-Kelompok 10-
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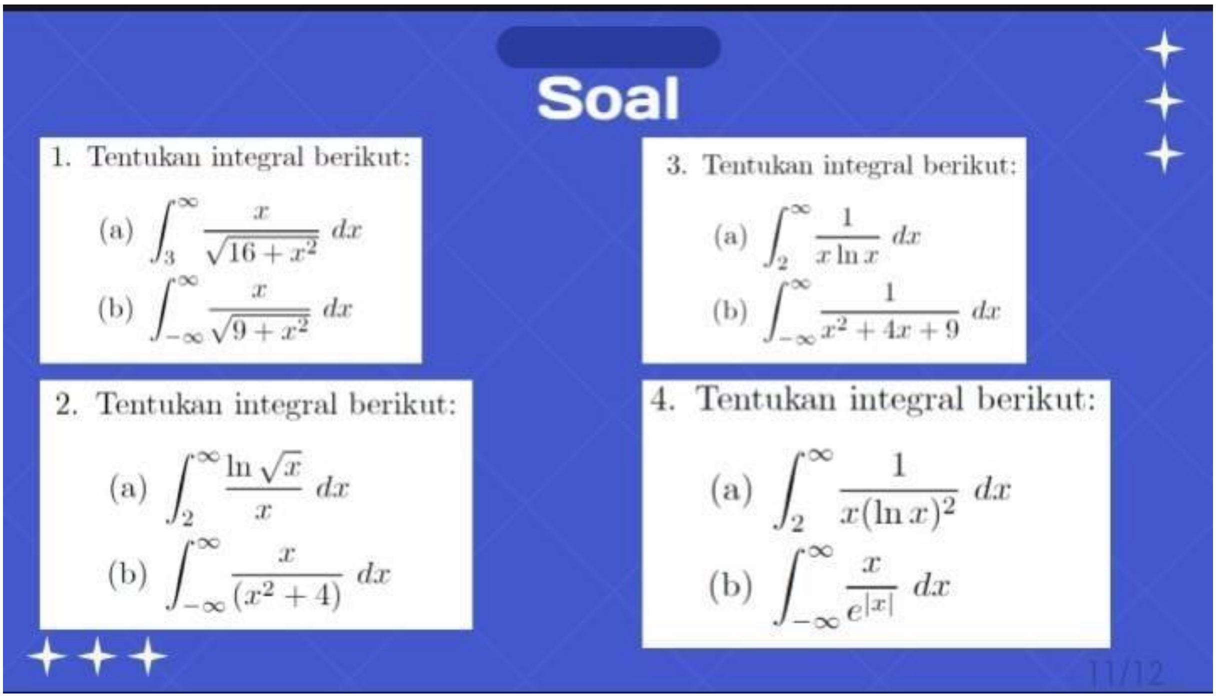

Jawab:

1.

1. Tentukan integral berikut :

a) $\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx$

$$= \lim_{b\to\infty}\int_3^b \frac{x}{\sqrt{16+x^2}}\,dx$$

Misal :

$\boldsymbol{u = 16 + x^2}$ $\boldsymbol{du = 2x\,dx}$

$\boldsymbol{\frac{du}{dx} = 2x}$ $\boldsymbol{\frac{du}{2} = x\,dx}$

$$= \lim_{b\to\infty}\int_3^b \frac{1}{2\sqrt{u}}\,du$$

$$= \frac{1}{2}\lim_{b\to\infty}\int_{25}^{16+b^2} u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2}\lim_{b\to\infty} (u^{1/2} \times 2|_{25}^{16+b^2}$$

$$= \frac{1}{2}\lim_{b\to\infty} 2\sqrt{16+b^2} - 2\sqrt{25}$$

$= \infty$ (**divergen**)

1b. $\int_{-\infty}^\infty \frac{x}{\sqrt{9+x^2}}\,dx$

$$= \lim_{a\to-\infty}\int_a^0 \frac{x}{\sqrt{9+x^2}}\,dx + \lim_{b\to\infty}\int_0^b \frac{x}{\sqrt{9+x^2}}\,dx$$

>>> misal u = 9 + x^2

$\frac{du}{dx}$ = 2x , maka du = 2x dx

$$= \lim_{a\to-\infty}\int_{9+a^2}^9 \frac{1}{2\sqrt{u}}\,du + \lim_{b\to\infty}\int_9^{9+b^2} \frac{1}{2\sqrt{u}}\,du$$

$$= \lim_{a\to-\infty}\int_{9+a^2}^9 \frac{1}{2}u^{-\frac{1}{2}}\,du + \lim_{b\to\infty}\int_9^{9+b^2} \frac{1}{2}u^{-\frac{1}{2}}\,du$$

$$= \lim_{a\to-\infty} u^{-\frac{1}{2}}\Big]_{9+a^2}^{9} + \lim_{b\to\infty} u^{-\frac{1}{2}}\Big]_{9}^{9+b^2}$$

$$= \sqrt{9} - \sqrt{9+a^2} + \sqrt{9+b^2} - \sqrt{9}$$

= (-∞+∞) DIVERGEN

2.

2a) $\int_2^{\infty} \frac{ln\sqrt{x}}{x} dx = \cdots$

$$\int_2^{\infty} \frac{ln\sqrt{x}}{x} dx = \lim_{b\to\infty} \int_2^{b} \frac{ln\sqrt{x}}{x} dx$$

Misal :

$$u = ln\sqrt{x}$$

$$du = \frac{1}{x} dx$$

$$\lim_{b\to\infty} \int_2^{b} u\, du = \lim_{b\to\infty} \frac{1}{2} u^2 \Big|_2^b$$

$$= \frac{1}{2}\left[\lim_{b\to\infty} (ln^2\sqrt{b} - ln^2\sqrt{2})\right]$$

$$= \frac{1}{2}(ln^2\sqrt{\infty} - ln^2\sqrt{2})$$

$= \infty$ (DIVERGEN)

b. $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\, dx$

misalkan u = $x^2 + 4$, du = 2x dx, $\frac{1}{2}$du = x dx

$= \lim_{m\to-\infty} \frac{1}{2} \int_{-\infty}^{4} \frac{1}{u}\, du + \lim_{n\to\infty} \frac{1}{2} \int_4^{\infty} \frac{1}{u}\, du$

$= \lim_{m\to-\infty} \frac{1}{2} (\ln u) \mid_{-\infty}^{4} + \lim_{n\to-\infty} \frac{1}{2} (\ln u) \mid_4^{\infty}$

$= \left(\frac{1}{2}(\ln 4 - \ln -\infty)\right) + \left(\frac{1}{2}(\ln \infty - \ln 4)\right)$

$= -\infty + \infty =$ divergen (tidak ada hasil)

3.

$$\int_2^{\infty} \frac{1}{x\ln(x)} dx$$

$$\lim_{t\to\infty} \int_2^{t} \frac{1}{x\ln(x)} dx$$

Misal $u = \ln(x)$, $du \frac{1}{x} dx, dan\; xdu = dx$

$$\lim_{t\to\infty} \int_{\ln(2)}^{\ln(t)} \frac{1}{u} du$$

$$\lim_{t\to\infty} \ln(|u|)\Big]_{\ln(2)}^{\ln(t)}$$

$$\lim_{t\to\infty} \ln(|\ln(t)|) - \ln(|\ln(2)|)$$

$$\lim_{t\to\infty} \ln\left(\frac{|\ln(t)|}{|\ln(2)|}\right)$$

∞ Divergen

b. $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\, dx$

misalkan u = $x^2 + 4$, du = 2x dx, $\frac{1}{2}$du = x dx

$= \lim_{m\to-\infty} \frac{1}{2} \int_{-\infty}^{4} \frac{1}{u}\, du + \lim_{n\to\infty} \frac{1}{2} \int_4^{\infty} \frac{1}{u}\, du$

$= \lim_{m\to-\infty} \frac{1}{2} (\ln u) \mid_{-\infty}^{4} + \lim_{n\to-\infty} \frac{1}{2} (\ln u) \mid_4^{\infty}$

$= \left(\frac{1}{2}(\ln 4 - \ln -\infty)\right) + \left(\frac{1}{2}(\ln \infty - \ln 4)\right)$

$= -\infty + \infty =$ divergen (tidak ada hasil)

4.

4A. $\int_2^\infty \frac{1}{x(lnx)^2} dx$

Jawab:

$$= \lim_{t\to-\infty} \int_2^t \frac{1}{x(lnx)^2} dx$$

Misal: u = ln(x), du = $\frac{1}{x}dx$, sehingga $xdu = dx$

$$= \lim_{t\to-\infty} \int_{\ln(2)}^{\ln(t)} \frac{1}{u^2} du$$

$$= \lim_{t\to-\infty} \int_{\ln(2)}^{\ln(t)} u^{-2} du$$

$$= \lim_{t\to-\infty} -u^{-1} \Big]_{\ln(2)}^{\ln(t)}$$

$$= \lim_{t\to-\infty} -ln^{-1}(t) + ln^{-1}(2)$$

$$= -\lim_{t\to-\infty} ln^{-1}(t) + \lim_{t\to-\infty} ln^{-1}(2)$$

$$= -0 + ln^{-1}(2)$$

$= \frac{1}{\ln(2)}$ (Konvergen)

4B. $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}} dx$

Jawab:

$$= \int_{-\infty}^{\infty} \frac{x}{e^{|x|}} dx$$

$$= \int_{-\infty}^{0} x.e^{-x} dx + \int_0^\infty x.e^x dx$$

$$= \lim_{a\to-\infty} \int_a^0 x.e^{-x} dx + \lim_{b\to\infty} \int_0^b x.e^x dx$$

$$= \left(\lim_{a\to-\infty} x.-e^{-x} \Big]_a^0 - \int_a^0 -e^{-x} dx \right) + \left(\lim_{b\to\infty} xe^x \Big]_0^b - \int_0^b e^x dx \right)$$

Misal: $u = x,\ du = dx, v = e^x, dv = e^x dx$ || $u = x,\ du = dx, v = e^{-x}, dv = e^{x-} dx$

$$= \left(\lim_{a\to-\infty} x.-e^{-x} \Big]_a^0 - \left(e^x \Big]_a^0 \right) \right) + \left(\lim_{b\to\infty} xe^x \Big]_0^b - \left(e^x \Big]_0^b \right) \right)$$

$$= (-1 - 0) + (0 - (-1)$$

$$= 0\ Konvergen$$