TUGAS KELOMPOK MINGGU 2
KALKULUS II

Kelompok 6:

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1. Tentukan integral berikut

   a. $\int_3^{\infty} \frac{x}{\sqrt{16+x^2}}\, dx$

   Misal:

   $u = (16 + x^2)^{1/2}$

   $u^2 = 16 + x^2$

   $2u\, du = 2x\, dx$

   $u\, du = x\, dx$

   $\int_3^{\infty} \frac{x}{\sqrt{16+x^2}}\, dx = \lim_{a\to\infty} \int_3^{a} \frac{x}{\sqrt{16+x^2}}\, dx = \lim_{a\to\infty} \int_3^{a} \frac{u}{u}\, du = \lim_{a\to\infty} u \Big|_3^a$

   $= \lim_{a\to\infty} (\sqrt{16+a^2} - \sqrt{16-3^2} = \infty$ (DIVERGEN)

   b. $\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}}\, dx$

   Misal:

   $u = (9 + x^2)^{1/2}$

   $u^2 = 9 + x^2$

   $2u\, du = 2x\, dx$

   $u\, du = x\, dx$

   $\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}}\, dx = \lim_{a\to-\infty} \int_a^{0} \frac{x}{\sqrt{9+x^2}}\, dx + \lim_{b\to\infty} \int_0^{b} \frac{x}{\sqrt{9+x^2}}\, dx$

   $= \lim_{a\to-\infty} \int_a^0 \frac{u}{u} du + \lim_{b\to\infty} \int_0^b \frac{u}{u} du = \lim_{a\to-\infty} u \Big|_a^0 + \lim_{b\to\infty} u \Big|_0^b$

   $= \lim_{a\to-\infty} \left(\sqrt{9+0} - \sqrt{9+a^2}\right) + \lim_{b\to\infty} \left(\sqrt{9+b^2} - \sqrt{9+0}\right) = \infty$ (DIVERGEN)

2. Tentukan integral berikut

   a. $\int_2^{\infty} \frac{\ln\sqrt{x}}{x}\, dx = \lim_{a\to\infty} \int_2^{a} \frac{\ln\sqrt{x}}{x}\, dx$

   Misal:

   $u = \ln\sqrt{x}$

$du = \frac{1}{x}\ dx$

$\lim_{a\to\infty} \int_2^a u\ du = \lim_{a\to\infty} \frac{1}{2} u^2 \Big|_2^a = \frac{1}{2}[\lim_{a\to\infty} (\ln^2 \sqrt{a} - \ln^2 \sqrt{2})] = \infty$ (DIVERGEN)

b. $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\ dx = \lim_{a\to-\infty} \int_a^0 \frac{x}{(x^2+4)}\ dx + \lim_{b\to\infty} \int_0^b \frac{x}{(x^2+4)}\ dx$

Misal:

$u = x^2 + 4$

$du = 2x\ dx$

$\frac{1}{2} du = x\ dx$

$\lim_{a\to-\infty} \int_a^0 \frac{1}{2u}\ du + \lim_{b\to\infty} \int_0^b \frac{1}{2u}\ du = \lim_{a\to\infty} \left(\frac{1}{2}\ln u\right)\Big|_a^0 + \lim_{a\to-\infty} \left(\frac{1}{2}\ln u\right)\Big|_0^b$

$= \frac{1}{2}\Big[(\ln(0^2+4) - \ln(-\infty)^2 + 4)] + \frac{1}{2}[(\ln(\infty^2+4) - \ln(0^2+4))\Big]$

$= \infty$ (DIVERGEN)

3. Tentukan integral berikut

a. $\int_2^{\infty} \frac{1}{x\ln x}\ dx = \lim_{a\to\infty} \int_2^a \frac{1}{x\ln x}\ dx$

Misal:

$u = \ln x$

$du = \frac{1}{x} dx$

$\lim_{a\to\infty} \int_2^a \frac{1}{u}\ du = \lim_{a\to\infty} \ln u \Big|_2^a = \frac{1}{2} \lim_{a\to\infty} (\ln(\ln a) - \ln(\ln 2) = \infty$ (DIVERGEN)

b. $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\ dx$

$= \lim_{a\to-\infty} \int_a^0 \frac{1}{x^2+4x+9}\ dx + \lim_{b\to\infty} \int_0^b \frac{1}{x^2+4x+9}\ dx$

$= \lim_{a\to-\infty} \int_a^0 \frac{1}{(x+2)^2+9}\ dx + \lim_{b\to\infty} \int_0^b \frac{1}{(x+2)^2+9}\ dx$

$= \lim_{a\to-\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Big|_a^0 + \lim_{b\to\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Big|_0^b$

$= \lim_{a\to-\infty} \left(\frac{\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}x+2\sqrt{5}}{5}\right)}{5}\right)\Bigg|_a^0 + \lim_{b\to\infty} \left(\frac{\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}x+2\sqrt{5}}{5}\right)}{5}\right)\Bigg|_0^b$

$= \frac{\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{5} - \frac{\sqrt{5}\tan^{-1}(-\infty)}{5} + \frac{\sqrt{5}\tan^{-1}(\infty)}{5} - \frac{\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{5}$

$= \frac{\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right) - \left(\sqrt{5}\left(-\frac{\pi}{2}\right)\right)}{5} + \frac{\sqrt{5}\left(\frac{\pi}{2}\right) - \sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{5}$

$= \left(\frac{2\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right) + \pi\sqrt{5}}{10}\right) + \left(\frac{\pi\sqrt{5} - 2\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{10}\right)$

$= \frac{2\sqrt{5}}{10}\pi = \frac{\sqrt{5}}{5}\pi$

4. Tentukan integral berikut

a. $\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$

Misal:

$u = \ln x$

$du = \frac{1}{x} dx$

$\int_2^{\infty} \frac{1}{x(\ln x)^2} dx = \lim_{a\to\infty} \int_2^{a} \frac{1}{u^2} du = \lim_{a\to\infty} \left(-\frac{1}{4}\right) \Big|_2^{a} = \lim_{a\to\infty} \left(-\frac{1}{a} + \frac{1}{\ln 2}\right) = \frac{1}{\ln 2}$

(KONVERGEN)

b. $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}} dx = \int_{-\infty}^{0} \frac{x}{e^{|x|}} dx + \int_{0}^{\infty} \frac{x}{e^{|x|}} dx = \lim_{a\to-\infty} \int_a^0 \frac{x}{e^{-x}} dx + \lim_{b\to\infty} \int_0^b \frac{x}{e^{x}} dx$

$= \lim_{a\to-\infty} \int_a^0 x.e^{x}\, dx + \lim_{b\to\infty} \int_0^b x.e^{-x}\, dx$

Misal:

$u = x \qquad v = \int e^{x}\, dx = e^{x} \qquad u = x \qquad v = \int e^{-x}\, dx = -e^{-x}$

$du = dx \qquad dv = e^{x}\, dx \qquad du = dx \qquad dv = e^{-x}\, dx$

$= \lim_{a\to-\infty} x.e^{x} - \int e^{x}\, dx \Big|_a^0 + \lim_{b\to\infty} x(-e^{-x}) - \int(-e^{-x})\, dx \Big|_0^b$

$= \lim_{a\to-\infty} x.e^{x} - e^{x} \Big|_a^0 + \lim_{b\to\infty} -x.e^{-x} - e^{-x} \Big|_0^b$

$= (0.e^{0} - e^{0}) - \left(\lim_{a\to-\infty} a.e^{a} - e^{a}\right) + \left(\lim_{b\to\infty} - b.e^{-b} - e^{-b}\right) - (-0.e^{-0} - e^{-0})$

$= -1 - (-1)$

$= 0$ (KONVERGEN)