)  # Flip it in time
[h_brev, z_brev] = RNN_forward(L_b, W_bc, W_br, b_b, h(inf,:), x_rev, T)
h_b = fliplr(h_brev)  # Flip back to straighten time
z_b = fliplr(z_brev)

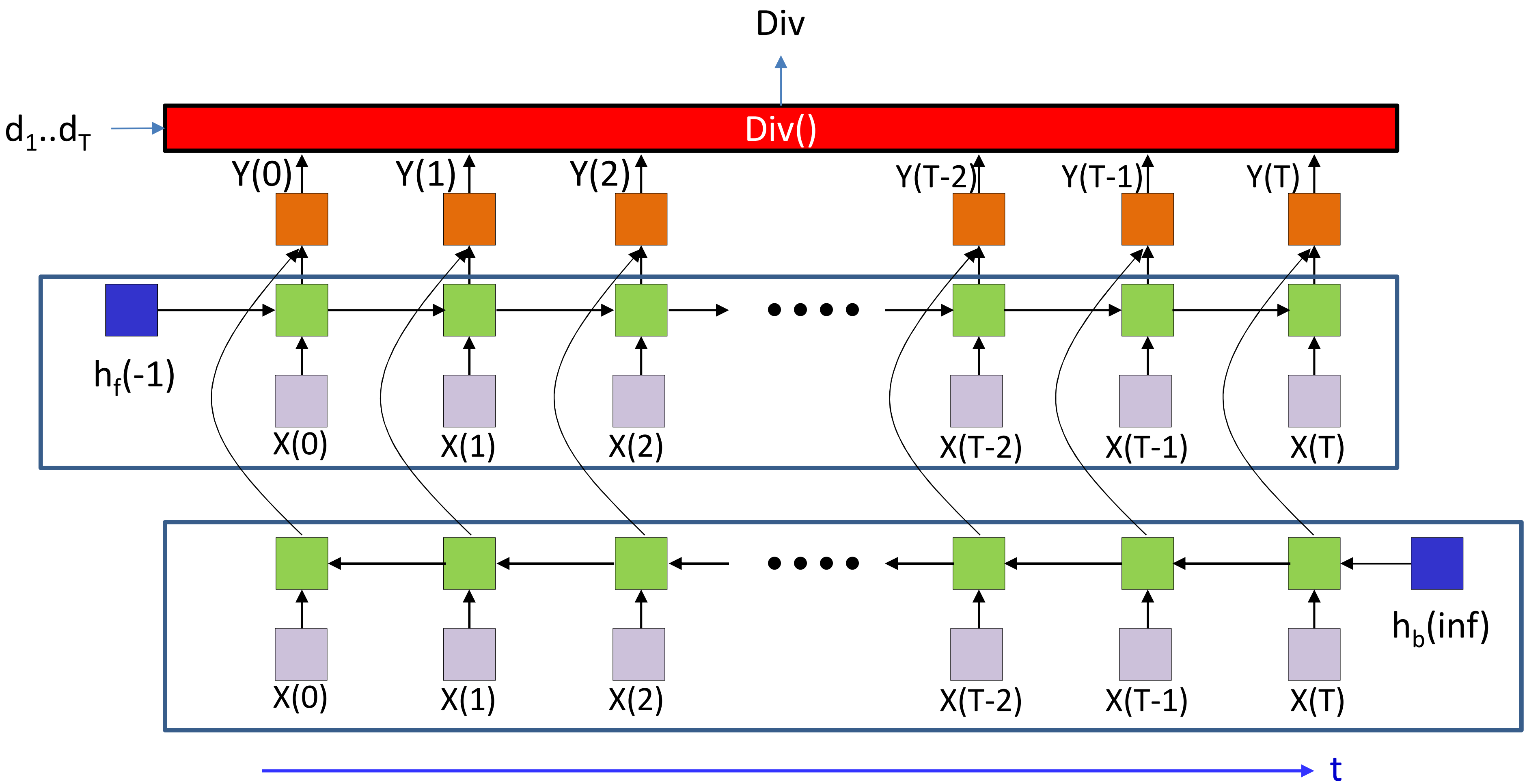
#combine the two for the output
for t = 0:T-1  # The output combines forward and backward
     z_o(t) = W_fo h_f(t,L_f) +  W_bo h_b(t,L_b) + b_o
     Y(t) = softmax( z_o(t) )
```

Backpropagation in BRNNs



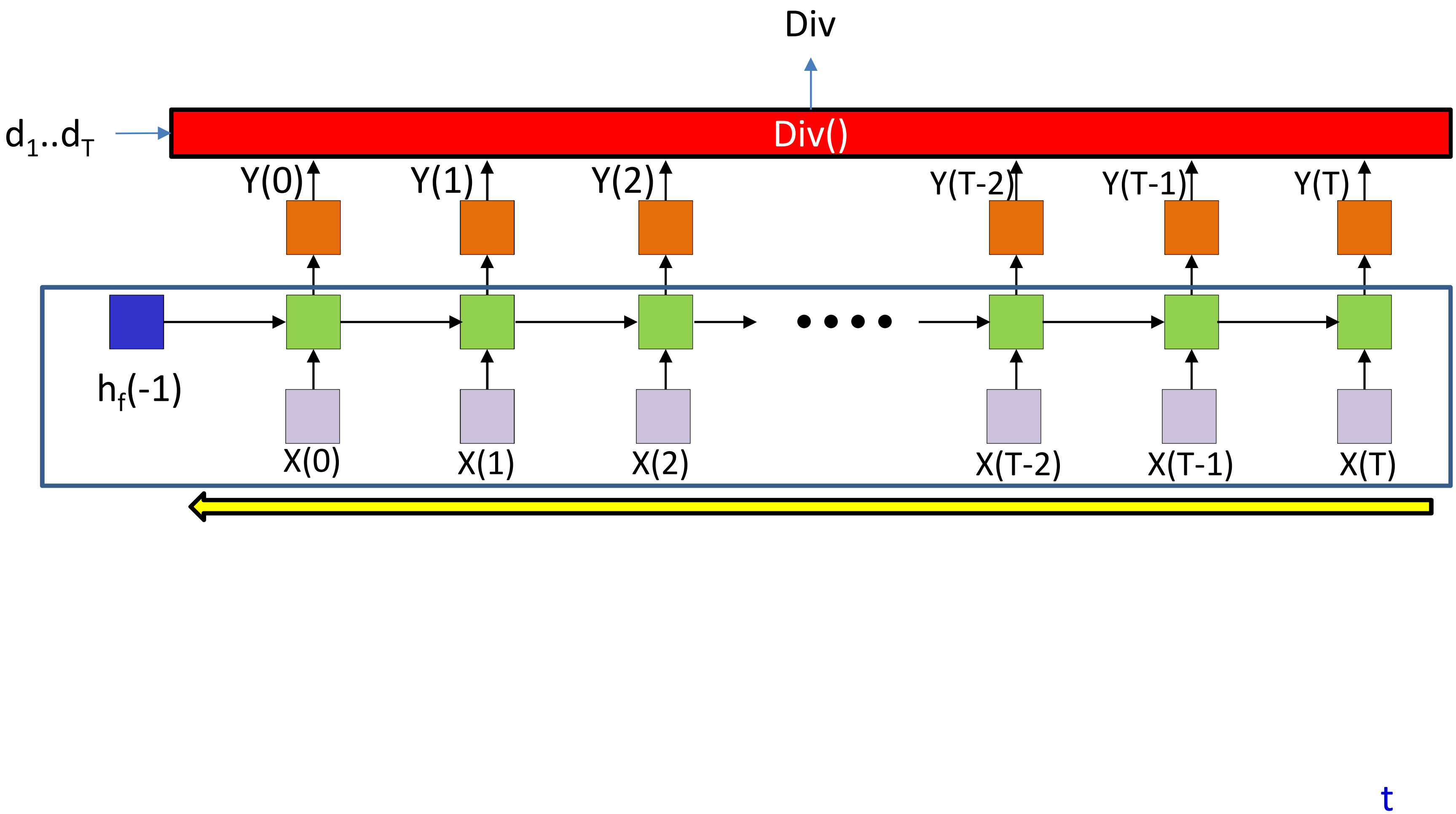
- Forward pass: Compute both forward and backward networks and final output

Backpropagation in BRNNs



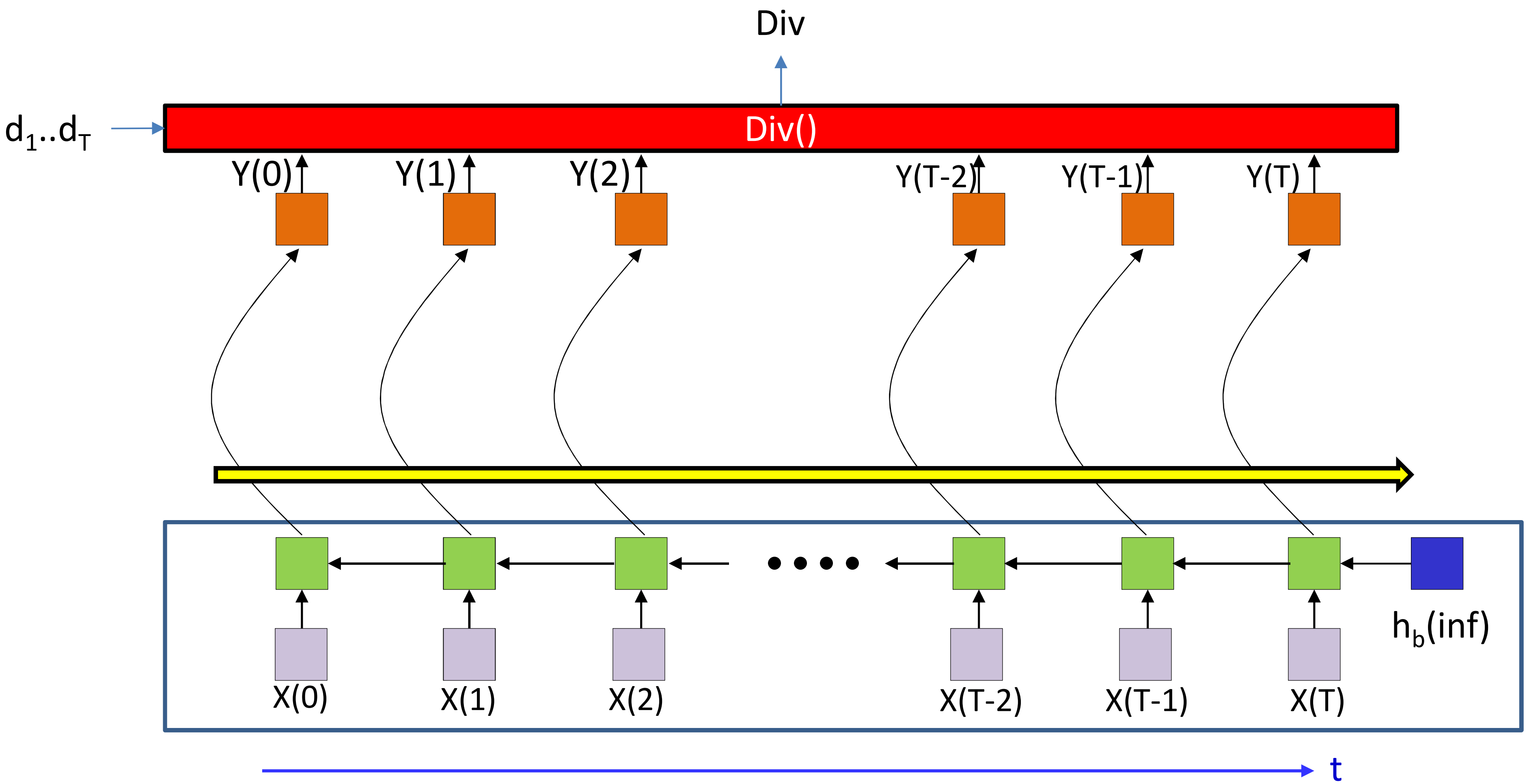
- Backward pass: Define a divergence from the desired output

Backpropagation in BRNNs



- Backward pass: Define a divergence from the desired output
- Separately perform back propagation on both nets
  - **From t=T down to t=0 for the forward net**

Backpropagation in BRNNs



- Backward pass: Define a divergence from the desired output
- Separately perform back propagation on both nets
  - From t=T down to t=0 for the forward net
  - **From t=0 up to t=T for the backward net**

Backpropagation: Pseudocode

- As before we will use a 2-step code:
  - A basic backprop routine that we will call
  - Two calls to the routine within a higher-level wrapper

First: backprop through a recurrent net

```
# Inputs:
#    (In addition to inputs used by L : Number of hidden layers
#    dh_top:  derivatives ddiv/dh_*(t,L) at each time (* may be f or b)
#    h, z:  h and z values returned by the forward pass
#    T: Length of input vector sequence
# Output:
#    dW_c, dW_b, db dh_init: derivatives w.r.t current and recurrent weights,
#                            biases, and initial h.
# Assuming all dz, dh, dW_c, dW_r and db are initialized to 0

function [dW_c,dW_r,db,dh_init] = RNN_bptt(L, W_c, W_r, b, hinit, x, T, dh_top, h, z)
    dh = zeros
    for t = T-1:downto:0  # Backward through time
        dh(t,L) += dh_top(t)
        for l = L:1  # Reverse through layers
            dz(t,l) = dh(t,l)Jacobian(h(t,l),z(t,l))
            dh(t,l-1) += dz(t,l) W_c(l)
            dh(t-1,l) += dz(t,l) W_r(l)

            dW_c(l) += h(t,l-1)dz(t,l)
            dW_r(l) += h(t-1,l)dz(t,l)
            db(l) += dz(t,l)

    return dWc, dWr, db, dh(-1)  # dh(-1) is actually dh(-1,1:L,:)
```

Bi-RNN gradient computatoin
Assuming time-synchronous output

```
# Subscript f represents forward net, b is backward net
# First compute derivatives that directly relate to dY(t) for all t,
# then pass the derivatives into RNN_bptt to compute forward and backward
# parameter derivatives

for t = 0:T-1  # The output combines forward and backward
    dz_o(t) = dY(t)Jacobian(Y(t),z_o(t))
    dh_fo(t) = dz_o(t)W_fo
    dh_bo(t) = dz_o(t)W_bo
    db_o  += dz_o(t)
    dW_fo += h_f(t,L)dz_o(t)
    dW_bo += h_b(t,L)dz_o(t)

#forward net
[dW_fc,dW_fr,db_f,dh_f(-1)] = RNN_bptt(L, W_fc, W_fr, b_f, h_f(-1), x, T, dh_fo, h_f, z_f)
#backward net
x_rev = fliplr(x)  # Flip it in time
[dW_bc,dW_br,db_b,dh_b(inf)] = RNN_bptt(L, W_bc, W_br, b_b, h_b(inf), x_rev, T, dh_bo, h_b, z_b)
```

Story so far

- Time series analysis must consider past inputs along with current input
- Recurrent networks look into the infinite past through a state-space framework
  - Hidden states that recurse on themselves
- Training recurrent networks requires
  - Defining a divergence between the actual and desired output *sequences*
  - Backpropagating gradients over the entire chain of recursion
    - Backpropagation through time
  - Pooling gradients with respect to individual parameters over time
- Bidirectional networks analyze data both ways, begin→end and end→beginning to make predictions
  - In these networks, backprop must follow the chain of recursion (and gradient pooling) separately in the forward and reverse nets

RNNs..

- Excellent models for time-series analysis tasks
  - Time-series prediction
  - Time-series classification
  - Sequence prediction..

So how did this happen

```
Naturalism and decision for the majority of Arab countries' capitalide was grounded
by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated
with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal
in the [[Protestant Immineners]], which could be said to be directly in Cantonese
Communication, which followed a ceremony and set inspired prison, training. The
emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom
of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known
in western [[Scotland]], near Italy to the conquest of India with the conflict.
Copyright was the succession of independence in the slop of Syrian influence that
was a famous German movement based on a more popular servicious, non-doctrinal
and sexual power post. Many governments recognize the military housing of the
[[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]],
that is sympathetic to be to the [[Punjab Resolution]]
(PJS)[http://www.humah.yahoo.com/guardian.
cfm/7754800786d17551963s89.htm Official economics Adjoint for the Nazism, Montgomery
was swear to advance to the resources for those Socialism's rule,
was starting to signing a major tripad of aid exile.]]
```

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```

More on this later..

RNNs..

- Excellent models for time-series analysis tasks
  - Time-series prediction
  - Time-series classification
  - Sequence prediction..
  - They can even simplify some problems that are difficult for MLPs
    - Next class..