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TUGAS MANDIRI

1. $\cos\pi, \frac{\cos 2\pi}{4}, \frac{\cos 3\pi}{9}, \frac{\cos 4\pi}{16}, \dots$

a) $\frac{\cos n\pi}{n^2}$ $n = 1, 2, 3, \dots$

↳ kekonvergenan

$-1 \leq \cos n\pi \leq 1$

$-\frac{1}{n^2} \leq \frac{\cos n\pi}{n^2} \leq \frac{1}{n^2}$

$\lim_{n\to\infty} -\frac{1}{n^2} = 0$ $\quad \downarrow \; 0 \quad \downarrow \quad \lim_{n\to\infty} \frac{1}{n^2} = 0$

konvergen ke 0

b) $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B

Buktikan $\{a_n + b_n\}$ konvergen ke A+B !

$\lim_{n\to\infty} a_n = A$ $\quad \lim_{n\to\infty} b_n = B$

$\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm b_n = A \pm B = A+B$

$|(a_n+b_n) - (A+B)| < \varepsilon$

$\{a_n\}$ konvergen ke A

$L = A$, akan dibuktikan

untuk tiap $\varepsilon > 0$ terdapat $N > 0$

sehingga $n \geq N$

$|a_n - L| < \frac{\varepsilon}{2}$

$|a_n - A| < \frac{\varepsilon}{2}$

$|(a_n+b_n) - (A+B)| \leq |a_n - A| + |b_n - B| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$ (terbukti)

c) $a_n = \sin \frac{n\pi}{4}$

$-1 \leq \sin\pi \leq 1$

Divergen (tidak ada limitnya)

alternating

bukan barisan monoton

2. a) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots$

$(-1)^{n+1}\left(\frac{1}{n}\right)$; $n = 1, 2, 3 \dots$

↳ kekonvergenan

$\left|(-1)^{n+1}\left(\frac{1}{n}\right)\right| \to \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$

konvergen ke 0

b) $a_n = \frac{3 - 8 \cdot 2^n}{5 + 4 \cdot 2^n}$

$\lim_{n\to\infty} \frac{3 - 16^n}{5 + 8^n} = -2$

konvergen ke -2

c) $a_n = \frac{\ln n}{n}$

↳ kemonotonan

$a'(n) = \frac{\frac{1}{n} \cdot n - 1 \cdot \ln n}{n^2}$

$= \frac{1 - \ln n}{n^2}$

bukan barisan monoton

↳ kekonvergenan

$\lim_{n\to\infty} \frac{\ln n}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{1/n}{1} = 0$

konvergen ke 0

3. a) $0{,}9,\ 0{,}99,\ 0{,}999,\ 0{,}9999$

$1 - \frac{1}{10^n}$; $n = 1, 2, 3 \dots$

$\lim_{n\to\infty} 1 - \frac{1}{10^n} = 1 - 0 = 1$

konvergen ke 1

b) $a_n = \frac{n+3}{3n-1}$

↳ kekonvergenan

$\lim_{n\to\infty} \frac{n+3}{3n-1} = \frac{1}{3}$

konvergen ke $1/3$

c) $a_n = \frac{n!}{10^n}$

↳ kemonotonan

$\frac{a_n}{a_{n+1}} = \frac{\frac{n!}{10^n}}{\frac{(n+1)!}{10^{(n+1)}}} = \frac{1.2.3 \dots n}{10.10 \dots 10^n} \cdot \frac{10.10 \dots 10^n}{1.2.3 \dots n}$

$= \frac{1}{n+1} \cdot 10^{n+1}$

$= \frac{10^{n+1}}{n+1} > 1$

monoton naik

↳ kekonvergenan

$a_n = \frac{n!}{10^n} = \frac{1.2.3 \dots n}{10.10 \dots 10^n} = \frac{\infty}{\infty}$ bentuk tak tentu tidak ada limit divergen