Tugas Individu - Cahyani Ayah Rofiana.

1a. Rumus Eksplisit & tentukan kekonvergenan.

$$\cos\pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}; \dots$$

b. Diket $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan (dgn definisi limit) $\{a_n + b_n\}$ konvergen A+B.

c. Tentukan kemonotonan, keterbatasan, dan limit (jika ada) barisan brkt.

$$a_n = \sin \frac{n\pi}{4}$$

— Jawab. —

1.a. Rumus : $\frac{\cos n\pi}{n^2}$; $n = 1, 2, 3 \dots$

$-1 \le \cos n\pi \le 1$

$$\frac{-1}{n^2} \le \frac{\cos n\pi}{n^2} \le \frac{1}{n^2}$$

$\lim_{n\to\infty} \frac{-1}{n^2} = 0 \qquad \downarrow 0 \qquad \lim_{n\to\infty} \frac{1}{n^2} = 0$

∴ konvergen ke ~~...~~ 0.

b. $\{a_n\}$ konvergen ke A & $\{b_n\}$ konvergen ke B.

$$\left.\begin{array}{l}\lim_{n\to\infty} a_n = A \\ \lim_{n\to\infty} b_n = B\end{array}\right\} \lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm b_n = A \pm B = A + B.$$

$|(a_n + b_n) - (A + B)| < \varepsilon$; $\{a_n\}$ konvergen ke A

$L = A$ ↓

Be diligent

$\varepsilon > 0$ terdapat $N > 0$

$\therefore n \geq N$

$|a_n - L| < \frac{\varepsilon}{2}$

$|a_n - A| \leq \frac{\varepsilon}{2}.$

• $\{b_n\}$ konvergen ke B

$L = B$; $\varepsilon > 0$ terdapat $N > 0$

$\therefore n \geq N$

$|b_n - L| < \varepsilon/2$

$|b_n - B| < \varepsilon/2$

$\rightarrow |(a_n + b_n) - (A+B)| \leq |a_n - A| + |b_n - B| < \varepsilon/2 + \varepsilon/2 = \varepsilon$

c. $a_n = \sin \frac{n\pi}{4}.$

$-1 \leq \sin n\pi \leq 1$

$-1/4 \leq \sin n\pi/4 \leq 1/4$ bukan monoton.

↳ divergen

↓ naik turun

2a) $1, -1/2, 1/3, -1/4, 1/5, 1/6$

Rumus : $\left| (-1)^{n+1} (1/n) \right| \rightarrow 1/n$

$$\lim_{n\to\infty} 1/n = 0$$

konvergen ke 0

b $a_n = \dfrac{3-8 \cdot 2^n}{5+4 \cdot 2^n}$

$$\lim_{n\to\infty} \frac{3-8\cdot 2^n}{5+4\cdot 2^n} = \lim_{n\to\infty} \frac{3-16^n}{5+8^n} = \frac{0-16}{0+8} = \frac{-16}{8} = -2$$

$$\hookrightarrow \frac{3/1^n - 16^n/1^n}{5/1^n + 8^n/1^n}$$

∴ Konvergen ke -2

c $a_n = \dfrac{\ln n}{n}$

★ Monoton . $a'(n) = \dfrac{\cancel{1/n \cdot n} - \ln . n . 1}{n^2}$

$= \dfrac{1-\ln n}{n^2}$ bukan monoton

★ Kekonvergenan $\lim_{n\to\infty} \dfrac{\ln . n}{n} = \lim_{n\to\infty} \dfrac{1/n}{1} = 0$

∴ Konvergen ke 0

3. a. 0.9 , 0.99 , 0.999

$\rightarrow 1 - 1/10^n ; n = 1,2,3$

$\lim_{n \to \infty} 1 - 1/10^n = 1 - 0 = 1$ Konvergen ke 1

b.) $a_n = \frac{n+3}{3n-1}$

Kekonvergenan

$\lim_{n \to \infty} \frac{n+3}{3n-1} = \frac{n/n + 3/n}{3n/n - 1/n} = \frac{1+0}{3-0} = \frac{1}{3}$. Konvergen ke 1/3

c) $a_n = \frac{n!}{10^n}$

Kemonotonan

$\frac{a_n}{a_{n+1}} = \frac{n!/10^n}{(n+1)!/10^{(n+1)}}$

$= \frac{\frac{1,2,3 \cdots n}{10 \cdot 10 \cdot 10 \cdots 10^n}}{\frac{1 \cdot 2 \cdot 3 \cdots n\ (n+1)}{10 \cdot 10 \cdot 10 \cdot 10^n \cdot 10^{n+1}}} = \frac{1}{n+1} \cdot 10^{n+1}$

$= \frac{10^{n+1}}{n+1} \geq 1 \rightarrow$ monoton naik

Be diligent

No.

Date.

kekonvergenan

$a_n = \frac{n!}{10^n}$

$= \frac{1 \cdot 2 \cdot 3 \ldots n}{10 \cdot 10 \cdot 10 \ldots 10^n} = \frac{\infty}{\infty}$, tak tentu

↳ limit tidak ada

∴ Divergen