Kelompok 2

Tugas Kelompok Minggu 5

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1. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

- kekonvergenan
- Sebut jenis ujibanding

<u>ANS:</u>

<u>Teorema ujibanding</u>

$\underbrace{\sum bn}_{\text{divergen}} \leq \underbrace{\sum an}_{\text{maka divergen}}$ atau $\underbrace{\sum an}_{\text{maka konvergen}} \leq \underbrace{\sum bn}_{\text{konvergen}}$

$\frac{3n+1}{n^2-4} \geq \frac{3n+1}{n^2}$

$\frac{3n+1}{n^2-4} \geq \frac{3n}{n^2}$

$\frac{3n+1}{n^2-4} \geq \frac{3}{n}$

$\sum \frac{3n+1}{n^2-4} \geq \underbrace{\sum \frac{3}{n}}$

↳ deret harmonik

↳ divergen

maka sesuai dng teorema

$\sum \frac{3n+1}{n^2-4}$ juga divergen //

2. $$\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3} \iff \frac{n}{n^2} \iff \frac{1}{n}$$ $\rightarrow$ deret harmonik (divergen)

$$\lim_{n\to\infty} \frac{ak}{bk} = \lim_{n\to\infty} \frac{n}{n^2+2n-3} \cdot n$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+2n-3}$$

$= 1 > 0$, maka divergen

(Uji Banding Limit)

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3. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

Jawab

$\lim_{n \to \infty} \left| \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!} \right|$

$\lim_{n \to \infty} \left| (n+1) \cdot \frac{n^{100}}{(n+1)^{100}} \right|$

$\lim_{n \to \infty} |n+1| \cdot \lim_{n \to \infty} \left| \frac{n}{n+1} \right|^{100}$

$= \infty \cdot 1$

$= \infty > 1$ divergen

* uji hasil bagi *deret biasa

(4) $\sum_{n=1}^{\infty} \frac{3^k + k}{k!}$

→ Uji Hasil Bagi

$\rho = \lim_{k \to \infty} \frac{3^{k+1} + (k+1)}{(k+1)!} \times \frac{k!}{3^k + k}$

$= \lim_{k \to \infty} \frac{3^k \cdot 3 + k + 1}{(k+1)} \frac{1}{(3^k + 3)} \times \frac{1/3^k}{1/3^k}$

$= \lim_{k \to \infty} \frac{3 + k/3^k + 1/3^k}{(k+1) + 3(k+1)/3^k}$

$= \frac{3 + 0 + 0}{\infty + 0}$

$= 0 < 1$

$\rho < 1$

$\therefore \sum_{n=1}^{\infty} \frac{3^k + k}{k!}$ konvergen karena $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} < 1$ //

Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan

5. $\displaystyle\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

Jawab

↳ untuk $n \geq 3$, $\dfrac{3n+1}{n^2-4} \geq \dfrac{3n+1}{n^2} \geq \dfrac{3n}{n^2} = \dfrac{3}{n}$

$\displaystyle\sum_{n=3}^{\infty} \frac{3}{n}$ merupakan deret harmonik sehingga deret $\displaystyle\sum_{n=3}^{\infty} \frac{3}{n}$ divergen.

Karena $\dfrac{3n+1}{n^2-4} \geq \dfrac{3}{n}$ untuk $n \geq 3$, dengan menggunakan

Teorema uji banding ditapat $\displaystyle\sum_{n=3}^{\infty} \frac{3n+1}{n^2-4}$ divergen.

Sehingga $\displaystyle\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$ merupakan deret yang divergen.

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6. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$

$$R = \lim_{n\to\infty} \left(\left(\frac{n}{3n+2}\right)^n\right)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n}{3n+2} = \frac{1}{3} < 1 \quad (\text{konvergen})$$

maka $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$ (konvergen) (uji Akar)

7. $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n$

Jawab:

Untuk $n \geq 2$ maka $a_n = \left(\frac{1}{\ln n}\right)^n$ positif. Karena

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \lim_{n\to\infty} \left(\left(\frac{1}{\ln n}\right)^n\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{\ln n} = 0 < 1$$

maka $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n$ konvergen

Tentukan status kekonvergenan deret ini

8) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$

- Uji deret ganti tanda

a.) Uji kemonotonan

$a_n = \frac{n}{n+1}$

$a_n - a_{n+1} = \frac{n}{n+1} - \frac{n+1}{n+2}$

$= \frac{(n^2+2n) - (n^2+2n+1)}{(n+1)(n+2)}$

$= \frac{-1}{(n+1)(n+2)} < 0, n \geq 1$

maka, $a_n - a_{n+1} < 0 \rightarrow$ Monoton naik untuk $\{a_n\}$

b.) Uji $\lim_{n \to \infty} a_n$

$\lim_{n \to \infty} \frac{n}{n+1} =$ Bentuk $\left(\frac{\infty}{\infty}\right)$

$\lim_{n \to \infty} \frac{n}{n+1} \overset{LH}{=} \lim_{n \to \infty} \frac{\frac{dn}{dn}}{\frac{d(n+1)}{dn}} = 1$

karena $\{a_n\}$ monoton naik dan $\lim_{n \to \infty} a_n \neq 0$, deret

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$ Divergen

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9) $\sum_{n=1}^{N} \sin \frac{n!}{n^2}$

Jawab :

$U_n = \sin \frac{n!}{n^2} \longrightarrow$ Berfluktuasi diantara $(-1, 1)$, sehingga divergen

$|U_n| = \left| \sin\left(\frac{n!}{n^2}\right) \right| \rightarrow$ Berfluktuasi diantara $(0, 1)$ sehingga divergen

Karena U_n dan $|U_n|$ keduanya divergen, maka $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$ divergen

(10) $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n \longrightarrow \sum_{n=1}^{\infty} (-1)^n \cdot \left(\frac{4}{3}\right)^n$

Uji Banding mutlak

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \rho$$

$$\rho = \lim_{n\to\infty} \frac{\left(\frac{4}{3}\right)^{n+1}}{\left(\frac{4}{3}\right)^n} = \lim_{n\to\infty} \left(\frac{4}{3}\right)^{n+1} \cdot \left(\frac{3}{4}\right)^n$$

$$= \lim_{n\to\infty} \left(\frac{4}{3}\right) \cdot \left(\frac{4}{3}\right)^n \cdot \left(\frac{3}{4}\right)^n$$

$$= \lim_{n\to\infty} \left(\frac{4}{3}\right) (1)^n = \frac{4}{3} > 1 \text{ (Divergen)}$$

Maka $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n$ Divergen //