-Kelompok 10-
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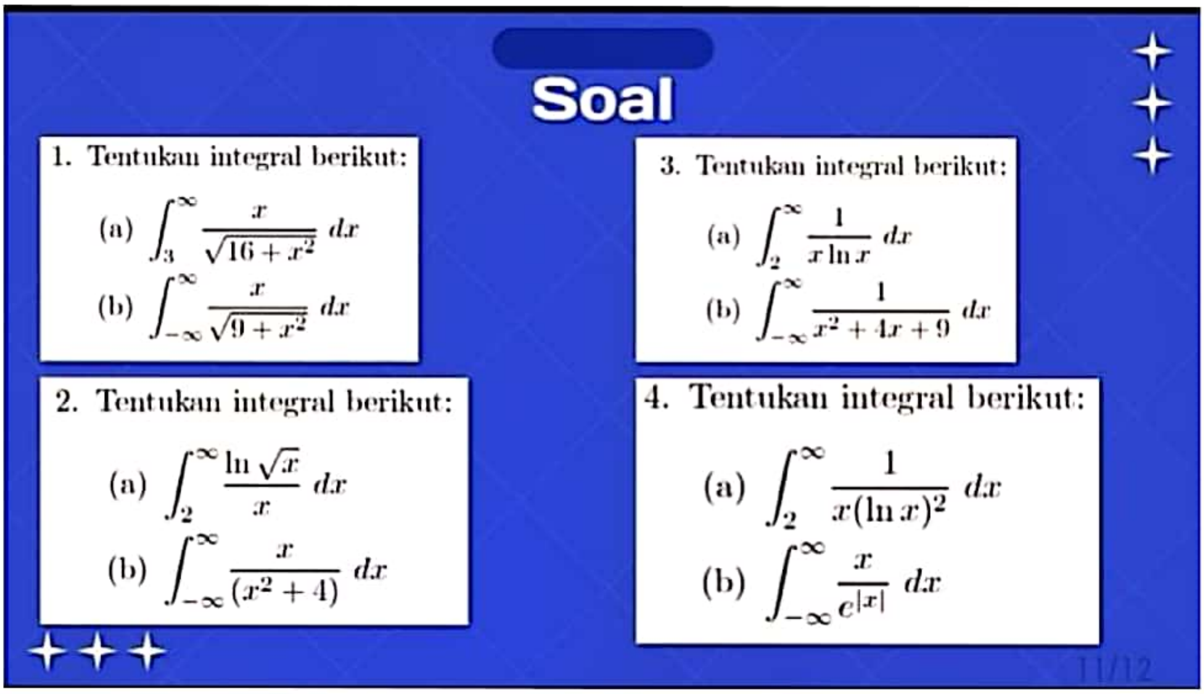

Jawab:

1.

1. Tentukan integral berikut :

a) $\int_3^\infty \frac{x}{\sqrt{16+x^2}}dx$

$= \lim_{b\to\infty}\int_3^b \frac{x}{\sqrt{16+x^2}}\,dx$

Misal :

$u = 16 + x^2 \qquad du = 2x\,dx$

$\frac{du}{dx} = 2x \qquad \frac{du}{2} = x\,dx$

$= \lim_{b\to\infty}\int_3^b \frac{1}{2\sqrt{u}}\,du$

$= \frac{1}{2}\lim_{b\to\infty}\int_{25}^{16+b^2} u^{-\frac{1}{2}}\,du$

$= \frac{1}{2}\lim_{b\to\infty}(u^{1/2} \times 2|_{25}^{16+b^2}$

$= \frac{1}{2}\lim_{b\to\infty} 2\sqrt{16+b^2} - 2\sqrt{25}$

$= \infty$ **(divergen)**

1b. $\int_{-\infty}^{\infty}\frac{x}{\sqrt{9+x^2}}\,dx$

$= \lim_{a\to-\infty}\int_a^0 \frac{x}{\sqrt{9+x^2}}\,dx + \lim_{b\to\infty}\int_0^b \frac{x}{\sqrt{9+x^2}}\,dx$

>>> misal u = $9 + x^2$

$\frac{du}{dx}$ = 2x , maka du = 2x dx

$= \lim_{a\to-\infty}\int_{9+a^2}^{9}\frac{1}{2\sqrt{u}}\,du + \lim_{b\to\infty}\int_9^{9+b^2}\frac{1}{2\sqrt{u}}\,du$

$= \lim_{a\to-\infty}\int_{9+a^2}^{9}\frac{1}{2}u^{-\frac{1}{2}}\,du + \lim_{b\to\infty}\int_9^{9+b^2}\frac{1}{2}u^{-\frac{1}{2}}\,du$

$= \lim_{a\to-\infty} u^{-\frac{1}{2}}\Big]_{9+a^2}^{9} + \lim_{b\to\infty} u^{-\frac{1}{2}}\Big]_{9}^{9+b^2}$

$= \sqrt{9} - \sqrt{9+a^2} + \sqrt{9+b^2} - \sqrt{9}$

= (-∞+∞) DIVERGEN

2.

2a) $\int_2^\infty \frac{ln\sqrt{x}}{x}dx = \cdots$

$$\int_2^\infty \frac{ln\sqrt{x}}{x}dx = \lim_{b\to\infty}\int_2^b \frac{ln\sqrt{x}}{x}dx$$

Misal :

$u = ln\sqrt{x}$

$du = \frac{1}{x}dx$

$$\lim_{b\to\infty}\int_2^b u\,du = \lim_{b\to\infty}\frac{1}{2}u^2\Big|_2^b$$

$$= \frac{1}{2}\left[\lim_{b\to\infty}(ln^2\sqrt{b} - ln^2\sqrt{2})\right]$$

$$= \frac{1}{2}(ln^2\sqrt{\infty} - ln^2\sqrt{2})$$

$= \infty$ (DIVERGEN)

b. $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\,dx$

misalkan u = $x^2 + 4$, du = 2x dx, $\frac{1}{2}$du = x dx

$$= \lim_{m\to-\infty}\frac{1}{2}\int_{-\infty}^{4}\frac{1}{u}\,du + \lim_{n\to\infty}\frac{1}{2}\int_{4}^{\infty}\frac{1}{u}\,du$$

$$= \lim_{m\to-\infty}\frac{1}{2}(\ln u)\mid_{-\infty}^{4} + \lim_{n\to-\infty}\frac{1}{2}(\ln u)\mid_{4}^{\infty}$$

$$= \left(\frac{1}{2}(\ln 4 - \ln -\infty)\right) + \left(\frac{1}{2}(\ln\infty - \ln 4)\right)$$

$= -\infty + \infty =$ divergen (tidak ada hasil)

3.

$$\int_2^\infty \frac{1}{x\ln(x)}dx$$

$$\lim_{t\to\infty}\int_2^t \frac{1}{x\ln(x)}dx$$

Misal $u = \ln(x)$, $du\frac{1}{x}dx, dan\ xdu = dx$

$$\lim_{t\to\infty}\int_{\ln(2)}^{\ln(t)}\frac{1}{u}du$$

$$\lim_{t\to\infty}\ln(|u|)\Big]_{\ln(2)}^{\ln(t)}$$

$$\lim_{t\to\infty}\ln(|\ln(t)|) - \ln(|\ln(2)|)$$

$$\lim_{t\to\infty}\ln\left(\frac{|\ln(t)|}{|\ln(2)|}\right)$$

∞ Divergen

b. $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\,dx$

misalkan u = $x^2 + 4$, du = 2x dx, $\frac{1}{2}$du = x dx

$$= \lim_{m\to-\infty}\frac{1}{2}\int_{-\infty}^{4}\frac{1}{u}\,du + \lim_{n\to\infty}\frac{1}{2}\int_{4}^{\infty}\frac{1}{u}\,du$$

$$= \lim_{m\to-\infty}\frac{1}{2}(\ln u)\mid_{-\infty}^{4} + \lim_{n\to-\infty}\frac{1}{2}(\ln u)\mid_{4}^{\infty}$$

$$= \left(\frac{1}{2}(\ln 4 - \ln -\infty)\right) + \left(\frac{1}{2}(\ln\infty - \ln 4)\right)$$

$= -\infty + \infty =$ divergen (tidak ada hasil)

4.

4A. $\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$

Jawab:

$$= \lim_{t \to -\infty} \int_2^t \frac{1}{x(lnx)^2} dx$$

Misal: u = ln(x), du = $\frac{1}{x} dx$, sehingga $xdu = dx$

$$= \lim_{t \to -\infty} \int_{\ln(2)}^{\ln(t)} \frac{1}{u^2} du$$

$$= \lim_{t \to -\infty} \int_{\ln(2)}^{\ln(t)} u^{-2} du$$

$$= \lim_{t \to -\infty} -u^{-1} \Big]_{\ln(2)}^{\ln(t)}$$

$$= \lim_{t \to -\infty} -ln^{-1}(t) + ln^{-1}(2)$$

$$= -\lim_{t \to -\infty} ln^{-1}(t) + \lim_{t \to -\infty} ln^{-1}(2)$$

$$= -0 + ln^{-1}(2)$$

$$= \frac{1}{\ln(2)}$$ (Konvergen)

4B. $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}} dx$

Jawab:

$$= \int_{-\infty}^{\infty} \frac{x}{e^{|x|}} dx$$

$$= \int_{-\infty}^{0} x.e^{-x} dx + \int_{0}^{\infty} x.e^{x} dx$$

$$= \lim_{a \to -\infty} \int_a^0 x.e^{-x} dx + \lim_{b \to \infty} \int_0^b x.e^{x} dx$$

$$= \left(\lim_{a \to -\infty} x.-e^{-x} \Big]_a^0 - \int_a^0 -e^{-x} dx \right) + \left(\lim_{b \to \infty} xe^{x} \Big]_0^b - \int_0^b e^{x} dx \right)$$

Misal: $u = x,\ du = dx, v = e^{x}, dv = e^{x} dx \mid\mid u = x,\ du = dx, v = e^{-x}, dv = e^{x-} dx$

$$= \left(\lim_{a \to -\infty} x.-e^{-x} \Big]_a^0 - \left(e^{x} \Big]_a^0 \right) \right) + \left(\lim_{b \to \infty} xe^{x} \Big]_0^b - \left(e^{x} \Big]_0^b \right) \right)$$

$$= (-1 - 0) + (0 - (-1)$$

$$= 0\ Konvergen$$

3b) $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9} dx = \cdots$

$$= \lim_{a \to -\infty} \int_a^0 \frac{1}{x^2+4x+9} dx + \lim_{b \to \infty} \int_0^{\infty} \frac{1}{x^2+4x+9} dx$$

$$= \lim_{a \to -\infty} \int_a^0 \frac{1}{(x+2)^2+5} dx + \lim_{b \to \infty} \int_0^{b} \frac{1}{(x+2)^2+5} dx$$

$$= \lim_{a \to -\infty} \int_a^0 \frac{1}{(x+2)^2+\left(\sqrt{5}\right)^2} dx + \lim_{b \to \infty} \int_0^{b} \frac{1}{(x+2)^2+\left(\sqrt{5}\right)^2} dx$$

Ket: Rumus dasar integral fungsi trigonometri invers

Rumus umum: $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$, **dengan** $a \neq 0$

$$= \lim_{a \to -\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_a^0 + \lim_{b \to \infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_0^b$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0+2}{\sqrt{5}}\right) - \lim_{a \to -\infty} \tan^{-1}\left(\frac{a+2}{\sqrt{5}}\right)\frac{1}{\sqrt{5}} + \lim_{b \to \infty} \frac{1}{\sqrt{5}} (\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

$$- \tan^{-1}(-\infty) + \tan^{-1}(\infty) + \tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

$$= \frac{1}{\sqrt{5}}\left(0 - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2}\right)$$

$$= \frac{1}{\sqrt{5}}(\pi)$$

$$= \frac{\pi}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\pi\sqrt{5}}{5}$$