Tugas Responsi 3 Kalkulus Kelompok 4

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No. Tgl.

Tugas Kelompok Pekan 3

Cari kekonvergenan dan kemonotonan :

1) $a_n = \frac{n}{3n-1}$

☆ kekonvergenan

$$\lim_{n\to\infty} \frac{n}{3n-1} \overset{L}{=} \lim_{n\to\infty} \frac{1}{3}$$

$$= \frac{1}{3} \quad \text{(konvergen)}$$

☆ kemonotonan

$$a_n - a_{n+1} = \frac{n}{3n-1} - \frac{n+1}{3(n+1)-1}$$

$$= \frac{n}{3n-1} - \frac{n+1}{3n+3-1}$$

$$= \frac{n}{3n-1} - \frac{n+1}{3n+2}$$

$$= \frac{\cancel{3n^2} + \cancel{2n} - (\cancel{3n^2} + \cancel{3n} - \cancel{n} - 1)}{9n^2 + 6n - 3n - 2}$$

$$= \frac{1}{9n^2 + 3n - 2} > 0 \quad \text{(turun)}$$

2) $a_n = \frac{n^3 + 3n^2 + 3n}{(n+1)^3}$

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★ kekonvergenan

$$\lim_{n \to \infty} \frac{n^3 + 3n^2 + 3n}{(n+1)^3}$$

$$\stackrel{L}{=} \lim_{n \to \infty} \frac{3n^2 + 6n + 3}{3(n+1)^2} \stackrel{L}{=} \lim_{n \to \infty} \frac{6n + 6}{6(n+1)}$$

$$\stackrel{L}{=} \lim_{n \to \infty} \frac{6}{6}$$

$$= 1 \text{ (konvergen)}$$

★ kemonotonan

$$a_n - a_{n+1} = \frac{n^3 + 3n^2 + 3n}{(n+1)^3} - \frac{(n+1)^3 + 3(n+1)^2 + 3(n+1)}{(n+1+1)^3}$$

$$= \frac{n^3 + 3n^2 + 3n}{(n+1)^3} - \frac{(n+1)^3 + 3(n+1)^2 + 3(n+1)}{(n+2)^3}$$

$$= \frac{(n^3 + 3n^2 + 3n)(n+2)^3 - [(n+1)^3 + 3(n+1)^2 + 3n+1](n+1)^3}{(n+1)^3 (n+2)^3}$$

$$= \frac{-3n^2 - 9n - 7}{(n^2 + 3n + 2)^3} < 0 \text{ (naik)}$$

3) $a_n = \frac{\cos(n\pi)}{n}$

★ kekonvergenan

$-1 < \cos(n\pi) < 1$

$-\frac{1}{n} < \cos(n\pi) < \frac{1}{n}$

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$\lim_{n\to\infty} -\frac{1}{n} = 0$, $\lim_{n\to\infty} \frac{1}{n} = 0$

$\frac{\cos(n\pi)}{n} = 0$ (konvergen)

★ kemonotonan

$a_n - a_{n+1} = \frac{\cos(n\pi)}{n} - \frac{\cos((n+1)\pi)}{n+1}$

$= \frac{\cos(n\pi)(n+1) \quad n\cos((n+1)\pi)}{n(n+1)}$

= (tidak naik dan tidak turun)

A) $a_n = e^{-n} \sin n$

★ kekonvergenan

$-1 < \sin n < 1$

$-e^{-n} < e^{-n} \sin n < e^{-n}$

$\lim_{n\to\infty} -e^{-n} = \lim_{n\to\infty} -\frac{1}{n}$, $\lim_{n\to\infty} e^{-n} = \lim_{n\to\infty} \frac{1}{e^n}$

$= 0$ $\qquad = 0$

$e^{-n} \sin n = 0$

No. Tgl.

★ kemonotonan

$$a_n - a_{n+1} = e^{-n} \sin n - e^{1-n} \sin(n+1)$$

$$= \frac{\sin n}{e^n} - \frac{\sin(n+1)}{e^{n+1}}$$

$$= \frac{e^{n+1} \sin n - e^n \sin(n+1)}{e^n (e^{n+1})}$$

$$= \frac{e^n (e \sin n - \sin(n+1))}{e^{2n}(e)}$$

$$= \frac{e \sin n - \sin(n+1)}{e^n (e)} > 0 \quad \text{(turun)}$$

5) $a_n = \frac{1}{n^3}$

★ kekonvergenan

$$\lim_{n \to \infty} \frac{1}{n^3} = 0$$

★ kemonotonan

$$a_n - a_{n+1} = \frac{1}{n^3} - \frac{1}{(n+1)^3}$$

$$= \frac{(n+1)^3 - n^3}{n^3 (n+1)^3}$$

$$= \frac{n^3 + 3n^2 + 3n + 1 - n^3}{n^3 (n+1)^3}$$

$$= \frac{3n^2 + 3n + 1}{n^3 (n+1)^3} > 0 \quad \text{(turun)}$$

No.

Tgl.

Cari rumus eksplisit dan kekonvergenan :

6) $\frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots$

★ rumus eksplisit

$U_n = a \cdot r^{n-1}$

$= \frac{1}{2^2} \left(\frac{1}{2}\right)^{n-1}$

$= \frac{1}{2^2} \cdot \frac{1}{2^{n-1}} = \frac{1}{2^{2+n-1}}$

$a_n = \frac{1}{2^{n+1}}$

★ kekonvergenan

$\lim\limits_{n \to \infty} \frac{1}{2^{n+1}} = 0$ (konvergen)

7) $\sin 1, 2 \sin \frac{1}{2}, 3 \sin \frac{1}{3}, 4 \sin \frac{1}{4}, \ldots$

★ rumus eksplisit

$a_n = n \sin \frac{1}{n}$; $n = 1, 2, 3, \ldots$

★ kekonvergenan

$\lim\limits_{n \to \infty} n \sin \frac{1}{n} \rightarrow$ bentuk $\infty \cdot 0$

$= \lim\limits_{n \to \infty} \frac{\sin(1/n)}{1/n} \rightarrow$ bentuk $\frac{0}{0}$

$\overset{L}{=} \lim\limits_{n \to \infty} \frac{-\cos\left(\frac{1}{n}\right) n^{-2}}{-n^{-2}} = \lim\limits_{n \to \infty} \cos(1/n) = 1$

(konvergen)

No. Tgl.

8) 0,1 , 0.11, 0.111, 0.1111, ...

☆ rumus eksplisit

$\frac{1}{9}(0,9 \ , \ 0,99, \ 0,999, \ldots)$

$= \frac{1}{9}(1-0,1 \ , \ 1-0,01 \ , \ 1-0,001, \ldots)$

$= \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)$

$a_n = \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)$

☆ kekonvergenan

$\lim_{n\to\infty} \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right) = \frac{1}{9}(1-0)$

$= \frac{1}{9}$ (konvergen)