Tugas Responsi 4 Kalkulus Kelompok 4

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1.

1. $\sum_{k=1}^{\infty} \left(\frac{1}{7}\right)^k$

Geometrik Series

$r = \frac{1}{7} < 1$ (konvergen)

$S = \frac{a}{1-r}$

$= \frac{\frac{1}{7}}{1-\frac{1}{7}} = \frac{1/7}{6/7} = \frac{1}{7} \cdot \frac{7}{6} = \frac{1}{6}$ //

2.

(2) $\sum_{k=1}^{\infty} \frac{k^2-5}{k+2}$

Menggunakan Uji Kedivergenan

$\lim_{k\to\infty} a_n = \lim_{k\to\infty} \frac{k^2-5}{k+2}$

$= \lim_{k\to\infty} \frac{k^2-5}{k+2} \cdot \frac{1}{k}$

$= \lim_{k\to\infty} \frac{k^2-5}{k^2+2k}$

$= 1$

Karena $\lim_{k\to\infty} a_n \neq 0$, maka deret divergen

3.

$$\sum_{k=1}^{\infty} \frac{2}{3k}$$

$$= \frac{2}{3} \cdot \sum \frac{1}{k}$$

→ (karena $\frac{1}{k}$ adalah deret harmonik dan selalu divergen sehingga $\sum_{k=1}^{\infty} \frac{2}{3k}$ Juga Divergen) (Divergen)

4.

4) $\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k-1}\right) \rightarrow \frac{k-1-k}{k^2-k}$

$$= \frac{-1}{k^2-k} \rightarrow \left\{-\frac{1}{2}, -\frac{1}{6}, -\frac{1}{12}, \dots\right\}$$

$$\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k-1}\right) = \left(\left(\frac{1}{2}-1\right) + \left(\frac{1}{3}-\frac{1}{2}\right) + \left(\frac{1}{4}-\frac{1}{3}\right) + \dots + \left(\frac{1}{k}-\frac{1}{k-1}\right)\right)$$

$$= \left(\left(-1+\frac{1}{2}\right) + \left(-\frac{1}{2}+\frac{1}{3}\right) + \left(-\frac{1}{3}+\frac{1}{4}\right) + \dots + \left(-\frac{1}{k-1} + \dots\right.\right.$$

$$= -1 + \frac{1}{k} \text{ atau } \frac{1}{k} - 1$$

$$\lim_{k \to \infty} \frac{1}{k} - 1 \rightarrow \lim_{k \to \infty} \frac{1}{\infty} - 1$$

$$= \lim_{k \to \infty} 0 - 1 = -1 \text{ (konvergen)}$$

5.

5) $\sum_{k=0}^{\infty} \frac{1}{k+3}$

$$\int_0^{\infty} \frac{1}{k+3}\,dk \Rightarrow \lim_{b\to\infty} \int_0^{b} \frac{1}{k+3}\,dk \quad \text{, misal : } u = k+3$$

$du = 1dk$

$dk = du$

$$= \lim_{b\to\infty} \int_3^{b+3} \frac{1}{u}\,du$$

$$= \lim_{b\to\infty} \left(\ln|u| \Big|_3^{b+3} \right.$$

$$= \lim_{b\to\infty} \left(\ln|b+3| - \ln|3| \right)$$

$$= \infty \text{ (divergen)}_{//}$$

Maka, $\sum_{k=0}^{\infty} \frac{1}{k+3}$ divergen.

PAPERLINE

6.

No. .... Tgl. - -

Tukel Pekan 4

6. $\sum_{k=1}^{\infty} \frac{2}{2k-3}$

$$\int_1^{\infty} \frac{2}{2k-3}\,dk = \lim_{b\to\infty} \int_1^{b} \frac{2}{2k-3}\,dk \quad \text{, misal, } 2k-3 = u$$

$2 = \frac{du}{dk}$

$2dk = du$

$$= \lim_{b\to\infty} \int_{-1}^{2b-3} \frac{du}{u}$$

$$= \lim_{b\to\infty} \int_{-1}^{2b-3} \frac{1}{u}\,du$$

$$= \lim_{b\to\infty} \left(\ln|u| \Big|_{-1}^{2b-3} \right.$$

$$= \lim_{b\to\infty} \ln|2b-3| - \ln|-1|$$

$$= \infty \text{ (divergen)}$$

karena $\int_1^{\infty} \frac{2}{2k-3}\,dx$ divergen, $\sum_{k=1}^{\infty} \frac{2}{2k-3}$ juga divergen

7.

$$\sum_{k=1}^{\infty} \frac{k}{k^2+3}$$

Jawab:

Misalkan: $f(x) = \frac{x}{x^2+3}$

Pada $[1, \infty)$ fungsi f bersifat kontinu,positif,taknaik?

$$\frac{df(x)}{dx} = \frac{\frac{d}{dx}(x) \times (x^2+3) - (x) \times \frac{d}{dx}(x^2+3)}{(x^2+3)^2}$$

$$\frac{df(x)}{dx} = \frac{(x^2+3) - x(2x)}{(x^2+3)^2}$$

$$\frac{df(x)}{dx} = -\frac{x^2-3}{(x^2+3)^2} < 0 (fungsi\ turun\ pada\ x > 1)$$

$\therefore$ Karena f tidak memenuhi syarat teorema maka deret tersebut divergen

Menghitung integral tak-wajar berikut:

$$\int_1^{\infty} \frac{x}{x^2+3}\, dx$$

misalkan: $u = x^2 + 3 \quad \leftrightarrow \quad du = 2x\, dx \quad \leftrightarrow dx = \frac{du}{2x}$ $.Sehingga\ integral\ dituliskan:$

$$\int_4^{\infty} \frac{1}{2u}\, du = \frac{1}{2}\int_4^{\infty} \frac{1}{u}\, du = \frac{1}{2}\lim_{b\to\infty}\int_4^{b} \frac{1}{u}\, du = \frac{1}{2}\lim_{b\to\infty}(\ln u|_4^b)$$

$$= \frac{1}{2}\lim_{b\to\infty} \ln b - \lim_{b\to\infty} \ln 4$$

$$= \infty\ (divergen)$$

8.

8). $\displaystyle\int_1^{\infty} \frac{3}{2x^2+1}\, dx$

Misalkan : $U = \sqrt{2}x$, $du = \sqrt{2}\, dx$, $dx = \frac{du}{\sqrt{2}}$

$$\int_1^{\infty} \frac{3}{2x^2+1}\, dx$$

$$= 3\int_1^{\infty} \frac{dx}{2x^2+1}$$

$$= \frac{3}{\sqrt{2}}\int_1^{\infty} \frac{du}{U^2+1}$$

$$= \frac{3}{\sqrt{2}} \lim_{b\to\infty} \int_{\sqrt{2}}^{\sqrt{2}b} \frac{du}{U^2+1^2}$$

$$= \frac{3}{\sqrt{2}} \lim_{b\to\infty} \left(\frac{1}{1}\tan^{-1}\left(\frac{U}{1}\right)\Big]_{\sqrt{2}}^{\sqrt{2}b}\right)$$

$$= \frac{3}{\sqrt{2}} \lim_{b\to\infty} (\tan^{-1}\sqrt{2}b - \tan^{-1}\sqrt{2})$$

$$= \frac{3}{\sqrt{2}}\left(\frac{\pi}{2} - \tan^{-1}\sqrt{2}\right)$$

$$= \frac{3(\pi - 2\tan^{-1}\sqrt{2})}{2\sqrt{2}}$$

$= 1,3$ (Konvergen)