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ⓐ Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya:

$$\cos \pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$$

Jawab :

$$\cos \pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$$

$$-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, \ldots \begin{array}{l} \to (-1)^n \\ \to (n^2) \end{array}$$

$$a_n = (-1)^n \cdot \frac{1}{n^2}$$

$$n = 1 \to (-1)^1 \cdot \frac{1}{1^2} = -1$$

$$n = 2 \to (-1)^2 \cdot \frac{1}{2^2} = \frac{1}{4}$$

$$n = 3 \to (-1)^3 \cdot \frac{1}{3^2} = -\frac{1}{9}$$

Konvergen

$$\lim_{n\to\infty} (-1)^n \cdot \frac{1}{n^2}$$

$$\lim_{n\to\infty} \left|\frac{1}{n^2}\right|$$

$$\lim_{n\to\infty} \frac{1}{n^2}$$

$$= 0 \quad \to \text{konvergen ke } 0$$

ⓑ Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan (dengan definisi limit) $\{a_n + b_n\}$ konvergen ke A+B

$$\underbrace{\lim_{n\to\infty} a_n = A \qquad \lim_{n\to\infty} b_n = B}_{\text{Teorema limit}}$$

$$\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

$$= A + B$$

- Untuk memberikan pembuktian, harus

$$|(a_n + b_n) - (A+B)| < \varepsilon$$

→ $\{a_n\}$ konvergen ke A

L = A, akan dibuktikan : untuk setiap $\varepsilon > 0$ terdapat $N > 0$ sedemikian sehingga $n \geq N$.

$$|a_n - L| < \frac{\varepsilon}{2} \; ; |a_n - A| < \frac{\varepsilon}{2}$$

→ $\{b_n\}$ konvergen ke B

L = B akan dibuktikan : untuk setiap $\varepsilon = 0$ terdapat $N > 0$ sedemikian sehingga $n \geq N$.

$$|b_n - L| < \frac{\varepsilon}{2} \; ; |b_n - B| < \frac{\varepsilon}{2}$$

- $|(a_n + b_n) - (A+B)| \leq |a_n - A| + |b_n - B| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$

Terbukti.

(c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut :

$$a_n = \sin \frac{n\pi}{4}$$

Jawab :

Kemonotonan

$$a_n = \sin \frac{n\pi}{4}$$

$$a_n - a_{n+1} = \sin \frac{n\pi}{4} - \sin \frac{(n+1)\pi}{4}$$

$$= \sin \frac{n\pi}{4} - \sin \frac{(n\pi + \pi)}{4}$$

→ -0.29, 0.29, 0.70, 0.70

maka a_n bukan barisan monoton

Keterbatasan

$$-1 \leq \sin \frac{n\pi}{4} \leq 1$$

maka, (-1) → batas bawah

1 → batas atas

Limit

$$\lim_{n \to \infty} \sin \frac{n\pi}{4}$$

Karena limit $\sin \frac{n\pi}{4}$ tidak mendekati suatu bilangan tetap,

maka limit $\sin \frac{n\pi}{4}$ divergen.

2. (a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya :

$$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$$

Jawab :

$$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6} \ldots$$

$$a_n = (-1)^{n+1} \cdot \left(\frac{1}{n}\right)$$

$$n \to 1 = (-1)^{1+1} \cdot \left(\frac{1}{1}\right) = 1$$

$$n \to 2 = (-1)^{2+1} \cdot \frac{1}{2} = -\frac{1}{2}$$

* konvergen

$$\lim_{n \to \infty} (-1)^{n+1} \cdot \left(\frac{1}{n}\right)$$

$$\lim_{n \to \infty} \left(\frac{(-1)^{n+1}}{n}\right)$$

$$\lim_{n \to \infty} \left(\frac{(-1)^n \cdot (-1)}{n}\right)$$

$= 0 \to$ konvergen ke 0

(b) Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen:

$$a_n = \frac{3-8 \cdot 2^n}{5+4 \cdot 2^n}$$

Jawab :

$$a_n = \frac{3-8 \cdot 2^n}{5+4 \cdot 2^n}$$

$$\lim_{n\to\infty} \frac{3-8.2^n}{5+4.2^n} \cdot \frac{\frac{1}{2^n}}{\frac{1}{2^n}}$$

$$\lim_{n\to\infty} \frac{\frac{3}{2^n}-8}{\frac{5}{2^n}+4}$$

$$= -\frac{8}{4}$$

$$= -2 \rightarrow \text{konvergen ke } -2$$

(C) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut:

$$a_n = \frac{\ln n}{n}$$

Jawab:

* Kemonotonan

$$a_n = \frac{\ln n}{n}$$

$$a_n - a_{n+1} = \frac{\ln n}{n} - \frac{\ln n+1}{n+1}$$

$$= \frac{\ln(n)}{n} - \frac{\ln(n)+1}{n+1}$$

$$= \frac{\ln(n)(n+1)}{n(n+1)} - \frac{(\ln(n)+1)n}{(n+1)n}$$

$$= \frac{\ln(n)(n+1) - (\ln(n)+1)n}{n(n+1)}$$

$$= \frac{\ln(n) - n}{n(n+1)} < 0 \Rightarrow \text{Barisan monoton naik}$$

$\rightarrow -0.5, -0.21, -0,15, -0.13, \ldots$

* Keterbatasan

$$1 \leq \ln(n) \leq \infty$$

$$\frac{1}{n} \leq \frac{\ln(n)}{n} \leq \frac{\infty}{n}$$

$$\frac{1}{n} \leq \frac{\ln(n)}{n} \leq \infty$$

$\frac{1}{2} \rightarrow$ batas bawah

$\infty \rightarrow$ batas atas

* Limit

$$\lim_{n\to\infty} \frac{\ln(n)}{n}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1}$$

$$\lim_{n\to\infty} \frac{1}{n}$$

$= 0 \rightarrow$ konvergen ke 0

3\. (a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya :

$$0.9, 0.99, 0.999, 0.9999, \ldots$$

Jawab.

$$0.9, 0.99, 0.999, 0.9999, \ldots$$

$$\frac{9}{10}, \frac{9}{100}, \frac{9}{1000}, \frac{9}{10000}, \ldots$$

$$a_n = 1 - \left(\frac{1}{10}\right)^n$$

$$n \to 1 : 1 - \left(\frac{1}{10}\right)^1 = \frac{9}{10} = 0.9$$

$$n \to 2 : 1 - \left(\frac{1}{10}\right)^2 = \frac{9}{100} = 0.99$$

Konvergen

$$\lim_{n\to\infty} 1 - \left(\frac{1}{10}\right)^n$$

$= 1 - 0$

$= 1 \rightarrow$ Konvergen ke 1

(b) Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen :

$$a_n = \frac{n+3}{3n-2}$$

Jawab:

$$a_n = \frac{n+3}{3n-2}$$

$$\lim_{n\to\infty} \frac{n+3}{3n-2}$$

$$\lim_{n\to\infty} \frac{n+3}{3n-2} \cdot \frac{1/n}{1/n}$$

$$\lim_{n\to\infty} \frac{1+\frac{3}{n}}{3-\frac{2}{n}}$$

$$= \frac{1}{3} \rightarrow \text{Konvergen ke } \frac{1}{3}$$

* $|a_n - L| < \varepsilon$

$$\left|\frac{n+3}{3n-2} - \frac{1}{3}\right| < \varepsilon$$

$$\left|\frac{3n+9-(3n-2)}{9n-6}\right| < \varepsilon$$

$$\left|\frac{11}{9n-6}\right| < \varepsilon \qquad n > N > 0$$

$$\left\{ \frac{11}{9n-6} \cdot \varepsilon \Leftrightarrow \frac{11}{\varepsilon} = 9N-6 \quad 9N = \frac{11}{\varepsilon} - 6 \right\}$$

$$= \frac{11}{9\left(\frac{11}{9\varepsilon} - \frac{6}{9}\right) - 6}$$

$$= \frac{11}{\frac{11}{\varepsilon}}$$

$$= \varepsilon$$

Terbukti.

(c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut :

$$a_n = \frac{n!}{10^n}$$

Jawab :

* Kemonotonan

$$a_n = \frac{n!}{10^n}$$

$$a_n - a_{n+1} = \frac{n!}{10^n} - \frac{(n+1)!}{10^{n+1}}$$

$$= \frac{n!}{10^n} - \frac{(n+1)n!}{10 \cdot 10^n}$$

$$= \frac{10n! - (n+1)n!}{10 \cdot 10^n}$$

$$= \frac{(10-(n+1)\,n!}{10^{n+1}}$$

$$= \frac{(9-n)\,n!}{10^{n+1}}$$

- Barisan naik : $n > 9$
- Barisan turun : $n \leq 9$

→ Barisan tak monoton

* Keterbatasan

$$0 \leq n! \leq \infty$$

$$\frac{0}{10^n} \leq \frac{n!}{10^n} \leq \frac{\infty}{10^n}$$ sehingga $0 \leq \frac{n!}{10^n} \leq 9$

$$0 \leq \frac{n!}{10^n} \leq \infty$$

* Limit

$$\lim_{n \to \infty} \frac{n!}{10^n}$$

$$= \infty$$

Karena limit $\frac{n!}{10^n}$ tidak mendekati suatu bilangan tetap,

maka limit $\frac{n!}{10^n}$ divergen.