KELOMPOK 5 KALKULUS II


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① $a_n = \dfrac{n}{3n-1}$

$$\lim_{n\to\infty} \frac{n}{3n-1} = \frac{1}{3}$$

$\boxed{\therefore \text{ Konvergen ke } \tfrac{1}{3}}$

* Kemonotonan

$a_n - a_{n+1}$

$$\frac{n}{3n-1} - \frac{(n+1)}{3(n+1)-1}$$

$$\frac{n}{3n-1} - \frac{(n+1)}{3n+2}$$

$$\frac{n(3n+2)-(3n-1)(n+1)}{(3n-1)(3n+2)}$$

$$= \frac{3n^2+2n-(-3n^2+2n-1)}{9n^2+3n-2}$$

$$= \frac{1}{9n^2+3n-2} > 0$$

$\therefore$ monoton turun

② $a_n = \dfrac{n^3+3n^2+3n}{(n+1)^3}$

$$\lim_{n\to\infty} = \frac{\frac{n^3+3n^2+3n}{n^3}}{\frac{n^3+\dots}{n^3}} = \frac{1}{1}$$

$\boxed{\therefore \text{ Konvergen ke } 1}$

* Kemonotonan.

$$\frac{n^3+3n^2+3n}{(n+1)^3} - \frac{(n+1)^3+3(n+1)^2+3(n+1)}{(n+1+1)^3}$$

$$= \frac{-3n^2-9n-7}{(n^2+3n+2)^2} < 0$$

$\therefore$ monoton naik

③ $a_n = \frac{\cos(n\pi)}{n}$

$$-1 \leq \cos(n\pi) \leq 1$$

$$\frac{-1}{n} \leq \frac{\cos(n\pi)}{n} \leq \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{-1}{n} = 0 \qquad \lim_{n\to\infty} \frac{1}{n} = 0$$

Otomatis 0 juga (menurut teorema apit).

$\boxed{\therefore \text{Konvergen ke } 0}$

* Kemonotonan

$$\frac{\cos n\pi}{n} - \frac{\cos (n+1)\pi}{n+1}$$

$$= \frac{(n+1)\cos n\pi - n\cos(n+1)\pi}{n^2+n}$$

$\therefore$ monoton tidak naik & tidak turun

4. $a_n = e^{-n} \sin n$

$a_n = \dfrac{\sin n}{e^n}$

$$\lim_{n \to \infty} \frac{\sin n}{e^n} = 0$$

$\boxed{\therefore \text{ konvergen ke } 0}$

* Kemonotonan

$a_n - a_{n+1} = (e^{-n} \sin n) - (e^{-(n+1)} \sin(n+1))$

$= \dfrac{e \sin(n) - \sin(n+1)}{e^{n+1}} > 0$

$\therefore$ monoton turun

5. $a_n = \dfrac{1}{n^3}$

$$\lim_{n \to \infty} \frac{1}{n^3} = 0$$

$\boxed{\therefore \text{ Konvergen ke } 0}$

* Kemonotonan

$a_n - a_{n+1} = \dfrac{1}{n^3} - \dfrac{1}{(n+1)^3}$

$= \dfrac{(n+1)^3 - n^3}{n^3 (n+1)^3} > 0$

$\therefore$ monoton turun

6. $\frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}$

$$\boxed{a_n = \frac{1}{2^{n+1}} \; ; \; n = 1, 2, 3, \ldots}$$ → rumus eksplisit

$$\lim_{n \to \infty} \frac{1}{2^{n+1}} = 0$$

$\therefore$ Konvergen ke 0

7. $\sin 1, 2 \sin \frac{1}{2}, 3 \sin \frac{1}{3}, 4 \sin \frac{1}{4}, \ldots$

$$\boxed{a_n = n \sin \frac{1}{n} \; ; \; n = 1, 2, 3, \ldots}$$ → rumus eksplisit

$$\lim_{n \to \infty} n \sin \frac{1}{n}$$

misal :

$n = \frac{1}{t}$ $\quad t = \frac{1}{n}$ $\quad n \to \infty$, $t \to 0$

$$\lim_{t \to 0} \frac{1}{t} \sin t = 1$$

$\therefore$ Konvergen ke 1

8. $0,1 , 0,11 , 0,111 , 0,1111 , \ldots$

$$\boxed{a_n = \frac{10^n - 1}{9 \cdot 10^n} \; ; \; n = 1, 2, 3, \ldots}$$ → rumus eksplisit

$$\lim_{n \to \infty} \frac{10^n - 1}{9 \cdot 10^n} = \frac{\frac{10^n - 1}{10^n}}{\frac{9 \cdot 10^n}{10^n}} = \frac{1}{9}$$

$\therefore$ Konvergen ke $\frac{1}{9}$