Tugas Responsi Pertemuan 3

Kalkulus 2

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Kalkulus II (R) - Pertemuan 3

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Latihan Soal Tugas Kelompok

1. $a_n = \frac{n}{3n-1}$

• Kekonvergenan

$\lim_{n\to\infty} \frac{n}{3n-1} = \frac{1}{3}$ → Konvergen ke $\frac{1}{3}$

• Kemonotonan

$$a_n - a_{n+1} = \frac{n}{3n-1} - \frac{n+1}{3(n+1)-1}$$

$$= \frac{n}{3n-1} - \frac{n+1}{3n+2}$$

$$= \frac{(n(3n+2)) - (n+1(3n-1))}{(3n-1)(3n+2)}$$

$$= \frac{(3n^2+2n) - (3n^2+3n-n-1)}{9n^2+6n-3n-2}$$

$$= \frac{1}{9n^2+3n-2} > 0$$ (Turun)

2. $a_n = \frac{n^3+3n^2+3n}{(n+1)^3}$

• Kekonvergenan

$\lim_{n\to\infty} \frac{n^3+3n^2+3n}{(n+1)^3}$

$\lim_{n\to\infty} \frac{3n^2+6n+3}{3(n+1)^2}$

$\lim_{n\to\infty} \frac{3n^2+6n+3}{3(n^2+2n+1)}$

$\lim_{n\to\infty} \frac{3n^2+6n+3}{3n^2+6n+3} = 1$

Konvergen ke 1

• Kemonotonan

$$\frac{n^3+3n^2+3n}{(n+1)^3} - \frac{(n+1)^3+3(n+1)^2+3(n+1)}{(n+1+1)^3}$$

$$= \frac{n^3+3n^2+3n}{(n+1)^3} - \frac{(n+1)^3+3(n+1)^2+3(n+1)}{(n+2)^3}$$

$$= \frac{\left((n^3+3n^2+3n)(n+2)^3\right) - \left(\left((n+1)^3+3(n+1)^2+3(n+1)\right)(n+1)^3\right)}{(n+1)^3(n+2)^3}$$

$$= \frac{-3n^2-9n-7}{(n^2+3n+2)^3} < 0$$ (Naik)

3. $a_n = \frac{\cos(n\pi)}{n}$

• Kekonvergenan

$-1 \leq \cos n\pi \leq 1$

$\frac{-1}{n} \leq \frac{\cos(n\pi)}{n} \leq \frac{1}{n}$

$\lim_{n\to\infty} -\frac{1}{n} = 0$

$\lim_{n\to\infty} \frac{1}{n} = 0$

Konvergen ke 0

• Kemonotonan

$$\frac{\cos n\pi}{n} - \frac{\cos(n+1)\pi}{n+1}$$

$$= \frac{(\cos n\pi(n+1)) - (\cos(n+1)\pi \cdot (n))}{n(n+1)}$$

→ Tidak naik dan tidak turun

4. $a_n = e^{-n} \cdot \sin n$

• Kekonvergenan

$-1 \leq \sin n \leq 1$

$-e^{-n} \leq \sin n \cdot e^{-n} \leq e^{-n}$

$\frac{-1}{e^n} \leq e^{-n} \sin n \leq \frac{1}{e^n}$

$\lim_{n\to\infty} -\frac{1}{e^n} = 0$

$\lim_{n\to\infty} \frac{1}{e^n} = 0$

Konvergen ke 0

• Kemonotonan

$a_n - a_{n+1} = (e^n \sin n) - (e^{(n+1)} \sin(n+1))$

$= \frac{e \sin(n) - \sin(n+1)}{e^{n+1}} > 0$

(Turun)

5. $a_n = \frac{1}{n^3}$

• Kekonvergenan

$\lim_{n\to\infty} \frac{1}{n^3} = 0$

Konvergen ke 0

• Kemonotonan

$\frac{a_n}{a_{n+1}} = \frac{\frac{1}{n^3}}{\frac{1}{(n+1)^3}}$

$= \frac{(n+1)^3}{n^3}$

$= \frac{n^3 + 3n^2 + 3n + 1}{n^3} > 1$ (Turun)

6. $\frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots$

• Rumus Eksplisit

$U_n = a \cdot r^{n-1}$

$= \frac{1}{2^2}\left(\frac{1}{2}\right)^{n-1}$

$= \frac{1}{2^2} \cdot \frac{1}{2^{n-1}}$

$= \frac{1}{2^{2+n-1}}$

$= \frac{1}{2^{n+1}}$

• Kekonvergenan

$\lim_{n\to\infty} \frac{1}{2^{n+1}} = 0$

Konvergen ke 0

7. $\sin 1, 2 \sin \frac{1}{2}, 3 \sin \frac{1}{3}, \ldots$

• Rumus Eksplisit

$a_n = n \sin \frac{1}{n}$

• Kekonvergenan

$\lim_{n\to\infty} n \sin \frac{1}{n}$

$\lim_{t\to 0} \frac{1}{t} \sin \frac{1}{t} = 1$

Konvergen ke 1

8. $0{,}1, 0{,}11, 0{,}111, 0{,}1111, \ldots$

• Rumus Eksplisit

$\frac{1}{9}(0.9, 0.99, 0.999, \ldots)$

$\frac{1}{9}(1-0.1, 1-0.01, 1-0.001, \ldots)$

$\frac{1}{9}\left[1 - \left(\frac{1}{10}\right)^n\right]$

• Kekonvergenan

$\lim_{n\to\infty} \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)$

$= \frac{1}{9}(1 - 0) = \frac{1}{9}$

Konvergen ke $\frac{1}{9}$