TUGAS KELOMPOK MINGGU 4

KALKULUS II

KEL 3

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Nomor 1

1. $\sum_{k=1}^{\infty} \left(\frac{1}{7}\right)^k$, $S_n = \frac{1}{7} + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{7}\right)^3 + \left(\frac{1}{7}\right)^4 + \cdots + \left(\frac{1}{7}\right)^k$

↳ $a = \frac{1}{7}$; $r = \frac{1}{7}$

karena $-1 < r < 1$, maka konvergen. sehingga:

↳ $\lim_{n\to\infty} \frac{a}{1-r} = \frac{1/7}{1-1/7} = \frac{1}{6}$

Nomor 2

2). $\sum_{k=1}^{\infty} \frac{k^2-5}{k+2}$

Jawab : mencari menggunakan uji kedivergenan

$$\lim_{k\to\infty} \frac{k^2-5}{k+2} = \lim_{k\to\infty} \left(\frac{\frac{k^2}{k} - \frac{5}{k}}{\frac{k}{k} + \frac{2}{k}} \right)$$

$$= \lim_{k\to\infty} \left(\frac{k - \frac{5}{k}}{1 + \frac{2}{k}} \right)$$

$$= \infty$$

karena nilai yang didapat dari $\lim_{n\to\infty} a_n \neq 0$, maka deret diatas adalah divergen

Nomor 3

③ $$\sum_{k=1}^{\infty} \frac{2}{3k} = \frac{2}{3} \sum_{k=1}^{\infty} \frac{1}{k}$$

$$= \frac{2}{3} \underbrace{\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots\right)}_{\text{Deret harmonik}}$$

$$= \infty$$

Karena nilai dari deret harmonik divergen, maka $\sum_{k=1}^{\infty} \frac{2}{3k}$ divergen //

Nomor 4

4. $$\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k-1}\right)$$

$$\sum_{k=2}^{n} \left(\frac{1}{k} - \frac{1}{k-1}\right) = \left(\cancel{\frac{1}{2}} - \frac{1}{1}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{3}}\right) + \dots + \left(\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n-2}}\right) + \left(\frac{1}{n} - \cancel{\frac{1}{n-1}}\right)$$

$$= -1 + \frac{1}{n}$$

$$= \lim_{n\to\infty} -1 + \frac{1}{n} = -1$$ Konvergen ke -1

Nomor 5

5. $$\sum_{k=0}^{\infty} \frac{1}{k+3}$$

$$\Rightarrow \int_0^{\infty} \frac{1}{k+3}\,dx = \lim_{b\to\infty} \int_0^b \frac{1}{k+3}\,dx = \lim_{b\to\infty} \int_0^b \frac{1}{u}\,du$$

misal $u = k+3$

$du = dx$

$$= \lim_{b\to\infty} (\ln u)\Big|_0^b$$

$$= \lim_{b\to\infty} \ln(b+3) - \ln(3)$$

$$= \infty \text{ (Divergen)}$$

Nomor 6

6). Kekonvergenan dengan uji integral.

$$\sum_{k=1}^{\infty} \frac{3}{2k-3}$$

$$\int_1^{\infty} \frac{3}{2k-3}\,dk = \lim_{a\to\infty} \int_1^{a} \frac{3}{2k-3}$$

$$= \lim_{a\to\infty} 3\int_1^{a} \frac{1}{2k-3}$$

misalkan : $U = 2k-3$

$du = 2\,dk \qquad dk = \frac{du}{2}.$

$$= \lim_{a\to\infty} 3\int_1^{a} \frac{1}{U}\cdot\frac{du}{2}.$$

$$= \lim_{a\to\infty} \frac{3}{2}\int_1^{a} \frac{du}{U}$$

$$= \lim_{a\to\infty} \frac{3}{2}\,(\ln(U)\Big|_1^{a}$$

$$= \lim_{a\to\infty} \frac{3}{2}(\ln(2k-3))\Big|_1^{a}$$

$$= \lim_{a\to\infty} \frac{3}{2}(\ln(2a-3)) - (\ln(2.1-3))$$

$= \infty$ – tdk terdefinisi (divergen).

karena nilai $\int_1^{\infty} \frac{3}{2k-3}$ divergen, maka deret $\sum_{k=1}^{\infty} \frac{3}{2k-3}$ divergen

Nomor 7

Date

7) $\displaystyle\sum_{k=0}^{\infty} \frac{k}{k^2+3}$

Bentuk $\displaystyle\sum_{k=0}^{\infty} \frac{k}{k^2+3}$ sama dengan $\displaystyle\sum_{x=0}^{\infty} \frac{x}{x^2+3}$

misal $u = x^2+3$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$\sum_{x=0}^{\infty} \frac{x}{x^2+3} = \lim_{b \to \infty} \int_0^b \frac{x}{x^2+3}\, dx$$

$$= \lim_{b \to \infty} \int_3^{b^2+3} \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$$

$$= \lim_{b \to \infty} \frac{1}{2} \int_3^{b^2+3} \frac{1}{u}\, du$$

$$= \lim_{b \to \infty} \frac{1}{2} \Big[\ln u \Big]_3^{b^2+3}$$

$$= \lim_{b \to \infty} \frac{1}{2} \Big(\ln(b^2+3) - \ln(3) \Big)$$

$$= \infty$$

Divergen

Nomor 8

No. ____
Date ____

(8) $$\sum_{k=1}^{\infty} \frac{3}{2k^2+1}, \quad f(x) = \frac{3}{2x^2+1}$$

$$\Rightarrow \int_1^{\infty} \frac{3}{2x^2+1} = \lim_{t\to\infty} \int_1^t \frac{3}{2x^2+1}\,dx$$

$$= \lim_{t\to\infty} \frac{3}{2}\int_1^t \frac{1}{x^2+\frac{1}{2}}\,dx$$

$$= \lim_{t\to\infty} \left(\frac{3}{2} \times \frac{1}{\sqrt{\frac{1}{2}}} \times \arctan\left(\frac{x}{\sqrt{\frac{1}{2}}}\right)\right)\Bigg|_1^t$$

$$= \lim_{t\to\infty} \left(\frac{3\sqrt{2}\arctan(\sqrt{2}x)}{2}\right)\Bigg|_1^t$$

$$= \lim_{t\to\infty} \left(\frac{3\sqrt{2}\arctan(\sqrt{2}t)}{2} - \frac{3\sqrt{2}\arctan(\sqrt{2})}{2}\right)$$

$$= \frac{3\sqrt{2} \times \frac{\pi}{2} - 3\sqrt{2}\arctan(\sqrt{2})}{2}$$

$$= \frac{3\sqrt{2}\pi - 6\sqrt{2}\arctan(\sqrt{2})}{4}$$

Ruas kanan diatas adalah konvergen.

Jika integral tak wajar $\int_1^{\infty} \frac{3}{2x^2+1}$ konvergen, maka $\sum_{k=1}^{\infty} \frac{3}{2k^2+1}$ konvergen