Pembahasan Soal Responsi ke-2

Infinite Limits of Integration;

Tentukan nilai integral berikut jika ada:

$$1.)\ \int_{100}^{\infty} e^x dx = [e^x]_{100}^{-\infty} = \infty - e^{100} = \infty\ (divergen) \blacksquare$$

$$2.)\ \int_{-\infty}^{-5} \frac{dx}{x^4} = \left[-\frac{1}{3x^3}\right]_{-\infty}^{-5} = -\frac{1}{3\cdot(-125)} - 0 = \frac{1}{375} \blacksquare$$

$$3.)\ \int_{-\infty}^{\infty} \frac{dx}{(x^2+16)^2} = \int_{-\infty}^{0} \frac{dx}{(x^2+16)^2} + \int_{0}^{\infty} \frac{dx}{(x^2+16)^2}$$

Lakukan pengintegralan dengan metode substitusi yaitu memisalkan $x = 4\tan\theta$, sehingga didapat $\int \frac{dx}{(x^2+16)^2} = \frac{1}{128}\tan^{-1}\left(\frac{x}{4}\right) + \frac{x}{32(x^2+16)}$.

$$\int_{-\infty}^{0} \frac{dx}{(x^2+16)^2} = \left[\frac{1}{128}\tan^{-1}\left(\frac{x}{4}\right) + \frac{x}{32(x^2+16)}\right]_{-\infty}^{0} = \frac{\pi}{256}$$

dan

$$\int_{0}^{\infty} \frac{dx}{(x^2+16)^2} = \left[\frac{1}{128}\tan^{-1}\left(\frac{x}{4}\right) + \frac{x}{32(x^2+16)}\right]_{0}^{\infty} = \frac{\pi}{256}$$

sehingga

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+16)^2} = \int_{-\infty}^{0} \frac{dx}{(x^2+16)^2} + \int_{0}^{\infty} \frac{dx}{(x^2+16)^2} = \frac{\pi}{256} + \frac{\pi}{256} = \frac{\pi}{128} \blacksquare$$

$$4.)\ \int_{-\infty}^{\infty} \frac{1}{x^2+2x+10}dx = \int_{-\infty}^{\infty} \frac{1}{(x+1)^2+9}dx = \int_{-\infty}^{0} \frac{1}{(x+1)^2+9}dx + \int_{0}^{\infty} \frac{1}{(x+1)^2+9}dx$$

Lakukan pengintegralan dengan metode substitusi yaitu memisalkan $x + 1 = 3\tan\theta$ sehinga didapat $\int \frac{1}{(x+1)^2+9}dx = \frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)$.

$$\int_{-\infty}^{0} \frac{1}{(x+1)^2+9}dx = \left[\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)\right]_{-\infty}^{0} = \frac{1}{6}\left(\pi + 2\tan^{-1}\left(\frac{1}{3}\right)\right)$$

dan

$$\int_{0}^{\infty} \frac{1}{(x+1)^2+9}dx = \left[\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)\right]_{0}^{-\infty} = \frac{1}{6}\left(\pi - 2\tan^{-1}\left(\frac{1}{3}\right)\right)$$

sehingga

$$\int_{-\infty}^{\infty} \frac{1}{x^2+2x+10}dx = \frac{1}{6}\left(\pi + 2\tan^{-1}\left(\frac{1}{3}\right)\right) + \frac{1}{6}\left(\pi - 2\tan^{-1}\left(\frac{1}{3}\right)\right) = \frac{\pi}{3} \blacksquare$$

Infinite Integrands;

Tentukan nilai integral berikut jika ada:

$$1.)\ \int_1^3 \frac{dx}{(x-1)^{1/3}} = \lim_{b\to 1^+}\left[\frac{3(x-1)^{2/3}}{2}\right]_b^3 = \frac{3}{2}\sqrt[3]{2^2} - \lim_{b\to 1^+}\frac{3(b-1)^{2/3}}{2} = \frac{3}{\sqrt[3]{2}} - 0 = \frac{3}{\sqrt[3]{2}}\ \blacksquare$$

$$2.)\ \int_0^9 \frac{dx}{\sqrt{9-x}} = \lim_{b\to 9^-}\left[-2\sqrt{9-x}\right]_0^b = \lim_{b\to 9^-} -2\sqrt{9-b} + 2\sqrt{9} = 6\blacksquare$$

$$3.)\ \int_{-1}^{128} x^{-\frac{5}{7}}\,dx = \lim_{b\to 0^-}\int_{-1}^{b} x^{-\frac{5}{7}}dx + \lim_{b\to 0^+}\int_{b}^{128} x^{-\frac{5}{7}}dx$$

$$= \lim_{b\to 0^-}\left[\frac{7}{2}x^{\frac{2}{7}}\right]_{-1}^{b} + \lim_{b\to 0^+}\left[\frac{7}{2}x^{\frac{2}{7}}\right]_{b}^{128} = 0 - \frac{7}{2} + \frac{7}{2}\cdot 4 - 0 = \frac{21}{2}\ \blacksquare$$

$$4.)\ \int_{-2}^{-1} \frac{dx}{(x+1)^{4/3}} = \lim_{b\to -1^-}\left[-\frac{3}{(x+1)^{\frac{1}{3}}}\right]_{-2}^{b} = -(-\infty) - 3 = \infty\ (diverges)\blacksquare$$

1. Tentukan integral berikut:

(a) $$\int_{3}^{\infty} \frac{x}{\sqrt{16+x^2}}\, dx$$

(b) $$\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}}\, dx$$

2. Tentukan integral berikut:

(a) $$\int_{2}^{\infty} \frac{\ln\sqrt{x}}{x}\, dx$$

(b) $$\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\, dx$$

3. Tentukan integral berikut:

(a) $$\int_{2}^{\infty} \frac{1}{x\ln x}\, dx$$

(b) $$\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\, dx$$

4. Tentukan integral berikut:

(a) $$\int_{2}^{\infty} \frac{1}{x(\ln x)^2}\, dx$$

(b) $$\int_{-\infty}^{\infty} \frac{x}{e^{|x|}}\, dx$$

Jawab:

30 Agustus 2022 | Tugas Responsi ke-2 | Nomor 1a: $\int_{3}^{\infty} \frac{x}{\sqrt{16+x^2}} dx = \left[\sqrt{x^2+16}\right]_{3}^{\infty} = \infty$; TIDAK ADA $\blacksquare$

Nomor 1b: $\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}} = \left[\sqrt{x^2+9}\right]_{-\infty}^{0} + \left[\sqrt{x^2+9}\right]_{0}^{\infty}$ = TIDAK ADA $\blacksquare$

Nomor 2a: $\int_{2}^{\infty} \frac{\ln\sqrt{x}}{x} dx$

Misalkan $u = \ln\sqrt{x}$ sehingga $du = \frac{1}{2x} dx$. $\int \frac{\ln\sqrt{x}}{x} dx = 2\int u\, du = u^2 + C = (\ln\sqrt{x})^2 \Rightarrow \frac{\ln^2(x)}{4}$

Jadi $\int_{2}^{\infty} \frac{\ln\sqrt{x}}{x} dx = \left[\frac{\ln^2(x)}{4}\right]_{2}^{\infty} =$ TIDAK ADA $\blacksquare$

Nomor 2b: $\int_{-\infty}^{\infty} \frac{x}{x^2+4} dx = \left[\frac{1}{2}\log(x^2+4)\right]_{-\infty}^{0} + \left[\frac{1}{2}\log(x^2+4)\right]_{0}^{\infty}$ = TIDAK ADA $\blacksquare$

Nomor 3a: $\int_{2}^{\infty} \frac{1}{x\ln x} dx = \left[\ln(\ln(x))\right]_{2}^{\infty}$ = TIDAK ADA $\blacksquare$

Nomor 3b: $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9} dx$. Misalkan $u = x+2$ sehingga $du = dx$. $\int \frac{1}{x^2+4x+9} dx = \int \frac{1}{(x+2)^2+5} dx = \int \frac{1}{u^2+5} du$

$= \frac{1}{5}\int \frac{1}{u^2/5+1} du$. Misalkan $s = \frac{u}{\sqrt{5}} \longrightarrow ds = \frac{1}{\sqrt{5}} du \longrightarrow \frac{1}{\sqrt{5}}\int \frac{1}{s^2+1} ds = \frac{\tan^{-1}(s)}{\sqrt{5}} + C = \frac{\tan^{-1}(\frac{u}{\sqrt{5}})}{\sqrt{5}} + C$

$= \frac{\tan^{-1}(\frac{x+2}{\sqrt{5}})}{\sqrt{5}} + C$. $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9} dx = \left[\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\right]_{-\infty}^{0} + \left[\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\right]_{0}^{\infty} = \frac{\sqrt{5}}{5}\pi$ $\blacksquare$

Nomor 4a: $\int_{2}^{\infty} \frac{1}{x(\ln x)^2} dx = \left[-\frac{1}{\ln(x)}\right]_{2}^{\infty} = \frac{1}{\ln(2)} + 0 = \frac{1}{\ln(2)}$ $\blacksquare$

Nomor 4b: $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}} dx$

$= \int_{-\infty}^{\infty} xe^{-|x|} dx = \int_{-\infty}^{0} xe^{-|x|} dx + \int_{0}^{\infty} xe^{-|x|} dx = \int_{-\infty}^{0} xe^{x} dx + \int_{0}^{\infty} xe^{-x} dx = -1+1$

$= 0$ $\blacksquare$