Backward Propagation in CNN: Matrix Form

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Forward Propagation in CNN

Input = X Z_1 =conv(X, f) H_1 = sigmoid(Z_1) $Z_2 = W^T . H_1 + b$ O = sigmoid(Z_2)

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Forward Propagation in CNN

Input = X $\quad Z_1 = conv(X, f)$ $\quad H_1 = sigmoid(Z_1)$ $\quad Z_2 = W^T . H_1 + b$ $\quad O = sigmoid(Z_2)$

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X = [100x22x22] $\quad f$ = [3 x 3] $\quad H_1$ = [100x20x20] $\quad H_1$ = [400 x 100] $\quad W^T$ = [1 x 400] $\quad O$ = [1 x 100]

Z_1 = [100x20x20] $\quad Z_2$ = [1 x 100]

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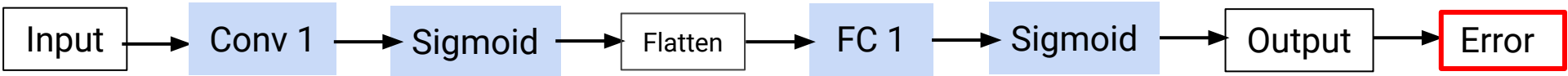
Backward Propagation in CNN



Input = X $\quad Z_1 = \text{conv}(X, f) \quad H_1 = \text{sigmoid}(Z_1) \quad Z_2 = W^T \cdot H_1 + b \quad O = \text{sigmoid}(Z_2) \quad E = \frac{(y - O)^2}{2}$

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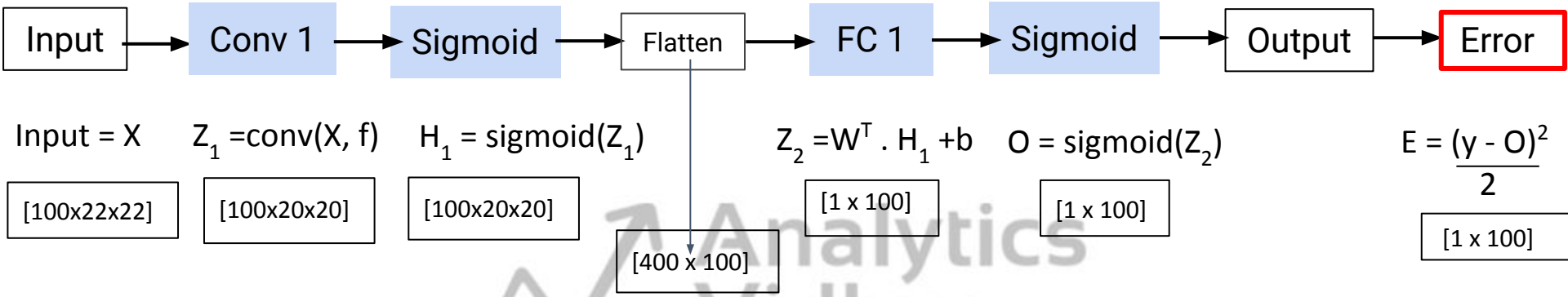
Backward Propagation in CNN



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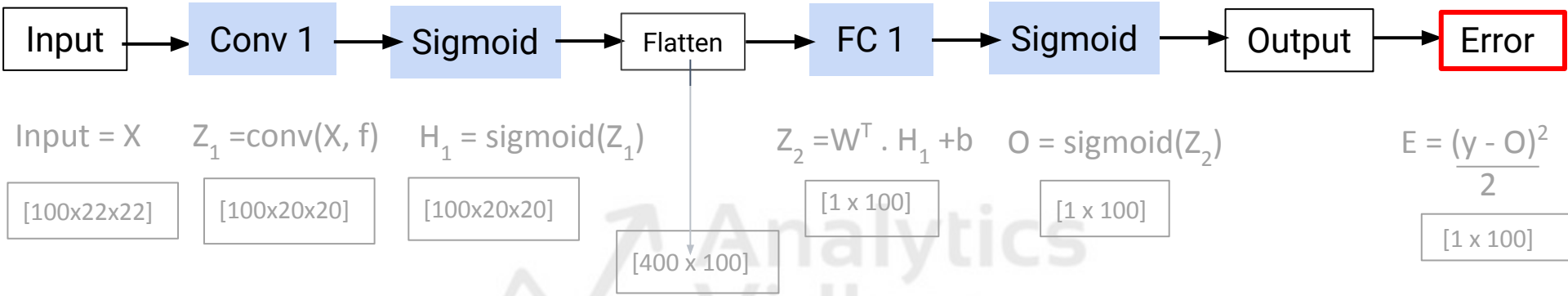

$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

$$\frac{dE}{db} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{db} = (O-Y) * O(1-O) * 1$$

$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1), X\right)$$

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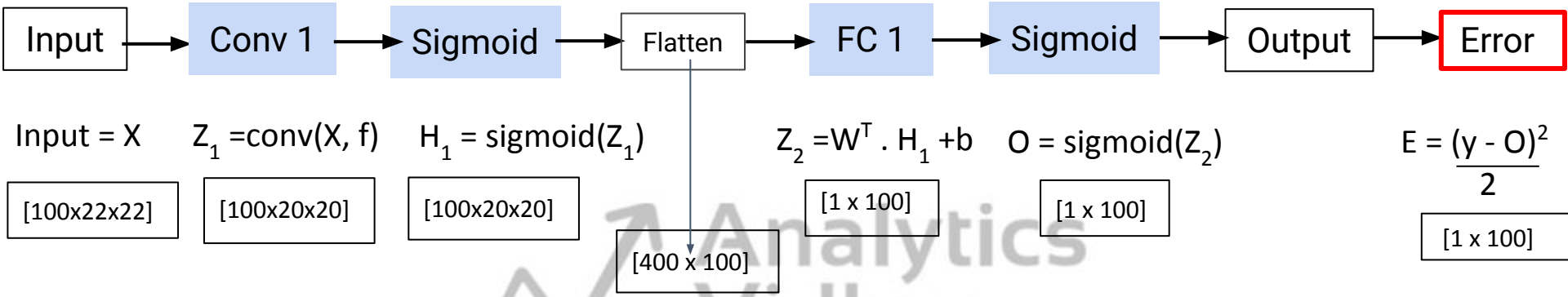

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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

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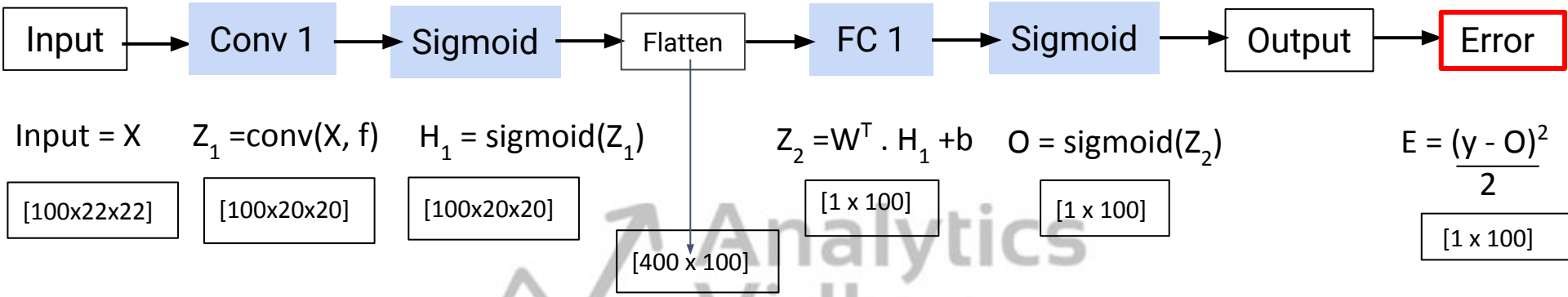

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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

$$= (1X100)$$

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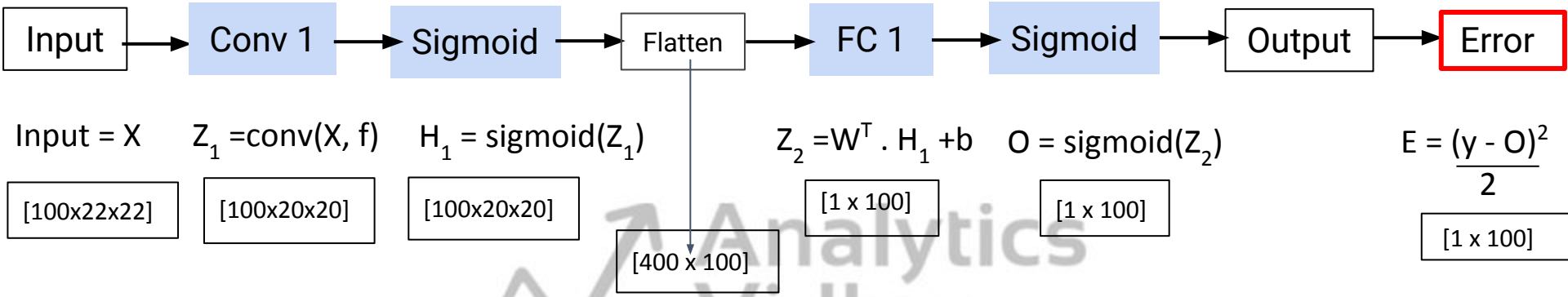

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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * \mathbf{O(1-O)} * H_1$$

$$= \mathbf{(1X100)} * \mathbf{(1X100)}$$

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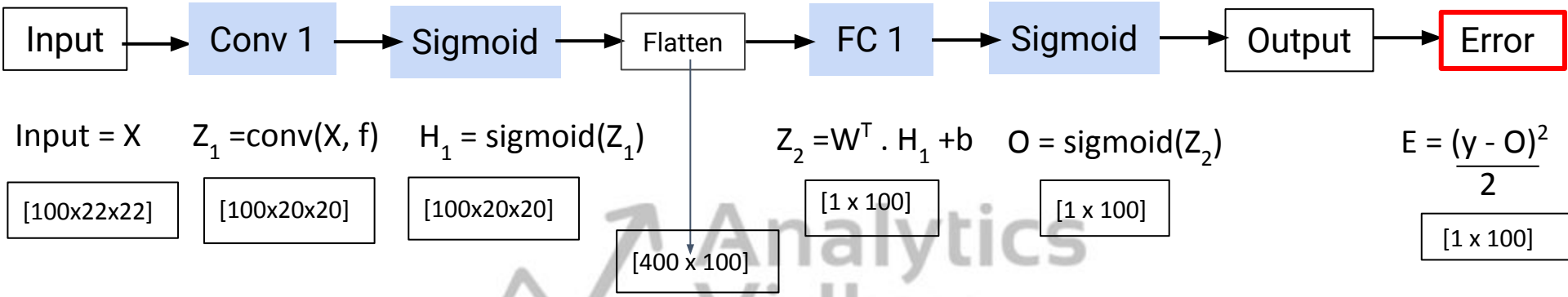
Backward Propagation in CNN



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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

$$= (1X100) * (1X100) * (400X100)$$

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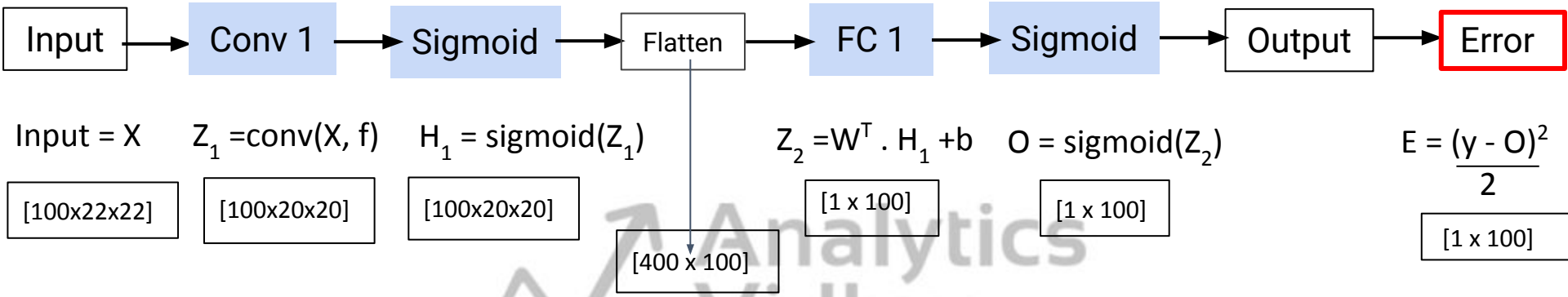
Backward Propagation in CNN



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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

$$= (1X100) * (1X100) * (400X100)$$

$$= \mathbf{(1X100)} * (400X100)$$

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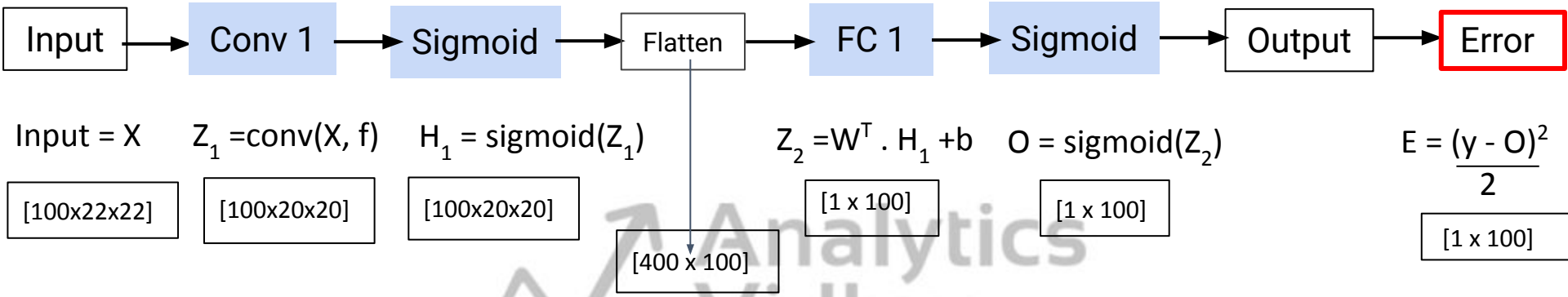
Backward Propagation in CNN



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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

(400X1)

$$= (1X100) * (1X100) * (400X100)$$

$$= \mathbf{(1X100)} * \mathbf{(400X100)}$$

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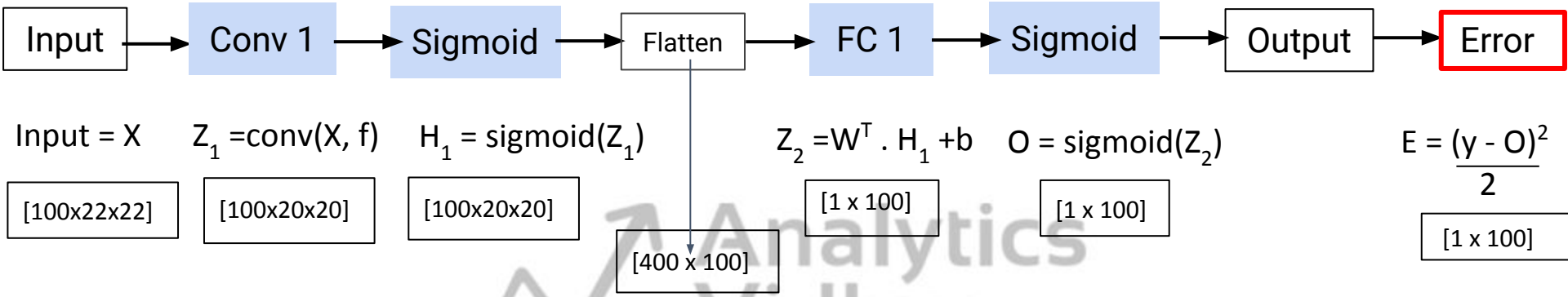
Backward Propagation in CNN



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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

$$\frac{dZ_2}{dW} X \left[\frac{dE}{dO} * \frac{dO}{dZ_2}\right]^T = \textbf{(400X100) * (100X1)}$$

(400X1)

$$= \underbrace{(1X100) * (1X100)}_{(1X100)} * (400X100)$$

$$= (1X100) * (400X100)$$

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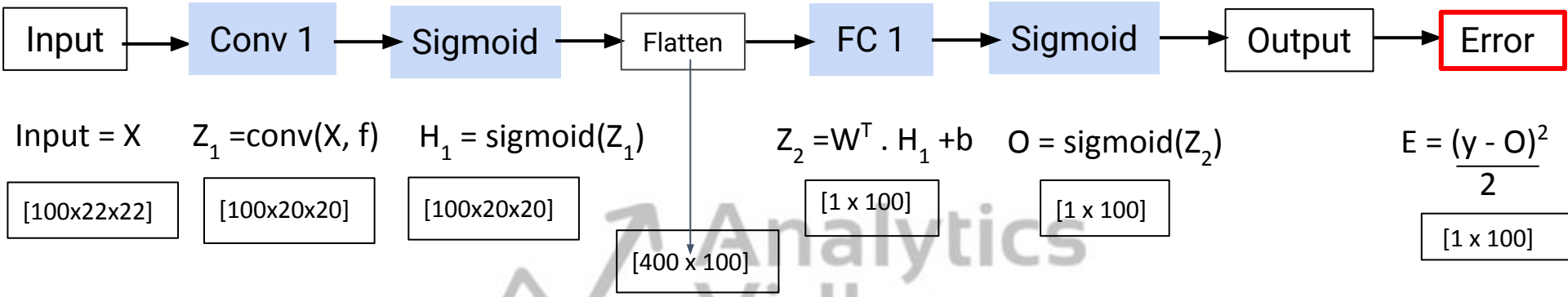

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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

$$\frac{dZ_2}{dW} X \left[\frac{dE}{dO} * \frac{dO}{dZ_2}\right]^T = (400X100) * (100X1)$$

(400X1)

$$= (1X100) * (1X100) * (400X100)$$

$$= (1X100) * (400X100)$$

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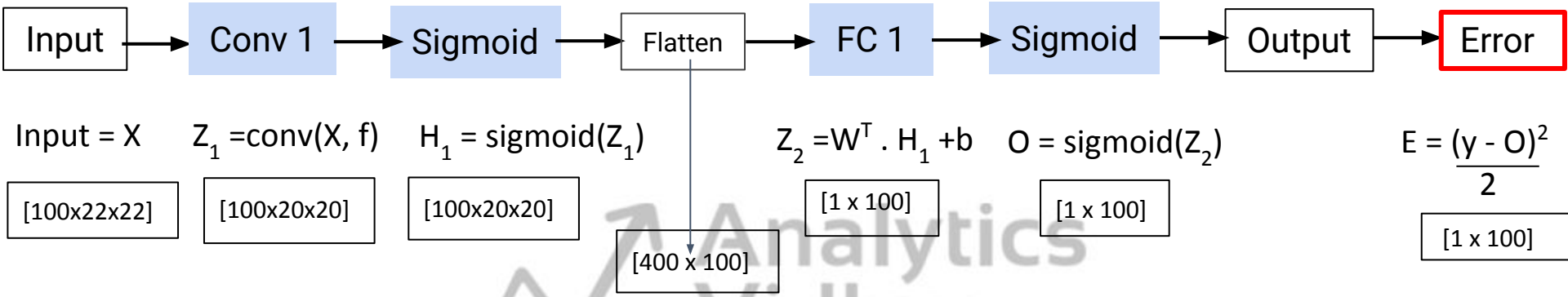

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$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * H_1$$

$$\frac{dZ_2}{dW} \; X \left[\frac{dE}{dO} * \frac{dO}{dZ_2}\right]^T = (400X100) * (100X1)$$

$$= \textbf{(400X1)}$$

(400X1)

$$= (1X100) * (1X100) * (400X100)$$

$$= (1X100) * (400X100)$$

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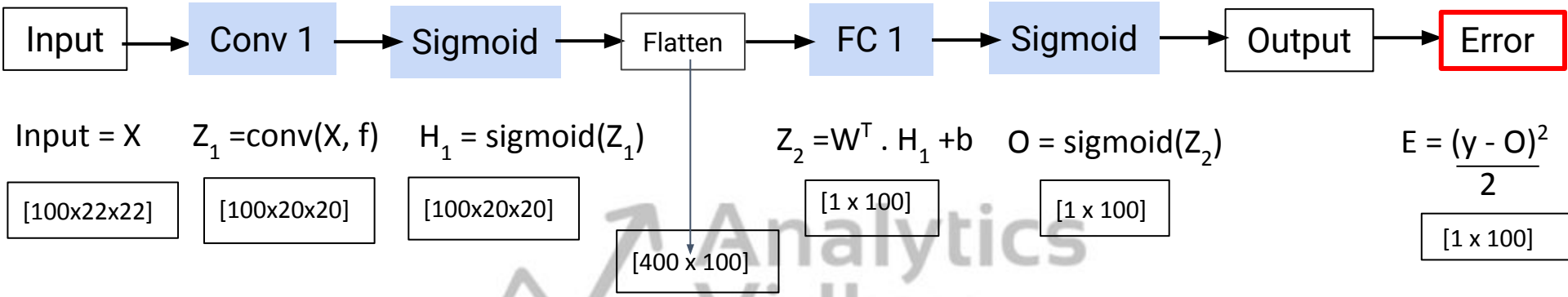

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$$\frac{dE}{dW} = \frac{dZ_2}{dW} X \left[\frac{dE}{dO} * \frac{dO}{dZ_2}\right]^T$$

$$\frac{dE}{db} = \frac{dZ_2}{db_{ho}} X \left[\frac{dE}{dO} * \frac{dO}{dZ_2}\right]^T$$

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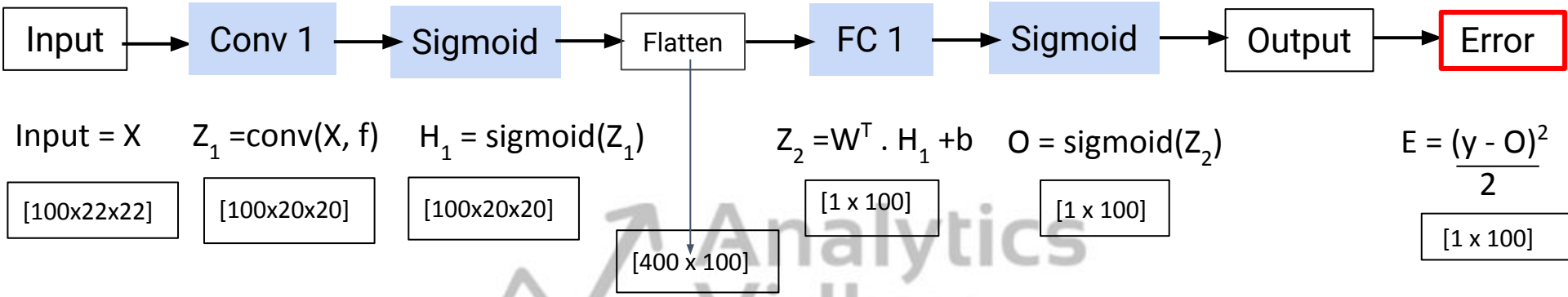

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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1),\ X\right)$$

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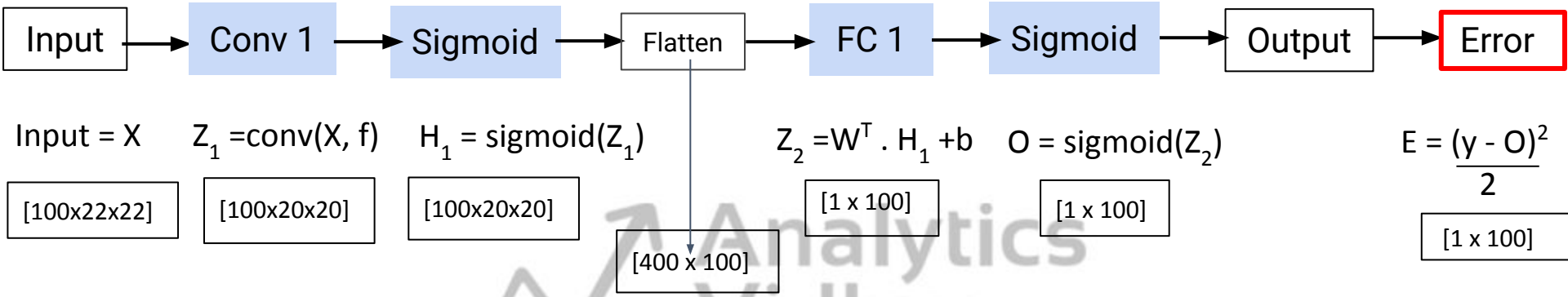
Backward Propagation in CNN



$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1),\ X\right)$$

$$= \quad (1X100)$$

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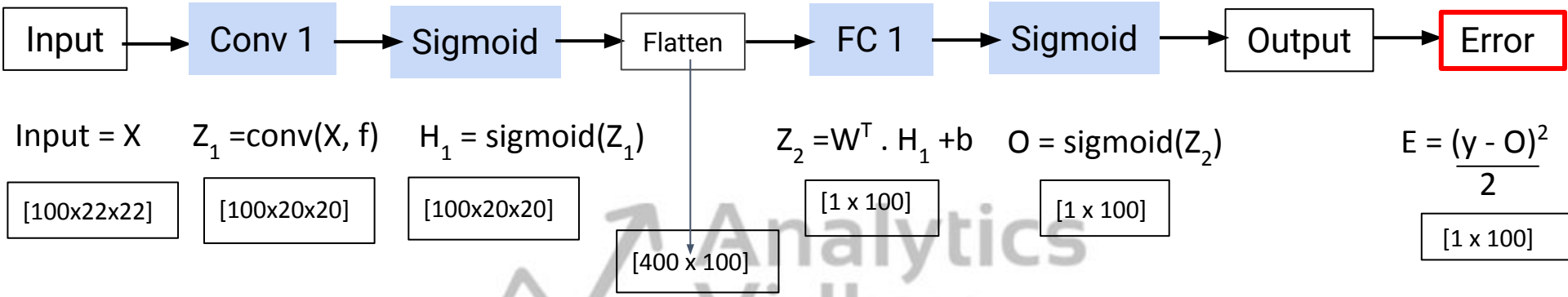
Backward Propagation in CNN



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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1),\ X\right)$$

$$= \quad (1X100) * (1X100)$$

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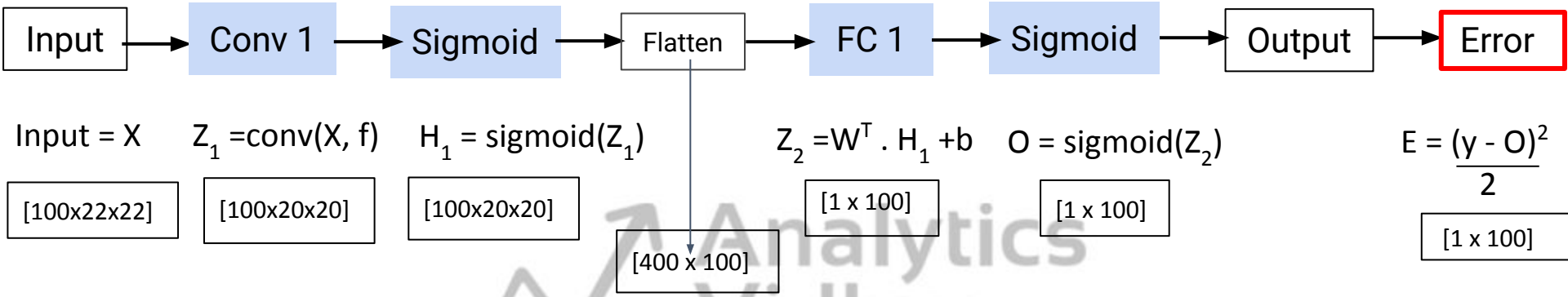

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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1), X\right)$$

$$= \quad (1X100) * (1X100) * (1X400)$$

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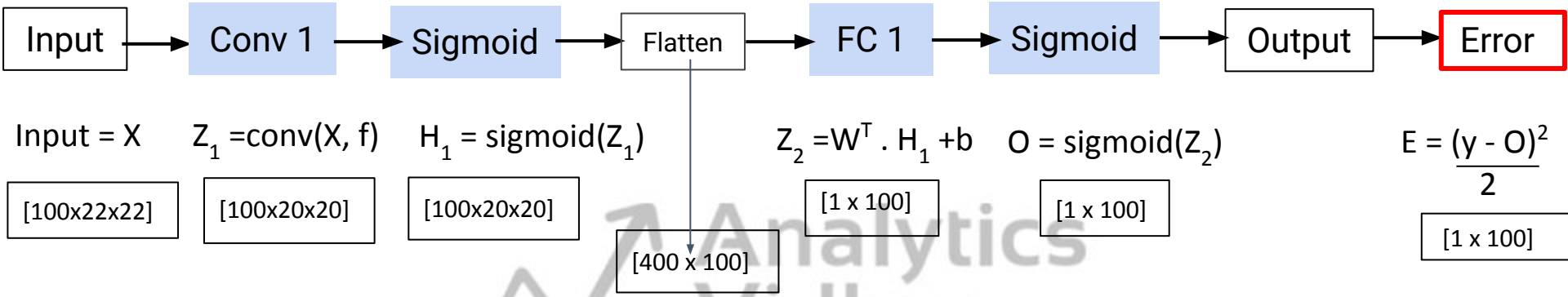

$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1), X\right)$$

$$= \quad (1X100) * (1X100) * (1X400) * (400X100)$$

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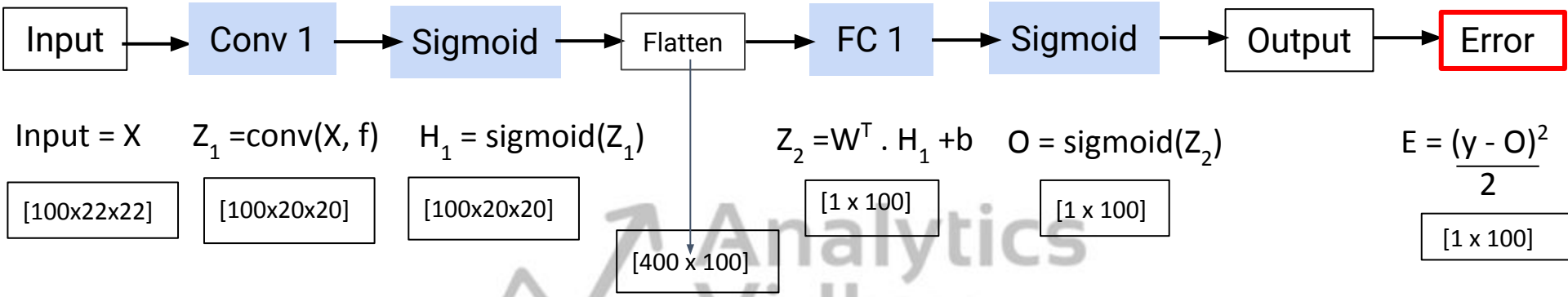

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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1),\ \mathbf{X}\right)$$

$$= \textbf{conv}((1X100) * (1X100) * (1X400) * (400X100), (100x22x22))$$

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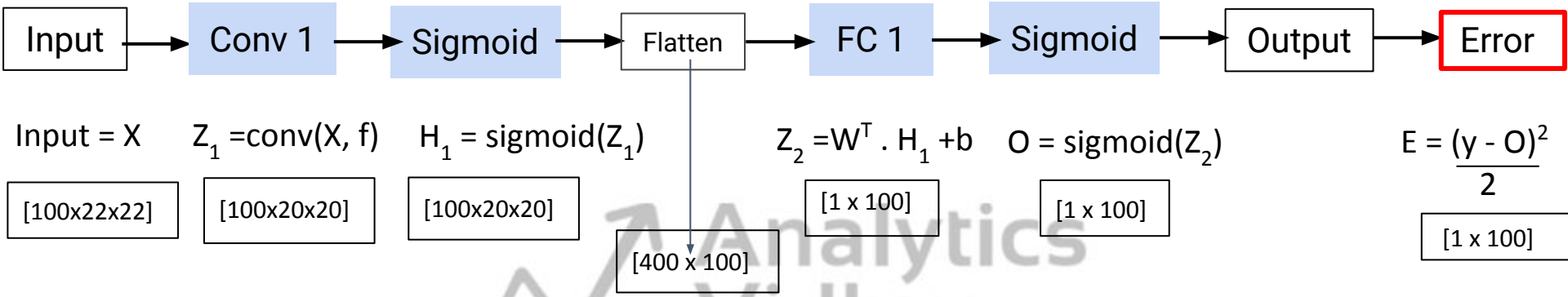

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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y)^* O(1-O)^* W^T * h_1(1-h_1),\ X\right)$$

$$= \text{conv}((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))$$

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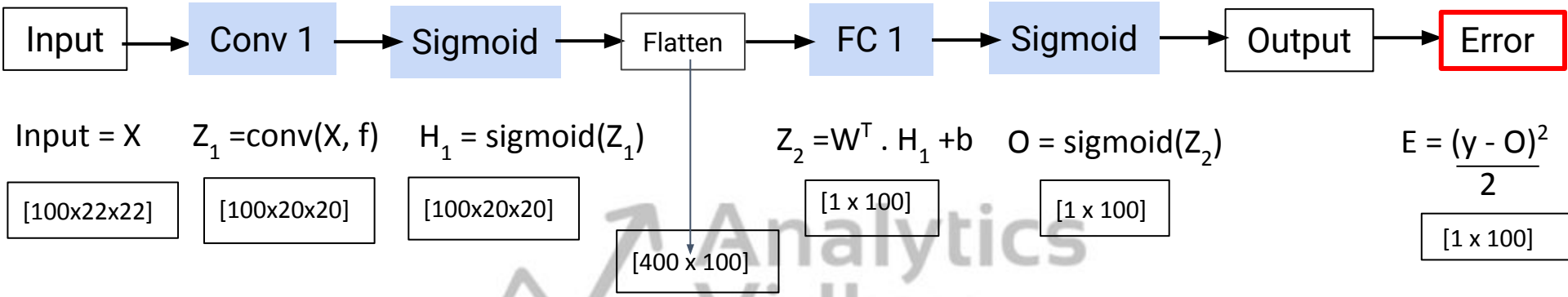

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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y)^* O(1-O)^* W^T * h_1(1-h_1),\ X\right)$$

$$= \text{conv}(\mathbf{(1X100)} * \mathbf{(1X100)} * (1X400) * (400X100), (100x22x22))$$

$$= \text{conv}\ \mathbf{((1X100)} * (1X400) * (400X100), (100x22x22))$$

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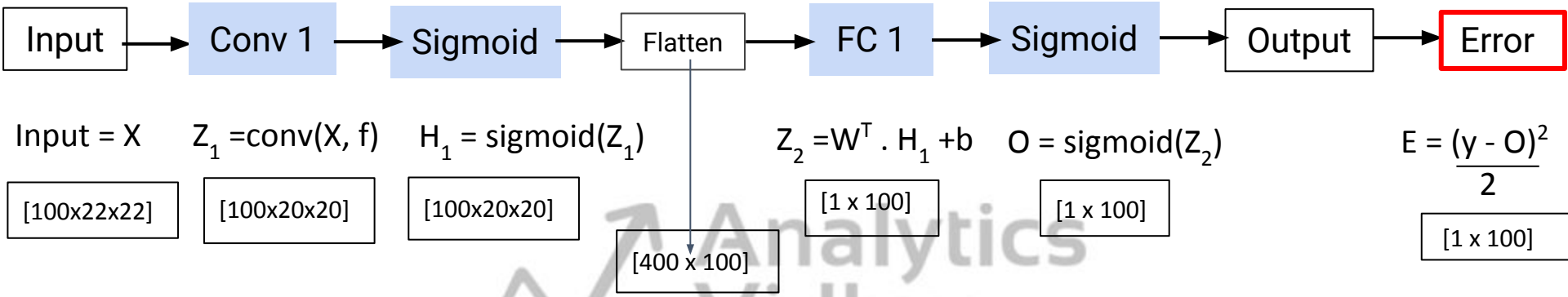

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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y)^* O(1-O)^* W^T * h_1(1-h_1),\ X\right)$$

$$= \text{conv}((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))$$

$$= \text{conv}((1X100) * (1X400)* (400X100), (100x22x22))$$

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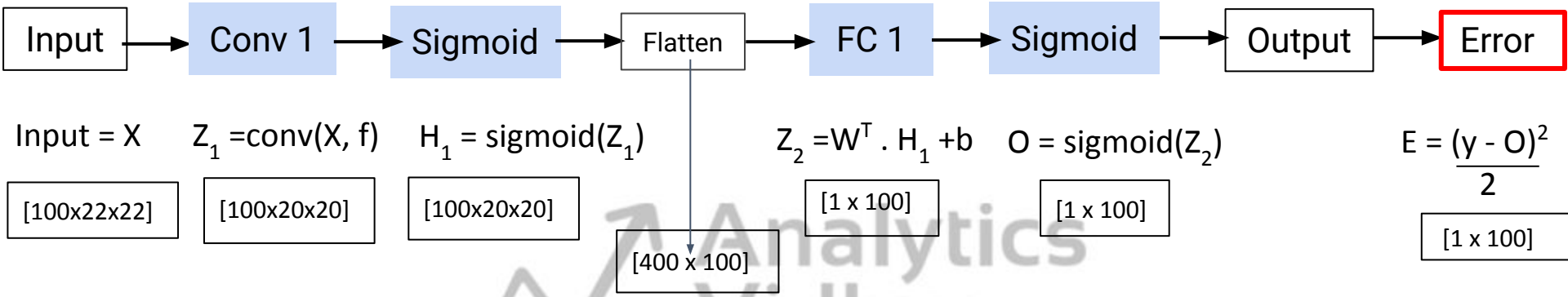

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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = conv\left((O-Y) * O(1-O) * W^T * h_1(1-h_1),\ X\right)$$

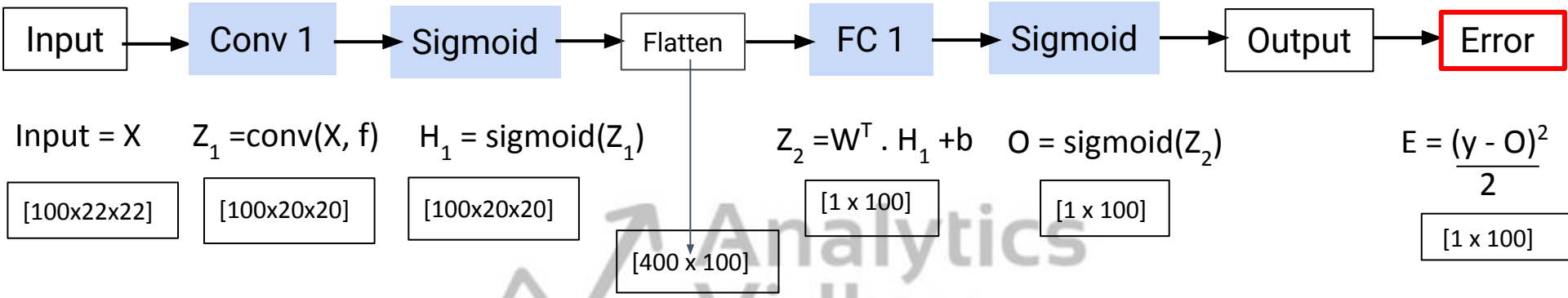
$$\left[\frac{dZ_2}{dH_1}\right]^T X \left[\frac{dE}{dO} * \frac{dO}{dZ_2}\right]$$

= conv((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))

= conv (**(400X1) * (1X100)** * (400X100), (100x22x22))

= conv **((400 x 100)** * (400X100), (100x22x22))

Backward Propagation in CNN



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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y)^* O(1-O)^* W^T * h_1(1-h_1),\ X\right)$$

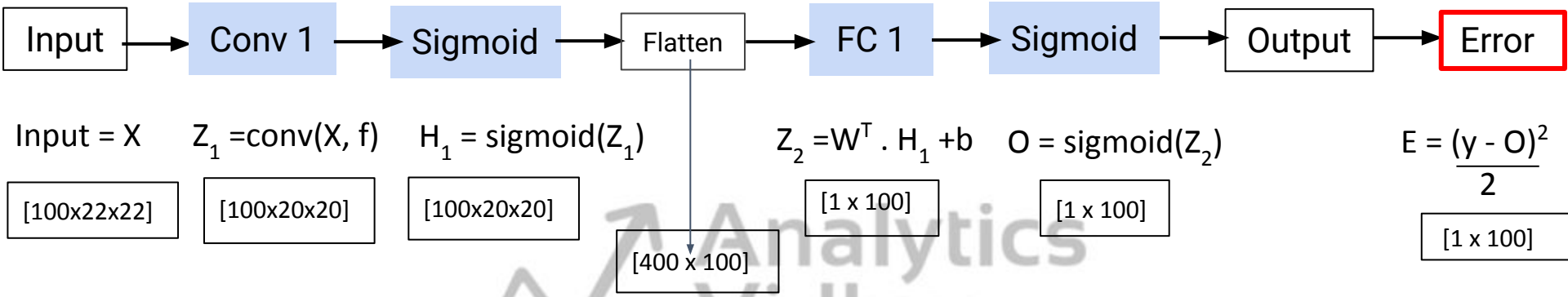
= conv((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))

= conv ((1X100) * (1X400)* (400X100), (100x22x22))

= conv **((400 x 100)** * (**400X100**), (100x22x22))

= conv (**(400 x 100)** , (100x22x22))

Backward Propagation in CNN



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$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1),\ X\right)$$

(3x3)

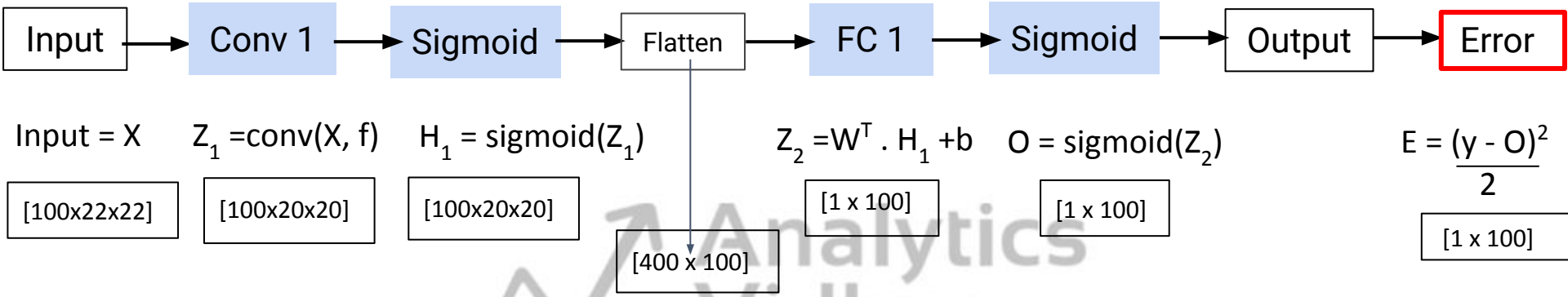
= conv((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))

= conv ((1X100) * (1X400)* (400X100), (100x22x22))

= conv ((400 x 100) * (400X100), (100x22x22))

= conv (**(400 x 100)** , (100x22x22))

Backward Propagation in CNN



Analytics Vidhya

$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y)^* O(1-O)^* W^T * h_1(1-h_1),\ X\right)$$

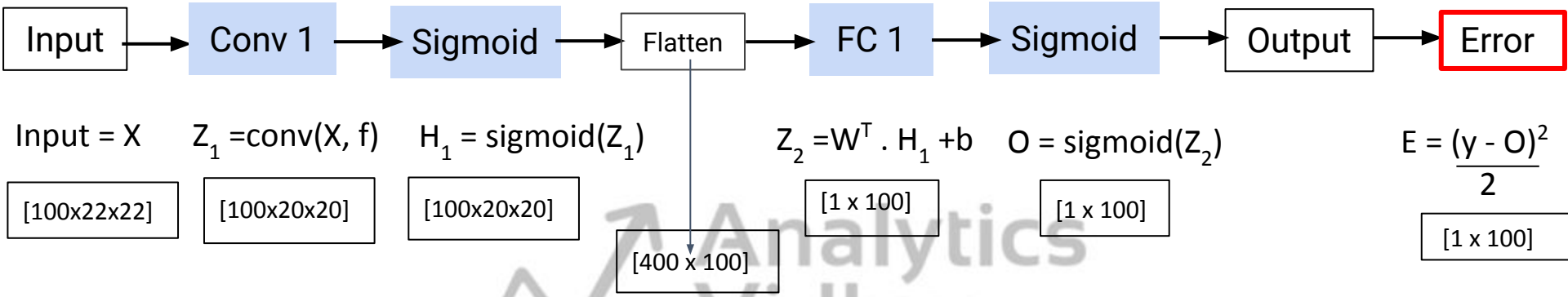
= conv((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))

= conv ((1X100) * (1X400)* (400X100), (100x22x22))

= conv ((400 x 100) * (400X100), (100x22x22))

Reshape = conv (**(20 x 20 x 100) , (22x22x100)**)

Backward Propagation in CNN



Analytics Vidhya

$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y)^* O(1-O)^* W^T * h_1(1-h_1),\ X\right)$$

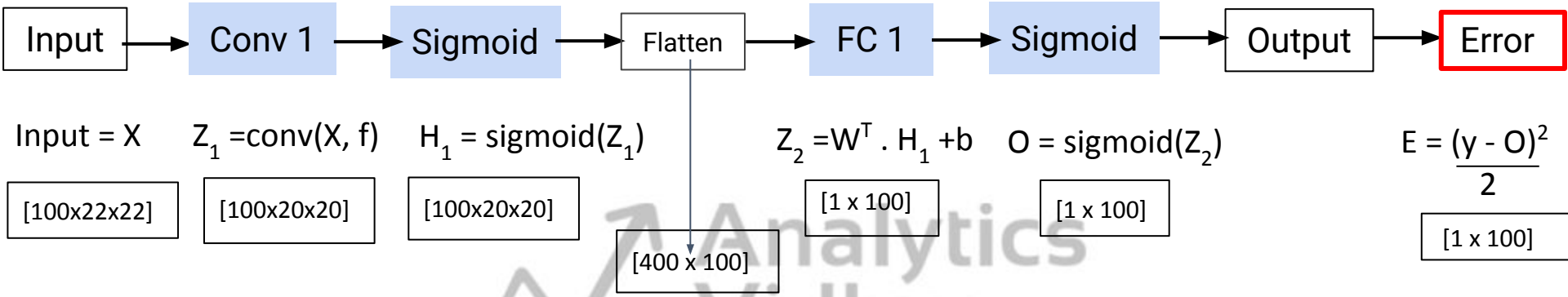
= conv((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))

= conv ((1X100) * (1X400)* (400X100), (100x22x22))

= conv ((400 x 100) * (400X100), (100x22x22))

= conv ((**20 x 20 x 100**) , (**22x22x100**))

Backward Propagation in CNN



Analytics Vidhya

$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y)^* O(1-O)^* W^T * h_1(1-h_1),\ X\right)$$

(3x3)

= conv((1X100) * (1X100) * (1X400)* (400X100), (100x22x22))

= conv ((1X100) * (1X400)* (400X100), (100x22x22))

= conv ((400 x 100) * (400X100), (100x22x22))

= conv ((22 x 22 x 100) , (20x20x100)) = **(3x3)**

Thank You!

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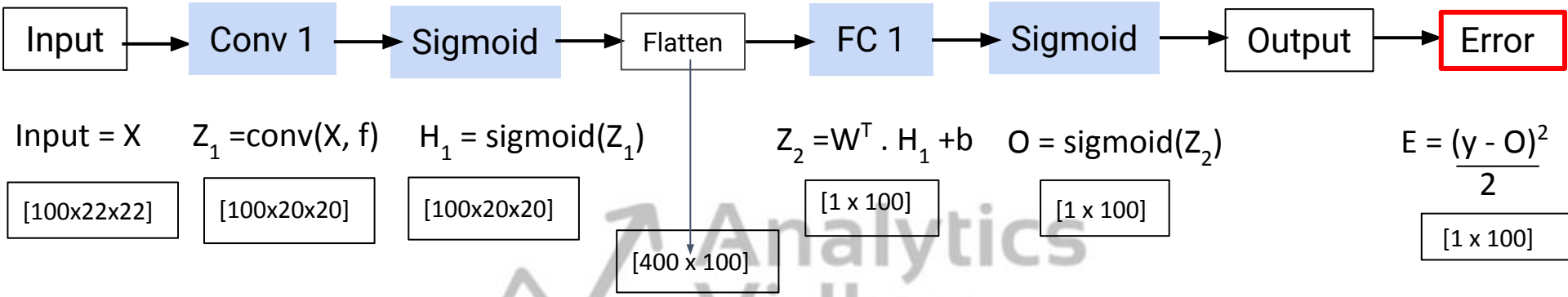
Backward Propagation in CNN



Analytics Vidhya

$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = \text{conv}\left((O-Y) * O(1-O) * W^T * h_1(1-h_1), X\right)$$

(3X3)

conv((20X20), 22 x 22)

= conv((1X100) * (1X100) * (400X100) * (400X100), * X)

= conv ((1X100) *

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Forward Propagation in CNN

Analytics Vidhya

Input = X

$Z_1 = X * f$

$H_1 = \text{sigmoid}(Z_1)$

$Z_2 = W^T . H_1 + b$

$O = \text{sigmoid}(Z_2)$

X = [100x22x22]

f = [5 x 3 x 3]

Z_1 = [100x20x20]

H_1 = [100x20x20x3]

H_1 = [1200 x 100]

W^T = [1200 x 1]

Z_2 = [1 x 100]

O = [1 x 100]

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Parameters to be Updated

Input = X $\quad Z_1 = X * f \quad H_1 = \text{sigmoid}(Z_1) \quad Z_2 = W^T \cdot H_1 + b \quad O = \text{sigmoid}(Z_2)$

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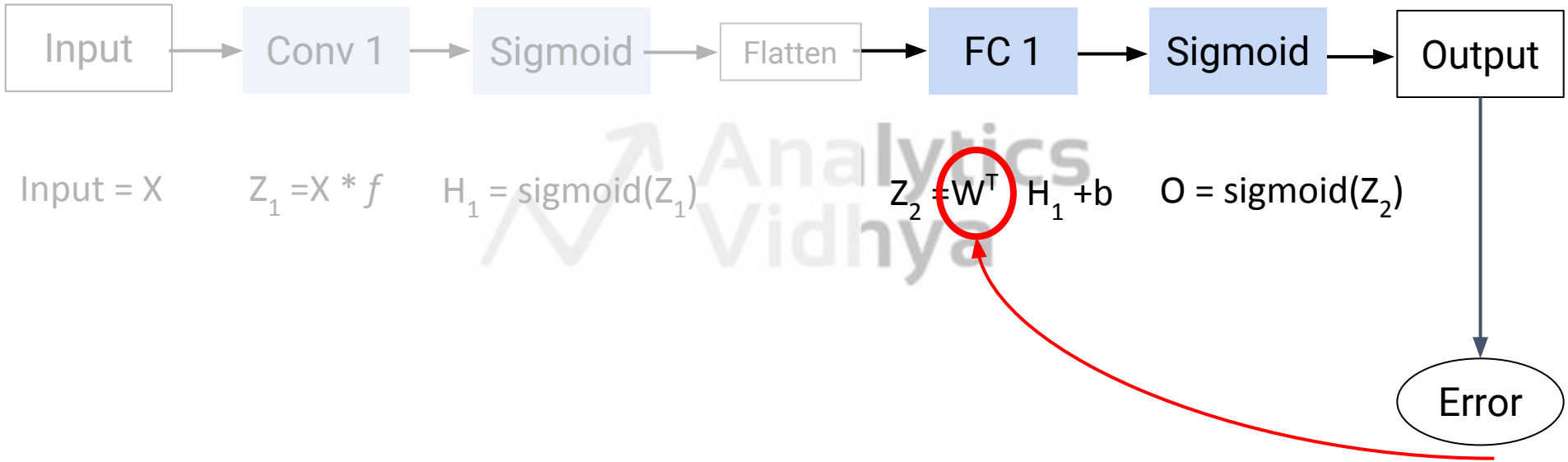
Backward Propagation in CNN: FC Layer



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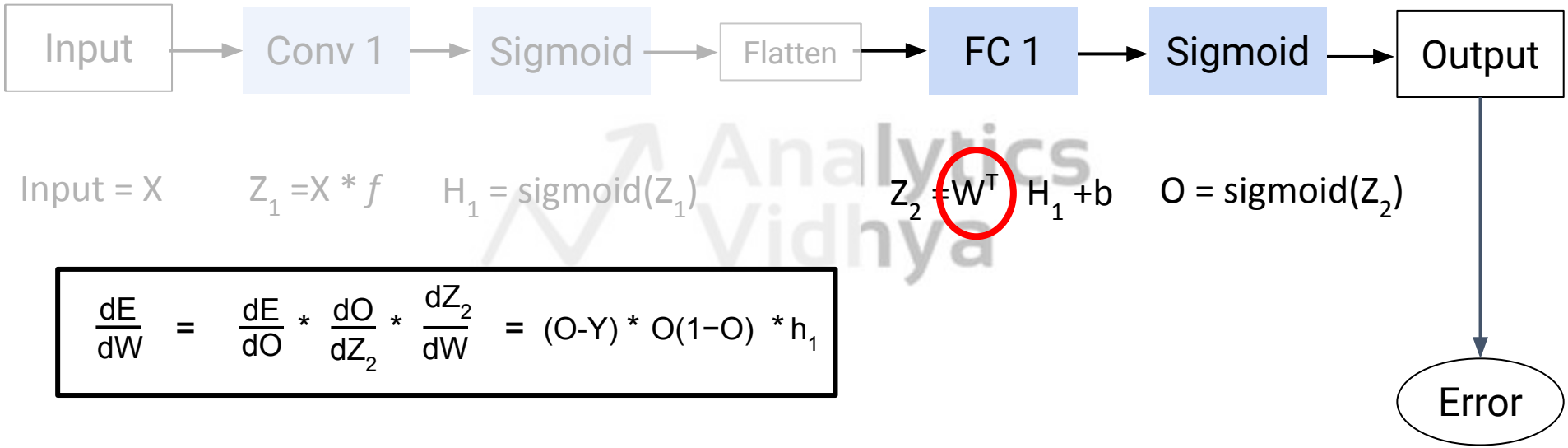
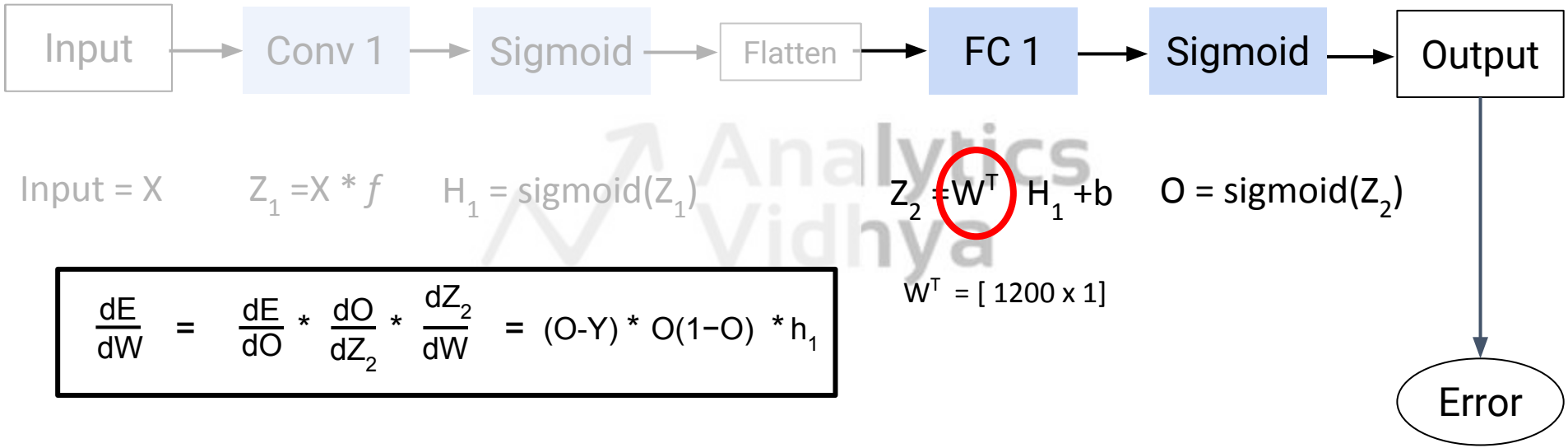
Backward Propagation in CNN: FC Layer

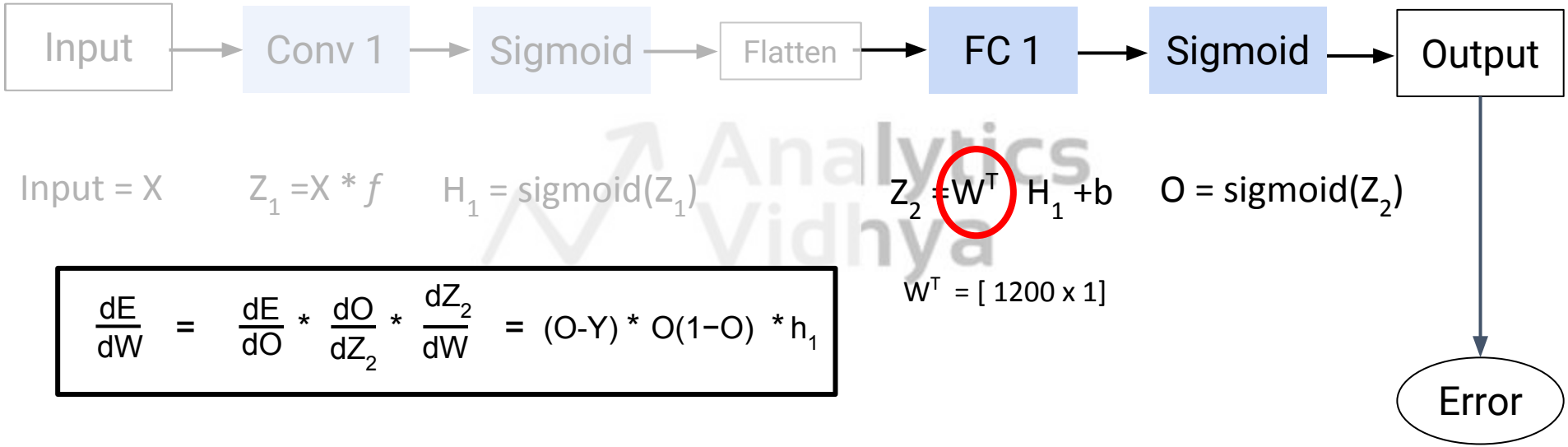


$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * h_1$$

Backward Propagation in CNN: FC Layer



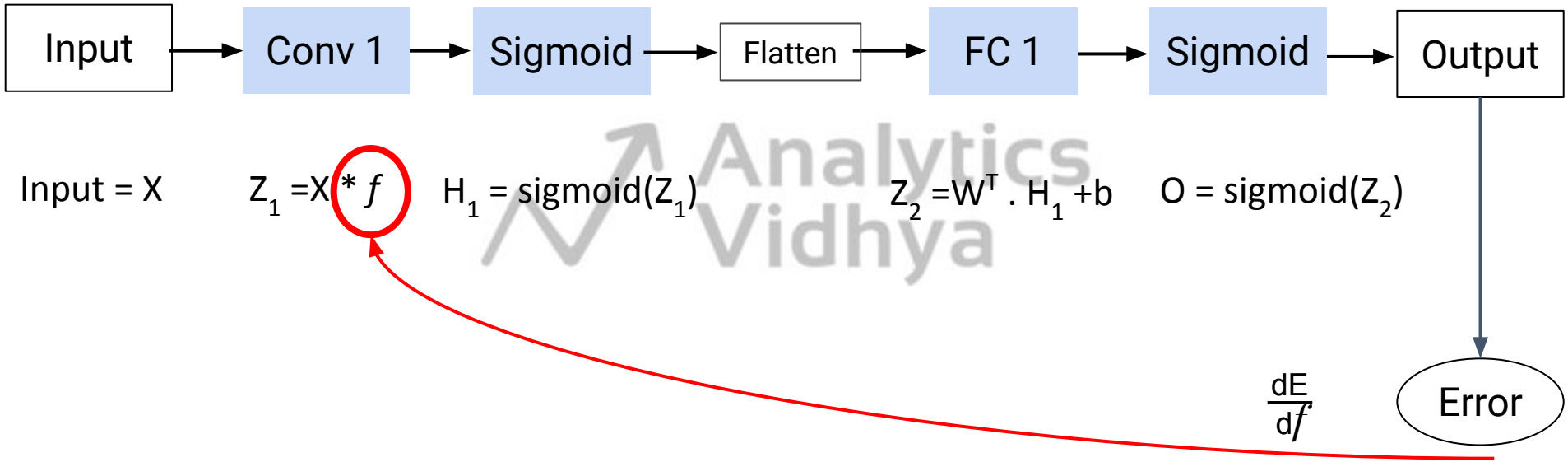
Backward Propagation in CNN: FC Layer



$$\frac{dE}{dW} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dW} = (O-Y) * O(1-O) * h_1$$

$$\frac{dE}{dW} = \frac{dZ_2}{dW_{ho}} X \left[\frac{dE}{dO} * \frac{dO}{dZ_2}\right]^T = [1200 \times 100] * [1 \times 100] = [1200 \times 100]$$

Backward Propagation in CNN: Conv Layer



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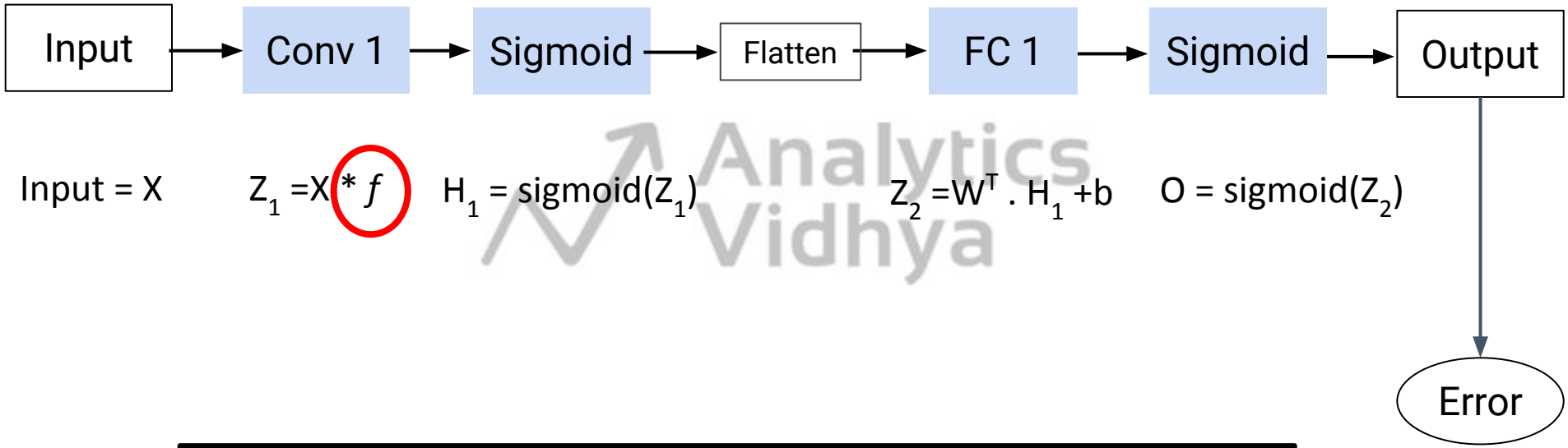
Backward Propagation in CNN: Conv Layer



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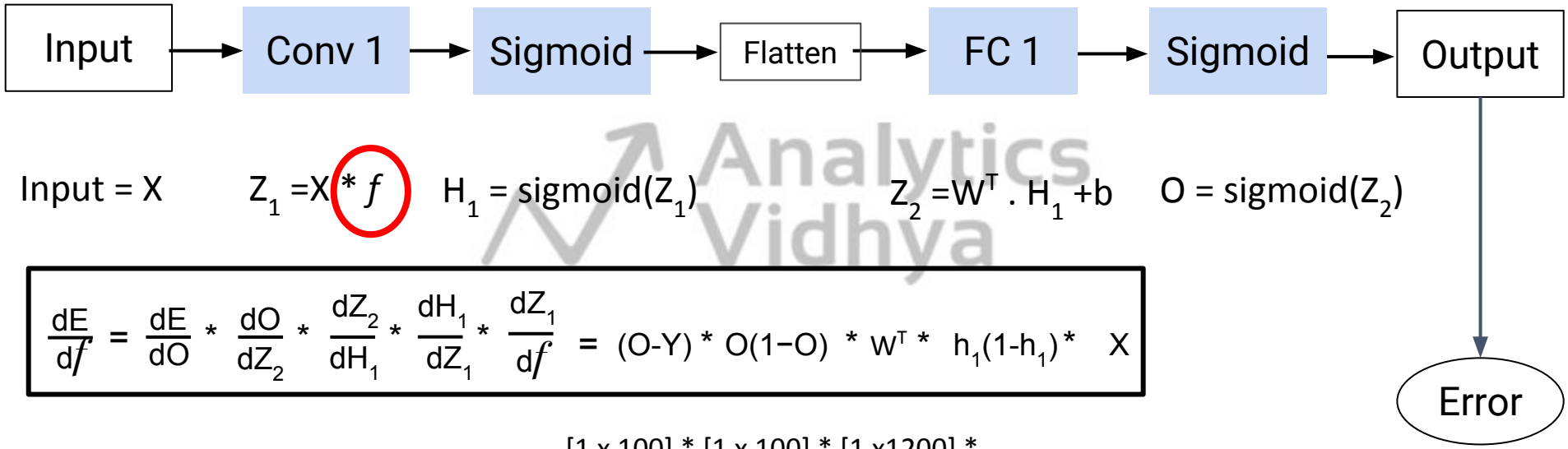
$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = (O-Y) * O(1-O) * W^T * h_1(1-h_1) * X$$

Backward Propagation in CNN: Conv Layer



Input = X $\quad Z_1 = X * f \quad H_1 = \text{sigmoid}(Z_1) \quad Z_2 = W^T \cdot H_1 + b \quad O = \text{sigmoid}(Z_2)$

Analytics Vidhya

$$\frac{dE}{df} = \frac{dE}{dO} * \frac{dO}{dZ_2} * \frac{dZ_2}{dH_1} * \frac{dH_1}{dZ_1} * \frac{dZ_1}{df} = (O-Y) * O(1-O) * W^T * h_1(1-h_1) * X$$

[1 x 100] * [1 x 100] * [1 x1200] *

Update equation

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