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1\. a) $\int_3^{\infty} \frac{x}{\sqrt{16+x^2}}\, dx = \lim_{b\to\infty} \left(\int_3^b \frac{x}{\sqrt{16+x^2}}\, dx \right)$

$= \lim_{b\to\infty} \left[\sqrt{16+x^2} \right]_3^b$

$= \lim_{b\to\infty} \left(\sqrt{16+a^2} - \sqrt{16+3^2} \right)$

$= \lim_{b\to\infty} \left(\sqrt{16+a^2} - 5 \right)$

$= +\infty$

Nilai integral divergen

$\boxed{1b}$. $\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}} = \int_{-\infty}^{0} \frac{x}{\sqrt{9+x^2}}\, dx + \int_0^{\infty} \frac{x}{\sqrt{9+x^2}}\, dx$

misal :
$u = 9 + x^2$
$du = 2x \cdot dx$
$dx = \frac{du}{2x}$

$= \lim_{a\to-\infty} \int_a^0 \frac{x}{\sqrt{9+x^2}}\, dx + \lim_{b\to\infty} \int_0^b \frac{x}{\sqrt{9+x^2}}\, dx$

$= \lim_{a\to-\infty} \int_{9+a^2}^{9} \cancel{x}\, u^{-1/2} \frac{du}{2\cancel{x}} + \lim_{b\to\infty} \int_9^{9+b^2} \cancel{x}\, u^{-1/2} \frac{du}{2\cancel{x}}$

$= \lim_{a\to-\infty} \frac{1}{\cancel{2}} \left[\cancel{2}\sqrt{u} \right]_{9+a^2}^{9} + \lim_{b\to\infty} \frac{1}{\cancel{2}} \left[\cancel{2}\sqrt{u} \right]_9^{9+b^2}$

$= \lim_{a\to-\infty} \left(\sqrt{9} - \sqrt{9+a^2} \right) + \lim_{b\to\infty} \left(\sqrt{9+b^2} - \sqrt{9} \right)$

$= -\infty + \infty$

$=$ divergen ||

2a)

2) a) $\displaystyle\int_{2}^{\infty} \frac{\ln\sqrt{x}}{x}\,dx$

$$= \lim_{b\to\infty} \int_{2}^{b} \frac{\ln\sqrt{x}}{x}\,dx$$

misal
$u = \ln x$
$\dfrac{du}{dx} = \dfrac{1}{x}$

$$= \lim_{b\to\infty} \int_{2}^{b} \frac{\ln(x)}{2x}\,dx$$

$$= \lim_{b\to\infty} \frac{1}{2} \int_{\ln 2}^{\ln b} \frac{u}{\cancel{x}} \cdot \cancel{x}\cdot du$$

$$= \lim_{b\to\infty} \frac{1}{2} \left[\frac{1}{2}u^2\right]_{\ln 2}^{\ln b}$$

$$= \lim_{b\to\infty} \frac{1}{4} \left[\ln^2(\infty) - \ln^2(2)\right] = \infty$$

= divergen

2b)

b) $\displaystyle\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\,dx$

misal
$u = x^2+4$
$\dfrac{du}{dx} = 2x$

$$= \lim_{a\to-\infty} \int_{-\infty}^{0} \frac{x}{(x^2+4)}\,dx + \lim_{b\to\infty} \int_{0}^{\infty} \frac{x}{(x^2+4)}\,dx$$

$$= \lim_{a\to-\infty} \int_{-\infty}^{4} \frac{\cancel{x}}{u}\,\frac{du}{2\cancel{x}} + \lim_{b\to\infty} \int_{4}^{\infty} \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\left(\lim_{a\to-\infty} \int_{-\infty}^{4} u^{-1}\,du + \lim_{b\to\infty} \int_{4}^{\infty} u^{-1}\,du\right)$$

$$= \frac{1}{2}\left(\lim_{a\to\infty} \int_{-\infty}^{4} \ln(x^2+4) + \lim_{b\to\infty} \int_{4}^{\infty} \ln(x^2+4)\right)$$

$$= \frac{1}{2}\left(\lim_{a\to\infty} \ln(x^2+4)\Big]_{-\infty}^{4} + \lim_{b\to\infty} \ln(x^2+4)\Big]_{4}^{\infty}\right)$$

$$= \infty + (-\infty)$$

= divergen

3a)

3a.) $\int_2^\infty \frac{1}{x\ln x}\,dx$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \tfrac{1}{x}\,dx \\ dx &= x \cdot du \end{aligned}}$$

$$= \lim_{b\to\infty} \int_2^b \frac{1}{x\ln x}\,dx$$

$$= \lim_{b\to\infty} \int_2^b \frac{1}{x\cdot u}\cdot x\cdot du$$

$$= \lim_{b\to\infty} \int_2^b \frac{du}{u}$$

$$= \left\{ \lim_{b\to\infty} \ln(\ln(x)) \right|_2^b \Rightarrow (\ln(\ln b)) - (\ln(\ln 2))$$

$$= (\ln(\ln\infty)) - (\ln(\ln 2))$$

$$= \infty - (\ln(\ln 2))$$

$$= \infty$$

3) b)

$$\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\,dx = \lim_{a\to-\infty}\int_a^0 \frac{1}{x^2+4x+9}\,dx + \lim_{b\to\infty}\int_0^b \frac{1}{x^2+4x+9}\,dx$$

$$= \lim_{a\to-\infty}\int_a^0 \frac{1}{(x+2)^2+(\sqrt{5})^2}\,dx + \lim_{b\to\infty}\int_0^b \frac{1}{(x+2)^2+(\sqrt{5})^2}\,dx$$

$$= \lim_{a\to-\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_a^0 + \lim_{b\to\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_0^b$$

$$= \lim_{a\to-\infty}\left(\frac{\sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right) - \sqrt{5}\arctan\left(\frac{\sqrt{5}a+2\sqrt{5}}{5}\right)}{5}\right) +$$

$$\lim_{b\to\infty}\left(\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}b+2\sqrt{5}}{5}\right) - \sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right)}{5}\right)$$

$$= \frac{\cancel{2\sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right)} + \sqrt{5}\pi}{10} + \frac{\sqrt{5}\pi - \cancel{2\sqrt{5}\arctan\left(\frac{2\sqrt{5}}{5}\right)}}{10}$$

$$= \frac{\sqrt{5}\pi + \sqrt{5}\pi}{10} = \frac{\sqrt{5}\pi}{5}$$

4a)

4(a) $\int_2^\infty \frac{1}{x(\ln x)^2} dx$

$= \lim_{b\to\infty} \int_2^b \frac{1}{x(\ln x)^2} dx$

misalkan:

$\boxed{u = \ln(x)}$

$du = \frac{1}{x} dx$

$\boxed{x.du = dx}$

* batas atas & bawah berubah!

$= \lim_{b\to\infty} \int_{\ln(2)}^{\ln(b)} \frac{\cancel{x}.du}{\cancel{x}.u^2}$

$= \lim_{b\to\infty} \int_{\ln(2)}^{\ln(b)} \frac{1}{u^2} \cdot du$

$= \lim_{b\to\infty} \left(-\frac{1}{u}\right]_{\ln(2)}^{\ln(b)}$

$= \lim_{b\to\infty} -\frac{1}{\ln(b)} - \left(-\frac{1}{\ln(2)}\right)$

$\approx -0 - (-1{,}4427)$

$\approx \underline{\underline{1{,}4427}}$

4b)

4b. $\displaystyle\int_{-\infty}^{\infty} \frac{x}{e^{|x|}}\, dx$

$= \displaystyle\lim_{a \to -\infty} \int_a^0 \frac{x}{e^{-x}}\, dx + \lim_{b \to \infty} \int_0^b \frac{x}{e^{x}}\, dx$

$= \displaystyle\lim_{a \to -\infty} \int_a^0 x \cdot e^{x} + \lim_{b \to \infty} \int_0^b x \cdot e^{-x}\, dx$

Integral parsial

* $\displaystyle\int_a^0 x \cdot e^{x} \to u = x \quad du = 1$

$dv = e^{x} \quad v = e^{x}$

$= \displaystyle\int_a^0 x \cdot e^{x} = x \cdot e^{x} - \int e^{x} \cdot 1$

$= x \cdot e^{x} - e^{x}$

$= e^{x}(x-1) \Big|_a^0$

* $\displaystyle\int_0^b x \cdot e^{-x} \longrightarrow u = x \quad du = 1$

$dv = e^{-x} \quad v = -e^{-x}$

$= \displaystyle\int_0^b x \cdot e^{-x} = -x \cdot e^{-x} - \int -1^{-x}$

$= -x \cdot e^{-x} + \int e^{-x}$

$= -x \cdot e^{-x} - e^{-x}$

$= e^{-x}(-x-1) \Big|_0^b$

$=$

$= \displaystyle\lim_{a \to -\infty} e^{x}(x-1) \Big|_a^0 + \lim_{b \to \infty} e^{-x}(-x-1) \Big|_0^b$

$= \left(e^{0}(0-1) - e^{-\infty}(-\infty-1)\right) + \left(e^{-\infty}(-\infty-1) - e^{0}(0-1)\right)$

$= 0$