Dynamic Mode Decomposition

JR

Dynamic Mode Decomposition (DMD) is a powerful data-driven approach to analyzing complex systems. DMD analyses the relationship between pairs of measurements from a dynamical system. The measurements, $\boldsymbol{x}_k$ and $\boldsymbol{x}_{k+1}$, where k indicates the temporal iteration. The dynamical system can be approximated by linearizing the equation. We also think of this a model reduction technique.

$$\boldsymbol{x}_{k+1} \approx \boldsymbol{A}\boldsymbol{x}_k$$

where $x \in R^n$ and $A \in R^{n\times n}$.

In order to approximate matrix A, we first collect data and transform this problem into a regression problem.

$$X = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix}$$

$$X' = \begin{bmatrix} | & | & & | \\ x_2 & x_3 & \cdots & x_m \\ | & | & & | \end{bmatrix}$$

Hence, we get

$$X' \approx AX$$

A can be computed by

$$A \approx X'X^{pseudo}$$

A is determined by minimizing the Frobenius norm of $|X' - AX|_F$.

In order to compute the efficiently, we try SVD decomposition.

$$X = U\Sigma V^* = [\tilde{U} \quad \tilde{U}_{rem}] \begin{bmatrix} \tilde{\Sigma} & 0 \\ 0 & \tilde{\Sigma}_{rem} \end{bmatrix} \begin{bmatrix} \tilde{V}^* \\ \tilde{V}^*_{rem} \end{bmatrix}$$

where $U \in R^{n\times n}$, $\Sigma \in R^{n\times m-1}$, $V^* \in R^{m-1\times m-1}$, $\tilde{U} \in R^{n\times r}$, $\tilde{\Sigma} \in R^{r\times r}$, $\tilde{V}^* \in R^{r\times m-1}$.

$$A \approx X'\tilde{V}^*\tilde{\Sigma}^{-1}\tilde{U}$$

Here, we will summarize the DMD computation procedure.

svd	$X = U\Sigma V^*, X' = \mathrm{A}U\Sigma V^*$
*	$U^*X'V\Sigma^{-1} = U^*\mathrm{A}U = \bar{A}$
eig	$\bar{A}W = W\Lambda$
*	$\Phi = X'V\Sigma^{-1}W$
*	$\hat{X}(k\Delta t) = \Phi\Lambda^{\mathrm{t}}b_0$

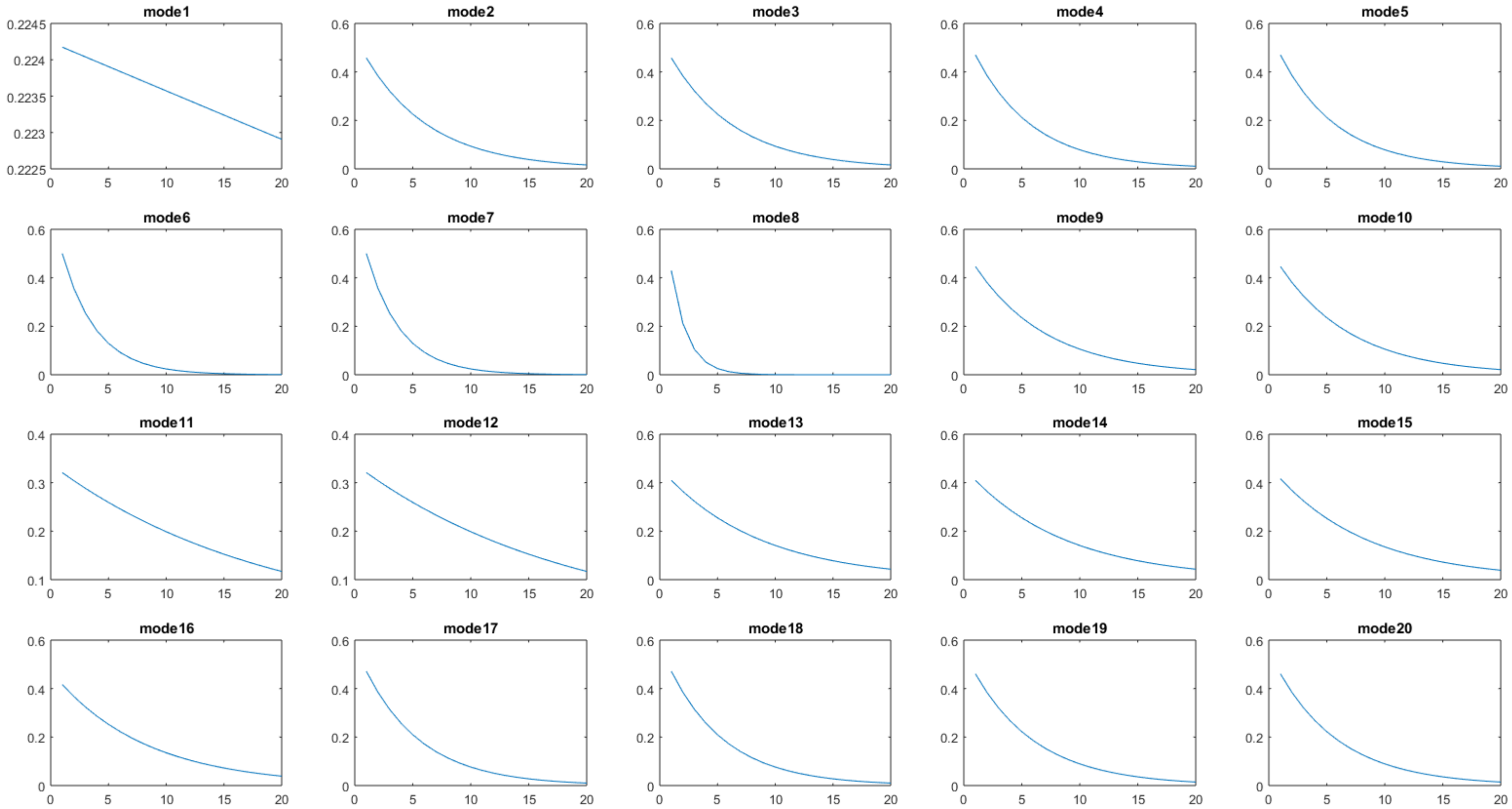
In this case, we will analyze the friction data. We will organize the data into matrices by sliding the window h. The size of h is defined by us.

Here, h = 30. We use write matlab scripts to analyze the limited data.

这里我们采用旋转形式里面的销盘数据进行分析。

1. 使用 matlab 导入数据，进行数据预处理得到.mat 数据；
2. 选取摩擦系数进行分析（DMD 的方法）；
3. 对比观察得到可能的结论。

常规塑料的数据结果



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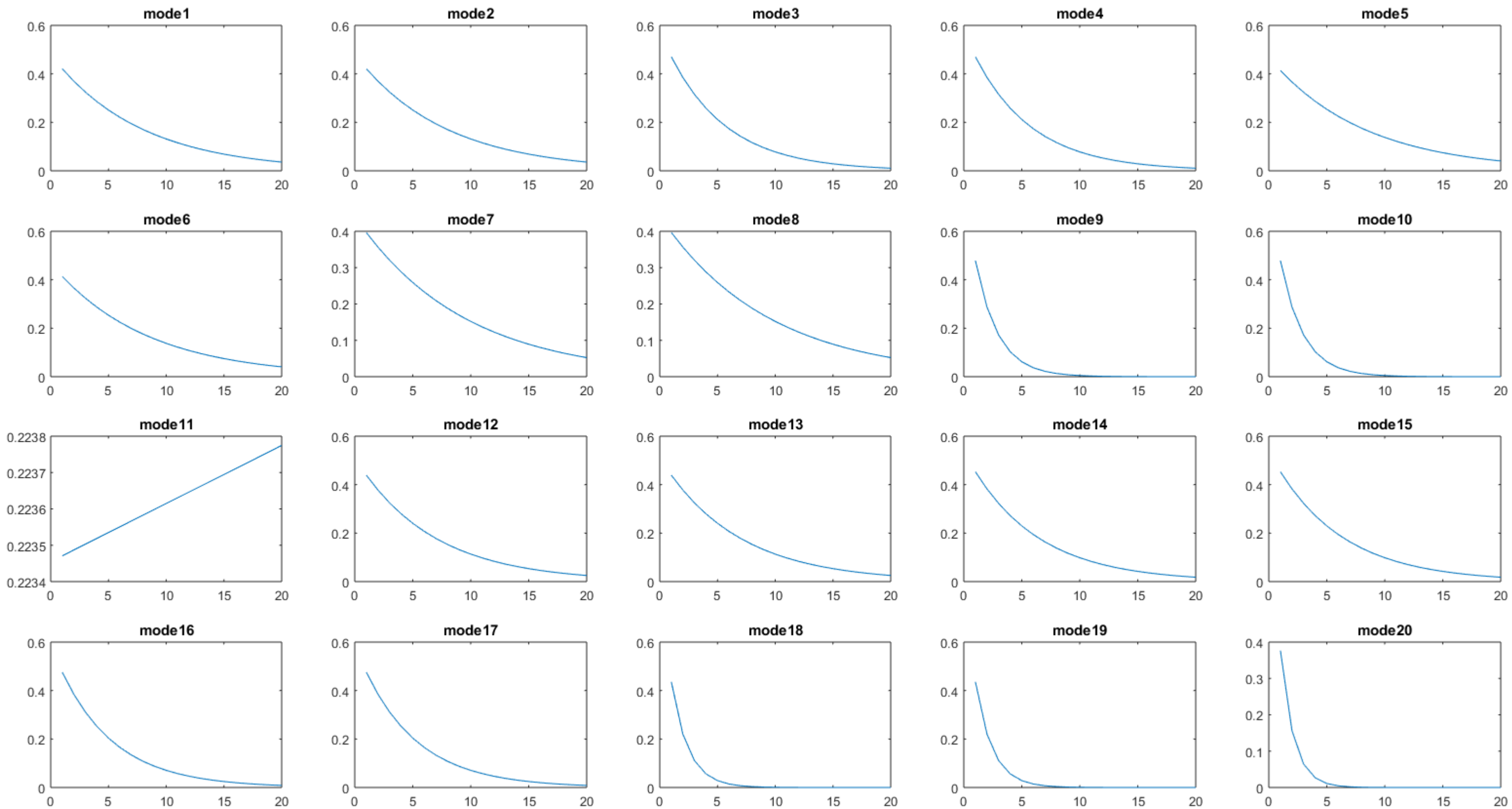
端面复合材料数据结果



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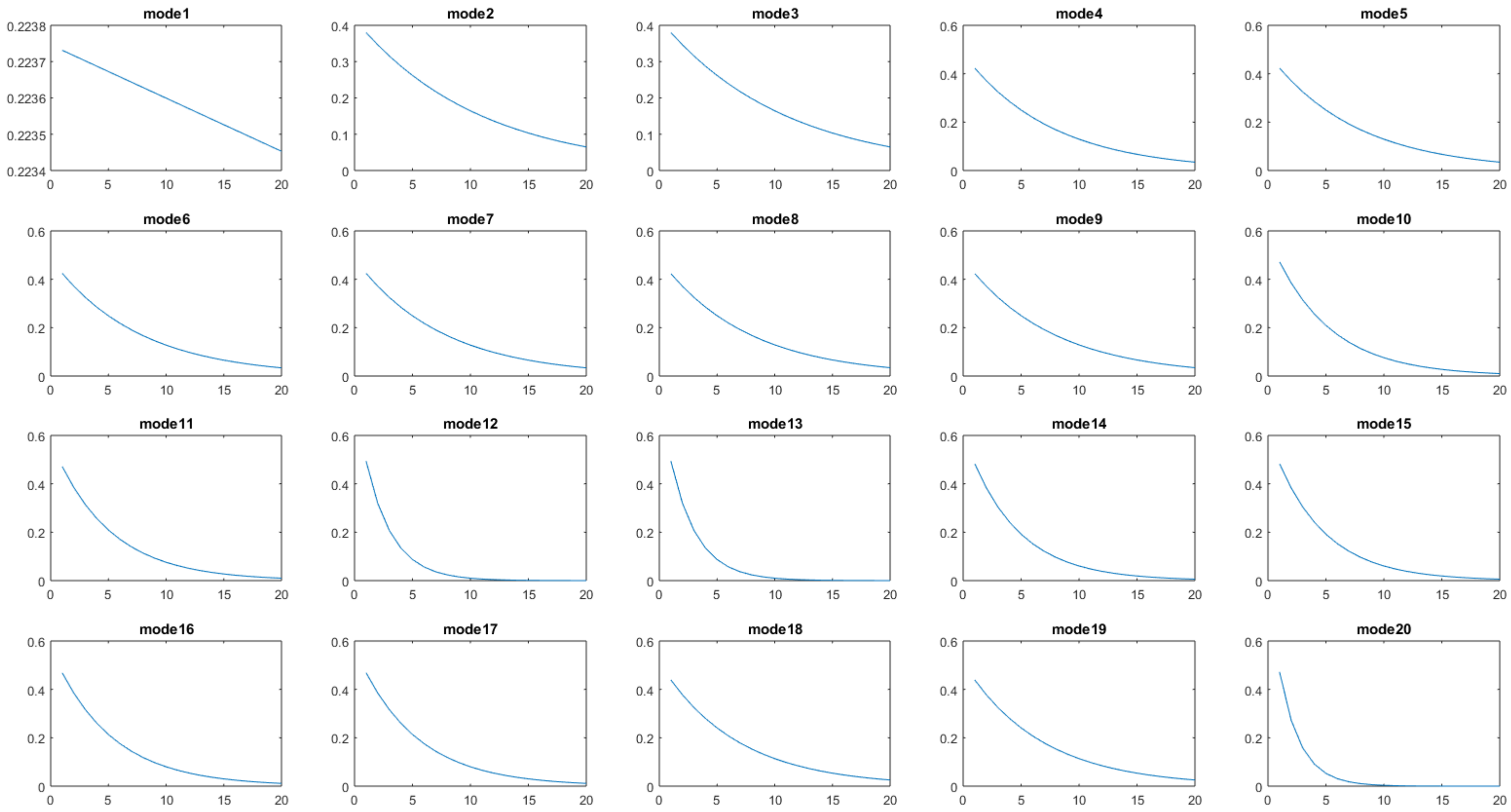
销盘载流 1 数据结果



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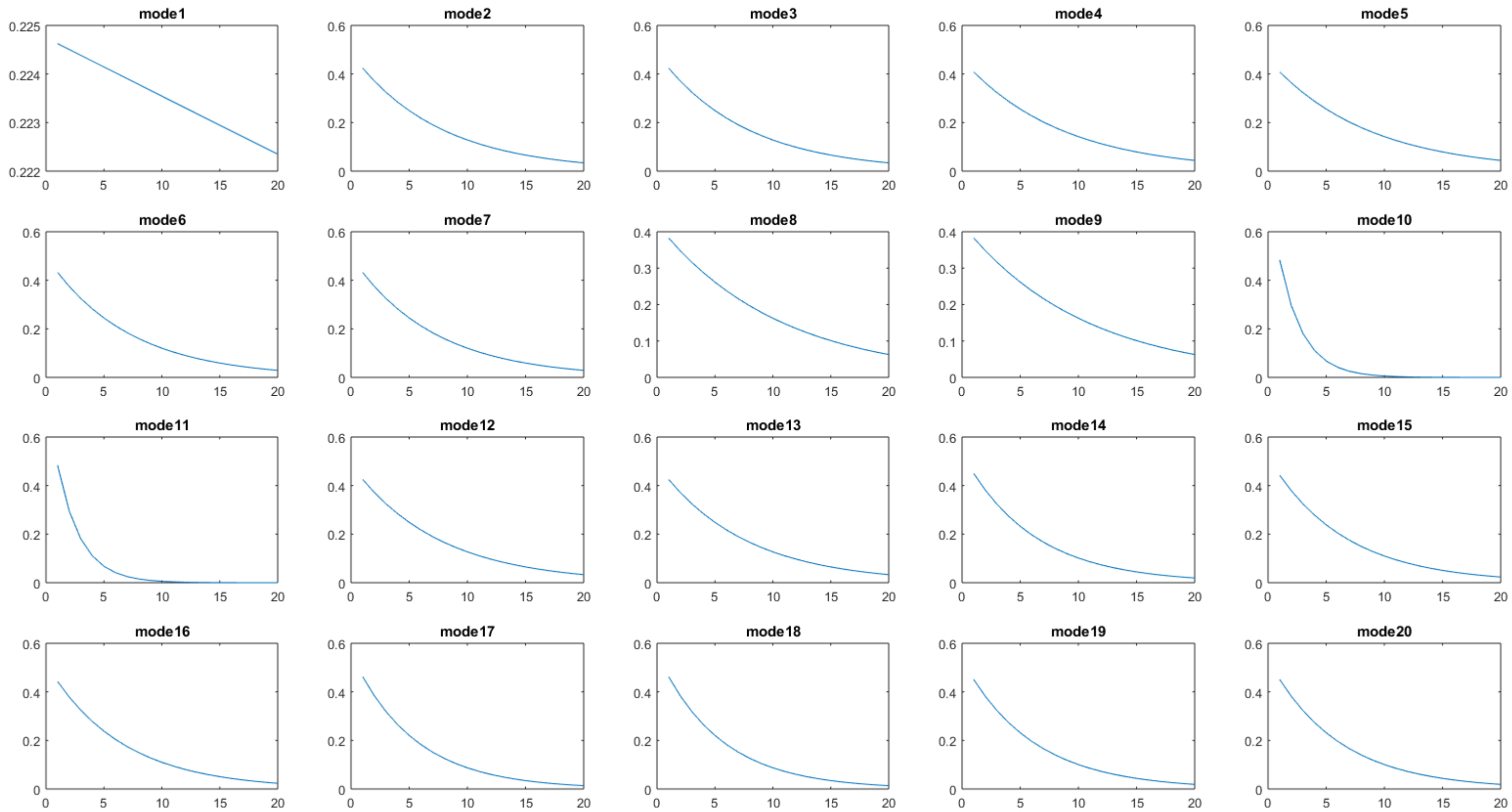
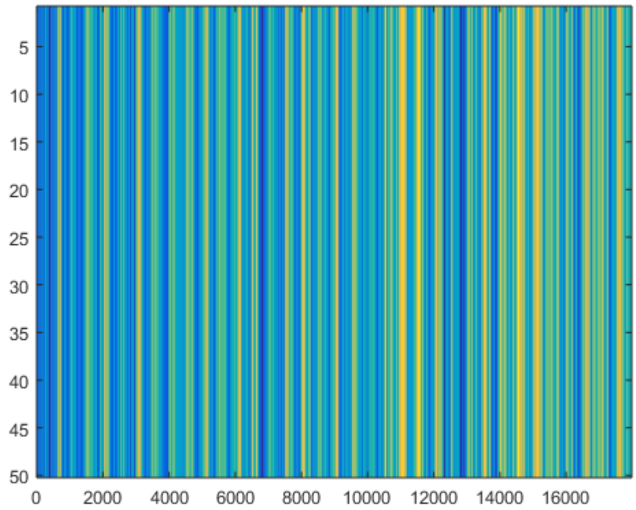
销盘载流 2 数据结果



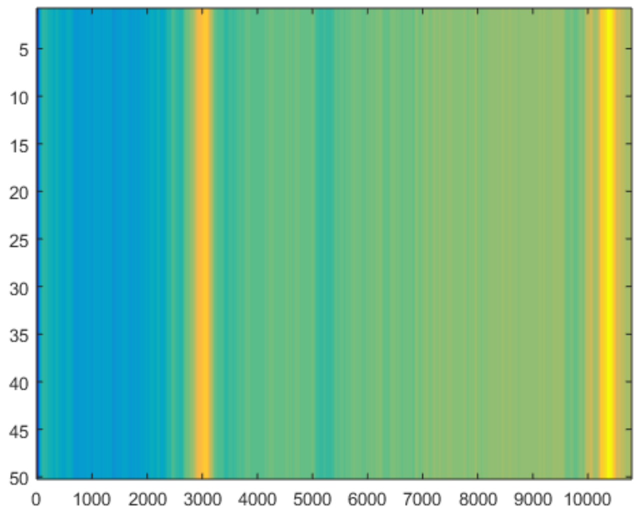
采用 DMD 进行分析，发现这几个实验下的摩擦系数的动态响应，分解得到的 mode 呈指数衰减的模式。并且各个实验下的数据有一定的相似模式。

采用短时傅里叶分析

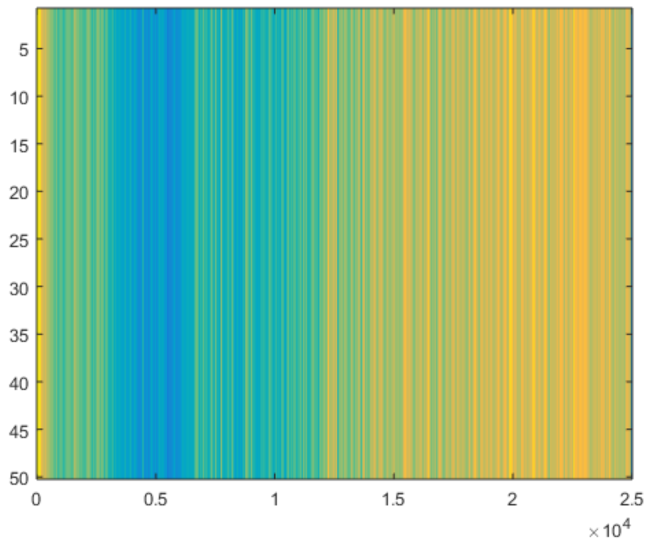
常规塑料



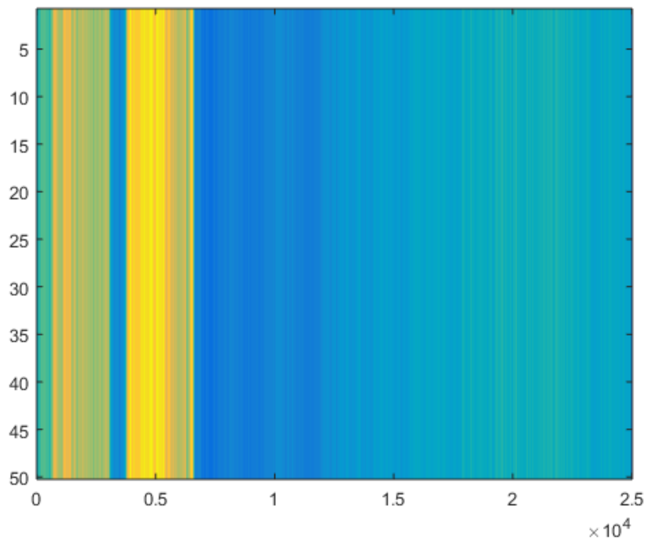
端面符合材料



销盘载流 1



销盘载流 2



短时傅里叶分析得到的数据，各个摩擦系数的数据之间的模式相似点很少。