-Kelompok 10-

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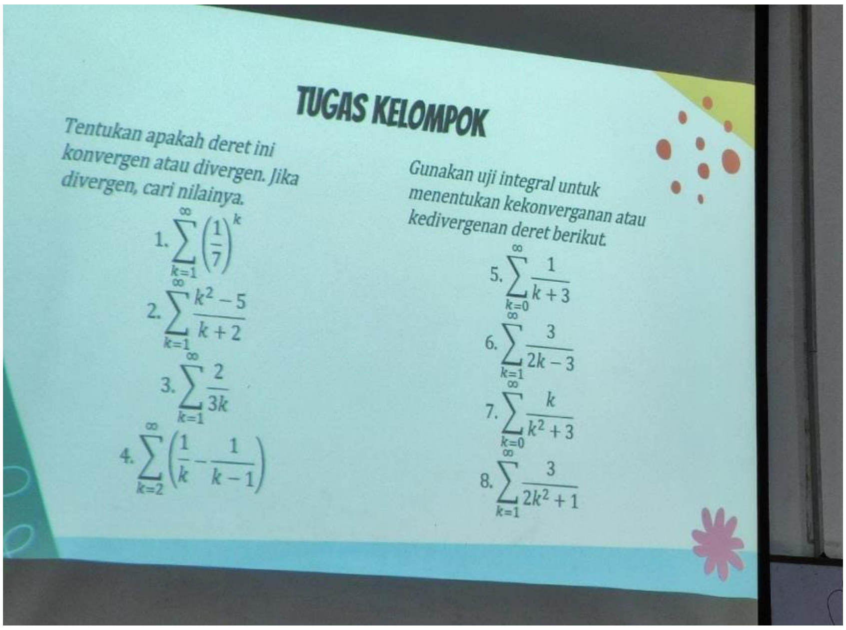

Jawab:

1.

1. $\sum_{k=1}^{\infty}\left(\frac{1}{7}\right)^k$

Deret di atas merupakan deret geometri dengan:

- $a=\frac{1}{7}$
- $r=\frac{1}{7}\in(-1,1)$, sehingga :
- $S_n=\frac{a}{1-r}=\frac{\frac{1}{7}}{1-\frac{1}{7}}=\frac{1}{6}$ dan $\sum_{k=1}^{\infty}\left(\frac{1}{7}\right)^k$ juga konvergen ke $\frac{1}{6}$

atau

- $S_n=\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\cdots+\frac{1}{7^n}$
- $\frac{1}{7}S_n=\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\cdots+\frac{1}{7^n}+\frac{1}{7^{(n+1)}}$

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- $\frac{6}{7}S_n=\frac{1}{7}-\frac{1}{7^{(n+1)}}$
- $S_n=\frac{1}{6}-\frac{6}{7^n}$
- $\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{1}{6}-\frac{6}{7^n}=\frac{1}{6}$
- Deret $\sum_{k=1}^{\infty}\left(\frac{1}{7}\right)^k$ juga konvergen menuju ke $\frac{1}{6}$

2.

$$\sum_{K=1}^{\infty}\frac{k^2-5}{k+2}$$

$$S_n=\frac{-4}{3}+\frac{-1}{4}+\frac{4}{5}+\cdots+\frac{k^2-5}{k+2}$$

$$\lim_{k\to\infty}\frac{k^2-5}{k+2}\to\frac{\infty}{\infty}$$

$$\lim_{k\to\infty}\frac{2k}{1}=\infty\neq 0$$

Karena $\lim_{k\to\infty}a_k\neq 0$ maka deret divergen

3.

3. $\sum_{k=1}^{\infty} \frac{2}{3k}$ = Konvergen atau divergen ? Jika divergen, cari nilainya.

$\frac{2}{3}\sum_{k=1}^{\infty}\frac{1}{k}$, Karena $\frac{1}{k}$ merupakan deret harmonik dan selalu divergen, maka $\sum_{k=1}^{\infty}\frac{2}{3k}$ juga divergen

Contoh pembuktian bahwa deret harmonik adalah divergen :

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$$

$$= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \cdots + \frac{1}{n}$$

$$= 1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \cdots + \frac{1}{n}$$

$$= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{n}$$

Untuk nilai n menuju tak hingga ($n \to \infty$)maka ruas kanan adalah divergen,
sehingga S_n divergen. Maka dari itu, dapat disimpulkan bahwa **deret harmonik** adalah **divergen**.

4.

$$\sum_{k=2}^{\infty}\left(\frac{1}{k} - \frac{1}{k-1}\right)$$

$$\text{Sn} = \left(\frac{1}{2} - 1\right) + \left(\frac{1}{3} - \frac{1}{2}\right) + \left(\frac{1}{4} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{k} - \frac{1}{k-1}\right)$$

$$\text{Sn} = -1 + \left(\frac{1}{2} - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{3}\right) + \ldots + \frac{1}{k}$$

$$\text{Sn} = -1 + \frac{1}{k}$$

$$\lim_{k\to\infty} -1 + \frac{1}{k} = -1 \text{ (Konvergen)}$$

Maka deret tersebut konvergen ke -1

5.

5. $\sum_{k=0}^{\infty}\frac{1}{k+3}$

$$S_n = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots\ldots + \frac{1}{k+3}$$

Syarat Uji Integral

- $f(x)$ *kontinu* $=$ *ya*
- $f(x)$ *positif* $=$ *ya*
- *tak naik* $=$ *ya*

Maka berlaku,

$$\lim_{b\to\infty}\int_0^b \frac{1}{x+3}$$

Misal,

$u = x + 3$

$du = dx$

$$\lim_{b\to\infty}\int_3^{b+3}\frac{1}{u}du$$

$$= \lim_{b\to\infty} (ln|_3^{b+3}$$

$$= [ln_{(b+3)} - ln3]$$

$$= \infty\ (divergen)$$

6.

6) $\sum_{k=1}^{\infty}\frac{3}{2k-3} = \int_1^{\infty}\frac{3}{2k-3}$

Misal:

$2k - 3 = u$

$dk = \frac{du}{2}$

$$\lim_{b\to\infty}\int_1^b \frac{3}{u}\frac{du}{2} = \lim_{b\to\infty}\frac{3}{2}(ln(u)|_1^b$$

$$= \frac{3}{2}\lim_{b\to\infty}(ln(2k-3)|_1^b$$

$$= \frac{3}{2}\lim_{b\to\infty}(ln(2b-3) - ln(1))$$

$$= \infty\ (DIVERGEN)$$

Karena $\int_1^{\infty}\frac{3}{2k-3}$ dx divergen, maka $\sum_{k=1}^{\infty}\frac{3}{2k-3}$ juga divergen

7.

$$\sum_{k=0}^{\infty}\frac{k}{k^2+3} = \int_0^{\infty}\frac{k}{k^2+3}dk$$

$$U = k^2 + 3, du = 2kdk, \frac{1}{2}du = kdk$$

$$= \int_3^{\infty}\frac{1}{u}\times\frac{1}{2}du$$

$$= \lim_{a\to\infty}\int_3^a \frac{1}{2u}du$$

$$= \lim_{a\to\infty}\frac{1}{2}\int_3^a \frac{1}{u}du$$

$$= \lim_{a\to\infty}\frac{1}{2}\ln(|u|)\Big]_3^a$$

$$= \frac{\lim_{a\to\infty}\ln(|a|) - \lim_{a\to\infty}\ln(|3|)}{\lim_{a\to\infty} 2}$$

$$= \infty\ (Divergen)$$

8.

$$\sum_{k=1}^{\infty}\frac{3}{2k^2+1} = \int_1^{\infty}\frac{3}{2k^2+1}dk$$

$$= \lim_{a\to\infty}\int_1^a \frac{3}{2(\frac{1}{2}+k^2)}dk$$

$$= \lim_{a\to\infty}\frac{3}{2}\int_1^a \frac{1}{\frac{1}{2}+k^2}dk$$

$$= \lim_{a\to\infty}\frac{3}{2}\int_1^a \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2+k^2}dk$$

$$= \lim_{a\to\infty}\frac{3}{2}\left(\frac{1}{\frac{1}{\sqrt{2}}}\arctan\left(\frac{k}{\frac{1}{\sqrt{2}}}\right)\Big]_1^a\right)$$

$$= \lim_{a\to\infty}\frac{3}{2}\left(\left(\sqrt{2}\arctan(k\sqrt{2})\right]_1^a\right)$$

$$= \lim_{a\to\infty}\frac{3\left(\sqrt{2}\arctan(a\sqrt{2}) - \sqrt{2}\arctan(\sqrt{2})\right)}{2}$$

$$= \lim_{a\to\infty}\frac{3}{2}\left(\sqrt{2}\frac{\pi}{2} - \sqrt{2}\arctan(\sqrt{2})\right)$$

$$= 1.3 \text{ (KONVERGEN)}$$