Angga Fathan Rofiqy _ G1401211006

1) a. Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya :

$$\cos \pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$$

↳ Rumus eksplisit

$$\boxed{a_n = \frac{\cos n\pi}{n^2}}$$

↳ kekonvergenan

$$\lim_{n \to \infty} \frac{\cos n\pi}{n^2} \Rightarrow \text{Pake teorema apit}$$

↳
$$-1 \leq \cos(n\pi) \leq 1$$

$$\frac{-1}{n^2} \leq \frac{\cos(n\pi)}{n^2} \leq \frac{1}{n^2}$$

$\lim_{n \to \infty} -\frac{1}{n^2} = \lim_{n \to \infty} \frac{1}{n^2} = 0$, ⇒ maka $\frac{\cos(n\pi)}{n^2}$ konvergen ke 0 //

b. Dik : $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B

Dit : buktikan (dng definisi limit) $\{a_n\} + \{b_n\}$ konvergen ke A+B

↳ karena a_n konvergen ke A dan b_n konvergen ke B , maka :

$$\left.\begin{aligned} \lim_{n \to \infty} a_n &= A \\ \lim_{n \to \infty} b_n &= B \end{aligned}\right\} \lim_{n \to \infty} (a_n \pm b_n) = \lim_{n \to \infty} a_n \pm \lim_{n \to \infty} b_n = A \pm B = A+B \; //$$

• Untuk setiap $\varepsilon > 0$ terdapat $N_1 > 0$ sedemikian sehingga $n \geq N_1$ berlaku: $|a_n - A| < \frac{1}{2}\varepsilon$

• Untuk setiap $\varepsilon > 0$ terdapat $N_2 > 0$ sedemikian sehingga $n \geq N_2$ berlaku: $|a_n - B| < \frac{1}{2}\varepsilon$

↳ pilih $N = \max\{N_1, N_2\}$

$$\begin{aligned} |a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\ &\leq |a_n - A| + |b_n - B| \\ &< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\ &= \varepsilon \; // \end{aligned}$$

↳ terbukti bahwa $\lim_{n \to \infty} (a_n + b_n) = A + B$. //

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c. Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut :

$$a_n = \sin\left(\frac{n\pi}{4}\right)$$

↳ kemonotonan

$a_n = \sin\left(\frac{n\pi}{4}\right)$

$a'(n) = \frac{\pi}{4} \cos\left(\frac{n\pi}{4}\right)$

- $x=1 \rightarrow a'(1) = \frac{1}{8}\sqrt{2}\pi$
- $x=2 \rightarrow a'(2) = 0$
- $x=3 \rightarrow a'(3) = -\frac{1}{8}\sqrt{2}\pi$

↳ tidak naik dan tidak turun (kadang (+) kadang (-))

↳ bukan barisan monoton //

↳ keterbatasan

$-1 \leq \sin\left(\frac{n\pi}{4}\right) \leq 1 \Rightarrow$ teorema apit tidak berlaku (divergen) //

2.) a. Tulis rumus eksplisit dan tentukan kekonvergenannya

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

↳ Rumus eksplisit

$$\boxed{a_n = \frac{(-1)^{n+1}}{n}}$$

↳ kekonvergenan

$\lim\limits_{n\to\infty} \frac{(-1)^{n+1}}{n} = 0 \Rightarrow$ konvergen ke 0 //

- atau bisa dengan teorema apit

$-1 \leq (-1)^{n+1} \leq 1$

$-\frac{1}{n} \leq \frac{(-1)^{n+1}}{n} \leq \frac{1}{n}$

↳ $\lim\limits_{n\to\infty} -\frac{1}{n} = \lim\limits_{n\to\infty} \frac{1}{n} = 0$, maka $\frac{(-1)^{n+1}}{n}$ konvergen ke 0 //

b. Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen :

$$a_n = \frac{3 - 8\cdot 2^n}{5 + 4\cdot 2^n}$$

↳ $\boxed{\text{Jika untuk setiap } \varepsilon > 0 \text{ terdapat } N > 0 \text{ sedemikian sehingga } n \geq N \text{ maka } |a_n - L| < \varepsilon}$

↳ $\lim\limits_{n\to\infty} \frac{3 - 16^n}{5 + 8^n} = \frac{3/_n - 16}{5/_n + 8} = \frac{0 - 16}{0 + 8} = -2$ (konvergen ke -2)

↳ $L = -2$

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$$|a_n - L| = \left| \frac{3-8\cdot 2^n}{5+4\cdot 2^n} + 2 \right| = \left| \frac{3-8\cdot 2^n + (10+8\cdot 2^n)}{5+4\cdot 2^n} \right|$$

$$= \frac{13}{5+4\cdot 2^n} \leq \frac{13}{5+4\cdot 2^N} \Rightarrow \text{analisis pemilihan nilai } N$$

$$\frac{13}{5+4\cdot 2^N} = \varepsilon \rightarrow \frac{13}{5+2^{N+2}} = \varepsilon$$

$$13 = 5\varepsilon + 2^{N+2}\varepsilon$$

$$2^{N+2} = \frac{13-5\varepsilon}{\varepsilon}$$

$$N+2 \ln(2) = \ln\left(\frac{13}{\varepsilon} - 15\right)$$

$$N+2 = \frac{\ln\left(\frac{13}{\varepsilon} - 15\right)}{\ln(2)}$$

$$N = \frac{\ln\left(\frac{13}{\varepsilon} - 15\right) - \ln(4)}{\ln(2)}$$

pilih $N = \dfrac{\ln\left(+\frac{13}{\varepsilon} - 5\right) - \ln 4}{\ln 2}$

$$\hookrightarrow \frac{13}{5+4\cdot 2^{\left(\frac{\ln\left(\frac{13}{\varepsilon}-5\right)-\ln 4}{\ln 2}\right)}} = \varepsilon \quad //$$

2.) c. kemonotonan, keterbatasan, limit (jika ada)

$$a_n = \frac{\ln n}{n}$$

↳ kemonotonan

- Definisikan fungsi a sbg:

$$a(x) = \frac{\ln x}{x}$$

$$a'(x) = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2}$$

↳ Didapat:

- $a'(x) < 0 \Leftrightarrow \dfrac{1-\ln x}{x^2} < 0$

$$\ln e < \ln x$$
$$e < x$$

↳ a turun pada (e, ∞)

- $a'(x) > 0 \Leftrightarrow \dfrac{1-\ln x}{x^2} > 0$

$$\ln e > \ln x$$
$$e > x$$

↳ a naik pada $(0, e)$

↳ maka a_n bukan barisan monoton pada N

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↳ <u>keterbatasan</u>

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln(n)}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{n} = \lim_{n\to\infty} \frac{1}{n^2} = 0$$

, $\frac{\ln(n)}{n}$ konvergen ke 0

dan terbatas sampai 0 //

3.) <u>a.</u> rumus eksplisit dan kekonvergenan

0,9 , 0,99 , 0,999 , 0,999 , ...

↳ rumus eksplisit

$$\boxed{a_n = \frac{10^n - 1}{10^n}}$$

↳ <u>kekonvergenan</u>

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{10^n - 1}{10^n} = \lim_{n\to\infty} \frac{\cancel{10^n}\left(1 - \frac{1}{10^n}\right)}{\cancel{10^n}} = 1$$

$a_n = \frac{10^n - 1}{10^n}$ konvergen ke 1 //

<u>b.</u> dengan definisi limit, buktikan $\{a_n\}$ konvergen

$$\boxed{a_n = \frac{n+3}{3n-2}}$$

↳ $\lim_{n\to\infty} \frac{n+3}{3n-2} = \frac{1}{3} \to L = \frac{1}{3}$ → konvergen ke $\frac{1}{3}$

↳ $$|a_n - L| = \left|\frac{n+3}{3n-2} - \frac{1}{3}\right|$$

$$= \left|\frac{n+3}{3n-2} - \frac{n - \frac{2}{3}}{3(n - \frac{2}{3})}\right|$$

$$= \left|\frac{(n+3) - (n - \frac{2}{3})}{3n-2}\right|$$

$$= \left|\frac{3 + \frac{2}{3}}{3n-2}\right| = \left|\frac{11}{3(3n-2)}\right| = \frac{11}{9n-6} \le \frac{11}{9N-6}$$

Analisis pemilihan nilai N

$$\frac{11}{9n-6} \le \frac{11}{9N-6} = \varepsilon$$

$$11 = 9N\varepsilon - 6\varepsilon$$

$$11 + 6\varepsilon = 9N\varepsilon$$

$$N = \frac{11 + 6\varepsilon}{9\varepsilon}$$

↳ pilih $N = \frac{11 + 6\varepsilon}{9\varepsilon}$

↳ $$\frac{11}{9\left(\frac{11 + 6\varepsilon}{9\varepsilon}\right) - 6} = \varepsilon \,//$$

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3.) c. kemonotonan, keterbatasan, limit (jika ada)

$$a_n = \frac{n!}{10^n}$$

↳ kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{n+1}}} = \frac{n!}{10^n} \cdot \frac{10^{n+1}}{(n+1)!} = \frac{\cancel{n!}}{\cancel{10^n}} \cdot \frac{\cancel{10^n} \cdot 10}{(n+1)\cancel{n!}} = \frac{10}{n+1}$$

$\frac{a_n}{a_{n+1}} < 1$; $n = 10, 11, 12, \ldots$ (naik)

$\frac{a_n}{a_{n+1}} \geq 1$; $n = 1, 2, \ldots 9$, (tak naik)

$\Big\}$ $a_n = \frac{n!}{10^n}$ bukan barisan monoton //

↳ keterbatasan

$$\lim_{n \to \infty} \frac{n!}{10^n} = \lim_{n \to \infty} \frac{n(n-1) \ldots 2 \cdot 1}{10 \cdot 10 \cdot \ldots 10}$$

$$= \lim_{n \to \infty} \frac{n}{10} \cdot \frac{(n-1)}{10} \cdot \frac{(n-2)}{10} \ldots \frac{1}{10}$$

$$= \infty$$

↳ $\{a_n\}$ divergen, sehingga $\{a_n\}$ tak terbatas //

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