JAWABAN TUGAS KELOMPOK KE-2

MAT 1211 KALKULUS II SEMESTER GANJIL 2022/2023

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Tugas Kelompok

1\. a. $\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx \rightarrow \lim_{a\to\infty} \int_3^a \frac{x}{\sqrt{16+x^2}}$, misal $u = 16+x^2 \rightarrow \frac{du}{dx} = 2x$

$\rightarrow du = 2x\,dx$

$$= \lim_{a\to\infty} \int_3^a \frac{1}{2\sqrt{u}}\,du$$

$$= \frac{1}{2} \lim_{a\to\infty} \int_{25}^{16+a^2} u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2} \lim_{a\to\infty} (u^{1/2} \cdot 2 \Big|_{25}^{16+a^2} = \frac{1}{2} \lim_{a\to\infty} 2\sqrt{16+a^2} - 2\sqrt{25}$$

$$= \infty \text{ (divergen)}$$

b. $\int_{-\infty}^\infty \frac{x}{\sqrt{9+x^2}}\,dx$

$$= \lim_{a\to-\infty} \int_a^0 \frac{x}{\sqrt{9+x^2}}\,dx + \lim_{b\to\infty} \int_0^b \frac{x}{\sqrt{9+x^2}}\,dx$$ $\rightarrow$ misal : $u = 9+x^2$

$\frac{du}{dx} = 2x$, $du = 2x\,dx$

$$= \lim_{a\to-\infty} \int_{9+a^2}^{9} \frac{1}{2}u^{-\frac{1}{2}}\,du + \lim_{b\to\infty} \int_9^{9+b^2} \frac{1}{2}u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2} \lim_{a\to-\infty} (2u^{1/2} \Big|_{9+a^2}^{9} + \frac{1}{2} \lim_{b\to\infty} (2u^{1/2} \Big|_9^{9+b^2}$$

$$= \lim_{a\to-\infty} \sqrt{9} - \sqrt{9+a} + \lim_{b\to\infty} \sqrt{9+b^2} - \sqrt{9}$$

$$= -\infty + \infty \text{ (divergen)}$$

2\. a. $\int_2^\infty \frac{\ln\sqrt{x}}{x}\,dx = \lim_{b\to\infty} \int_2^b \frac{\ln\sqrt{x}}{x}\,dx$ $\rightarrow$ misal : $t = \ln x$ $\quad x = 2 \rightarrow t = \ln(2)$

$dt = \frac{1}{x}\,dx \quad x = b \rightarrow t = \ln(b)$

$$= \lim_{b\to\infty} \int_2^b \frac{\ln x^{\frac{1}{2}}}{x}\,dx$$

$$= \frac{1}{2} \lim_{b\to\infty} \int_2^b \frac{\ln x}{x}\,dx$$

$$= \frac{1}{2} \lim_{b\to\infty} \int_{\ln(2)}^{\ln(b)} t\,dt$$

$$= \frac{1}{2} \lim_{b\to\infty} \frac{t^2}{2} \Big|_{\ln(2)}^{\ln(b)}$$

$$= \frac{1}{4} \lim_{b\to\infty} \ln(b)^2 - \ln(2)^2 = +\infty \text{ (divergen)}$$

b. $\int_{-\infty}^{\infty} \frac{x}{(x^2+1)}\, dx$

$= \lim_{a \to -\infty} \int_a^0 \frac{x}{x^2+1}\, dx + \lim_{b \to \infty} \int_0^b \frac{x}{x^2+1}\, dx \rightarrow$ misal : $u = x^2+1$, $du = 2x\, dx$ $\rightarrow \frac{1}{2} du = x\, dx$

$= \lim_{a \to -\infty} \int_a^0 \frac{1}{2u}\, du + \lim_{b \to \infty} \int_0^b \frac{1}{2u}\, du$

$= \lim_{a \to -\infty} \left(\frac{1}{2} \ln u \right|_a^0 + \lim_{b \to \infty} \left(\frac{1}{2} \ln u \right|_0^b$

$= \frac{1}{2}\left[\left(\ln(0^2+1) - \ln(-\infty^2+1)\right)\right] + \frac{1}{2}\left[\left(\ln(\infty^2+1) - \ln(0^2+1)\right)\right]$

$= \infty$ (divergen)

3. a. $\int_2^{\infty} \frac{1}{x \ln x}\, dx = \lim_{b \to \infty} \int_2^b \frac{1}{x \ln x}\, dx \rightarrow$ misal : $u = \ln x \Leftrightarrow du = \frac{1}{x} dx$

$\Leftrightarrow \lim_{b \to} \int_{\ln 2}^{\ln b} \frac{1}{u}\, du = \lim_{b \to \infty} \left(\ln u \right|_{\ln 2}^{\ln b}$

$= (\ln(\ln \infty)) - (\ln(\ln 2))$

$= \infty$ (divergen)

b. $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\, dx$

$= \lim_{a \to -\infty} \int_a^0 \frac{1}{x^2+4x+9}\, dx + \lim_{b \to \infty} \int_0^b \frac{1}{x^2+4x+9}\, dx$

$= \lim_{a \to -\infty} \int_a^0 \frac{1}{(x+2)^2+(\sqrt{5})^2}\, dx + \lim_{b \to \infty} \int_0^b \frac{1}{(x+2)^2+(\sqrt{5})^2}\, dx$

$= \lim_{a \to -\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right) \Big|_a^0 + \lim_{b \to \infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right) \Big|_0^b$

$= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0+2}{\sqrt{5}}\right) - \lim_{a \to -\infty} \tan^{-1}\left(\frac{a+2}{\sqrt{5}}\right) \cdot \frac{1}{\sqrt{5}} + \lim_{b \to \infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{b+2}{\sqrt{5}}\right) - \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0+2}{\sqrt{5}}\right)$

$= \frac{1}{\sqrt{5}} \left(\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \tan^{-1}(-\infty) + \tan^{-1}(\infty) - \tan^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$

$= \frac{1}{\sqrt{5}} \left(0 - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2}\right) = \frac{\pi\sqrt{5}}{5}$

4. a. $\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx \rightarrow$ misal : $u = \ln(x)$, $du = \frac{1}{x} dx$ $\rightarrow x\, du = dx$

$= \lim_{b \to \infty} \int_2^b \frac{1}{x(\ln x)^2}\, dx$

$= \lim_{b \to \infty} \int_2^b \frac{1}{x u^2}\, x\, du$

$= \lim_{b \to \infty} \int_2^b u^{-2}\, du$

$= \lim_{b \to \infty} \left(\frac{u^{-1}}{-1} \right|_2^b$

$= \lim_{b \to \infty} -\frac{1}{\ln 2} \Big|_2^b$

$= \lim_{b \to \infty} -\left(\frac{1}{\ln(b)} - \frac{1}{\ln(2)}\right)$

$= \lim_{b \to \infty} \left(0 - \frac{1}{\ln(2)}\right)$

$= \lim_{b \to \infty} \frac{1}{\ln(2)} \approx 1{,}442\, x$ (konvergen)

b. $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}} \, dx$

$= \int_{-\infty}^{0} \frac{x}{e^{|x|}} \, dx + \int_{0}^{\infty} \frac{x}{e^{|x|}} \, dx$

$= \lim_{a \to -\infty} \left(\int_{a}^{0} \frac{x}{e^{|x|}} \, dx \right) + \lim_{b \to \infty} \left(\int_{0}^{b} \frac{x}{e^{|x|}} \, dx \right)$

$= \lim_{a \to -\infty} \left(-1 - a e^{a} + e^{a} \right) + \lim_{b \to \infty} \left(- \frac{b+1}{e^{b}} + 1 \right)$

$= -1 + 1$

$= 0$ (konvergen)