Tugas Mandiri

1. a. Rumus eksplisit dari $\cos\pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$ dan kekonvergenan

b. Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan (definisi limit) $\{a_n + b_n\}$ konvergen ke A+B

c. Tentukan kemonotonan, keterbatasan dan limit (jika ada) $a_n = \sin \frac{n\pi}{4}$

Jawab:

a. $U_n = \frac{\cos n\pi}{n^2}$

$$\lim_{n\to\infty} \frac{\cos n\pi}{n^2} =$$

$$\underbrace{\frac{-1}{n^2}}_{0} < \underbrace{\frac{\cos n\pi}{n^2}}_{0} < \underbrace{\frac{1}{n^2}}_{0}$$

// (konvergen)

b) $\{a_n\} \to A \qquad n_1 > N_1 \to |a_n - A| < \frac{1}{2}\varepsilon$

$\lim_{n\to\infty} a_n = A$

$\{b_n\} \to B \qquad n_2 > N_2 \to |b_n - B| < \frac{1}{2}\varepsilon$

$\lim_{n\to\infty} b_n \to B$

$$\begin{aligned} |a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\ &\leq |a_n - A| + |b_n - B| \\ &< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\ &= \varepsilon \text{ //} \end{aligned}$$

$\lim_{n\to\infty} (a_n + b_n) = A + B$ ∎

c. * Kemonotonan

$a_n = \sin \frac{n\pi}{4}$

$$\frac{a_n}{a_{n+1}} = \frac{\sin \frac{n\pi}{4}}{\underbrace{\sin \frac{n\pi + \pi}{4}}_{\text{bisa positif/negatif}}}$$

(tidak monoton)

* Keterbatasan

$\{a_n\}$ tidak memiliki keterbatasan

* limit

$\lim_{n\to\infty} \sin \frac{n\pi}{4}$ = tidak ada (divergen)

2. a. Tulis rumus eksplisit & kekonvergenannya

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

b. Dengan definisi limit, buktikan kekonvergenan

$$a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$$

c. Tentukan kemonotonan, keterbatasan, & limit (jika ada)

$$a_n = \frac{\ln n}{n}$$

Jawab:

a. $a_n = \frac{(-1)^{n+1}}{n}$ atau $\frac{(-1)^{n-1}}{n}$ $\quad \lim_{n\to\infty} \frac{(-1)^{n+1}}{n} =$

$$\underbrace{\frac{-1}{n}}_{0} \leq \frac{(-1)^{n+1}}{n} \leq \underbrace{\frac{1}{n}}_{0}$$

$\downarrow$

0 // (konvergen)

b. $a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\frac{3}{2^n} - 8}{\frac{5}{2^n} + 4} = \frac{-8}{4} = -2$$

$L = -2$

$\cdot\!>$ Pilih $N = \frac{\ln\left(\frac{13}{\varepsilon} - 5\right) - \ln 4}{\ln 2}$

$$|a_n - L| = \left| \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n} + 2 \right|$$

$$= \left| \frac{3 - 8 \cdot 2^n + 2(5 + 4 \cdot 2^n)}{5 + 4 \cdot 2^n} \right|$$

$$= \frac{13}{5 + 4 \cdot 2^n} < \frac{13}{5 + 4 \cdot 2^N} = \frac{13}{5 + 4\,\frac{\left(\frac{13}{\varepsilon} - 5\right)}{4}} = \varepsilon \,/\!/ \blacksquare$$

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c. * Kemonotonan

$$a_n = \frac{\ln n}{n}$$

$$a(x) = \frac{\ln x}{x}$$

$$a'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$\cdot > a'(x) < 0 \leftrightarrow 1 - \ln x < 0$

$\ln e < \ln x$

$e < x$

turun pada (e, ∞)

$\cdot > a'(x) > 0 \leftrightarrow 1 - \ln x > 0$

$\ln e > \ln x$

$e > x$

naik pada $(0, e)$

$\therefore$ $\{a_n\}$ bukan barisan monoton

* limit

$$\lim_{n\to\infty} \frac{\ln n}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0 \text{ (konvergen)}$$

* Keterbatasan

$$a_1 = \frac{\ln 1}{1} = 0$$

$$a_2 = \frac{\ln 2}{2} = 0{,}34657$$

$$a_3 = \frac{\ln 3}{3} = 0.3662$$

Jadi $\{a_n\}$ terbatas di bawah oleh 0

terbatas di atas oleh a_3

3. a. Tulis rumus eksplisit barisan berikut & kekonvergenannya

$0.9, 0.99, 0.999, 0.9999, \ldots$

b. Dengan definisi limit, buktikan $\{a_n\}$ konvergen

$$a_n = \frac{n+3}{3n-2}$$

c. Tentukan kemonotonan, keterbatasan, & limit

$$a_n = \frac{n!}{10^n}$$

Jawab:

a. $0.9, 0.99, 0.999, \ldots$

$= (1-0{,}1,\ 1-0{,}01,\ 1-0{,}001, \ldots)$

$= 1 - \left(\frac{1}{10}\right)^n$

$a_n = 1 - \left(\frac{1}{10}\right)^n$

$\lim\limits_{n\to\infty} 1 - \left(\frac{1}{10}\right)^n = 1 - 0 = 1$ (konvergen)

b. $a_n = \frac{n+3}{3n-2}$

$\lim\limits_{n\to\infty} \frac{n+3}{3n-2} = \frac{1}{3} \rightarrow L = \frac{1}{3}$

→ Pilih $N = \frac{8-6\varepsilon}{9\varepsilon}$

$$|a_n - L| = \left|\frac{n+3}{3n-2} - \frac{1}{3}\right|$$

$$= \left|\frac{3n+9-(3n+1)}{9n-6}\right|$$

$$= \frac{8}{9n-6} < \frac{8}{9N-6} = \frac{8}{9\left(\frac{8-6\varepsilon}{9\varepsilon}\right)-6} = \varepsilon \ \blacksquare$$

No.

Date:

c. * Kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{n+1}}} = \frac{n!\,(10^{n+1})}{10^n\,(n+1)!} = \frac{10}{n+1} \geq 1 \quad \text{untuk } n = 1, 2, \ldots, 9$$

(tak naik)

$$< 1 \quad \text{untuk } n = 10, 11, \ldots$$

(naik)

* $\lim_{n \to \infty} \frac{n!}{10^n} = +\infty$ (divergen) //

* keterbatasan

$\{a_n\}$ terbatas di bawah oleh a_9 dan a_{10}

tapi tidak terbatas di atas