ECON90055 Computational Economics: Assignment 3

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1 Problem 1

1.1 Solutions to part(a) and part(b)

Since when we solve for results of the matrix, all three methods provided consistent convergence to 0_n . This part will focus on the display of code that solve the problem, and the efficiency of each method, i.e. how many iterations does each method take for the convergence. Each method has similar set up, the difference is at where it iterates to get new updates of x's. For example, Gauss-Seidel use updated value of x_1 into calculation of next x_2 . While, Jacobi's Method does not use updated value into calculation straight away until the current iteration ends. In each iteration, it uses input value of x's to generate a complete vector of updated x's. Successive Over-Relaxation(SOR) Method is similar to Gauss-Seidel as it takes the updated value of each x (e.g. x_1) back into the calculation for the next x (e.g. x_2). The difference is that when iterate value for a certain x, SOR Method includes a ratio to take into account the effect of old value of x on the new value of to be updated(x'). The ratio depends on the value of x.

The following three figures show my code on each of methods:

Listing 1: Code for Gauss-Seidel Method

```
2
 % ECON90055 Assignment 3 - "Gaussian-Seidel"
3
4
 % Coder: Zelin Chen (797036)
5
 6
 % Cx = b, C = constant, b = results, err = stopping
 function term = gaussei03(C,b,err)
 % create the argument A
9
 A = [C b];
 % row of A: column is assumed to be (n+1)
 n = size(A,1);
```

```
13
14 % define a starting value of x
15 \mid x_{val} = zeros(n,1);
16 | for m=1:n
17
        x_val(m) = 1;
18
   end
19
20 | % define errors
21 \mid x_{err} = x_{val};
22 | % define a temporary storage for x's during iteration
23 \mid x_{tem} = zeros(n,1);
  % iteration count
25
   term = 0;
26
   % iteration continue until hit the stopping rule
27
28
   while max(abs(x_err))>err
29
        term = 1 + term;
30
31
        % iterate value for each x, given the value of all
            other x.
32
        for j =1:n %row
            x_{val}(j) = A(j,n+1);
34
            for k =1:n %col
                 if k^{-}=j
                 x_val(j) = x_val(j) - A(j,k)*x_val(k);
36
37
38
            end
39
            % solve for updated x'
40
            x_{val}(j) = x_{val}(j)/A(j,j);
            \% solve for error btw x' and x
41
            x_{err}(j) = x_{val}(j) - x_{tem}(j);
42
43
        end
        % save temporary results for later error
44
            calculation
45
        x_{tem} = x_{val};
46
   end
47
   end
```

Listing 2: Code for Jacobi's Method

```
7 \mid %  Cx = b, C = constant, b = results, err = stopping
  function term = jacobi(C,b,err)
9 % create the argument A
10 | A = [C b];
  |\% row of A: column is assumed to be (n+1)
11
12 \mid n = size(A,1);
13
  % define x starting values = 1
14
15 \mid x_{old} = zeros(n,1);
16 | for m=1:n
17
        x_old(m,1) = 1;
18
   end
19
20 % space for storing new value of x in iteration
21 \mid x_{new} = zeros(n,1);
22 | % define space for storing errors
23 \mid x_{err} = x_{new} - x_{old};
24
25 | % term count
26 | term = 0;
27
28
  |% iteration continue until it hit stopping rule
   while max(abs(x_err))>err
30
        term = 1 + term;
31
32
        % iterate value for each x, given the value of all
            other x.
        for j =1:n %row
            x_new(j) = A(j,n+1);
34
35
            for k =1:n %col
36
                 if k^=j
                 x_new(j) = x_new(j) - A(j,k)*x_old(k);
37
38
                 end
39
            end
40
            % solve for updated x'
41
            x_{new}(j) = x_{new}(j)/A(j,j);
42
            % solve for error btw x' and x
43
            x_{err}(j) = x_{new}(j) - x_{old}(j);
44
        end
45
        % update
46
        x_old = x_new;
47
   end
48
   end
```

Listing 3: Code for SOR

```
| \( \langle \) 
  2 | % ECON90055 Assignment 3 - "SOR Metohod"
  4
        % Coder: Zelin Chen (797036)
        6
        |% Cx = b, C = constant, b = results, err = stopping
                   rule
        function term = SOR(C,b,k,err)
  9
        % determine value of w.
        w = 0.1*k;
11
12 % create the argument A
13 A = [C b];
14 |% row of A: column is assumed to be (n+1)
15 \mid n = size(A,1);
16
17 \mid \% define a starting value of x
18 \mid x_{val} = zeros(n,1);
19
        for m=1:n
20
                      x_val(m) = 1;
21
        end
22
23 |% define errors
24 \mid x_{err} = x_{val};
25 | % define a temporary storage for x's during iteration
26 \mid x_{tem} = x_{val};
        % iteration count
28
        term = 0;
29
30 | % iteration continue until the maxmium element in
                    errors is lower than 0.0001
        while max(abs(x_err))>err
32
                      term = 1 + term;
34
                      % iterate value for each x, given the value of all
                                   other x.
                      for j =1:n %row
                                   x_{val}(j) = A(j,n+1) + x_{tem}(j)*((1-w)/w)*A(j,j)
36
37
38
                                   for k =1:n
39
                                               if k^{-}=j
                                               x_{val}(j) = x_{val}(j) - A(j,k)*x_{val}(k);
40
```

```
41
                  end
42
             end
             % solve for updated x'
             x_{val}(j) = (x_{val}(j)/A(j,j)) * w;
44
45
             % solve for error btw x' and x
             x_{err}(j) = x_{val}(j) - x_{tem}(j);
47
        end
48
        % update temporary x
49
        x_{tem} = x_{val};
50
   end
51
    end
```

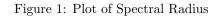
I wrote a main script to call the function automatically under sample size n = 5,10,20,50 and stopping error $= 0.01,\,0.001$ and 0.0001. The result is displayed in table 1 in appendix.

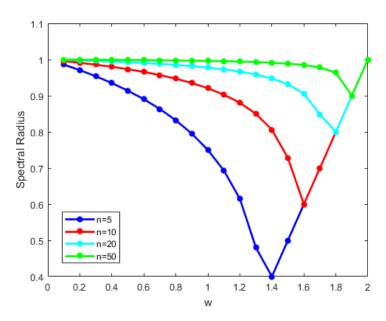
1.2 Solutions to part(c)

Spectral radius of matrix change with the size of the matrix, i.e. n = 5, 10, 20, and 50. The matlab function 'eig' solve for eigenvalues of a given matrix. Then the spectral radius equals to the element with largest absolute values. With the change of size of matrix (n), figure 1 produces a four different lines indicating the relation between spectral radius and the choice of w. Its code for finding and drawing spectal radius are shown below separately:

Listing 4: Code for Finding Spectral Radius

```
2
  % ECON90055 Assignment 3 - "spec_radius"
3
4
  % Coder: Zelin Chen (797036)
  5
  function spec = spec_radius(C,k)
6
7
  % storage space
8
  spec = zeros(1,k);
9
  w_val = zeros(1,k);
  % fn LDU decomposite C into L, D and U
  [L,D,U] = LDU(C);
12
  for i = 1:k
      w_{val}(i) = 0.1 * i;
14
      % spectral radius = maximal eigenvalue of
        expression inside the bracket.
      spec(i) = max(abs(eig(((1/w_val(i) * D + L)^(-1)
        * ( (1/w_val(i) -1) * D - U )))));
16
  end
  end
```





Listing 5: Code for Drawing Spectral Radius

```
1
  2
  % ECON90055 Assignment 3 - "Q1-c-figure"
3
4
  % Coder: Zelin Chen (797036)
  6
  clear all;
8
  \% err = 0.1 to the power of 2, 3 and 4
9
  err_power = [2,3,4];
  |% sample size = 5, 10, 20 and 50
  n_{size} = [5,10,20,50];
12
  len_n = length(n_size);
13
14
  % w = 0.1 * k
  k = 20;
  w_val = zeros(1,k);
16
17
  for i = 1:k
18
      w_val(i) = 0.1 * i;
19
  end
20
21
  % store value for each spectral radius
22 | spec = zeros(len_n,k);
```

```
for i=1:len_n
24
        n = n_size(i);
25
        C = An(n);
26
        spec(i,:) = spec_radius(C,k);
27
28
29
   figure
   plot(w_val, spec(1,:), 'b*-', w_val, spec(2,:), 'r*-', w_val
30
       , spec(3,:), 'c*-', w_val, spec(4,:), 'g*-', 'LineWidth'
       , 2)
   legend('n=5','n=10','n=20','n=50')
   xlabel('w')
32
   ylabel('Spectral Radius')
   saveas(gcf, 'spectral_radius_plot.png')
34
```

1.3 Solutions to part(d)

¹ Under stopping rule of 0.01, the choice on best w for SOR is consistent for different n. When w = 0.9, SOR solve four different sizes of matrix by 6 rounds. When n = 5, it produce the same performance as Gauss-Seidel method (SOR(w = 1)=6). Under the other three sizes of n, they are all one term shorter than Gauss-Seidel method (SOR(w = 1)=7).

When increase preciseness of our answer to 0.001, the result shifted. Under n=5, w=1.3 reach the limit with lowest terms(10), while Guass-Seidel method (SOR(w=1)) takes 14 terms to converge. When n=10, w increases to 1.5 with only 18 terms, leave where SOR(w=1) takes 20 terms. For n=20, SOR has four values of w produce the shortest terms, including SOR(w=1). The result is same for n=50 as w=1,1.1,1.2, and 1.3 all have the same number of times (17) to converge.

The advantage of SOR method emergs when limit rises up to 0.0001. For n=5, the quickest SOR (w=1.3and1.4) takes only 13 terms, which is 9 terms quicker than Gauss-Seidel method. For n=10, $\mathrm{SOR}(w=1.6)$ takes the 22 terms, where Gauss-Seidel method takes more than doubled (48 terms). For n=20, $\mathrm{SOR}(w=1.7)$ produces the quickest result with only 38 iterations, where it takes Gauss-Seidel method to run up to 57 rounds. Finally n=50, $\mathrm{SOR}(w=1.6,1.7)$ produce the quickest convergence with 46 period of times, on the contrast, Gauss-Seidel method takes 50 terms.

¹please refer to table 1 in appendix

2 Problem 2

Table 2^2 displays the result of convergence for different methods: Bisection, False Position, Secant and Newton's method. In order to produce the table, the stopping rule has been set to be equal to iteration < 20, instead of error > 0.0005, because 20 the largest iteration among that four methods to stop at error of 0.00005. From table 2, we see that Bisection method takes the longest iteration to converge to a limit, 20 iterations. False Position method takes only 14 terms to converge. Comparatively, Secant and Newton's methods are more efficient, as Secant method takes 7 terms and Newton's method takes only 6 terms to converge. The code of each method is shown as following. (It also includes a main script to call each function and build the result into a latex table)

Listing 6: Code for Bisection Method

```
2
  % ECON90055 Assignment 3 - "Bisection Method"
3
  % Coder: Zelin Chen (797036)
  5
6
  % function: f(x) = x^3 -2
  % starting interval for search: [1,2]
9
  % middle point = b - w(b-a), where w assumed to be
  function [vm,fm] = bisection()
11
  a = 1;
12
  b = 2;
  w = 0.5;
13
14
15
  % define space to store temporary value of errors and
     median points
16
  error = 1;
17
  m = 1:
18
19
  % space to store result for each iteration, first two
     iterations set as
20
  % defined
21
  vm = zeros(15,1);
22
  vm(1) = a;
23
  vm(2) = b;
  fm = zeros(15,1);
  fm(1) = f(vm(1));
  fm(2) = f(vm(2));
```

 $^{^2}$ please find it in appendix

```
27
28
   term = 2;
29
30
   while term < 20 % error > 0.00005
        % update new point
32
        term = term + 1;
        m = b - w*(b-a);
34
        vm(term) = m;
        error = abs(m - vm(term-1));
36
37
        % sign: if >0 TRUE=1, if <0 FALSE=0
38
        fm_sign = (f(m)>0);
39
        a_sign = (f(a)>0);
40
41
        % update interval
42
        if fm_sign==a_sign
43
            a = m;
44
        else
45
            b = m;
46
        end
47
        % store results
48
        fm(term) = f(m);
        vm(term) = m;
49
50
   end
51
   end
```

Listing 7: Code for False Position

```
1
2
  | % ECON90055 Assignment 3 - "False Position"
3
  % Coder: Zelin Chen (797036)
4
  6
 \frac{1}{2} function: f(x) = x^3 - 2
7
 % starting interval for search: [1,2]
9
  function [vm,fm] = false_position()
10 \mid a = 1;
 b = 2;
11
12
 error = 1;
13
14
 % space to store result for each iteration, first two
    iterations set as
 % defined
16 | vm = zeros(15,1);
17 | vm(1) = a;
```

```
18 | vm(2) = b;
19
  fm = zeros(15,1);
20 \mid fm(1) = f(vm(1));
21
  fm(2) = f(vm(2));
22
23
  term = 2;
24
25
  while term < 20 % error > 0.00005
26
        term = term + 1;
27
        % update new point
28
        m = (a*f(b) - b*f(a))/(f(b) - f(a));
29
        error = abs(m - vm(term-1));
30
        % update interval
31
        if f(m)*f(a)<0
32
            b = m;
33
        else
34
            a = m;
        end
36
        % store result
37
        vm(term) = m;
38
        fm(term) = f(m);
39
   end
40
41
   end
```

Listing 8: Code for Secant Method

```
\(\frac{1}{2}\) \(\frac{1}\) \(\frac{1}{2}\) \(\frac{1}2\) \(\frac{1}2\) \(\frac{1}2\) \(\frac{1}2\) \(\frac{1
             % ECON90055 Assignment 3 - "Secant Method"
   3 | %-----
            % Coder: Zelin Chen (797036)
            6
            \% function: f(x) = x^3 - 2
            % starting interval for search: [1,2]
            function [vm,fm] = secant()
            a = 1;
            b = 2;
11
12
13 | error = 1;
14 \% space to store result for each iteration, first two
                               iterations set as
15 % defined
16 | vm = zeros(15,1);
17 \mid vm(1) = a;
18 \ | vm(2) = b;
```

```
19 | fm = zeros(15,1);
  fm(1) = f(vm(1));
21
  fm(2) = f(vm(2));
22
23
  term = 2;
24
25
  while term < 20 % error > 0.00005
26
       term = term + 1;
27
       % update values
       vm(term) = vm(term-2) - ((vm(term-1)-vm(term-2))
28
           /(f(vm(term-1))-f(vm(term-2))) ) *f(vm(term-2))
29
       fm(term) = f(vm(term));
       error = abs(vm(term) - vm(term-1));
30
31 end
   end
```

Listing 9: Code for Newton's Method

```
|% ECON90055 Assignment 3 - "Newton's Method"
  3
  | % Coder: Zelin Chen (797036)
\% function: f(x) = x^3 - 2
  % starting interval for search: [1,2]
9 |% Derivative: f'(x) = 3*x^2
10 | function [vm,fm] = newton()
11
  a = 1;
12 | b = 2;
13
  error = 1;
14
15 |% space to store result for each iteration, first two
    iterations set as
  % defined
16
  vm = zeros(15,1);
17
  vm(1) = a;
19
  vm(2) = b;
20
  fm = zeros(15,1);
21 \mid fm(1) = f(vm(1));
  fm(2) = f(vm(2));
23
24 \mid \text{term} = 2;
25
26 | while term < 20 % error > 0.00005
```

Listing 10: Main Script for Question 2

```
\( \langle \) \(
   1
   2
            % ECON90055 Assignment 3 - "Q2-main script-tables"
   3
             % Coder: Zelin Chen (797036)
   4
   5
            6
   7
           % call function
           [b_vm,b_fm] = bisection();
   8
           [f_vm,f_fm] = false_position();
  9
10 \mid [s_{vm}, s_{fm}] = secant();
11 \mid [n_{vm}, n_{fm}] = newton();
12
13 |% create table
14
           table.data = [b_vm b_fm f_vm f_fm s_vm s_fm n_vm n_fm
15
16
           | table.tableColLabels = \{' x_n ', ' f(x_n) ', ' x_n ', ' f
                              (x_n), x_n, x_n,
               table.tableRowLabels ={'1','2','3','4','5','6','7','8'
17
                               ,'9','10','11','12','13','14','15','16','17','18','
                              19','20'};
18
19
            table.tableCaption = 'Table1: Comparison of Various
                              Methods for Finding Zeros of f(x)=x^3-2;
20
              if ~isfield(table, 'dataNanString'), table.dataNanString
                                  = 'NaN'; end
21
22
           table.dataFormat = {'%.5f'};
23
24 | table.makeCompleteLatexDocument = 1;
25
26 | % generate LaTex code
27 | latex = latexTable(table);
```

```
28
29 % save LaTex code as file
30 fid=fopen('Q2_table.tex','w');
31 [nrows,ncols] = size(latex);
32 for row = 1:nrows
        fprintf(fid,'%s\n',latex{row,:});
34 end
35 fclose(fid);
4 fprintf('\n... your LaTex code has been saved as ''
        Q2_table.tex'' in your working directory\n');
```

Appendices

Listing 11: Code for creating matrix C in Q1

```
% ECON90055 Assignment 3 - "Create Matrix An"
3
  % Coder: Zelin Chen (797036)
5
  function [C,b] = An(n)
9
  % create vector for b
  b = zeros(n,1);
  % create square matrix n*n
11
12
  C = zeros(n,n);
13
14
  % putting values into A_n
15
      for j = 1:n \%row
16
         for k = 1:n \% column
17
             if k==j
18
                C(j,k) = 2;
19
             elseif k==(j-1)
20
                C(j,k) = 1;
             elseif k==(j+1)
22
                C(j,k) = 1;
23
             else
                C(j,k) = 0;
24
25
             end
26
         end
27
      end
28
  end
```

Listing 12: Code for function f(x) in Q2

```
function y = f(x)
y = x^3 - 2;
end
```

Listing 13: Code for derivative of function f(x) in Q2

```
1 function y = df(x)
2      y = 3*x^2;
3 end
```

0.0001NaN0.001NaNTable 1: Number of Iterations of three methods for different Stopping Rule and Size n $\frac{1}{8}$ $\frac{21}{20}$ NaN0.01 \square 0.0001 NaN 38 $\frac{5}{2}$ n=200.001NaN $\frac{21}{20}$ NaN0.01 $\frac{18}{18}$ 0.0001NaNn = 100.001 NaN $\frac{21}{20}$ NaNΠ 0.0001NaN $\frac{22}{71}$ 0.001NaN \Box NaN 0.01 ∞ SOR(2.0)SOR(1.4)SOR(1.6)SOR(0.2)SOR(0.3)SOR(0.4)SOR(0.9)SOR(1.2)SOR(1.3)SOR(1.5)SOR(1.7)SOR(1.8) $\overline{SOR(1.9)}$ SOR(0.1)SOR(0.5)SOR(0.6)SOR(0.7)SOR(0.8)SOR(1.0)SOR(1.1)Method Gauss Jacobi

-1.000006.000001.375000.00482 0.00000 0.00000 0.000000.00000Table 2: Table 1: Comparison of Various Methods for Finding Zeros of $f(x) = x^3 - 2$ 0.178280.00000 0.00000 0.00000 0.00000 0.000000.00000.00.000000.000000.000000.000000.0000.0 $\overline{f}(x_n)$ Newton 1.259921.259921.259921.259921.259921.000001.296301.259921.259921.259921.259921.259921.259922.000001.500001.260931.259921.259921.259921.25992-1.00000-0.50729-0.22986-0.00100-0.00000 6.000000.02447 0.00000 0.00000 0.00000 NaN NaN $f(x_n)$ NaN NaNNaN NaNNaN NaNNaNNaNSecant 1.142861.000001.209681.265042.000001.25971 1.259921.259921.259921.25992NaN NaN NaN NaNNaN NaNNaNNaNNaNNaN-0.00294-0.22986-0.09874-0.04143-0.01722 -0.00712-1.00000-0.5072900000.9-0.00121 -0.00009 -0.00004-0.00000-0.00000 -0.00000-0.00050-0.00001-0.00001-0.00000 -0.00021 $f(x_n)$ False Position 1.000001.142861.251161.259302.000001.209681.238841.256301.258421.259821.259881.259901.259921.259921.259921.259921.259671.259911.259921.25992-0.00016-1.00000-0.04688-0.01002 -0.000740.00042-0.000010.000026.000001.375000.103300.027290.008570.001590.000130.0000.00.0000.00.599610.260990.00391 $f(x_n)$ Bisection 1.25977 1.000001.28125 1.259951.259921.259922.00000 1.500001.250001.375001.312501.265631.257811.261721.260741.260251.260011.259891.259931.259932010 12 13 14 15 1617 $\frac{\infty}{2}$ 19 1 ∞ ロ വ 9 CJ က 6