TUGAS KELOMPOK

MINGGU 9

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IPB UNIVERSITY

DEPARTEMEN STATISTIKA 2022

1) Tentukan titik puncak, fokus, & direktris dari parabola berikut, serta gambar grafiknya!

a. $(x+2)^2 = 8(y-1)$

- $h = -2$ · $p = 2$ · $K = 1$

→ fokus = $(h, p+K)$

$= (-2, 2+1)$

$= (-2, 3)$

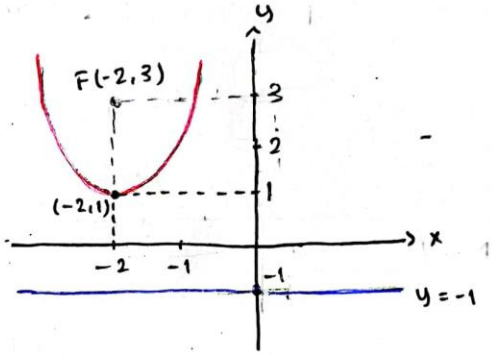
→ direktris = $y = K - p$

$= 1 - 2$

$= -1$

→ puncak = (h, K)

$= (-2, 1)$



b. $4x^2 + 16x - 16y + 32 = 0$

$4(x^2 + 4x) = 16y - 32$

$4(x^2 + 4x) + 16 = 16y - 32 + 16$

$4(x^2 + 4x + 4) = 16y - 16$

$4(x+2)^2 = 16(y-1)$

$(x+2)^2 = 4(y-1)$

- $h = -2$ · $p = 1$ · $K = 1$

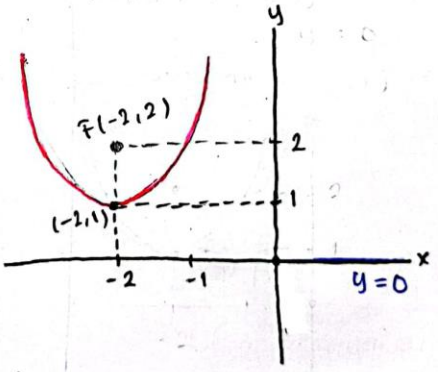
→ fokus = $(h, p+K)$

$= (-2, 1+1) = (-2, 2)$

→ direktris = $y = K - p$

$= 1 - 1 = 0$

→ puncak = $(h, K) = (-2, 1)$



2.a $\frac{(x+3)^2}{4} + \frac{(y+2)^2}{16} = 1$

$\frac{(x+3)^2}{2^2} + \frac{(y+2)^2}{4^2} = 1$

$a = 4 \quad c = \sqrt{16-4} = \sqrt{12}$

$b = 2 \quad c = \sqrt{12}$

* Puncak

$(h, k \pm a)$

$(-3, -2 \pm 4)$

$(-3, 2)$ & $(-3, -6)$

* Fokus

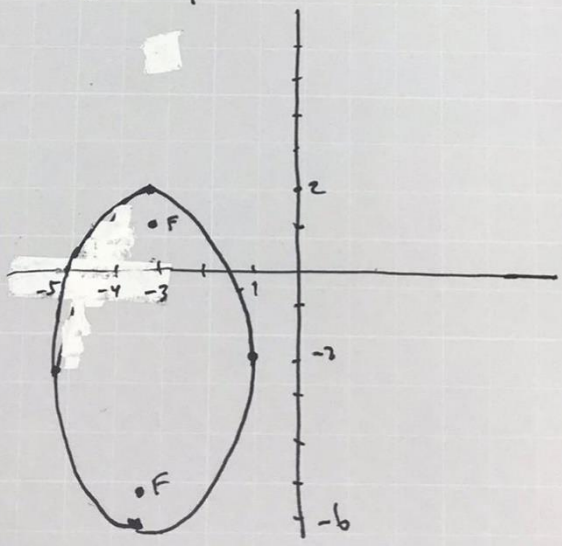
$(h, k \pm c)$

$(-3, -2 \pm \sqrt{12})$

$(-3, -2 + \sqrt{12})$ & $(-3, -2 - \sqrt{12})$

* eksentrikan

$e = \frac{c}{a} = \frac{\sqrt{12}}{4} = \frac{2\sqrt{3}}{4} = \frac{1}{2}\sqrt{3}$



2.b. $x^2 + 4y^2 - 2x + 16y + 1 = 0$

$x^2 - 2x + 4y^2 + 16y + 1 = 0$

$(x-1)^2 - 1 + 4((y+2)^2 - 4) + 1 = 0$

$(x-1)^2 + 4(y+2)^2 = 16$

$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{4} = 1 \rightarrow \frac{(x-1)^2}{4^2} + \frac{(y+2)^2}{2^2} = 1$

$a = 4$

$b = 2$

$c = \sqrt{16 - 4}$

$= \sqrt{12}$

* puncak $(h \pm a, k)$

$(1 \pm 4, -2)$

$(5, -2)$ & $(-3, -2)$

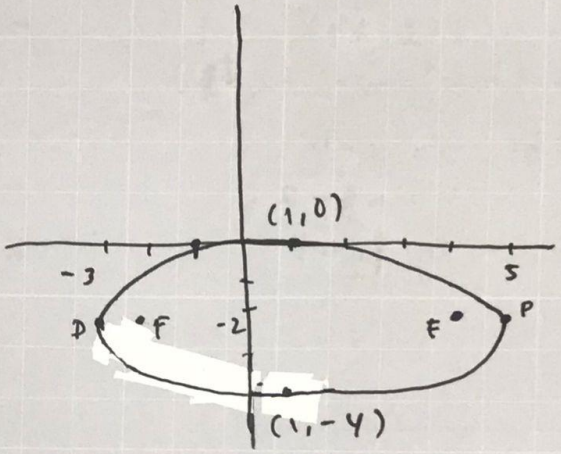
* fokus $(h + c, k)$

$(1 \pm \sqrt{12}, -2)$

$(1 + \sqrt{12}, -2)$ & $(1 - \sqrt{12}, -2)$

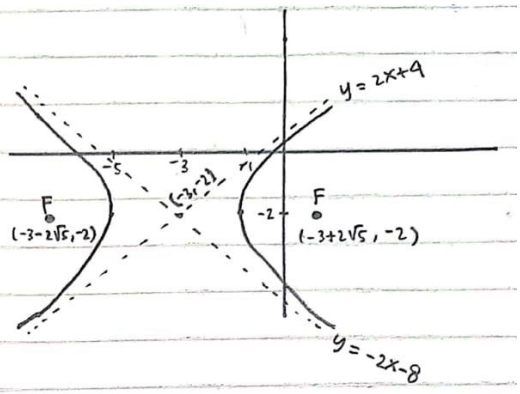
* eksentrikan

$e = \frac{c}{a} = \frac{\sqrt{12}}{4} = \frac{2\sqrt{3}}{4} = \frac{1}{2}\sqrt{3}$



3. Tentukan titik puncak, fokus, dan garis asimtot hiperbola berikut, serta gambarkan grafiknya.

a. $\frac{(x+3)^2}{9} - \frac{(y+2)^2}{16} = 1$



$h = -3, k = -2, a = 2, b = 4$

$c^2 = a^2 + b^2$

$= 4 + 16 = 20$

$c = \sqrt{20} = 2\sqrt{5}$

* Titik puncak : $(-3 \pm 2, -2)$

: $(-5, -2)$ dan $(-1, -2)$

* Titik fokus : $(-3 \pm 2\sqrt{5}, -2)$

: $(-3-2\sqrt{5}, -2)$ dan $(-3+2\sqrt{5}, -2)$

* Garis asimtot : $y+2 = \pm (4/2)(x+3)$

$y+2 = \pm 2(x+3)$

$y+2 = \pm 2x \pm 6$

$y = \pm 2x \pm 6 - 2$

$y = -2x - 8$ dan $y = 2x + 4$

③ b) $9x^2 - 16y^2 + 54x + 64y - 127 = 0$

$9x^2 + 54x - 16y^2 + 64y = 127$

$9(x^2 + 6x) - 16(y^2 - 4y) = 127$

$9(x+3)^2 - 81 - 16(y-2)^2 + 64 = 127$

$9(x+3)^2 - 16(y-2)^2 = 144$

$\frac{(x+3)}{16} - \frac{(y-2)^2}{9} = 1$

$a = 4 \quad b = 3 \quad c = \sqrt{16+9} = 5$

$h = -3 \quad K = 2$

Titik puncak : $(h \pm a, K)$

$(1, 2)$ & $(-7, 2)$

Fokus : $(h \pm c, K)$

$(2, 2)$ & $(-8, 2)$

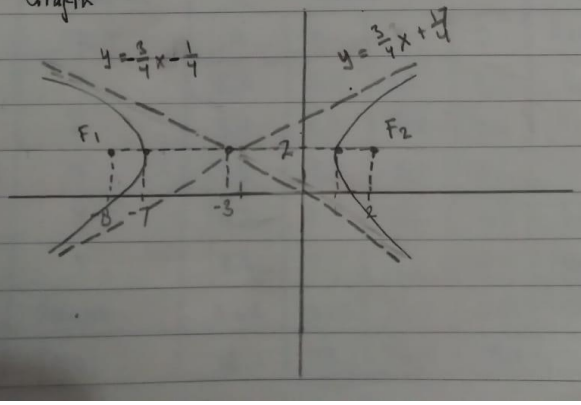
asimtot : $y - K = \pm (\frac{b}{a})(x-h)$

$y - 2 = \pm (\frac{3}{4})(x+3)$

$y = \frac{3}{4}x + \frac{17}{4}$

$y = -\frac{3}{4}x + \frac{1}{4}$

Grafik



4) a. Persamaan parabola dengan puncak di (2,3) dan fokus di (2,5)

Puncak = (h,k)
Puncak = (2,3)
h = 2
k = 3

Fokus = (h, p+k)
Fokus = (2,5)
h = 2
p+k = 5 → p = 2

Persamaan Parabola:
$(x-h)^2 = 4p(y-k)$
$(x-2)^2 = 4.2(y-3)$
$(x-2)^2 = 8(y-3)$

NO. :

4. (b). Tentukan persamaan parabola dengan sumbu parabola vertikal, serta melalui titik (-2,3), (0,3), (1,9)

Jawab

Jika sumbu parabola vertikal, persamaan parabolanya
$(x-h)^2 = 4p(y-k)$

■ Titik (-2,3)
$(-2-h)^2 = 4p(3-k)$
$h^2+4h+4 = 12p-4pk$ ...(i)

■ Titik (0,3)
$(0-h)^2 = 4p(3-k)$
$h^2 = 12p-4pk$ (ii)

■ Titik (1,9) ⇒ $h=-1$
$(1-h)^2 = 4p(9-k)$
$4 = 36p-4pk$ (iii)

* substitusi pers i ke pers ii
$h^2+4h+4 = h^2$
$4h = -4$
$h = -1$

* substitusi $h=-1$ ke pers ii
$1 = 12p-4pk$
$4pk = 12p-1$ .... (iv)

* Substitusi Pers iv ke pers iii
$4 = 36p-12p+1$
$3 = 24p$
$p = \frac{1}{8}$

* Substitusi $p = 1/8$ ke pers iv
$4.\frac{1}{8}.k = 12.\frac{1}{8} - 1$
$\frac{1}{2}k = \frac{3}{2} - 1$
$1/2k = 1/2$
$k = 1$

∴ Maka persamaan parabolanya
$(x+1)^2 = 4.\frac{1}{8}(y-1)$
$(x+1)^2 = \frac{1}{2}(y-1)$

9c. Elips dengan fokus $(\pm 2, 2)$ dan melalui titik asal.

fokus $(h \pm c, k)$

$K = 2$ $\quad h + c = 2$

$h - c = -2$ (−)

$2c = 4$

$c = 2,\ h = 0$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \rightarrow \frac{(x-0)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$

Karena melewati $(0,0) \rightarrow \frac{0}{a^2} + \frac{(0-2)^2}{b^2} = 1$

$\hookrightarrow b^2 = 4$

$a^2 = b^2 + c^2$

$= 4 + 4 = 8$

$\rightarrow \frac{x^2}{8} + \frac{(y-2)^2}{4} = 1$

4. Tentukan persamaan insan kerucut berikut:

(a). Hiperbola dengan puncak di $(0,0)$ dan $(0,6)$ dan dengan sebuah fokus di $(0,8)$

Jawab: titik puncak = $(0,0)$ dan $(0,6)$

$(h, k \pm a) = (0,0)$ dan $(0,6) \rightarrow h = 0$

* $k + a = 6$

$k - a = 0$ (+)

$2k = 6$

$k = 3$ //

* $k + a = 6$

$3 + a = 6$

$a = 3$ //

titik fokus = $(h, k \pm c) \rightarrow (0,8)$

* $k + c = 8$

$3 + c = 8$

$c = 5$ //

$c^2 = a^2 + b^2$

$5^2 = 3^2 + b^2$

$25 = 9 + b^2$

$b^2 = 16$

Pers. Hiperbola:

$\frac{(y-3)^2}{9} - \frac{x^2}{16} = 1 \rightarrow \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$