Praktikum 10. Aljabar Matriks

# TRANSFORMASI LINEAR

## Definisi

Misal $U$dan $V$ adalah ruang vektor, $T: U \to V$ dinamakan transformasi linear apabila:
$T(\underline{u} + \underline{v}) = T(\underline{u}) + T(\underline{v})$
$T(k\underline{u}) = kT(\underline{u})$

## Contoh Soal

1. Tunjukkan bahwa $T: R^2 \to R^3$ dimana $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ -x \\ y \end{pmatrix}$

   Jawab:

   Misal $\underline{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ dan $\underline{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ maka

   - Akan dibuktikan bahwa $T(\underline{u} + \underline{v}) = T(\underline{u}) + T(\underline{v})$

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     $T\begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix} = T\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + T\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

     $\begin{pmatrix} u_1 + v_1 - (u_2 + v_2) \\ -(u_1 + v_1) \\ u_2 + v_2 \end{pmatrix} = \begin{pmatrix} u_1 - u_2 \\ -u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 - v_2 \\ -v_1 \\ v_2 \end{pmatrix}$

     $\begin{pmatrix} u_1 + v_1 - u_2 - v_2 \\ -u_1 - v_1 \\ u_2 + v_2 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 - u_2 - v_2 \\ -u_1 - v_1 \\ u_2 + v_2 \end{pmatrix}$

     **Terbukti**

   - Akan dibuktikan bahwa $T(k\underline{u}) = kT(\underline{u})$

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     $T\begin{pmatrix} ku_1 \\ ku_2 \end{pmatrix} = kT\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

     $\begin{pmatrix} ku_1 - ku_2 \\ -ku_1 \\ ku_2 \end{pmatrix} = k\begin{pmatrix} u_1 - u_2 \\ -u_1 \\ u_2 \end{pmatrix}$

$$\begin{pmatrix} ku_1 - ku_2 \\ -ku_1 \\ ku_2 \end{pmatrix} = \begin{pmatrix} ku_1 - ku_2 \\ -ku_1 \\ ku_2 \end{pmatrix}$$

**Terbukti**

Karena tertutup terhadap pada operasi penjumlahan dan perkalian skalar maka T merupakan tranformasi linear.

2. Tunjukkan $T: R^3 \to R^2$ dimana $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^2 \\ y \end{pmatrix}$!

Jawab:

Misal $\underline{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ dan $\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ maka

- Akan dibuktikan bahwa $T(\underline{u} + \underline{v}) = T(\underline{u}) + T(\underline{v})$

$$T(\underline{u} + \underline{v}) = T(\underline{u}) + T(\underline{v})$$

$$T\begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{pmatrix} = T\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + T\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$\begin{pmatrix} (u_1 + v_1)^2 \\ u_2 + v_2 \end{pmatrix} = \begin{pmatrix} {u_1}^2 \\ u_2 \end{pmatrix} + \begin{pmatrix} {v_1}^2 \\ v_2 \end{pmatrix}$$

$$\begin{pmatrix} (u_1 + v_1)^2 \\ u_2 + v_2 \end{pmatrix} \neq \begin{pmatrix} {u_1}^2 + {v_1}^2 \\ u_2 + v_2 \end{pmatrix}$$

**Tidak terbukti**

Karena tidak tertutup terhadap pada operasi penjumlahan maka T bukan tranformasi linear.

**Sifat:**

Jika T: V → W adalah transformasi linier, maka:

- T(0) = 0
- T(-v) = - T(v) untuk semua v di V
- T(v-w) = T(v) – T(w) untuk semua v dan w di V

## Gram-Schmidt

Tujuan: Mengubah baris $\{x_1, x_2, \ldots, x_n\}$ menjadi:

1. Basis orthogonal $\{y_1, y_2, \ldots, y_n\}$
2. Basis orthonormal $\{z_1, z_2, \ldots, z_n\}$

**Rumus:**

$\underline{y}_1 = \underline{x}_1$

$\underline{y}_2 = \underline{x}_2 - p\underline{y}_1\underline{x}_2$

$\underline{y}_3 = \underline{x}_3 - p\underline{y}_1\underline{x}_3 - p\underline{y}_2\underline{x}_3$

Proyeksi $x_2$ ke $y_1$:

$$p\underline{y}_1\underline{x}_2 = \frac{\underline{y}_1{}'x_2}{||\underline{y}_1||^2}\,\underline{y}_1$$

**Contoh Soal:**

1. Lakukan proses orthogonalisasi Gram-Schmidt

$$\underline{x}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}, \underline{x}_2 = \begin{pmatrix}6\\4\\5\end{pmatrix}, \underline{x}_3 = \begin{pmatrix}3\\6\\9\end{pmatrix}$$

Jawab:

$$\underline{y}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$p\underline{y}_1\underline{x}_2 = \frac{(1 \quad 1 \quad 1)\begin{pmatrix}6\\4\\5\end{pmatrix}}{1^2 + 1^2 + 1^1}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{15}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} = 5\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}5\\5\\5\end{pmatrix}$$

$$\underline{y}_2 = \begin{pmatrix}6\\4\\5\end{pmatrix} - \begin{pmatrix}5\\5\\5\end{pmatrix} = \begin{pmatrix}1\\-1\\0\end{pmatrix}$$

$$p\underline{y}_1\underline{x}_3 = \frac{(1 \quad 1 \quad 1)\begin{pmatrix}3\\6\\9\end{pmatrix}}{1^2 + 1^2 + 1^1}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{18}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} = 6\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}6\\6\\6\end{pmatrix}$$

$$p\underline{y}_2\underline{x}_3 = \frac{(1 \quad -1 \quad 0)\begin{pmatrix}3\\6\\9\end{pmatrix}}{1^2 + (-1)^2 + 0^1}\begin{pmatrix}1\\-1\\0\end{pmatrix} = \frac{-3}{2}\begin{pmatrix}1\\-1\\0\end{pmatrix}$$

$$\underline{y}_3 = \begin{pmatrix}3\\6\\9\end{pmatrix} - \begin{pmatrix}6\\6\\6\end{pmatrix} - \frac{-3}{2}\begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}-3/2\\-3/2\\3\end{pmatrix}$$

Kita ubah agar othogonal:

$$\underline{z}_1{}' = \left\{ {}^{1}\!/_{\sqrt{3}}\, ,\, {}^{1}\!/_{\sqrt{3}}\, ,\, {}^{1}\!/_{\sqrt{3}} \right\}$$

$$\underline{z}_2{}' = \left\{ {}^{1}\!/_{\sqrt{2}}\, ,\, {}^{-1}\!/_{\sqrt{2}}\, ,\, 0 \right\}$$

$$\underline{z}_1{}' = \left\{ {}^{-1}\!/_{9}\, ,\, {}^{-1}\!/_{9}\, ,\, {}^{2}\!/_{9} \right\}$$