TUGAS INDIVIDU

Nama : Syifa Khairunnisa
NIM : G1401211012

① $\cos\pi, \dfrac{\cos 2\pi}{4}, \dfrac{\cos 3\pi}{9}, \dfrac{\cos 4\pi}{16}, \ldots$

a) $\boxed{\dfrac{\cos n\pi}{n^2} \; ; n = 1, 2, 3, \ldots}$ → rumus eksplisit

✧ Kekonvergenan

$$-1 \leq \cos n\pi \leq 1$$

$$\frac{-1}{n^2} \leq \frac{\cos n\pi}{n^2} \leq \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{-1}{n^2} = 0 \qquad = 0 \qquad \lim_{n\to\infty} \frac{1}{n^2} = 0$$

∴ Konvergen ke 0.

b) $\{a_n\}$ konvergen ke A & $\{b_n\}$ konvergen ke B

Buktikan $\{a_n + b_n\}$ Konvergen ke $A + B$!

$$\left.\begin{aligned} \lim_{n\to\infty} a_n &= A \\ \lim_{n\to\infty} b_n &= B \end{aligned}\right\} \lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm b_n = A \pm B = A + B$$

— teorema —

$$|(a_n + b_n) - (A + B)| < \varepsilon$$

- $\{a_n\}$ Konvergen ke A

$L = A$, akan dibuktikan :

Untuk tiap $\varepsilon > 0$ terdapat $N > 0$

sehingga $n \geq N$

$|a_n - L| < \frac{\varepsilon}{2}$

$|a_n - A| < \frac{\varepsilon}{2}$

- $\{b_n\}$ Konvergen ke B

$L = B$, akan dibuktikan:

Untuk tiap $\varepsilon > 0$ terdapat $N > 0$

sehingga $n \geq N$

$|b_n - L| < \frac{\varepsilon}{2}$

$|b_n - B| < \frac{\varepsilon}{2}$

$\Rightarrow |(a_n + b_n) - (A+B)| \leq |a_n - A| + |b_n - B| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$ (terbukti).

C) $a_n = \sin \frac{n\pi}{4}$

$-1 \leq \sin n\pi \leq 1$

$-\frac{1}{4} \leq \sin \frac{n\pi}{4} \leq \frac{1}{4}$

↳ divergen (ga ada limitnya)

↳ alternating

$\therefore$ bukan barisan monoton

② a) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

$\boxed{(-1)^{n+1}\left(\frac{1}{n}\right) \; ; \; n = 1, 2, 3, \ldots}$ → rumus eksplisit

◇ Kekonvergenan

$* \left|(-1)^{n+1}\left(\frac{1}{n}\right)\right| \longrightarrow \frac{1}{n}$

kalo dimutlakin jadi 1

$\lim_{n \to \infty} \frac{1}{n} = 0$

∴ Konvergen ke 0.

dibagi dengan pangkat tertinggi (n)

$$\frac{\frac{3}{1^n} - \frac{16^n}{1^n}}{\frac{5}{1^n} + \frac{8^n}{1^n}}$$

b) $a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$

$$\lim_{n \to \infty} \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n} = \lim_{n \to \infty} \frac{3 - 16^{\cancel{n}}}{5 + 8^{\cancel{n}}} = \frac{0 - 16}{0 + 8} = -\frac{16}{8} = -2$$

∴ Konvergen ke −2

c) $a_n = \frac{\ln n}{n}$

◇ Kemonotonan

$$a'(n) = \frac{\frac{1}{\cancel{n}} \cdot \cancel{n} - \ln n \cdot 1}{n^2}$$

$$= \frac{1 - \ln n}{n^2}$$ ⟹ bukan barisan monoton

◇ Kekonvergenan

$$\lim_{n \to \infty} \frac{\ln n}{n} \overset{*}{=} \lim_{n \to \infty} \frac{\frac{1}{n}}{1} = 0$$

∴ Konvergen ke 0

3) a) 0,9 , 0,99 , 0,999 , 0,9999

$$\boxed{1-\frac{1}{10^n} \;;\; n = 1,2,3,\ldots} \rightarrow \text{rumus ekrplisit}$$

$$\lim_{n\to\infty} 1-\frac{1}{10^n} = 1-0 = 1$$

$\therefore$ Konvergen ke 1

b) $$a_n = \frac{n+3}{3n-1}$$

◊ Kekonvergenan

$$\lim_{n\to\infty} \frac{n+3}{3n-1} = \frac{\frac{n}{n}+\frac{3}{n}}{\frac{3n}{n}-\frac{1}{n}} = \frac{1+0}{3-0} = \frac{1}{3}$$

$\therefore$ Konvergen ke $\frac{1}{3}$

c) $$a_n = \frac{n!}{10^n}$$

◊ Kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{(n+1)}}}$$

$$= \frac{\frac{\cancel{1\cdot 2\cdot 3\cdot\ldots\cdot n}}{\cancel{10\cdot 10\cdot 10\cdot\ldots\cdot 10^n}}}{\frac{\cancel{1\cdot 2\cdot 3\cdot\ldots\cdot n}\cdot(n+1)}{\cancel{10\cdot 10\cdot 10\cdot\ldots\cdot 10^n}\cdot 10^{n+1}}} = \frac{1}{n+1}\cdot 10^{n+1}$$

$$= \frac{10^{n+1}}{n+1} > 1 \longrightarrow \text{monoton naik}$$

◊ Kekonvergenan

$$a_n = \frac{n!}{10^n}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot \ldots \cdot n}{10 \cdot 10 \cdot 10 \cdot \ldots \cdot 10^n} = \frac{\infty}{\infty}$$ = bentuk tak tentu

↳ ga ada limitnya

∴ divergen