Kalkulus II

RESPONSI 02

KELOMPOK 9

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1. Tentukan integral berikut:

a.) $\int_3^\infty \frac{x}{\sqrt{16+x^2}}\,dx$

$\lim_{a\to\infty} \left(\sqrt{16+a^2} - \sqrt{16+9}\right)\Big|_3^\infty$

$\lim_{a\to\infty} \left(\sqrt{16+a^2} - 5\right)$

$= \infty$ // [DIVERGEN]

b.) $\int_{-\infty}^\infty \frac{x}{\sqrt{9+x^2}}\,dx$

$\int_{-\infty}^0 \frac{x}{\sqrt{9+x^2}}\,dx + \int_0^\infty \frac{x}{\sqrt{9+x^2}}\,dx$

$= \lim_{a\to-\infty} \int_a^0 \frac{x}{\sqrt{9+x^2}}\,dx + \lim_{b\to\infty} \int_0^b \frac{x}{\sqrt{9+x^2}}\,dx$

$= \lim_{a\to-\infty} \left(3 - \sqrt{9+a^2}\right) + \lim_{b\to\infty} \left(\sqrt{9+a^2} - 3\right)$

$= -\infty + \infty$ // [DIVERGEN]

2. Tentukan integral berikut:

a.) $\int_2^\infty \frac{\ln\sqrt{x}}{x}\,dx$

$= \lim_{b\to\infty} \int_2^b \frac{\ln\sqrt{x}}{x}\,dx \rightarrow \lim_{b\to\infty} \int_2^b \frac{\ln x^{1/2}}{x}\,dx$

$= \lim_{b\to\infty} \int_2^b \frac{1/2 \ln x}{x}\,dx \rightarrow \lim_{b\to\infty} \frac{1}{2} \int_2^b \frac{\ln x}{x}\,dx$

$= \lim_{b\to\infty} \left[\frac{1}{2}\cdot\frac{1}{2}(\ln x)^2\right]_2^b \rightarrow \lim_{b\to\infty} \left[\frac{1}{4}(\ln x)^2\right]_2^b$

$= \lim_{b\to\infty} \left[\frac{1}{4}(\ln b)^2\right] - \left[\frac{1}{4}(\ln 2)^2\right]$

$= \infty - 0{,}120 = \infty$ // [DIVERGEN] b.) $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)}\,dx$

$\int_{-\infty}^{0} \frac{x}{(x^2+4)}\,dx + \int_{0}^{\infty} \frac{x}{(x^2+4)}\,dx$

Misalkan: $u = x^2 + 4 \rightarrow \frac{du}{dx} = 2x \rightarrow dx = \frac{du}{2x}$

$$= \int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}} \rightarrow \frac{1}{2}\int \frac{1}{u}\cdot du$$

$$= \frac{1}{2}\ln u \rightarrow \frac{1}{2}\ln(x^2+4)$$

$$\rightarrow \lim_{a\to-\infty} \int_a^0 \frac{x}{x^2+4}\,dx + \lim_{b\to\infty}\int_0^b \frac{x}{x^2+4}\,dx$$

$$= \lim_{a\to-\infty}\left(\frac{1}{2}\ln(0^2+4) - \frac{1}{2}\ln(a^2+4)\right) + \lim_{b\to\infty}\left(\frac{1}{2}\ln(b^2+4) - \frac{1}{2}\ln(0^2+4)\right)$$

$$= \lim_{a\to-\infty}\left(\ln(2) - \frac{1}{2}\ln(a^2+4)\right) + \lim_{b\to\infty}\left(\frac{1}{2}\ln(b^2+4) - \ln(2)\right)$$

$$= -\infty + \infty \;//$$ [DIVERGEN]

3. Tentukan integral berikut:

a.) $\int_2^{\infty} \frac{1}{x \cdot \ln x}\,dx$

Misalkan: $u = \ln x$

$\frac{du}{dx} = \frac{1}{x} \rightarrow dx = du \cdot x$

$$= \lim_{b\to\infty}\int_{\ln 2}^{b} \frac{du}{u}$$

$$= \lim_{b\to\infty} \ln u \Big|_{\ln 2}^{b}$$

$$= \ln(\infty) - \ln(\ln 2)$$

$$= \infty \;//$$ [DIVERGEN]

b.) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+4x+9}\,dx$

$$\int_{-\infty}^{0} \frac{1}{x^2+4x+9}\,dx + \int_{0}^{\infty} \frac{1}{x^2+4x+9}\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{x^2+4x+9}\,dx + \lim_{b\to\infty} \int_{0}^{b} \frac{1}{x^2+4x+9}\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{(x+2)^2+5}\,dx + \lim_{b\to\infty} \int_{0}^{b} \frac{1}{(x+2)^2+5}\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{(x+2)^2+(\sqrt{5})^2}\,dx + \lim_{b\to\infty} \int_{0}^{b} \frac{1}{(x+2)^2+(\sqrt{5})^2}\,dx$$

$$= \lim_{a\to-\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_{a}^{0} + \lim_{b\to\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)\Bigg|_{0}^{b}$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0+2}{\sqrt{5}}\right) - \lim_{a\to-\infty} \tan^{-1}\left(\frac{a+2}{\sqrt{5}}\right)\cdot\frac{1}{\sqrt{5}}$$

$$+ \lim_{b\to\infty} \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{b+2}{\sqrt{5}}\right) - \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0+2}{\sqrt{5}}\right)$$

$$= \frac{1}{\sqrt{5}}\left(\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \tan^{-1}(-\infty) + \tan^{-1}(\infty) - \tan^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$$

$$= \frac{1}{\sqrt{5}}\left(0 - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2}\right)$$

$$= \frac{\pi}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\pi\sqrt{5}}{5} \;//$$

4. Tentukan integral berikut :

a) $\displaystyle\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx$

$$\hookrightarrow \int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx = \lim_{b\to\infty} \int_2^{b} \frac{1}{x(\ln x)^2}\, dx$$

Misalkan : $u = \ln x \rightarrow \frac{du}{dx} = \frac{1}{x} \rightarrow \boxed{dx = du \cdot x}$

$$\lim_{b\to\infty} \int_2^{b} \frac{1}{\cancel{x} \cdot u^2} \cdot du \cdot \cancel{x} = \lim_{b\to\infty} \int_2^{b} \frac{1}{u^2}\, du$$

$$= \lim_{b\to\infty} -\frac{1}{u}\Big]_2^b \rightarrow \lim_{b\to\infty} -\frac{1}{\ln x}\Big|_2^b$$

$$= \lim_{b\to\infty} -\frac{1}{\ln(b)} - \left(-\frac{1}{\ln(2)}\right)$$

$$= \lim_{b\to\infty} -\frac{1}{\ln(b)} + \frac{1}{\ln(2)} = \frac{1}{\ln(2)} = 1{,}4427 \,//$$

b) $\displaystyle\int_{-\infty}^{\infty} \frac{x}{e^{|x|}}\, dx$

$$\hookrightarrow \lim_{a\to-\infty} \int_a^0 \frac{x}{e^{-x}}\, dx + \lim_{b\to\infty} \int_0^b \frac{x}{e^{x}}\, dx$$

$$= \lim_{a\to-\infty} \int_a^0 x \cdot e^x\, dx + \lim_{b\to\infty} \int_0^b x \cdot e^{-x}\, dx$$

~Integral Parsial~

$\displaystyle\int_a^0 x \cdot e^x \rightarrow u = x \quad du = 1$

$dv = e^x \quad v = e^x$

$$= \int_a^0 x \cdot e^x = x \cdot e^x - \int e^x \cdot 1$$

$$= x \cdot e^x - e^x$$

$$= e^x (x-1)\Big|_a^0$$

$$\int_0^b x \cdot e^{-x} \longrightarrow \quad u = x \qquad du = 1$$
$$dv = e^{-x} \qquad v = e^{-x}$$

$$= \int_0^b x \cdot e^{-x} = -x \cdot e^{-x} - \int -1^{-x}$$

$$= -x \cdot e^{-x} + \int e^{-x}$$

$$= -x \cdot e^{-x} - e^{-x}$$

$$= e^{-x}(-x-1)\Big|_0^b$$

$$= \lim_{a \to -\infty} e^{x}(x-1)\Big|_a^0 + \lim_{b \to \infty} e^{-x}(-x-1)\Big|_0^b$$

$$= \left(e^0(0-1) - e^{-\infty}(-\infty-1)\right) + \left(e^{-\infty}(-\infty-1) - e^0(0-1)\right)$$

$$= 0 \,//$$