TUMMAN

R3 KALDU

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① a) $\cos\pi$, $\frac{\cos 2\pi}{4}$, $\frac{\cos 3\pi}{9}$, $\frac{\cos 4\pi}{16}$, ...

- Rumus eksplisit :

$$a_n = \frac{\cos n\cdot\pi}{n^2} \quad ; \quad n = 1,2,3,4,\dots$$

- Kekonvergenan

$$-1 \le \cos n\cdot\pi \le 1$$
$$\frac{-1}{n^2} \le \frac{\cos n\pi}{n^2} \le \frac{1}{n^2}$$
$$\lim_{n\to\infty} \frac{-1}{n^2} = 0 \qquad \lim_{n\to\infty} \frac{1}{n^2} = 0$$

Konvergen menuju 0

b) $\{a_n\}$ konvergen ke A dan $\{b_n\}$ konvergen ke B. Buktikan $\{a_n + b_n\}$ konvergen ke A + B

$$\lim_{n\to\infty} a_n = A \longrightarrow |a_n - L| < \tfrac{1}{2}\varepsilon$$
$$|a_n - A| < \tfrac{1}{2}\varepsilon$$
$$\lim_{n\to\infty} b_n = B \longrightarrow |b_n - L| < \tfrac{1}{2}\varepsilon$$
$$|b_n - B| < \tfrac{1}{2}\varepsilon$$
$$|a_n + b_n - (A+B)| = |(a_n - A) + (b_n - B)|$$
$$\le |(a_n - A) + (b_n - B)|$$
$$< \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon$$
$$= \varepsilon \qquad \text{(TERBUKTI)}$$

c) $a_n = \sin\frac{n\pi}{4} \Rightarrow -1 \le \sin\frac{n\pi}{4} \le 1$

↳ divergen (tidak memiliki limit), tidak memiliki batas, bukan barisan monoton.

② a) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots$

- Rumus eksplisit :

$$a_n = \frac{(-1)^{n+1}}{n} \quad ; \quad n = 1,2,3,\dots$$

- Kekonvergenan

$$\lim_{n\to\infty} (-1)^{n+1}\cdot\frac{1}{n} = \lim_{n\to\infty} \left|(-1)^{n+1}\cdot\frac{1}{n}\right|$$
$$= \lim_{n\to\infty} \frac{1}{n} = 0$$

Konvergen menuju 0

b) $a_n = \frac{3-8\cdot 2^n}{5+4\cdot 2^n}$

$$\lim_{n\to\infty} \frac{3-8\cdot 2^n}{5+4\cdot 2^n} = \lim_{n\to\infty} \frac{3-16^n}{5+8^n}$$
$$= \frac{-16}{8} = -2$$

Konvergen menuju -2

c) $a_n = \frac{\ln n}{n}$

- Kemonotonan

$$a'(n) = \frac{\frac{1}{n}\, n - 1\cdot \ln n}{n^2}$$
$$= \frac{1 - \ln n}{n^2} \quad \text{(bukan barisan monoton)}$$

- Keterbatasan

$$\lim_{n\to\infty} \frac{\ln n}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$$

Konvergen ke 0

(3) a) 0,9 , 0,99 , 0,999 , 0,9999 , ...

- Rumus eksplisit:

$a_n = 1 - \frac{1}{10^n}$; $n = 1, 2, 3, \ldots$

- Kekonvergenan :

$\lim_{n\to\infty} 1 - \frac{1}{10^n} = 1 - 0 = 1$

Konvergen menuju 1

b) $a_n = \frac{n+3}{3n-2} \Rightarrow \lim_{n\to\infty} \frac{n+3}{3n-2} = \lim_{n\to\infty} \frac{1+3/n}{3-2/n}$

$= \lim_{n\to\infty} \frac{1+0}{3-0} = \frac{1}{3}$

Konvergen ke $\frac{1}{3}$

c) $a_n = \frac{n!}{10^n} = \frac{1\cdot 2\cdot 3\cdot 4\cdot \ldots \cdot n}{10\cdot 10\cdot 10\cdot \ldots \cdot 10^n}$

- Kemonotonan :

$\frac{a_{n+1}}{a_n} = \frac{(n+1)!\,10^n}{10^{n+1}\cdot n!} = \frac{n+1}{10}$

$\frac{a_{n+1}}{a_n} \leq 1$; $\{a_n\}$ tak naik untuk $n = 1, 2, \ldots, 9$

$\frac{a_{n+1}}{a_n} \geq 1$; $\{a_n\}$ naik untuk $n = 10, 11, 12, \ldots$

- Keterbatasan

$\lim_{n\to\infty} \frac{n!}{10^n} = \lim_{n\to\infty} \frac{1\cdot 2\cdot 3\cdot \ldots \cdot n}{10\cdot 10\cdot 10\cdot \ldots \cdot 10^n}$

$= \frac{\infty}{\infty}$ (Divergen)