No. :

# Tugas ~~Kelompok~~ Mandiri

(a) Tulis rumus eksplisit berikut dan tentukan kekonvergenannya.

~~Jawab~~: $\cos \pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$

Jawab:

$a_n = \frac{\cos(n\pi)}{n^2}$ $\qquad \lim_{n\to\infty} \left(\frac{-1}{n^2}\right) = 0 \; ; \; \lim_{n\to\infty} \left(\frac{1}{n^2}\right) = 0$

Cek kekonvergenan:

$-1 \leq \cos(n\pi) \leq 1$

$\frac{-1}{n^2} \leq \frac{\cos(n\pi)}{n^2} \leq \frac{1}{n^2}$

Dengan menggunakan teorema apit, deret $\frac{\cos(n\pi)}{n^2}$ konvergen ke 0.

(b). Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B.
Buktikan (dengan definisi limit) $\{a_n + b_n\}$ konvergen ke A+B

Jawab:

$\{a_n\}$ konvergen ke A, artinya : $\lim_{n\to\infty} a_n = A$

$\{b_n\}$ konvergen ke B, artinya : $\lim_{n\to\infty} b_n = B$

$a_n + b_n = \lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n = A + B$ (Terbukti)

(c). Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut $a_n = \sin\left(\frac{n\pi}{4}\right)$

Jawab:

$a_n - a_{n+1} = \sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{(n+1)\pi}{4}\right) \rightarrow$ Tidak naik dan tidak turun

Batas:

$-1 \leq a_n \leq 1$

$\lim_{n\to\infty} \sin\left(\frac{n\pi}{4}\right) =$ Tidak ada (Divergen)

182 mm X 257 mm

KIKY

No. :

② $|a_n - A| < \varepsilon$

$|b_n - B| < \varepsilon$

$|a_n - A + b_n - B| \leq |a_n - A| + |b_n - B|$

$|a_n - A| + |b_n - B| < \varepsilon + \varepsilon$

$|a_n - A| + |b_n - B| < 2\varepsilon$

$|a_n - A| + |b_n - B| < \varepsilon$ (terbukti

Karena

$\varepsilon > 0$

$2\varepsilon > 0$

(a) Tulis rumus eksplisit barisan berikut, dan tentukan kekonvergenannya

$$1, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{4}, \frac{1}{5}, \frac{-1}{6}, \ldots$$

Jawab:

$a_n = (-1)^{n+1} \cdot \frac{1}{n}$ Jika $\lim_{n\to\infty} |a_n| = 0 \to \lim_{n\to\infty} a_n = 0$

$\lim_{n\to\infty} \left|(-1)^{n+1} \cdot \frac{1}{n}\right| = \lim_{n\to\infty} |(-1)^{n+1}| \left|\frac{1}{n}\right| = \lim_{n\to\infty} \frac{1}{n} = 0$ (konvergen ke 0)

(b) Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen

$$a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$$

Jawab:

$$\lim_{n\to\infty} \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$$

$$= \lim_{n\to\infty} \frac{3 - 2^3 \cdot 2^n}{5 + 2^2 \cdot 2^n}$$

$$= \lim_{n\to\infty} \frac{3 - 2^{n+3}}{5 + 2^{n+2}}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{0 - \ln(2) \cdot 2^{n+3}}{0 + \ln(2) \cdot 2^{n+2}}$$

$$= \lim_{n\to\infty} \frac{-2^{n+3}}{2^{n+2}}$$

$$= \lim_{n\to\infty} -2 = -2 \;//$$

$$|a_n - L| < \varepsilon$$

$$\left|\frac{3 - 2^{n+3}}{5 + 2^{n+2}} + 2\right| < \varepsilon$$

$$\left|\frac{3 - 2^{n+3} + 10 + 2^{n+3}}{5 + 2^{n+2}}\right| < \varepsilon$$

$$\frac{13}{5 + 2^{n+2}} < \varepsilon$$

$$\boxed{\frac{\frac{13}{\varepsilon} - 5}{4} < 2^n}$$

$$\frac{13}{5 + 2^2\left(\frac{13}{\varepsilon} - 5\right)} = \varepsilon \quad \text{(Terbukti)}$$

No :

(c). Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut $a_n = \frac{\ln n}{n}$

Jawab:

$$a_n - a_{n+1} = \frac{\ln n}{n} - \frac{\ln(n+1)}{n+1}$$

$$= \frac{\ln n}{n} - \frac{\ln n \cdot \ln 1}{n+1}$$

$$= \frac{\ln n}{n} - \frac{\ln n \cdot 0}{n+1}$$

$$\rightarrow = \frac{\ln n}{n} - 0 = \frac{\ln n}{n}$$

$$\geq 0$$

Barisan tak naik

$$\lim_{n \to \infty} \frac{\ln n}{n} \rightarrow \frac{\infty}{\infty}$$

$$\stackrel{LH}{=} \lim_{n \to \infty} \frac{\frac{1}{n}}{1}$$

$$\stackrel{LH}{=} \lim_{n \to \infty} \frac{1}{n}$$

$= 0 \rightarrow$ konvergen ke 0

Keterbatasan :

$a_n \geq 0$

No. :

(a). Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya

0.9, 0.99, 0.999, 0.9999, ...

Jawab :

$a_n = 1 - \left(\frac{1}{10}\right)^n$

$\lim_{n \to \infty} \left(1 - \left(\frac{1}{10}\right)^n\right)$

$= \lim_{n \to \infty} \left(1 - \frac{1}{10^n}\right)$

$\to = 1 - 0$

$= 1$

Baris tersebut konvergen ke 1

(b). Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen

$a_n = \frac{n+3}{3n-2}$

Jawab :

Jawab :

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \left(\frac{n+3}{3n-2}\right) = \boxed{\frac{1}{3}} \to L$

$\left| a_n - \frac{1}{3} \right| < \varepsilon$

$\left| \frac{n+3}{3n-2} - \frac{1}{3} \right| < \varepsilon$

$\left| \frac{3n+9-3n+2}{9n-6} \right| < \varepsilon$

$\left| \frac{11}{9n-6} \right| < \varepsilon$

$\to \frac{11}{9n-6} < \varepsilon$

$11 < \varepsilon(9n-6)$

$\frac{11}{\varepsilon} < (9n-6)$

$9n > \frac{11}{\varepsilon} + 6$

$n > \frac{11}{9\varepsilon} + \frac{2}{3}$

$\to$ Pilih $N$ sehingga

$N = \left\lceil \frac{11}{9\varepsilon} + \frac{2}{3} \right\rceil \in \mathbb{N}$

No. :

Sebut saja $\varepsilon > 0$

N kita definisikan sebagai : $N = \left\lceil \frac{11}{9\varepsilon} + \frac{2}{3} \right\rceil$

Sebut saja $n \geq N$

Maka :

$$|S_n - L| = \left| \frac{n+3}{3n-2} - \frac{1}{3} \right|$$

Karena $n \geq N > \frac{11}{9\varepsilon} + \frac{2}{3}$

$$= \frac{11}{9n-6} < \frac{11}{9\left(\frac{11}{9\varepsilon} + \frac{2}{3}\right) - 6}$$

$$\frac{11}{9n-6} < \frac{11}{\frac{11}{\varepsilon}} =$$

$$\boxed{\frac{11}{9n-6} < \varepsilon}$$ Terbukti

No. :

(c). Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan berikut $a_n = \frac{n!}{10^n}$

Jawab :

$\frac{a_n}{a_{n+1}} \leq 1$

$\frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{n+1}}} \leq 1$

$\frac{n!}{n+1!} \cdot \frac{10^{n+1}}{10^n} \leq 1$

$\frac{n!}{(n+1)n!} \cdot \frac{10 \cdot 10^n}{10^n} \leq 1$

$\frac{10}{n+1} \leq 1$

$10 \leq n+1$

$\boxed{n \geq 9 \to \text{Barisan tidak monoton}}$

untuk $0 < n < 9$, barisan $a_n$ turun . Pada $n \geq 9$, barisan $a_n$ tidak turun

$\lim_{n \to \infty} \frac{n!}{10^n} = +\infty$ (Divergen)

180 mm x 257 mm

KIKY