

- $\int_0^t K(t-\tau) \nu(\tau) d\tau ; \quad \nu(\tau) = \dot{z}(\tau)$
- $K(s) \nu(s) ; \quad K(s)$ can be written as $C_r^T (S \underline{I} - \underline{A}r)^{-1} b_r = \frac{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$
here $b_n = 0$
- $\dot{z}_r(t) = A_r z_r(t) + B_r \nu(t)$
- $(S \underline{I} - \underline{A}r) Z_r(s) = B_r \nu(s)$
- $Z_r(s) = (S \underline{I} - \underline{A}r)^{-1} B_r \nu(s)$
- $C_r^T Z_r(s) = \underbrace{C_r^T (S \underline{I} - \underline{A})^{-1} B_r}_{K(s)} \nu(s)$
- $C_r^T Z_r(t) = k(t) * \nu(t) = \int_0^t K(t-\tau) \nu(\tau) d\tau = y_r(t)$

① radiation force

$$F_{rad}(t) = -m_\infty \ddot{z}(t) - \int_0^t K(t-\tau) \nu(\tau) d\tau$$

m_∞ : added mass at infinite frequency; $K(t)$: radiation impulse.

② wave excitation force:

$$F_{exc}(t) = \text{Real} \left\{ \frac{H}{2} F_{exc}(\omega, \theta) e^{j\omega t} \right\}$$

③ hydrostatic restoring force:

$$F_h(t) = \rho g V_i = \rho g (V_0 - S z(t)) ; \quad \text{set } V_0 = 0$$

Let hydrostatic stiffness $S_h = \rho g S \Rightarrow F_h(t) = -S_h z(t)$

$$m \ddot{z}(t) = F_{rad} + F_{exc} + F_{pro} + F_h$$

$$m \ddot{z}(t) = -m_\infty \ddot{z}(t) - \int_0^t K(t-\tau) \dot{z}(\tau) d\tau + F_{exc}(t) + F_{pro}(t) - S_h z(t) ; \quad \nu(t) = \dot{z}(t)$$

$$(m_\infty + m) \ddot{z}(t) + S_h z(t) + \underbrace{\int_0^t K(t-\tau) \dot{z}(\tau) d\tau}_{C_r^T Z_r(t)} = F_{exc}(t) + F_{pro}(t)$$

$$(m_\infty + m) \ddot{z}(t) + S_h z(t) + C_r^T Z_r(t) = F_{exc}(t) + F_{pro}(t)$$

$$\ddot{z}(t) = -\frac{1}{m+m_\infty} S_h z(t) - \frac{1}{m+m_\infty} C_r^T Z_r(t) + \frac{1}{m+m_\infty} F_{exc}(t) + \frac{1}{m+m_\infty} F_{pro}(t)$$

$$\text{Let: } \underline{x}(t) = (x_1(t) \ x_2(t) \ x_3(t))^T = (z(t) \ \dot{z}(t) \ Z_r(t))^T$$