# TUGAS KELOMPOK MINGGU 5

KALKULUS II

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Periksa kekonvergenan deret yang diberikan dan sebutkan jenis uji yang digunakan:

1. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

   Uji Banding

   $$\frac{3n+1}{n^2} \leq \frac{3n+1}{n^2-4} \leq \cdots$$

   $$\sum_{n=1}^{\infty} \frac{3n+1}{n^2} \leq \sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}\ (DIVERGEN)$$

   Uji Banding Limit: $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4} \leftrightarrow \frac{3n+1}{n^2} \rightarrow DIVERGEN$

   $$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

   $$L = \lim_{n\to\infty} \frac{3n+1}{n^2-4} \times \frac{n^2}{3n+1}$$

   $$L = \lim_{n\to\infty} \frac{n^2}{n^2-4}$$

   $$L = 1 > 0\ (Bersama - sama\ Divergen)$$

2. $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$

   Uji Banding Limit

   $a_n = \frac{n}{n^2+2n-3}$

   $b_n = n^{p-q} = n^{1-2} = \frac{1}{n}$ $\longrightarrow$ Deret harmonik (divergen)

   $L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n^2+2n-3} \times \frac{n}{1} = 1$ $\longrightarrow$ $0 < \text{L} < \infty$

Maka $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$ dan $\sum_{n=1}^{\infty} \frac{n}{n^2+2n-3}$ bersama-sama divergen

3. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

   Menggunakan teorema uji hasil bagi yaitu $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = p$

$$\lim_{n\to\infty} \frac{\frac{(n+1)!}{(n+1)^{100}}}{\frac{n!}{n^{100}}} = \lim_{n\to\infty} \frac{(n+1)! \times n^{100}}{n! \times (n+1)^{100}} = \lim_{n\to\infty} \frac{n^{100}}{(n+1)^{99}} = \infty$$

   Menurut teorema uji hasil bagi, jika $p$ lebih dari 1 maka deret tersebut divergen.

4. $\sum_{n=1}^{\infty} \frac{3^k+k}{k!}$

   Menggunakan uji hasil bagi

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$$

$$\rho = \lim_{k\to\infty} \frac{3^{k+1}+(k+1)}{(k+1)!} \times \frac{k!}{3^k+k} = \lim_{k\to\infty} \frac{3^{k+1}+(k+1)}{(k+1)} \times \frac{1}{3^k+k} =$$

$$\lim_{k\to\infty} \frac{3^k.3+k+1}{3^k.k+k^2+3^k+k} \times \frac{\frac{1}{3^k}}{\frac{1}{3^k}} = \lim_{k\to\infty} \frac{3+\frac{k}{3^k}+\frac{1}{3^k}}{k+\frac{k^2}{3^k}+1+\frac{k}{3^k}} =$$

$$\lim_{k\to\infty} \frac{3+0+0}{k+0+1+0} = \lim_{k\to\infty} \frac{3}{k+1} = 0$$

   Karene $\rho < 1$, maka $\sum_{n=1}^{\infty} \frac{3^k+k}{k!}$ Konvergen.

5. $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4}$

   Uji Banding

$$\frac{3n+1}{n^2} \leq \frac{3n+1}{n^2-4} \leq \cdots$$

$$\sum_{n=1}^{\infty} \frac{3n+1}{n^2} \leq \sum_{n=1}^{\infty} \frac{3n+1}{n^2-4} \ (DIVERGEN)$$

   Uji Banding Limit: $\sum_{n=1}^{\infty} \frac{3n+1}{n^2-4} \leftrightarrow \frac{3n+1}{n^2} \to DIVERGEN$

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

$$L = \lim_{n\to\infty} \frac{3n+1}{n^2-4} \times \frac{n^2}{3n+1}$$

$$L = \lim_{n\to\infty} \frac{n^2}{n^2-4}$$

$$L = 1 > 0\ (Bersama - sama\ Divergen)$$

6. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n$

   JAWAB:

   Dengan menggunakan uji akar:

   $$a_n = \left(\frac{n}{3n+2}\right)^n$$

   $$R = \lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \lim_{n\to\infty} \left(\left(\frac{n}{3n+2}\right)^n\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{3n+2} = \frac{1}{3} < 1$$

   $$R < 1\ maka\ \sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^n\ konvergen$$

7. $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n$

   Teorema Uji akar

   Karena suku suku pada deret positif

   $$\sum_{n=2}^{\infty} \left(\frac{1}{\ln n}\right)^n$$

   Maka dapat menggunakan

   $$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = R$$

   $$\lim_{n\to\infty} \left(\left(\frac{1}{\ln n}\right)^n\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{\ln n} = 0$$

   Karena R < 1 maka konvergen

Tentukan apakah deret yang diberikan adalah konvergen mutlak, konvergen bersyarat, atau divergen.

8. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$

   Uji ganti tanda

   a. Uji kemonotonan

$$a_n = \frac{n}{n+1}$$

$$a_n - a_{n+1} = \frac{n}{n+1} - \frac{n+1}{n+2}$$

$$= \frac{(n^2+2n)-(n^2+2n+1)}{(n+1)(n+2)} = \frac{-1}{(n+1)(n+2)} < 0\,, n \geq 1$$

Maka, $a_n - a_{n+1} < 0$ monoton naik untuk $\{a_n\}$

b. Uji limit $a_n$

$$\lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$$

$\sum u_n$ divergen menurut uji ganti tanda.

9. $\sum_{n=1}^{\infty} \sin\frac{n!}{n^2}$

$U_n$ = $\sin\frac{n!}{n^2}$

$|U_n|$ = $\left|\sin n!\ \frac{1}{n^2}\right|$ = $\frac{1}{n^2}\,|\sin n!\ | \leq \frac{1}{n^2}$

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ merupakan deret p yang konvergen karena p = 2 > 1, maka deret $\sum_{n=1}^{\infty}|U_n|$ konvergen.

Akibatnya deret $\sum_{n=1}^{\infty} U_n$ konvergen mutlak

10. $\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^n$

Uji akar :

$\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^n$

$$R = \lim_{n\to\infty}(a_n)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty}\left(\left(-\frac{4}{3}\right)^n\right)^{\frac{1}{n}}$$

$$= -\frac{4}{3}$$

❖ $-\frac{4}{3} < 1$ ; Konvergen

$\sum_{n=1}^{\infty}\left|\left(-\frac{4}{3}\right)^n\right|$

$\sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n = \frac{4}{3} > 1$

❖ Divergen

Jadi, $\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^{n}$ ; Konvergen bersyarat