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① a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya

$\cos \pi$, $\frac{\cos 2\pi}{4}$, $\frac{\cos 3\pi}{9}$, $\frac{\cos 4\pi}{16}$, . .

$a_n = \cos \frac{\pi n}{\pi^2}$ Kekonvergenan : $-1 \leq \cos n\pi \leq 1$

$$\frac{-1}{n^2} \leq \frac{\cos n\pi}{n^2} \leq \frac{1}{n^2}$$

$$\lim_{n\to\infty} -\frac{1}{n^2} = 0 \qquad 0 \qquad \lim_{n\to\infty} \frac{1}{n^2} = 0$$

∴ kekonvergenan menuju 0

b) Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B.
Buktikan dengan definisi limit $\{a_n + b_n\}$ konvergen ke A+B

$\left.\begin{array}{l} \lim_{n\to\infty} a_n A \\ \\ \lim_{n\to\infty} a_n B \end{array}\right\}$ $\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n = A \pm B : A+B$ //

untuk pembuktian, maka $|(a_n + b_n) - (A+B)| < \varepsilon$

$\{a_n\}$ konvergen ke A. L=A dibuktikan :

u/ tiap $\varepsilon > 0$ terdapat $N > 0$

sedemikian sehingga $n \geq N$

$|a_n - L| < \varepsilon/2$

$|a_n - A| < \varepsilon/2$

$\{b_n\}$ konvergen ke B. L=B dibuktikan :

u/ tiap $\varepsilon > 0$ thd $N > 0$ sedemikian sehingga $n \geq N$

$|b_n - L| < \varepsilon/2$

$|b_n - B| < \varepsilon/2$

$$|(a_n + b_n) - (A+B)| \leq |a_n - A| + |b_n - B| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

Terbukti //

c) Tentukan kemonotonan, keterbatasan, limit (jika ada) :

$a_n = \frac{\sin n\pi}{4}$

Kemonotonan :

$a_n = \sin \frac{n\pi}{4}$

$a'(n) = \frac{\pi}{4} \cos n\pi/4$

tak naik & tak turun

- $x = 1 \rightarrow a'(1) : 1/8 \sqrt{2}\, \pi$
- $x = 2 \rightarrow a'(2) = 0$
- $x = 3 \rightarrow a'(3) : -\frac{1}{8} \sqrt{2}\, \pi$

Keterbatasan

$-1 \leq \sin \frac{n\pi}{4} \leq 1$

teorema apit tidak berlaku (divergen)

② a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya.

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

$a_n = (-1)^{n+1} \cdot \frac{1}{n}$

Kekonvergenan :

$\lim_{n\to\infty} \left|(-1)^{n+1} \frac{1}{n}\right| = \lim_{n\to\infty} \left|(-1)^{n+1}\right| \left|\frac{1}{n}\right| \quad \lim_{n\to\infty} \left|\frac{1}{n}\right| = 0$

∴ ~~ke~~konvergen ke 0

b) Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen :

$a_n = \frac{3-8 \cdot 2^n}{5+4 \cdot 2^n}$

$\lim_{n\to\infty} \frac{3-8\ 2^n}{5+4\ 2^n} \quad \lim_{n\to\infty} \frac{3-16^n}{5+8^n}$

$\lim_{n\to\infty} \frac{3/1^n - 16}{5/1^n + 8}$

$\frac{0-16}{0+8} = -2$

Konvergen ke -2

c) Tentukan kemonotonan, keterbatasan, limit (jika ada) barisan :

$a_n = \frac{\ln n}{n}$

Kemonotonan :

$a'(n) = \frac{\frac{1}{\cancel{n}} \cdot \cancel{n} - 1 \cdot \ln n}{n^2} = \frac{1 - \ln n}{n^2} \rightarrow$ tak naik & tak turun

Keterbatasan

$\lim_{n\to\infty} \frac{\ln n}{n}$ LH, $\lim_{n\to\infty} \frac{1/n}{1} = 0$ konv ke 0

③ a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya.

$0{,}9 \; ; \; 0{,}99 \; ; \; 0{,}999 \; ; \; 0{,}9999$.

↳ $(1 - 0{,}1) \; ; \; (1 - 0{,}01) \; ; \; (1 - 0{,}0001)$

$1 - \frac{1}{10} \; ; \; 1 - \frac{1}{100} \; ; \; 1 - \frac{1}{1000}$

$a_n = 1 - \frac{1}{10^n}$

Kekonvergenan

$a_n = 1 - \frac{1}{10^n}$

$\lim_{n\to\infty} = 1 - \frac{1}{10^n} = 1 - 0 = 1 \rightarrow$ konv ke 1

b) Dengan definisi limit, buktikan barisan $\{a_n\}$ konvergen.

$a_n = \frac{n+3}{3n-2}$

$\lim_{n\to\infty} a_n = L$

$\lim_{n\to\infty} \frac{n+3}{3n-2} = \lim_{n\to\infty} \frac{1 + 3/n}{3 - 2/n}$

$\lim_{n\to\infty} \frac{1+0}{3-0} = \frac{1}{3} \rightarrow$ konv ke $\frac{1}{3}$

c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada):

$a_n = \frac{n!}{10n} \rightarrow a_n = \frac{1}{10}, \frac{2}{10}, \frac{3}{10} \ldots \frac{n}{10^n}$

Kemonotonan :

$\frac{a_n}{a_{n+1}} = \frac{\frac{1 \cdot 2 \cdot 3 \cdot a \ldots n}{10 \cdot 10 \cdot 10 \ldots 10^n}}{1 \cdot 2 \cdot 3 \cdot 4 \ldots n+1 / 10 \cdot 10 \ldots 10^{n+1}}$

$$\frac{a_n}{a_{n+1}} = \frac{1.2.3.4 \ldots n \,/\, 1.2.3.4 \ldots n \,.\, n+1}{10.10.10 \ldots 10^n \,/\, 10.10.10 \ldots 10^n . 10^{n+1}}$$

$$= \frac{1}{n+1} \times \frac{10^{n+1}}{1} = \frac{10^{n+1}}{n+1} > 0 \text{ (naik)}$$

Keterbatasan :

$$a_n = \frac{n!}{10^n} = \frac{1.2.3.4 \ldots n}{10.10 \ldots 10^n}$$

$$= \frac{\infty}{\infty} \text{ (divergen)}_{//}$$