A31. (f)

$$\phi(t) = A_d(t) = \sum_{n=0}^{+\infty} \frac{A_c^n t^n}{n!} \quad ; \quad B_d(t) = \sum_{n=0}^{+\infty} \frac{A_c^n t^{n+1}}{(n+1)!} B_c$$

$x(k+1) = A_d(t)\, x(k) + B_d(t) \cdot u(k)$; $u(k)$ hat $\frac{T}{2}$ Verzögerung

benutzen: $U_{previous}$, $k^T x(k)$

$x(k)$ $\quad$ $x(k+\frac{1}{2})$ $\quad$ $x(k+1)$

$$x(k+1) = A_d(\tfrac{T}{2})\, x(k+\tfrac{1}{2}) + B_d(\tfrac{T}{2})\, k^T x(k)$$

$$x(k+\tfrac{1}{2}) = A_d(\tfrac{T}{2})\, x(k) + B_d(\tfrac{T}{2})\, u(\text{previous})$$

$$z(k) = \begin{pmatrix} x(k) \\ u(\text{previous}) \end{pmatrix} ; \quad z(k+1) = \begin{pmatrix} x(k+1) \\ u(\text{current}) \end{pmatrix} \quad u_{current} = u(k);\ u_{previous} = u(k-1)$$

$$x(k+1) = A_d^2(\tfrac{T}{2})\, x(k) + A_d(\tfrac{T}{2}) B_d(\tfrac{T}{2})\, u(k-1) + B_d(\tfrac{T}{2})\, k^T x(k)$$

$$x(k+1) = \left(A_d^2(\tfrac{T}{2}) + B_d(\tfrac{T}{2}) k^T\right) x(k) + A_d(\tfrac{T}{2}) B_d(\tfrac{T}{2})\, u(k-1)$$

$$u(k) = k^T x(k)$$

$$\Rightarrow \begin{pmatrix} x(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} A_d^2(\frac{T}{2}) + B_d(\frac{T}{2}) k^T & A_d(\frac{T}{2}) B_d(\frac{T}{2}) \\ k^T & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ u(k) \end{pmatrix}$$

$$k^T = \left(-\tfrac{1}{4} \quad -\tfrac{7}{8}\right); \quad A_c = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad B_c = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A_c^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow A_d(t=\tfrac{T}{2}) = \sum_{n=0}^{+1} \frac{A_c^n (\frac{T}{2})^n}{n!} = I_{2\times2} + A_c^1 (\tfrac{T}{2})^1 = \begin{pmatrix} 1 & \frac{T}{2} \\ 0 & 1 \end{pmatrix}$$

$$B_d(t=\tfrac{T}{2}) = \sum_{n=0}^{+1} \frac{A_c^n t^{n+1}}{(n+1)!} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \left(I_{22} \times \tfrac{T}{2} + \tfrac{A_c}{2} \tfrac{T^2}{4}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$B_d(t=\tfrac{T}{2}) = \left(\begin{pmatrix} \frac{T}{2} & 0 \\ 0 & \frac{T}{2} \end{pmatrix} + \begin{pmatrix} 0 & 0.5 \\ 0 & 0 \end{pmatrix} \tfrac{T^2}{4}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{8}T^2 \\ \frac{1}{2}T \end{pmatrix}$$

$$\begin{pmatrix} \begin{pmatrix} 1 & \frac{T}{2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{T}{2} \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{8}T^2 \\ \frac{1}{2}T \end{pmatrix}(k_1 \quad k_2) & \begin{pmatrix} 1 & \frac{T}{2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{8}T^2 \\ \frac{1}{2}T \end{pmatrix} \\ (k_1 \quad k_2) & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{8}k_1T^2 & \frac{1}{8}k_2T^2 \\ \frac{1}{2}k_1T & \frac{1}{2}k_2T \end{pmatrix} & \begin{pmatrix} \frac{1}{8}T^2 + \frac{1}{4}T^2 \\ \frac{1}{2}T \end{pmatrix} \\ (k_1 \quad k_2) & 0 \end{pmatrix}$$

$u(k)$ nicht kommt wegen der Verzögerung $\quad$ $u(k)$ kommt

$x(k)$ $\quad$ $x(k+\frac{1}{2})$ $\quad$ $x(k+1)$

benutzen
$u(k-1)$