TUGAS KELOMPOK MINGGU 2

KALKULUS II

Kelompok 3 :

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Nomor 1a.

1. $$\int_3^{\infty} \frac{x}{\sqrt{16+x^2}}\,dx = \lim_{b\to\infty} \int_3^{b} \frac{x}{\sqrt{16+x^2}}\,dx$$

misal : $u = 16 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \lim_{b\to\infty} \frac{1}{2}\int_{25}^{\infty} \frac{du}{\sqrt{u}} = \lim_{b\to\infty} \frac{1}{2}\left(2\sqrt{u}\,\Big|_{25}^{\infty}\right)$$

$$= \frac{1}{2}\left(2\sqrt{\infty} - 2\sqrt{25}\right)$$

$$= \infty \quad \text{(Divergen)}$$

Nomor 1b.

(1b) $$\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}}\,dx$$

$$= \int_{-\infty}^{0} \frac{x}{\sqrt{9+x^2}}\,dx + \int_{0}^{\infty} \frac{x}{\sqrt{9+x^2}}\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{x}{\sqrt{9+x^2}}\,dx + \lim_{a\to\infty} \int_{0}^{a} \frac{x}{\sqrt{9+x^2}}\,dx$$

$$= \lim_{a\to-\infty} \left(\sqrt{9+x^2}\,\Big|_a^0 + \lim_{a\to\infty} \left(\sqrt{9+x^2}\,\Big|_0^a\right.\right.$$

$$= \lim_{a\to-\infty} \left(3 - \sqrt{9+a^2}\right) + \lim_{a\to\infty} \sqrt{9+a^2} - 3$$

$$= -\infty + \infty$$

$$= \text{divergen} \;//$$

Notes !

$$\int \frac{x}{\sqrt{9+x^2}}\,dx = \int \frac{1}{\sqrt{u}} \cdot \frac{1}{2}\,du$$

misal

$u = 9+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \int \frac{1}{2\sqrt{u}}\,du$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{\cancel{2}}\left(\cancel{2}\sqrt{u}\right)$$

$$= \sqrt{u}$$

$$= \sqrt{9+x^2}$$

Nomor 2a.

2) a. $\int_2^\infty \frac{\ln\sqrt{x}}{x} = \lim_{a\to\infty} \int_2^a \frac{\ln\sqrt{x}}{x}$

$= \lim_{a\to\infty} \int_2^a \frac{\ln x^{\frac{1}{2}}}{x}$

$= \lim_{a\to\infty} \frac{1}{2} \int_2^a \frac{\ln x}{x}$

$\downarrow$

Misal $U = \ln x$

$\frac{dU}{dx} = \frac{1}{x}$

$dU = \frac{1}{x}\, dx$

$\downarrow$

$= \lim_{a\to\infty} \frac{1}{2} \int_{\ln 2}^a U\, dU$

$= \lim_{a\to\infty} \frac{1}{2} \frac{1}{2}U^2 \Big|_{\ln 2}^a$

$= \lim_{a\to\infty} \frac{1}{4} (a^2)^2 - (\ln 2)^2$

$= \infty$ (Divergen)

Nomor 2b.

3 (b) $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\,dx = \int_{-\infty}^{0} \frac{x}{(x^2+4)}\,dx + \int_{0}^{\infty} \frac{x}{(x^2+4)}\,dx$

$= \lim_{a \to -\infty} \int_a^0 \frac{x}{(x^2+4)}\,dx + \lim_{b \to \infty} \int_0^b \frac{x}{(x^2+4)}\,dx$

Misalkan $u = x^2+4 \Leftrightarrow du = 2x\,dx$

$= \lim_{a \to -\infty} \int_a^0 \frac{x}{u} \cdot \frac{du}{2x} + \lim_{b \to \infty} \int_0^b \frac{x}{u} \cdot \frac{du}{2x}$

$= \lim_{a \to -\infty} \frac{1}{2} \int_a^0 \frac{1}{u}\,du + \lim_{b \to \infty} \frac{1}{2} \int_0^b \frac{1}{u}\,du$

$= \lim_{a \to -\infty} \frac{1}{2} (\ln(|u|) \Big|_a^0 + \lim_{b \to \infty} \frac{1}{2} (\ln(|u|) \Big|_0^b$

$= \lim_{a \to -\infty} \frac{1}{2} (\ln(x^2+4) \Big|_a^0 + \lim_{b \to \infty} \frac{1}{2} (\ln(x^2+4) \Big|_0^b$

$= \lim_{a \to -\infty} \left(\frac{1}{2} \ln(0^2+4) - \frac{1}{2} \ln(a^2+4)\right) + \lim_{b \to \infty} \left(\frac{1}{2} \ln(b^2+4) - \frac{1}{2} \ln(0^2+4)\right)$

$= \lim_{a \to -\infty} \left(\ln(2) - \frac{1}{2} \ln(a^2+4)\right) + \lim_{b \to \infty} \left(\frac{1}{2} \ln(b^2+4) - \ln(2)\right)$

$= -\infty + \infty$

= divergen

Nomor 3a.

$$\int_2^{\infty} \frac{1}{x \ln x} dx = \lim_{b \to \infty} \int_2^{b} \frac{1}{x \ln x} dx.$$

misal $u = \ln x$

$$du = \frac{1}{x} dx$$

$$= \lim_{b \to \infty} \int_{\ln 2.}^{\ln b} \frac{du}{u}$$

$$= \lim_{b \to \infty} \int_{\ln 2}^{\ln b} \ln u$$

$$= \lim_{b \to \infty} \ln u \Big|_{\ln 2.}^{\ln b}$$

$$= \lim_{b \to \infty} \ln(\ln b) - \ln(\ln 2)$$

$$= \infty \quad \text{(divergen)}$$

Nomor 3b.

3b) $$\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}$$

$$\hookrightarrow \int_{-\infty}^{0} \frac{1}{x^2+4x+9} + \int_{0}^{\infty} \frac{1}{x^2+4x+9}$$

$$\hookrightarrow \lim_{a \to -\infty} \left(\int_a^0 \frac{1}{x^2+4x+9} dx \right) + \lim_{b \to \infty} \left(\int_0^b \frac{1}{x^2+4x+9} dx \right)$$

$$\hookrightarrow \lim_{a \to -\infty} \left(\int_a^0 \frac{1}{(x+2)^2+5} dx \right) + \lim_{b \to \infty} \left(\int_0^b \frac{1}{(x+2)^2+5} dx \right)$$

$$\hookrightarrow \lim_{a \to -\infty} \left(\int_a^0 \frac{1}{(x+2)^2+(\sqrt{5})^2} dx \right) + \lim_{b \to \infty} \left(\int_0^b \frac{1}{(x+2)^2+(\sqrt{5})^2} dx \right)$$

$$\hookrightarrow \lim_{a \to -\infty} \left(\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right) \Big|_a^0 \right) + \lim_{b \to \infty} \left(\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right) \Big|_0^b \right)$$

$$\hookrightarrow \lim_{a \to -\infty} \left(\frac{\sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right) - \sqrt{5}\arctan\left(\frac{\sqrt{5}a+2\sqrt{5}}{5}\right)}{5} \right) + \lim_{b \to \infty} \left(\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}b+2\sqrt{5}}{5}\right) - \sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right)}{5} \right)$$

$$\hookrightarrow \frac{\cancel{2\sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right)} + \sqrt{5}\pi}{10} + \frac{\sqrt{5}\pi - \cancel{2\sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right)}}{10}$$

$$\hookrightarrow \frac{\sqrt{5}\pi + \sqrt{5}\pi}{10} = \frac{2\sqrt{5}\pi}{10} = \frac{\sqrt{5}\pi}{5}$$

Nomor 4a.

Date

4) a. $\int_2^\infty \frac{1}{x(\ln x)^2}\,dx$

$= \lim_{b\to\infty} \int_2^b \frac{1}{x(\ln x)^2}\,dx$

misal $t = \ln x$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x}\,dx$

$= \lim_{b\to\infty} \int_2^b \frac{1}{t^2}\,dt$

$= \lim_{b\to\infty} \left[-\frac{1}{t}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{1}{\ln x}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{1}{\ln b} - \left(-\frac{1}{\ln 2}\right)\right]$

$= -\frac{1}{\ln\infty} + \frac{1}{\ln 2}$

$= 0 + \frac{1}{\ln 2}$

$\approx 1{,}4427$ // Konvergen

Nomor 4b.

TUGAS KELOMPOK

8. $\int_{-\infty}^{\infty} \frac{u}{e^{|u|}}\, du = \lim_{a\to-\infty} \int_a^0 \frac{u}{e^{|u|}}\, du + \lim_{b\to\infty} \int_0^b \frac{u}{e^{|u|}}\, du$

↳ $\lim_{u\to-\infty} \int_a^0 \frac{u}{e^{-u}}\, du$ (gunakan integral parsial)

$= \int \frac{u}{e^{-u}}\, du = \int u.e^{u}\, du$ → $uv : \int v\, du$ (integral parsial)

- $u = u \Rightarrow du = du$

$dv = e^{u}\, du \Rightarrow v = e^{u}$

$= u.e^{u} - \int e^{u}\, du$

$= u.e^{u} - e^{u}$

$\lim_{a\to-\infty} (u.e^{u} - e^{u}\Big|_a^0 = \lim_{a\to-\infty} (-1) - (ae^{a} - e^{a}) = \lim_{a\to-\infty} -1 - ae^{a} + e^{a} = -1$

↳ $\lim_{b\to\infty} \int_0^b \frac{u}{e^{u}}\, du$ (gunakan integral parsial)

$= \int u.e^{-u}\, du \Rightarrow uv - \int v\, du$

$u = u \Rightarrow du = du$

$dv = e^{-u}\, du \Rightarrow v = -e^{-u}$

$= u.-e^{-u} - \int -e^{-u}\, du$

$= -ue^{-u} + \int e^{-u}\, du$

$= -ue^{-u} - e^{-u}$

$\lim_{b\to\infty} (-ue^{-u} - e^{-u}\Big|_0^b = \lim_{b\to\infty} -be^{-b} - e^{-b} - (-1) = \lim_{b\to\infty} -be^{-b} - e^{-b} + 1 = 1$

↳ $\lim_{a\to-\infty} \int_a^0 \frac{u}{e^{|u|}}\, du + \lim_{b\to\infty} \int_0^b \frac{u}{e^{|u|}}\, du = -1 + 1 = 0$