Tugas Responsi 5 Kalkulus Kelompok 4

Kelompok 4:

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1.

1). $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

Jawab : (menggunakan uji banding)

$\frac{3n}{n^2} \leq \frac{3n+1}{n^2-4}$

$\frac{3}{n} \leq \frac{3n+1}{n^2-4}$

$\sum \frac{3}{n} \leq \sum \frac{3n+1}{n^2-4}$

$3 \sum \frac{1}{n} \leq \sum \frac{3n+1}{n^2-4}$

$\hookrightarrow$ deret harmonik, sehingga $\sum \frac{3n+1}{n^2-4}$ divergen

2.

(2) $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$

$\rightarrow$ Uji Banding Limit

$\Rightarrow \sum_{n=1}^{\infty} \frac{n}{n^2+2n-3} \Leftrightarrow \frac{n}{n^2} \Leftrightarrow \frac{1}{n}$ $\rightarrow (b_n)$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n^2+2n-3} \cdot \frac{n}{1}$

$= \lim_{n\to\infty} \frac{n^2}{n^2+2n-3}$

$= 1 \rightarrow 1 > 0$

$\therefore$ Karena $\sum b_n$ divergen, maka $\sum a_n$ divergen.

3.

3.) $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

$\Rightarrow \rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

$= \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!}$

$= \lim_{n\to\infty} n+1 \cdot \frac{n^{100}}{(n+1)^{100}}$

$= \infty > 1$

Menurut uji hasil bagi (rasio), deret $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ divergen.

4.

4./ $\sum_{n=1}^{\infty} \frac{3^k + k}{k!}$

$\Rightarrow$ (menggunakan uji hasil bagi rasio)

$$\rho = \frac{\lim_{n\to\infty} a_{n+1}}{a_n}$$

$$= \lim_{k\to\infty} \frac{3^{(k+1)} + (k+1)}{(k+1)!} \cdot \frac{k!}{3^k + k}$$

$$= \lim_{k\to\infty} \frac{3^{(k+1)} + (k+1)}{(k+1)(3^k + k)}$$

$$= \lim_{k\to\infty} \frac{3^k \cdot 3 + k + 1}{3^k \cdot k + k^2 + 3^k + k} \times \left(\frac{\frac{1}{3^k}}{\frac{1}{3^k}} \right)$$

$$= \lim_{k\to\infty} \frac{3 + 0 + 0}{k + 0 + 1 + 0}$$

$$= \lim_{k\to\infty} \frac{3}{k+1} \Rightarrow 0$$

Karena $\rho = 0 < 1$, maka <u>konvergen</u>.

5.

5. $$\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$$

Uji banding limit

$$a_n = \frac{3n+1}{n^2-4} \qquad b_n = \frac{1}{n} \rightarrow \text{divergen}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{3n+1}{n^2-4}}{1/n}$$

$$\Leftrightarrow \lim_{n\to\infty} \frac{3n+1}{n^2-4} \cdot n$$

$$\Leftrightarrow \lim_{n\to\infty} \frac{3n^2+n}{n^2-4} = 1 > 0$$

$\therefore \sum a_k$ divergen

6.

6. $$\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$$

untuk $n \geq 1$, $a_n = \left(\frac{n}{3n+2}\right)^n$ positif

menggunakan uji akar

$$\lim_{n\to\infty} (a_n)^{1/n} = R$$

$$R = \lim_{n\to\infty} \left(\left(\frac{n}{3n+2}\right)^n\right)^{1/n}$$

$$= \lim_{n\to\infty} \frac{n}{3n+2}$$

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{1}{3}$$

$$R = \frac{1}{3}$$

karena $R = \frac{1}{3} < 1$, maka $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$ adalah konvergen.

7.

Tgl.

7.

$$\sum_{n=2}^{\infty}\left(\frac{1}{\ln n}\right)^{n}$$

untuk $n \geq 2$, $a_n = \left(\frac{1}{\ln n}\right)^{n}$ positif

$\lim_{n\to\infty} (a_n)^{1/n} = R$ menggunakan Uji Akar

$$R = \lim_{n\to\infty}\left(\left(\frac{1}{\ln n}\right)^{n}\right)^{1/n}$$

$$= \lim_{n\to\infty}\frac{1}{\ln n}$$

$$R = 0$$

karena $R = 0 < 1$, maka $\sum_{n=2}^{\infty}\left(\frac{1}{\ln n}\right)^{n}$ adalah konvergen.

8.

DATE ____/____

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$$

Jawab

$U_n = (-1)^{n+1}\frac{n}{n+1}$ dan $a_n = \frac{n}{n+1}$

$$\rho = \lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = \lim_{n\to\infty}\left|\frac{n+1}{n+2}\cdot\frac{n+1}{n}\right| = \lim_{n\to\infty}\frac{n^2+2n+1}{n^2+n}$$

$$= 1$$

Karena $\rho = 1$ maka uji banding mutlah ~~maka~~ Tidak memberikan kesimpulan. Oleh karena itu, diperlukan uji Lain yaitu uji deret ganti tanda.

$a_n = \frac{n}{n+1}$

$\lim_{n\to\infty}\frac{n}{n+1} = 1 \neq 0$ {divergen} dan $\sum |U_n| = \sum \frac{n}{n+1}$

$\lim_{n\to\infty}\frac{n}{n+1} = 1$ {konvergen}

Karena $\sum |U_n|$ konvergen dan $\sum U_n$ divergen, maka $\sum U_n$ konvergen bersyarat

9.

(9) $\sum_{n=1}^{\infty} \sin \frac{n!}{n^2}$

$U_n = \sin \frac{n!}{n^2}$

$|U_n| = \left| \sin \frac{n!}{n^2} \right|$

$= \left| \sin n! \times \frac{1}{n^2} \right|$

$= \frac{1}{n^2} |\sin n!|$

$= \frac{1}{n^2} |\sin n!| \leq \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow$ deret p

$p = 2 > 1$

sehingga deret $\sum_{n=1}^{\infty} |U_n|$ konvergen, maka deret $\sum_{n=1}^{\infty} U_n$ konvergen mutlak

10.

10.) $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)^n$

$R = \lim_{n \to \infty} (a_n)^{1/n}$

$= \lim_{n \to \infty} \left(\left(-\frac{4}{3}\right)^n\right)^{1/n}$

$= \lim_{n \to \infty} -\frac{4}{3}$

$= -\frac{4}{3} \neq 0$ (divergen)

Teorama = Jika $\sum \left(-\frac{4}{3}\right)^n$ divergen maka $\sum \left| \left(-\frac{4}{3}\right)^n \right|$ divergen