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1) a Tulis rumus eksplisit barisan berikut dan tentukan ke-konvergenannya :

$$\cos\pi,\ \frac{\cos 2\pi}{4},\ \frac{\cos 3\pi}{9},\ \frac{\cos 4\pi}{16},\ \ldots$$

b. Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan (dengan definisi limit) $\{a_n + b_n\}$ konvergen ke $A + B$

c. Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut :

$$a_n = \sin \frac{n\pi}{4}$$

Jawab :

1a) rumus eksplisit : $\dfrac{\cos n\pi}{n^2}$

konvergen : $-1 \le \cos n\pi \le 1$

$$: \quad \frac{-1}{n^2} \le \frac{\cos n\pi}{n^2} \le 1$$

$$\lim_{n \to \infty} \frac{-1}{n^2} = \frac{-1}{\infty^2} = 0 \quad \Big| \quad \lim_{n \to \infty} \frac{1}{n^2} = \frac{1}{\infty^2} = 0$$

Maka, konvergen ke 0 , $\displaystyle\lim_{n \to \infty} \frac{\cos n\pi}{n^2} = 0$

1b) • $\{a_n\}$ konvergen ke A maka $\displaystyle\lim_{n \to \infty} a_n = A$. Maka setiap $\varepsilon_1 > 0$ terdapat $N_1 > 0$ sehingga $n > N_1$, berlaku :

$$|a_n - A| < \tfrac{1}{2}\varepsilon$$

• $\{b_n\}$ konvergen ke B maka $\displaystyle\lim_{n \to \infty} b_n = B$. Maka setiap $\varepsilon_2 > 0$ terdapat $N_2 > 0$ sehingga $n > N_2$, berlaku :

$$|b_n - B| < \tfrac{1}{2}\varepsilon$$

• $N = \max\{N_1, N_2\}$. Diperoleh :

$$\begin{aligned} |a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\ &\le |a_n - A| + |b_n - B| \\ &< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\ &= \varepsilon \end{aligned}$$

Maka, dapat dibuktikan bahwa $\displaystyle\lim_{n \to \infty} (a_n + b_n) = A + B$

1c) • kemonotonan

$$a_n - a_{n+1} = \sin \frac{n\pi}{4} - \frac{\sin (n+1)\pi}{4} = \sin \frac{n\pi - n\pi - \pi}{4}$$

$$= \sin \frac{-\pi}{4} < 0 \text{ (barisan monoton naik)}$$

• keterbatasan

$$-1 \leq \sin \frac{n\pi}{4} \leq 1 \quad \text{batas} : \{-1, 1\}$$

• limit

$$\lim_{n \to \infty} \sin \frac{n\pi}{4}$$ divergen (batas atas ≠ batas bawah)

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2a) Tentukan rumus eksplisit dan konvergennya

$$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6} \dots$$

b) dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen.

$$a_n = \frac{3-8.2^n}{5+4.2^n}$$

c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan :

$$a_n = \frac{\ln n}{n}$$

JAWAB :

2a) rumus eksplisit : $a_n = \dfrac{(-1)^{n+1}}{n} =$

konvergen : menggunakan teori apit

$$-1 \leq (-1)^{n+1} \leq 1 \Leftrightarrow -\frac{1}{n} \leq \frac{(-1)^{n+1}}{n} \leq \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{-1}{n} = \frac{-1}{\infty} = 0 \quad \Big| \quad \lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

maka, $\displaystyle\lim_{n\to\infty} \frac{(-1)^{n+1}}{n} = 0$

2b) $$\lim_{n\to\infty} \frac{3-8.2^n}{5+4.2^n} = \lim_{n\to\infty} \frac{3-8.2^n}{5+4.2^n} \cdot \frac{\frac{1}{2^n}}{\frac{1}{2^n}} = \lim_{n\to\infty} \frac{\frac{3}{2^n} - 8}{\frac{5}{2^n} + 4}$$

$$= \frac{0-8}{0+4} = -2$$

$$\Rightarrow |a_n - L| = \left|\frac{3-8.2^n}{5+4.2^n} + 2\right| = \frac{13}{5+4.2^n} < \frac{13}{5+4.2^N}$$

$$\Rightarrow \frac{13}{5+4.2^N} = \varepsilon$$

$$\Rightarrow 5+4.2^N = \frac{13}{\varepsilon} \Leftrightarrow 4.2^N = \frac{13}{\varepsilon} - 5$$

$$\Rightarrow N = \frac{\ln(^{13}/\varepsilon - 5) - \ln(4)}{\ln(2)}$$

maka,

$$|a_n - L| = \left|\frac{3-8.2^n}{5+4.2^n} + 2\right| \Leftrightarrow \frac{13}{5+4.2^n} < \frac{13}{5+4.2^N}$$

$$= \frac{13}{5+4.2^N} \Longleftrightarrow \frac{13}{5+4.2^{\frac{(\ln^{13}/\varepsilon - 5) - \ln 4)}{\ln(2)}}} \Longleftrightarrow \varepsilon$$

2C) $a_n = \dfrac{\ln n}{n}$

⇨ kemonotonan

$$a'(n) = \frac{1/n \cdot n - \ln n}{n^2} = \frac{1 - \ln(n)}{n^2}$$

$= a'(n) > 0$	$= a'(n) < 0$
$= \dfrac{1-\ln(n)}{n^2} > 0$	$= \dfrac{1-\ln(n)}{n^2} < 0$
$= 1 - \ln(n) > 0$	$= 1 - \ln(n) < 0$
$= \ln(e) - \ln(n) > 0$	$= \ln(e) < \ln(n)$
$= \ln(e) > \ln(n)$	$= e < n$
$= e > n$	
(naik pada $0, e$)	(turun pada e, ∞)

⇨ keterbatasan :

melakukan pengecekan pada a_1, a_2, a_3

- $a_1 = \dfrac{\ln(1)}{1} = 0$
- $a_2 = \dfrac{\ln(2)}{2} \approx 0{,}34657$
- $a_3 = \dfrac{\ln(3)}{3} \approx 0{,}3662$

$$= \lim_{n\to\infty} \frac{\ln n}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{1}{n} = 0$$

= maka $\{a_n\}$ terbatas dibawah oleh 0 dan terbatas diatas oleh a_3

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3a) Tulis rumus eksplisit & tentukan kekonvergenannya :
0,9 , 0,99 , 0,999 . 0,9999 , ---

b) Dengan definisi limit , buktikan barisan $\{a_n\}$ konvergen :
$$a_n = \frac{n+3}{3n-2}$$

c) Tentukan kemotonon , keterbatasan , dan limit (jika ada) barisan :
$$a_n = \frac{n!}{10^n}$$

Jawab :

3a) rumus eksplisit = $a_n = \left(1 - \frac{1}{10^n}\right)$

konvergen = $\lim_{n \to \infty} 1 - \frac{1}{10^n} = 1 - \frac{1}{10^\infty} = 1 - 0 = 1$

b) $a_n = \frac{n+3}{3n-2} = \lim_{n \to \infty} \frac{n+3}{3n-2} = \frac{1}{3}$ (pangkat terbesar)

- misal $\varepsilon > 0$
$|a_n - L| = \left|\frac{n+3}{3n-2} - \frac{1}{3}\right| = \left|\frac{3n+9-(3n-2)}{9n-6}\right| = \frac{11}{9n-6}$

- untuk $n > N$
$\frac{11}{9n-6} < \frac{11}{9N-6} = \varepsilon$, $\frac{11}{9N-6} = \varepsilon \Leftrightarrow 9N-6 = \frac{11}{\varepsilon} \Leftrightarrow N = \frac{\frac{11}{\varepsilon}+6}{9}$

- maka, $|a_n - L| = \left|\frac{n+3}{3n-2} - \frac{1}{3}\right| = \frac{11}{9n-6} < \frac{11}{9N-6}$
$= \frac{11}{9N-6}$
$= \frac{11}{9\left(\frac{11/\varepsilon+6}{9}\right)-6} = \frac{11}{(11/\varepsilon+6)-6} = \frac{11}{11/\varepsilon}$
$= \varepsilon$

c) $a_n = \frac{n!}{10^n}$

→ kemonotonan

$a_n = \frac{n!}{10^n} \iff \frac{a_{n+1}}{a_n} = \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \frac{n+1}{10}$

Maka,

- $\frac{a_{n+1}}{a_n} \leq 1$ dengan $\{a_n\}$ tak naik pada $n = 1, 2, 3, \ldots 9$
- $\frac{a_{n+1}}{a_n} > 1$ dengan $\{a_n\}$ naik pada $n = 10, 11, 12, \ldots$

→ keterbatasan

- $\{a_n\}$ tak naik pada $n = 1, 2, 3, \ldots 9$, yakni :

  $a_1 \geq a_2 \geq a_3 \geq \ldots \geq 9$

  Maka :

  $a_9 = \frac{9!}{10^9} \approx 3.6288 \times 10^{-4}$ (batas bawah)

  $a_{10} = \frac{10!}{10^{10}} = \frac{10 \cdot 9!}{10 \cdot 10^9} = \frac{9!}{10^9}$ (batas bawah juga)

- $\{a_n\}$

  $\lim_{n \to \infty} \frac{n!}{10^n} = \lim_{n \to \infty} \frac{n(n-1)(n-2) \ldots 3.2.1}{10.10.10 \ldots 10}$

  $= \lim_{n \to \infty} \frac{n}{10} \cdot \frac{n-1}{10} \cdot \frac{n-2}{10} \ldots \frac{3}{10} \cdot \frac{2}{10} \cdot \frac{1}{10}$

  $= \frac{\infty}{10} \cdot \frac{\infty - 1}{10} \cdot \frac{\infty - 2}{10} \ldots \frac{3}{10} \cdot \frac{2}{10} \cdot \frac{1}{10}$

  $= \infty$ (batas atas tak hingga)