TUGAS KELOMPOK MINGGU 3

KALKULUS II

KEL 3

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Nomor 1

1. $a_n = \frac{n}{3n-1} \Rightarrow \lim_{n\to\infty} \frac{n}{3n-1} = \frac{1}{3}$

$$a_n - a_{n+1} = \frac{n}{3n-1} - \frac{n+1}{3n+2}$$

$$= \frac{3n^2+2n-3n^2-2n+1}{9n^2+3n-2}$$

$$= \frac{1}{9n^2+3n-2} > 0 \quad \text{(turun)}$$

Nomor 2

2). $a_n = \frac{n^3+3n^2+3n}{(n+1)^3}$

⇨ mencari kekonvergenan :

$= \lim_{n \to \infty} \frac{n^3+3n^2+3n}{(n+1)^3} \overset{LH'}{=} \lim_{n \to \infty} \frac{3n^2+6n+3}{3(n+1)^2}$

$= \lim_{n \to \infty} \frac{3n^2+6n+3}{3n^2+6n+3}$

$= \frac{3}{3} = 1$ (pangkat tertinggi)

⇨ mencari kemonotonan :

$a_n - a_{n+1}$

$= \frac{n^3+3n^2+3n}{(n+1)^3} - \frac{(n+1)^3+3(n+1)^2+3(n+1)}{((n+1)+1)^3}$

$= \frac{n^3+3n^2+3n}{(n+1)^3} - \frac{(n+1)^3+3(n+1)^2+3(n+1)}{(n+2)^3}$

$= \frac{(n^3+3n^2+3n).(n+2)^3 - ((n+1)^3+3(n+1)^2+3(n+1).(n+1)^3)}{(n+1)^3\ (n+2)^3}$

$= \frac{-3n^2-9n-7}{(n^2+3n+2)^3} < 0$ (barisan monoton naik)

Nomor 3

3. $a_n = \frac{\cos n\pi}{n} \Rightarrow -1 \le \cos n\pi \le 1$

$$\frac{-1}{n} \le \frac{\cos n\pi}{n} \le \frac{1}{n} \Rightarrow \text{konvergen ke } 0$$

$$\hookrightarrow \frac{\cos n\pi}{n} - \frac{\cos (n+1)\pi}{n+1}$$

$$= \frac{(n+1)\cos n\pi - n\cos(n+1)\pi}{n^2+n} \Rightarrow \text{tidak naik \& tidak turun}$$

Nomor 4

4 $a_n = e^{-n} \sin n$

Kekonvergenan

$$-e^{-n} \le e^{-n}\sin n \le e^{-n}$$

$$\lim_{n\to\infty} -e^{-n} \le \lim_{n\to\infty} e^{-n}\sin n \le \lim_{n\to\infty} e^{-n}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$0 \qquad \boxed{\text{Teorema apit}} \qquad 0 \qquad 0$$

$$\lim_{n\to\infty} e^{-n}\sin n = 0 \quad \text{Konvergen ke } 0$$

Kemonotonan

$$\frac{a_{n+1}}{a_n} = \frac{e^{-n-1}\sin(n+1)}{e^{-n}\sin(n)} = \frac{\sin(n)\cos(1) + \sin(1)\cos(n)}{e\sin(n)}$$

$$= \frac{\cos(1)}{e} + \frac{\sin(1)\cot(n)}{e}$$

cot memiliki batas $-\infty$ dan ∞ maka bukan barisan monoton

Nomor 5

5. Kekonvergenan

$$a_n = \frac{1}{n^3}$$

$$\lim_{n\to\infty} \frac{1}{n^3} = 0$$

Kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{\frac{1}{n^3}}{\frac{1}{(n+1)^3}}$$

$$= \frac{(n+1)^3}{n^3}$$

$$= \frac{n^3 + 3n^2 + 3n + 1}{n^3}$$

$$= 1 + \left(\frac{3n^2 + 3n + 1}{n^3}\right) > 1$$

(turun)

Nomor 6

6. $\frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots$

* Rumus eksplisit

$$a_n = \frac{1}{2^{n+1}}$$

* Kekonvergenan

$\lim_{n\to\infty} \frac{1}{2^{n+1}} = 0$ (konvergen ke 0)

* Kemonotonan

$$\frac{a_n}{a_{n+1}} \Rightarrow \frac{\frac{1}{2^{n+1}}}{\frac{1}{2^{n+2}}}$$

$$= \frac{2^{n+2}}{2^{n+1}}$$

$$= \frac{2^n \cdot 4}{2^n \cdot 2}$$

$$= 2$$

$\frac{a_n}{a_{n+1}} > 1$, maka monoton turun

Nomor 7

7) $\sin 1$, $2 \sin \frac{1}{2}$, $3 \sin \frac{1}{3}$, $4 \sin \frac{1}{4}$.

rumus eksplisit $\rightarrow$ $a_n = n \sin \frac{1}{n}$

Konvergen

$$\lim_{n \to \infty} a_n \rightarrow \lim_{n \to \infty} n . \sin \frac{1}{n}$$

misal $\frac{1}{n} = t$; $n = \frac{1}{t}$.

$$= \lim_{t \to 0} \frac{\sin t}{t}$$

$= 1.$

a_n konvergen menuju ke 1 //

Nomor 8

8) 0.1, 0.11, 0.111, 0.1111, ...

* Rumus eksplisit :

$\frac{1}{9}(0{,}9,\ 0{,}99,\ 0{,}999,\ \ldots)$

$= \frac{1}{9}(1-0{,}1,\ 1-0{,}01,\ 1-0{,}001,\ \ldots)$

$= \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^1,\ 1-\left(\frac{1}{10}\right)^2,\ 1-\left(\frac{1}{10}\right)^3,\ \ldots\right)$

$= \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)$ //

* Kekonvergenan

$\lim_{n\to\infty} \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right) = \frac{1}{9}(1-0) = \frac{1}{9}$

Konvergen menuju 1/9 //