TUGAS KELOMPOK MINGGU 5

KALKULUS II

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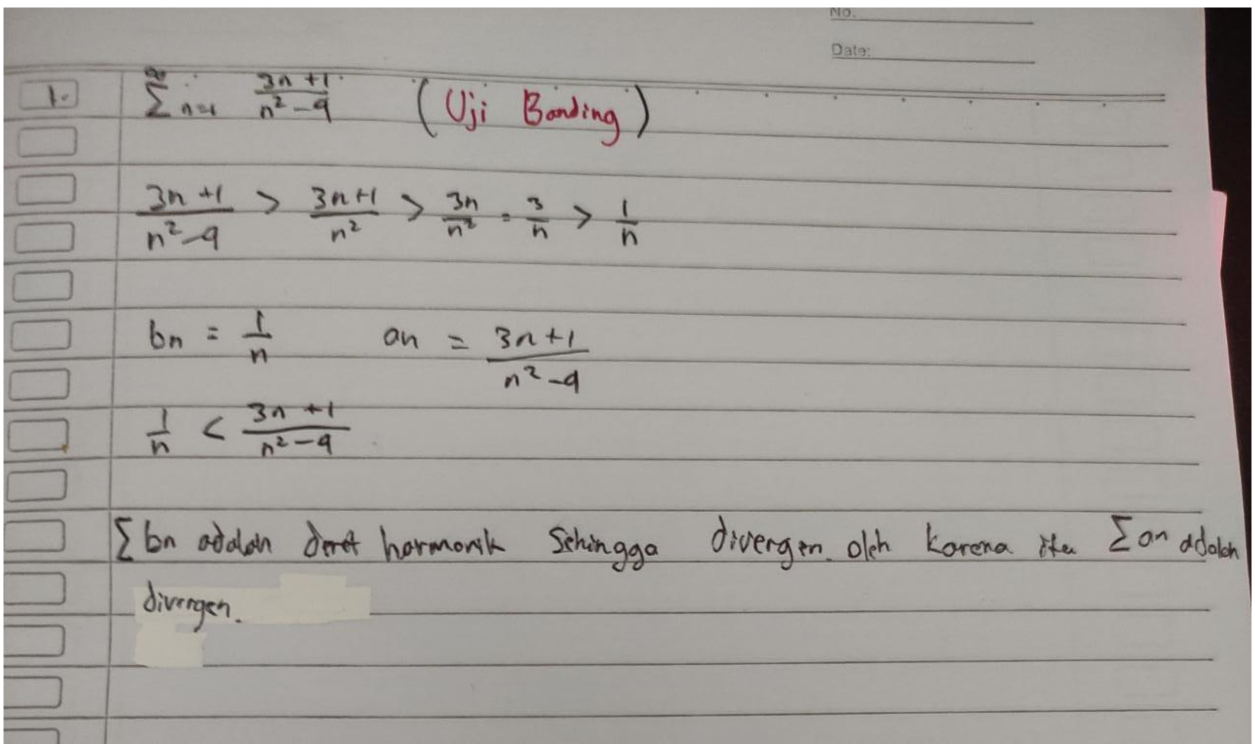
Nomor 1



1. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-9}$ (Uji Banding)

$\frac{3n+1}{n^2-9} > \frac{3n+1}{n^2} > \frac{3n}{n^2} = \frac{3}{n} > \frac{1}{n}$

$b_n = \frac{1}{n}$ $a_n = \frac{3n+1}{n^2-9}$

$\frac{1}{n} < \frac{3n+1}{n^2-9}$

$\sum b_n$ adalah deret harmonik sehingga divergen oleh karena itu $\sum a_n$ adalah divergen.

Nomor 2

2) $$\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$$

↳ gunakan uji banding limit :

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L \quad , \text{ dimana } \frac{n}{n^2+2n-3} > \frac{n}{n^2+2n} > \frac{n}{n^2} > \frac{1}{n}$$

$$= \lim_{n\to\infty} \frac{\frac{n}{n^2+2n-3}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+2n-3} = 1 > 0$$

sehingga, menurut uji banding limit $a_n = \frac{n}{n^2+2n-3}$ divergen

Nomor 3

3. Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan.

$$\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$$

$$P = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!}$$

$$= \lim_{n\to\infty} \frac{n+1}{(n+1)^{100}} \cdot n^{100}.$$

$$= \lim_{n\to\infty} \frac{n^{100}}{(n+1)^{99}} \rightarrow \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{100.\ n^{99}}{99\ (n+1)^{98}} \rightarrow \frac{\infty}{\infty}$$

dilakukan aturan L'Hopital berulang

$$\overset{LH}{=} \lim_{n\to\infty} \frac{100!\ n^{1}}{99!(n+1)^{0}}$$

$$= \lim_{n\to\infty} \frac{100.\ 99!\ .\ n}{99!}$$

$$= \lim_{n\to\infty} 100.n$$

$$= \infty$$

$$P = \infty > 1$$

$\therefore$ $P>1$, maka $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ divergen

Nomor 4

4) $$\sum_{n=1}^{\infty} \frac{3^k + k}{k!}$$

Uji hasil bagi

$$\rho = \lim_{k \to \infty} \frac{3^{k+1} + (k+1)}{(k+1)!} \cdot \frac{k!}{3^k + k}$$

$$= \lim_{k \to \infty} \frac{3^k . 3 + k + 1}{(k+1)} \cdot \frac{1}{(3^k + k)} \cdot \frac{1/3^k}{1/3^k}$$

$$= \lim_{k \to \infty} \frac{3 + k/3^k + 1/3^k}{(k+1)(3^k + k)(1/3^k)}$$

$$= \frac{3 + 0 + 0}{\infty}$$

$$= 0 < 1$$

$$\rho < 1$$

$\therefore \sum_{n=1}^{\infty} \frac{3^k + k}{k!}$ konvergen karena $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} < 1$.

Nomor 5

5. $$\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$$

↳ Uji Banding

$$\frac{3n}{n^2} \leq \frac{3n+1}{n^2-4}$$

$$\frac{3}{n} \leq \frac{3n+1}{n^2-4}$$

$$3 \cdot \frac{1}{n} \leq \frac{3n+1}{n^2-4}$$

$$\underbrace{\sum_{n=1}^{\infty} 3 \cdot \frac{1}{n}}_{\text{Deret harmonik (divergen)}} \leq \sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$$

Maka, berdasarkan uji banding, deret $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ divergen.

Nomor 6

6. $$\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$$

↳ UJI AKAR

$$R = \lim_{n \to \infty} (a_n)^{1/n}$$

$$= \lim_{n \to \infty} \left(\left(\frac{n}{3n+2}\right)^n\right)^{1/n}$$

$$= \lim_{n \to \infty} \frac{n}{3n+2}$$

$$= \frac{1}{3} \qquad (1/3 < 1)$$

∴ menurut teorema uji akar, karena $R < 1$, maka $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$ konvergen

Nomor 7

Date

(7) $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n$

Jawab

↳ untuk $n \geq 2$ maka $a_n = \left(\frac{1}{\ln n}\right)^n$ positif.

↳ $\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \lim_{n\to\infty} \left(\left(\frac{1}{\ln n}\right)^n\right)^{\frac{1}{n}}$

$= \lim_{n\to\infty} \left(\frac{1}{\ln n}\right)$

$= 0$

↳ karena $0<1$, maka menurut uji akar (i), deret $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n$ konvergen //

Nomor 8

8. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$?

Menggunakan UDGT :

$a_n = \frac{n}{n+1}$, * $a_n > a_{n+1}$

$\frac{n}{n+1} > \frac{n+1}{n+2}$

$n^2+2n \not> n^2+2n+1$ (tidak terbukti)

* nilai limit

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1} = 1$ (ΣU_n Divergen)

Nomor 9

9) $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$

Jawaban :

- $U_n = \sin \frac{n!}{n^2}$ ⇒ fluktuasi diantara $(-1,1)$, maka divergen
- $|U_n| = \left| \sin \left(\frac{n!}{n^2} \right) \right|$ ⇒ fluktuasi diantara $(0,1)$, maka divergen

∴ karena didapatkan U_n dan $|U_n|$ yang keduanya divergen, maka $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$ merupakan divergen

Nomor 10

10. $\sum_{n=1}^{\infty} \left(-\frac{4}{3} \right)^n = \sum_{n=1}^{\infty} -1^n \cdot (4/3)^n$

↳ Uji ganti tanda

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \left(\frac{4}{3} \right)^n = \infty \neq 0$$

↳ menurut uji ganti tanda, karena $\lim_{n \to \infty} a_n \neq 0$, maka $\sum_{n=1}^{\infty} \left(-\frac{4}{3} \right)^n$ divergen