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1. a.) Tulis Rumus Eksplisit dan Kekonvergenan barisan berikut

$$\cos \pi,\ \frac{\cos 2\pi}{4},\ \frac{\cos 3\pi}{9},\ \frac{\cos 4\pi}{16},\ \ldots$$

Maka $a_n = \dfrac{\cos n\pi}{n^2}$ (Rumus Eksplisit)

Menentukan kekonvergenan

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\cos n\pi}{n^2}.$$

$$-1 \le \cos n\pi \le 1$$

$$-\frac{1}{n^2} \le \frac{\cos n\pi}{n^2} \le \frac{1}{n^2}$$

$$\lim_{n\to\infty} -\frac{1}{n^2} \le \lim_{n\to\infty} \frac{\cos n\pi}{n^2} \le \lim_{n\to\infty} \frac{1}{n^2}$$

$$0 \le \lim_{n\to\infty} \frac{\cos n\pi}{n^2} \le 0$$

maka $\lim_{n\to\infty} \dfrac{\cos n\pi}{n^2} = 0$ (konvergen)

b.) Diketahui $\{a_n\}$ konvergen ke $A$ dan $\{b_n\}$ konvergen ke $B$. Buktikan (dengan definisi limit) $\{a_n + b_n\}$ konvergen ke $A+B$..

maka ada $n \ge N > 0$ sehingga $|a_n - A| < \varepsilon_1$ dan $|b_n - B| < \varepsilon_2$
untuk setiap $\varepsilon_1, \varepsilon_2 > 0$

$$|a_n + b_n - (A+B)| = |(a_n - A) + (b_n - B)|$$

$$< |a_n - A| + |b_n - B| \quad \text{(pertidaksamaan segitiga)}$$

$$< \varepsilon_1 + \varepsilon_2 = \varepsilon_3$$

Maka, terbukti $\{a_n + b_n\}$ konvergen ke $A+B$

c.) Tentukan kemonotonan, keterbatasan, dan limit dari $a_n = \sin \frac{n\pi}{4}$

Kemonotonan $\rightarrow a_{n+1} - a_n = \sin\left(\frac{n\pi}{4} + \frac{\pi}{4}\right) - \sin\left(\frac{n\pi}{4}\right)$

$$= \sin\frac{n\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{n\pi}{4} - \sin\frac{n\pi}{4}$$

untuk $n = 0 \rightarrow a_{n+1} - a_n > 0$
& $n = 4 \rightarrow a_{n+1} - a_n < 0$
sehingga bukan barisan monoton

Keterbatasan $\rightarrow$ $n = 0 \rightarrow a_n = 0$
$n = 1 \rightarrow a_n = \frac{1}{2}\sqrt{2}$
$n = 2 \rightarrow a_n = 1$
$n = 3 \rightarrow a_n = \frac{1}{2}\sqrt{2}$
$n = 4 \rightarrow a_n = 0$
$n = 5 \rightarrow a_n = -\frac{1}{2}\sqrt{2}$
dan $-1 \le a_n \le 1$

Konvergenan → $\lim_{n\to\infty} \sin \frac{n\pi}{4}$ = tidak ada (karena nilainya berayun antara positif dan negatif)

2. a.) Rumus Eksplisit dari $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots$

$$a_n = (-1)^{n+1} \cdot \frac{1}{n}$$

Kekonvergenan
Berdasarkan teorema, jika $\lim_{n\to\infty} |a_n| = 0 \rightarrow \lim_{n\to\infty} a_n = 0$

$\lim_{n\to\infty} \left| (-1)^{n+1} \frac{1}{n} \right| = \lim_{n\to\infty} \frac{1}{n} = 0 \rightarrow$ maka $\lim_{n\to\infty} a_n = 0$ (konvergen)

b.) Buktikan barisan berikut konvergen menggunakan limit epsilon delta.

$$a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n} = \lim_{n\to\infty} \frac{\frac{3}{2^n} - 8}{\frac{5}{2^n} + 4} = -2$$

$$|a_n + 2| = |a| + 2 < \varepsilon$$

∴ terbukti

c. $a_n = \frac{\ln n}{n}$

monoton → $a_{n+1} - a_n = \frac{\ln(n+1)}{n+1} - \frac{\ln n}{n} = \frac{n \ln(n+1) - (n+1) \ln n}{(n+1)(n)}$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{\ln(n+1)}{n+1}}{\frac{\ln(n)}{n}} = {}^n\log(n+1) \cdot \frac{n}{n+1}$$

limit $\lim_{n\to\infty} \frac{\ln n}{n}$

$\overset{LH}{=} \lim_{n\to\infty} \frac{1}{n} = 0$

(konvergen)

Untuk $n = 2 \rightarrow a_{n+1} - a_n > 0$

$n \geq 3 \rightarrow a_{n+1} - a_n < 0$

maka barisan tidak monoton

Keterbatasan → mulai $n = 3$ tidak naik dan turun

a. Rumus eksplisit

$$a_n = 1 - \frac{1}{10^n}$$

Konvergen

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1 - \frac{1}{10^n} = 1$$

b.) $$\left| \frac{n+3}{3n-2} + \left(-\frac{1}{3}\right) \right| \leq \left| \frac{n+3}{3n-2} \right| + \frac{1}{3} \leq \varepsilon$$

$\therefore$ terbukti

c.) $$a_n = \frac{n!}{10^n}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!}{10^{n+1}}}{\frac{n!}{10^n}} = \frac{n+1}{10} \rightarrow \text{Bukan barisan monoton}$$

untuk n dari 1 sampai 8 itu an ~~naik~~ turun → n = 9 ke 10 tetap

Setelahnya naik

Kekonvergenan

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1 \times 2 \times 3 \times \cdots n}{10 \times 10 \times 10 \times 10 \times \ldots 10} = \infty$$