KELOMPOK 8

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Tugas Responsi

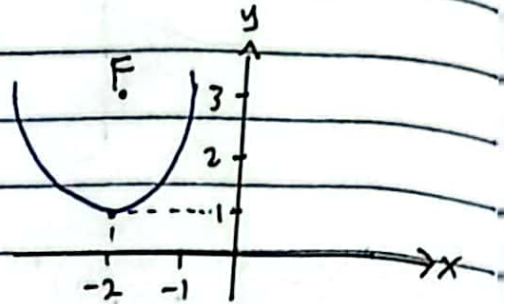**1.** Tentukan titik puncak, fokus, dan direktriks dari parabola, serta gambarlah grafiknya.

a. $(x+2)^2 = 8(y-1)$

$h = -2$; $t = 1$; $8 = 4p \rightarrow p = 2$

titik puncak: $(h, t) = (-2, 1)$

fokus : $(h, p+t) = (-2, 3)$

direktriks : $y = t - p = 1 - 2 = -1$

b. $4x^2 + 16x - 16y + 32 = 0$

$4(x^2 + 4x) - 16y + 32 = 0$

$4[(x+2)-4] - 16y + 32 = 0$

$4(x+2) - 16 - 16y + 32 = 0$

$4(x+2) - 16y = -16$

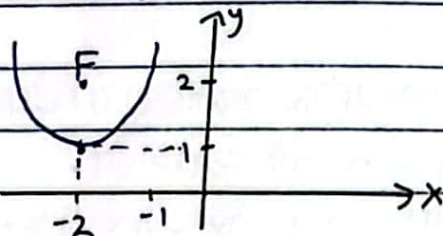$4(x+2)^2 = -16 + 16y$

$(x+2)^2 = 4y - 4$

$(x+2)^2 = 4(y-1)$

$h = -2$; $k = 1$; $p = 1$

fokus : $(h, p+k) = (-2, 2)$

titik puncak : $(h, t) = (-2, 1)$

direktriks : $y = t - p = 1 - 1 = 0$

2. Tentukan titik puncak, fokus, dan keeksentrikan dari elips berikut, serta gambarlah grafiknya.

a. $\dfrac{(x+3)^2}{4} + \dfrac{(y+2)^2}{16} = 1$

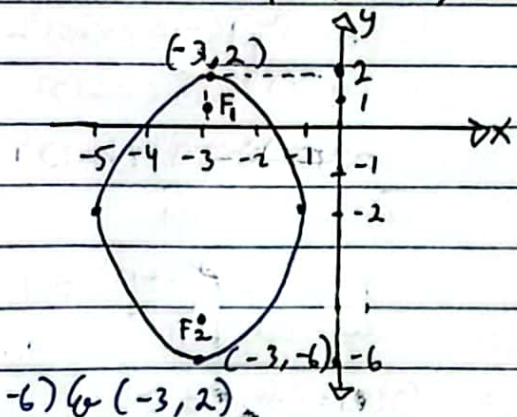$h = -3$; $t = -2$; $a = 4$; $b = 2$

$c = \sqrt{a^2 - b^2} = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$

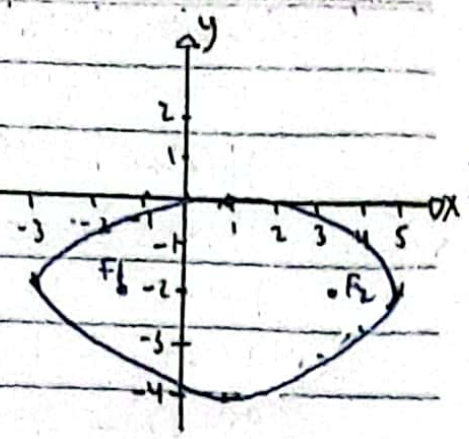fokus : $(h, t \pm c) = (-3, -2 \pm 2\sqrt{3})$

puncak: $(h, k \pm a) = (-3, -2 \pm 4) = (-3, -6) \, \& \, (-3, 2)$

keeksentrikan : $e = \dfrac{c}{a} = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

lebar : $(h \pm b, t) = (-3 \pm 2, -2)$

$= (-5, -2) \, \& \, (-1, -2)$

b. $x^2 + 4y^2 - 2x + 16y + 1 = 0$

$x^2 - 2x + 4y^2 + 16y + 1 = 0$

$(x-1)^2 - 1 + 4[(y+2)^2 - 4] + 1 = 0$

$(x-1)^2 - 1 + 4(y+2)^2 - 16 + 1 = 0$

$(x-1)^2 + 4(y+2)^2 = 16$

$\dfrac{(x-1)^2}{16} + \dfrac{(y+2)^2}{4} = 1$

$h = 1 \; ; \; k = -2 \; ; \; a = 4 \; ; \; b = 2 \; ; \; C = 2\sqrt{3}$
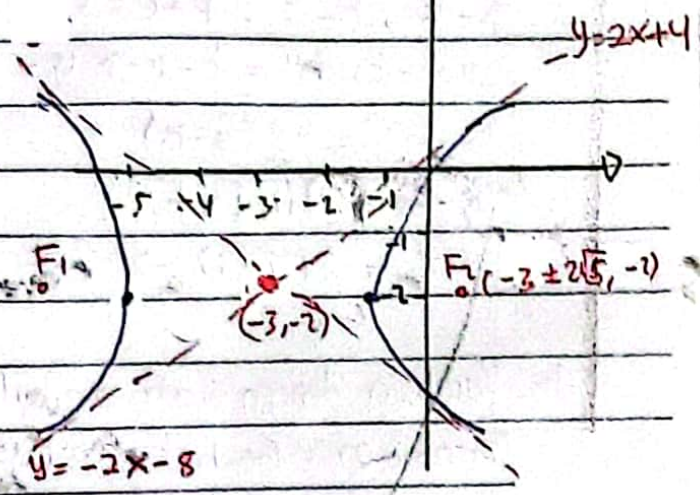
Fokus : $(h \pm C, t) = (1 \pm 2\sqrt{3}, -2) = (1 + 2\sqrt{3}, -2)$ & $(1 - 2\sqrt{3}, -2)$

Puncak : $(h \pm a, k) = (1 \pm 4, -2) = (-3, -2)$ & $(5, -2)$

Keesentrikon : $e = \dfrac{c}{9} = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

lebar : $(h, k \pm b) = (1, -2 \pm 2) = (1, -4)$ & $(1, 0)$

3. Tentukan titik puncak, fokus, dan garis asimtot hiperbola, serta gambarlah grafiknya.

a. $\dfrac{(x+3)^2}{4} - \dfrac{(y+2)^2}{16} = 1$

$h = -3 \; ; \; k = -2 \; ;$

$a = 2 \; ; \; b = 4 \; ; \; C = \sqrt{20} = 2\sqrt{5}$

fokus $= (h \pm c, k)$

$\quad = (-3 + 2\sqrt{5}, -2)$ & $(-3 - 2\sqrt{5}, -2)$

titik puncak $= (h \pm a, k)$

$\quad = (-5, -2)$ & $(-1, -2)$

Asimtot : $y - k = \pm \left(\dfrac{b}{a}\right)(x - h)$

$\quad y + 2 = \pm \left(\dfrac{4}{2}\right)(x + 3)$

$\quad y = \pm 2(x + 3) - 2$

$\quad y = 2x + 4 \;$ dan $\; y = -2x - 8$

b. $9x^2 - 16y^2 + 54x + 64y - 127 = 0$

$9x^2 + 54x - 16y^2 + 64y - 127 = 0$

$9(x^2 + 6x) - 16(y^2 - 4y) - 127 = 0$

$9[(x+3)^2 - 9] - 16[(y-2)^2 - 4] - 127 = 0$

$9(x+3)^2 - 81 - 16(y-2)^2 + 64 - 127 = 0$

$9(x+3)^2 - 16(y-2)^2 = 144$

$\dfrac{(x+3)^2}{16} - \dfrac{(y-2)^2}{9} = 1$

$a = 4 ; b = 3 ; c = 5$

$h = -3 ; k = 2$

Puncak $= (h \pm a, k)$

$= (-7, 2)$ dan $(1, 2)$

fokus $= (h \pm c, k)$
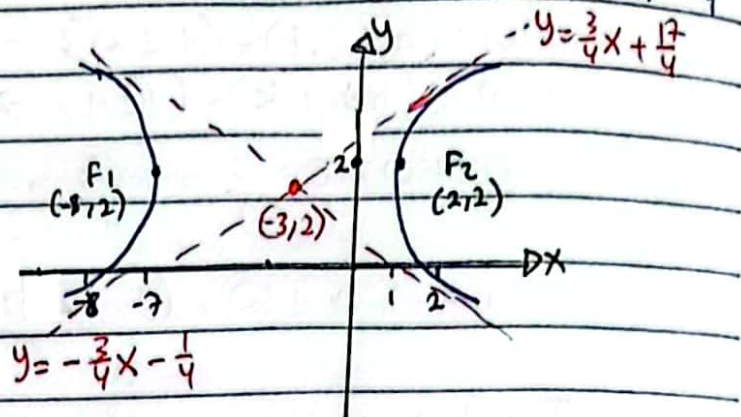
$= (-8, 2)$ dan $(2, 2)$

asimtot: $y - k = \pm\left(\dfrac{b}{a}\right)(x - h)$

$y - 2 = \pm\left(\dfrac{3}{4}\right)(x + 3)$

$\Rightarrow y = \dfrac{3}{4}x + \dfrac{17}{4}$ $\Rightarrow y = -\dfrac{3}{4}x - \dfrac{1}{4}$

$y = -\dfrac{3}{4}x - \dfrac{1}{4}$

4. Tentukan persamaan irisan kerucut berikut:

a. Parabola dengan puncak di $(2, 3)$ dan fokus di $(2, 5)$

$h = 2 ; k = 3 ; p + k = 5 \to p = 2$

direktriks : $y = k - p \to y = 3 - 2 \to y = 1$

$(x - h)^2 = 4p(y - k)$

$(x - 2)^2 = 4(2)(y - 3)$

$(x-2)^2 = 8(y-3) \to x^2 - 4y + 4 - 8y + 24 = 0 \to x^2 - 12y + 28 = 0$

b. Parabola dengan sumbu vertikal, serta melalui titik $(-2, 3)$ $(0, 3)$ $(1, 9)$.

Persamaan : $(x - h)^2 = 4p(y - k)$

$\Rightarrow (-2, 3) \to (-2 - h)^2 = 4p(3 - k)$

$\qquad h^2 + 4h + 4 = 12p - 4pk \ldots (1)$

$\Rightarrow (0, 3) \to (0 - h)^2 = 4p(3 - k)$

$\qquad h^2 = 12p - 4pk \ldots (2)$

$\Rightarrow (1, 9) \to (1 - h)^2 = 4p(9 - k)$

$\qquad h^2 - 2h + 1 = 36p - 4pk \ldots (3)$

#Eliminasi (1) dan (2)

$h^2 + 4h + 4 = 12p - 4pk$

$h^2 \qquad\quad = 12p - 4pk \ -$

$4h = -4$

$h = -1$

 # Subtitusi $h=1$ ke (2)

$(-1)^2 = 12p - 4pk$

$1 = 12p - 4pk \dots (4)$

 # eliminasi (4) dan (5)

$36p - 4pk = 4$

$12p - 4pk = 1$ —

$\overline{}$

$\qquad 24p = 3$

$\qquad p = \dfrac{1}{8}$

$h = -1 \; ; \; t = 1 \; ; \; p = \dfrac{1}{8}$

$(x-h)^2 = 4p(y-t)$

$(x+1)^2 = 4\left(\dfrac{1}{8}\right)(y-1)$

$(x+1)^2 = \dfrac{y-1}{2}$ —▷ $x^2 + 2x + 1 = \dfrac{y-1}{2}$ —▷ $2x^2 + 4x + 2 = y - 1$

$\qquad\qquad\qquad\qquad\qquad —▷ \; 2x^2 + 4x - y + 3 = 0$

 # Subtitusi $h=1$ ke (3)

$(-1)^2 - 2(-1) + 1 = 36p - 4pk$

$\qquad\qquad 4 = 36p - 4pk \dots (5)$

 # subtitussi ke (3)

$36p - 4pk = 4$

$4p(9 - k) = 4$

$4 \cdot \dfrac{1}{8}(9 - k) = 4$

$(9 - k) = 8 \; —▷ \; k = 1$

c. Elips dengan fokus $(\pm 2, 2)$ dan yang melalui titik asal

fokus $(h \pm c, t) \; —▷ \; h = 0 \; ; \; c = 2 \; ; \; k = 2$

melalui titik asal $(0, 0)$

$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-t)^2}{b^2} = 1$

$\dfrac{(0-0)^2}{a^2} + \dfrac{(0-2)^2}{b^2} = 1$

$b^2 = 4 \; —▷ \; b = \pm 2$

$c = \sqrt{a^2 - b^2}$

$2 = \sqrt{a^2 - 4}$

$4 = a^2 - 4$

$a = \sqrt{4 + 4} \; —▷ \; a = \sqrt{8}$

Persamaan : $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-t)^2}{b^2} = 1$

$\dfrac{(x-0)^2}{(\sqrt{8})^2} + \dfrac{(y-2)^2}{2^2} = 1 \; —▷ \; \dfrac{x^2}{8} + \dfrac{(y-2)^2}{4} = 1$

d. Hiperbola dengan puncak di $(0,0)$ dan $(0,6)$, dan dengan sebuah fokus di $(0,8)$

Puncak $= (h, k \pm a)$; $h = 0$; $k - a = 0$

$$k + a = 6 \quad +$$
$$2k = 6$$
$$k = 3 \quad \to k - a = 0 \to a = 3$$

→ fokus di $(0, 8) \to (h, k \pm c)$
$$k \pm c = 8 \to 3 \pm c = 8 \to \pm c = 5$$

→ $c^2 = a^2 + b^2$ → Persamaan :
$$5^2 = 3^2 + b^2$$
$$b^2 = 25 - 9$$ $$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$b = \sqrt{16}$$
$$b = 4$$ $$\frac{(y-3)^2}{3^2} - \frac{(x-0)^2}{4^2} = 1$$

$$\frac{(y-3)^2}{9} - \frac{x^2}{16} = 1$$