Dinda Khamila Nurfatimah
G1401211035

No.

Date

TUGAS MANDIRI RESPONSI 3

1. a) Tentukan rumus eksplisit dan kekonvergenannya :

$\cos\pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$

- Rumus eksplisit

$a_n = \frac{\cos n\pi}{n^2}, \ n \geq 1$

- Kekonvergenan

$-1 \leq \cos n\pi \leq 1$ $\qquad \lim_{n\to\infty} \left(-\frac{1}{n^2}\right) = 0$

$-\frac{1}{n^2} \leq \frac{\cos n\pi}{n^2} \leq \frac{1}{n^2}$ $\qquad \lim_{n\to\infty} \left(\frac{1}{n^2}\right) = 0$

$\therefore \{a_n\}$ konvergen ke 0 //

b) Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan $\{a_n + b_n\}$ konvergen ke A+B.

$\lim_{n\to\infty} a_n = A$, untuk setiap $\varepsilon > 0$ terdapat $N_1 > 0$ sedemikian sehingga $n \geq N_1$ berlaku

$|a_n - A| < \frac{1}{2}\varepsilon$

$\lim_{n\to\infty} b_n = B$, untuk setiap $\varepsilon > 0$ terdapat $N_2 > 0$ sedemikian sehingga $n \geq N_2$ berlaku

$|b_n - B| < \frac{1}{2}\varepsilon$

- Pilih $N = \max\{N_1, N_2\}$, diperoleh :

$$
\begin{aligned}
|a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\
&\leq |a_n - A| + |b_n - B| \\
&< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\
&= \varepsilon
\end{aligned}
$$

$\therefore$ Terbukti bahwa $\lim_{n\to\infty} (a_n + b_n) = A + B$.

c) $a_n = \sin \frac{n\pi}{4}$

- Kemonotonan

$a_n - a_{n+1} = \sin\frac{n\pi}{4} - \sin\frac{(n+1)\pi}{4} = \sin\frac{n\pi}{4} - \sin\frac{(n\pi + \pi)}{4}$

↳ Barisan tidak memiliki kemonotonan karena nilainya bisa + dan −.

- Keterbatasan

Karena barisan tidak monoton, maka tidak memiliki keterbatasan

- Limit

$-1 \leq \sin\frac{n\pi}{4} \leq 1 \Rightarrow$ divergen (tidak memiliki limit).

Dinda Khamila Nurfatimah
G1401211035

No.
Date

2a)
$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

∘) Rumus eksplisit
$a_n = (-1)^{n+1} \cdot \frac{1}{n}, n \geq 1$

∘) Kekonvergenan
$\lim_{n\to\infty} \left|(-1)^{n+1} \cdot \frac{1}{n}\right| = \lim_{n\to\infty} \left|(-1)^{n+1}\right|\left|\frac{1}{n}\right| = \lim_{n\to\infty} \frac{1}{n} = 0$ //

2b) Buktikan barisan $\{a_n\}$ konvergen :
$a_n = \frac{3-8.2^n}{5+4.2^n}$
$= \lim_{n\to\infty} \frac{3-8.2^n}{5+4.2^n} \overset{LH}{=} \lim_{n\to\infty} -\frac{\cancel{8}^{2} n.2^{n-1}}{\cancel{4} n.2^{n-1}} = \lim_{n\to\infty} -2 = -2$ //
∴ $\{a_n\}$ konvergen ke -2

2c) $a_n = \frac{\ln n}{n}$

∘) Kemonotonan
$a'(n) = \frac{\frac{1}{n} \cdot n - \ln n \cdot 1}{n^2} = \frac{1-\ln n}{n^2}$ ⇒ bukan barisan monoton karena nilainya bisa (+) dan (−).

∘) Keterbatasan
Tidak ada keterbatasan karena bukan barisan monoton

∘) Kekonvergenan
$\lim_{n\to\infty} \frac{\ln n}{n} \overset{LH}{=} \frac{1/n}{1} = \frac{1}{n} = 0$ //

3a) 0.9, 0.99, 0.999, 0.9999, ...

∘) Rumus eksplisit
(1−0,1), (1−0,01), (1−0,001), ...
$a_n = 1 - \frac{1}{10^n}, n \geq 1$

∘) Kekonvergenan
$\lim_{n\to\infty} 1 - \frac{1}{10^n} = \lim_{n\to\infty} 1 - \frac{1}{10^n} = 1 - 0 = 1$

∴ $\{a_n\}$ konvergen ke 1 //

Dinda Khamila Nurfatimah
G1401211035

No.

Date

3b) $a_n = \frac{n+3}{3n-2}$

◦> Kekonvergenan

$$\lim_{n\to\infty} \frac{n+3}{3n-2} \overset{LH}{=} \lim_{n\to\infty} \frac{1}{3} = \frac{1}{3}$$

∴ Konvergen ke $\frac{1}{3}$.

3c) $a_n = \frac{n!}{10^n}$

◦> Kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{n!/10^n}{(n+1)!/10^{n+1}} = \frac{n!}{\cancel{10^n}} \times \frac{10.\cancel{10^n}}{(n+1)!} = \frac{10\,n!}{(n+1)!} > 1 \text{ (monoton naik).}$$

◦> Keterbatasan

$$\lim_{n\to\infty} \frac{n!}{10^n} = \frac{1\times2\times3\times\ldots\times n}{\underbrace{10\times10\times10\times\ldots\times10}_{n}} \Rightarrow \{a_n\} \text{ tak terbatas}$$

◦> Kekonvergenan

$$\lim_{n\to\infty} \frac{n!}{10^n} = \frac{1\times2\times3\times\ldots\times n}{\underbrace{10\times10\times10\times\ldots\times10}_{n}} \Rightarrow \frac{\infty}{\infty} \Rightarrow \text{bentuk taktentu, maka divergen}$$

tidak ada limit.