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1. $\cos\pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$

=> Rumus Eksplisit

$a_n = \frac{\cos n\pi}{n^2}$; $n = 1, 2, 3, \ldots$

=> Kekonvergenan

Teorema apit

$-1 \leq \cos n\pi \leq 1$

$-\frac{1}{n^2} \leq \frac{\cos n\pi}{n^2} \leq \frac{1}{n^2}$

$\lim_{n\to\infty} -\frac{1}{n^2} = 0$ & $\lim_{n\to\infty} \frac{1}{n^2} = 0$

sehingga $a_n = \frac{\cos n\pi}{n^2}$ juga konvergen menuju 0.

2. $\{a_n\}$ konvergen ke A

$\{b_n\}$ konvergen ke B

Buktikan $\{a_n + b_n\}$ konvergen ke A + B.

=> $\lim_{n\to\infty} a_n = A$

=> $\lim_{n\to\infty} b_n = B$

=> $\lim_{n\to\infty} (a_n + b_n) = A + B$

=> $|(a_n + b_n) - (A+B)| < \varepsilon$

$\{a_n\}$ konv. ke A, L = A

akan dibuktikan untuk tiap $\varepsilon > 0$ terdapat $N > 0$ shg $n \geq N$

=> $|a_n - L| < \varepsilon/2$

$|a_n - A| < \varepsilon/2$

$|(a_n + b_n) - (A+B)| \leq$

$|a_n - A| + |b_n - B| < \varepsilon/2 + \varepsilon/2 = \varepsilon$

(Terbukti)

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(3) $a_n = \frac{\sin n\pi}{4}$

$a_n - a_{n+1} = \frac{\sin n\pi}{4} - \frac{\sin (n+1)\pi}{4}$

$= \frac{\sin n\pi - (\sin(n+1)\pi)}{4}$

$\Rightarrow$ Tak naik tak turun

o> $\lim_{n \to \infty} \frac{\sin n\pi}{4}$ = tidak terdefinisi (Divergen)

o> $\{a_n\}$ tidak memiliki batas.

(4.) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

o> Rumus Eksplisit

$\left(\frac{1}{n}\right)(-1)^{n-1} = \frac{(-1)^{n-1}}{n}$

o> $\lim_{n \to \infty} (1/n)(-1)^{n-1} = \lim_{n \to \infty} \frac{n^{-1}}{(-1)^{1-n}} = 0$ (konvergen)

(5.) $a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$

$\lim_{n \to \infty} \frac{3-8\cdot 2^n}{5+4\cdot 2^n} = \lim_{n \to \infty} \frac{3-8\cdot 2^n}{5+4\cdot 2^n} \cdot \frac{1/2^n}{1/2^n} = \lim_{n \to \infty} \frac{3/2^n - (8\cdot 2^n/2^n)}{5/2^n + (4\cdot 2^n/2^n)}$

$= \frac{0-8}{0+4} = -2$ (konvergen)

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(6.) $a_n = \frac{\ln n}{n}$

∘) Kemonotonan

$a(x) = \frac{\ln x}{x}$

$a'(x) = \frac{1 - \ln x}{x^2}$

∘) $a'(x) < 0 \Rightarrow \frac{1 - \ln x}{x^2} < 0$

$\ln e < \ln x$

$e < x$ (TURUN)

∘) $a'(x) > 0 \Rightarrow \frac{1 - \ln x}{x^2} > 0$

$\ln e > \ln x$

$e > x$ (NAIK)

$\{a_n\}$ Tidak monoton.

∘ $\lim_{n \to \infty} \frac{\ln n}{n} \Rightarrow \infty/\infty$

$\overset{LH}{=} \lim_{n \to \infty} \frac{1/n}{1} = 0$ (konvergen)

(7.) 0,9 , 0.99, 0.999, . . .

∘) Rumus eksplisit

$a_n = 1 - \left(\frac{1}{10}\right)^n$

∘) $\lim_{n \to \infty} 1 - \left(\frac{1}{10}\right)^n = 1 - 0 = 1$ (konvergen)

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8. $a_n = \dfrac{n+3}{3n-2}$

$\lim\limits_{n \to \infty} \dfrac{n+3}{3n+2} \overset{LH}{=} \lim\limits_{n \to \infty} \dfrac{1}{3} = \dfrac{1}{3}$ (konvergen)

9. $a_n = \dfrac{n!}{10^n}$

o) Kemonotonan

$\dfrac{a_n}{a_{n+1}} = \dfrac{n!}{10^n} \times \dfrac{10^{n+1}}{(n+1)!}$

$= \dfrac{1}{1} \times \dfrac{10}{n+1}$

$= \dfrac{10}{n+1}$ (tak naik tak turun)

o) $\{a_n\}$ adalah barisan yang tidak monoton.

o) $\lim\limits_{n \to \infty} \dfrac{n!}{10^n} = \infty$ (Divergen)

o) $\{a_n\}$ tidak memiliki batas