Farhan Abdillah Harahap - G1401211007

1.) a) rumus eksplisit dan kekonvergenan.

$$\cos\pi,\ \frac{\cos 2\pi}{4},\ \frac{\cos 3\pi}{9},\ \frac{\cos 4\pi}{16},\ \dots$$

$$\boxed{a_n = \frac{\cos n\pi}{n^2} \ ;\ n = 1, 2, 3, \dots}$$

kekonvergenan

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\cos n\pi}{n^2}$$ → menggunakan teorema apit

$$-1 \leq \cos(n\pi) \leq 1$$

$$\frac{-1}{n^2} \leq \frac{\cos(n\pi)}{n^2} \leq \frac{1}{n^2}$$

$\lim\limits_{n\to\infty} -\frac{1}{n^2} = \lim\limits_{n\to\infty} \frac{1}{n^2} = 0$, maka $\frac{\cos(n\pi)}{n^2}$ konvergen ke 0 //

b.) $\{a_n\} \to A$

$\{b_n\} \to B$

Buktikan (def limit) bahwa $\{a_n + b_n\} \to A + B$

$\Rightarrow \{a_n\} \to A \rightarrow \lim\limits_{n\to\infty} a_n = A.$ untuk setiap $\varepsilon > 0$ terdapat $N_1 > 0$ sedemikian sehingga untuk $n > N_1$ berlaku $\boxed{|a_n - A| < \frac{1}{2}\varepsilon}$

$\varepsilon > 0$ sembarang, maka $\frac{1}{2}\varepsilon > 0$ juga sembarang

$\Rightarrow \{b_n\} \to B \rightarrow \lim\limits_{n\to\infty} b_n = B.$ untuk setiap $\varepsilon > 0$ terdapat $N_2 > 0$ sedemikian sehingga untuk $n > N_2$ berlaku $\boxed{|b_n - B| < \frac{1}{2}\varepsilon}$

$\Rightarrow$ Pilih $N = \max\{N_1, N_2\}$

$$\begin{aligned} |a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\ &\leq |a_n - A| + |b_n - B| \\ &< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\ &= \varepsilon \ // \end{aligned}$$

∴ Terbukti bahwa $\lim\limits_{n\to\infty} (a_n + b_n) = A + B$ //

1.) c.) Tentukan kemonotonan, keterbatasan, limit (jika ada)

$$a_n = \sin\left(\frac{n\pi}{4}\right)$$

kemonotonan

$a_n = \sin\left(\frac{n\pi}{4}\right)$

$a'_n = \frac{\pi}{4}\cos\left(\frac{n\pi}{4}\right)$

turun: $\cos\left(\frac{n\pi}{4}\right) < \cos(k\pi)$

$\frac{n\pi}{4} < \frac{4k\pi}{4}$

$\frac{n\pi - 4k\pi}{4} < 0$

$\pi(n-4k) < 0$

$n < 4k \; ; k = 1,2,3,\ldots$

naik: $a'_n > 0$

$\frac{\pi}{4}\cos\left(\frac{n\pi}{4}\right) > 0$

$\cos\left(\frac{n\pi}{4}\right) > \cos(k\pi)$

$\frac{n\pi}{4} > k\pi$

$\frac{n\pi}{4} > \frac{4k\pi}{4}$

$\frac{n\pi - 4k\pi}{4} > 0$

$\pi(n-4k) > 0$

$n > 4k \; ; k = 1,2,3,\ldots$

$a_n = \sin\left(\frac{n\pi}{4}\right)$ bukan barisan monoton //

keterbatasan

$-1 \le \sin\left(\frac{n\pi}{4}\right) \le 1$

↳ teorema apit tak berlaku (divergen) //

2.) a.) rumus eksplisit, kekonvergenan

$$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}$$

$$a_n = \frac{(-1)^{n+1}}{n}$$

$-1 \le (-1)^{n+1} \le 1$

$-\frac{1}{n} \le \frac{(-1)^{n+1}}{n} \le \frac{1}{n}$

$\lim\limits_{n\to N} \frac{(-1)^{n+1}}{n} = 0$ atau dengan teorema apit

$\lim\limits_{n\to N} -\frac{1}{n} = \lim\limits_{n\to N} \frac{1}{n} = 0$ maka $\frac{(-1)^{n+1}}{n}$ konvergen ke 0. //

b.) dengan definisi limit, buktikan an konvergen

Jika untuk setiap $\varepsilon > 0$ terdapat $N > 0$ sedemikian sehingga $n \ge N$ berlaku $|a_n - L| < \varepsilon$

$$a_n = \frac{3 - 8\cdot 2^n}{5 + 4\cdot 2^n}$$

$$\lim_{n\to N} \frac{2^n\left(\frac{3}{2^n} - 8\right)}{2^n\left(\frac{5}{2^n} + 4\right)} = -2 \to L = -2$$

↓ konvergen ke -2 //

$$|a_n - L| = \left|\frac{3 - 8\cdot 2^n}{5 + 4\cdot 2^n} + 2\right|$$

$$= \left|\frac{3 - 8\cdot 2^n + 2(5 + 4\cdot 2^n)}{5 + 4\cdot 2^n}\right|$$

$$= \frac{13}{5 + 4\cdot 2^n} \le \frac{13}{5 + 4\cdot 2^N} = \frac{13}{5 + 4\cdot 2^{\frac{\ln\left(\frac{13}{\varepsilon} - 5\right) - \ln 4}{\ln(2)}}} = \varepsilon \;//$$

pilih $N = \dfrac{\ln\left(\frac{+13}{\varepsilon} - 5\right) - \ln 4}{\ln 2}$

analisis pemilihan nilai N

$$\frac{13}{5 + 4\cdot 2^N} = \varepsilon$$

$$\frac{13}{5 + 2^{N+2}} = \varepsilon$$

$$13 = 5\varepsilon + 2^{N+2}\varepsilon$$

$$2^{N+2} = \frac{13 - 5\varepsilon}{\varepsilon}$$

$$N+2 \ln(2) = \ln\left(\frac{13}{\varepsilon} - 5\right)$$

$$N+2 = \frac{\ln\left(\frac{13}{\varepsilon} - 5\right)}{\ln(2)}$$

$$N = \frac{\ln\left(\frac{13}{\varepsilon} - 5\right) - \ln(4)}{\ln(2)}$$

2.) c.) kemonotonan, keterbatasan, limit (jika ada)

$$a_n = \frac{\ln(n)}{n}$$

kemonotonan

$$a_n' = \frac{\frac{d}{dn}(\ln(n)) \cdot n - \frac{d}{dn}(n) \cdot \ln(n)}{n^2}$$

$$a_n' = \frac{\frac{1}{n} \cdot n - \ln(n)}{n^2} = \frac{1 - \ln(n)}{n^2}$$

a_n naik ketika $a_n' > 0$

$$\frac{1 - \ln(n)}{n^2} > 0$$

$$1 - \ln(n) > 0$$

$$\ln(n) < 1$$

$$\ln(n) < \ln(e)$$

$$n < e$$

a_n turun ketika $a_n' < 0$

$$\frac{1 - \ln(n)}{n^2} < 0$$

$$1 - \ln(n) < 0$$

$$\ln(n) > 1$$

$$\ln(n) > \ln(e)$$

$$n > e$$

a_n naik ketika $n < e$ dan a_n turun ketika $n > e$ berarti a_n bukan barisan monoton //

keterbatasan

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{\ln(n)}{n} \overset{LH}{=} \lim_{n \to \infty} \frac{\frac{1}{n}}{n} = \lim_{n \to \infty} \frac{1}{n^2} = 0$$

$\frac{\ln(n)}{n}$ konvergen ke 0 dan terbatas sampai 0 //

3.) a.) rumus eksplisit dan kekonvergenan

0,9, 0,99, 0,999, 0,9999, ...

$$\boxed{a_n = \frac{10^n - 1}{10^n}}$$

$n = 1, 2, 3, \ldots$

kekonvergenan

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{10^n - 1}{10^n} = \lim_{n \to \infty} \frac{\cancel{10^n}\left(1 - \frac{1}{10^n}\right)}{\cancel{10^n}} = 1$$

$a_n = \frac{10^n - 1}{10^n}$ konvergen ke 1 //

3.) b.) dengan definisi limit, buktikan $\{a_n\}$ konvergen

$$a_n = \frac{n+3}{3n-2}$$

Jika untuk setiap $\varepsilon > 0$ terdapat $N > 0$ sedemikian sehingga $n \geq N$ berlaku $|a_n - L| < \varepsilon$

$$\lim_{n\to\infty} \frac{n+3}{3n-2} = \frac{1}{3} \rightarrow L = \frac{1}{3}$$

konvergen ke $\frac{1}{3}$ //

$$|a_n - L| = \left|\frac{n+3}{3n-2} - \frac{1}{3}\right|$$

$$= \left|\frac{n+3}{3n-2} - \frac{n-\frac{2}{3}}{3(n-\frac{2}{3})}\right|$$

$$= \left|\frac{(n+3)-(n-\frac{2}{3})}{3n-2}\right|$$

$$= \left|\frac{3+\frac{2}{3}}{3n-2}\right| = \left|\frac{11}{3(3n-2)}\right| = \frac{11}{9n-6} \leq \frac{11}{9N-6}$$

analisis pemilihan nilai N

$$\frac{11}{9n-6} \leq \frac{11}{9N-6} = \varepsilon$$

$$11 = 9N\varepsilon - 6\varepsilon$$

$$11 + 6\varepsilon = 9N\varepsilon$$

$$N = \frac{11+6\varepsilon}{9\varepsilon}$$

pilih $N = \frac{11+6\varepsilon}{9\varepsilon}$

$$= \frac{11}{9\left(\frac{11+6\varepsilon}{9\varepsilon}\right)-6} = \varepsilon \quad //$$

c.) kemonotonan, keterbatasan, limit

$$a_n = \frac{n!}{10^n}$$

kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{n+1}}} = \frac{n!}{10^n} \cdot \frac{10^{n+1}}{(n+1)!} = \frac{\cancel{n!}}{10^n} \cdot \frac{10^n \cdot 10}{(n+1)\cancel{n!}} = \frac{10}{n+1}$$

$\frac{a_n}{a_{n+1}} < 1$; $n = 10, 11, 12, \ldots$ (naik)

$\frac{a_n}{a_{n+1}} \geq 1$; $n = 1, 2, \ldots, 9$ (tak naik)

$\therefore a_n = \frac{n!}{10^n}$ bukan barisan monoton //

keterbatasan

$$\lim_{n\to\infty} \frac{n!}{10^n} = \lim_{n\to\infty} \frac{n(n-1)\ldots 2 \cdot 1}{10 \cdot 10 \cdot \ldots \cdot 10}$$

$$= \lim_{n\to\infty} \frac{n}{10} \frac{n-1}{10} \frac{n-2}{10} \ldots \frac{2}{10} \frac{1}{10}$$

$$= \infty$$

$\therefore a_n = \frac{n!}{10^n}$ tak berbatas di atas (divergen) //