Notation

a	Symbols in this font are real scalars.
$\mathbf{a}$	Symbols in this font are real column vectors.
$\mathbf{A}$	Symbols in this font are real matrices.
$\mathcal{A}$	Symbols in this font are sigma-points used in the Unscented Transformtion, except $\mathcal{N}$, $\mathcal{F}$, and $\mathcal{O}$, which have special meanings.
$p(\mathbf{a})$	The probability density of **a**.
$p(\mathbf{a}\|\mathbf{b})$	The probability density of **a** given **b**.
$\sim \mathcal{N}(\mathbf{a}, \mathbf{B})$	Normally distributed with mean **a** and covariance **B**.
$\mathcal{O}$	Observability matrix.
$(\cdot)_k$	The value of a quantity at timestep k.
$(\cdot)_{k_1:k_2}$	The set of values of a quantity from timestep k_1 to timestep k_2, inclusive.
$\mathbf{a}^{(m)}$	Superscript (m) is the index of a sample drawn from a density.
$\underrightarrow{\mathcal{F}}_a$	A vectrix representing a reference frame in three dimensions.
$\underrightarrow{a}$	A vector quantity in three dimensions.
$(\cdot)^\times$	The cross-product operator which produces a skew-symmetrix matrix from a column.
$\mathbf{1}$	The identity matrix.
$\mathbf{0}$	The zero matrix.

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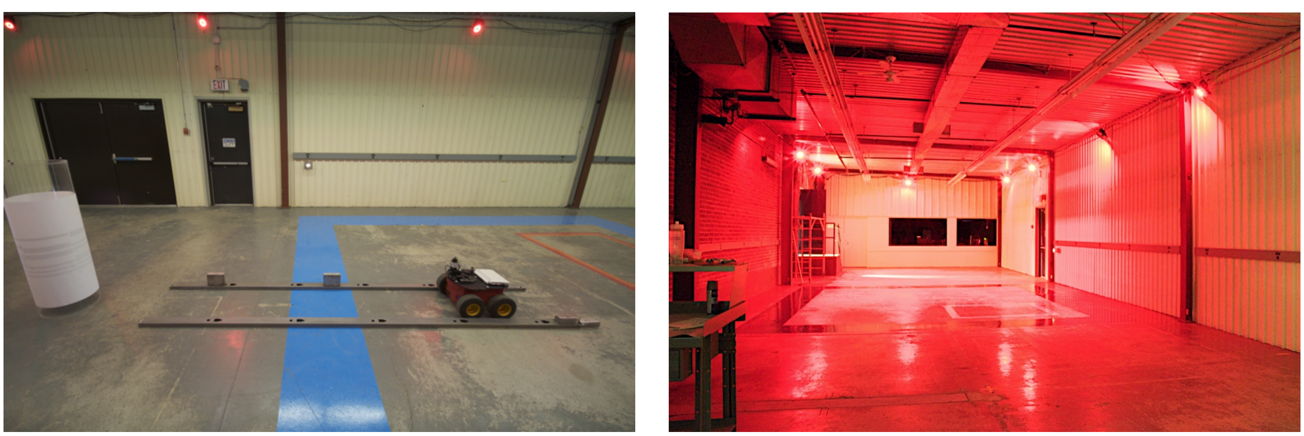
Assignment 1

'Giant Glass of Milk Dataset'

In this first assignment we'll investigate a linear one-dimensional problem consisting of a robot driving back and forth in a straight line. We wish to estimate the position of the robot along this line throughout its ≈ 280 m journey. We'll use the batch linear-Gaussian estimator to fuse speed measurements coming from wheel odometry with range measurements coming from a laser rangefinder.

1.1 Experimental Setup

The setup consists of a mobile robot driving back and forth between two fixed rails as depicted in Figure 1.1. The robot, the rails, and the large cylinder were equipped with reflective markers and tracked using a ten-camera motion capture system. This motion capture system is able to provide the position of each reflective marker to within a few



(a) Mobile robot driving between rails. Robot has an odometer to measure translational speed and a laser rangefinder to measure range to the large cylinder. Both sensors are noisy.

(b) Vicon motion capture lab. Ten cameras work together to track markers on the robot and provide groundtruth position/orientation.

Figure 1.1: Setup for Dataset 1.

millimeters. Position estimates from this motion capture system are considered to be quite a bit more accurate than estimates based on the robot's onboard sensors and thus it serves as the benchmark/groundtruth in our experiment.

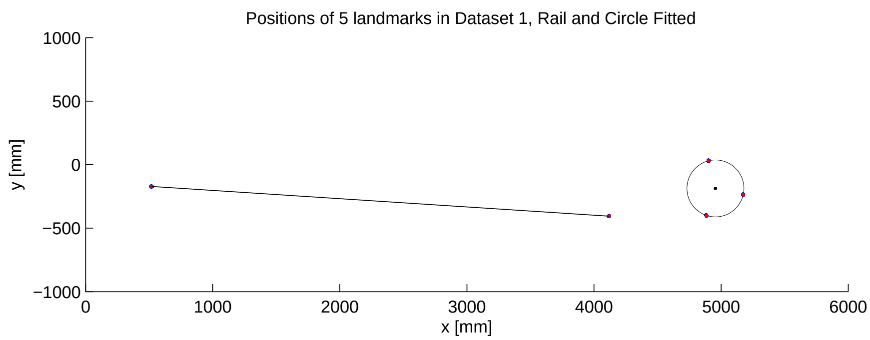


Figure 1.2: Circle and Rail for Dataset 1.

There were two reflective markers placed on the rail to the robot's right and three along the circumference of the large cylinder. The positions of these were measured using the motion capture system for 20 minutes and then averaged. A circle was fit to the points on the cylinder and its center found. A reference line was drawn between the two points on the rail. The output of these steps can be seen in Figure 1.2.

The mobile robot was driven back and forth for 20 minutes and three streams of data were logged:

– laser rangefinder scans consisting of 681 range measurements spread over a 240° horizontal field of view centered on straight ahead (Hokuyo URG-04LX sensor), logged at 10 Hz

– robot's speed based on a wheel odometry, logged at 10 Hz

– ground position of a marker on the laser rangefinder origin, logged at 70 Hz

Figure 1.3 shows the robot's path over the 20 minute trial. We see that the robot's path is very straight and consistent. We may therefore think of this as a one-dimensional estimation problem for this assignment.

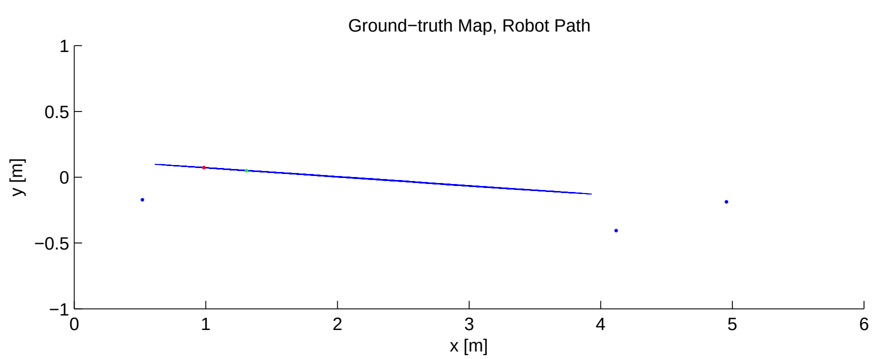


Figure 1.3: Robot Path for Dataset 1. Two points on rail and cylinder center shown for reference.

As with most real datasets, our data streams do not arrive synchronously or with evenly-spaced timesteps. Figure 1.4 shows the variability in the sampling periods of the nominally 10 Hz laser scans. For our purposes, we will

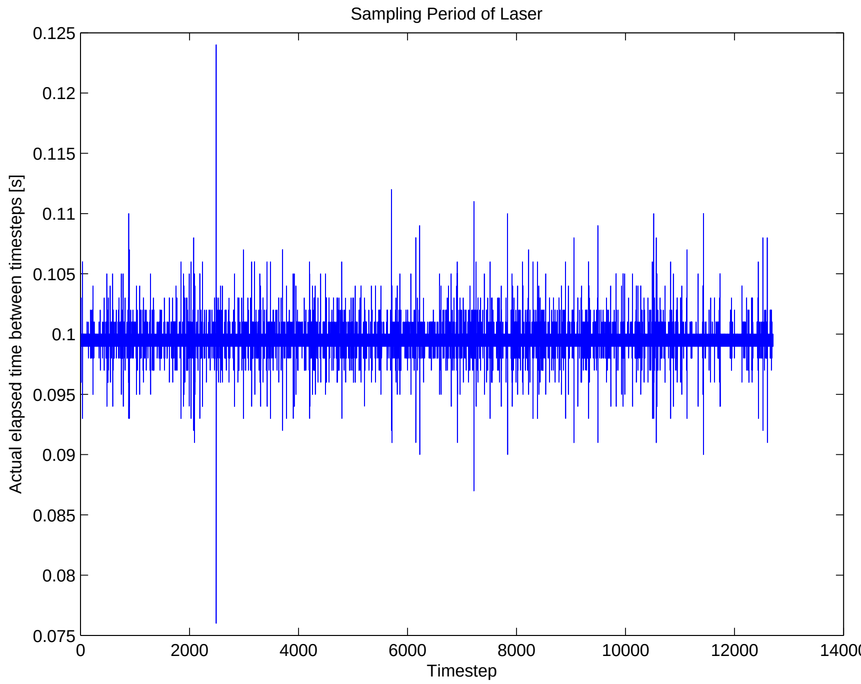


Figure 1.4: Sampling periods of the 10 Hz laser rangefinder scans. We see that the period does hover around 0.1 seconds.

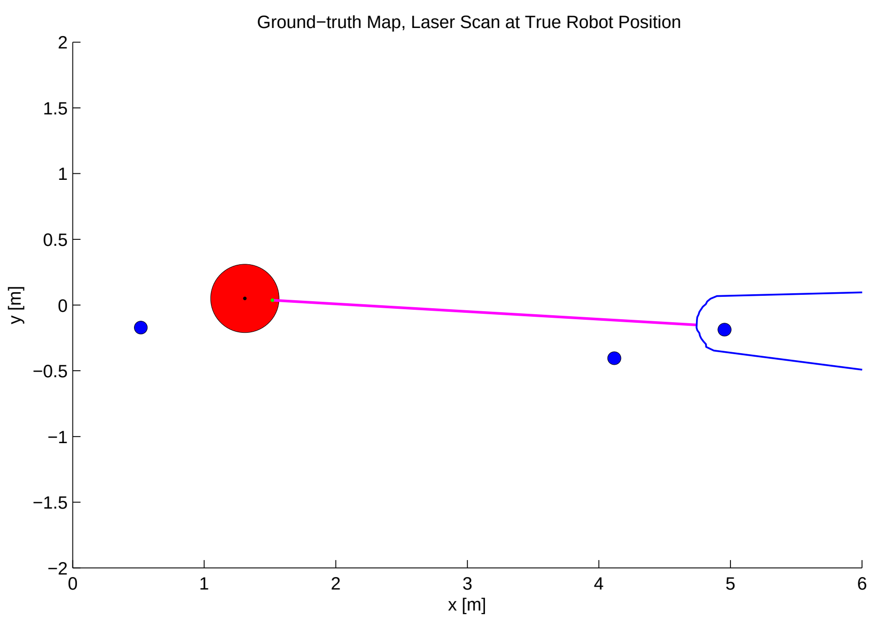


Figure 1.5: The purple line shows a range measurement from the robot's laser scanner to the cylinder's closest edge. The cylinder's radius is added to this to get the actual range to the cylinder.

assume that the data was acquired with a uniform 0.1 second sampling period. To avoid having to deal with the speed measurements and groundtruth data arriving asynchronously with the laser rangefinder scans, these measurements were linearly interpolated at the laser rangefinder timestamps. Thus, the data to be used in this assignment is synchronous with a uniform sampling period. Moreover, to avoid having to process the raw laser rangefinder scans, the range between the robot and the cylinder was extracted from each scan by using the cylinder's circular shape. Figure 1.5 shows how a range measurement was fit to a laser scan.

Another important issue worth mentioning is the noise on the sensor readings. Because we have a very accurate motion capture system, we may use the groundtruth positions of everything to determine the true robot-to-cylinder range readings and the true robot speed at each timestep. Figure 1.6 shows histograms of the errors in the range and speed measurements for the 20 minute dataset.

The file containing the final processed data for this assignment is called `dataset1.mat`, which is a Matlab binary file. You can view the contents by typing

```
load dataset1.mat
who
```

at the Matlab (or Octave) command prompt. The following seven Matlab variables will be listed:

`t` : a 12709×1 array containing the data timestamps [s]

`x_true` : a 12709×1 array containing the true position, x_k, of the robot (one-dimensional, along the rail) [m]

`l` : the scalar true position, x_c, of the cylinder's center (distance along the rail) [m]

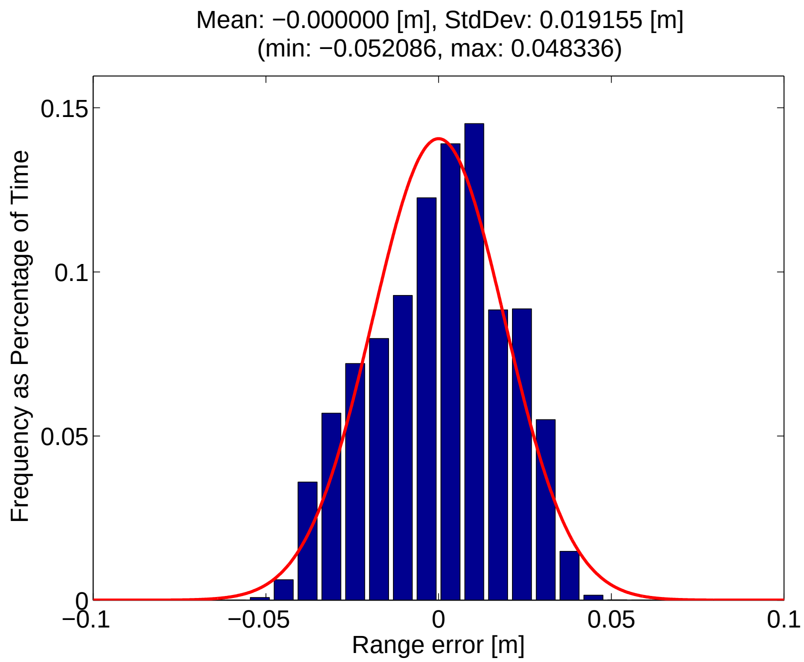
`r` : a 12709×1 array containing the range, r_k, between the robot and the cylinder's center as measured by the laser rangefinder sensor [m]

`r_var` : the variance of the range readings (based on groundtruth) [m^2]

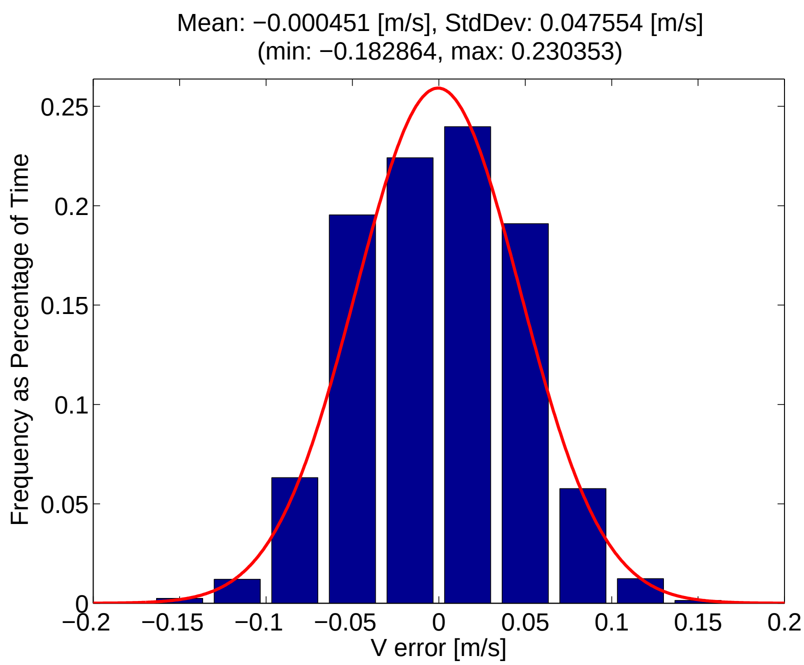
`v` : a 12709×1 array containing the speed, u_k, of the robot as measured by the robot's odometer [m/s]

`v_var` : the variance of the speed readings (based on groundtruth) [m^2/s^2]

The main idea is to estimate the robot's position (along the rails) without using `x_true` and then use this as a benchmark to gauge performance.



(a) Range errors.



(b) Speed errors.

Figure 1.6: Histograms of sensor errors. Red curves are Gaussians with standard deviation fit to the data. Both range and speed have close to zero-mean errors with a spread.

1.2 Motion and Observation Models

For this simple one-dimensional problem, the vehicle and sensor models are quite straightforward:

$$\text{motion:} \quad x_k = x_{k-1} + T u_k + w_k \tag{1.1a}$$

$$\text{observation:} \quad y_k := x_c - r_k = x_k + n_k \tag{1.1b}$$

where x_k is the robot's position along the rail, u_k is the robot's speed (derived from odometer), w_k is the process noise, T is the sampling period, r_k is the range to the cylinder (derived from the laser rangefinder), x_c is the position of the cylinder's center (along the rail), and n_k is the exteroceptive sensor noise. We make a small transformation to the observation model to create a different sensor output, $y_k = x_c - r_k$. Both the motion and observation models are linear.

1.3 Assignment

For the assignment we'll use the batch linear-Gaussian algorithm to estimate the robot's position using both the odometry and laser measurements. You might wonder, why do we need to use both? In this simple example we could use either datastream to estimate the robot's position. Figure 1.7 shows the errors we would obtain if we use each datastream individually. We see the odometry error grows without bound over time (probably due to a very small bias). The position error derived from range remains bounded, but in a real situation, we might not have so much range data; we'll look at using only a small portion of this to effectively correct the odometry data.

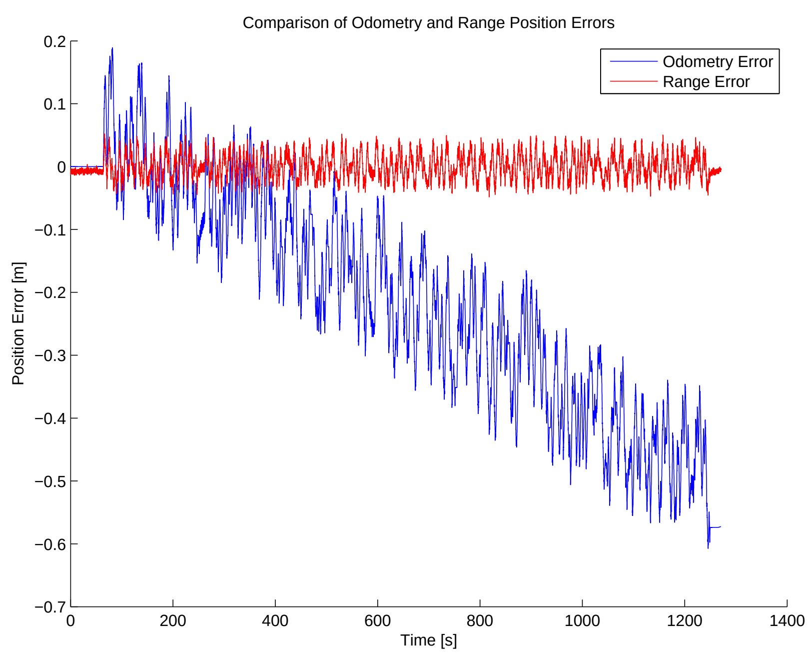


Figure 1.7: Estimation errors based on using either wheel odometry or laser range (and landmark position). Note that the odometry error is growing without bound over time.

Answer the following questions in a short typeset report (marks in the margin - 15 total):

2 1. Based on the data above, is the assumption of zero-mean Gaussian noise reasonable? What values of the variances, σ_q^2 and σ_r^2, should we use?

$$w_k \sim \mathcal{N}(0, \sigma_q^2), \qquad n_k \sim \mathcal{N}(0, \sigma_r^2) \tag{1.2}$$

3 2. Write out an expression for the batch linear-Gaussian objective function that we will seek to minimize:

$$J\left(x_{1:K} \,|\, u_{1:K}, y_{1:K}\right), \tag{1.3}$$

where K is the index of the maximum time.

2 3. Derive an expression for the optimal position estimates,

$$x^*_{1:K} = \arg\min_x \; J\left(x_{1:K} \,|\, u_{1:K}, y_{1:K}\right) \tag{1.4}$$

using the method of least squares.

3 4. This dataset has $K = 12709$. It may be computationally intractable to solve for all K positions using a batch method. Suppose, however, we only want to solve for a subset of these poses,

$$x_\delta, x_{2\delta}, x_{3\delta}, \ldots, x_K, \tag{1.5}$$

where $\delta \geq 1$. We will only use the laser rangefinder reading at the corresponding timesteps,

$$r_\delta, r_{2\delta}, r_{3\delta}, \ldots, r_K, \tag{1.6}$$

but want to use all of the odometry speed measurements. Modify the objective function and expression for the optimal position estimates to handle this case.

5 5. Write a Matlab or Octave script to solve for the robot positions for

$$\delta = 1000, 100, 10, 1. \tag{1.7}$$

Make the following figures and write some short comments about your findings. Note, the true robot position, $x_{k\delta}$, is contained in the `x_true` array.

   (a) Plot the following quantities on the same axes:
      – the error, $x^*_{k\delta} - x_{k\delta}$ vs. $t_{k\delta}$, as a solid line.
      – the uncertainty envelope, $\pm 3\sigma_{x_{k\delta}}$ vs. $t_{k\delta}$, as two dotted lines. Note, the variance, $\sigma^2_{x_{k\delta}}$, can be extracted from the least-squares solution (see the notes).

   Make one plot for each value of δ.

   (b) Make a histogram of the errors $x^*_{k\delta} - x_{k\delta}$, for each value of δ. How does this correspond to the predicted uncertainty from the least-squares formulation?

Attach your script in an Appendix.