KELOMPOK 8

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Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan :

1. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ $\cdot$ $\frac{3n}{n^2} \leq \frac{3n+1}{n^2-4}$ [Uji Banding]

$$\frac{3}{n} \leq \frac{3n+1}{n^2-4}$$

$$\sum \frac{3}{n} = 3 \boxed{\sum \frac{1}{n}} \rightarrow \text{harmonik}$$

$$= \text{divergen}$$

$\therefore$ karena $\frac{3n}{n^2} \leq \frac{3n+1}{n^2-4}$ dan $\frac{3n}{n^2}$ merupakan deret divergen maka $\frac{3n+1}{n^2-4}$ merupakan deret divergen.

2. $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$ [Uji Banding Limit]

$a_n \leftarrow \frac{n}{n^2+2n-3} \Longleftrightarrow \frac{n}{n^2+2n} \Longleftrightarrow \frac{n}{n^2} = \frac{1}{n} \rightarrow$ konvergen

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n^2+2n-3} \cdot \frac{n}{1}$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+2n-3}$$

$$= 1 \Rightarrow 0 < L < \infty$$

$\therefore$ $\sum a_n$ dan $\sum b_n$ keduanya konvergen / divergen, karena $\sum b_n$ divergen maka $\sum a_n$ divergen

3. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!}$$

$$= \lim_{n\to\infty} n+1 \cdot \frac{n^{100}}{(n+1)^{100}}$$ [Uji Hasil Bagi]

$$= \lim_{n\to\infty} \frac{n^{100}}{(n+1)^{99}} \to \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{100\,n^{99}}{99\,(n+1)^{98}} \to \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{100!\,n^{1}}{99!\,(n+1)^{0}} = \lim_{n\to\infty} \frac{100 \cdot 99! \cdot n}{99!} = 100 \cdot \infty = \infty$$

$p = \infty > 1$ $\quad \therefore p > 1$, maka $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ divergen

4. $\sum_{n=1}^{\infty} \frac{3^k + k}{k!}$ [Uji Rasio]

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{k+1} + (k+1)}{(k+1)!} \cdot \frac{k!}{3^k + k}$$

$$= \lim_{n\to\infty} \frac{3^k \cdot 3 + k + 1}{(k+1)(3^k + k)}$$

$$= \lim_{n\to\infty} \frac{3^k \cdot 3 + k + 1}{3^k \cdot k + k^2 + 3^k + k} : \frac{3^k}{3^k}$$

$$= \lim_{n\to\infty} \frac{3 + \frac{k}{3^k} + \frac{1}{3^k}}{k + k^2/3^k + 1 + \frac{k}{3^k}}$$

$$= \frac{3 + 0 + 0}{\infty + 0 + 1 + 0}$$

$$= \frac{3}{\infty} = 0 < 1 \to \text{konvergen}$$

5. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ [Uji Banding Limit]

$a_n = \frac{3n+1}{n^2-4}$ $\qquad b_n = \frac{3n}{n^2} = \frac{3}{n} \to$ deret harmonik

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3n+1/n^2-4}{\frac{3}{n}} = \lim_{n\to\infty} \frac{3n+1}{n^2-4} \cdot \frac{n}{3} = \lim_{n\to\infty} \frac{3n^2+n}{3n^2-12}$$

$\overset{LH}{=} \lim_{n\to\infty} \frac{6n+1}{6n} = 1$

karena $\sum b_n = \sum \frac{3}{n}$ merupakan deret harmonik sehingga divergen, maka menurut uji banding limit; $\sum a_n$ divergen.

6.) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$ [Uji Akar]

$\lim_{n\to\infty} \left(\left(\frac{n}{3n+2}\right)^n\right)^{1/n} = \lim_{n\to\infty} \frac{n}{3n+2}$

$\overset{LH}{=} \lim_{n\to\infty} \frac{1}{3}$

$= \frac{1}{3} < 1$ (konvergen)

7.) $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n = \lim_{n\to\infty} \left(\left(\frac{1}{\ln n}\right)^n\right)^{1/n}$ [Uji Akar]

$= \lim_{n\to\infty} \frac{1}{\ln n} = 0 < 1$ (konvergen)

Tentukan apakah deret yang diberikan adalah konvergen mutlak, konvergen bersyarat, atau divergen.

8.) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n}{n+1}$

$U_n = (-1)^{n+1} \cdot \frac{n}{n+1}$, $a_n = \frac{n}{n+1}$

$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$= \lim_{n\to\infty} \left|\frac{n+1}{(n+1)+1} \cdot \frac{n+1}{n}\right|$

$= \lim_{n\to\infty} \frac{(n+1)^2}{n \cdot (n+2)} = \lim_{n\to\infty} \frac{n^2+2n+1}{n^2+2n} \div \frac{n}{n^2}$

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$$= \lim_{n\to\infty} \frac{1+\frac{2}{n}+\frac{1}{n^2}}{1+\frac{2}{n}} = 1$$

„ uji pembanding

∴ karena $P = 1$ maka mutlak tidak memberikan kesimpulan. Oleh karena itu, diperlukan uji lainnya, yaitu menggunakan uji deret ganti tanda

$a_n = \frac{n}{n+1} \to \lim_{n\to\infty} \frac{n}{n+1} = 1$ (divergen) $1 \neq 0$

$$\sum_{n=1}^{\infty} |U_n| = \left| (-1)^{n+1} \cdot \frac{n}{n+1} \right| = \sum_{n=1}^{\infty} \frac{n}{n+1}$$

$\lim_{n\to\infty} \frac{n}{n+1} = 1$ (divergen)

karena $|\sum U_n|$ divergen, maka $\sum U_n$ divergen juga.

9. $$\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$$

$U_n = \sin \frac{n!}{n^2}$ → berflutuasi diantara $(-1,1)$ sehingga divergen

$|U_n| = \left| \sin\left(\frac{n!}{n^2}\right) \right|$ → berflutuasi diantara $(0,1)$ sehingga divergen

karena U_n dan $|U_n|$ keduanya divergen, maka $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$ divergen.

10. $$\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n$$

$$R = \lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \left(\left(-\frac{4}{3}\right)^n\right)^{1/n}$$

$$= \lim_{n\to\infty} -\frac{4}{3}$$

$-\frac{4}{3} \neq 0$ (divergen)

Teorema: Jika $\sum \left(-\frac{4}{3}\right)^n$ divergen maka $\sum \left|\left(-\frac{4}{3}\right)^n\right|$ divergen.

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