TUGAS INDIVIDU P3

(1) ⓐ $\cos\pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \ldots$

$a_n = \frac{\cos n\pi}{n^2}$ dengan $n = 1, 2, \ldots$

* kekonvergenan

$\lim_{n\to\infty} \frac{\cos n\pi}{n^n}$

$= 0$

maka konvergen ke 0 //

ⓑ $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan dg definisi limit $\{a_n+b_n\}$ konvergen ke A+B.

$\Rightarrow \lim_{n\to\infty} a_n = A \rightarrow |a_n - A| < \frac{1}{2}\varepsilon$

$\lim_{n\to\infty} b_n = B \rightarrow |b_n - B| < \frac{1}{2}\varepsilon$

$\Rightarrow |a_n + b_n - (A+B)| = |(a_n - A) + (b_n - B)|$

$\leq |a_n - A| + |b_n - B|$

$< \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon$

$= \varepsilon$

maka $\lim_{n\to\infty} (a_n + b_n) = A+B$ //

ⓒ $a_n = \sin\left(\frac{n\pi}{4}\right)$

* kemonotonan

☆ $\frac{a_n}{a_{n+1}}$

$\Rightarrow \frac{\sin(n\pi/4)}{\sin[(n\pi+\pi)/4]}$

$\Rightarrow \frac{\sin(n\pi/4)}{-\sin(n\pi/4)} < 1$

☆ $a_n - a_{n+1}$

$\sin\frac{(n\pi)}{4} - \sin\frac{(n\pi+\pi)}{4}$

$2\sin\left(\frac{n\pi}{4}\right) \geq 0$

maka a_n bukan barisan monoton

* limit

$\lim_{n\to\infty} \frac{\sin n\pi}{4}$

= tdk ada limit (divergen)

* keterbatasan

karena alternating maka tidak memiliki batas.

(2) ⓐ $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots$

$a_n = (-1)^{n+1}\left(\frac{1}{n}\right)$

* konvergen

$\lim_{n\to\infty} \left|(-1)^{n+1}\left(\frac{1}{n}\right)\right| = \lim_{n\to\infty} 1 \cdot \left(\frac{1}{n}\right)$

$= 0$

maka konvergen ke 0 //

ⓑ $a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$

$\lim_{n\to\infty} = \frac{3/2^n - 8}{5/2^n + 4}$

$= \frac{-8}{4} = -2$

konvergen ke -2 //

ⓒ $a_n = \frac{\ln n}{n}$

* kemonotonan

☆ $\frac{a_n}{a_{n+1}}$

$\Rightarrow \frac{\ln n / n}{\ln(n+1)/(n+1)}$

$\frac{(n+1)\ln n}{n \ln(n+1)} > 1$

☆ $a_n - a_{n+1}$

$\frac{\ln n}{n} - \frac{\ln(n+1)}{(n+1)}$

$\frac{(n+1)\ln n - n\ln(n+1)}{n^2+n} < 0$

Bukan barisan monoton //

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* limit

$\lim_{n\to\infty} \frac{\ln n}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{1/n}{1} = 0$ Konvergen ke 0

* keterbataran

Karena an bukan barisan monoton, sehingga tidak terbatas

* limit

$\lim_{n\to\infty} \frac{n!}{10^n} = \frac{1.2.3\ldots n}{10.10\ldots10^n} = \frac{\infty}{\infty}$ = bentuk tak tentu (divergen)

* keterbataran

Karena divergen maka tidak ada batas atas

(3) (a) 0,9; 0,99; 0,999; 0,9999;

$a_n = 1 - 0{,}1^n$

* konvergen

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1 - 0{,}1^n$

$= 1$

maka konvergen ke 1

(b) $a_n = \frac{n+3}{3n-2}$

$\lim_{n\to\infty} \frac{n+3}{3n-2} = \frac{1}{3}$

konvergen ke $\frac{1}{3}$

(c) $a_n = \frac{n!}{10^n}$

* kemonotonan

$\frac{a_n}{a_{n+1}}$

$\frac{n!/10^n}{(n+1)!/10^{n+1}} < 1$

$a_n - a_{n+1}$

$\frac{n!}{10^n} - \frac{(n+1)!}{10^{n+1}} < 0$

barisan monoton naik

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