Tugas Responsi Pertemuan 5
Kalkulus 2

Kelompok 7:

Nabil Naufal	G1401211008
Muhammad Rizky Fajar	G1401211009
Salsabila Dwi Rahmi	G1401211026
Adisti Suci Rahmah	G1401211027
Farrel Gilbran	G1401211057
Vita Rizkyana Anggraeni	G1401211065
Kamilah Nurul Azizah	G1401211073
Septiranny Rizqika Putri	G1401211083
Hanifa Rahmacindia Nasution	G1401211094

Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

1.

$$\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$$

1) $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

• Uji Banding Limit

$b_n = \frac{3}{n}$ (Divergen) → Deret harmonik

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3n+1}{n^2-4} \cdot \frac{n}{3} = \lim_{n\to\infty} \frac{3n^2+n}{3n^2-12} = 1 \rightarrow 0 < L < \infty$$

Menurut teori Uji Banding Limit, karena $\sum b_n$ divergen maka $\sum a_n$ divergen.

Deret $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ divergen.

2.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$$

2) $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$

Uji Banding

$$\frac{n}{n^2+2n} \leq \frac{n}{n^2+2n-3} \leq \ldots$$

$$\frac{1}{n+2} \leq \frac{n}{n^2+2n-3}$$

$$\lim_{n\to\infty} \frac{1}{n+2} = 0 \rightarrow \sum_{n=1}^{\infty} \frac{1}{n+2} = \text{konvergen}$$

$$\downarrow$$

$$\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3} = \text{konvergen}$$

3.

$$\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$$

3. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

$\Rightarrow$ menggunakan Uji Hasil Bagi / Uji Rasio :

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!}$$

$$= \lim_{n\to\infty} (n+1) \left(\frac{n}{n+1}\right)^{100}$$

$$= \lim_{n\to\infty} (n+1) \cdot \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{100}$$

$$= \infty \cdot (1)^{100} = \infty > 1$$ Deret Divergen.

4.

$$\sum_{n=1}^{\infty} \frac{3^k + k}{k!}$$

4. $\sum_{k=1}^{\infty} \frac{3^k + k}{k!}$

UJI HASIL BAGI (RASIO)

$$\rho = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$$

$$= \lim_{k\to\infty} \frac{3^{k+1}+(k+1)}{(k+1)!} \cdot \frac{k!}{3^k+k}$$

$$= \lim_{k\to\infty} \frac{3 \cdot 3^k + (k+1)}{(k+1)(3^k+k)}$$

$$= \lim_{k\to\infty} \frac{\frac{3 \cdot 3^k}{k+1} + 1}{3^k + k}$$

$$= \lim_{k\to\infty} \frac{\frac{3}{k+1} + \frac{1}{3^k}}{1 + \frac{k}{3^k}}$$

$$= \frac{0+0}{1+0} = 0 < 1$$, maka $\sum_{k=1}^{\infty} \frac{3^k+k}{k!}$ konvergen

5.

$$\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$$

5) $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

- Uji Banding Limit

$b_n = \frac{3}{n}$ (Divergen) $\rightarrow$ Deret harmonik

$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3n+1}{n^2-4} \cdot \frac{n}{3} = \lim_{n\to\infty} \frac{3n^2+n}{3n^2-12} = 1 \rightarrow 0 < L < \infty$

Menurut teori Uji Banding Limit, karena Σb_n divergen maka Σa_n divergen

Deret $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ divergen.

6.

$$\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$$

(6) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$

$\lim_{n\to\infty} \left(\frac{n}{3n+2}\right)^n$

$R = \lim_{n\to\infty} \left(\left(\frac{n}{3n+2}\right)^n\right)^{1/n}$

$= \lim_{n\to\infty} \frac{n}{3n+2}$

$= \lim_{n\to\infty} \frac{n}{n\left(3+\frac{2}{n}\right)}$

$= \lim_{n\to\infty} \frac{1}{3+\frac{2}{n}}$

$= \frac{1}{3} < 1 \rightarrow$ konvergen

7.

$$\sum_{n=2}^{\infty}\left(\frac{n}{\ln n}\right)^{n}$$

7. $\sum_{n=2}^{\infty}\left(\frac{1}{\ln n}\right)^{n}$

* menggunakan uji akar $\lim_{n\to\infty} (a_n)^{\frac{1}{n}}$

$a_n = \left(\frac{1}{\ln n}\right)^{n}$

$\Rightarrow \lim_{n\to\infty}\left(\left(\frac{1}{\ln n}\right)^{n}\right)^{\frac{1}{n}}$

$= \lim_{n\to\infty}\frac{1}{\ln n} = 0$

$R = 0$, $0 < 1$, maka $\sum_{n=1}^{\infty}\left(\frac{1}{\ln n}\right)^{n}$ konvergen

Tentukan apakah deret yang diberikan adalah konvergen mutlak, konvergen bersyarat, atau divergen.

8.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$$

8. $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n}{n+1}$

Jawab :

Uji Ganti Tanda untuk $\sum U_n$

$a_n = \frac{n}{n+1}$

• Cek apakah $\sum a_n$ turun:

$a_{n+1} < a_n$

$\frac{(n+1)}{n+2} < \frac{n}{n+1}$

$(n^2+2n+1) < n^2+2n$ (salah)

• Cek nilai limitnya:

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$

$\therefore$ $\sum U_n$ divergen menurut uji ganti tanda.

Uji kedivergenan pada $\sum |U_n|$

$\lim_{n\to\infty} |U_n|$

$= \lim_{n\to\infty} \left| (-1)^{n+1} \cdot \frac{n}{n+1} \right|$

$= \lim_{n\to\infty} \frac{n}{n+1}$

$= 1 \neq 0$

$\therefore$ $\sum |U_n|$ divergen menurut uji kedivergenan.

Karena $\sum |U_n|$ divergen dan $\sum U_n$ divergen, maka $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n}{n+1}$ divergen.

9.

$$\sum_{n=1}^{\infty} sin\frac{n!}{n^2}$$

9. $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$

$$\sum_{n=1}^{\infty} |u_n| = \sum_{n=1}^{\infty} \left| \sin \frac{n!}{n^2} \right|$$

$$= \sum_{n=1}^{\infty} \left| \sin \frac{\cancel{n} \cdot (n-1)!}{n^{\cancel{2}}} \right|$$

$$= \sum_{n=1}^{\infty} \left| \sin \frac{(n-1)!}{n} \right|$$

$$= 1 \quad \text{(Konvergen mutlak)}$$

10.

$$\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^{n}$$

10.) $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n = \sum_{n=1}^{\infty} (-1)^n \cdot \left(\frac{4}{3}\right)^n$

$U_n = (-1)^n \cdot \left(\frac{4}{3}\right)^n \quad ; \quad |U_n| = a_n = \left(\frac{4}{3}\right)^n$

∘) Uji kekonvergenan $\sum |U_n|$ dengan uji hasil bagi (rasio)

$\sum_{n=1}^{\infty} |U_n| = \sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$

$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

$= \lim_{n\to\infty} \left(\frac{4^{n+1}}{3^{n+1}}\right) : \left(\frac{4^n}{3^n}\right)$

$= \lim_{n\to\infty} \frac{4 \cdot \cancel{4^n} \cdot \cancel{3^n}}{3 \cdot \cancel{3^n} \cdot \cancel{4^n}}$

$\rho = \frac{4}{3} > 1$

maka, $\sum_{n=1}^{\infty} |U_n|$ divergen.

∘) Uji kekonvergenan $\sum U_n$ dengan uji deret ganti tanda

$\sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} (-1)^n \cdot \left(\frac{4}{3}\right)^n$

• cek kemonotonan

$\frac{a_{n+1}}{a_n} = \left(\frac{4^{n+1}}{3^{n+1}}\right) \cdot \left(\frac{3^n}{4^n}\right) = \frac{4 \cdot 4^n \cdot 3^n}{3 \cdot 3^n \cdot 4^n} = \frac{4}{3} > 1 \rightarrow$ naik

• cek nilai limit

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{4}{3}\right)^n = \infty \neq 0$

maka, $\sum_{n=1}^{\infty} U_n$ divergen.

$\therefore \sum^{\infty} (-1)^n \cdot \left(\frac{4}{3}\right)^n = \sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n$ divergen