Tugas Responsi 2 Kalkulus Kelompok 4

Kelompok 4:

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1. Tentukan integral berikut:

(a) $\displaystyle\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{x}{\sqrt{9+x^2}}\,dx$

Jawab:

Tgl.

Tukel 2

1. a) $\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx$

$= \lim_{b\to\infty} \int_3^b \frac{x}{\sqrt{16+x^2}}\,dx$, misal $u = 16+x^2$ $\quad du = 2x\,dx$

$\frac{du}{dx} = 2x \quad \frac{du}{2} = x\,dx$

$= \lim_{b\to\infty} \int_3^b \frac{1}{2\sqrt{u}}\,du$

$= \frac{1}{2} \lim_{b\to\infty} \int_{25}^{16+b^2} u^{-\frac{1}{2}}\,du$

$= \frac{1}{2} \lim_{b\to\infty} \left(u^{1/2} \cdot 2 \right|_{25}^{16+b^2}$

$= \frac{1}{2} \cdot \lim_{b\to\infty} 2\sqrt{16+b^2} - 2\sqrt{25}$

$= \infty$ (divergen)

b) $\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}}\,dx$

$= \lim_{a\to-\infty} \int_a^0 \frac{x}{\sqrt{9+x^2}}\,dx + \lim_{b\to\infty} \int_0^b \frac{x}{\sqrt{9+x^2}}\,dx$

No.

mis, $9+x^2 = u$

$2x = \frac{du}{dx}$, $du = 2x\,dx$

$= \lim_{a \to -\infty} \int_{9+a^2}^{9} \frac{1}{2} u^{-\frac{1}{2}}\, du + \lim_{b \to \infty} \int_{9}^{9+b^2} \frac{1}{2} u^{-\frac{1}{2}}\, du$

$= \frac{1}{2} \lim_{a \to -\infty} \left(2u^{\frac{1}{2}} \right|_{9+a^2}^{9} + \frac{1}{2} \lim_{b \to \infty} \left(2u^{\frac{1}{2}} \right|_{9}^{9+b^2}$

$= \lim_{a \to -\infty} \sqrt{9} - \sqrt{9+a^2} + \lim_{b \to \infty} \sqrt{9+b^2} - \sqrt{9}$

$=$ divergen $(-\infty + \infty)$

2. Tentukan integral berikut:

(a) $\displaystyle\int_2^{\infty} \frac{\ln \sqrt{x}}{x}\, dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\, dx$

Jawab:

2. a) $\displaystyle\int_2^{\infty} \frac{\ln \sqrt{x}}{x}\, dx$

$\displaystyle = \lim_{b \to \infty} \int_2^{b} \frac{\ln x^{\frac{1}{2}}}{x}\, dx$

$\displaystyle = \lim_{b \to \infty} \int_2^{b} \frac{1}{2} \cdot \frac{\ln x}{x}\, dx$ misal, $\ln x = u$

$\displaystyle \frac{1}{x} = \frac{du}{dx}$

$\displaystyle = \frac{1}{2} \lim_{b \to \infty} \int_{\ln}^{\ln b} u\, du$ $\displaystyle du = \frac{dx}{x}$

$\displaystyle = \frac{1}{2} \cdot \lim_{b \to \infty} \left(\frac{u^2}{2} \right|_{\ln 2}^{\ln b}$

$\displaystyle = \frac{1}{2} \lim_{b \to \infty} \frac{(\ln b)^2}{2} - \frac{(\ln 2)^2}{2}$

= divergen

b) $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)} \, dx$

$$= \lim_{a \to -\infty} \int_{a}^{0} \frac{x}{(x^2+4)} \, dx + \lim_{b \to \infty} \int_{0}^{b} \frac{x}{(x^2+4)} \, dx$$

misal, $x^2+4 = u$

$2x = \frac{du}{dx}$

$\frac{du}{2} = x \, dx$

$$= \lim_{a \to -\infty} \int_{a^2+4}^{4} \frac{du}{2u} + \lim_{b \to \infty} \int_{4}^{b^2+4} \frac{du}{2u}$$

$$= \frac{1}{2} \lim_{a \to \infty} \left(\ln u \right|_{a^2+4}^{4} + \frac{1}{2} \lim_{b \to \infty} \left. (\ln u) \right|_{4}^{b^2+4}$$

$$= \frac{1}{2} \lim_{a \to -\infty} \left(\ln 4 - \ln(a^2+4) \right) + \frac{1}{2} \lim_{b \to \infty} \left(\ln(b^2+4) - \ln 4 \right)$$

$$= \infty$$

3. Tentukan integral berikut:

(a) $\displaystyle\int_2^{\infty} \frac{1}{x \ln x}\, dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2 + 4x + 9}\, dx$

Jawab:

No. 3

a) $\displaystyle\int_2^{\infty} \frac{1}{x \ln x}\, dx$

$= \displaystyle\lim_{b \to \infty} \int_2^{b} \frac{1}{x \ln x}\, dx$

misal, $u = \ln x$

$\dfrac{du}{dx} = \dfrac{1}{x}$

$= \displaystyle\lim_{b \to \infty} \int_{\ln 2}^{\ln b} \frac{du}{u}$

$= \displaystyle\lim_{b \to \infty} \left(\ln u \right|_{\ln 2}^{\ln b}$

$= \displaystyle\lim_{b \to \infty} \ln(\ln b) - \ln(\ln 2)$

$= \infty$ (divergen)

b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\, dx$

$= \displaystyle\lim_{a \to -\infty} \int_a^0 \frac{1}{x^2+4x+9}\, dx + \lim_{b \to \infty} \int_0^{\infty} \frac{1}{x^2+4x+9}\, dx$

$$= \lim_{a \to -\infty} \int_a^0 \frac{1}{(x+2)^2+5}\,dx + \lim_{b \to \infty} \int_0^b \frac{1}{(x+2)^2+5}\,dx$$

$$= \lim_{a \to -\infty} \int_a^0 \frac{1}{(x+2)^2+(\sqrt{5})^2}\,dx + \lim_{b \to \infty} \int_0^b \frac{1}{(x+2)^2+(\sqrt{5})^2}\,dx$$

$$= \lim_{a \to -\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_a^0 + \lim_{b \to \infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_0^b$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0+2}{\sqrt{5}}\right) - \lim_{a \to -\infty} \tan^{-1}\left(\frac{a+2}{\sqrt{5}}\right) \cdot \frac{1}{\sqrt{5}} +$$

$$\lim_{b \to \infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{b+2}{\sqrt{5}}\right) - \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0+2}{\sqrt{5}}\right)$$

$$= \frac{1}{\sqrt{5}} \left(\cancel{\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)} - \tan^{-1}(-\infty) + \tan^{-1}(\infty) - \cancel{\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)} \right)$$

$$= \frac{1}{\sqrt{5}} \left(0 - \left(\frac{-\pi}{2}\right) + \frac{\pi}{2} \right)$$

$$= \frac{\pi}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\pi\sqrt{5}}{5}$$

4. Tentukan integral berikut:

(a) $\displaystyle\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{x}{e^{|x|}}\, dx$

Jawab:

Tgl.

a) $\displaystyle\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx$

mis : $u = \ln x$,

$\dfrac{du}{dx} = \dfrac{1}{x}$ $\quad du = \dfrac{dx}{x}$

$= \displaystyle\lim_{b\to\infty} \int_{\ln 2}^{\ln b} \frac{du}{u^2}$

$= \displaystyle\lim_{b\to\infty} \left(-u^{-1} \Big|_{\ln 2}^{\ln b}\right.$

$= \displaystyle\lim_{b\to\infty} -\frac{1}{\ln b} + \frac{1}{\ln 2}$

$= 0 + \dfrac{1}{\ln 2}$

$= \dfrac{1}{\ln 2}$ (konvergen)

b) $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}} dx$

$$= \int_{-\infty}^{0} \frac{x}{e^{|x|}} dx + \int_{0}^{\infty} \frac{x}{e^{|x|}} dx \qquad |x| \begin{cases} x ; & x > 0 \\ -x ; & x < 0 \end{cases}$$

$$= \lim_{a \to -\infty} \int_{a}^{0} \frac{x}{e^{-x}} dx + \lim_{b \to \infty} \int_{0}^{b} \frac{x}{e^{x}} dx$$

$$= \lim_{a \to -\infty} \left(xe^{x} - e^{x} \right|_{a}^{0} + \lim_{b \to \infty} \left(- \frac{x+1}{e^{x}} \right|_{0}^{b}$$

$$= \lim_{a \to -\infty} \left(-1 - ae^{a} + e^{a} \right) + \lim_{b \to \infty} \left(- \frac{b+1}{e^{b}} + 1 \right)$$

$$= \left(-1 - (-0) + 0 \right) + \left(-0 + 1 \right)$$

$$= 0 \text{ (konvergen)}$$