DSR-A32: $A=\begin{pmatrix}1&1\\0&1\end{pmatrix}$; $b=\begin{pmatrix}1\\1\end{pmatrix}$; $c^T=(1\quad 0)$; $d=0$

(a): und (b): $(zI-A)=\begin{pmatrix}z-1&-1\\0&z-1\end{pmatrix}$; $\det(zI-A)=(z-1)^2=0$

$z_{p1}=z_{p2}=+1$; zweimal Polstelle $\geq 1 \Rightarrow$ instabil

(c): steuerbar und beobachtbar:

$Ab=\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}+2\\+1\end{pmatrix}$; $Q_S=[b\quad Ab]=\begin{pmatrix}+1&+2\\+1&+1\end{pmatrix}$

$\det(Q_S)=1-2=-1\neq 0 \Rightarrow$ steuerbar

$c^TA=(1\quad 0)\begin{pmatrix}1&1\\0&1\end{pmatrix}=(1\quad 1)$; $Q_B=\begin{pmatrix}c^T\\c^TA\end{pmatrix}=\begin{pmatrix}1&0\\1&1\end{pmatrix}$;

$\det(Q_B)=+1\neq 0 \Rightarrow$ beobachtbar

andere Methode: Markov-Parameter: $H=\begin{pmatrix}c^Tb & c^TAb\\ c^TAb & c^TA^2b\end{pmatrix}=\begin{pmatrix}c^T\\c^TA\end{pmatrix}(b\quad Ab)=Q_BQ_S$

$\det(H)\neq 0 \Rightarrow \det(Q_S)\det(Q_B)\neq 0 \Rightarrow$ system sind steuerbar und beobachtbar

$c^Tb=(1\quad 0)\begin{pmatrix}1\\1\end{pmatrix}=+1$; $c^TAb=(1\quad 1)\begin{pmatrix}1\\1\end{pmatrix}=+2$

$c^TA\,Ab=(1\quad 1)\begin{pmatrix}+2\\+1\end{pmatrix}=3$; $\det(H)=\det\begin{pmatrix}+1&+2\\+2&+3\end{pmatrix}=-1\neq 0$

(d): $\det(zI-A)=(z-1)^2$; $\operatorname{adj}(zI-A)=\begin{pmatrix}z-1&+1\\0&z-1\end{pmatrix}$

$c^T\operatorname{adj}(zI-A)\,b=(z-1\quad +1)\begin{pmatrix}1\\1\end{pmatrix}=z$; $G(z)=\dfrac{z}{(z-1)^2}$

(e): $\hat{x}(k+1)=A\hat{x}(k)+bu(k)+h(\hat{y}_{mess}(k)-y_{mess})$

(f): $x(k+1)=Ax(k)+bu(k)$

(g): $e(k+1)=\hat{x}(k+1)-x(k+1)=A\hat{x}(k)-Ax(k)+hc^T(\hat{x}(k)-x(k))$

$\Rightarrow e(k+1)=(A+hc^T)\,e(k)$

$A_e=A+hc^T=\begin{pmatrix}1&1\\0&1\end{pmatrix}+\begin{pmatrix}-0.4\\-0.4\end{pmatrix}(1\quad 0)=\begin{pmatrix}1&1\\0&1\end{pmatrix}+\begin{pmatrix}-0.4&0\\-0.4&0\end{pmatrix}=\begin{pmatrix}+0.6&+1\\-0.4&+1\end{pmatrix}$

$(zI-A_e)=\begin{pmatrix}z-0.6&-1\\+0.4&z-1\end{pmatrix}$; $\det(zI-A_e)=(z-1)(z-0.6)+0.4=0$

$z^2-1.6z+1=0 \Rightarrow z^2-1.6z+0.64=(\pm 0.6j)^2$

$z_{p1,2}=+0.8\pm 0.6j$; $|z_{p1,2}|=+1 \Rightarrow$ Beobachter instabil.

(h): $\begin{pmatrix}1&1\\0&1\end{pmatrix}+\begin{pmatrix}h_1\\h_2\end{pmatrix}(1\quad 0)=\begin{pmatrix}1+h_1&1\\h_2&1\end{pmatrix}=A_e$

$(zI-A_e)=\begin{pmatrix}z-(1+h_1)&-1\\-h_2&z-1\end{pmatrix}$

$\det(zI-A_e)=(z-(1+h_1))(z-1)-h_2\overset{!}{=}(z-0.3)^2=z^2-0.6z+0.09$

$=z^2-(2+h_1)z+(1+h_1)-h_2\overset{!}{=}z^2-0.6z+0.09$

$2+h_1=0.6$; $h_1-h_2+1=0.09$

$\Rightarrow h_1=-1.4$; $h_2=-0.49 \Rightarrow \underline{h}=\begin{pmatrix}-1.4\\-0.49\end{pmatrix}$