TUGAS KELOMPOK MINGGU 2

KALKULUS II

Kelompok 6:

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SOAL LATIHAN

1. $a_n = \dfrac{n}{3n-1}$

∘> Kekonvergenan

$$\lim_{n\to\infty} \frac{n}{3n-1} \Rightarrow \text{bentuk } \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{1}{3} = \frac{1}{3} //$$

Barisan $\{a_n\}$ konvergen ke $\frac{1}{3}$.

∘> Kemonotonan

$$a_n - a_{n+1} = \frac{n}{3n-1} - \frac{n+1}{3(n+1)-1} = \frac{n}{3n-1} - \frac{n+1}{3n+2} = \frac{n(3n+2)-(3n-1)(n+1)}{(3n-1)(3n+2)}$$

$$= \frac{3n^2+2n-3n^2-2n+1}{9n^2+3n-2} = \frac{1}{9n^2+3n-2} > 0$$

$a_n > a_{n+1}$, maka barisan monoton turun //

(2) $a_n = \dfrac{n^3+3n^2+3n}{(n+1)^3}$

$\lim\limits_{n\to\infty} \dfrac{n^3+3n^2+3n}{(n+1)^3} \overset{LH}{=} \lim\limits_{n\to\infty} \dfrac{3n^2+6n+3}{3(n+1)^2} \overset{LH}{=} \lim\limits_{n\to\infty} \dfrac{6n+6}{6(n+1)} = 1$ Konvergen menuju 1

Kemonotonan $\quad u = n^3+3n^2+3n \quad v = (n+1)^3$

$a_n < a_{n+1} \quad du = 3n^2+6n+3 \quad dv = 3(n+1)^2$

Pembuktian : $a'(x) = \dfrac{(3n^2+6n+3)(n+1)^3 - (3(n+1)^2)(n^3+3n^2+3n)}{((n+1)^3)^2}$

$= \dfrac{(3n^2+6n+3)(n^3+3n^2+3n+1) - (3(n^2+2n+1))(n^3+3n^2+3n)}{(n+1)^6}$

$= \dfrac{(3n^2+6n+8)((n^3+3n^2+3n+1) - (n^3+3n^2+3n)}{(n+1)^6}$

n^2+2n+1

$= \dfrac{1}{(n+1)^4} > 0$ naik

3). $a_n = \dfrac{\cos(n\pi)}{n}$

$\Rightarrow -1 \leq \cos n\pi \leq 1$

$-\dfrac{1}{n} \leq \dfrac{\cos n\pi}{n} \leq \dfrac{1}{n}$

$\lim\limits_{n\to\infty} -\dfrac{1}{n} = 0$

$\lim\limits_{n\to\infty} \dfrac{1}{n} = 0$

konvergen ke 0

* Kemonotonan

$\dfrac{\cos n\pi}{n} - \dfrac{\cos(n+1)\pi}{n+1}$

$= \dfrac{(n+1)\cos n\pi - n\cos(n+1)\pi}{n^2+n}$

$\Rightarrow$ tidak naik dan tidak turun

4. $a_n = e^{-n} \cdot \sin(n)$

$= \lim_{n \to \infty^+} \left(e^{-n} \cdot \sin(n) \right)$

$= \lim_{n \to \infty^+} \left(\frac{\sin(n)}{e^n} \right)$

$= 0$ (KONVEREGEN)

$a_n - a_{n+1} = (e^{-n} \sin n) - (e^{-n+1} \cdot \sin(n+1))$

$= \frac{e \sin(n) - \sin(n+1)}{e^{n+1}} > 0$

$=$ (TURUN)

5. $a_n = \frac{1}{n^3}$

- kekonvergenan

$\lim_{n \to \infty} \frac{1}{n^3} = 0$

- Kemonotonan

$\frac{a_n}{a_{n+1}} = \frac{\frac{1}{n^3}}{\frac{1}{(n+1)^3}}$

$\frac{a_n}{a_{n+1}} = \frac{(n+1)^3}{n^3}$

$= \frac{n^3 + 3n^2 + 3n + 1}{n^3}$

$= 1 + \left(\frac{3n^2 + 3n + 1}{n^3} \right) > 1$ (Turun)

6. $\frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots$

$U_n = a \cdot r^{n-1}$

$= \frac{1}{2^2} \left(\frac{1}{2} \right)^{n-1}$

$= \frac{1}{2^2} \cdot \frac{1}{2^{n-1}}$

$= \frac{1}{2^{2+n-1}} = \frac{1}{2^{n+1}}$

$\lim_{n \to \infty} \frac{1}{2^{n+1}} = 0$ konvergen

7. $\sin 1,\ 2\sin\frac{1}{2},\ 3\sin\frac{1}{3},\ 4\sin\frac{1}{4},\ \ldots$

↳ rumus eksplisit

$a_n = n\sin\frac{1}{n}$; $n = 1, 2, 3, \ldots$

↳ kekonvergenan

$\lim_{n\to\infty} n\sin\frac{1}{n} \rightarrow \lim_{t\to 0} \frac{1}{t}\sin t = 1$

Sehingga $a_n = n\sin\frac{1}{n}$ konvergen ke 1.

8. $0.1,\ 0.11,\ 0.111,\ 0.1111, \ldots$

↳ $\frac{1}{9}(0.9,\ 0.99,\ 0.999, \ldots)$

$\frac{1}{9}(1-0.1,\ 1-0.01,\ 1-0.001, \ldots)$

$\frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)$

* Rumus eksplisit

$\frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]$,,

* kekonvergenan

$\lim_{n\to\infty} \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)$

$\frac{1}{9}(1-0)$

$\frac{1}{9}(1)$

konvergen menuju $\frac{1}{9}$,,