TUGAS RESPONSI 3
KELOMPOK 2 KALKULUS 2

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NOMOR 1

(1) $a_n = \frac{n}{3n-1}$

$\Rightarrow \lim_{n \to \infty} \frac{n}{3n-1} = \frac{1}{3}$

maka a_n konvergen ke $\frac{1}{3}$

* kemonotonan

$a_n - a_{n+1} = \frac{n}{3n-1} - \left(\frac{n+1}{3(n+1)-1}\right)$

$= \frac{n}{3n-1} - \frac{(n+1)}{3n+2}$

$= \frac{n(3n+2) - (n+1)(3n-1)}{(3n-1)(3n+2)}$

$\Rightarrow = \frac{3n^2+2n - 3n^2+n-3n+1}{9n^2+6n-3n-2}$

$= \frac{1}{9n^2+3n-2} \geq 0$ (turun)

~~* kemonotonan~~

2. $a_n = \dfrac{n^3+3n+3n}{(n+1)^3}$

diubah

$a_n = \dfrac{(n+1)^3-1}{(n+1)^3}$

$= 1 - \dfrac{1}{(n+1)^3}$

Cehek kekonvergenan

$\lim\limits_{n \to \infty} : 1 - \dfrac{1}{(n+1)^3} = 1$ (konverge)

Cheke kemonontonan $a_n - a_{n+1}$

=

$\left(1 - \dfrac{1}{(n+1)^3}\right) - \left(1 - \dfrac{1}{(n+2)^3}\right)$

$\dfrac{1}{(n+2)^3} - \dfrac{1}{(n+1)^3}$

$\dfrac{(n+1)^3-(n+2)^3}{(n+2)^3(n+1)^3}$

$(n+1)^3 - (n+2)^3$ akan selalu < 0

makadari itu $a_n < a_{n+1}$

(barisan naik) //

NOMOR 2

NOMOR 3

(3) $a_n = \frac{\cos(n\pi)}{n}$

$\frac{-1}{n} < \cos n\pi < \frac{1}{n}$

$\lim_{n \to \infty} -\frac{1}{n} = 0$

$\lim_{n \to \infty} \frac{1}{n} = 0$

maka $\lim_{n \to \infty} \frac{\cos(n\pi)}{n}$ konvergen ke 0

3. $\frac{a_{n+1}}{a_n} = \frac{\frac{\cos(n\pi + \pi)}{n+1}}{\frac{\cos(n\pi)}{n}} = \frac{\frac{-\cancel{\cos(n\pi)}}{(n+1)}}{\frac{\cancel{\cos(n\pi)}}{n}} = -\left(\frac{n}{n+1}\right) < 1$

Maka $\to a_n$ Bukan Barisan monoton

NOMOR 4

4. $a_n = e^{-n} \sin n$

$-e^{-n} \leq e^{-n} \sin n \leq e^{-n}$

Menggunakan Teorema Apit

maka

$$\lim_{n \to \infty} e^{-n} \sin n = 0$$

Kekonvergenan

$$\frac{a_{n+1}}{a_n} = \frac{e^{-n-1} \sin(n+1)}{e^{-n} \sin(n)} = \frac{\sin(n)\cos(1) + \sin(1)\cos(n)}{e \sin(n)}$$

$$= \frac{\cos(1)}{e} + \frac{\sin(1) \cot(n)}{e}$$

Karena fungsi cot periodik dan menghasilkan nilai antara minus tak hingga sampai tak hingga. maka, an bukan barisan monoton

Kemonoton

NOMOR 5

5) $a_n = \frac{1}{n^3}$

$\rightarrow \lim_{n \to \infty} \frac{1}{n^3}$

$= 0 \rightarrow$ konvergen ke 0

* kemonotonan

$\frac{a_n}{a_{n+1}} > 1$

$= \frac{\frac{1}{n^3}}{\frac{1}{(n+1)^3}} >$

$= \frac{(n+1)^3}{n^3}$

$= \frac{n^3 + 3n^2 + 3n + 1}{n^3}$

$1 + \left(\frac{3n^2 + 3n + 1}{n^3}\right) > 1$

maka (monoton turun)

(4) $\frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}$

+ Rumus eksplisit

$$a_n = \frac{1}{2^{n+1}}$$

+ kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{1/2^{n+1}}{1/2^{n+2}}$$

$$= \frac{2^{n+2}}{2^{n+1}}$$

$$= \frac{2^n \cdot 2^2}{2^n \cdot 2}$$

$$= 2 > 1$$

maka monoton turun //

+ konvergen

$$\lim_{n\to\infty} \frac{1}{2^{n+1}} = \frac{1}{2^\infty} = 0$$

konvergen ke 0

NOMOR 6

NOMOR 7

(7) $\sin 1, 2\sin\frac{1}{2}, 3\sin\frac{1}{3}, 4\sin\frac{1}{4}$

$a_n = n\sin\frac{1}{n}$

* kekonvergenan

$\lim_{n\to\infty} n\sin\frac{1}{n}$

$= \lim_{n\to\infty} \frac{\cos(1/n)\cdot(-1/n^2)}{(-1/n^2)} = 1$ ∴ konvergen ke 1 //

(8) $0{,}1 ; 0{,}11 ; 0{,}111 ; 0{,}1111 ; \ldots$

$0{,}1^2 + 0{,}1^2 + 0{,}1^3 + 0{,}1^4 + \ldots$

deret geometri

$$\frac{a(r^n - 1)}{r-1}$$

$$= \frac{0{,}1\,(0{,}1^n - 1)}{0{,}1 - 1}$$

$$= \frac{\cancel{1/10}\left[(1/10)^n - 1\right]}{-9/\cancel{10}} \times \frac{10^n}{10^n}$$

$$= \frac{1^n - 10^n}{10^n} \times -\frac{1}{9}$$

$$a_n = \boxed{\frac{10^n - 1}{9 \times 10^n}}$$

* kekonvergenan

$$\lim_{n \to \infty} \frac{10^n - 1}{9 \times 10^n}$$

$$= \lim_{n \to \infty} \frac{1}{9} - \frac{1}{9 \times 10^n}$$

$$= \frac{1}{9} - 0$$

maka konvergen ke $\frac{1}{9}$ //

NOMOR 8