Tugas Responsi Pertemuan 2
Kalkulus 2

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1. Tentukan integral berikut
   a. $\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx$
   Jawab :

$\boxed{1.a}$ $$\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx$$

$$= \lim_{b\to\infty} \int_3^b \frac{x}{\sqrt{16+x^2}}\,dx$$

misal $u = 16 + x^2$

$\frac{du}{dx} = 2x$

$$= \lim_{b\to\infty} \int_{25}^b \frac{1}{2} u^{-1/2}\,du$$

$$= \lim_{b\to\infty} u^{1/2} \Big|_{25}^b$$

$$= \lim_{b\to\infty} \sqrt{16+x^2} \Big|_3^b$$

$$= \lim_{b\to\infty} \sqrt{16+b^2} - 5 = \infty$$

Divergen

b. $\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}}\, dx$

Jawab :

1. b. $\int_{-\infty}^{\infty} \frac{x}{\sqrt{9+x^2}}\, dx$

$= \lim_{a\to-\infty} \int_a^0 \frac{x}{\sqrt{9+x^2}}\, dx + \lim_{b\to\infty} \int_0^b \frac{x}{\sqrt{9+x^2}}\, dx$

misalkan $9+x^2 = u$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \lim_{a\to-\infty} \int_{9+a^2}^{9} \frac{1}{2\sqrt{u}}\, du + \lim_{b\to\infty} \int_9^{9+b^2} \frac{1}{2\sqrt{u}}\, du$

$= \lim_{a\to-\infty} \int_{9+a^2}^{9} \frac{1}{2} u^{-\frac{1}{2}}\, du + \lim_{b\to\infty} \int_9^{9+b^2} \frac{1}{2} u^{-1/2}\, du$

$= \lim_{a\to-\infty} u^{\frac{1}{2}} \Big|_{9+a^2}^{9} + \lim_{b\to\infty} u^{\frac{1}{2}} \Big|_{9}^{9+b^2}$

$= \sqrt{9} - \sqrt{9+a^2} + \sqrt{9+b^2} - \sqrt{9}$

$= (-\infty + \infty)$ divergen //

2. Tentukan integral berikut

a. $\int_2^{\infty} \frac{\ln\sqrt{x}}{x}\, dx$

Jawab :

2). a. $\int_2^{\infty} \frac{\ln\sqrt{x}}{x}\, dx$

Jawab :

$\int_2^{\infty} \frac{\ln\sqrt{x}}{x}\, dx$

$= \lim_{a\to\infty} \int_2^a \frac{\ln(x)^{1/2}}{x}\, dx$

$= \frac{1}{2} . \lim_{a\to\infty} \int_2^a \frac{\ln x}{x} . dx$

misalkan

$u = \ln x \to du = \frac{1}{x}\, dx$

B. atas : $\ln\infty = \infty$

B. bawah : $\ln 2$

$= \frac{1}{2} \lim_{a\to\infty} \int_{\ln 2}^{a} u . du$

$= \frac{1}{2} \lim_{a\to\infty} \left(\frac{u^2}{2} \right]_{\ln 2}^{a}$

$= \frac{1}{4} \lim_{a\to\infty} (a^2 - (\ln 2)^2)$

Divergen

b. $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\, dx$

Jawab :

2.) b. $\int_{-\infty}^{\infty} \frac{x}{x^2+4}\, dx$

$= \int_{-\infty}^{0} \frac{x}{x^2+4}\, dx + \int_{0}^{\infty} \frac{x}{x^2+4}\, dx$

$= \lim_{a \to -\infty} \int_{a}^{0} \frac{x}{x^2+4}\, dx + \lim_{b \to \infty} \int_{0}^{b} \frac{x}{x^2+4}\, dx$

misal $x^2+4 = u$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$

$\lim_{a \to \infty} \int_{a}^{4} \frac{1}{2u}\, du + \lim_{b \to \infty} \int_{4}^{b} \frac{1}{2u}\, du$

$= \lim_{a \to \infty} \left[\frac{\ln u}{2}\right]_{a}^{4} + \lim_{b \to \infty} \left[\frac{\ln u}{2}\right]_{4}^{b}$

= divergen

3. Tentukan integral berikut

a. $\int_{2}^{\infty} \frac{1}{x \ln x}\, dx$

Jawab :

3.) a.) $\int_{2}^{\infty} \frac{1}{x \ln x}\, dx = \lim_{a \to \infty} \int_{2}^{a} \frac{1}{x \ln x}\, dx = \lim_{a \to \infty} \int_{2}^{a} \frac{1}{t}\, dt = \lim_{a \to \infty} \left[\ln(|t|)\right]_{2}^{a}$

$= \lim_{a \to \infty} \left[\ln(|\ln x|)\right]_{2}^{a}$

$= \lim_{a \to \infty} \left[\ln(|\ln a|) - \ln(|\ln 2|)\right]$

misalkan $t = \ln(x)$, $t' = \frac{1}{x}$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x}\, dx$

(Divergen)

b. $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\, dx$

Jawab :

$$\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\, dx$$

$$\int_{-\infty}^{0} \frac{1}{x^2+4x+9}\, dx + \int_{0}^{\infty} \frac{1}{x^2+4x+9}\, dx$$

$$= \lim_{a\to -\infty} \int_{a}^{0} \frac{1}{x^2+4x+9}\, dx + \lim_{b\to\infty} \int_{0}^{b} \frac{1}{x^2+4x+9}\, dx$$

$$= \lim_{a\to -\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Big|_a^0 + \lim_{b\to\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Big|_0^b$$

$$= \lim_{a\to -\infty} \left(\frac{\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}x+2\sqrt{5}}{5}\right)}{5}\right)\Bigg|_a^0 + \lim_{b\to\infty} \left(\frac{\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}x+2\sqrt{5}}{5}\right)}{5}\right)\Bigg|_0^b$$

$$= \frac{\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{5} - \frac{\sqrt{5}\tan^{-1}(-\infty)}{5} + \frac{\sqrt{5}\tan^{-1}(\infty)}{5} - \frac{\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{5}$$

$$= \frac{\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right) - \left(\sqrt{5}\left(-\frac{\pi}{2}\right)\right)}{5} + \frac{\sqrt{5}\left(\frac{\pi}{2}\right) - \sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{5}$$

$$= \left(\frac{2\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right) + \pi\sqrt{5}}{10}\right) + \left(\frac{\pi\sqrt{5} - 2\sqrt{5}\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)}{10}\right)$$

$$= \frac{2\sqrt{5}}{10}\pi = \frac{\sqrt{5}}{5}\pi \quad \text{konvergen.}$$

4. Tentukan integral berikut

a. $\int_{2}^{\infty} \frac{1}{x\,(\ln x)^2}\, dx$

Jawab :

4. a. $$\int_{2}^{\infty} \frac{1}{x(\ln x)^2}\, dx$$

$t = \ln x$

$\frac{dt}{dx} = \frac{1}{x}$

$\frac{dx}{x} = dt$

$$\lim_{a\to\infty} \left(\int_2^a \frac{1}{x(\ln x)^2}\, dx\right)$$

$$\lim_{a\to\infty} \left(\int_2^a \frac{1}{t^2}\, dt\right)$$

$$\lim_{a\to\infty} \left(-\frac{1}{t}\Big|_2^a\right)$$

$$\lim_{a\to\infty} \left(-\frac{1}{\ln(x)}\Big|_2^a\right)$$

$$\lim_{a\to\infty} \left(-\frac{1}{\ln(a)} + \frac{1}{\ln(2)}\right)$$

$$\frac{1}{\ln(2)}$$ **konvergen**

b. $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}}\, dx$

Jawab :

4b. $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}}\, dx = \int_{-\infty}^{0} \frac{x}{e^{|x|}}\, dx + \int_{0}^{\infty} \frac{x}{e^{|x|}}\, dx$

$= \lim_{a \to -\infty} \int_{a}^{0} \frac{x}{e^{-x}}\, dx + \lim_{b \to \infty} \int_{0}^{b} \frac{x}{e^{x}}\, dx$

$= \lim_{a \to -\infty} \int_{a}^{0} \underline{x \cdot e^{x}\, dx} + \lim_{b \to \infty} \int_{0}^{b} \underline{x \cdot e^{-x}\, dx}$

① Misal : $u = x$ $\quad dv = e^{x} dx$

$du = dx \quad v = e^{x}$

② Misal : $u = x$ $\quad dv = e^{-x} dx$

$du = dx \quad v = e^{-x}$

Integral Parsial $\rightarrow \int u\, dv = uv - \int v\, du$

$= \lim_{a \to -\infty} \left(x \cdot e^{x} - e^{x} \right|_{a}^{0} + \lim_{b \to \infty} \left(x \cdot e^{-x} - e^{-x} \right|_{0}^{b}$

$= \left[\left(0 \cdot e^{0} - e^{0} \right) - \left(\lim_{a \to -\infty} a \cdot e^{a} - e^{a} \right) \right] + \left[\left(\lim_{b \to \infty} b \cdot e^{-b} - e^{-b} \right) - \left(0 \cdot e^{0} - e^{0} \right) \right]$

$= (-1 - 0) + (0 - (-1)) = -1 + 1 = 0$ (Konvergen)