TUGAS MANDIRI

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① a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya

$\cos\pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$

↳ rumus eksplisit

$$a_n = \frac{\cos n\pi}{n^2}$$

↳ kekonvergenannya

$-1 \le \cos n\pi \le 1$

$\frac{-1}{n^2} \le \frac{\cos n\pi}{n^2} \le \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{-1}{n^2} = 0$ dan $\lim_{n\to\infty} \frac{1}{n^2} = 0$

∴ maka baris $\frac{\cos(n\pi)}{n^2}$ konvergen ke 0

b) Diketahui $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan dgn definisi limit $\{a_n + b_n\}$ konvergen ke A + B

↳ $\{a_n\} \to A \Rightarrow \lim_{n\to\infty} a_n = A$. untuk setiap $\varepsilon > 0$ terdapat $N_1 > 0$ sedemikian sehingga untuk $n > N_1$ berlaku $\boxed{|a_n - A| < \frac{1}{2}\varepsilon}$

$\varepsilon > 0$ sembarang
$\frac{1}{2}\varepsilon > 0$ juga sembarang

↳ $\{b_n\} \to B \Rightarrow \lim_{n\to\infty} b_n = B$. untuk setiap $\varepsilon > 0$ terdapat $N_2 > 0$ sedemikian sehingga untuk $n > N_2$ berlaku $\boxed{|b_n - B| < \frac{1}{2}\varepsilon}$

↳ Pilih $N = \max\{N_1, N_2\}$

$$\begin{aligned} |a_n + b_n - (A+B)| &= |(a_n - A) + (b_n - B)| \\ &\le |a_n - A| + |b_n - B| \\ &< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon \\ &= \varepsilon \end{aligned}$$

∴ terbukti bahwa $\lim_{n\to\infty} (a_n + b_n) = A + B$

c) tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan $a_n = \sin\frac{n\pi}{4}$

↳ kemonotonan

$a_n = \frac{\sin n\pi}{4}$

$a'(n) = \frac{\pi}{4}\cos\frac{n\pi}{4}$

- $n=1 \to a'(1) = \frac{1}{8}\sqrt{2}\pi$
- $n=2 \to a'(2) = 0$
- $n=3 \to a'(3) = -\frac{1}{8}\sqrt{2}\pi$

↳ tdk naik dan tdk turun karena kadang (-) dan (+)

↳ keterbatasan

$-1 \le \sin \frac{n\pi}{4} \le 1$ → teorema apit tidak dpt digunakan

↳ limit

$$\lim_{n\to\infty} \frac{\sin n\pi}{4} \overset{LH}{=} \lim_{n\to\infty} \frac{\pi}{4} \cos n\pi = \infty$$ DIVERGEN

(2) a) Tulis rumus eksplisit barisan berikut dan tentukan kekonvergenannya

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$

↳ rumus eksplisit : $a_n = \frac{(-1)^{n+1}}{n}$

↳ kekonvergenan

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{n} \right| = \lim_{n\to\infty} |(-1)^{n+1}| \left|\frac{1}{n}\right| = \lim_{n\to\infty} \left|\frac{1}{n}\right| = 0$$ konvergen ke 0

b) Dengan definisi limit, buktikan barisan $\{a_n\}$ berikut konvergen:

$$a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$$

↳ $$\lim_{n\to\infty} \frac{3 - 8\cdot 2^n}{5 + 4\cdot 2^n} \overset{LH}{=} \lim_{n\to\infty} \frac{-8\cdot 2^n}{4 \cdot 2^n} = -2$$

maka $L = -2$

↳ $$|a_n - L| = \left| \frac{3 - 8\cdot 2^n}{5 + 4\cdot 2^n} + 2 \right|$$
$$= \left| \frac{3 - 8\cdot 2^n + 2(5 + 4\cdot 2^n)}{5 + 4 \cdot 2^n} \right|$$
$$= \frac{13}{5 + 4\cdot 2^n} \le \frac{13}{5 + 4 \cdot 2^N} = \frac{13}{5 + 4\left(\frac{13/\varepsilon - 5}{4}\right)} = \varepsilon$$

↓

terbukti

Pilih $N = \frac{\ln\left(\frac{13}{\varepsilon} - 5\right) - \ln 4}{\ln 2}$

$2^N = \frac{13/\varepsilon - 5}{4}$

analisis pemilihan nilai N

$$\frac{13}{5 + 4\cdot 2^N} = \varepsilon$$

$$\frac{13}{5 + 2^{N+2}} = \varepsilon$$

$$13 = 5\varepsilon + 2^{N+2}\varepsilon$$

$$2^{N+2} = \frac{13 - 5\varepsilon}{\varepsilon}$$

$$N + 2 \ln(2) = \ln\left(\frac{13}{\varepsilon} - 5\right)$$

$$N + 2 = \frac{\ln\left(\frac{13}{\varepsilon} - 5\right)}{\ln(2)}$$

$$N = \frac{\ln\left(\frac{13}{\varepsilon} - 5\right) - \ln(4)}{\ln(2)}$$

c) Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan

$$a_n = \frac{\ln n}{n}$$

↳ kemonotonan

$$a'_n = \frac{\frac{d}{dn}(\ln(n)).n - \frac{d}{dn}(n).\ln(n)}{n^2}$$

$$a'_n = \frac{\frac{1}{n}.n - \ln(n)}{n^2} = \frac{1-\ln(n)}{n^2}$$

↳ keterbatasan

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln(n)}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{n}$$

$$= \lim_{n\to\infty} \frac{1}{n^2} = 0$$

$\therefore \frac{\ln(n)}{n}$ konvergen ke 0

a_n naik ketika $a_n' > 0$

$$\frac{1-\ln(n)}{n^2} > 0$$

$$1-\ln(n) > 0$$

$$\ln(n) < 1$$

$$\ln(n) < \ln(e)$$

$$n < e$$

a_n turun ketika $a_n' < 0$

$$\frac{1-\ln(n)}{n^2} < 0$$

$$1-\ln(n) < 0$$

$$\ln(n) > 1$$

$$\ln(n) > \ln(e)$$

$$n > e$$

a_n naik ketika $n<e$ dan a_n turun ketika $n>e$ berarti a_n bukan barisan monoton

3) a) rumus eksplisit dan kekonvergenan

0.9, 0.99, 0.999, 0.9999, ...

↳ rumus eksplisit

$$a_n = \frac{10^n - 1}{10^n}$$

↳ kekonvergenan

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{10^n - 1}{10^n} = \lim_{n\to\infty} \frac{10^n\left(1-\frac{1}{10^n}\right)}{10^n} = 1$$

$a_n = \frac{10^n - 1}{10^n}$ konvergen ke 1

b) dengan definisi limit, buktikan $\{a_n\}$ konvergen

$$a_n = \frac{n+3}{3n-2}$$

$$\lim_{n\to\infty} \frac{n+3}{3n-2} = \frac{1}{3} \to L = \frac{1}{3}$$

$$|a_n - L| = \left| \frac{n+3}{3n-2} - \frac{1}{3} \right|$$
$$= \left| \frac{n+3}{3n-2} - \frac{n-\frac{2}{3}}{3(n-\frac{2}{3})} \right|$$
$$= \left| \frac{(n+3)-(n-\frac{2}{3})}{3n-2} \right|$$
$$= \left| \frac{3+\frac{2}{3}}{3n-2} \right| = \left| \frac{11}{3(3n-2)} \right| = \frac{11}{9n-6} \leq \frac{11}{9N-6}$$
$$= \frac{11}{9\left(\frac{11+6\varepsilon}{9\varepsilon}\right)-6}$$
$$= \varepsilon$$

juga untuk setiap $\varepsilon > 0$ terdapat $N > 0$ sedemikian sehingga $n \geq N$ berlaku $|a_n - L| < \varepsilon$

analisis pemilihan nilai n

$$\frac{11}{9n-6} \leq \frac{11}{9N-6} = \varepsilon$$
$$11 = 9N\varepsilon - 6\varepsilon$$

$$11 + 6\varepsilon = 9N\varepsilon$$
$$N = \frac{11+6\varepsilon}{9\varepsilon}$$

Pilih $N = \frac{11+6\varepsilon}{9\varepsilon}$

c) kemonotonan, keterbatasan, limit

$$a_n = \frac{n!}{10^n}$$

↳ kemonotonan

$$\frac{a_n}{a_{n+1}} = \frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{n+1}}} = \frac{n!}{10^n} \cdot \frac{10^{n+1}}{(n+1)!} = \frac{\cancel{n!}}{10^n} \cdot \frac{10^n \cdot 10}{(n+1)\cancel{n!}} = \frac{10}{n+1}$$

$\frac{a_n}{a_{n+1}} < 1$; $n = 10, 11, 12, \ldots$ (naik)

$\frac{a_n}{a_{n+1}} \geq 1$: $n = 1, 2, \ldots, 9$ (tak naik)

$a_n = \frac{n!}{10^n}$ bukan barisan monoton

↳ keterbatasan

$$\lim_{n\to\infty} \frac{n!}{10^n} = \lim_{n\to\infty} \frac{n(n-1)\ldots 2.1}{10.10.10\ldots10}$$
$$= \lim_{n\to\infty} \frac{n.(n-1).(n-2)\ldots 2.1}{10.10.10\ldots 10.0}$$
$$= \infty$$

$a_n = \frac{n!}{10^n}$ tak terbatas di atas (divergen)