DSR32

(i): $A_G = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} + \begin{pmatrix}1\\ 1\end{pmatrix}(k_1 \;\; k_2) = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} + \begin{pmatrix}k_1 & k_2\\ k_1 & k_2\end{pmatrix} = \begin{pmatrix}k_1+1 & k_2+1\\ k_1 & k_2+1\end{pmatrix}$

$\det(zI - A_G) = \begin{pmatrix} z-(k_1+1) & -(k_2+1)\\ -k_1 & z-(k_2+1)\end{pmatrix} = (z-(k_1+1))(z-(k_2+1)) - k_1(k_2+1) \overset{!}{=} (z-0.6)^2$

$\Rightarrow z^2 - (k_1+k_2+2)z + (k_1+1)(k_2+1) - k_1k_2 - k_1 \overset{!}{=} z^2 - 1.2z + 0.36$

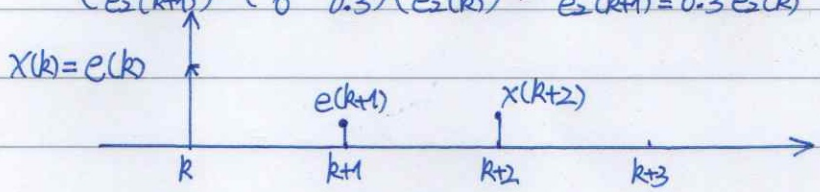
$z^2 - (k_1+k_2+2)z + k_2 + 1 = z^2 - 1.2z + 0.36$

$k_2 + 1 = 0.36; \quad k_1 + k_2 + 2 = 1.2 \Rightarrow k_2 = -0.64; \quad k_1 = -0.16; \quad k^T = (-0.16 \;\; -0.64)$

(j): $A_e = \begin{pmatrix}0.3 & 0\\ 0 & 0.3\end{pmatrix}; \quad A_G = \begin{pmatrix}0.6 & 0\\ 0 & 0.6\end{pmatrix}$

$\begin{pmatrix}x_1(k+1)\\ x_2(k+1)\end{pmatrix} = \begin{pmatrix}0.6 & 0\\ 0 & 0.6\end{pmatrix}\begin{pmatrix}x_1(k)\\ x_2(k)\end{pmatrix} \Rightarrow \begin{matrix} x_1(k+1) = 0.6\,x_1(k)\\ x_2(k+1) = 0.6\,x_2(k)\end{matrix}$

$\begin{pmatrix}e_1(k+1)\\ e_2(k+1)\end{pmatrix} = \begin{pmatrix}0.3 & 0\\ 0 & 0.3\end{pmatrix}\begin{pmatrix}e_1(k)\\ e_2(k)\end{pmatrix} \Rightarrow \begin{matrix} e_1(k+1) = 0.3\,e_1(k)\\ e_2(k+1) = 0.3\,e_2(k)\end{matrix}$



$e(k+1) = 0.3\,e(k); \quad x(k+2) = 0.6^2 x(k) = 0.36\,x(k)$

$0.36\,x(k) > 0.3\,e(k) \Rightarrow$ mindestens doppelt schnell

(k): $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} + \begin{pmatrix}h_1\\ h_2\end{pmatrix}(1 \;\; 0) = \begin{pmatrix}1+h_1 & 1\\ h_2 & 1\end{pmatrix}$

$(z-(h_1+1))(z-1) - h_2 \overset{!}{=} z^2 \Rightarrow \begin{matrix} h_1 + 2 = 0\\ h_1 - h_2 + 1 = 0\end{matrix} \Rightarrow \begin{pmatrix}h_1\\ h_2\end{pmatrix} = \underline{h} = \begin{pmatrix}-2\\ -1\end{pmatrix} = \underline{h}_{Luenberger}$

(l): $\bar{h} = A h \Rightarrow \bar{h}_{Luenberger} = A\, h_{Kalman}$

$h_{Kalman} = A^{-1} \bar{h}_{Luenberger} = \begin{pmatrix}+1 & -1\\ 0 & +1\end{pmatrix}\begin{pmatrix}-2\\ -1\end{pmatrix} = \begin{pmatrix}-1\\ -1\end{pmatrix}$