KALKULUS II - RESPONSI PEKAN 4

KELOMPOK 9

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1. $\sum_{k=1}^{\infty} \left(\frac{1}{7}\right)^k$

$\hookrightarrow S_n = \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \dots + \frac{1}{7^n}$

$\frac{1}{7} S_n = \qquad \frac{1}{7^2} + \frac{1}{7^3} + \dots + \frac{1}{7^n} + \frac{1}{7^{n+1}}$ $-$

$\frac{6}{7} S_n = \frac{1}{7} - \frac{1}{7^{n+1}}$

$S_n = \left(\frac{1}{7} - \frac{1}{7^{n+1}}\right) \cdot \frac{7}{6}$

$= \left(1 - \frac{1}{7^n}\right) \frac{1}{6}$

$\rightarrow \lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(1 - \frac{1}{7^n}\right) \frac{1}{6}$

$= (1 - 0) \frac{1}{6}$

$= \frac{1}{6}$ [KONVERGEN]

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$$\sum_{k=1}^{\infty} \frac{k^2-5}{k+2}$$

$$\lim_{k \to \infty} \frac{k^2-5}{k+2} = \infty \to \infty \neq 0$$

↳ Divergen

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3. $\sum_{k=1}^{\infty} \frac{2}{3k}$

- $f(x) = \frac{2}{3x}$

$$= \int_1^{\infty} \frac{2}{3x}\,dx = \lim_{a \to \infty} \int_1^a \frac{2}{3x}\,dx$$

$$= \lim_{a \to \infty} \frac{2}{3} \cdot \ln(|a|)$$

$$= \lim_{a \to \infty} \frac{2}{3} \times \ln a$$

$$= \infty \text{ (Divergen)}$$

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$\boxed{4}$ $\sum_{k=2}^{\infty}\left(\frac{1}{k}-\frac{1}{k-1}\right)$

$\hookrightarrow S_n = \left(\frac{1}{2}-1\right)+\left(\frac{1}{3}-\frac{1}{2}\right)+\left(\frac{1}{4}-\frac{1}{3}\right)+\dots+\left(\frac{1}{k}-\frac{1}{k-1}\right)$

$S_n = -1\left(\cancel{\frac{1}{2}-\frac{1}{2}}\right)+\left(\cancel{\frac{1}{3}-\frac{1}{3}}\right)+\left(\cancel{\frac{1}{4}-\frac{1}{4}}\right)+\dots+\left(-\frac{1}{k-1}\right)$

$S_n = -1-\frac{1}{k-1}$

$\lim_{n\to\infty} S_n = -1$ (Konvergen ke -1)

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$\boxed{5}$ $\sum_{k=0}^{\infty}\frac{1}{k+3}$ $\quad S_n = \frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\dots+\frac{1}{k+3}$

Syarat uji integral	Maka berlaku
$\cdot f(u)$ kontinu = ya	$\lim_{b\to\infty}\int_0^b \frac{1}{u+3}$, misal $U = u+3$
$\cdot f(u)$ positif = ya	$du = du$
$\cdot$ tak naik = ya	

$\lim_{b\to\infty}\int_3^{b+3}\frac{1}{u}\,du$

$\Rightarrow \lim_{b\to\infty}\left(\ln u\Big|_3^{b+3}\right.$

$= [\ln(b+3)-\ln 3]$

$= \infty$ divergen

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6.) $\sum_{k=1}^{\infty} \frac{3}{2k-3}$

Jawab:

Misal $u = 2k-3$

$\frac{du}{dx} = 2 \rightarrow dx = \frac{du}{2}$

$$\int_1^{\infty} \frac{3}{2k-3}\,dx = \lim_{t\to\infty} \int_1^t \frac{3}{2} \cdot \frac{1}{u} \cdot du$$

$$= \lim_{b\to\infty} \frac{3}{2} (\ln u)\Big|_1^b$$

$$= \lim_{b\to\infty} \frac{3}{2} \left(\ln(2b-3) - \ln(1)\right)$$

$$= \infty \quad [\text{DIVERGEN}]$$

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7. $\sum_{k=0}^{\infty} \frac{k}{k^2+3}$

Syarat uji integral

f(x) kontinu = ya

f(x) positif = ya

tak naik = ya

$$= \int_0^{\infty} \frac{k}{k^2+3} = \lim_{b\to\infty} \int_0^b \frac{k}{k^2+3}\,dx = \lim_{b\to\infty} \int_3^{b^2+3} \frac{1}{2u}\,du$$

$$= \lim_{b\to\infty} \frac{1}{2} (\ln u)\Big|_3^{b^2+3}$$

$$= \lim_{b\to\infty} \frac{1}{2} (\ln b^2+3) - \ln 3)$$

$$= \infty \text{ (Divergen)}$$

misal : $u : k^2+3$

$du : 2k\,dx$

$\frac{1}{2} du : k\,dx$

8. $\sum_{k=1}^{\infty} \frac{3}{2k^2+1}$, $f(x) = \frac{3}{2x^2+1}$

$$\Rightarrow \int_1^{\infty} \frac{3}{2x^2+1} = \lim_{t \to \infty} \int_1^t \frac{3}{2x^2+1}\,dx$$

$$= \lim_{t \to \infty} \frac{3}{2} \int_1^t \frac{1}{x^2+\frac{1}{2}}\,dx$$

$$= \lim_{t \to \infty} \left(\frac{3}{2} \times \frac{1}{\sqrt{1/2}} \times \arctan\left(\frac{x}{\sqrt{1/2}}\right) \right) \Bigg|_1^t$$

$$= \lim_{t \to \infty} \left(\frac{3\sqrt{2}\arctan(\sqrt{2}x)}{2} \right) \Bigg|_1^t$$

$$= \lim_{t \to \infty} \left(\frac{3\sqrt{2}\arctan(\sqrt{2}t)}{2} - \frac{3\sqrt{2}\arctan(\sqrt{2})}{2} \right)$$

$$= \frac{3\sqrt{2} \times \pi/2 - 3\sqrt{2}\arctan(\sqrt{2})}{2}$$

$$= \frac{3\sqrt{2}\pi - 6\sqrt{2}\arctan(\sqrt{2})}{4}$$

Jika integral tak wajar $\int_1^{\infty} \frac{3}{2x^2+1}$ konvergen, maka $\sum_{k=1}^{\infty} \frac{3}{2k^2+1}$ konvergen