Understanding Backward Propagation Mathematically

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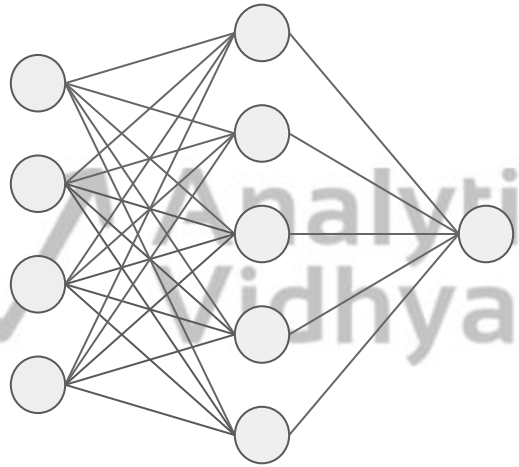
Forward Propagation Recap



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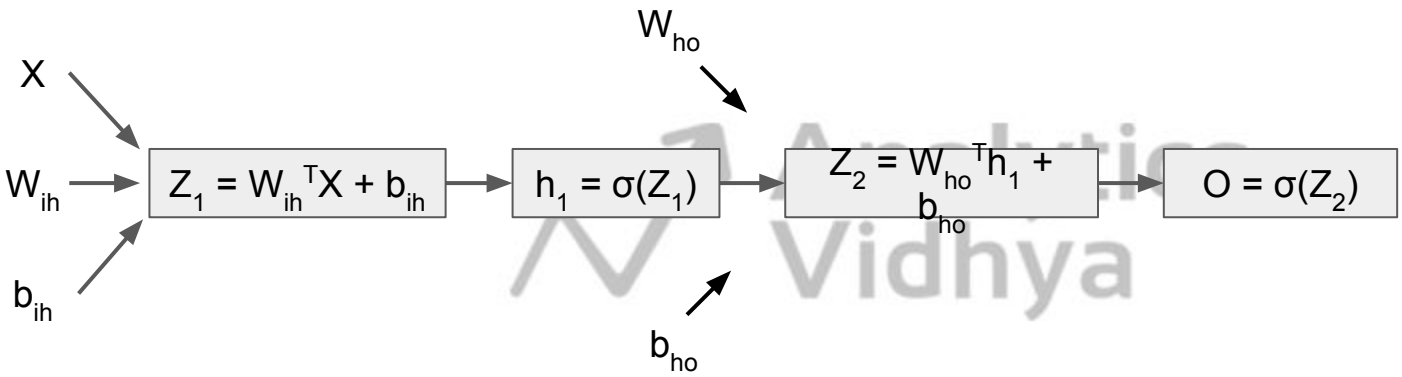
Forward Propagation: Computation Graph



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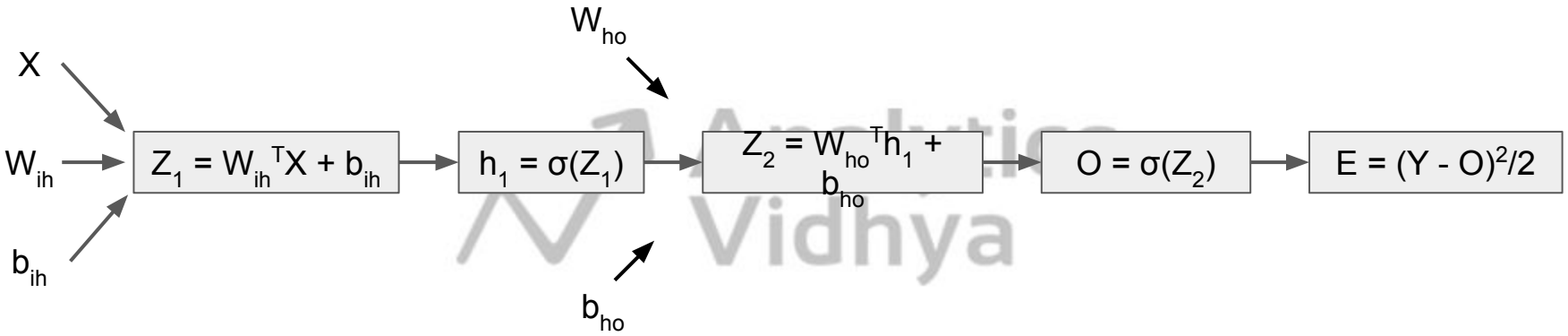
Backward Propagation: Calculating the error



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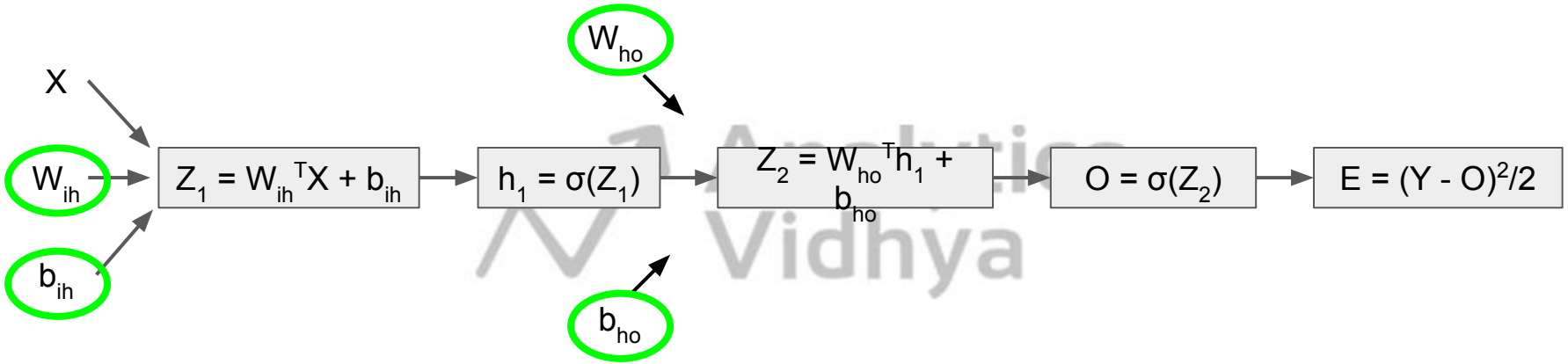
Backward Propagation: Calculating the error



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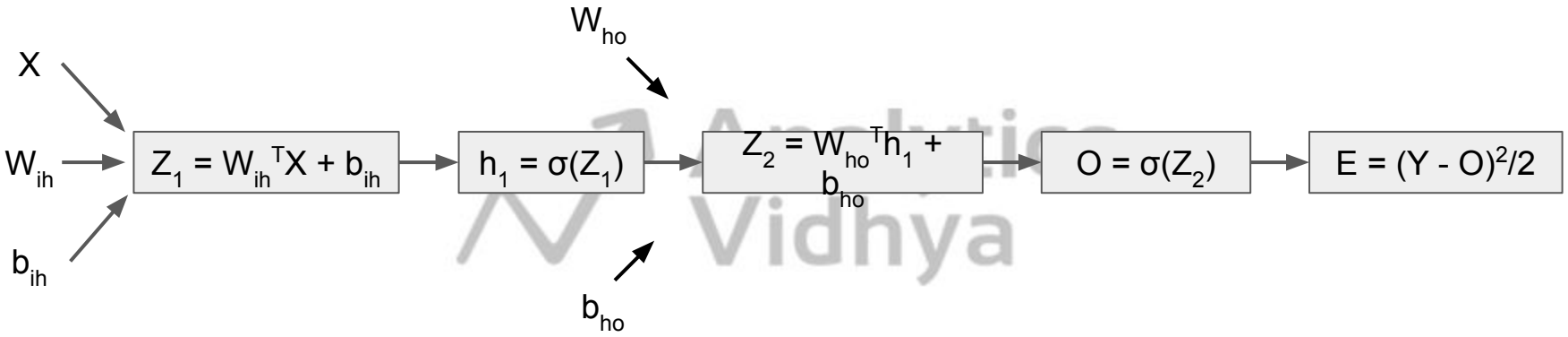

$$\frac{dE}{dW_{ho}} \quad \frac{dE}{db_{ho}} \quad \frac{dE}{dW_{ih}} \quad \frac{dE}{db_{ih}}$$

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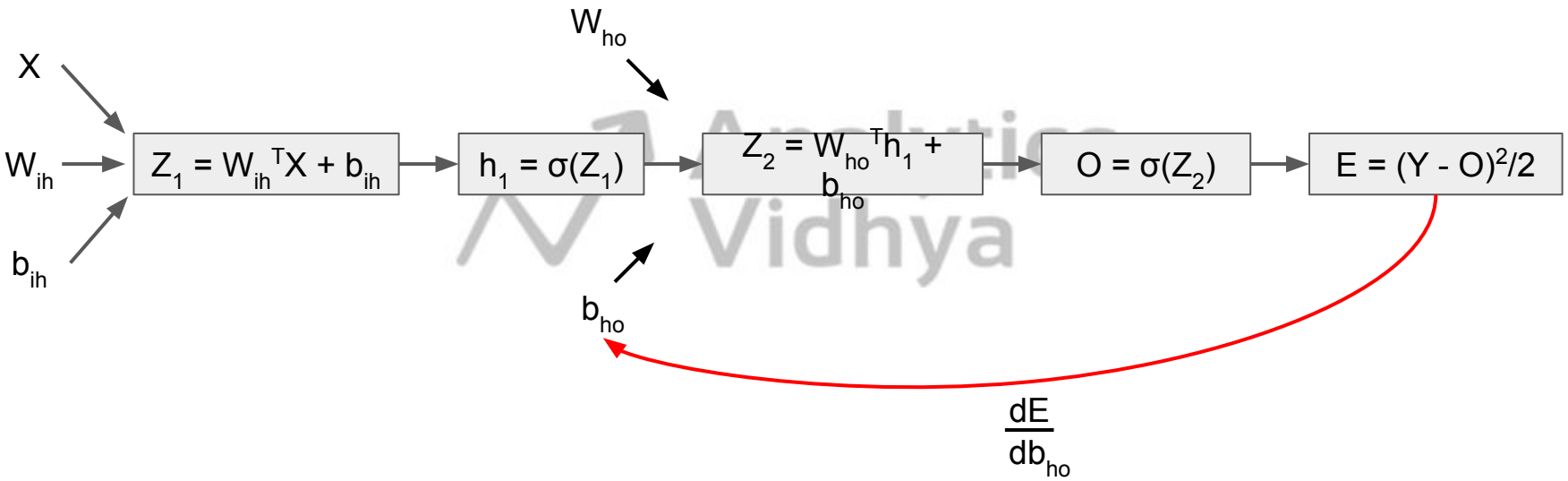

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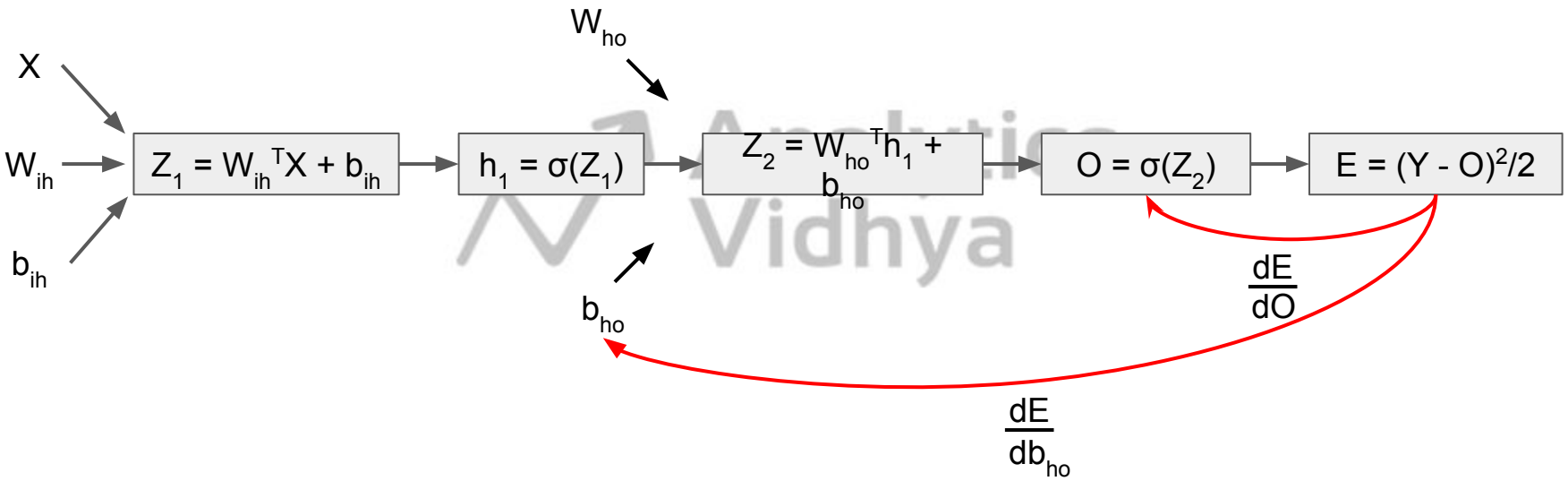

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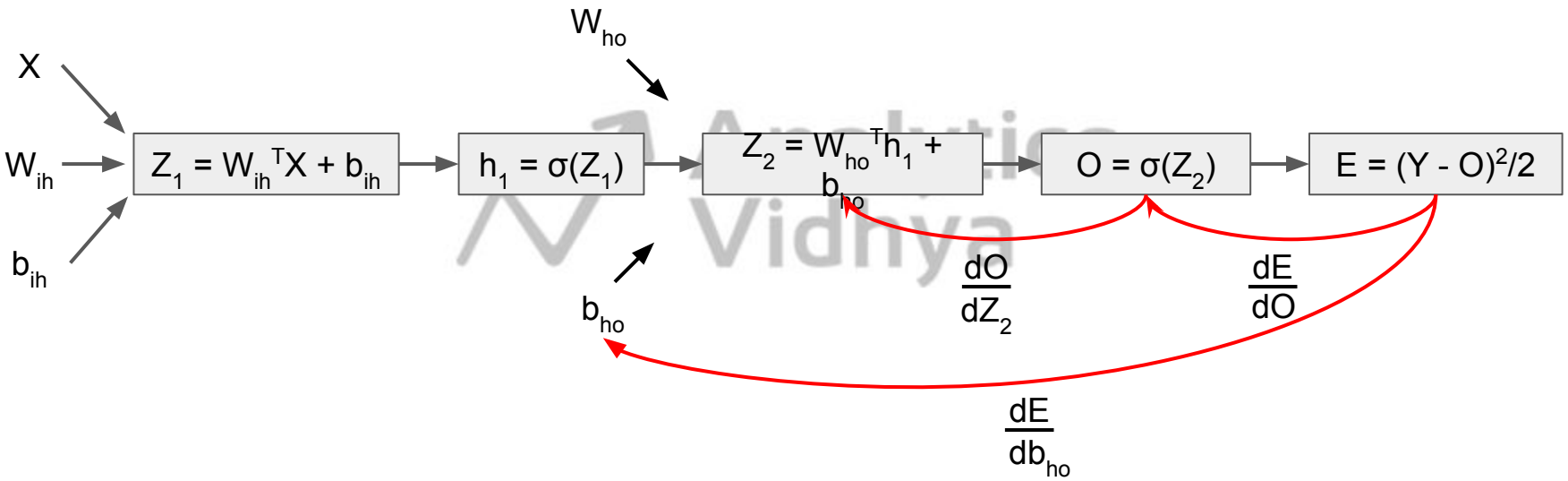

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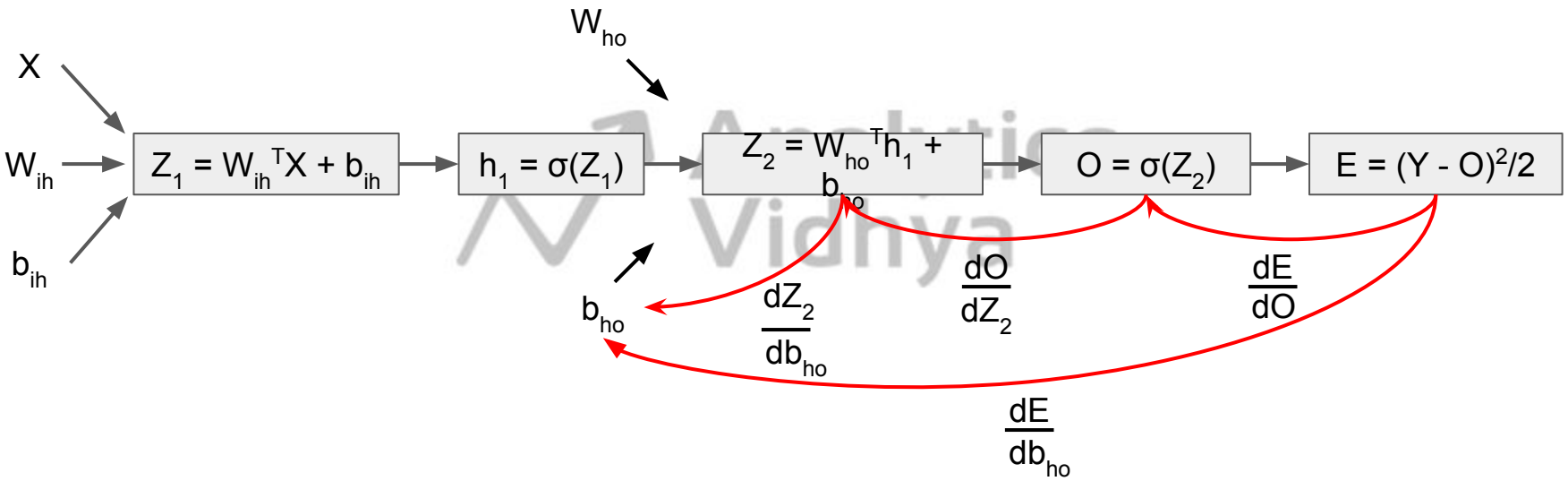

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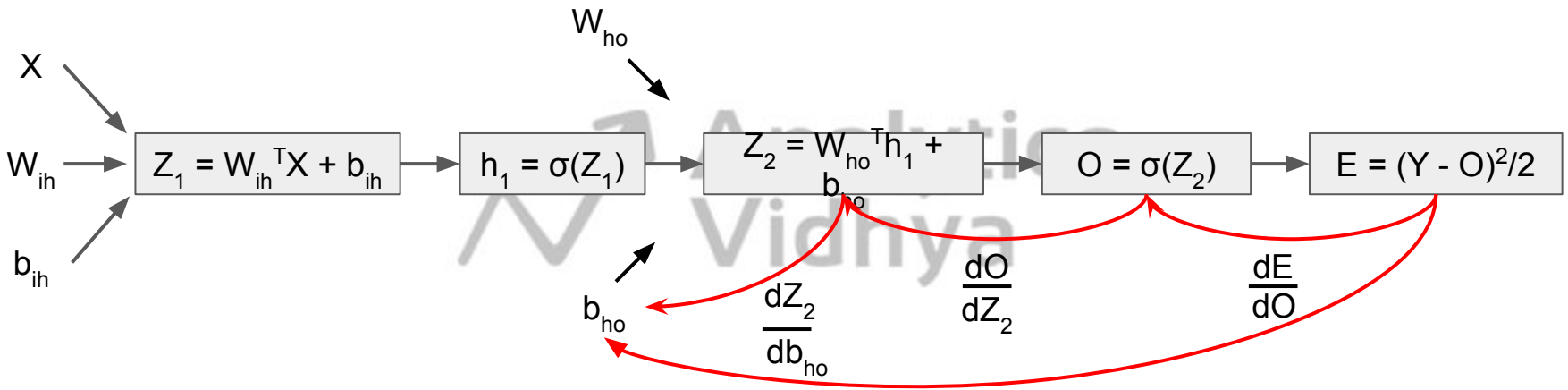

$$\frac{dE}{db_{ho}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{db_{ho}}$$

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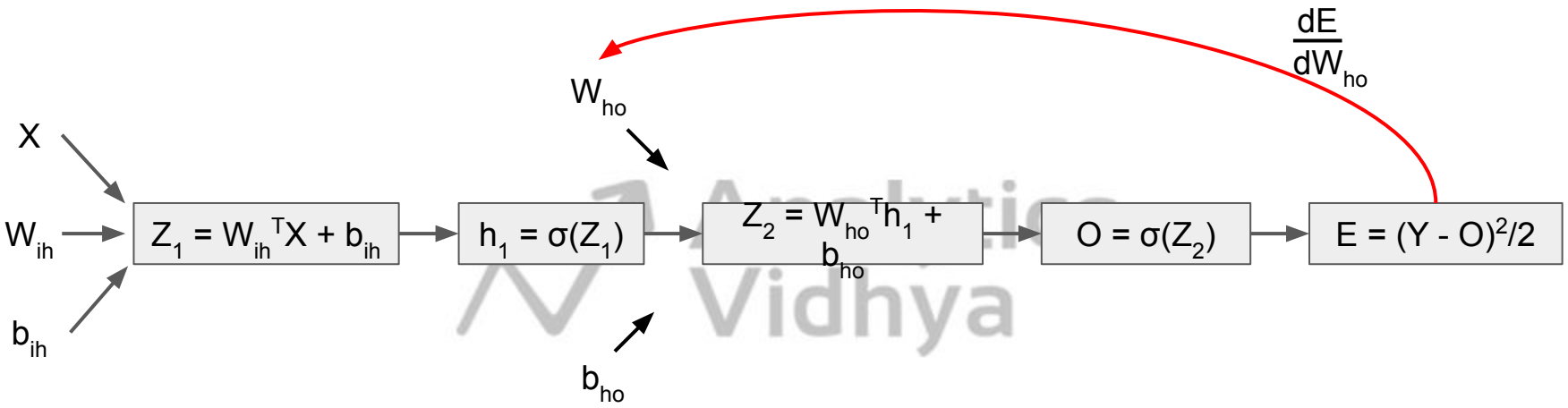

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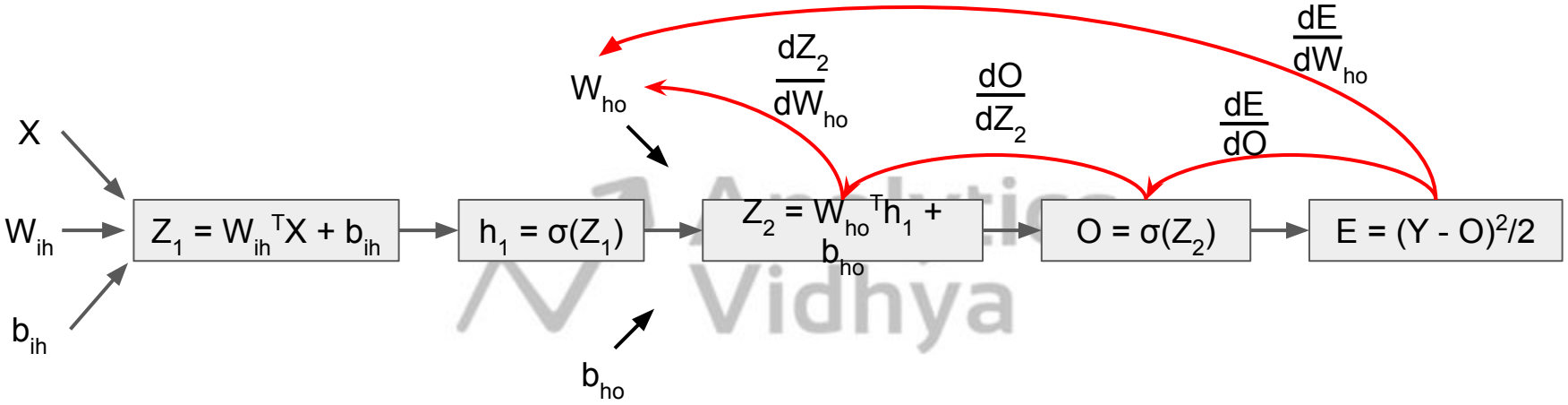

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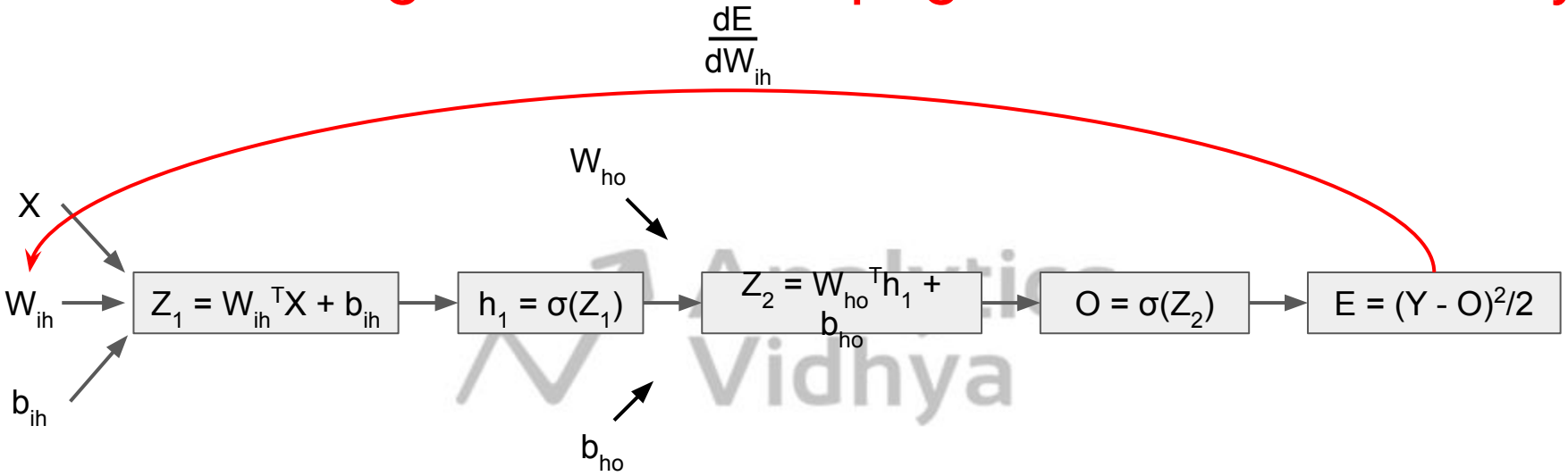

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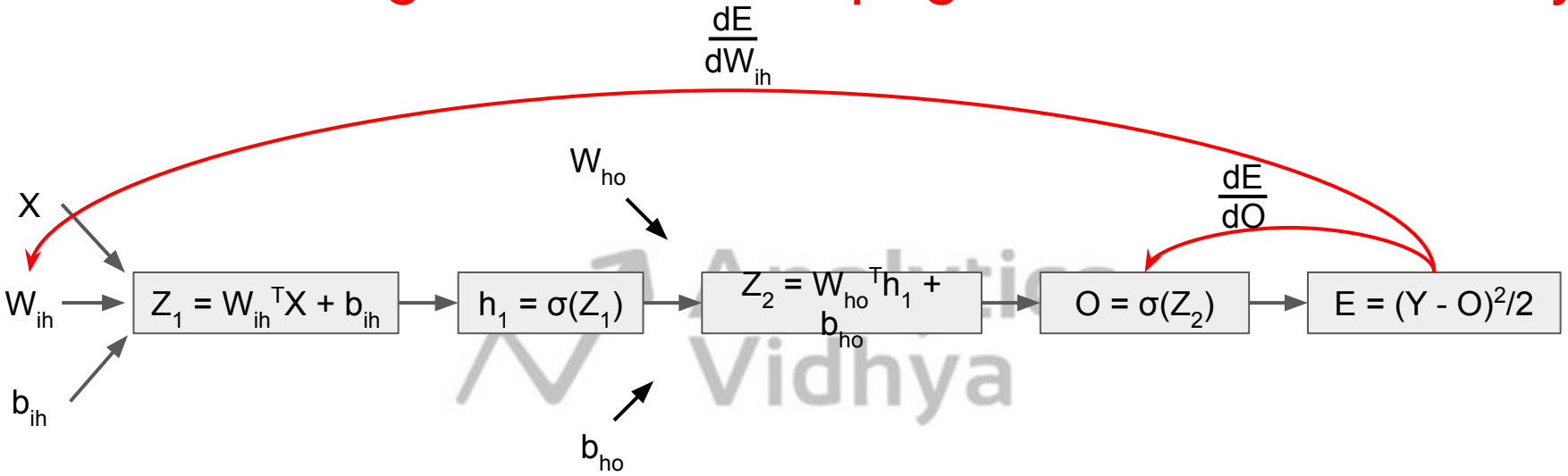

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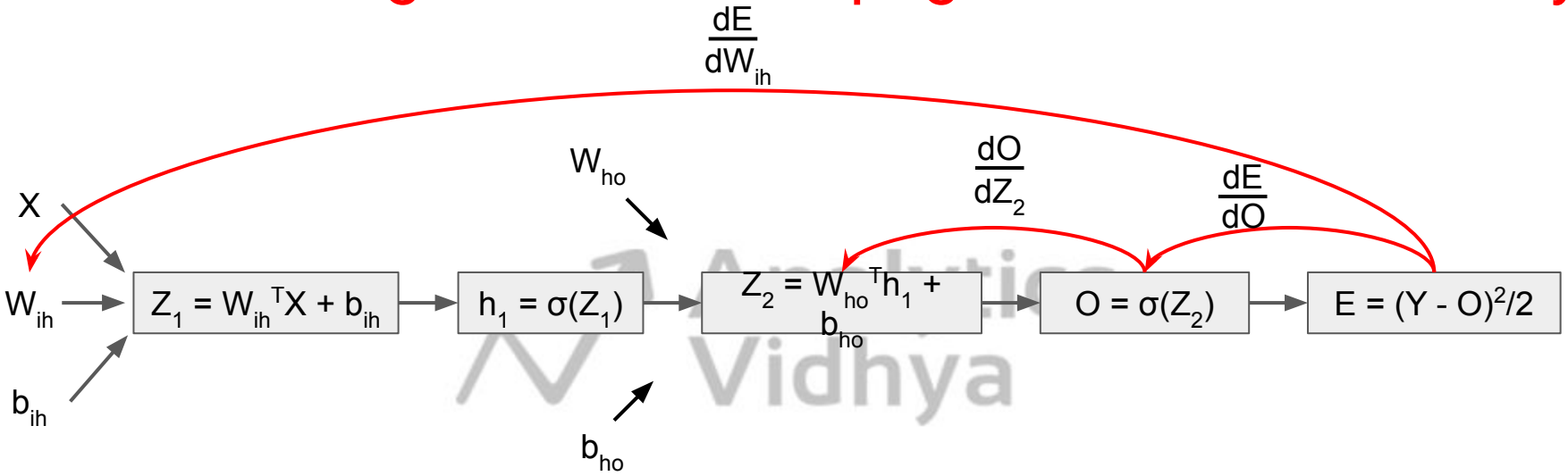

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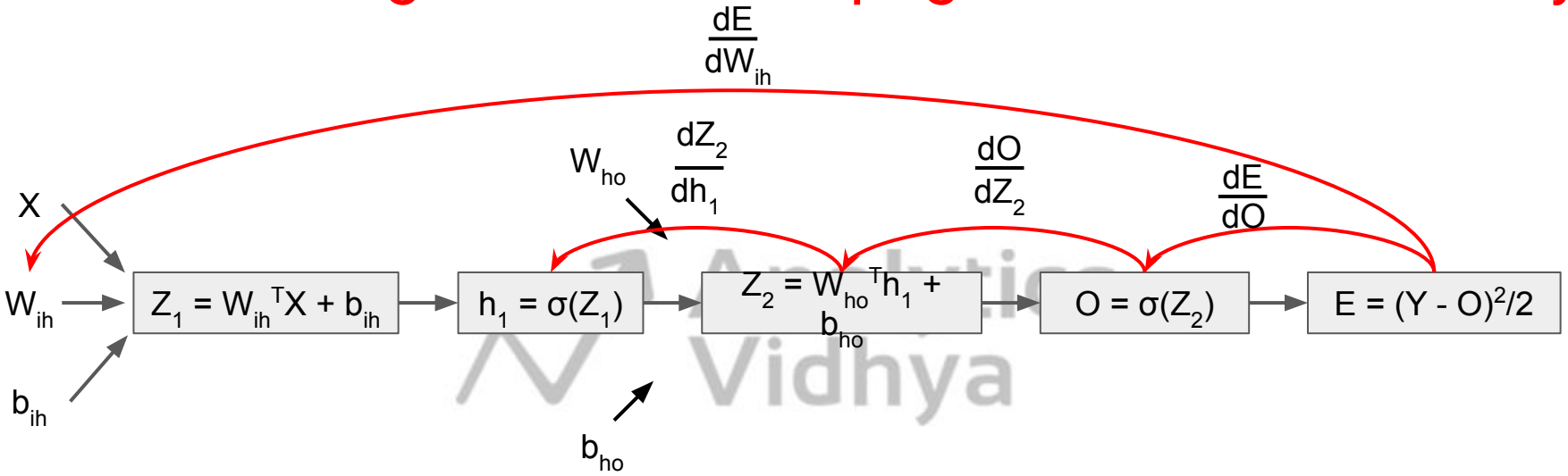

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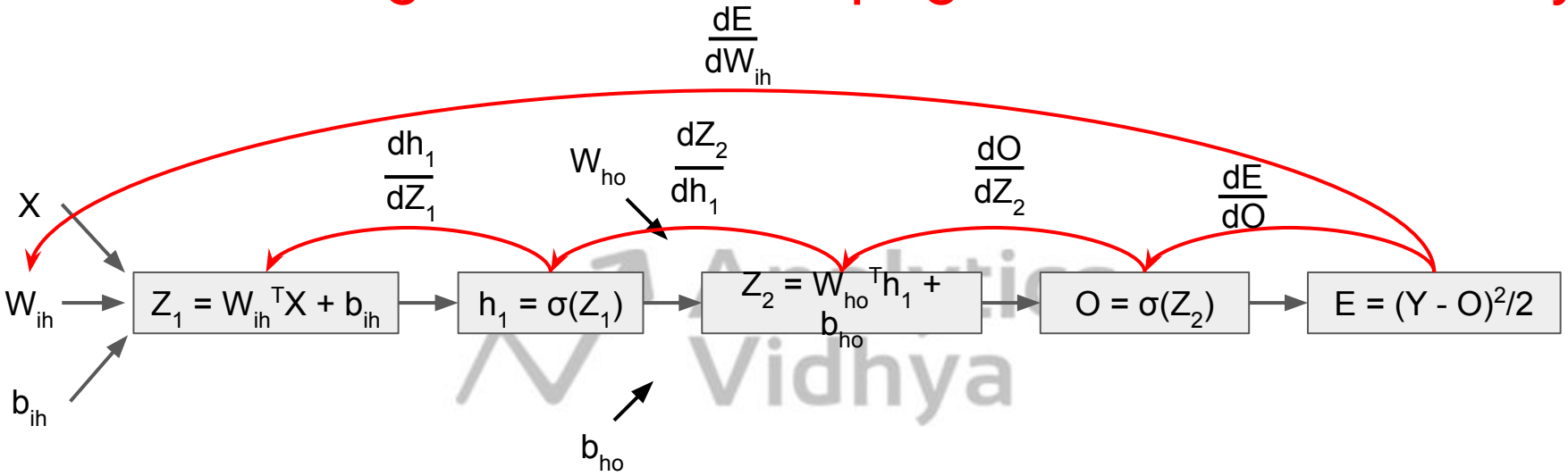

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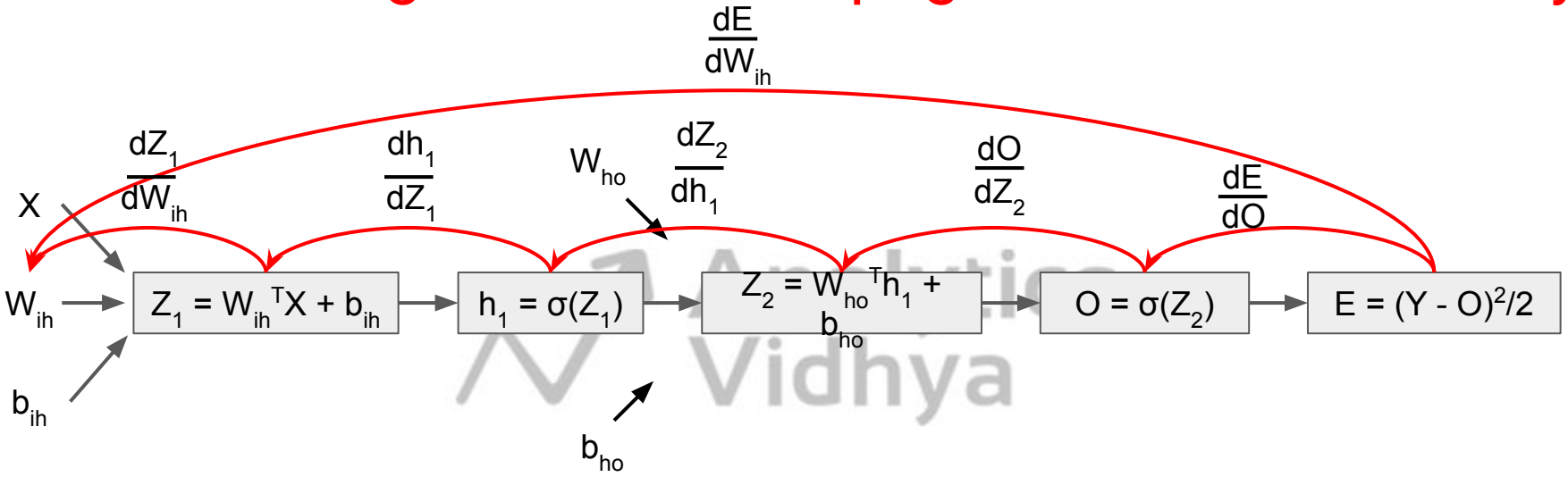

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$$\frac{dE}{dW_{ih}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dh_1} * \frac{dh_1}{dZ_1} * \frac{dZ_1}{dW_{ih}}$$

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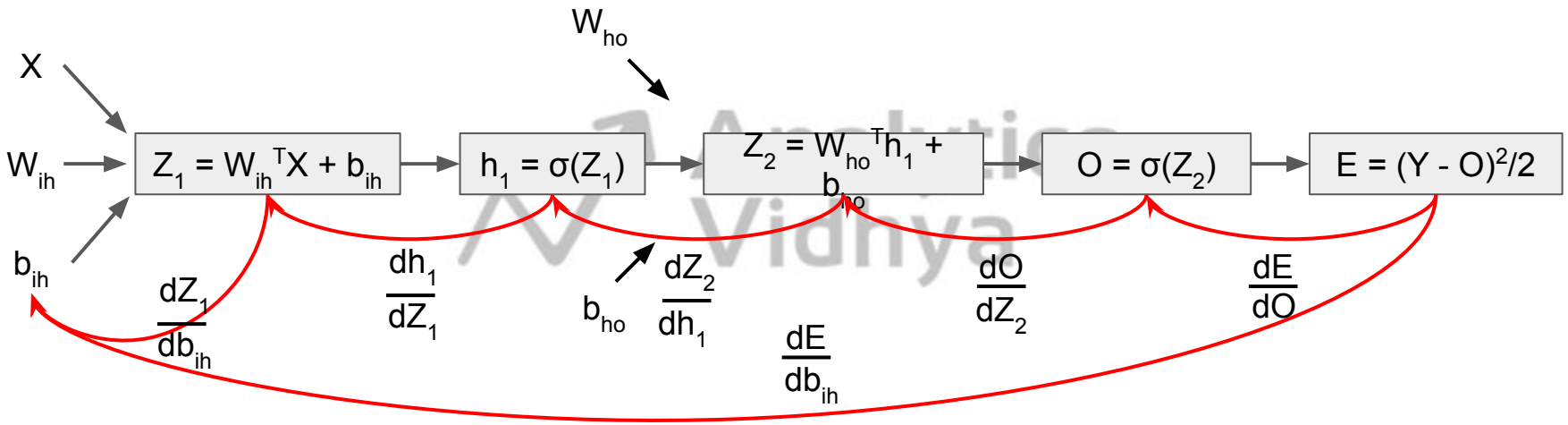

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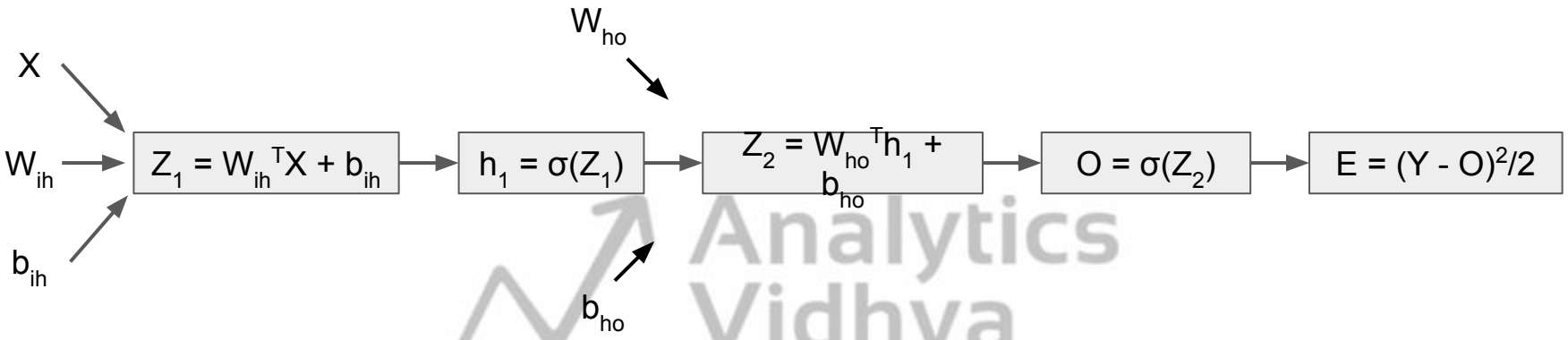

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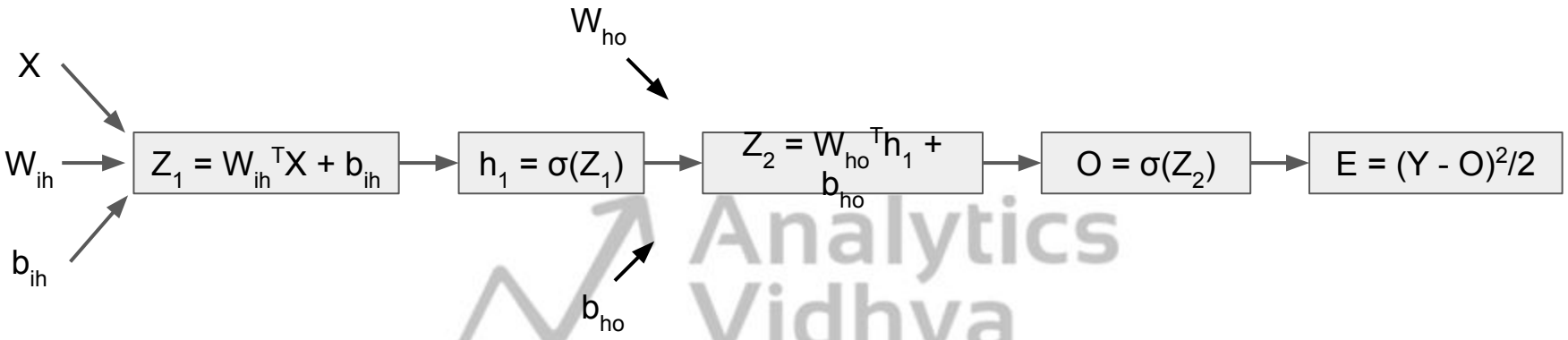

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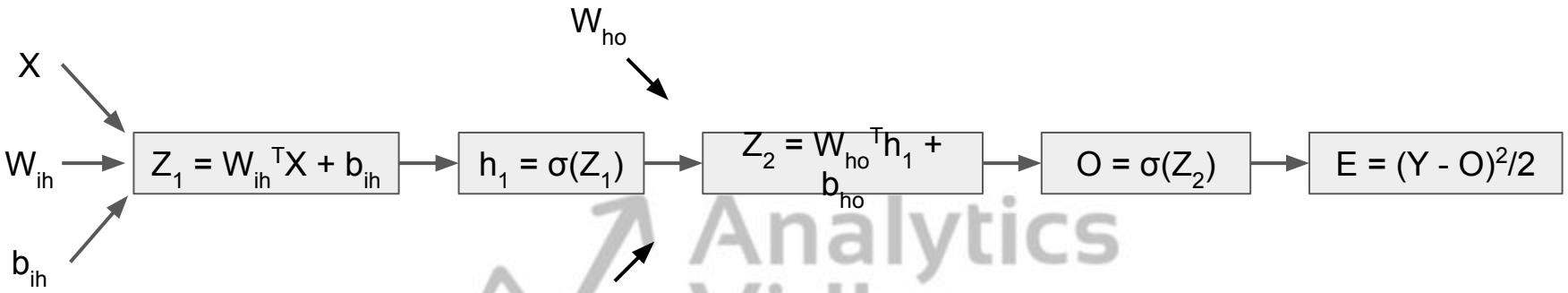

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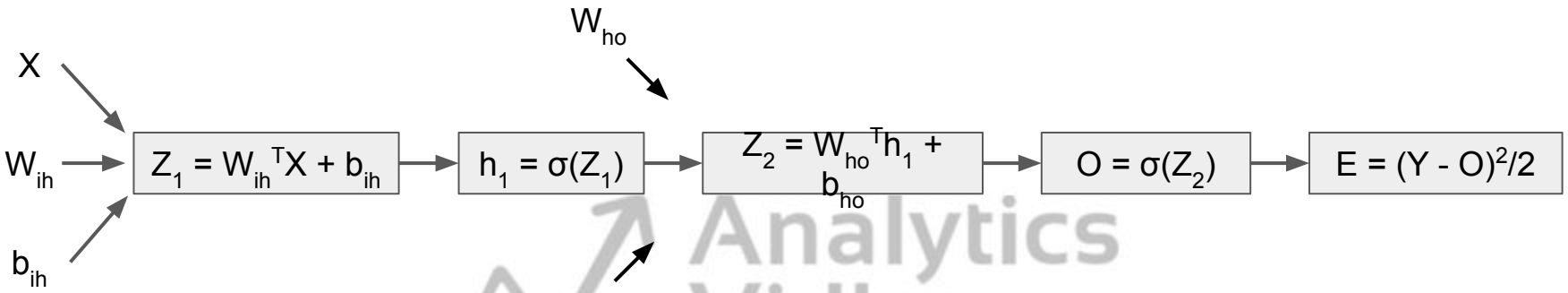

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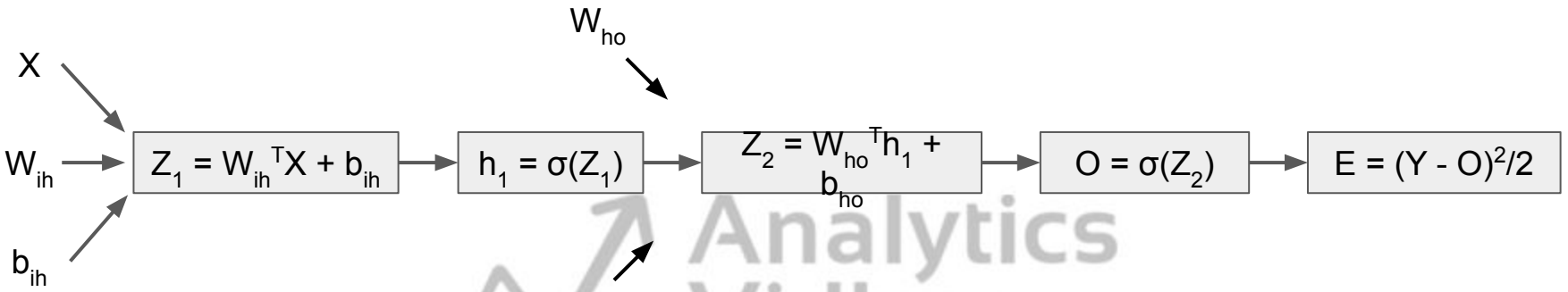

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$$E = (Y - O)^2/2$$

$$\frac{dE}{dO} = \frac{2*(Y-O)*(-1)}{2}$$

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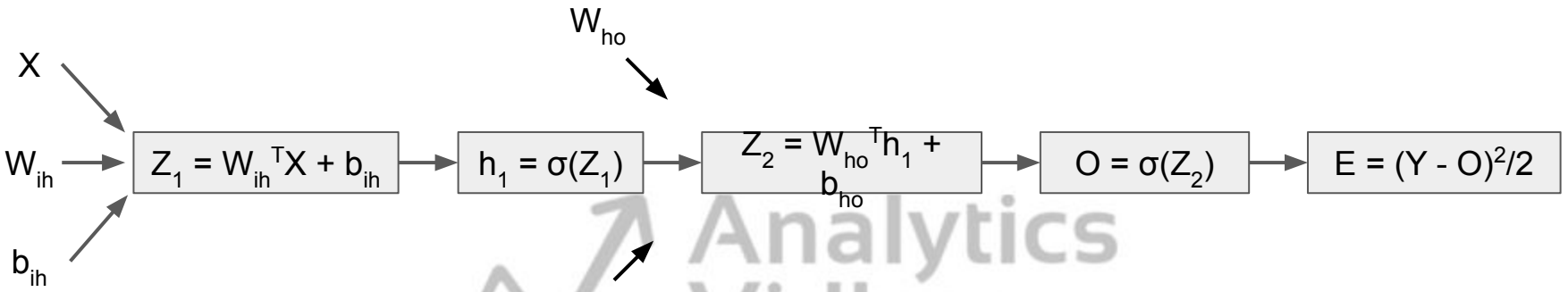

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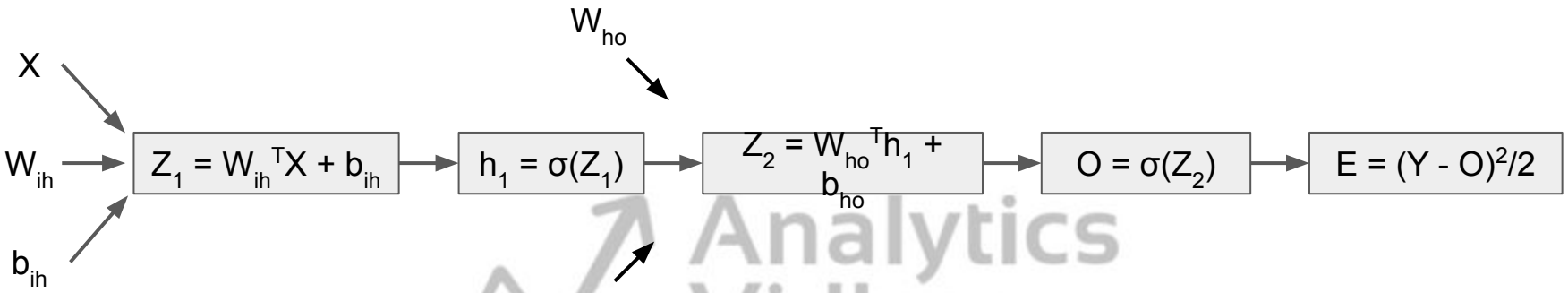

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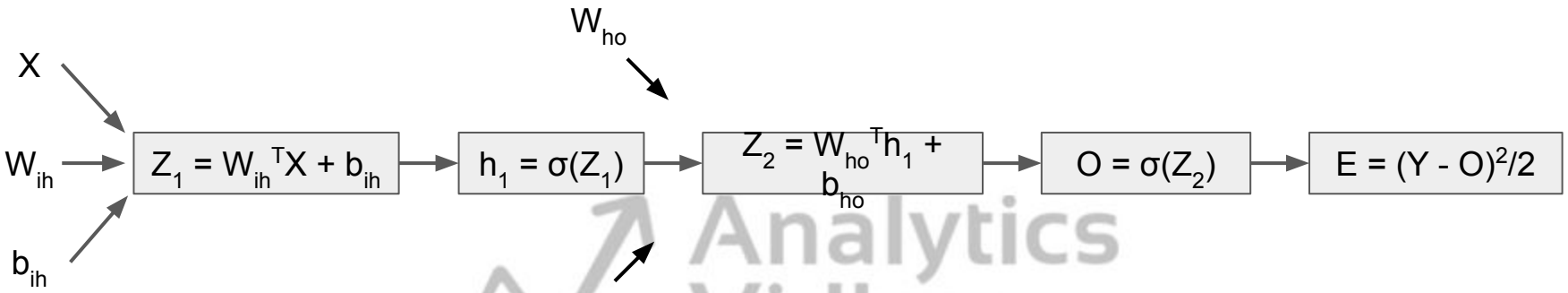

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$$O = \sigma(Z_2)$$

$$\frac{dO}{dZ_2}$$

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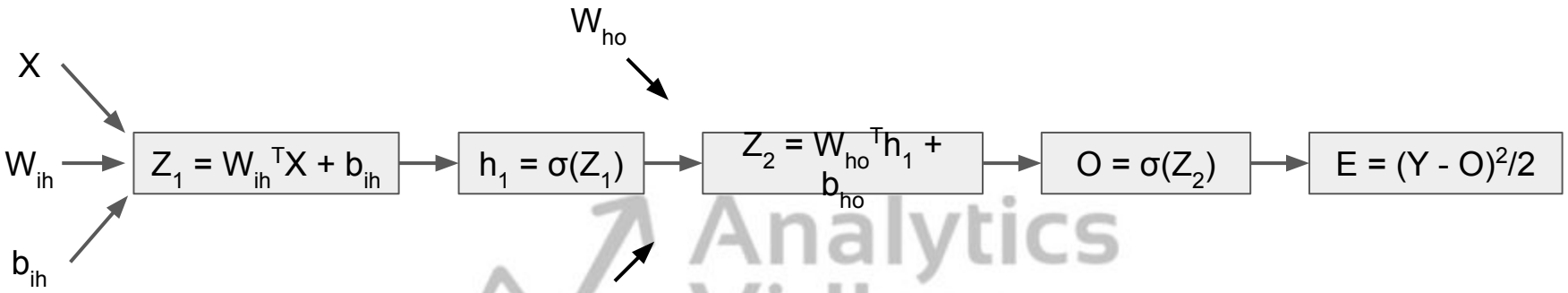

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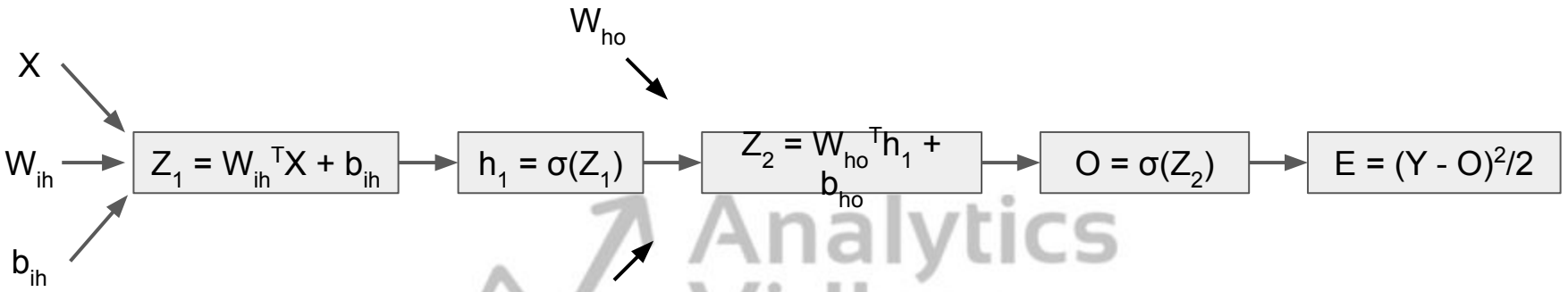

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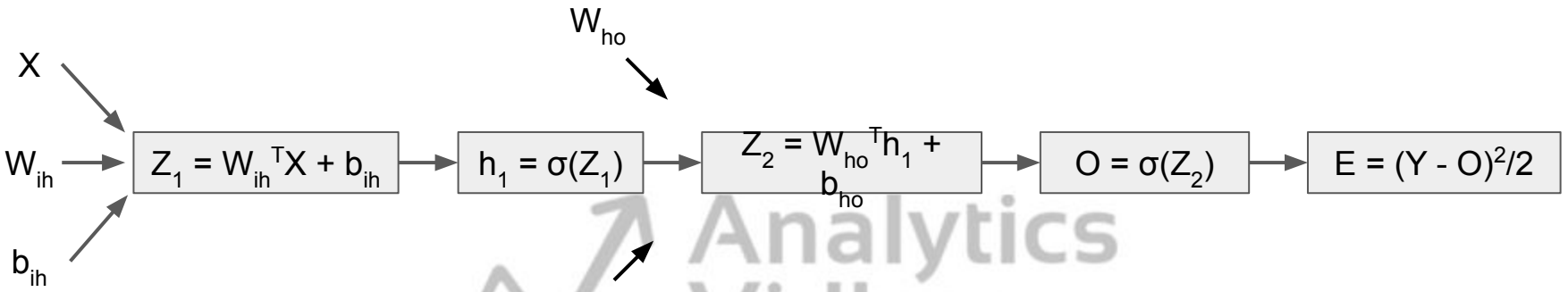

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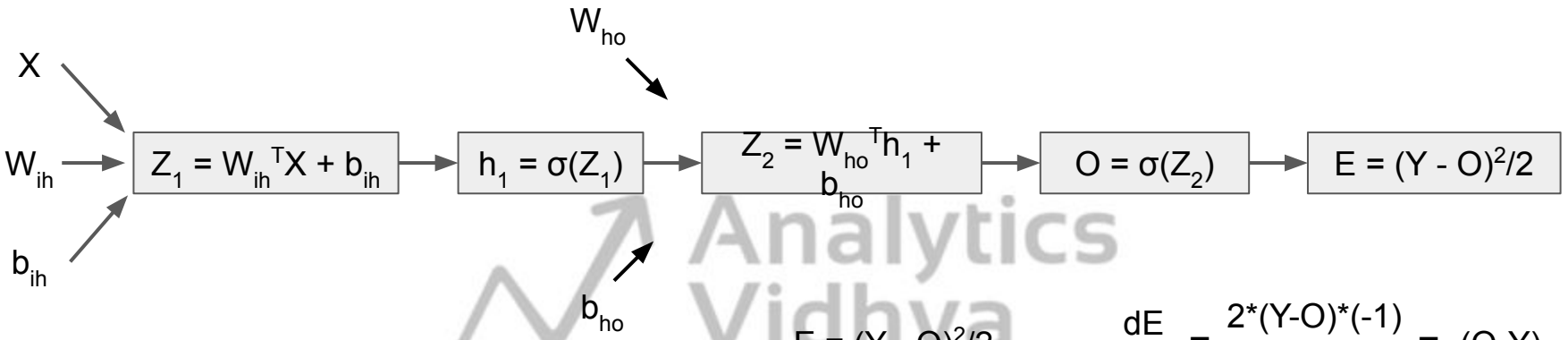

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$Z_2 = W_{ho}^T h_1 + b_{ho}$

$$\frac{dZ_2}{db_{ho}}$$

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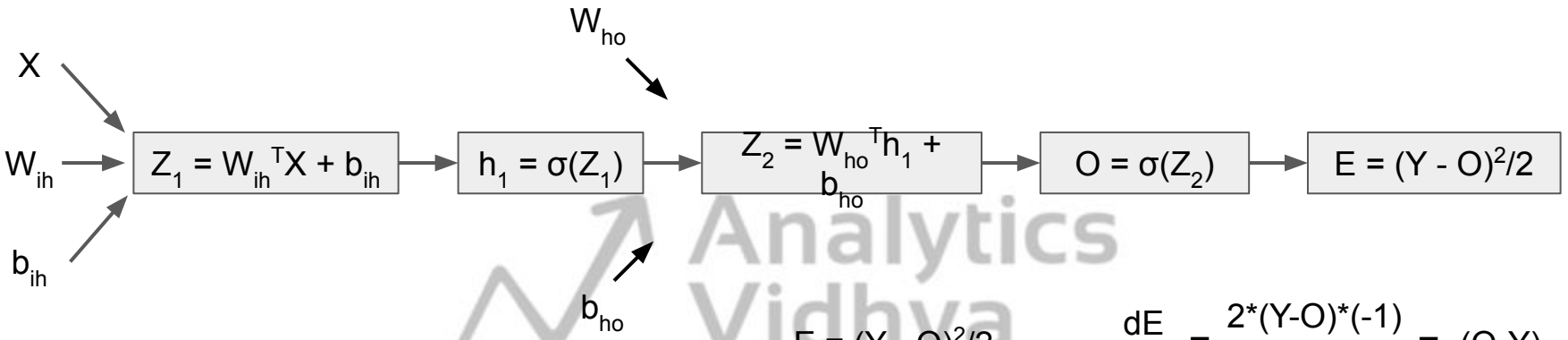

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$Z_2 = W_{ho}^T h_1 + b_{ho}$ $\quad \frac{dZ_2}{db_{ho}} = 1$

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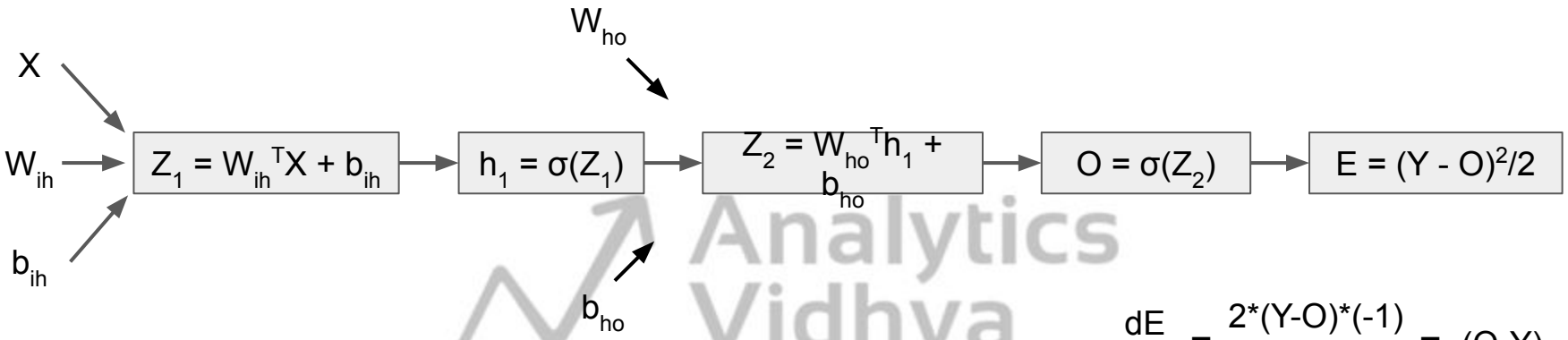

Analytics Vidhya

$$\frac{dE}{db_{ho}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{db_{ho}} = (O-Y) * O(1-O) * 1$$

$$\frac{dE}{dO} = \frac{2*(Y-O)*(-1)}{2} = (O-Y)$$

$$\frac{dO}{dZ_2} = \sigma(Z_2)(1-\sigma(Z_2)) = O(1-O)$$

$$\frac{dZ_2}{db_{ho}} = 1$$

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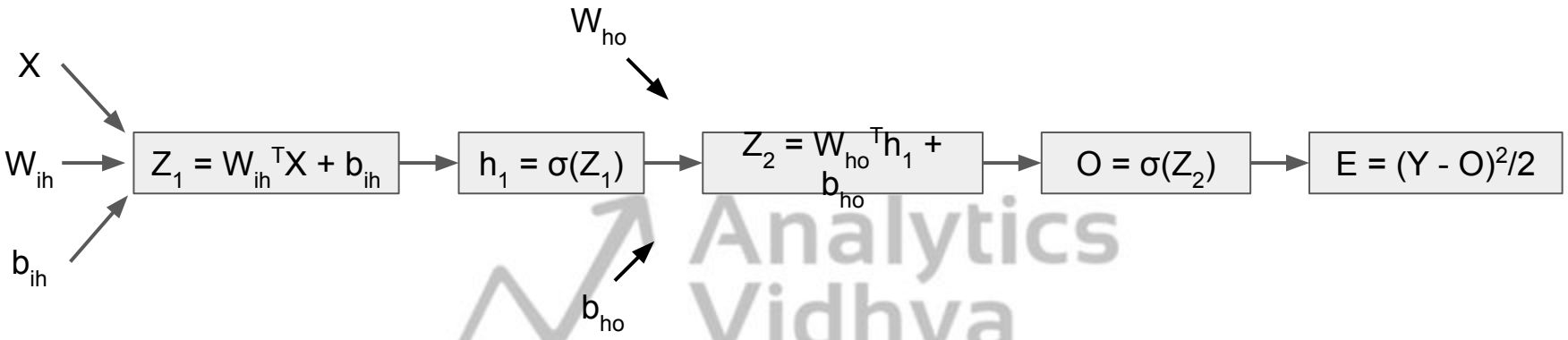

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$$\frac{dE}{db_{ho}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{db_{ho}} = (O-Y) * O(1-O) * 1$$

$$\frac{dE}{dW_{ho}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW_{ho}}$$

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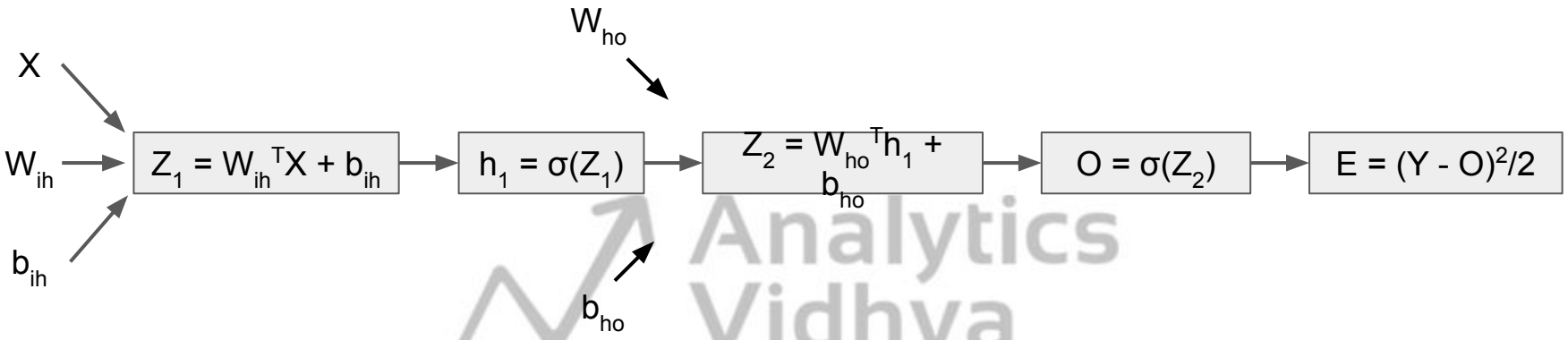

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$$\frac{dE}{db_{ho}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{db_{ho}} = (O-Y) * O(1-O) * 1$$

$$\frac{dE}{dW_{ho}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW_{ho}} = (O-Y) * O(1-O) * h_1$$

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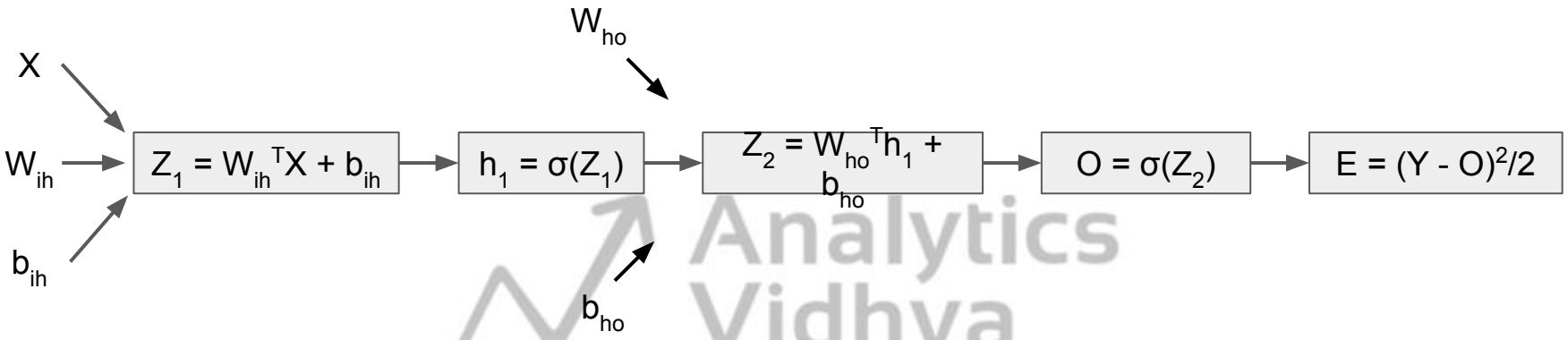

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$$\frac{dE}{dW_{ih}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dh_1} * \frac{dh_1}{dZ_1} * \frac{dZ_1}{dW_{ih}}$$

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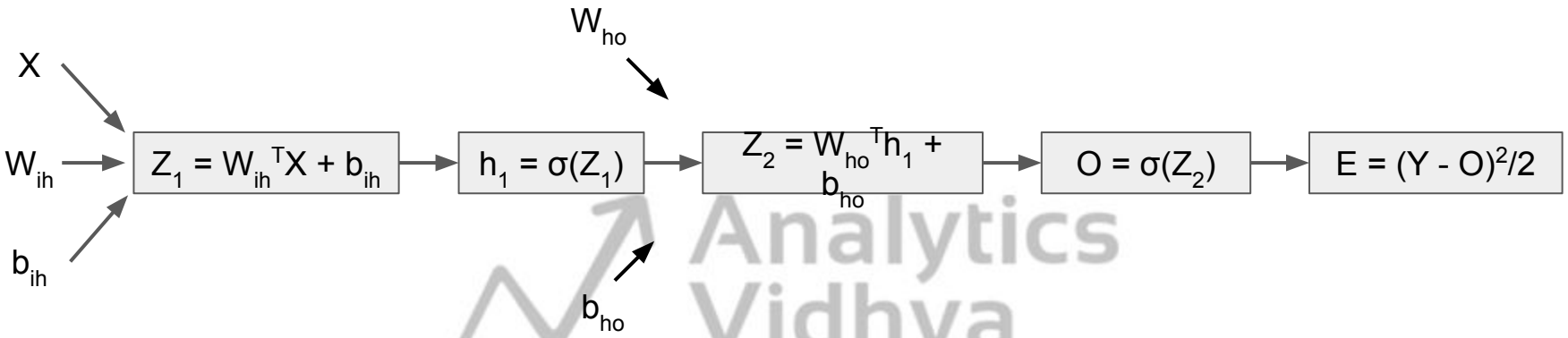

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$$\frac{dE}{dW_{ih}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dh_1} * \frac{dh_1}{dZ_1} * \frac{dZ_1}{dW_{ih}} = (O-Y) * O(1-O) * W_{ho} * h_1(1-h_1) * X$$

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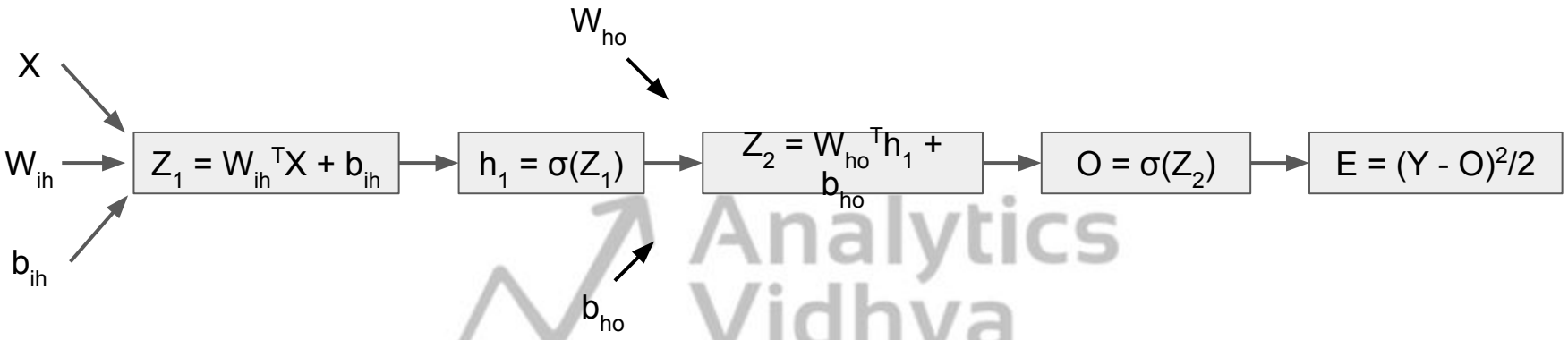

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$$\frac{dE}{dW_{ih}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dh_1} * \frac{dh_1}{dZ_1} * \frac{dZ_1}{dW_{ih}} = (O-Y) * O(1-O) * W_{ho} * h_1(1-h_1) * X$$

$$\frac{dE}{db_{ih}} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dh_1} * \frac{dh_1}{dZ_1} * \frac{dZ_1}{db_{ih}} = (O-Y) * O(1-O) * W_{ho} * h_1(1-h_1) * 1$$

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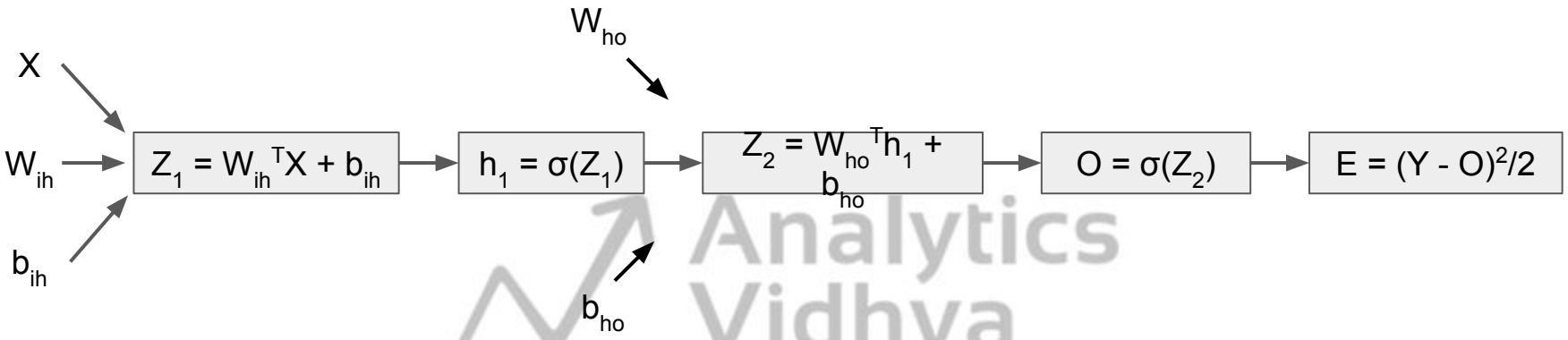
Backward Propagation: Updating the parameters



$$W_{ho} = W_{ho} - \alpha * \frac{dE}{dW_{ho}}$$

$$W_{ih} = W_{ih} - \alpha * \frac{dE}{dW_{ih}}$$

$$b_{ho} = b_{ho} - \alpha * \frac{dE}{db_{ho}}$$

$$b_{ih} = b_{ih} - \alpha * \frac{dE}{db_{ih}}$$

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Thank You

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