



X

Kalkulus 2 - Responsi 5

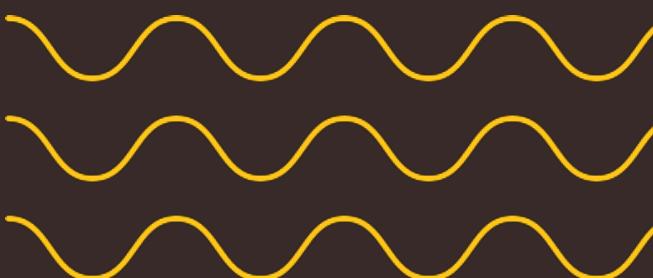


+

# UJI KEKONVERGENAN DERET TAKHINGGA

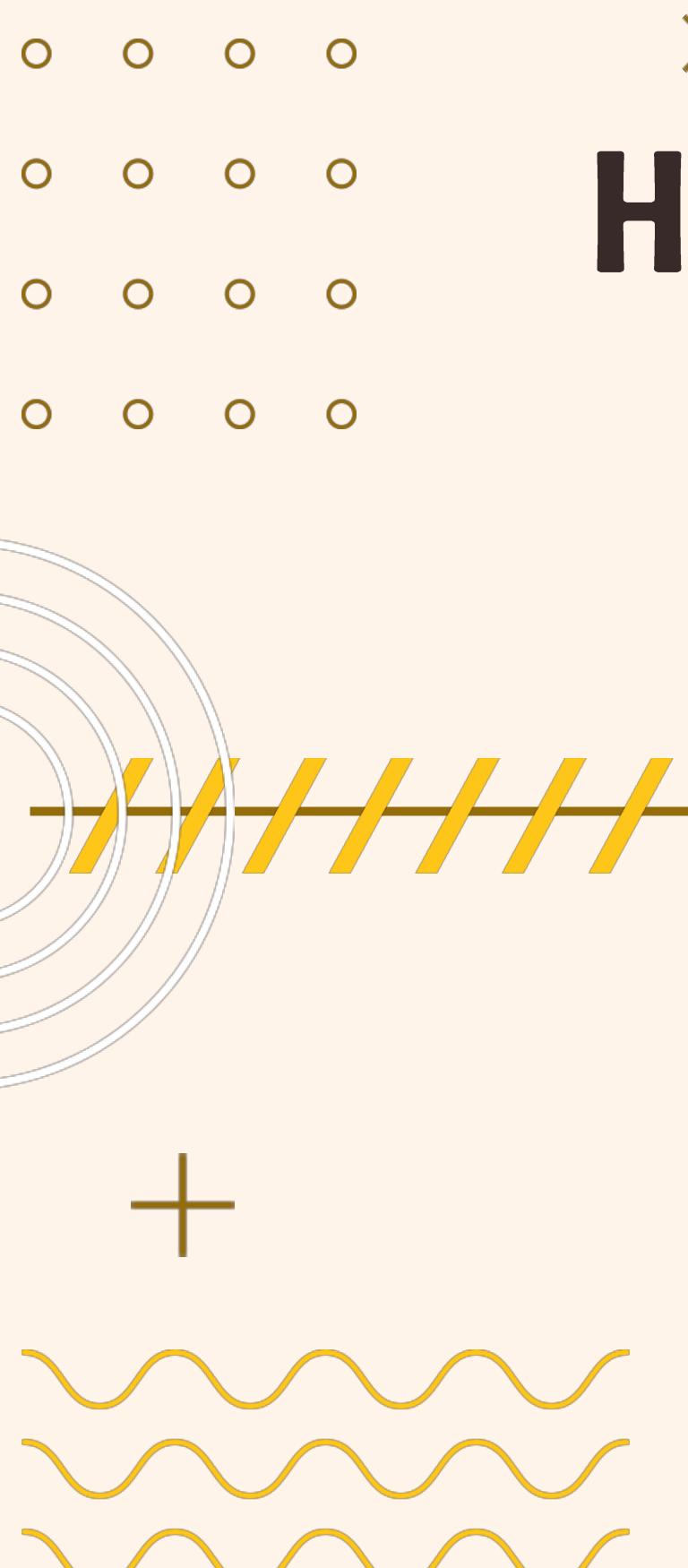


Wulan dan Yeremia

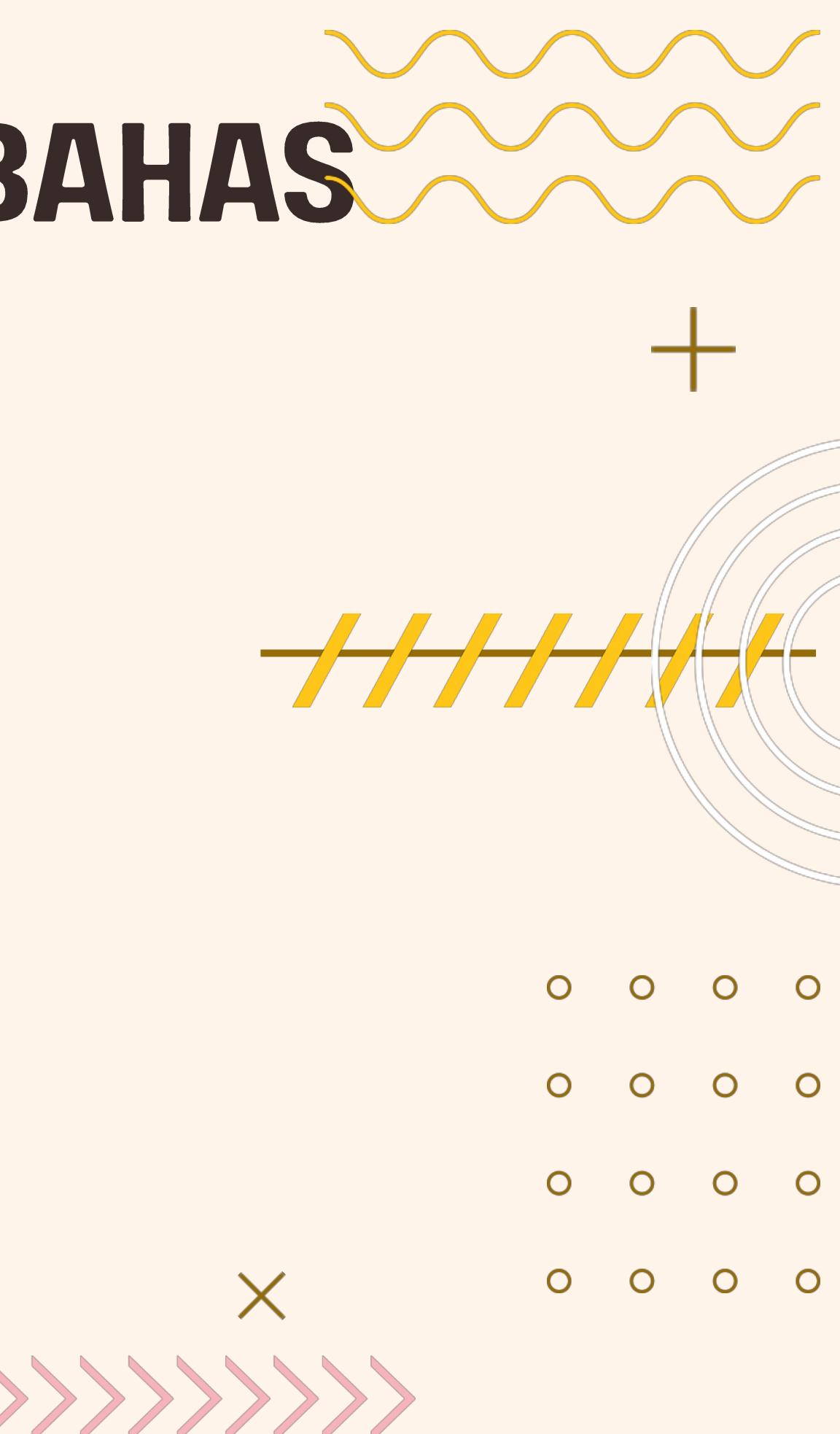


+

X



# HAL YANG AKAN DIBAHAS



- Uji Perbandingan
- Deret Berganti Tanda
- Kekonvergenan Mutlak
- Uji Rasio
- Uji Akar



# Deret Positif

## 1. UJI BANDING

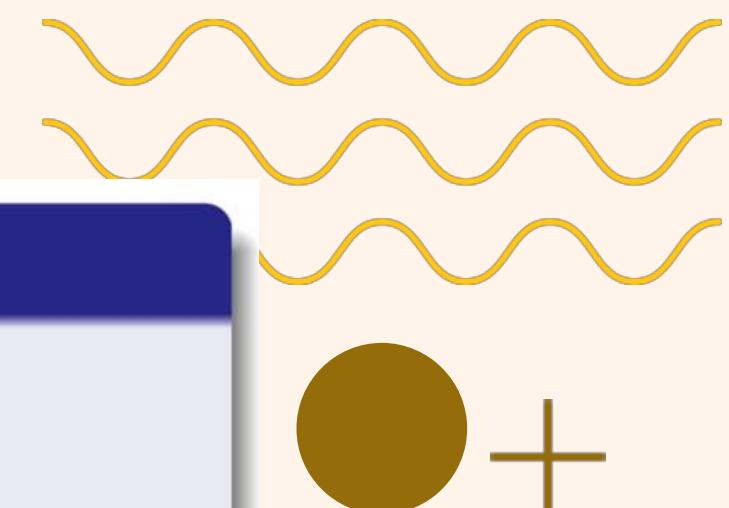
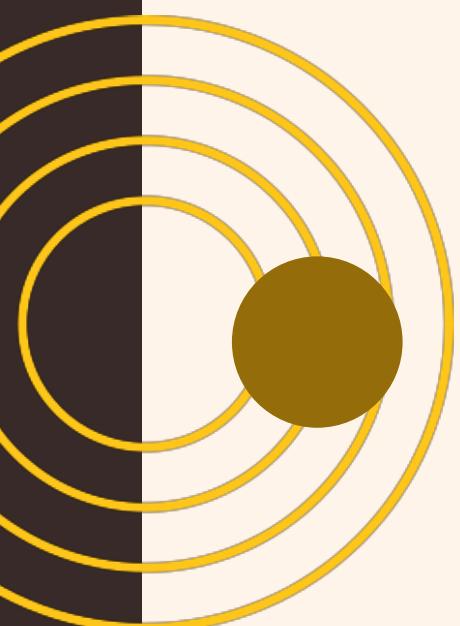
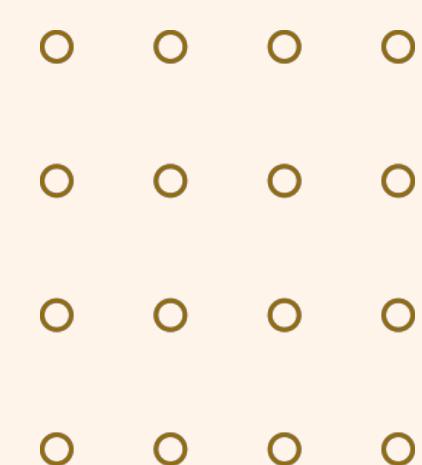
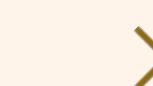
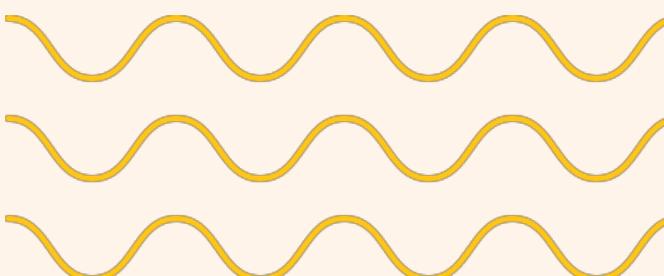
### Teorema (Uji banding)

Misalkan untuk  $n \geq N$  berlaku  $0 \leq a_n \leq b_n$ .

- 1 Jika  $\sum_{n=1}^{\infty} b_n$  konvergen maka  $\sum_{n=1}^{\infty} a_n$  konvergen.
- 2 Jika  $\sum_{n=1}^{\infty} a_n$  divergen maka  $\sum_{n=1}^{\infty} b_n$  divergen.

$a_n \leq b_n$  untuk  $n \geq N$  :  $\sum a_n$  deret kecil,  $\sum b_n$  deret besar.

- Jika deret yang besar konvergen, maka deret yang kecil juga konvergen
- Jika deret yang kecil divergen, maka deret yang besar juga divergen



## Langkah awal: Menentukan deret pembanding

$$1. \sum_{k=1}^{\infty} \frac{k}{4k^2 - 5}$$

$$\frac{k}{4k^2} \leq \frac{k}{4k^2 - 5} \leq \dots$$

$$\frac{1}{4k} \leq \frac{k}{4k^2 - 5}$$

$$\sum_{k=1}^{\infty} \frac{1}{4k}$$

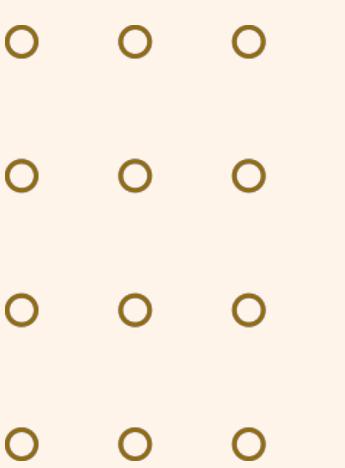
konvergen

$$2. \sum_{k=1}^{\infty} \frac{1}{2+\sqrt{k}}$$

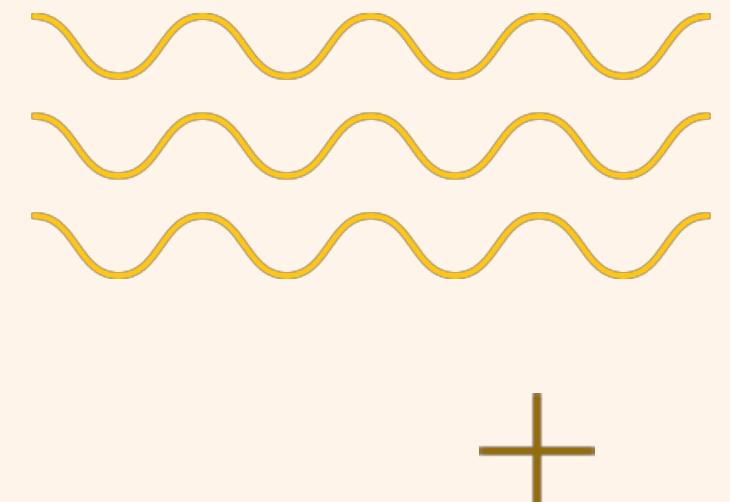
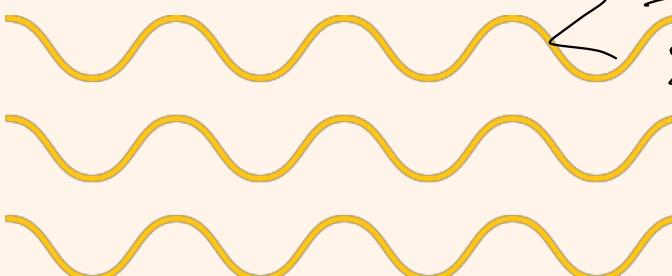
$$\frac{1}{2+\sqrt{k}} \leq \frac{1}{\sqrt{k}}$$

$$\sum_{k=1}^{\infty} \frac{1}{2+\sqrt{k}} = \sum_{k=1}^{\infty} \frac{1}{k^{1/2}}$$

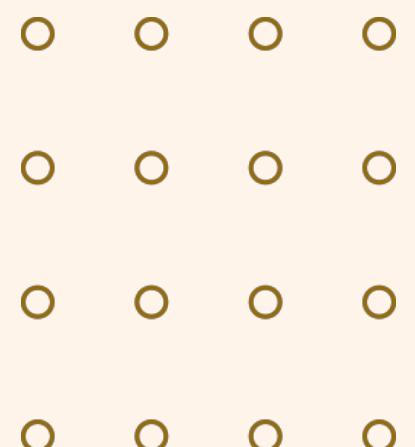
$p = \frac{1}{2} < 1$  (divergen)



+



+



×

## 2. UJI BANDING LIMIT

- Uji banding limit biasanya bekerja dengan baik dalam membandingkan deret aljabar dengan deret-p. **Pilih deret pembanding dengan derajat sama.**
- ...
- ...
- ...



### Teorema (Uji banding limit)

Misalkan  $a_n \geq 0, b_n \geq 0$  dan

$$\lim_{n \rightarrow \infty} \left( \frac{a_n}{b_n} \right) = L.$$

1 Jika  $0 < L < \infty$ , maka  $\sum_{n=1}^{\infty} a_n$  dan  $\sum_{n=1}^{\infty} b_n$  bersama-sama konvergen

atau bersama-sama divergen.

2 Jika  $L = 0$  dan  $\sum_{n=1}^{\infty} b_n$  konvergen maka  $\sum_{n=1}^{\infty} a_n$  konvergen.

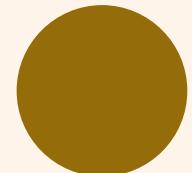
- ...
- ...
- ...
- ...



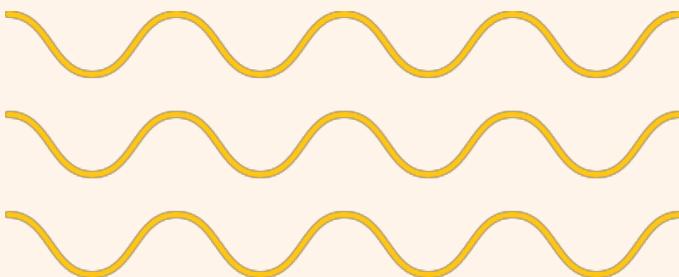
## Contoh:

$$1. \sum_{k=1}^{\infty} \frac{k}{\sqrt{k^3 + 3k^2 - 5}} \Leftrightarrow \frac{k}{k^{3/2}} \Leftrightarrow \frac{1}{k^{1/2}} \xrightarrow{b_k} \text{divergen}$$

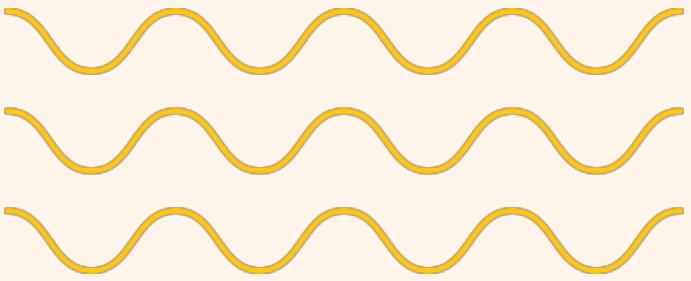
$$\begin{aligned} L &= \lim_{k \rightarrow \infty} \frac{a_k}{b_k} \\ &= \lim_{k \rightarrow \infty} \frac{k}{\sqrt{k^3 + 3k^2 - 5}} \cdot \frac{k^{1/2}}{1} \\ &= \lim_{k \rightarrow \infty} \sqrt{\frac{k}{k^3 + 3k^2 - 5}} \\ &= 1 > 0 \end{aligned} \quad \text{maka, } \sum a_k \text{ divergen.}$$



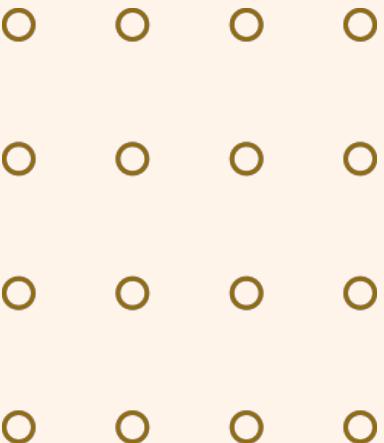
+

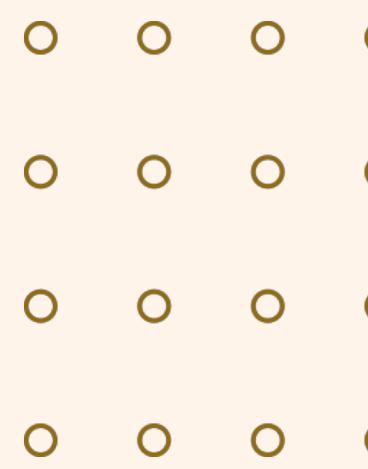


×



+





**Contoh:**

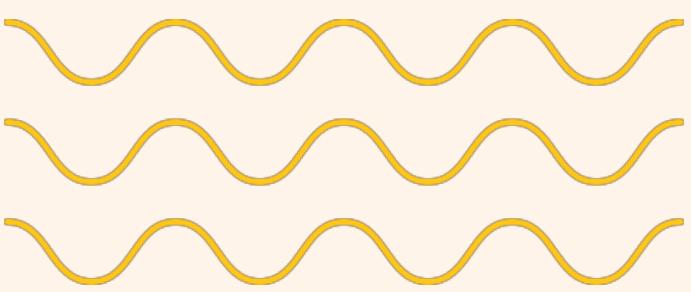
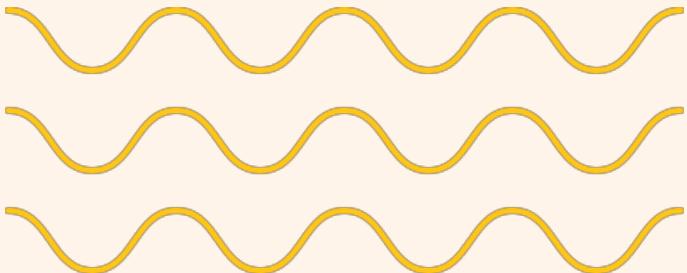
$$\sum_{k=1}^{\infty} \frac{k}{4k^2 + 5} \quad \text{p. } a_k$$

$\Leftrightarrow \frac{k}{4k^2} \Leftrightarrow \frac{1}{k}$  berdivergen

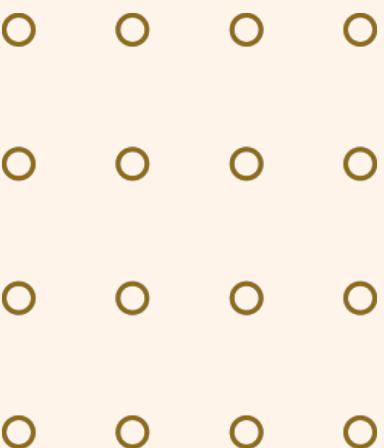
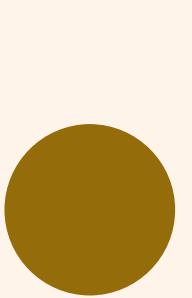
$$\begin{aligned}\lim_{k \rightarrow \infty} \frac{a_k}{b_k} &= \lim_{k \rightarrow \infty} \frac{k}{4k^2 + 5} \cdot \frac{k}{1} \\ &= \lim_{k \rightarrow \infty} \frac{k^2}{4k^2 + 5} \\ &= \frac{1}{4} > 0 \\ L &= \frac{1}{4}\end{aligned}$$



+



+



×

### 3. UJI HASIL BAGI (RASIO)



X

Uji rasio biasanya digunakan untuk memeriksa kekonvergenan deret yang mengandung suku  $k!$  atau  $r^k$ .

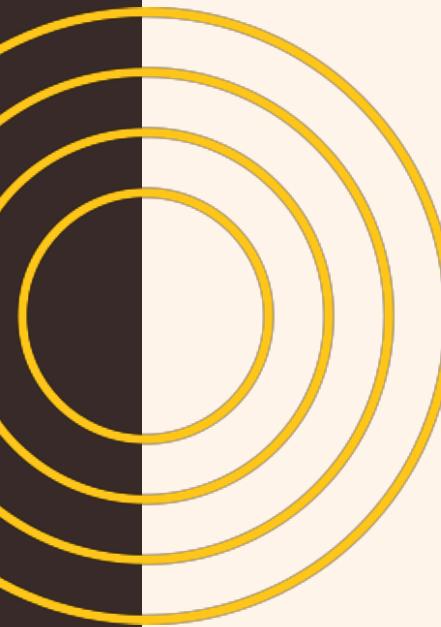
+

#### Teorema (Uji hasil bagi)

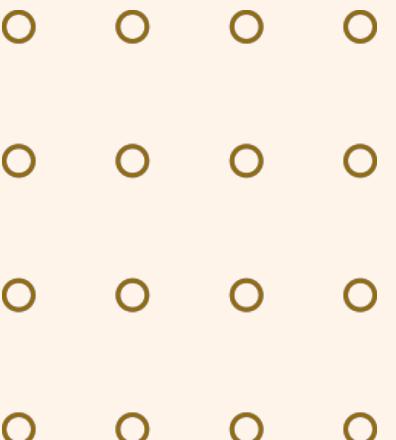
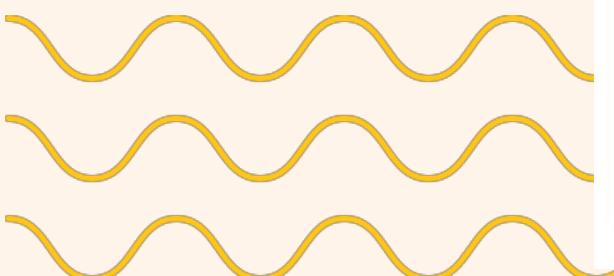
Misalkan  $\sum_{n=1}^{\infty} a_n$  adalah deret yang suku-sukunya positif dan misalkan pula

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \rho.$$

- 1 Jika  $\rho < 1$ , maka  $\sum_{n=1}^{\infty} a_n$  konvergen.
- 2 Jika  $\rho > 1$ , maka  $\sum_{n=1}^{\infty} a_n$  divergen.
- 3 Jika  $\rho = 1$ , maka uji ini tidak memberi kesimpulan (diperlukan uji lainnya).



+



+

+

X

## Contoh:

1.  $\sum_{k=1}^{\infty} \frac{5^k}{k!}$  positif

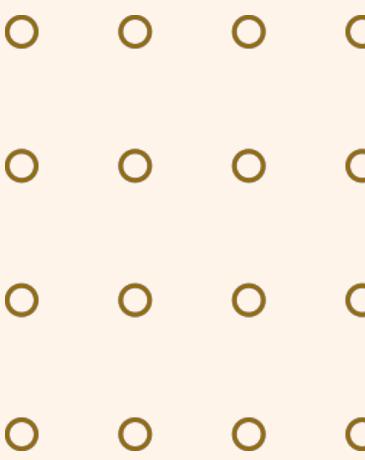
$$\begin{aligned} l &= \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} \\ &= \lim_{k \rightarrow \infty} \frac{5^{k+1}}{(k+1)!} \cdot \frac{k!}{5^k} \\ &= \lim_{k \rightarrow \infty} \frac{5}{k+1} = 0 \end{aligned}$$

$\hookrightarrow l = 0 < 1$ , konvergen

2.  $\sum_{k=1}^{\infty} \frac{k!}{10^k}$

$$\begin{aligned} l &= \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} \\ &= \lim_{k \rightarrow \infty} \frac{(k+1)!}{10^{k+1}} \cdot \frac{10^k}{k!} \\ &= \lim_{k \rightarrow \infty} \frac{k+1}{10} \\ &= \infty > 1 \end{aligned}$$

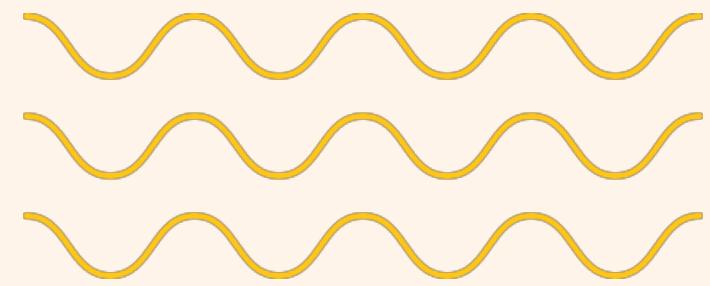
maka, divergen



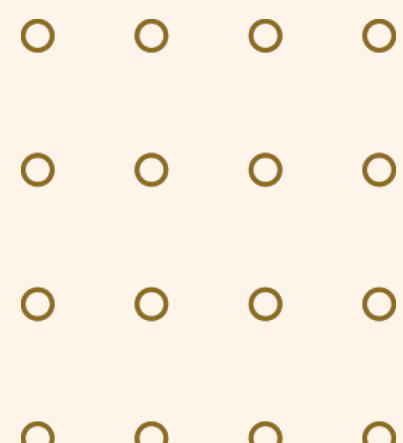
+



×



+



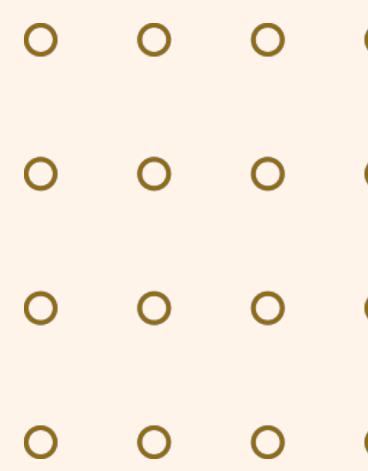
## 4. UJI AKAR (ROOT TEST)

### Teorema (Uji akar)

Misalkan  $\sum_{n=1}^{\infty} a_n$  adalah deret yang suku-sukunya positif dan misalkan pula

$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = R.$$

- 1 Jika  $R < 1$ , maka  $\sum_{n=1}^{\infty} a_n$  konvergen.
- 2 Jika  $R > 1$ , maka  $\sum_{n=1}^{\infty} a_n$  divergen.
- 3 Jika  $R = 1$ , maka uji ini tidak memberi kesimpulan (diperlukan uji lainnya).

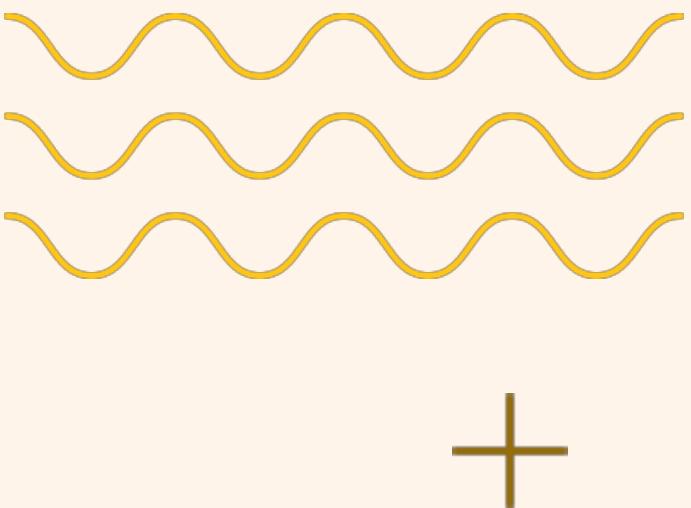


Contoh:

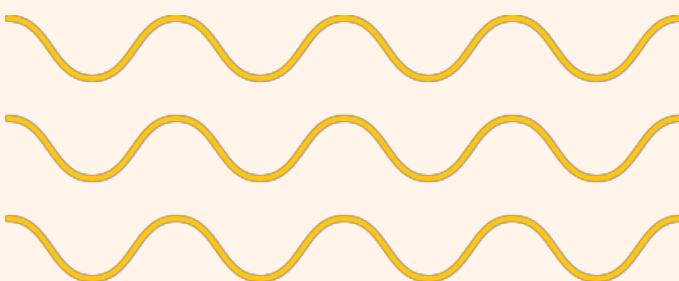
$$1. \sum_{k=1}^{\infty} \left( \frac{1}{2} + \frac{k^2}{k^2+1} \right)^k a_k$$

$$\begin{aligned} R &= \lim_{k \rightarrow \infty} (a_k)^{\frac{1}{k}} \\ &= \lim_{k \rightarrow \infty} \left( \frac{1}{2} + \frac{k^2}{k^2+1} \right) \end{aligned}$$

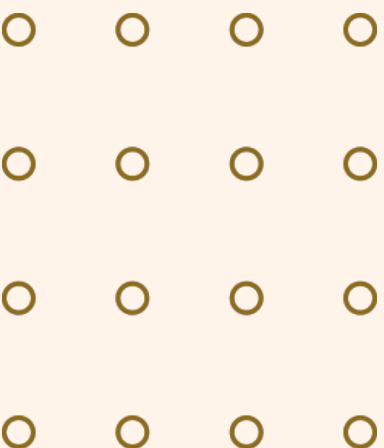
$$R = \frac{3}{2} > 1 \text{ maka } \sum a_k \text{ divergen}$$



$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$$



×



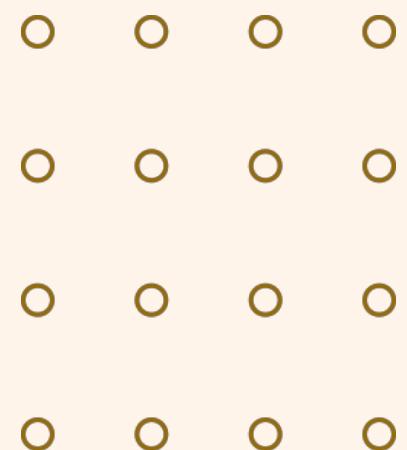
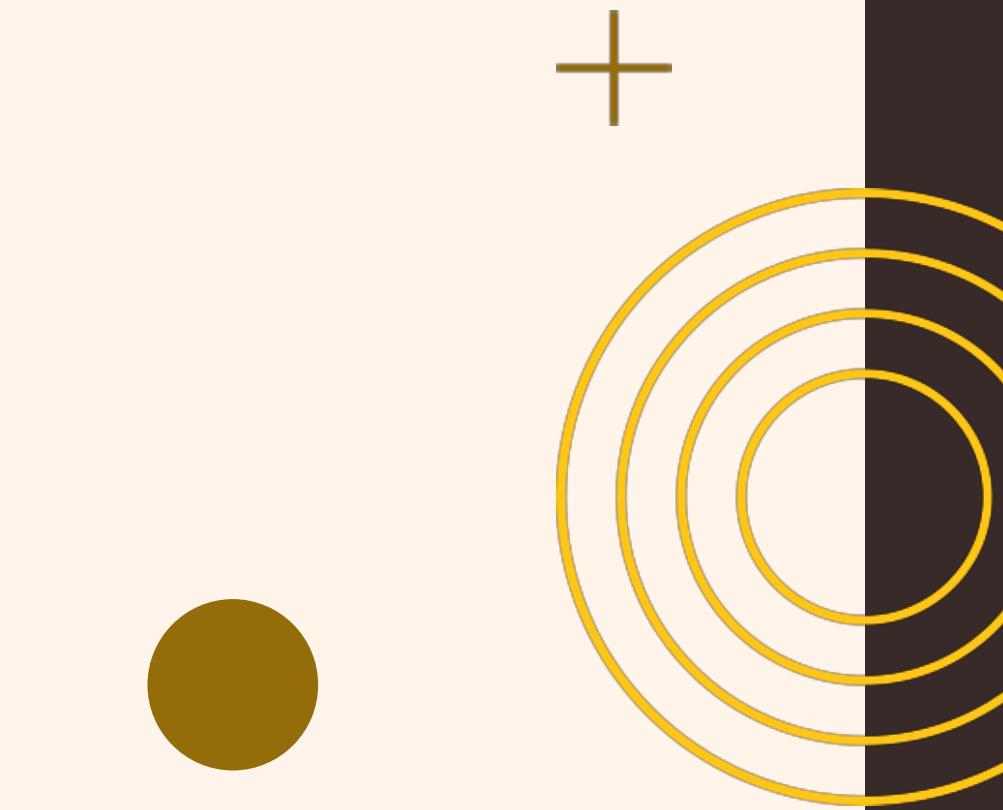
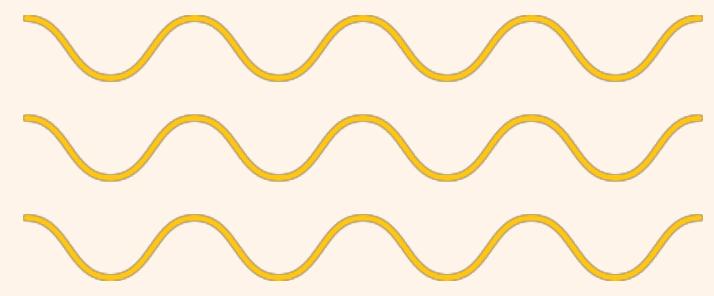
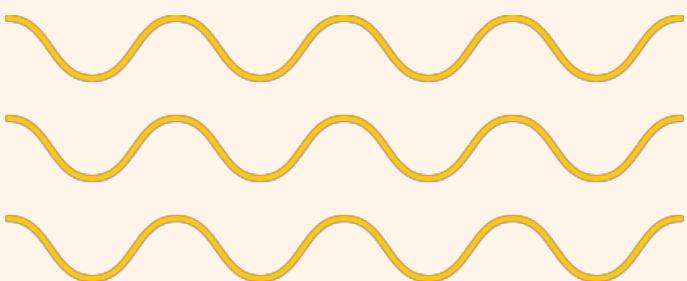
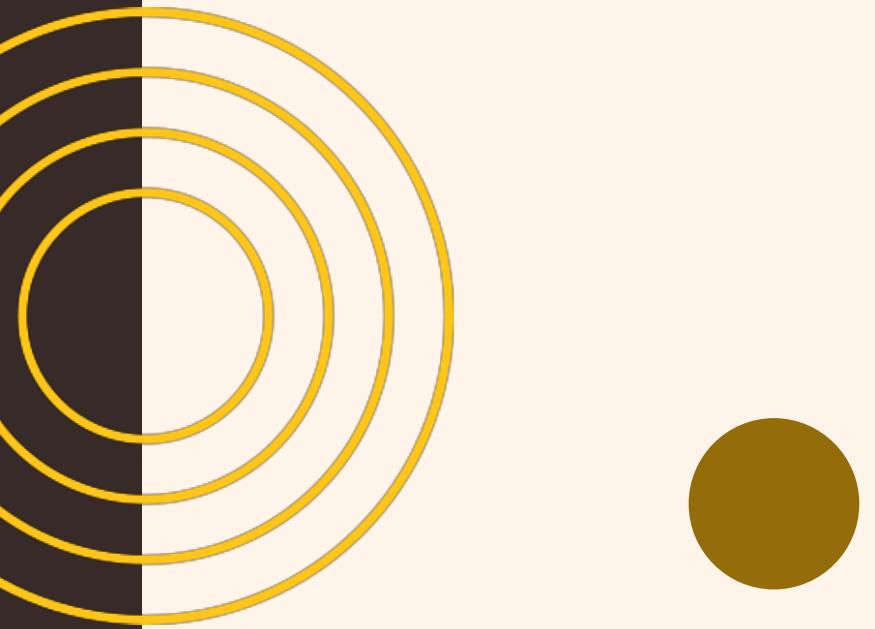
+



**Contoh:**

$$1. \sum_{k=1}^{\infty} \left( \frac{1}{\ln k} \right)^k$$

$$\begin{aligned} R &= \lim_{k \rightarrow \infty} (a_k)^{1/k} \\ &= \lim_{k \rightarrow \infty} \left( \frac{1}{\ln k} \right) \\ &= 0 < 1 \rightarrow \text{konvergen} \end{aligned}$$



# RINGKASAN

## Ringkasan:

Untuk menguji apakah deret  $\sum_{n=1}^{\infty} a_n$  dengan suku-suku positif adalah konvergen atau divergen, perhatikan  $a_n$  dengan seksama.

- 1 Jika  $\lim_{n \rightarrow \infty} a_n \neq 0$ , maka menurut *uji kedivergenen suku ke-n*,  $\sum_{n=1}^{\infty} a_n$  adalah divergen.
- 2 Jika  $a_n$  mengandung  $n!$ ,  $c^n$  dengan  $c$  adalah konstanta, atau  $n^n$  coba gunakan *uji hasil bagi*.
- 3 Jika  $a_n$  hanya mengandung pangkat  $n^c$  dan konstanta  $c$ , maka gunakan *uji banding limit*. Kluasusnya jika  $a_n$  merupakan fungsi rasional dari  $n$ , maka pilih  $b_n = n^{p-q}$  dengan  $p$  adalah pangkat tertinggi pembilang dan  $q$  adalah pangkat tertinggi penyebut pada  $a_n$ .
- 4 Jika  $a_n$  berbentuk  $(f(n))^n$  dengan  $f$  adalah suatu fungsi, maka gunakan *uji akar*.
- 5 Sebagai usaha terakhir, cobalah *uji banding*, *uji integral* atau *uji jumlah terbatas*.
- 6 Beberapa deret mensyaratkan *manipulasi bijak* atau *trik tertentu* untuk menentukan kekonvergenan atau kedivergenannya.

# DERET BERGANTI TANDA (ALTERNATING SERIES)

## Definisi

Misalkan  $\{a_n\}$  adalah barisan bilangan nyata tak-negatif. Yang dimaksud dengan deret ganti tanda (alternating series) adalah deret yang memiliki bentuk umum

$$\begin{aligned}\sum_{n=1}^{\infty} u_n &= \sum_{n=1}^{\infty} (-1)^n a_n \\ &= -a_1 + a_2 - a_3 + a_4 - a_5 + \dots\end{aligned}$$

atau

$$\begin{aligned}\sum_{n=1}^{\infty} u_n &= \sum_{n=1}^{\infty} (-1)^{n+1} a_n \\ &= a_1 - a_2 + a_3 - a_4 + a_5 - \dots\end{aligned}$$

## Deret Berganti Tanda (alternating series)

# 1. UJI DERET GANTI TANDA

### Teorema (Uji deret ganti tanda)

Misalkan  $\sum_{n=1}^{\infty} (-1)^n a_n$  atau  $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$  adalah deret ganti tanda dengan  $a_n > a_{n+1} \geq 0$  untuk semua bilangan asli  $n$ .

- 1 Jika  $\lim_{n \rightarrow \infty} a_n = 0$ , maka deret ganti tanda di atas konvergen.
- 2 Jika jumlah  $S$  diaproksimasi dengan jumlah  $n$  suku pertama  $\tilde{S}_n$ , maka kesalahan yang dibuat tidak akan melebihi  $a_{n+1}$ .

+ Jika  $\lim_{n \rightarrow \infty} a_n \neq 0$ , maka deret ganti tanda divergen (Uji kedivergenan).

## 2. KONVERGEN MUTLAK DAN KONVERGEN BERSYARAT

### Definisi (Konvergen mutlak dan konvergen bersyarat)

- 1 Suatu deret  $\sum_{n=1}^{\infty} u_n$  disebut konvergen mutlak jika  $\sum_{n=1}^{\infty} |u_n|$  adalah konvergen.
- 2 Suatu deret  $\sum_{n=1}^{\infty} u_n$  disebut konvergen bersyarat jika  $\sum_{n=1}^{\infty} u_n$  konvergen tetapi  $\sum_{n=1}^{\infty} |u_n|$  adalah divergen.

### Teorema

Jika  $\sum_{n=1}^{\infty} |u_n|$  konvergen maka  $\sum_{n=1}^{\infty} u_n$  adalah konvergen.

Untuk memeriksa kekonvergenan  $\sum |u_k|$  dapat digunakan uji-uji kekonvergenan deret positif.

Jika  $\sum |u_k|$  konvergen, maka  $\sum u_k$  konvergen.

Jika  $\sum |u_k|$  divergen, gunakan uji-uji ganti tanda untuk memeriksa kekonvergenan  $\sum u_k$ .

$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k} = -1 + \frac{1}{2} - \frac{1}{3} + \dots \quad (\text{konvergen})$$

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \dots \quad (\text{divergen})$$

Contoh:

$$\sum_{k=1}^{\infty} (-1)^k \left( \frac{1}{k} \right) \Rightarrow u_k, \quad a_k = \frac{1}{k}$$

$$\begin{aligned} \frac{a_{k+1}}{a_k} &= \frac{1}{k+1} \cdot \frac{k}{1} \\ &= \frac{k}{k+1} < 1 \Leftrightarrow a_k > a_{k+1} \quad (\text{basing turun}) \end{aligned}$$

$$\frac{1}{k+1} < \frac{1}{k} \Rightarrow a_k > a_{k+1}.$$

Menurut uji deret ganti tanda,  $\sum u_k$  konvergen

$$\lim_{k \rightarrow \infty} a_k = \lim_{k \rightarrow \infty} \frac{1}{k} = 0$$

$$|S - S_n| \leq 0.01$$

$$\sum_{k=0}^{\infty} (-1)^k \frac{1}{k} = S$$

$$\sum_{k=0}^{n-1} (-1)^k \frac{1}{k} = S_n$$

$$|S - S_n| \leq a_{n+1}$$

$$a_{n+1} = 0.01 \Leftrightarrow \frac{1}{n+1} = 0.01 \Leftrightarrow n+1 = 100 \Leftrightarrow n = 99$$

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$$

$$\sum_{k=1}^{\infty} u_k = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} \quad \text{konvergen.}$$

$$\sum_{k=1}^{\infty} |u_k| = \sum_{k=1}^{\infty} \left| (-1)^{k+1} \frac{1}{k} \right|$$

$$= \sum_{k=1}^{\infty} \frac{1}{k} \quad \text{divergen}$$

Jadi,  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$  konvergen bersyarat

### 3. UJI BANDING MUTLAK

#### Teorema (Uji pembanding mutlak)

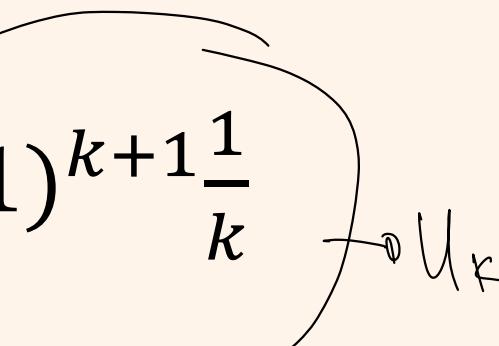
Misalkan  $\sum_{n=1}^{\infty} u_n$  adalah deret yang suku-sukunya taknol, dan misalkan pula

$$\lim_{n \rightarrow \infty} \frac{|u_{n+1}|}{|u_n|} = \rho.$$

- 1 Jika  $\rho < 1$ , maka  $\sum_{n=1}^{\infty} |u_n|$  konvergen.
- 2 Jika  $\rho > 1$ , maka  $\sum_{n=1}^{\infty} |u_n|$  divergen.
- 3 Jika  $\rho = 1$ , maka uji ini tidak memberi kesimpulan (diperlukan uji lainnya).

**Contoh:**

$$1. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$$



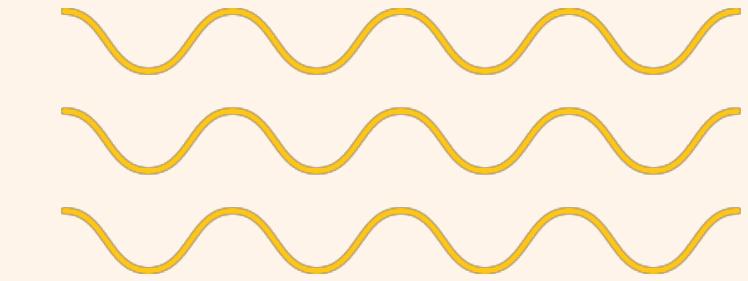
$$, a_k = \frac{1}{k}$$

$$|u_k| = \left( (-1)^{k+1} \frac{1}{k} \right)$$

$$a_k = \frac{1}{k}$$

$$u_k = \boxed{(-1)^{k+1}} \frac{1}{k}$$

$$a_k = u_k = \frac{1}{k}$$



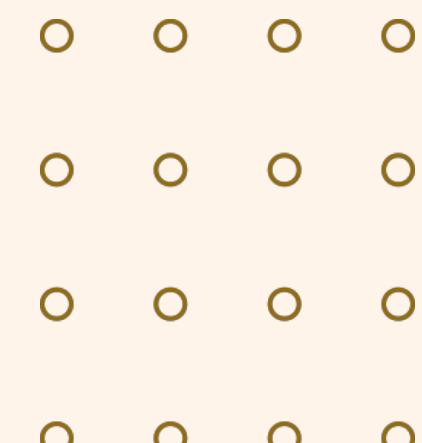
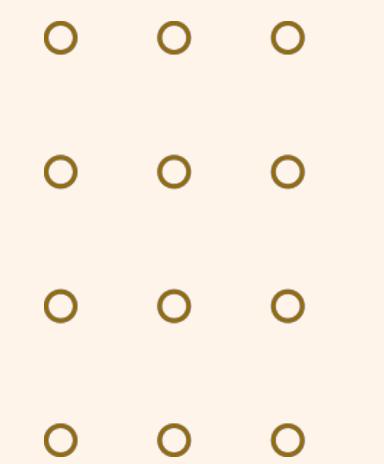
+



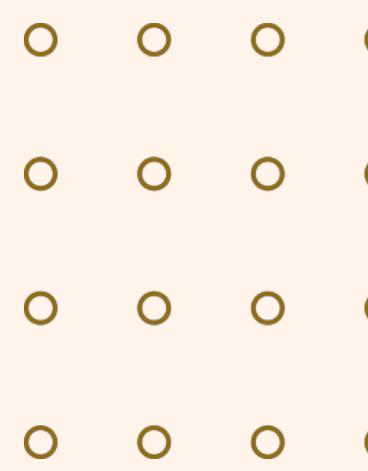
+

$$= 1$$

Jadi,  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$ , konvergen berpasang.



×



Contoh:

2.  $\sum_{k=1}^{\infty} \frac{\sin k!}{k^2} u_k$

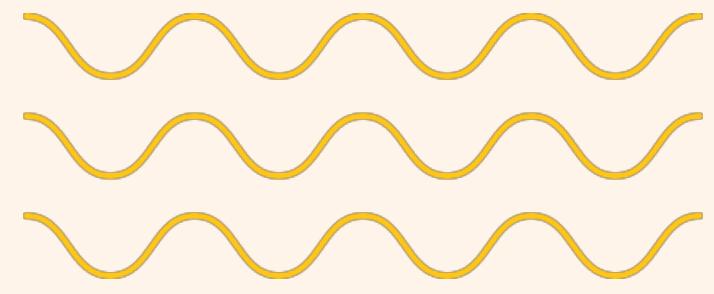
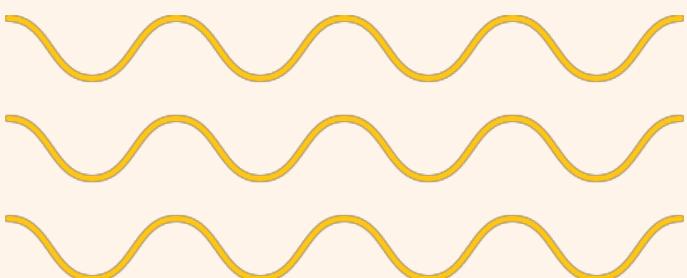
$$\lesssim |u_k| = \left( \frac{\sin k!}{k^2} \right)$$

$$= \frac{|\sin k!|}{k^2}$$

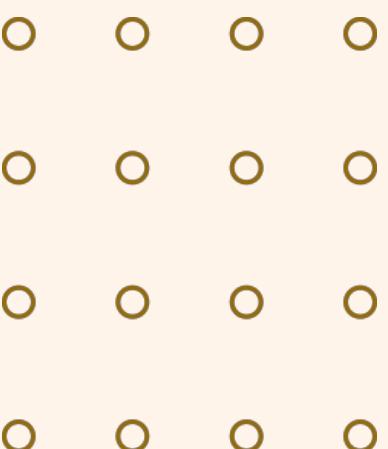
$$\lesssim \frac{1}{k^2} \rightarrow \text{konvergen.}$$

$\sum |u_k|$  konvergen.

konvergen mutlak.



+



×

## Contoh:

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{4^k}{k!}, \quad a_k = \frac{4^k}{k!} ; |U_k| = a_k = \frac{4^k}{k!}$$

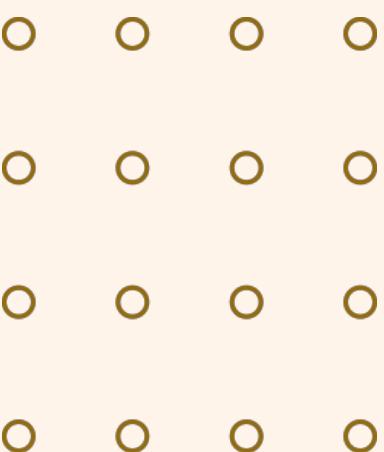
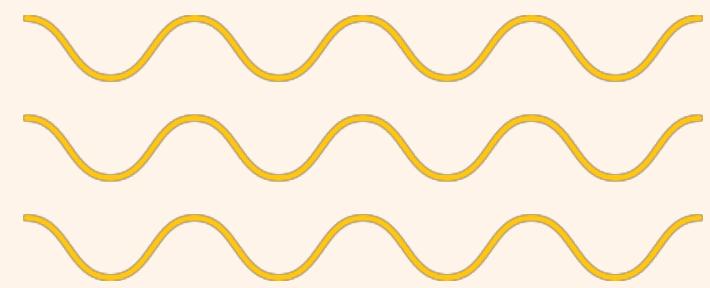
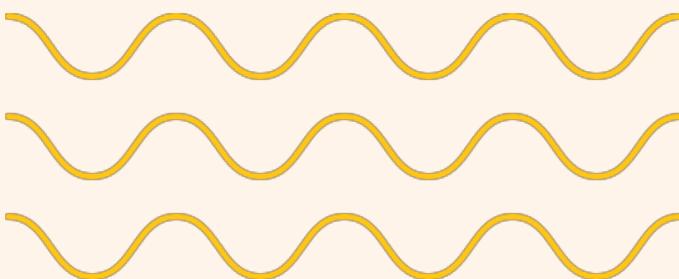
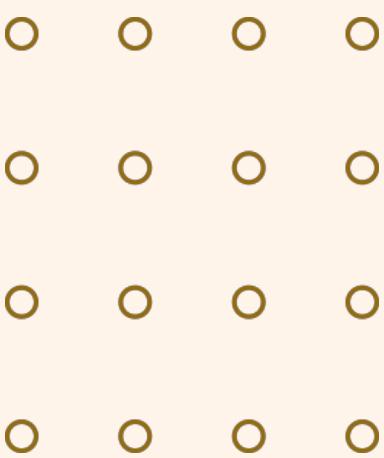
$$f = \lim_{k \rightarrow \infty} \frac{|U_{k+1}|}{|U_k|}$$

$$= \left( \frac{4^{k+1}}{(k+1)!} \cdot \frac{k!}{4^k} \right)$$

$$= \lim_{k \rightarrow \infty} \frac{4}{k+1}$$

$$= 0 < 1$$

Karena  $f = 0 < 1$  maka deret  $\sum |U_k|$  konvergen ( $\sum U_k$  konvergen juga).  
sehingga  $\sum U_k$  konvergen mutlak.



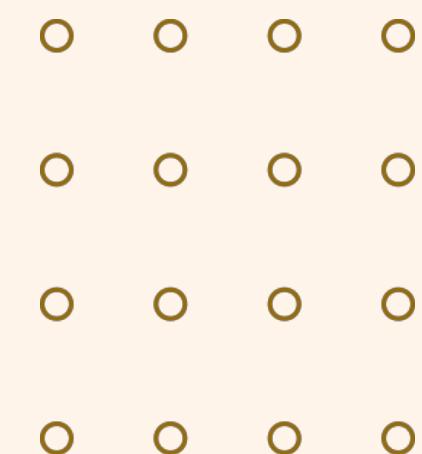
## ○ ○ Tugas Kelompok:

- ○ Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

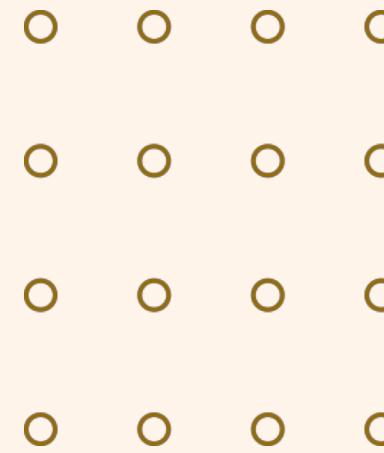
1.  $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$
2.  $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$
3.  $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$
4.  $\sum_{n=1}^{\infty} \frac{3^k+k}{k!}$
5.  $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$
6.  $\sum_{n=1}^{\infty} \left( \frac{n}{3n+2} \right)^n$
7.  $\sum_{n=2}^{\infty} \left( \frac{1}{\ln n} \right)^n$

Tentukan apakah deret yang diberikan adalah konvergen mutlak, konvergen bersyarat, atau divergen.

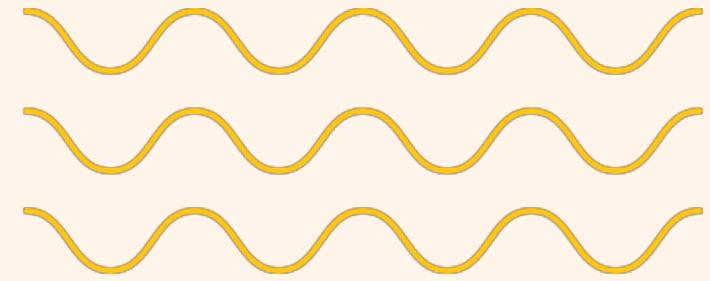
8.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$
9.  $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$
10.  $\sum_{n=1}^{\infty} \left( -\frac{4}{3} \right)^n$



**THANK YOU**



X



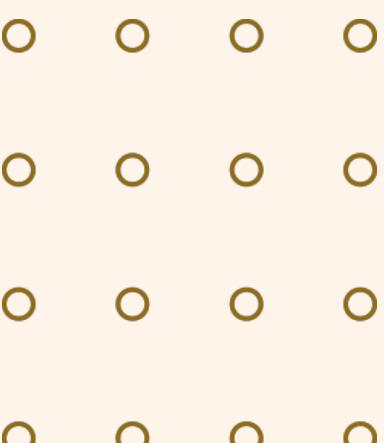
+



+



X



+

