-Kelompok 10-
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Tugas Kelompok:

Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

1. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$
2. $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$
3. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$
4. $\sum_{n=1}^{\infty} \frac{3^k+k}{k!}$
5. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$
6. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$
7. $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n$

Tentukan apakah deret yang diberikan adalah konvergen mutlak, konvergen bersyarat, atau divergen.

8. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$
9. $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$
10. $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n$

Jawab:

1.

1. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

Uji banding

$$a_n = \lim_{n\to\infty} \frac{3n+1}{n^2-4}$$

$$b_n = \lim_{n\to\infty} \frac{3n}{n^2} = \frac{3}{n}$$

Jika $\sum_{n=1}^{\infty} b_n$ *konvergen maka* $\sum_{n=1}^{\infty} a_n$ *konvergen*

atau

Jika $\sum_{n=1}^{\infty} a_n$ *divergen maka* $\sum_{n=1}^{\infty} b_n$ *divergen*

$$\lim_{n\to\infty} \frac{3}{n} = \frac{3}{\infty} = 0$$

Maka $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ merupakan divergen

2.

2. $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$

; $a_n = \frac{n}{n^2+2n-3}$

; $b_n = \frac{n}{n^2} = \frac{1}{n}$ (merupakan deret harmonik sehingga b_n Divergen)

UJI BANDING LIMIT

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

$$= \lim_{n\to\infty} \frac{n}{n^2+2n-3} \times \frac{n}{1}$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+2n-3} = 1$$

Karena $0 < L < \infty$ maka $\sum a_n$ dan $\sum b_n$ sama-sama bersifat Divergen.

3.

$\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

Uji Hasil Bagi (Rasio)

$$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{100}} \times \frac{n^{100}}{n!}$$

$$\lim_{n\to\infty} \frac{(n+1) \times n^{100}}{(n+1)^{100}}$$

$$\lim_{n\to\infty} \frac{n^{100}}{(n+1)^{99}} = \infty$$

Karena $\infty > 1$ maka $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ divergen

4.

4.

$$\sum_{n=1}^{\infty} \frac{3^k + k}{k!}$$

Uji hasil bagi (rasio)

$$\lim_{k\to\infty} \frac{a_n + 1}{a_n}$$

$$\lim_{k\to\infty} \frac{3^{(k+1)} + (k+1)}{(k+1)!} \times \frac{k!}{3^k + k}$$

$$\lim_{k\to\infty} \frac{3^k \times 3 + k + 1}{(k+1)(3^k + k)} \times \frac{1/3^k}{1/3^k}$$

$$\lim_{k\to\infty} \frac{3 + k/3^k + 1/3^k}{(k+1) + 3(k+1)/3^k} = \frac{3+0+0}{k+0} = 0 \; (konvergen\ karena\ \lim_{k\to\infty} \frac{a_n + 1}{a_n} < 1)$$

5.

5) $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

Uji Banding Limit:

$$a_n = \frac{3n+1}{n^2-4}$$

$$b_n = \frac{3n}{n^2} = \frac{3}{n} \rightarrow Deret\ Harmonik\ (DIVERGEN)$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{3n+1}{n^2-4}}{\frac{3}{n}}$$

$$= \lim_{n\to\infty} \frac{3n+1}{n^2-4} \times \frac{n}{3}$$

$$= \lim_{n\to\infty} \frac{3n^2+n}{3n^2-12}$$

$$= 1$$

6.

Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

6) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n = \cdots$

Untuk n ≥ 1, $a_n = \left(\frac{n}{3n+2}\right)^2$ bernilai positif menggunakan uji akar.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = R$$

$$R = \lim_{n\to\infty} \left(\left(\frac{n}{3n+2}\right)^n\right)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n}{3n+2} \text{ (menggunakan Dalil L'Hospital atau dibagi dengan pangkat tertinggi)}$$

$$= \lim_{n\to\infty} \frac{1}{3}$$

$$R = \frac{1}{3}$$

Karena R bernilai $\frac{1}{3}$ yang mana < 1, sehingga $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$ adalah **konvergen**.

7.

7).

$$\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n = \lim_{n\to\infty} \left(\left(\frac{1}{\ln n}\right)^n\right)^{1/n}$$

$$\lim_{n\to\infty} \left(\frac{1}{\ln n}\right)$$

$$= \frac{1}{\infty}$$

$$= 0$$

karena r<1 maka konvergen

menggunakan uji akar (root test)

8.

8. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$

Uji ganti tanda

a. Uji kemonotonan

$$a_n = \frac{n}{n+1}$$

$$a_n - a_{n+1} = \frac{n}{n+1} - \frac{n+1}{n+2}$$

$$= \frac{(n^2+2n)-(n^2+2n+1)}{(n+1)(n+2)} = \frac{-1}{(n+1)(n+2)} < 0,\ n \geq 1$$

Maka, $a_n - a_{n+1} < 0$ monoton naik untuk $\{a_n\}$

b. Uji limit a_n

$$\frac{n}{n+1} = 1 \neq 0$$

$\sum u_n$ divergen menurut uji ganti tanda.

9.

10. $\sum_{n=1}^{\infty} \sin\left(\frac{n!}{n^2}\right)$

$U_n = \lim_{n \to x} \sin\left(\frac{n!}{n^2}\right)$ *Fluktuasi dari* -1 *sampai* 1 *(divergen)*

$|U_n| = \lim_{n \to x} |\sin\left(\frac{n!}{n^2}\right)|$ *Fluktuasi dari* 0 *sampai* 1 *(divergen)*

Sehingga $\sum_{n=1}^{\infty} \sin\left(\frac{n!}{n^2}\right)$ *(Divergen)*

10.

10. $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n$

Uji akar

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = R$$

$$U_n = \lim_{n \to \infty} \left(-\frac{4}{3}\right)^n$$

$$R = \lim_{n \to \infty} -\frac{4}{3}$$

$$-\frac{4}{3} < 1 \ (Konvergen)$$

$$|U_n| = \lim_{n \to \infty} \left|\left(-\frac{4}{3}\right)^n\right|$$

$$= \lim_{n \to \infty} \left|(-1)^n \times \left(\frac{4}{3}\right)^n\right|$$

$$= \lim_{n \to \infty} \left|\left(\frac{4}{3}\right)^n\right|$$

$$R = \lim_{n \to \infty} \frac{4}{3}|$$

$$\frac{4}{3} > 1 \ (Divergen)$$

(Konvergen bersyarat)