MAT211 Kalkulus II
Kunci Jawaban Sesi UTS
Kuis 1

WSBP & YB
Paralel 3

29 September 2022

Paket 1
Tentukan limit-limit berikut:

1. $\lim_{x\to 0+}\left(\frac{1}{x}\right)^{sinx}$
   Jawab:
   $$\lim_{x\to 0+}\left(\frac{1}{x}\right)^{sinx}=\infty^0$$
   $$y=\left(\frac{1}{x}\right)^{sinx}$$
   $$\ln y = sinx\ln\left(\frac{1}{x}\right)$$
   $$y=e^{sinx\ln\left(\frac{1}{x}\right)}$$
   $$\lim_{x\to 0+}y=e^{\lim_{x\to 0+}sinx\ln\left(\frac{1}{x}\right)}$$
   $$\lim_{x\to 0+}sinx\ln\left(\frac{1}{x}\right)=0.\infty$$
   $$\lim_{x\to 0+}\frac{\ln\left(\frac{1}{x}\right)}{\frac{1}{sinx}}=\frac{\infty}{\infty}$$
   $$\overset{\mathrm{H}}{=}\lim_{x\to 0+}\frac{-\frac{1}{x}}{-\frac{cosx}{sinx}\frac{1}{sinx}}=\lim_{x\to 0+}\frac{\sin^2 x}{xcosx}\overset{\mathrm{H}}{=}\lim_{x\to 0+}\frac{\sin(2x)}{\cos x+xsin\,x}=0$$
   $$\lim_{x\to 0+}y=e^0=1$$

2. $\lim_{x\to 0}\frac{1-\cos x}{x}$
   Jawab:
   $$\lim_{x\to 0}\frac{1-\cos x}{x}=\frac{0}{0}$$
   $$\overset{\mathrm{H}}{=}\lim_{x\to 0}\frac{\sin x}{1}=\frac{1}{0}=0$$

3. $\lim_{x\to\infty}\frac{\sin x}{x}$
   Jawab:
   $$-1\le\sin x\le 1$$
   $$\frac{-1}{x}\le\frac{\sin x}{x}\le\frac{1}{x}$$

$$\lim_{x\to\infty} \frac{-1}{x} = \frac{1}{x} = 0$$
Maka, $\lim_{x\to\infty} \frac{\sin x}{x} = 0$

4.Tentukan integral tak wajar berikut:

$$\int_3^\infty \frac{dx}{(x-2)^{3/2}}$$

Jawab:

Misalkan $u = x - 2$

$$\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty} \int_3^t (x-2)^{-3/2}\,dx = \lim_{t\to\infty} \left[-2\,(x-2)^{-1/2}\right]_3^t$$

$$= \lim_{t\to\infty} \left(\frac{-2}{\sqrt{t-2}} + \frac{2}{\sqrt{1}}\right) = 0 + 2 = 2.$$

Deret konvergen.

5.Diberikan barisan $\{a_n\}$ dengan empat suku pertama diberikan oleh

$$-\frac{1}{3}, \frac{4}{9}, -\frac{9}{27}, \frac{16}{81}, \dots$$

Tentukan rumus eksplisit, konvergen atau divergen, dan jika konvergen $\{a_n\}$ konvergen ke mana, serta tentukan batasnya?

Jawab:

$$a_n = (-1)^n \frac{n^2}{3^n}$$

$$\lim_{x\to\infty} \frac{n^2}{3^n} = \lim_{x\to\infty} \frac{2n}{3^n \ln 3} = \lim_{x\to\infty} \frac{2}{3^n (\ln 3)^2} = 0$$

Menggunakan L'Hopital; konvergen ke 0.

Paket 2

Tentukan limit-limit berikut:

1. $\lim_{x\to 0} \frac{1-\cos x}{2x^2+5x}$

   Jawab:

   $$\lim_{x\to 0} \frac{1-\cos x}{2x^2+5x} = \frac{0}{0}$$
   $$\overset{\mathrm{H}}{=} \lim_{x\to 0} \frac{\sin x}{4x+5} = \frac{0}{5} = 0$$

2. $\lim_{x\to 0} \frac{\sin 4x}{\tan x}$

   Jawab:

   $$\lim_{x\to 0} \frac{\sin 4x}{\tan x} = \frac{0}{0}$$
   $$\overset{\mathrm{H}}{=} \lim_{x\to 0} \frac{4\cos 4x}{\sec^2 x} = 4$$

3. $\lim_{x\to 0} x^3 \cot x$

Jawab:

$$\lim_{x\to 0} x^3 \cot x = 0.\infty$$

$$\lim_{x\to 0} x^3 \cot x = \lim_{x\to 0} \frac{x^3 \cos x}{\sin x} = \frac{0}{0}$$

$$\overset{\mathrm{H}}{=} \lim_{x\to 0} \frac{3x^2 cosx}{cosx} = 0$$

4.Tentukan integral tak wajar berikut:

$$\int_1^\infty \frac{1}{(2x+1)^3} dx$$

Jawab:

$$\int_1^\infty \frac{1}{(2x+1)^3}\, dx = \lim_{t\to\infty} \int_1^t \frac{1}{(2x+1)^3}\, dx$$

$$= \lim_{t\to\infty} \left[-\frac{1}{4(2x+1)^2}\right]_1^t$$

$$= \lim_{t\to\infty} \left[-\frac{1}{4(2t+1)^2} + \frac{1}{36}\right] = 0 + \frac{1}{36}.$$

Deret konvergen

5. Diberikan barisan $\{a_n\}$ dengan lima suku pertama diberikan oleh

$$2,1,\frac{2^3}{3^2},\frac{2^4}{4^2},\frac{2^5}{5^2},\ldots$$

Tentukan rumus eksplisit, konvergen atau divergen, dan jika konvergen $\{a_n\}$ konvergen ke mana, serta tentukan batasnya?

Jawab:

$$a_n = \frac{2^n}{n^2}$$

$$\lim_{x\to\infty} \frac{2^n}{n^2} = \lim_{x\to\infty} \frac{2^n \ln 2}{2n} = \lim_{x\to\infty} \frac{2^n (\ln 3)^2}{2} = \infty$$

Divergen.

Paket 3

Tentukan limit-limit berikut:

1. $\lim_{x\to 0+} \frac{\cot x}{\ln x}$

   Jawab:

   $$\lim_{x\to 0+} \frac{\cot x}{\ln x} = \frac{\infty}{\infty}$$

   $$\overset{\mathrm{H}}{=} \lim_{x\to 0+} - \csc^2 x.x = 0$$

2. $\lim_{x\to\infty} \frac{x^2}{x^2+1}$

   Jawab:

   $$\lim_{x\to\infty} \frac{x^2}{x^2+1} = \frac{\infty}{\infty}$$

   $$\overset{\mathrm{H}}{=} \lim_{x\to\infty} \frac{2x}{2x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2}{2} = 1$$

3. $\lim_{x\to 0} \frac{cosx}{sinx} - \frac{1}{x}$

Jawab:

$$\lim_{x\to 0} \frac{cosx}{sinx} - \frac{1}{x} = \infty - \infty$$

$$\lim_{x\to 0} \frac{cosx}{sinx} - \frac{1}{x} = \lim_{x\to 0} \frac{xcosx - sinx}{xsinx} = \frac{0}{0}$$

$$\stackrel{\text{H}}{=} \lim_{x\to 0} \frac{cosx - xsinx + (-cosx)}{sinx + xcosx} = \frac{0}{0}$$

$$\stackrel{\text{H}}{=} \lim_{x\to 0} \frac{-sinx - xcosx}{cosx + cosx - xsinx} = \frac{0}{1 + 1 - 0} = 0$$

4.Tentukan integral tak wajar berikut:

$$\int_0^\infty \frac{1}{(3x+1)^2} dx$$

Jawab:

$$\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} = \lim_{t\to -2^+} \int_t^{14} (x+2)^{-1/4}\, dx$$

$$= \lim_{t\to -2^+} \left[\frac{4}{3}(x+2)^{3/4}\right]_t^{14}$$

$$= \frac{4}{3} \lim_{t\to -2^+} \left[16^{3/4} - (t+2)^{3/4}\right]$$

$$= \tfrac{4}{3}(8-0) = \tfrac{32}{3}.$$

Deret konvergen.

5.Diberikan barisan $\{a_n\}$ dengan empat suku pertama diberikan oleh

$$\frac{1}{2^2}, \frac{2}{2^3}, \frac{3}{2^4}, \frac{4}{2^5}, \cdots$$

Tentukan rumus eksplisit, konvergen atau divergen, dan jika konvergen $\{a_n\}$ konvergen ke mana, serta tentukan batasnya?

Jawab:

Misalkan $n + 1 = x; n = x - 1$

$$a_n = \frac{n}{2^{n+1}}$$

$$\lim_{x\to\infty} \frac{x-1}{2^x} = \lim_{x\to\infty} \frac{1}{2^x \ln 2} = 0$$

Menggunakan L'Hopital, maka

$\lim_{x\to\infty} \frac{n}{2^{n+1}} = 0$; konvergen ke 0.

Paket 1

Tentukanlah deret yang diberikan merupakan konvergen atau divergen dan sebutkan jenis uji yang digunakan.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n+1}}$

   Jawab:

   $$a_n=\frac{1}{n\sqrt{n+1}}=\frac{1}{\sqrt{n^3+n^2}};b_n=\frac{1}{n^{3/2}}$$

   $$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{n^{3/2}}{\sqrt{n^3+n^2}}=\lim_{n\to\infty}\sqrt{\frac{n^3}{n^3+n^2}}=\lim_{n\to\infty}\sqrt{\frac{1}{1+\frac{1}{2}}}=1;0<1<\infty$$

   $\sum_{n=1}^{\infty}b_n$ konvergen → $\sum_{n=1}^{\infty}a_n$ konvergen. (menggunakan uji limit)

2. $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{2^n}$

   Jawab:

   $$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\ln(n+1)2^n}{2^{n+1}\ln n}=\lim_{n\to\infty}\frac{\ln(n+1)}{2\ln n}$$

   Gunakan L'Hopital

   $\overset{\mathrm{H}}{=}\lim_{n\to\infty}\frac{\frac{1}{n+1}}{\frac{2}{n}}=\lim_{n\to\infty}\frac{n}{2(n+1)}=\lim_{n\to\infty}\frac{1}{2+\frac{2}{n}}=\frac{1}{2}<1;$ menggunakan uji rasio menunjukkan bahwa konvergen.

Ekspresikan bilangan berikut dalam bentuk pecahan.

3. $2.\overline{516}=2.516516516\,...$

Jawab:

$$2.\overline{516}=2+\frac{516}{10^3}+\frac{516}{10^6}+\cdots$$

Ini adalah deret geometrik dengan $a=\frac{516}{10^3}$ dan $r=\frac{1}{10^3}$. Deret ini konvergen menuju $S=\frac{a}{1-r}=\frac{\frac{516}{10^3}}{1-\frac{1}{10^3}}=\frac{516}{999}$. Karena itu

$$2.\overline{516}=2+\frac{516}{999}=\frac{2514}{999}=\frac{838}{333}\blacksquare$$

Tentukan apakah deret geometrik ini konvergen atau divergen

4. $3-4+\dfrac{16}{3}-\dfrac{64}{9}+\cdots$

Jawab:

Terlihat jelas bahwa deret geometrik ini memiliki rasio $r = -\frac{4}{3}$. Karena $|r| = \frac{4}{3} > 1$, deret ini divergen.■

Tentukan jari-jari kekonvergenan dan interval kekonvergenannya

5. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

Jawab:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)^4\,4^{n+1}}\cdot\frac{n^4\,4^n}{x^n}\right| = \lim_{n\to\infty}\left|\frac{n^4}{(n+1)^4}\cdot\frac{x}{4}\right|$$

$$= \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^4\frac{|x|}{4} = 1^4\cdot\frac{|x|}{4} = \frac{|x|}{4}$$

Dengan uji banding mutlak, deret ini konvergen ketika $\frac{|x|}{4} < 1 \Leftrightarrow |x| < 4$, jadi didapat jari-jari kekonvergenan $R = 4$. Ketika $x = 4$, maka deret $\sum_{n=1}^{\infty}\frac{1}{n^4}$ konvergen karena deret-p dimana $p = 4 > 1$. Ketika $x = -4$, deret $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}$ konvergen dengan uji deret ganti tanda. Oleh karena itu, interval kekonvergenannya adalah $[-4,4]$■

Ubahlah fungsi berikut ini menjadi deret pangkat dan tentukan interval kekonvergenannya.

6. $f(x) = \dfrac{x-1}{x+2}$

Jawab:

$$f(x) = 1 - \frac{3}{x+2} = 1 - \frac{\frac{3}{2}}{\frac{x}{2}+1} = 1 - \frac{3}{2}\cdot\frac{1}{1-\left(-\frac{x}{2}\right)} = 1 - \frac{3}{2}\sum_{n=0}^{\infty}\left(-\frac{x}{2}\right)^n \; atau \; -\frac{1}{2} - \sum_{n=1}^{\infty}\frac{(-1)^n 3x^n}{2^{n+1}}.$$

Deret geometrik $\sum_{n=0}^{\infty}\left(-\frac{x}{2}\right)^n$ konvergen ketika $\left|-\frac{x}{2}\right| < 1 \Leftrightarrow |x| < 2$, jadi jari-jari kekonvergenan $R = 2$ dan interval kekonvergenannya $(-2,2)$■

Paket 2

Tentukanlah deret yang diberikan merupakan konvergen atau divergen dan sebutkan jenis uji yang digunakan.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+1}{3^n}$

   Jawab:

   $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{[(n+1)^2+1]3^n}{3^{n+1}(n^2+1)} = \lim_{n\to\infty}\frac{\mathrm{n}^2+2\mathrm{n}+2}{3n+3} = \lim_{n\to\infty}\frac{1+\frac{2}{n}+\frac{2}{n^2}}{3+\frac{3}{n^2}} = \frac{1}{3} < 1$; menggunakan uji rasio menunjukkan bahwa konvergen.

2. $\sum_{n=1}^{\infty}\left(\frac{1}{2}+\frac{1}{n}\right)^{n}$

Jawab:

$\lim_{n\to\infty}(a_n)^{1/n}=\lim_{n\to\infty}\left[\left(\frac{1}{2}+\frac{1}{n}\right)^{n}\right]^{1/n}=\lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{n}\right)=\frac{1}{2}<1;$ menggunakan uji akar menunjukkan bahwa konvergen.

Ekspresikan bilangan berikut dalam bentuk pecahan.

3. $1.234\overline{567}$

Jawab:

$$1.234\overline{567}=1.234+\frac{567}{10^6}+\frac{567}{10^9}+\cdots$$

Ini adalah deret geometrik dengan $a=\frac{567}{10^6}$ dan $r=\frac{1}{10^3}$. Deret ini konvergen ke $S=\frac{a}{1-r}=\frac{567/10^6}{1-1/10^3}=\frac{567}{999000}$. Karena itu

$$1.234\overline{567}=1.234+\frac{567}{999000}=\frac{45679}{37000}\ \blacksquare$$

Tentukan apakah deret geometrik ini konvergen atau divergen

4. $10-2+0.4-0.08+\cdots$

Jawab:

Terlihat jelas bahwa deret geometrik ini memiliki rasio $r=-\frac{2}{10}$. Karena $|r|=\frac{1}{5}<1$, deret ini konvergen. $\blacksquare$

Tentukan jari-jari kekonvergenan dan interval kekonvergenan

5. $\sum_{n=1}^{\infty}\frac{x^n}{2n-1}$

Jawab:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{x^{n+1}}{2n+1}\cdot\frac{2n-1}{x^n}\right|=\lim_{n\to\infty}\left(\frac{2n-1}{2n+1}|x|\right)$$

$$=\lim_{n\to\infty}\left(\frac{2-1/n}{2+1/n}|x|\right)=|x|$$

Dengan uji banding mutlak, deret $\sum_{n=1}^{\infty}\frac{x^n}{2n-1}$ konvergen ketika $|x|<1$, sehingga $R=1$. Ketika $x=1$, deret $\sum_{n=1}^{\infty}\frac{1}{2n-1}$ divergen karena perbandingan dengan $\sum_{n=1}^{\infty}\frac{1}{2n}$ karena $\frac{1}{2n-1}>\frac{1}{2n}$ dan $\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$ divergen karena merupakan perkalian konstan dari deret harmonik. Ketika $x=-1$, deret $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}$ konvergen dengan uji deret ganti tanda. Karena itu, interval kekonvergenannya adalah $[-1,1)$. $\blacksquare$

Ubahlah fungsi berikut ini menjadi deret pangkat dan tentukan interval kekonvergenannya.

6. $f(x) = \dfrac{x^2}{x^4 + 16}$

Jawab:

$$f(x) = \frac{x^2}{16}\left(\frac{1}{1 + \frac{x^4}{16}}\right) = \frac{x^2}{16} \cdot \left(\frac{1}{1 - \left[-\left(\frac{x}{2}\right)\right]^4}\right) = \frac{x^2}{16}\sum_{n=0}^{\infty}\left[-\left(\frac{x}{2}\right)^4\right]^n \; atau \; \sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{2^{4n+4}}.$$

Deret geometrik $\sum_{n=0}^{\infty}\left[-\left(\frac{x}{2}\right)^4\right]^n$ konvergen ketika $\left|-\left(\frac{x}{2}\right)^4\right| < 1 \Leftrightarrow |x| < 2$, jadi jari-jari kekonvergenan $R = 2$ dan interval kekonvergenannya $(-2,2)$■

MAT211 Kalkulus II
Kunci Jawaban Sesi UTS
Pertemuan 1

WSBP & YB
Paralel 3

1. $\lim\limits_{x \to 1} \frac{\sin \pi x}{\ln x}$

   Jawab:

   $$\lim_{x \to 1} \frac{\sin \pi x}{\ln x} = \frac{0}{0}$$

   $$\overset{\text{H}}{=} \lim_{x \to 1} \frac{\pi \cos \pi x}{1/x} = \lim_{x \to 1} x\pi \cos \pi x = \pi \cos \pi = \pi(-1) = -3.14$$

2. $\lim\limits_{x \to \infty} \sqrt{x} \sin \frac{1}{x}$

   Jawab:

   $$\lim_{x \to \infty} \sqrt{x} \sin \frac{1}{x} = \infty . 0$$

   $$= \lim_{x \to \infty} \frac{\sin \frac{1}{x}}{\frac{1}{\sqrt{x}}} = \frac{0}{0}$$

   $$\overset{\text{H}}{=} \lim_{x \to \infty} \frac{cos \frac{1}{x}\left(-\frac{1}{x^2}\right)}{-\frac{1}{2x^{3/2}}} = \lim_{x \to \infty} 2x^{\frac{3}{2}} \frac{1}{x^2} cos \frac{1}{x} = \lim_{x \to \infty} \frac{2 \cos \frac{1}{x}}{\sqrt{x}} = 0$$

3. $\lim\limits_{x \to 0+} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

   Jawab:

   $$\lim_{x \to 0+} \left(\frac{1}{\sin x} - \frac{1}{x}\right) = \infty - \infty$$

   $$= \lim_{x \to 0+} \left(\frac{x - sinx}{xsinx}\right) \overset{\text{H}}{=} \lim_{x \to 0+} \left(\frac{1 - cosx}{sinx + xcosx}\right) \overset{\text{H}}{=} \lim_{x \to 0+} \left(\frac{sinx}{2cosx - xsinx}\right) = 0$$

4. $\lim_{x\to\infty}(1+3x)^{\frac{1}{2\ln x}}$

Jawab:

$y = (1+3x)^{1/2\ln x}$

$y = \exp\left(\frac{1}{2\ln x}\ln(1+3x)\right)$

$\lim_{x\to\infty} y = e^{\lim_{x\to\infty}\frac{1}{2\ln x}\ln(1+3x)}$

$$\lim_{x\to\infty}\frac{1}{2\ln x}\ln(1+3x) = \frac{1}{2}\lim_{x\to\infty}\frac{\ln(1+3x)}{\ln x} \underset{\mathrm{H}}{=} \frac{1}{2}\lim_{x\to\infty}\frac{\frac{3}{1+3x}}{\frac{1}{x}} = \frac{1}{2}\lim_{x\to\infty}\frac{3}{3} = \frac{1}{2}$$

$\lim_{x\to\infty} y = e^{1/2} = \sqrt{e}$

MAT211 Kalkulus II

Kunci Jawaban Sesi UTS

Pertemuan 2

WSBP & YB

Paralel 3

Infinite Limits of Integration;

Tentukan nilai integral berikut jika ada:

$$1.)\quad \int_{100}^{\infty} e^x dx = [e^x]_{100}^{-\infty} = \infty - e^{100} = \infty\ (divergen)\blacksquare$$

$$2.)\quad \int_{-\infty}^{-5}\frac{dx}{x^4} = \left[-\frac{1}{3x^3}\right]_{-\infty}^{-5} = -\frac{1}{3\cdot(-125)} - 0 = \frac{1}{375}\ \blacksquare$$

$$3.)\quad \int_{-\infty}^{\infty}\frac{dx}{(x^2+16)^2} = \int_{-\infty}^{0}\frac{dx}{(x^2+16)^2} + \int_{0}^{\infty}\frac{dx}{(x^2+16)^2}$$

Lakukan pengintegralan dengan metode substitusi yaitu memisalkan $x = 4\tan\theta$, sehingga didapat $\int\frac{dx}{(x^2+16)^2} = \frac{1}{128}\tan^{-1}\left(\frac{x}{4}\right) + \frac{x}{32(x^2+16)}$.

$$\int_{-\infty}^{0}\frac{dx}{(x^2+16)^2} = \left[\frac{1}{128}\tan^{-1}\left(\frac{x}{4}\right) + \frac{x}{32(x^2+16)}\right]_{-\infty}^{0} = \frac{\pi}{256}$$

dan

$$\int_{0}^{\infty}\frac{dx}{(x^2+16)^2} = \left[\frac{1}{128}\tan^{-1}\left(\frac{x}{4}\right) + \frac{x}{32(x^2+16)}\right]_{0}^{\infty} = \frac{\pi}{256}$$

sehingga

$$\int_{-\infty}^{\infty}\frac{dx}{(x^2+16)^2} = \int_{-\infty}^{0}\frac{dx}{(x^2+16)^2} + \int_{0}^{\infty}\frac{dx}{(x^2+16)^2} = \frac{\pi}{256} + \frac{\pi}{256} = \frac{\pi}{128}\blacksquare$$

$$4.)\quad \int_{-\infty}^{\infty}\frac{1}{x^2+2x+10}dx = \int_{-\infty}^{\infty}\frac{1}{(x+1)^2+9}dx$$

$$= \int_{-\infty}^{0}\frac{1}{(x+1)^2+9}dx + \int_{0}^{\infty}\frac{1}{(x+1)^2+9}dx$$

Lakukan pengintegralan dengan metode substitusi yaitu memisalkan $x+1=3\tan\theta$ sehinga didapat $\int \frac{1}{(x+1)^2+9}dx=\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)$.

$$\int_{-\infty}^{0}\frac{1}{(x+1)^2+9}dx=\left[\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)\right]_{-\infty}^{0}=\frac{1}{6}\left(\pi+2\tan^{-1}\left(\frac{1}{3}\right)\right)$$

dan

$$\int_{0}^{\infty}\frac{1}{(x+1)^2+9}dx=\left[\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)\right]_{0}^{-\infty}=\frac{1}{6}\left(\pi-2\tan^{-1}\left(\frac{1}{3}\right)\right)$$

sehingga

$$\int_{-\infty}^{\infty}\frac{1}{x^2+2x+10}dx=\frac{1}{6}\left(\pi+2\tan^{-1}\left(\frac{1}{3}\right)\right)+\frac{1}{6}\left(\pi-2\tan^{-1}\left(\frac{1}{3}\right)\right)=\frac{\pi}{3}\ \blacksquare$$

Infinite Integrands;

Tentukan nilai integral berikut jika ada:

1.) $$\int_{1}^{3}\frac{dx}{(x-1)^{1/3}}=\lim_{b\to1^+}\left[\frac{3(x-1)^{2/3}}{2}\right]_{b}^{3}=\frac{3}{2}\sqrt[3]{2^2}-\lim_{b\to1^+}\frac{3(b-1)^{2/3}}{2}=\frac{3}{\sqrt[3]{2}}-0$$
$$=\frac{3}{\sqrt[3]{2}}\ \blacksquare$$

2.) $$\int_{0}^{9}\frac{dx}{\sqrt{9-x}}=\lim_{b\to9^-}\left[-2\sqrt{9-x}\right]_{0}^{b}=\lim_{b\to9^-}-2\sqrt{9-b}+2\sqrt{9}=6\blacksquare$$

3.) $$\int_{-1}^{128}x^{-\frac{5}{7}}\,dx=\lim_{b\to0^-}\int_{-1}^{b}x^{-\frac{5}{7}}dx+\lim_{b\to0^+}\int_{b}^{128}x^{-\frac{5}{7}}dx$$
$$=\lim_{b\to0^-}\left[\frac{7}{2}x^{\frac{2}{7}}\right]_{-1}^{b}+\lim_{b\to0^+}\left[\frac{7}{2}x^{\frac{2}{7}}\right]_{b}^{128}$$
$$=0-\frac{7}{2}+\frac{7}{2}\cdot4-0=\frac{21}{2}\ \blacksquare$$

4.) $$\int_{-2}^{-1}\frac{dx}{(x+1)^{4/3}}=\lim_{b\to-1^-}\left[-\frac{3}{(x+1)^{\frac{1}{3}}}\right]_{-2}^{b}=-(-\infty)-3=\infty\ (diverges)\blacksquare$$

1. Tentukan integral berikut:

(a) $\displaystyle\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{x}{\sqrt{9+x^2}}\,dx$

2. Tentukan integral berikut:

(a) $\displaystyle\int_2^\infty \frac{\ln\sqrt{x}}{x}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{x}{(x^2+4)}\,dx$

3. Tentukan integral berikut:

(a) $\displaystyle\int_2^\infty \frac{1}{x\ln x}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2+4x+9}\,dx$

4. Tentukan integral berikut:

(a) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^2}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{x}{e^{|x|}}\,dx$

Jawab:

30 Agustus 2022 | Tugas Responsi ke-2 | Nomor 1a: $\int_3^\infty \frac{x}{\sqrt{16+x^2}}dx = \left[\sqrt{x^2+16}\right]_3^\infty = \infty$; TIDAK ADA ■

Nomor 1b: $\int_{-\infty}^\infty \frac{x}{\sqrt{9+x^2}} = \left[\sqrt{x^2+9}\right]_{-\infty}^0 + \left[\sqrt{x^2+9}\right]_0^\infty$ = TIDAK ADA ■ Nomor 2a: $\int_2^\infty \frac{\ln\sqrt{x}}{x}dx$

Misalkan $u = \ln\sqrt{x}$ sehingga $du = \frac{1}{2x}dx$. $\int \frac{\ln\sqrt{x}}{x}dx = 2\int u\,du = u^2 + C = (\ln\sqrt{x})^2 \Rightarrow \frac{\ln^2(x)}{4}$

Jadi $\int_2^\infty \frac{\ln\sqrt{x}}{x}dx = \left[\frac{\ln^2(x)}{4}\right]_2^\infty$ = TIDAK ADA ■ Nomor 2b: $\int_{-\infty}^\infty \frac{x}{x^2+4}dx = \left[\frac{1}{2}\log(x^2+4)\right]_{-\infty}^0$

$+ \left[\frac{1}{2}\log(x^2+4)\right]_0^\infty$ = TIDAK ADA ■ Nomor 3a: $\int_2^\infty \frac{1}{x\ln x}dx = \left[\ln(\ln(x))\right]_2^\infty$ = TIDAK ADA ■

Nomor 3b: $\int_{-\infty}^\infty \frac{1}{x^2+4x+9}dx$. Misalkan $u = x+2$ sehingga $du = dx$. $\int \frac{1}{x^2+4x+9}dx = \int \frac{1}{(x+2)^2+5}dx = \int \frac{1}{u^2+5}du$

$= \frac{1}{5}\int \frac{1}{u^2/5+1}du$. Misalkan $s = \frac{u}{\sqrt{5}} \longrightarrow ds = \frac{1}{\sqrt{5}}du \longrightarrow \frac{1}{\sqrt{5}}\int \frac{1}{s^2+1}ds = \frac{\tan^{-1}(s)}{\sqrt{5}} + C = \frac{\tan^{-1}\left(\frac{u}{\sqrt{5}}\right)}{\sqrt{5}} + C$

$= \frac{\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)}{\sqrt{5}} + C$. $\int_{-\infty}^\infty \frac{1}{x^2+4x+9}dx = \left[\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\right]_{-\infty}^0 + \left[\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\right]_0^\infty = \frac{\sqrt{5}}{5}\pi$ ■

Nomor 4a: $\int_2^\infty \frac{1}{x(\ln x)^2}dx = \left[-\frac{1}{\ln(x)}\right]_2^\infty = \frac{1}{\ln(2)} + 0 = \frac{1}{\ln(2)}$ ■ Nomor 4b: $\int_{-\infty}^\infty \frac{x}{e^{|x|}}dx$

$= \int_{-\infty}^\infty xe^{-|x|}dx = \int_{-\infty}^0 xe^{-|x|}dx + \int_0^\infty xe^{-|x|}dx = \int_{-\infty}^0 xe^x dx + \int_0^\infty xe^{-x}dx = -1+1$

$= 0$ ■

MAT211 Kalkulus II

Kunci Jawaban Sesi UTS

Soal Latihan - Pertemuan 3

WSBP & YB

Paralel 3

Tentukan kekonvergenan dan kemonotonan baris berikut:

1. $a_n = \frac{n}{3n-1}$

   Jawab:

   $a_1 = \frac{1}{2}, a_2 = \frac{2}{5}, a_3 = \frac{3}{8}, a_4 = \frac{4}{11}$

   $\lim\limits_{x\to\infty} \frac{n}{3n-1} = \lim\limits_{x\to\infty} \frac{1}{3-\frac{1}{n}} = \frac{1}{3}$; konvergen

   Kemonotonan:

   $\frac{a_n}{a_{n+1}} = \frac{1/n^3}{1/(n+1)^3} = \frac{(n+1)^3}{n^3} = \frac{n^3+3n^2+3n+1}{n^3} > 1$ (turun)

2. $a_n = \frac{n^3+3n^2+3n}{(n+1)^3}$

Jawab:

$a_1 = \frac{7}{8}, a_2 = \frac{26}{27}, a_3 = \frac{63}{64}, a_4 = \frac{124}{125}$

$\lim_{x\to\infty} \frac{n^3+3n^2+3n}{(n+1)^3} = \lim_{x\to\infty} \frac{n^3+3n^2+3n}{n^3+3n^2+3n+1} = \frac{1+\frac{3}{n}+\frac{3}{n^2}}{1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}} = 1$; konvergen

Kemonotonan:

$a_n - a_{n+1} = \frac{n^3 + 3n^2 + 3n}{(n+1)^3} - \frac{(n+1)^3 + 3(n+1)^2 + 3(n+1)}{(n+1+1)^3}$

$= \frac{((n^3 + 3n^2 + 3n)(n+2)^3) - (((n+1)^3 + 3(n+1)^2 + 3(n+1))(n+1)^3)}{(n+1)^3(n+2)^3}$

$= \frac{-3n^2-9n-7}{(n^2+3n+2)^3} < 0$ (naik)

3. $a_n = \frac{\cos(n\pi)}{n}$

Jawab:

$a_1 = -1, a_2 = \frac{1}{2}, a_3 = -\frac{1}{3}, a_4 = \frac{1}{4}$

$\cos(n\pi) = (-1)^n$, jadi $-\frac{1}{n} \leq \frac{\cos(n\pi)}{n} \leq \frac{1}{n}$

$\lim_{x\to\infty} -\frac{1}{n} = \lim_{x\to\infty} \frac{1}{n} = 0$; Berdasarkan Teorema Apit, konvergen ke 0.

Kemonotonan:

$a_n - a_{n+1} = \frac{cosn\pi}{n} - \frac{\cos\,(n+1)\pi}{n+1}$

$= \frac{\cos n\pi(n+1)-(\cos(n+1)\pi.n}{n(n+1)}$ (tidak naik dan tidak turun)

4. $a_n = e^{-n}\sin n$

Jawab:

$a_1 = e^{-1}sin1 \approx 0.3096, a_2 = e^{-2}\sin 2 \approx 0.1231, a_3 = e^{-3}\sin 3 \approx 0.0070$

$a_4 = e^{-4}\sin 4 \approx -0.0139$

Jawab:

$-1 \leq \sin n \ \leq 1$ untuk semua n,

$-e^{-n} \leq e^{-n}\sin n \leq e^{-n}$

$\lim_{x\to\infty} -e^{-n} = \lim_{x\to\infty} e^{-n} = 0$; Berdasarkan Teorema Apit, konvergen ke 0.

Kemonotonan:

$a_n - a_{n+1} = e^{-n}\sin n - e^{-(n+1)}\sin n + 1$

$= \frac{e\sin n - \sin(n+1)}{e^{n+1}} > 0$ (turun)

5. $a_n = \frac{1}{n^3}$

Jawab:

$a_1 = 1, a_2 = \frac{1}{8}, a_3 = \frac{1}{27}, a_4 = \frac{1}{256}$

$\lim_{x\to\infty} \frac{1}{n^3} = 0$; konvergen ke 0.

Kemonotonan:

$\frac{a_n}{a_{n+1}} = \frac{\frac{1}{n^3}}{\frac{1}{(n+1)^3}} = \frac{(n+1)^3}{n^3} > 1$ (turun)

Carilah rumus eksplisit dan tentukan kekonvergenannya

6. $\frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots$

   Jawab:

   $a_n = \frac{1}{2^{n+1}}$

   $\lim_{n\to\infty} \frac{1}{2^{n+1}} = 0$; konvergen

7. $\sin 1, 2\sin\frac{1}{2}, 3\ siin\frac{1}{3}, 4\sin\frac{1}{4}, \ldots$

   Jawab:

   $a_n = \frac{n}{(n+1) - \frac{1}{n+1}} = \frac{n(n+1)}{(n+1)^2 - 1} = \frac{n^2+n}{n^2+2n}$

   $\lim_{n\to\infty} \frac{n^2+n}{n^2+2n} = \lim_{x\to\infty} \frac{1+\frac{1}{n}}{1+\frac{2}{n}} = 1$; konvergen

8. $0.1, 0.11, 0.111, 0.1111, \ldots$

   Jawab:

   $a_n = \frac{1}{9}\left[1 - \left(\frac{1}{10}\right)^n\right]$

   $\lim_{n\to\infty} \frac{1}{9}\left[1 - \left(\frac{1}{10}\right)^n\right] = \frac{1}{9}(1-0) = \frac{1}{9}$; konvergen

MAT211 Kalkulus II
Kunci Jawaban Sesi UTS
Tugas Mandiri - Pertemuan 3

WSBP & YB
Paralel 3

1. (a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya:

   $$cos\pi, \frac{cos2\pi}{4}, \frac{cos3\pi}{9}, \frac{cos4\pi}{16}, \ldots$$

   Jawab:

   $a_n = \frac{cosn\pi}{n^2}$

   Berdasarkan teorema apit:

   $-1 \le \cos n\pi \le 1$

   $-\frac{1}{n^2} \le \frac{cosn\pi}{n^2} \le \frac{1}{n^2}$

   $\lim_{n\to\infty} -\frac{1}{n^2} = \lim_{n\to\infty} \frac{1}{n^2} = 0$; konvergen

(b) Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan (dengan definisi limit) $\{a_n + b_n\}$ konvergen ke A+B.

Jawab:

$\lim_{n\to\infty} a_n = A$

$\lim_{n\to\infty} b_n = B$

$\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n = A + B$

$|(a_n + b_n) - (A + B)| < \varepsilon$

$a_n\ konvergen\ ke\ A$

$L = A$; akan dibuktikan untuk setiap $\varepsilon > 0$ terdapat $N > 0$ sedemikian sehingga $n \geq N$

$|a_n - L| < \frac{1}{2}\varepsilon$

$|a_n - A| < \frac{1}{2}\varepsilon$

$b_n\ konvergen\ ke\ B$

$L = B$; akan dibuktikan untuk setiap $\varepsilon > 0$ terdapat $N > 0$ sedemikian sehingga $n \geq N$

$|b_n - B| < \frac{1}{2}\varepsilon$

$|(a_n + b_n) - (A + B)| \leq |a_n - A| + |b_n - B|$

$|(a_n + b_n) - (A + B)| < \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon$

$|(a_n + b_n) - (A + B)| < \varepsilon$

(c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut:

$$a_n = \sin\frac{n\pi}{4}$$

Jawab:

Tidak memiliki limit, keterbatasan, dan bukan barisan monoton.

2. (a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya:

$$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5} \ldots$$

Jawab:

$a_n = (-1)^{n+1}\left(\frac{1}{n}\right)$

$\lim_{n\to\infty} (-1)^{n+1}\left(\frac{1}{n}\right) = \lim_{n\to\infty} \left|(-1)^{n+1}\left(\frac{1}{n}\right)\right| = \lim_{n\to\infty} \frac{1}{n} = 0$; konvergen

(b) Buktikan bahwa $\{a_n\}$ berikut konvergen:

$$a_n = \frac{3 - 8.2^n}{5 + 4.2^n}$$

Jawab:

$\lim_{n\to\infty} \frac{3-8.2^n}{5+4.2^n} = \frac{\frac{3}{2^n}-8}{\frac{5}{2^n}+4} = -2$; konvergen

(c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut:

$$a_n = \frac{\ln n}{n}$$

Jawab:

$\frac{a_n}{a_{n+1}} = \frac{\frac{\ln n}{n}}{\frac{\ln(n+1)}{n+1}} = \frac{(\mathrm{n}+1)\ln n}{n\ln(n+1)} > 1$ (bukan barisan monoton)

3. (a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya:

$$0.9, 0.99, 0.999, 0.9999, 0.99999 \ldots$$

Jawab:

$a_n = 1 - \frac{1}{10^n}$

$\lim_{n\to\infty} 1 - \frac{1}{10^n} = 1 - 0 = 1$; konvergen

(b) Buktikan bahwa $\{a_n\}$ berikut konvergen:

$$a_n = \frac{n+3}{3n-2}$$

Jawab:

$\lim_{n\to\infty} \frac{n+3}{3n-2} = \lim_{n\to\infty} \frac{1+\frac{3}{n}}{3-\frac{2}{n}} = \frac{1}{3}$; konvergen

(c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut:

$$a_n = \frac{n!}{10^n}$$

Jawab:

Bukan barisan monoton.

MAT211 Kalkulus II

Kunci Jawaban Sesi UTS

Pertemuan 4

WSBP & YB

Paralel 3

Tentukan apakah deret ini konvergen atau divergen. Jika divergen, cari nilainya.

$$1.\ \sum_{k=1}^{\infty}\left(\frac{1}{7}\right)^k$$

Jawab:

Ini adalah deret geometrik dengan $a = \frac{1}{7}$ dan $r = \frac{1}{7}$ sehingga nilai deret in adalah:

$$S = \frac{1/7}{1-1/7} = \frac{1}{6} \blacksquare$$

$$2.\ \sum_{k=1}^{\infty}\frac{k^2-5}{k+2}$$

Jawab:

Karena nilai $\lim_{k\to\infty} \frac{k^2-5}{k+2} = \infty \neq 0$ maka deret ini divergen■

$$3.\ \sum_{k=1}^{\infty} \frac{2}{3k} = divergen\blacksquare$$

Jawab:

$$\sum_{k=1}^{\infty} \frac{2}{3k} = \frac{2}{3}\sum_{k=1}^{\infty} \frac{1}{k} \text{ adalah divergen karena } \sum_{k=1}^{\infty} \frac{1}{k} \text{ divergen}$$

$$4.\ \sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k-1}\right)$$

Jawab:

Deret ini merupakan *collapsing series*.

$$S_n = \left(\frac{1}{2} - 2\right) + \left(\frac{1}{3} - \frac{1}{2}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n-2}\right) + \left(\frac{1}{n} - \frac{1}{n-1}\right) = -1 + \frac{1}{n}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} -1 + \frac{1}{n} = -1, \text{jadi } \sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k-1}\right) = -1\blacksquare$$

Gunakan uji integral untuk menentukan kekonverganan atau kedivergenan deret berikut.

$$5.\ \sum_{k=0}^{\infty} \frac{1}{k+3}$$

Jawab:

$\int_0^\infty \frac{1}{x+3} dx = \infty$. Jadi deret ini divergen■

$$6.\ \sum_{k=1}^{\infty} \frac{3}{2k-3}$$

Jawab:

$$\int_2^\infty \frac{3}{2x-3} dx = \left[\frac{3}{2} \ln|2x-3|\right]_2^\infty = \infty$$

Jadi deret ini divergen■

$$7.\ \sum_{k=0}^{\infty} \frac{k}{k^2+3}$$

Jawab:

$$\int_2^\infty \frac{x}{x^2+3} dx = \infty$$

Jadi deret ini divergen■

$$8.\ \sum_{k=1}^{\infty} \frac{3}{2k^2+1}$$

Jawab:

$$\int_1^\infty \frac{3}{2x^2+1} dx = \frac{3}{\sqrt{2}}\left(\frac{\pi}{2} - \tan^{-1}\sqrt{2}\right) < \infty$$

Jadi deret ini konvergen■

MAT211 Kalkulus II
Kunci Jawaban Sesi UTS
Tugas Kelompok - Pertemuan 5

WSBP & YB
Paralel 3

Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

1. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

   Jawab:

   $$a_n = \frac{3n+1}{n^2-4} =; b_n = \frac{1}{n}$$

   $$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3n^2+n}{n^2-4} = \lim_{n\to\infty} \frac{3+\frac{1}{n}}{1-\frac{4}{n^2}} = 3; 0 < 3 < \infty$$

   $\sum_{n=1}^{\infty} b_n$ konvergen → $\sum_{n=1}^{\infty} a_n$ konvergen. (menggunakan uji limit)

2. $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$

   Jawab:

   $$a_n = \frac{n}{n^2+2n-3} =; b_n = \frac{1}{n}$$

   $$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2}{n^2+2n-3} = \lim_{n\to\infty} \frac{1}{1+\frac{2}{n}-\frac{3}{n^2}} = 1; 0 < 1 < \infty$$

   $\sum_{n=1}^{\infty} b_n$ divergen → $\sum_{n=1}^{\infty} a_n$ divergen. (menggunakan uji limit)

3. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

   Jawab:

   $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!n^{100}}{(n+1)^{100}n!} = \lim_{n\to\infty} \frac{n^{100}}{(n+1)^{99}} = \lim_{n\to\infty} \frac{n}{\left(\frac{n+1}{n}\right)^{99}} = \infty$; divergen (menggunakan uji rasio)

4. $\sum_{n=1}^{\infty} \frac{3^n+n}{n!}$

   Jawab:

   $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(3^{n+1}+n+1)n!}{(n+1)!(3^n+n)} = \lim_{n\to\infty} \frac{3^{n+1}+n+1}{n3^n+3^n+n^2+n} = 0 < \infty$;konvergen (menggunakan uji rasio)

5. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

   Jawab:

$$a_n = \frac{3n+1}{n^2-4} =; b_n = \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3n^2+n}{n^2-4} = \lim_{n\to\infty} \frac{3+\frac{1}{n}}{1-\frac{4}{n^2}} = 3; 0 < 3 < \infty$$

$\sum_{n=1}^{\infty} b_n$ konvergen → $\sum_{n=1}^{\infty} a_n$ konvergen. (menggunakan uji limit)

6. $\sum_{n=1}^{\infty}\left(\frac{n}{3n+2}\right)^n$
   Jawab:
   $\lim_{n\to\infty}(a_n)^{1/n} = \lim_{n\to\infty}\left[\left(\frac{n}{3n+2}\right)^n\right]^{1/n} = \lim_{n\to\infty}\left(\frac{n}{3n+2}\right) = 1/3 < 1$; menggunakan uji akar menunjukkan bahwa konvergen.

7. $\sum_{n=1}^{\infty}\left(\frac{1}{\ln n}\right)^n$
   Jawab:
   $\lim_{n\to\infty}(a_n)^{1/n} = \lim_{n\to\infty}\left[\left(\frac{1}{\ln n}\right)^n\right]^{1/n} = \lim_{n\to\infty}\left(\frac{1}{\ln n}\right) = 0 < 1$; menggunakan uji akar menunjukkan bahwa konvergen.

Tentukan apakah deret yang diberikan adalah konvergen mutlak, konvergen bersyarat, atau divergen:

8. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$
   Jawab:

$$U_n = (-1)^{n+1}\frac{n}{n+1}; a_n = \frac{n}{n+1}$$

$$\rho = \lim_{n\to\infty}\frac{|U_n+1|}{|U_n|}$$

$$= \lim_{n\to\infty}\frac{\left|(-1)^{n+1+1}\frac{n+1}{n+1+1}\right|}{\left|(-1)^{n+1}\frac{n}{n+1}\right|}$$

$$= \lim_{n\to\infty}\frac{\frac{n+1}{n+1+1}}{\frac{n}{n+1}}$$

$$= \lim_{n\to\infty}\frac{(n+1)(n+1)}{(n+1+1)(n)} = 1$$

Karena $\rho = 1$, maka uji banding mutlak tidak dapat digunakan untuk menyimpulkan. Dan akan dibuktikan melalui uji deret ganti tanda:

$$\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{n+1+1}}{\frac{n}{n+1}} = \frac{(n+1)^2}{n(n+1)} > 1; a_{n+1} > a_n; (naik)$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n}{n+1} \neq 0$$

$\lim_{n\to\infty} a_n \neq 0$; (divergen)

Maka, deret $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$ adalah divergen

9. $\sum_{n=1}^{\infty} sin\frac{n!}{n^2}$

   Jawab:

   Deret divergen

10. $\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^n$

    Jawab:

$$\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^n = \sum_{n=1}^{\infty}(-1)^n\left(\frac{4}{3}\right)^n$$

Uji banding mutlak

$$\rho = \lim_{n\to\infty}\frac{|U_n+1|}{|U_n|}$$

$$= \lim_{n\to\infty}\frac{\left|\left(-\frac{4}{3}\right)^{n+1}\right|}{\left|\left(-\frac{4}{3}\right)^{n}\right|}$$

$$= \lim_{n\to\infty}\left(\frac{4}{3}\right)^{n+1}\left(\frac{3}{4}\right)^{n}$$

$$= \lim_{n\to\infty}(1)\ \frac{4}{3}$$

$= \frac{4}{3} > 1$ (divergen)

Uji deret ganti tanda

$$\frac{a_{n+1}}{a_n} = \frac{\left(\frac{4}{3}\right)^{n+1}}{\left(\frac{4}{3}\right)^{n}} = \frac{4}{3} > 1; (naik)$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(\frac{4}{3}\right)^n = \infty \neq 0$$

$\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^n$ divergen.