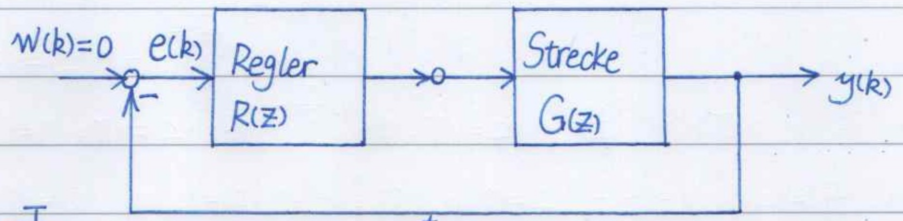
A2-1



$$e^{-\frac{T}{T_1}} = 0.5; \quad h(t) = 2(1 - e^{-\frac{t}{T_1}})1(t)$$

$$h(k) = h(kT) = 2(1 - e^{-k\frac{T}{T_1}})1(kT) = 2(1 - 0.5^k)1(kT)$$

(b): $g(k) = h(k) - h(k-1) = 2(1-0.5^k)1(k) - 2(1-0.5^{k-1})1(k-1)$

$$g(k) = \begin{cases} 0 \; ; k \leq 0 \\ 2(1-0.5^k)1(k) - 0 = 1 \; ; k=1 \\ 2(1-0.5^k)1(k) - 2(1-0.5^{k-1})1(k-1) \; ; k \geq 2 \Rightarrow (2\times 0.5^{k-1} - 2\times 0.5^k) \; k \geq 2 \end{cases}$$

$$2\times 0.5^{k-1} - 2\times 0.5^k = 0.5^{-1}\times 0.5^{k-1} - 0.5^{-1}\times 0.5^k = 0.5^{k-2} - 0.5^{k-1} = (0.5^{-1} - 1)0.5^{k-1} = 0.5^{k-1}$$

$$g(k) = \{0;\, 1;\, \tfrac{1}{2};\, \tfrac{1}{4};\, \dots\}, \quad \underline{\underline{(\tfrac{1}{2})^{k-1} 1(k) = g(k)}}$$

(c): $H(z) = \sum_{k=0}^{+\infty} h(k) z^{-k} = 2 \sum_{k=0}^{+\infty} (1 - (\frac{1}{2})^k) z^{-k})$

$$H(z) = 2\frac{1}{1-z^{-1}} - 2\frac{1}{1-\frac{1}{2}z^{-1}} = 2\left(\frac{z}{z-1} - \frac{z}{z-\frac{1}{2}}\right) = \frac{z}{(z-1)(z-\frac{1}{2})}$$

$$G(z) = (1-z^{-1})H(z) = \frac{z(1-z^{-1})}{(z-1)(z-\frac{1}{2})} = \frac{1}{z-\frac{1}{2}}$$

$u(k) = K_R(w(k) - y(k))$; hier $w(k) = 0$

$$U(z) = K_R(W(z) - Y(z)) \; ; \; Y(z) = \frac{1}{z-\frac{1}{2}}U(z)$$

$$Y(z) = \frac{(W(z)-Y(z))K_R}{z-\frac{1}{2}} \Rightarrow \left(1 + \frac{K_R}{z-\frac{1}{2}}\right)Y(z) = \frac{K_R}{z-\frac{1}{2}}W(z)$$

$$G_W(z) = \frac{Y(z)}{W(z)} = \frac{K_R}{z + K_R - \frac{1}{2}} \; ; \; z_p = \frac{1}{2} - K_R$$

$$-1 < \frac{1}{2} - K_R < 1 \Rightarrow -\frac{1}{2} < K_R < \frac{3}{2}$$

(d): $z + K_R - \frac{1}{2} \overset{!}{=} z \Rightarrow K_R - \frac{1}{2} = 0 \Rightarrow K_R = \frac{1}{2}$

(e): $U_I(k) = U_I(k-1) + Te(k) \Rightarrow U_I(1-z^{-1}) = T(W(z) - Y(z))$

$$G(z) = \frac{1}{z-\frac{1}{2}} \quad ; \; G_W(z) = \frac{K(z)G(z)}{K(z)G(z)+1} = \frac{\frac{Tz}{z-1}\frac{1}{z-\frac{1}{2}}K_R}{1 + K_R\frac{Tz}{(z-1)(z-\frac{1}{2})}}$$

$$G_W(z) = \frac{K_R T z}{(z-1)(z-\frac{1}{2}) + K_R T z}$$