



ISE 562: Value and Decision Theory, Spring 2020

Investment in Real Estate or Stocks

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Problem Statement

David is a senior software engineer working in Los Angeles. He is 40 years old and lives with his wife. They have a family with two kids. David has worked for 15 years, his family has saved some money for the kids' education. As his children would grow up and go to university 10 years later, he plans to use \$1 million for investment in a ten years period which may have much higher return rates than depositing the money in the bank. However, he doesn't have any experience in investment, and he is confused about whether to invest in real estate or stocks. To sum up, he hopes to get the highest profits from investment and reduce the risk of losing money as much as possible.

Analysis Framework

Decision Maker

David is the decision-maker and stakeholder in this problem. Our team would offer different investment options. David can choose the one he prefers and makes the final decision.

Alternatives and Objectives

Table 1: Alternatives and Objectives

	Alternatives	Objectives
1	Single House/ Apartment in LA Area	Maximize Return
2	Vanguard Real Estate ETF (VNQ)	Minimize Risk
3	S&P 500 Index	Marketability / Liquidity
4	Alphabet, Inc. (GOOGL)	Flexibility of Use

Alternatives:**1. Single House / Apartment in LA Area**

Los Angeles is one of the most widely recognized locations in the world. It attracts millions of travelers each year. Buying a house or apartment in Los Angeles can be a good investment due to the high demand for rental units and strong economy. Real estate prices tend to rise each year and the owner can live in the house or rent it to get some rent payments.

2. Vanguard Real Estate ETF (VNQ)

Unlike buying a house, the vanguard real estate ETF focuses on real estate investment trusts (REITs), investors can avoid the headaches about developing and maintaining the property, providing landlord services and collecting rent payments. Vanguard real estate ETF is conveniently packaged to shares that can be easily bought and sold.

3. S&P 500 Index

S&P 500 is a stock market index that measures the stock performance of 500 large companies. Research has proven that buying stocks and holding them for long periods of time has been the greatest wealth creator in history. Stocks are liquid, which are also easily bought and sold, investors can rely on them in case of emergencies. However, stocks are more volatile in the short-term. The price can have extreme fluctuations.

4. Alphabet, Inc. (GOOGL)

Alphabet, Inc. (GOOGL) is one of the most successful stocks, it is \$50 a share in 2004 and \$1160 a share in 2019. As Google is the biggest search engine in the world and its Android service is widely used in the mobile phone, the stock has great potential to get high return rates for the investors. However, it may also have extreme fluctuations in the short term.

Objectives:**1. Return**

The most important objective is maximizing the return rate of the investment.

2. Risk

The investment risk is the probability or likelihood of occurrence of losses relative to the expected return. We hope to reduce the risk as much as possible to avoid big loss.

3. Liquidity

Liquidity is based on whether we can buy or sell the assets or equity easily. Good liquidity can help investors avoid risk in case of emergency.

4. Flexibility of Use

Flexibility of use depends on whether the assets or equity can be used by the owner. Good flexibility of use is preferred by David.

Data and Expert Judgement

Data Sources

We collected monthly data of stock prices and property values over a 15-year period to perform the analysis. Historical data of S&P 500 index (^GSPC), Google (GOOGL) and REITs (VNQ), from Sep 01, 2004 to Sep 01, 2019, were downloaded for Yahoo Finance. Home values of houses with 4 bedrooms in Los Angeles County, from September 2004 to September 2019, were retrieved from Zillow Research.

Consequence Table (Estimates)

- Return (% Return)
 - Values depend on the results generated by the probabilistic simulation in the Analysis and Evaluation section. Refer to Consequence Table (Complete).
 - Current Return (% Return) values are deterministic estimates calculated from historical data
- Risk (Annual Volatility)
 - Risk is defined by the annual volatility of the returns. We calculate this risk measure by calculating the monthly standard deviation of each investment and multiplying by $\sqrt{12}$. $\sigma_{annually} = \sigma_{monthly}\sqrt{12}$
- Flexibility of Use
 - Flexibility of use is defined by the option of using the investment in other ways. For example, the property investment can be used by the owner for residency or renting out to generate revenue. While the stock options do not have any other uses when compared to the property investment.
- Marketability
 - Marketability is defined to be the time it takes to liquidate the asset (i.e. selling of stocks or property)
 - Marketability for Property Investment in California is approximately 77 days. That's 42 days to get an offer, plus the typical 35-day closing period.
 - <https://listwithclever.com/real-estate-blog/average-time-to-sell-a-house-in-california/>
 - Marketability for stocks is calculated by the number of days in a year divided by the number of trading days in a year. $365 \text{ days} / 252 \text{ trading days} = 1.45 \text{ days}$

Table 2: Consequence Table (Estimates)

	Return (% Return)	Risk (Annual volatility)	Flexibility of Use	Marketability (Day)
Property Investment	54.48%	0.0292	Yes	77
REITs (VNQ)	203.36%	0.2275	No	1.45
Stock Index (S&P 500)	177.36%	0.1382	No	1.45
Stock Shares (GOOGL) (Expert Elicitation)	299.45%	0.3079	No	1.45

Expert Judgement (Visual Fit)

Using the historical data from GOOGL stock, we found that the mean and standard deviation of the annual return to be very optimistic (0.259, 0.378). Thus, we wanted to get a second opinion from an expert to get a more accurate distribution that will represent the GOOGL stock in the near future.

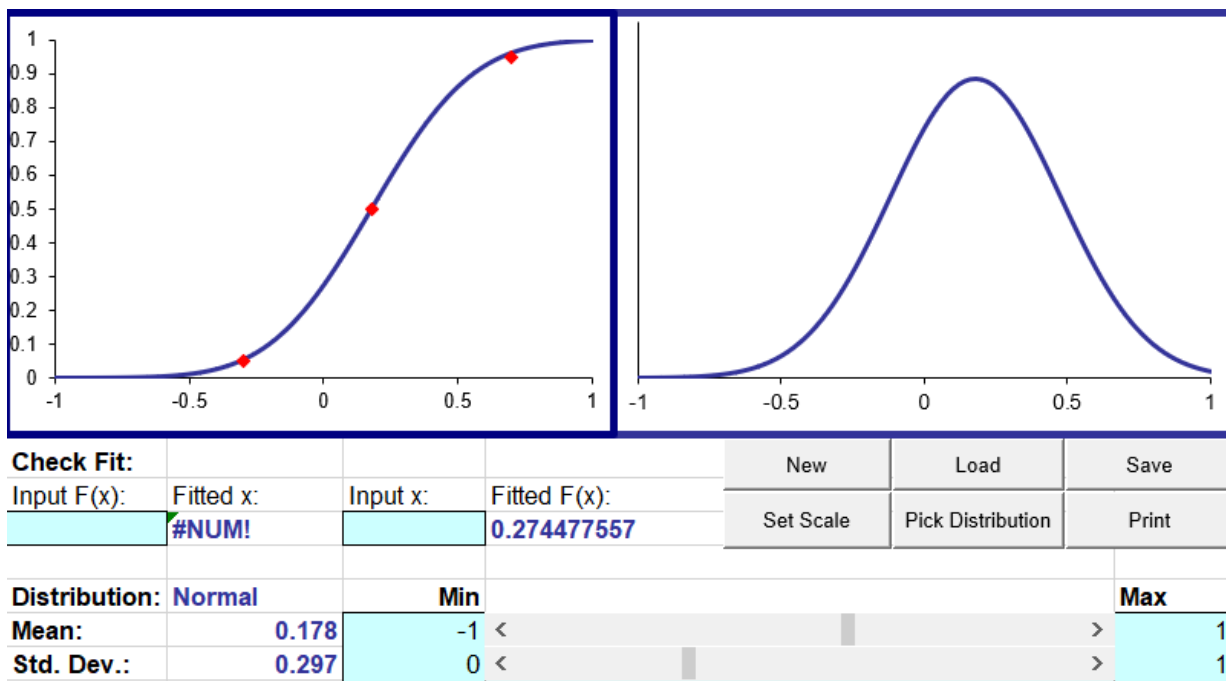


Figure 1: Expert Elicitation of GOOGL stock annual gains using VisualFit

We enlisted the help from a Master of Financial Engineering student to perform a probability elicitation of the GOOGL stock using VisualFit. The result is shown above, with the mean at 0.178 and standard deviation at 0.297. We came to the same conclusion as the expert, in which the GOOGL mean annual returns should be higher than stocks like VNQ and S&P 500 but with a higher standard deviation as GOOGL is more volatile.

Expert Judgement (Probability Distributions of Annual Return Rate)

After finishing the VisualFit for GOOGL stock, we turned to find the probability distributions of annual return rate of other alternatives based on past 15 years' data. In the Jupyter notebook, we tried several distributions like normal distribution, beta distribution and triangular distribution. After calculating the sum of squared residuals, we found the normal distribution is the best fit. Hence, we simply calculated the mean and standard deviation of all the past 15 years' return rates of property investment, VNQ, S&P 500, and used these distributions to generate the return rate of future 10 years.

Table 3: Probability distributions of annual return rate of all the alternatives

Stock Shares (GOOGL) (Expert Elicitation)	Property Investment	REITs (VNQ)	Stock Index (S&P 500)
1.441820771	0.239563367	0.258556068	0.102487132
0.269986797	0.152274916	0.25774271	0.087108594
0.411470476	0.011861184	0.040393363	0.142905287
-0.293951712	-0.099080753	-0.107693698	-0.236050444
0.238015552	-0.05189108	-0.264683877	-0.093693225
0.060381228	0.053118593	0.307105992	0.079577703

Table 3 (continued): Probability distributions of annual return rate of all the alternatives

Stock Shares (GOOGL) (Expert Elicitation)	Property Investment	REITs (VNQ)	Stock Index (S&P 500)
-0.020445506	0.003410448	0.012159587	-0.008569845
0.464934791	0.044624126	0.325183363	0.27332908
0.160914544	0.167997065	0.056610912	0.167199982
0.342196234	0.074529453	0.131090403	0.172899992
0.084906824	0.061843738	0.09296237	-0.026497122
0.259551677	0.063693877	0.199793869	0.12928964
0.211004121	0.061378975	0.001461562	0.161921755
0.23965821	0.062676915	0.015187018	0.156634961
0.009880443	0.02367528	0.156290482	0.018237344
RANDNORMAL(0.178,0.297)	RANDNORMAL(0.058,0.083)	RANDNORMAL(0.099,0.161)	RANDNORMAL(0.075,0.127)

Analysis and Evaluation

Probabilistic Simulation

Using probabilistic simulations, we can generate the potential returns of each investment option by sampling from normal distributions defined from historical data and expert elicitation. With SimVoi, we are able to run a Monte Carlo Simulation with 10,000 trials to simulate the potential returns from each investment option under random assumptions of the future. In each of the 10 year period, we sample an annual return from a normal distribution defined through historical data

or expert elicitation. To calculate the total profit from that outcome we sum up the returns and deduct taxable income (20%). An example is shown below:

% Returns	Google % Return	
	Inputs	Total Profit After Gains
Year 1	0.730228769	\$1,730,229
Year 2	-0.425267576	\$994,419
Year 3	0.083865203	\$1,077,816
Year 4	0.309090728	\$1,410,959
Year 5	0.493638307	\$2,107,462
Year 6	-0.145623451	\$1,800,566
Year 7	0.919920823	\$3,456,944
Year 8	0.324160746	\$4,577,549
Year 9	0.434733424	\$6,567,563
Year 10	-0.157141969	\$5,535,523
Initial Investment	\$1,000,000	
Total Returns	\$4,316,529	
Taxable Income (20%)	\$863,305.78	
Total Profit	\$3,540,133	
Return, Inflation Rate 2%	\$1,218,994	
Return Rate	290.41%	

Figure 2: Probabilistic Simulation of GOOGL in SimVoi

Probabilistic Simulation for property investment is similar to the method discussed above, but with several additions to calculate the total profit. For property investments, we assumed that David would be renting the house out to generate income. Assuming that the monthly rent income for the house will be \$5,000 and property management takes 10% of that, David will have a monthly gain of \$4,500. In addition to rent, owning a property provides the owner with tax deduction, which is up to \$10,000 per year. Thus, the total profit of property investment is generated by the summation of the total returns from increase in property value, rent income of 10 years (\$540,000), and tax deduction of 10 years (\$100,000). An example is shown below:

Property Investment % Return			
% Returns	Inputs		Total Profit After Gains
Year 1	0.009529012	RANDNORMAL(Mean, Std)	\$1,009,529
Year 2	-0.064000862	RANDNORMAL(Mean, Std)	\$944,918
Year 3	-0.13104254	RANDNORMAL(Mean, Std)	\$821,094
Year 4	-0.024678179	RANDNORMAL(Mean, Std)	\$800,831
Year 5	0.096818563	RANDNORMAL(Mean, Std)	\$878,366
Year 6	-0.039968961	RANDNORMAL(Mean, Std)	\$843,259
Year 7	0.04364961	RANDNORMAL(Mean, Std)	\$880,067
Year 8	0.195435192	RANDNORMAL(Mean, Std)	\$1,052,062
Year 9	0.094779806	RANDNORMAL(Mean, Std)	\$1,151,777
Year 10	0.0534568	RANDNORMAL(Mean, Std)	\$1,213,347
Initial Investment	\$1,000,000		
Rent Income (\$4,500/ month)	\$540,000	Total rent income	
Tax Deduction (\$10,000 / yr)	\$100,000	Total tax deduction	
Total Returns	\$534,353		
Taxable Income (20%)	\$106,870.53		
Total Profit	\$614,392		
Return, Inflation Rate 2%	\$1,218,994		
Return Rate	50.40%		

Figure 3: Probabilistic Simulation of Property Investment in SimVoi

Probabilistic Simulation (Results)

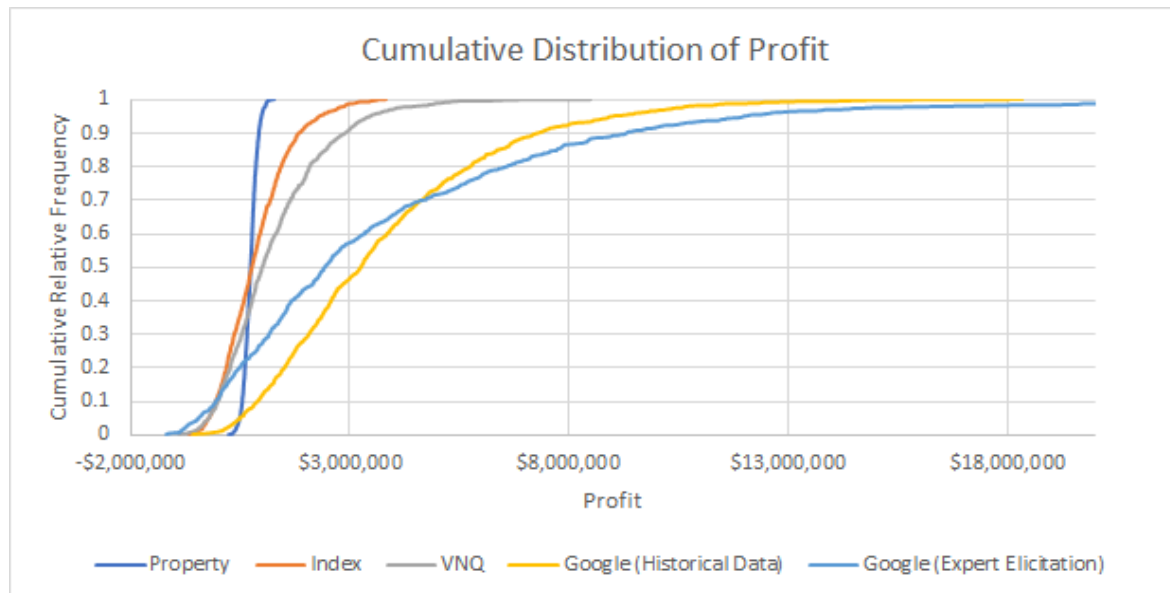


Figure 4: Cumulative Distribution of Profit from Probabilistic Simulation Trials

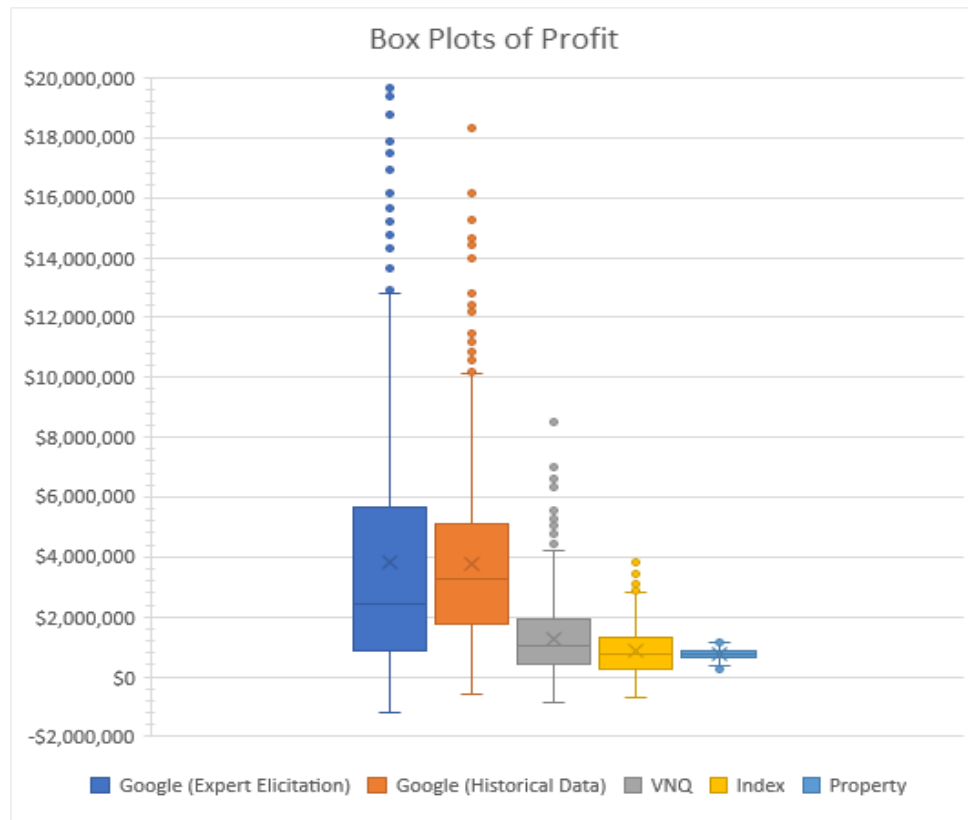


Figure 5: Box Plots of Profit from Probabilistic Simulation Trials

Results from the probabilistic simulations are shown above in the Cumulative Distribution and Box Plots. The results aligned with our initial assumptions, in which the GOOGL stock provided the highest average returns but also contained a larger risk that can end with a negative return. In addition, we can see that property investment has a very small range of outcomes which signifies the low risk that property investment presents to the investor.

Consequence Table (Complete)

- Return (% Return)
 - Using the results from the probabilistic simulations, we are able to calculate the mean profits generated by the Monte Carlo Simulation. Thus, we can use the mean profits to then generate the average percent return of each investment option.
 - Taxable income has been detected when calculating the return rate. Since the net income of all the alternatives has exceeded \$434,551, the Long-term capital gains tax rate is 20%.
 - The decision maker plans to lease his property and the monthly income after paying the management fees is about \$4,500. The tax rate of property in Los Angeles is about 0.755% annually and the tax deduction for real estate is up to \$10,000 per year.

Table 4: Consequence Table (Complete)

	Return (% Return)	Risk (Annual volatility)	Flexibility of Use	Marketability (Day)
Property Investment	60.57%	0.0292	Yes	77
REITs (VNQ)	110.74%	0.2275	No	1.45
Stock Index (S&P 500)	69.68%	0.1382	No	1.45
Stock Shares (GOOGL) (Expert Elicitation)	317%	0.3079	No	1.45

Sensitivity Analysis

We carried out sensitivity analysis after obtaining the consequence table, in which, return rate, flexibility and marketability are viewed as utility. In order to get the weighted utility, we calculate it based on the decision maker's preference. In our case, David doesn't care much about the flexibility and marketability and he's pretty interested in the return rate. Hence, the normalized weight of return rate is 71%, the normalized weight of flexibility and marketability are 14%. Then, we got the weighted utility of all the alternatives as follows.

Single-attribue Values			Alternatives			
Benefit	Objectives	Measures	Property Investment	REITs (VNQ)	Stock Index (S&P500)	Stock shares(Google)
	Return	%	0.00	20	4	100
	Risk (Variance)		100	29	61	0
	Flexibility	Yes/No	100	0	0	0
	Marketability	Day	0	100	100	100
	Unweighted Utility		50	37	41	50
	Weighted Utility		33	28	27	67

Figure 6: Weighted utility of all the alternatives

The relationship between the weighted utility and risk of all the alternatives can be intuitively seen from the following graph.

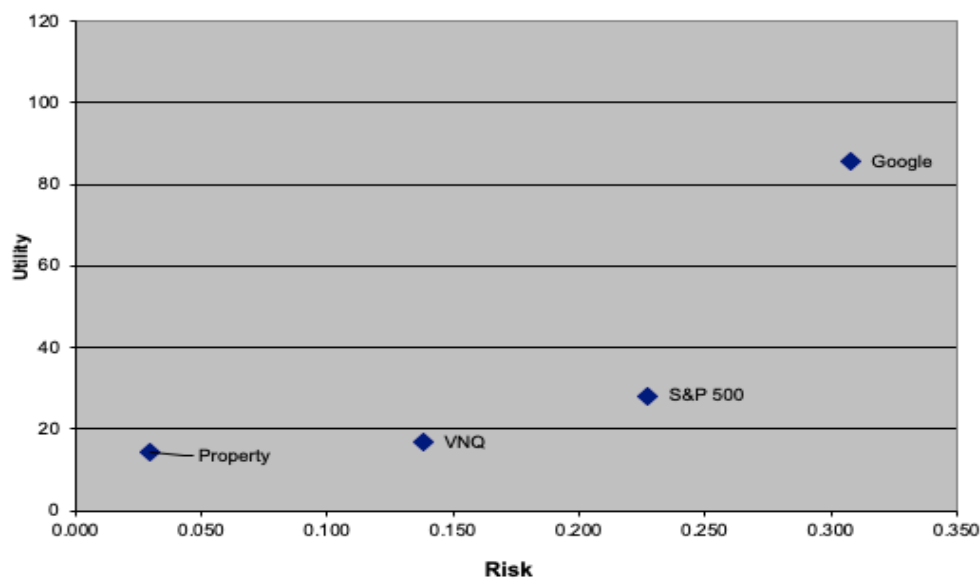


Figure 7: Relationship between risk and utility

It's clear that Google stock possesses the highest utility and risk at the same time while the property investment is the opposite. The VNQ and S&P 500 have moderate utility and risk. Generally speaking, this graph follows the rule that high risk, high return. To study the influence of risk on making the decision, we drew the following graph.

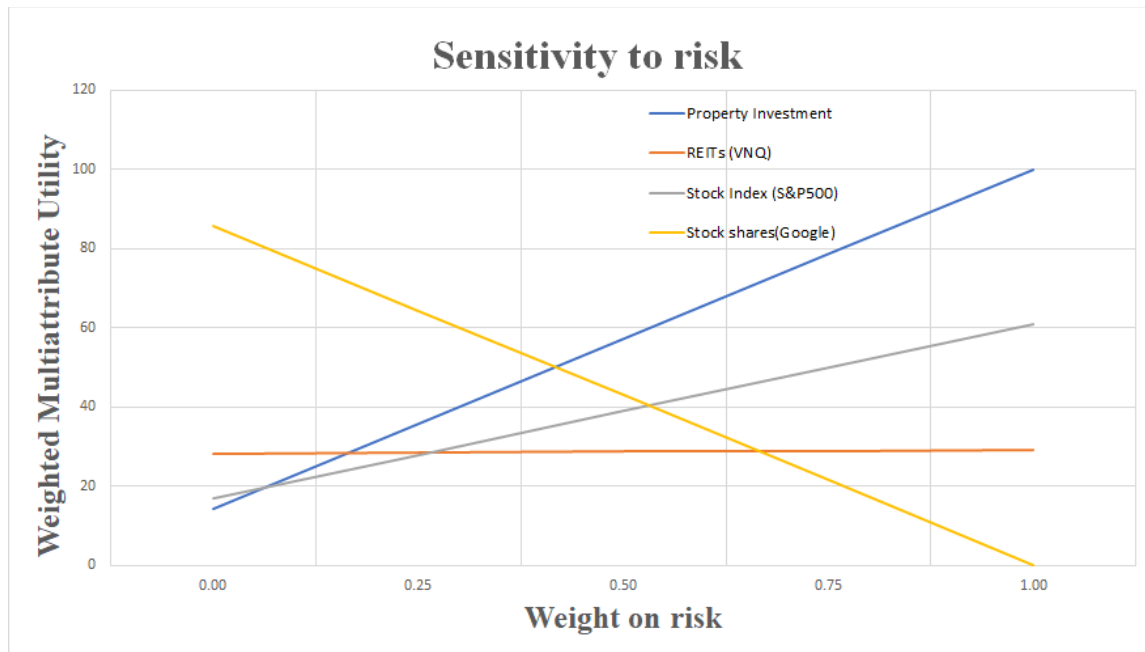


Figure 8: Sensitivity analysis between weighted utility and risk

As the weight on risk rises, the weighted score of Google stock decreases dramatically while the weighted score of property investment and S&P 500 increase significantly, which means all of these three alternatives are sensitive to the change of weight on risk. There are several intersections between different alternatives, which represent the demarcation point of risk seeking and risk averse. In order to balance the weight of risk and utility, we studied the intersections between 0.4 and 0.6. Among the two points we found, one of them is the intersection of Google stock and S&P 500. Since we planned to figure out whether to invest in stock or real estate, but both Google stock and S&P 500 are stock, hence, we drop this point. And another intersection of Google stock and property investment has a 0.42 weight on risk.

Conclusion and Recommendation

Based on the analysis, we offer three investment recommendations, which are risk seeking, risk neutral and risk averse respectively, for David. If David would like to take short-term risks to achieve the maximum return, he could invest all his money into Google stocks. The return could be 5 times more than other alternatives, while risk would also be much higher. If David wanted to balance the risk and the return, he could invest half of his money into Google stock and purchase a house with the other half of the money. With this option, even if he lost money or was not satisfied with the return on Google stock, he still is very likely to make considerable profit on the property. If David was not willing to take risks and preferred stable profits, we recommend him to invest all his money into property. The risk could be only one ten of that of other options and the profit is also acceptable. This option also provides David more flexibility, while he could choose to rent out the house to pay off taxes and get some extra profits, or he could move in the house with his family.