LUTFI SYAHREZA LUBIS G1401211003

1.

a) rumus eksplisit dan kekonvergenan dari

$\cos \pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$

* rumus eksplisit

$a_n = \frac{\cos n\pi}{n^2}$

* kekonvergenan

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\cos n\pi}{n^2}$

$-1 \leq \cos n\pi \leq 1$

$\frac{-1}{n^2} \leq \frac{\cos n\pi}{n^2} \leq \frac{1}{n^2}$

$\therefore \lim_{n\to\infty} -\frac{1}{n^2} = 0 \quad \therefore \lim_{n\to\infty} \frac{1}{n^2} = 0$

sehingga

$\lim_{n\to\infty} \frac{\cos n\pi}{n^2} = 0$ (konvergen)

b) $\{a_n\}$ konvergen ke A

$\{b_n\}$ konvergen ke B

buktikan dengan definisi limit $\{a_n + b_n\}$ konvergen ke A dan B!

$\therefore \{a_n\}$ konvergen ke A

$\lim_{n\to\infty} a_n = A$

$\forall \varepsilon > 0, N_1 > 0, n > N_1$

$\varepsilon >$ sembarang

$\therefore \{b_n\}$ konvergen ke B

$\lim_{n\to\infty} b_n = B$

$\forall \varepsilon > 0, N_2 > 0, n > N_2$

ingin ditunjukkan

$\forall \varepsilon > 0, \exists K[\varepsilon] \in \mathbb{N} \ni \forall n \geq K(\varepsilon) \to |(a_n + b_n) - (A+B)| < \varepsilon$

$|a_n - A| < \frac{1}{2}\varepsilon \quad , |a_n - B| < \frac{1}{2}\varepsilon$

Jika $N = \max\{N_1, N_2\}$,

$$\begin{aligned} |a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\ &\leq |(a_n - A| + |b_n - B| \\ &< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\ &= \varepsilon \quad \text{(terbukti)} \end{aligned}$$

c) kemonotonan, keterbatasan, dan limit

$a_n = \sin\left(\frac{n\pi}{4}\right)$

* kemonotonan

$a_n - a_{n+1} = \sin \frac{n\pi}{4} - \sin \frac{(n+1)\pi}{4}$

$=$ tak tentu, sehingga tak monoton

* keterbatasan

Jika $\{a_n\}$ konvergen, $\{a_n\}$ terbatas

$\lim_{n\to\infty} \sin \frac{n\pi}{4} =$ tidak tentu (divergen)

sehingga $\{a_n\}$ tak terbatas

* limit

$\lim_{n\to\infty} \frac{\sin \pi}{4} =$ tidak tentu

2.

a) rumus eksplisit dan kekonvergenan

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

* rumus eksplisit

$a_n = (-1)^{n+1}\left(\frac{1}{n}\right)$

* kekonvergenan

$$\begin{aligned} \lim_{n\to\infty} (-1)^{n+1}\left(\frac{1}{n}\right) &= \lim_{n\to\infty} |(-1)^{n+1}|\left|\frac{1}{n}\right| \\ &= \lim_{n\to\infty} \frac{1}{n} \\ &= 0 \text{ (konvergen)} \end{aligned}$$

b) kekonvergenan $\{a_n\}$ dengan def limit.

$a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$

$= \lim_{n\to\infty} \frac{3/2^n - (8 \cdot 2^n)/2^n}{5/2^n + (4 \cdot 2^n)/2^n}$

$= \lim_{n\to\infty} \frac{3/2^n - 8}{5/2^n + 4} = \frac{-8}{4} = -2$

$|a_n - L|$

$= \left| \dfrac{3 - 8.2^n}{5 + 4.2^n} - (-2) \right|$

$= \left| \dfrac{3 - 2^{3+n}}{5 + 2^{2+n}} + 2 \right|$

$= \left| \dfrac{3 - 2^{3+n} + 10 + 2^{3+n}}{5 + 2^{2+n}} \right|$

$= \left| \dfrac{13}{5 + 2^{2+n}} \right| < \dfrac{13}{5 + 4.2^N}$

$n > N > 0$

$$\left\{ \dfrac{13}{5+4.2^N} = \varepsilon \iff \dfrac{13}{\varepsilon} = 5 + 4.2^N \qquad 2^N = \dfrac{\frac{13}{\varepsilon} - 5}{4} \right\}$$

$= \dfrac{13}{5 + 4\left(\dfrac{\frac{13}{\varepsilon} - 5}{4}\right)}$

$= \dfrac{13}{\frac{13}{\varepsilon}} = \varepsilon$

c) kemonotonan, keterbatasan, dan limit

$a_n = \dfrac{\ln n}{n}$

* kemonotonan

$a_n - a_{n+1} = \dfrac{\ln n}{n} - \dfrac{\ln(n+1)}{n+1}$

$= \dfrac{\ln n}{n} - \left(\dfrac{\ln n \times \ln 1}{n+1}\right)$

$= \dfrac{\ln n}{n} - \dfrac{\ln n \times 0}{n+1}$

$= \dfrac{\ln n}{n} \geq 0$ (tak naik)

* keterbatasan

$\lim_{n\to\infty} \dfrac{\ln n}{n} \to \dfrac{\infty}{\infty}$

$\stackrel{L}{=} \lim_{n\to\infty} \dfrac{\frac{1}{n}}{1}$

$= \lim_{n\to\infty} \dfrac{1}{n} = 0$ (konvergen)

maka tak terbatas $a \geq 0$

* Limit

$\lim_{n\to\infty} \dfrac{\ln n}{n} = 0$

3.

a) rumus eksplisit dan kekonvergenan

$0{,}9,\ 0.99,\ 0.999,\ 0.9999, \ldots$

* rumus eksplisit

$1 - 0{,}1,\quad 1 - 0.01,\quad 1 - 0.001, \ldots$

$= 1 - \dfrac{1}{10},\quad 1 - \dfrac{1}{100},\quad 1 - \dfrac{1}{1000} \ldots$

$= 1 - \dfrac{1}{10^1},\quad 1 - \dfrac{1}{10^2} \ldots$

$a_n = 1 - \dfrac{1}{10^n}$

* kekonvergenan

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1 - \dfrac{1}{10^n}$

$= 1 - 0$

$= 1$ (konvergen)

b) buktikan $\{a_n\}$ konvergen dengan definisi limit

$a_n = \dfrac{n+3}{3n-2}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{n+3}{3n-2}$

$\stackrel{L}{=} \lim_{n\to\infty} \dfrac{1}{3}$

$= \dfrac{1}{3}$ (konvergen)

$$|a_n - L| < \varepsilon$$

$$\left| \frac{n+3}{3n-2} - \frac{1}{3} \right| < \varepsilon$$

$$\left| \frac{3n+9-3n+2}{9n-6} \right| < \varepsilon$$

$$\left| \frac{11}{9n-6} \right| < \varepsilon$$

$$n > N > 0$$

$$\frac{11}{9N-6} = \varepsilon \Leftrightarrow \frac{11}{\varepsilon} = 9N-6$$

$$9N = \frac{11}{\varepsilon} - 6$$

$$= \frac{11}{9\left(\frac{11}{9\varepsilon} - \frac{6}{9}\right) - 6}$$

$$= \frac{11}{\frac{11}{\varepsilon}}$$

$= \varepsilon$ (Terbukti)

c) kemonotonan, keterbatasan, Limit

$$a_n = \frac{n!}{10^n}$$

* kamonotonan

$$\frac{a_n}{a_{n+1}} \leq 1$$

$$\frac{\left(\frac{n!}{10^n}\right)}{\frac{(n+1)!}{10^{n+1}}} = \frac{n!}{10^n} \cdot \frac{10^{n+1}}{(n+1)!}$$

$$= \frac{n!}{(n+1)n!} \cdot \frac{10^n \cdot 10}{10^n}$$

$$= \frac{10}{n+1}$$

$$\frac{10}{n+1} \leq 1$$

$$10 \leq n+1$$

$9 \leq n$ (tak monoton)

* keterbatasan

turun pada $0 < n < 9$

tidak turun pada $n \geq 9$

* limit

$\lim\limits_{n \to \infty} \frac{n!}{10^n} = \infty$ (divergen)