Nama = Nazuwa Alia
NIM = 6149211033

No
Date

Tugas Minggu ke 3

1a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya :

$\cos \pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$

- Rumus Eksplisit

$a_n = \frac{\cos n\pi}{n^2}$

- Kekonvergenan

$a_n = \frac{\cos(n\pi)}{n^2}$

$\lim_{n\to\infty} \frac{\cos(n\pi)}{n^2}$

$-1 \leq \frac{\cos(n\pi)}{n^2} \leq 1$

$\frac{-1}{n^2} \leq \frac{\cos(n\pi)}{n^2} \leq \frac{1}{n^2}$

- $\lim_{n\to\infty} \frac{-1}{n^2} = 0$
- $\lim_{n\to\infty} \frac{\cos n\pi}{n^2} = 0$
- $\lim_{n\to\infty} \frac{1}{n^2} = 0$

$\Rightarrow$ Sehingga Konvergen ke 0

1b). - Karena $\{a_n\}$ konvergen ke A maka $\lim_{n\to\infty} a_n = A$, setiap $\varepsilon > 0$, $N_1 > 0$ untuk $n > N_1$ berlaku $|a_n - A| < \frac{1}{2}\varepsilon$

- Karena $\{b_n\}$ konvergen ke B maka $\lim_{n\to\infty} b_n = B$, setiap $\varepsilon > 0$, $N_2 > 0$ untuk $n > N_2$ berlaku $|b_n - B| < \frac{1}{2}\varepsilon$

- $$\begin{aligned} |a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\ &\leq |a_n - A| + |b_n - B| \\ &< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\ &= \varepsilon \end{aligned}$$

- Terbukti bahwa $\lim_{n\to\infty} (a_n + b_n) = A + B$

1c) $a_n = \sin \frac{n\pi}{4}$

- Kemonotonan

$a_n = \frac{\sin n\pi}{4}$

$\frac{\sin(n\pi)}{4} - \frac{\sin(n+1)\pi}{4} = \frac{\sin(n\pi)}{4} - \frac{-\sin(n\pi)}{4}$

$= \frac{\sin(n\pi)}{4} + \frac{\sin(n\pi)}{4} = \frac{2\sin(n\pi)}{4}$

$= \frac{\sin(n\pi)}{2}$

$a_1 = \frac{\sqrt{2}}{2}$; $a_2 = 1$; $a_3 = \frac{\sqrt{2}}{2}$; $a_4 = 0$

$\Rightarrow$ Barisan tidak naik dan tidak turun

No
Date

Keterbatasan

$-1 \le \sin \frac{n\pi}{4} \le 1$, $\lim_{n\to\infty} \sin \frac{n\pi}{4}$ → tidak ada

Teorema apit tidak berlaku sehingga hasilnya divergen.

2 a). Tuliskan rumus eksplisit barisan berikut dan tentukan kekonvergenannya

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots$

- Rumus Eksplisit

$a_n = \frac{(-1)^{n+1}}{n} = \frac{(-1)^{n-1}}{n}$

- Kekonvergenan

$a_n = \frac{(-1)^{n+1}}{n}$

$\lim_{n\to\infty} \frac{(-1)^{n+1}}{n}$

$-1 \le \frac{(-1)^{n+1}}{n} \le 1$ → $\frac{-1}{n} \le \frac{(-1)^{n+1}}{n} \le \frac{1}{n}$

$\lim_{n\to\infty} \frac{-1}{n} = 0$, $\lim_{n\to\infty} \frac{1}{n} = 0$

$\Rightarrow \frac{(-1)^{n+1}}{n}$, konvergen ke 0

2 b). Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen :

$a_n = \frac{3-8.2^n}{5+4.2^n}$

$\lim_{n\to\infty} \frac{3-8.2^n}{5+4.2^n}$

$\lim_{n\to\infty} \frac{\frac{3}{2^n} - 8}{\frac{5}{2^n} + 4}$

$= \frac{0-8}{0+4}$

$= -2$

Sehingga, $\frac{3-8.2^n}{5+4.2^n}$ konvergen ke -2

No
Date

2c). Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut :

$a_n = \dfrac{\ln n}{n}$

$\Rightarrow a_n = \dfrac{\ln n}{n}$

$\lim\limits_{n\to\infty} \dfrac{\frac{1}{n}}{1} = 0$, konvergen ke 0
keterbatasan ke 0

• Kemonotonan

$a'(n) = \dfrac{\frac{1}{n}\cdot n - 1 . \ln n}{n^2} = \dfrac{1 - \ln n}{n^2}$

Tidak naik dan tidak turun,

3a). Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya

0.9, 0.99, 0.999, 0.9999, ...

• Rumus Eksplisit :

0.9, 0.99, 0.999, 0.9999, ...

(1-0,1) ; (1-0,01) ; (1-0,001)

$1 - \frac{1}{10}, \; 1 - \frac{1}{100}, \; 1 - \frac{1}{1000}$

$a_n = 1 - \dfrac{1}{10^n}$

• Kekonvergenan

$a_n = 1 - \dfrac{1}{10^n}$

$\lim\limits_{n\to\infty} 1 - \dfrac{1}{10^n} = 1 - 0 = 1$, konvergen ke 1

3b). Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen :

$a_n = \dfrac{n+3}{3n-2}$

$\Rightarrow a_n = \dfrac{n+3}{3n-2} \quad = \dfrac{1 + \frac{3}{\infty}}{3n - \frac{2}{\infty}}$

$\lim\limits_{n\to\infty} \dfrac{n+3}{3n-2} \quad = \dfrac{1+0}{3-0}$

$\lim\limits_{n\to\infty} \dfrac{\frac{n}{n} + \frac{3}{n}}{\frac{3n}{n} \cdot \frac{2}{n}} \quad = \dfrac{1}{3}$ (konvergen)

$|a_n - L| = \left| \dfrac{n+3}{3n-2} - \dfrac{1}{3} \right|$

$= \left| \dfrac{n + 3(3) - 1(3n-2)}{(3n-2)(3)} \right|$

No
Date

$= \left| \frac{11}{9n-6} \right|$

$\leq \frac{11}{9N-6}$

Nilai N :

$= \frac{11}{9\left(\frac{11+6\varepsilon}{9\varepsilon}\right)-6} = \varepsilon$ (terbukti)

$\frac{11}{9N-6} = \varepsilon$

$11 = 9N\varepsilon - 6\varepsilon$

$9N\varepsilon = 11 + 6\varepsilon$

$N = \frac{11+6\varepsilon}{9}$

3c). Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut :

$a_n = \frac{n!}{10^n}$

- Kemonotonan

$\frac{a_n}{a_{n+1}} = \frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{n+1}}}$

$= \frac{1}{n+1} \cdot 10^{n+1}$

$= \frac{10^{n+1}}{n+1} > 1$, barisan monoton naik

- Kekonvergenan

$\lim_{n\to\infty} \frac{n!}{10^n}$

$= \frac{1,2,3,\ldots,n}{10,10,10,\ldots 10}$

$= \frac{\infty}{\infty}$ (divergen)

JOYKO 36 Lines, 6 mm

Scanned by TapScanner