Tugas Responsi Pertemuan 4
Kalkulus 2

Kelompok 7:

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Untuk soal no 1-4. Tentukan apakah deret ini konvergen atau divergen. Jika divergen, cari nilainya.

1.

$$\sum_{k=1}^{\infty} \left(\frac{1}{7}\right)^{k}$$

1. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{7}\right)^{k}$

$$f(x) = \left(\frac{1}{7}\right)^{x}$$

$$\int_{1}^{\infty} \left(\frac{1}{7}\right)^{x} dx$$

$$\lim_{a \to \infty} \left(\int_{1}^{a} \left(\frac{1}{7}\right)^{x} dx \right)$$

$$\lim_{a \to \infty} \left(-\frac{1}{\ln(7) \times 7^{a}} + \frac{1}{7\ln(7)} \right)$$

$$\frac{1}{7\ln(7)} \quad \text{(konvergen)} \; //.$$

2.

$$\sum_{k=1}^{\infty} \frac{k^2-5}{k+2}$$

2. $\sum_{k=1}^{\infty} \frac{k^2-5}{k+2}$

→ bagi pangkat tertinggi

→ menurut teorema

$\lim_{n\to\infty} \frac{k^2-5}{k+2} = \frac{1}{0} = \infty \neq 0$ maka divergen

3.

$$\sum_{k=1}^{\infty} \frac{2}{3k}$$

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3. $\sum_{k=1}^{\infty} \frac{2}{3k}$.

$= \frac{2}{3} \sum_{k=1}^{\infty} \frac{1}{k} \longrightarrow$ Deret Harmonik

※ Deret Harmonik $\Rightarrow$ Divergen.

$\sum \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ ⇝ hasilnya akan terus naik menuju ∞

Jadi, $\sum_{k=1}^{\infty} \frac{2}{3k}$ Divergen.

4.

$$\sum_{k=2}^{\infty}\left(\frac{1}{k}-\frac{1}{k-1}\right)$$

$$4.)\ \sum_{k=2}^{\infty}\left(\frac{1}{k}-\frac{1}{k-1}\right) = \left(-\frac{1}{2}\right)+\left(-\frac{1}{6}\right)+\left(-\frac{1}{12}\right)+\left(-\frac{1}{20}\right)+\dots$$

$$= \left(-\frac{1}{1\cdot 2}\right)+\left(-\frac{1}{2\cdot 3}\right)+\left(-\frac{1}{3\cdot 4}\right)+\left(-\frac{1}{4\cdot 5}\right)+\dots$$

$$= \sum_{n=1}^{\infty}\left(-\frac{1}{n(n+1)}\right)$$

$$\lim_{n\to\infty}\left(-\frac{1}{n(n+1)}\right) = 0 \quad \text{(konvergen)}$$

Untuk soal no 5-8. Gunakan uji integral untuk menentukan kekonvergenan atau kedivergenan deret berikut.

5.

$$\sum_{k=0}^{\infty} \frac{1}{k+3}$$

$$\sum_{k=0}^{\infty} \frac{1}{k+3}$$

$= \int_0^{\infty} \frac{1}{k+3}\,dx$, misal $u = k+3$, $du = dx$

$$= \lim_{b\to\infty} \int_0^{\infty} \frac{1}{k+3}\,dx$$

$$= \lim_{b\to\infty} \int_0^{\infty} \frac{1}{u}\,du$$

$$= \lim_{b\to\infty} (\ln u)\Big|_0^b = \lim_{b\to\infty} (\ln(b+3) + \ln(0+3))$$

$$= \lim_{b\to\infty} (\ln(b+3) - \ln(3))$$

$= \infty$ (Divergen)

6.

$$\sum_{k=1}^{\infty} \frac{3}{2k-3}$$

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$6.\ \sum_{k=1}^{\infty} \frac{3}{2k-3}$

misalkan $a_k = f(k) = \frac{3}{2k-3}$.

$$\int_1^{\infty} \frac{3}{2k-3}\,dk = \lim_{a\to\infty} \int_1^{a} \frac{3}{2k-3}\,dk$$

$$= \lim_{a\to\infty} \frac{3}{2} \cdot \ln(2k-3) \Big|_1^a$$

$$= \lim_{a\to\infty} \left(\frac{3}{2} \cdot \ln(2a-3) - \frac{3}{2} \cdot \ln(-1) \right)$$

= divergen.

karena $\int_1^{\infty} \frac{3}{2k-3}\,dk$ divergen, maka deret $\sum_{k=1}^{\infty} \frac{3}{2k-3}$ divergen

7.

$$\sum_{k=0}^{\infty} \frac{k}{k^2+3}$$

7. $\displaystyle\sum_{k=0}^{\infty} \frac{k}{k^2+3}$

$$= \int_0^{\infty} \frac{k}{k^2+3}\, dx$$

$$= \lim_{b \to \infty} \int_0^{b} \frac{k}{k^2+3}\, dx$$

misal:

$\Rightarrow u = k^2+3$

$\frac{du}{dx} = 2k$

$k \cdot dx = \frac{du}{2}$

$$= \lim_{b \to \infty} \int_3^{b} \frac{1}{2u}\, du$$

$$= \lim_{b \to \infty} \frac{1}{2} \ln u \Big|_3^b$$

$$= \lim_{b \to \infty} \frac{1}{2} \ln(k^2+3) \Big|_0^b$$

$$= \lim_{b \to \infty} \frac{1}{2} \left(\ln(b^2+3) - \ln 3 \right)$$

$$= \infty$$

Divergen //

8.

$$\sum_{k=0}^{\infty} \frac{3}{2k^2+1}$$

$$8. \sum_{K=1}^{\infty} \frac{3}{2K^2+1}$$

$$\int_1^{\infty} \frac{3}{2K^2+1}\,dx = \lim_{b\to\infty} \int_1^b \frac{3}{2K^2+1}$$

$$= \lim_{b\to\infty} \left. \frac{3\sqrt{2}\tan^{-1}\sqrt{2}K}{2} \right|_1^b$$

$$= \lim_{b\to\infty} \frac{3\sqrt{2}\tan^{-1}\sqrt{2}b}{2} - \frac{3\sqrt{2}\tan^{-1}\sqrt{2}(1)}{2}$$

$$= \frac{3\sqrt{2}\pi - 6\sqrt{2}\tan^{-1}\sqrt{2}}{4} \quad \text{(Konvergen)}$$