JAWABAN TUGAS KELOMPOK **R5**

MAT 1211 KALKULUS II SEMESTER GANJIL 2022/2023

Dosen:

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Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

1) $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4} \rightarrow a_n$

Uji Banding Limit

$\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4} \longleftrightarrow \frac{3n}{n^2} \longleftrightarrow \frac{3}{n} \rightarrow 3\,\frac{1}{n} \rightarrow$ deret harmonik (divergen)

a_n ; b_n

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

$$= \lim_{n\to\infty} \frac{3n+1}{n^2-4} \cdot \frac{n}{3}$$

$$= \lim_{n\to\infty} \frac{3n^2+n}{3n^2-12}$$

$$= \frac{\frac{3n^2}{n^2} + \frac{n}{n^2}}{\frac{3n^2}{n^2} - \frac{12}{n^2}} = \frac{3+0}{3+0}$$

$$= \frac{3}{3} = 1 > 0$$

$\therefore$ Menurut uji banding limit, a_n divergen

2. $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$

Gunakan uji banding limit

$a_n = \frac{n}{n^2+2n-3}$, $b_n = \frac{1}{n}$ (deret harmonik (divergen))

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

$$\lim_{n\to\infty} \frac{\frac{n}{n^2+2n-3}}{1/n}$$

$$\lim_{n\to\infty} \frac{n^2}{n^2+2n-3} = 1 > 0$$

$\therefore$ menurut uji banding limit, a_n divergen

3. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

=) Uji rasio (hasil bagi)

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!}$$

$$= \lim_{n\to\infty} \frac{n^{100}}{(n+1)^{99}} = \infty \text{ (divergen)}$$

$\therefore$ karena $\rho > 1$, maka $\sum_{n=1}^{\infty} n!/n^{100}$ divergen

4 $\sum_{n=1}^{\infty} \frac{3^k+k}{k!}$ $\quad p = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$ | $p < 1$: konv, $p > 1$: div, $p = 1$: –

$$p = \lim_{k\to\infty} \frac{3^{(k+1)} + (k+1)}{(k+1)(3^k+k)}$$

$$p = \lim_{k\to\infty} \frac{3^k \cdot 3 + k + 1}{3^k \cdot k + k^2 + 3^k + k}$$

$$p = \lim_{k\to\infty} \frac{3^k \ln(3) \cdot 3 + 1}{3^k \cdot k + k^2 + 3^k + k}$$

$$p = \lim_{k\to\infty} \frac{\cancel{3^k}\left(\ln(3)\cdot 3 + \frac{1}{3^k}\right)}{\cancel{3^k}\left(k + \frac{k^2}{3^k} + \frac{1}{3^k} + \frac{k}{3^k}\right)}$$

$$p = \lim_{k\to\infty} \frac{\cancel{k}\left(\frac{\ln(3)\cdot 3}{k} + \frac{1}{3^k \cdot k}\right)}{\cancel{k}\left(1 + \frac{k}{3^k} + \frac{1}{k\cdot 3^k} + \frac{1}{3^k}\right)}$$

$$p = \frac{0}{1} = 0$$

$p < 1 \rightarrow$ konvergen

Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

① $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

Uji Banding

Untuk $n \geq 3$ berlaku $0 \leq a_n \leq b_n$

maka $\frac{3n+1}{n^2-4} > \frac{3n}{n^2} \rightarrow \underbrace{\frac{3n+1}{n^2-4}}_{b_n} > \underbrace{\frac{3}{n}}_{a_n}$

$\sum_{n=1}^{\infty} \frac{3}{n} = 3 \underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}$

Deret harmonik $\rightarrow$ divergen

Karena $\sum_{n=1}^{\infty} \frac{3}{n}$ adalah deret divergen, maka $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ adalah deret divergen

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6 $\sum_{n=1}^{\infty} \left(n/_{3n+2} \right)^n$

→ uji akar

$R = \lim_{n \to \infty} (a_n)^{1/n}$

$= \lim_{n \to \infty} \left(\left(n/_{3n+2} \right)^n \right)^{1/n}$

$= \lim_{n \to \infty} \frac{n}{3n+2}$ LH $\lim_{n \to \infty} \frac{1}{3} \to \frac{1}{3} < 1$

Menurut teorema uji akar karena $R < 1$, maka $\sum_{n=1}^{\infty} \left(n/_{3n+2} \right)^n$ konvergen //

2.) $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n} \right)^n$; $a_n = \left(\frac{1}{\ln n} \right)^n$

* Uji akar $\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = R$

→ $\lim_{n \to \infty} \left(\left(\frac{1}{\ln n} \right)^n \right)^{\frac{1}{n}}$

$\lim_{n \to \infty} \frac{1}{\ln \cdot n} = 0$

∴ $0 < 1$, maka $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n} \right)^n$

konvergen.

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8. $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n}{n+1}$

Jawab :

•) Menggunakan Uji Deret Ganti Tanda (UDGT) untuk $\sum U_n$

$a_n = \frac{n}{n+1}$

1) Cek Apakah $a_n > a_{n+1}$ ($\sum a_n$ turun)

$$a_n > a_{n+1}$$
$$\frac{n}{n+1} > \frac{n+1}{n+2}$$
$$n^2+2n \not> n^2+2n+1 \text{ (Tidak terbukti)}$$

2) Cek nilai limitnya

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$$

∴ Menurut Uji Deret Ganti Tanda, $\sum U_n$ divergen

•) Menggunakan Uji Kedivergenan $\sum |U_n|$

$$\lim_{n\to\infty} |U_n|$$
$$\lim_{n\to\infty} \left| (-1)^{n+1} \cdot \frac{n}{n+1} \right| = \lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$$

∴ Menurut Uji Kedivergenan, $\sum |U_n|$ divergen

Sehingga, karena $\sum U_n$ dan $\sum |U_n|$ divergen, maka $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n}{n+1}$ divergen.

9. $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2} \rightarrow a_n = \frac{n!}{n^2}$

$|a_n| = \left| \sin \frac{n!}{n^2} \right| \rightarrow$ Berfluktuasi diantara nilai 0 sampai 1, sehingga diverger

karena $|a_n| \rightarrow$ divergen, maka deret $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$ juga divergen.

Tugas Kelompok : Responsi 5

(10) $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n$

$$\sum_{n=1}^{\infty} |U_n| = \sum_{n=1}^{\infty} \left|\left(-\frac{4}{3}\right)^n\right| = \sum_{n=1}^{\infty} \left|(-1)^n (4/3)^n\right|$$

$$= \sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$$

$$= \lim_{n\to\infty} \left(\frac{4}{3}\right)^{n/n}$$

$$= \lim_{n\to\infty} \frac{4}{3}$$

$$= \frac{4}{3} \rightarrow R > 1 \rightarrow \text{divergen}$$

$$\sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n = \lim_{n\to\infty} \left(-\frac{4}{3}\right)^{n/n} = \lim_{n\to\infty} -\frac{4}{3} \rightarrow R < 1 \rightarrow \text{konvergen}$$