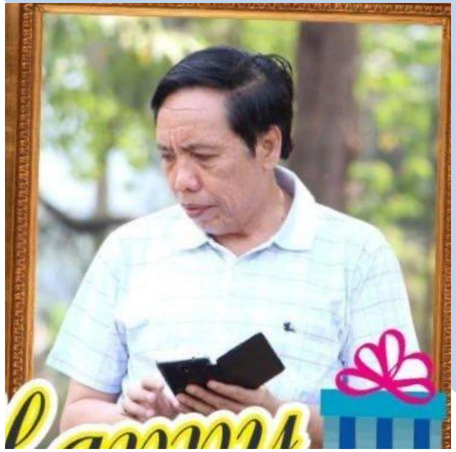

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Sebaran Kepekatan Peluang Normal
(*Normal Distribution*)

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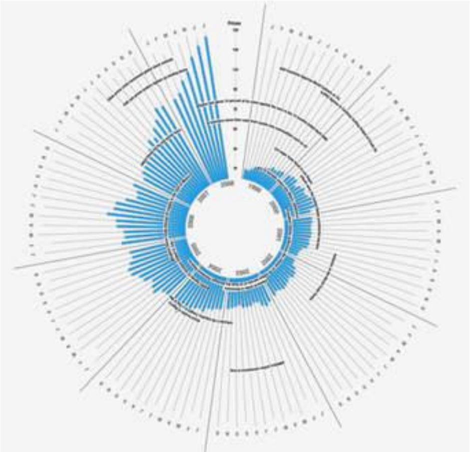
Program Studi Statistika dan Sains Data



Outline

1. Peubah Acak Kontinu
2. Sebaran Peluang Normal
3. Sebaran Peluang Normal Baku
4. Penghitungan Peluang Normal

Sebaran Peluang Peubah Acak Kontinu

R

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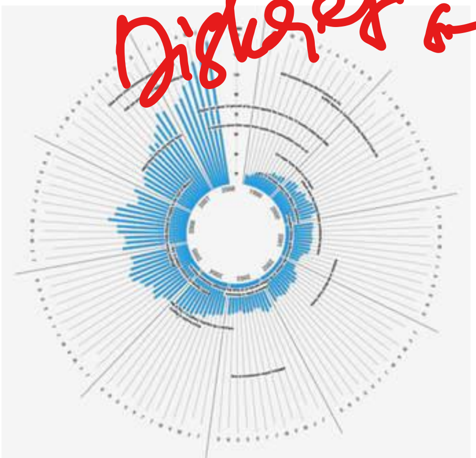
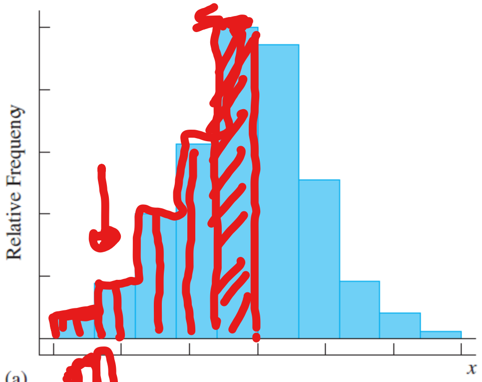
Diskret ← $P(X=x) = ?$ → Kontinu $P(X=x) = 0$



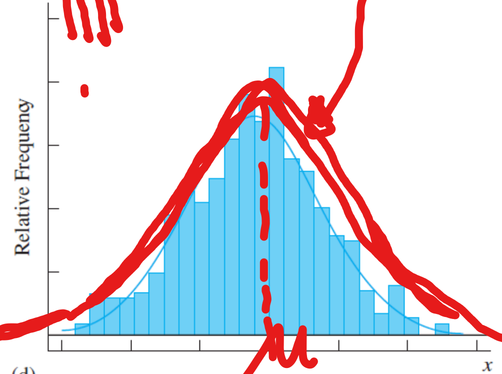
- **Peubah acak kontinu**. Dari contoh acak diukur tinggi dan bobot badan, keduanya merupakan peubah acak kontinu. Untuk peubah acak kontinu maka maka peluangnya pada satu titik selalu sama dengan nol.
- Mengapa? Karena $P(a < X < b)$ adalah integral fungsi kepekatan $f(X)$ dari titik *a* samapai ke titik *b*.

1, 2, 3 4, 5, 6 → Kontinu

- Sebaran peluang peubah acak diskret dapat didekati (aproksimasi) dengan sebaran kontinu.
- Seperti nampak pada gambar, histogram frekuensi relatif yang menggambarkan sebaran data dapat didekati dengan fungsi kontinu.
- Jika jumlah pengukurannya besar sekali maka selang kelas menjadi semakin sempit dan histogram semakin mulus.
- Kurva yang mulus dalam histogram itu (lihat histogram yang di bawah) mencerminkan **sebaran peluang dari peubah acak kontinu.**



(a)



(d)

Fungsi Kepekatan Peluang

Sebaran Peluang Peubah Acak Kontinu

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$\mathbb{R}$

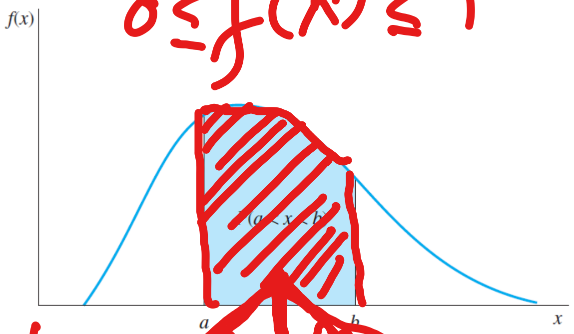
$-\infty$ ——— ∞

- Bagaimana model dari sebaran peluang tersebut?
- Nilai peubah acak kontinu adalah semua bilangan nyata.
- Sebaran peluangnya dibangun sedemikian sehingga membentuk fungsi seperti nampak dalam gambar.
- Kepekatan peluang (***probability density***), dituliskan dengan rumus matematika $f(x)$ yang dikenal sebagai sebaran peluang (***probability distribution***) atau fungsi kepekatan peluang untuk peubah acak *x*.

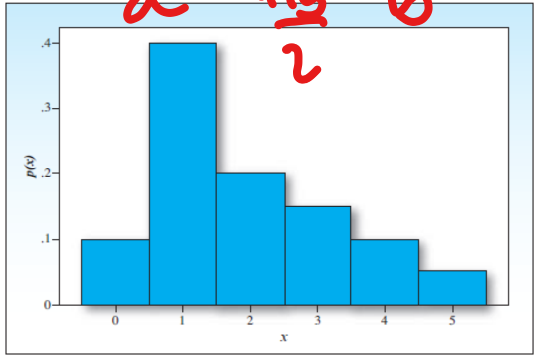
Σ $\qquad f(x) = P(X \le x) = \int_{-\infty}^{\infty}$

- Ingat bahwa untuk peubah acak diskret berlaku:
  1. Jumlah seluruh peluangnya yaitu $p(x)$ sama dengan 1
  2. Peluang *x* ada di dalam suatu selang merupakan jumlah dari peluang yang ada dalam selang tersebut.

$0 \le f(x) \le 1$



$a \qquad \frac{a+b}{2} \qquad b$



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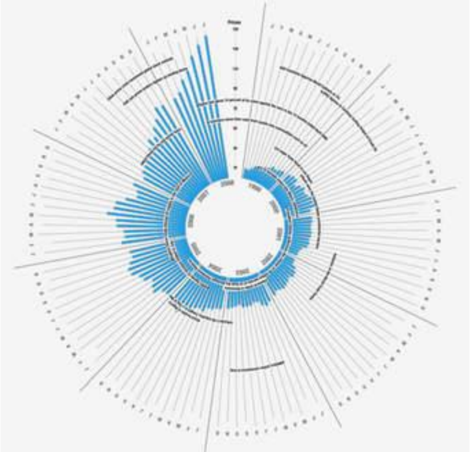
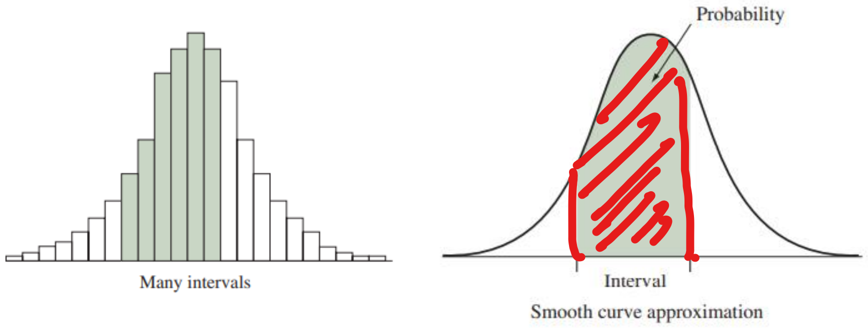
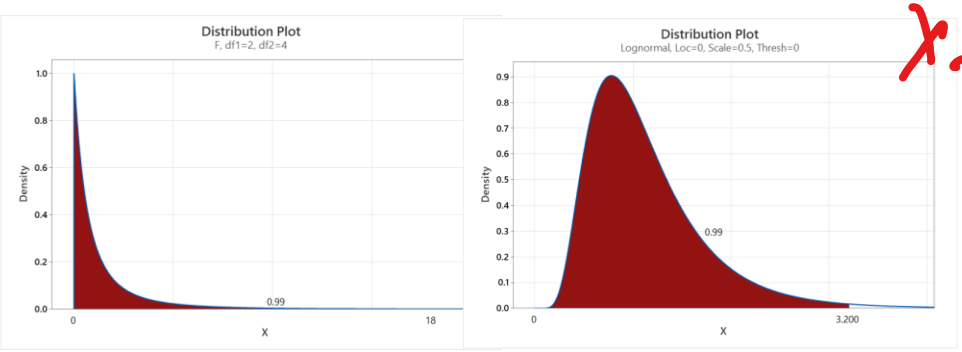




- Setiap interval memiliki peluang diantara 0 s.d 1
- Peluang dari suatu interval diberikan dari luas di bawah kurva
- Total peluang semua interval = total luas di bawah kurva = 1



X → 1 — 10 menit   0 – 180

1 menit

25 menit

- Contoh:
  - ✓ X = Lamanya waktu tunggu dalam suatu antrian
  - ✓ Y = Lamanya waktu menelpon seseorang → 1 → 2 jam
  - ✓ Z = nilai UTS Metode Statistika mahasiswa FKH

Sebaran Peluang Peubah Acak Kontinu

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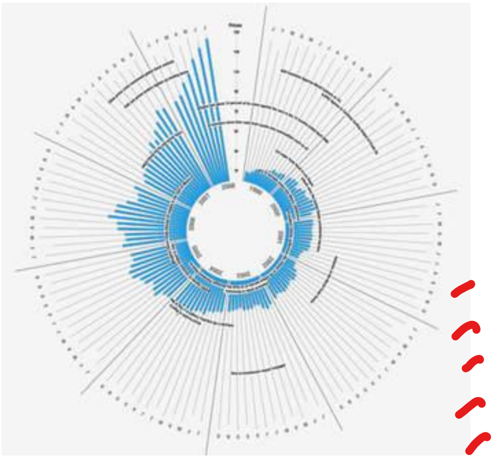


$P(X \le x) \rightarrow P(X \ge 85) = 1 - P(X < 85)$

- Nilai UTS untuk 200 mahasiswa

Diskret

$P(X \le x) \ne P(X < x)$

Stem-and-leaf of UTS N = 200

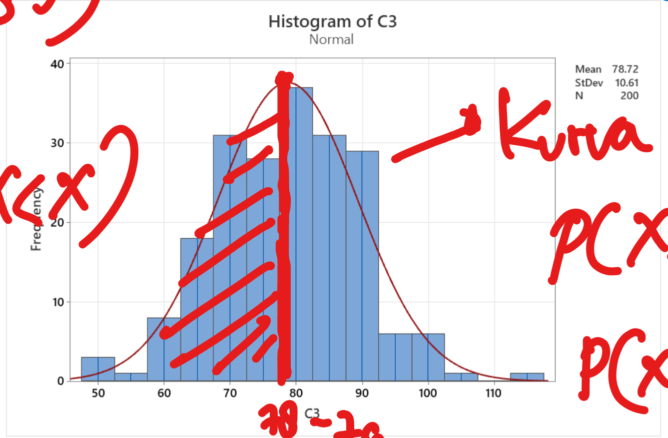
1	4	8
4	5	014
7	5	799
18	6	00222222344
42	6	556666667777888899999999
73	7	0000000000111111222233444444444
(34)	7	5555566666666777778888888899999999
93	8	00000111111111111222223333333344
61	8	55555556666666667777777788888888889999999
20	9	011222334
11	9	566779
5	10	011
2	10	6
1	11	3

Leaf Unit = 1

$P(X < x) = P(X \le x)$

Kontinu

$P(X = x) = 0$

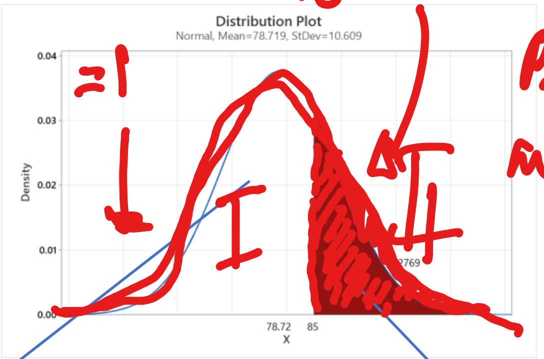


Kurva Normal

78-79

$P(X > 85)$
$=$
$P(X \ge 85)$

Berapa?
prop nilai
≥ 85



$= 1$

Luas area di bawah kurva = 1

Peluang nilai UTS ≥ 85 → P(X≥85) dapat diperoleh dengan menghitung luas di bawah kurva pada selang ≥ 85

- Misalkan peubah acak X = nilai UTS mahasiswa
- Berapa P(Xsama dengan 85

$\int f(x)\,dx$

Sebaran Peluang Peubah Acak Kontinu

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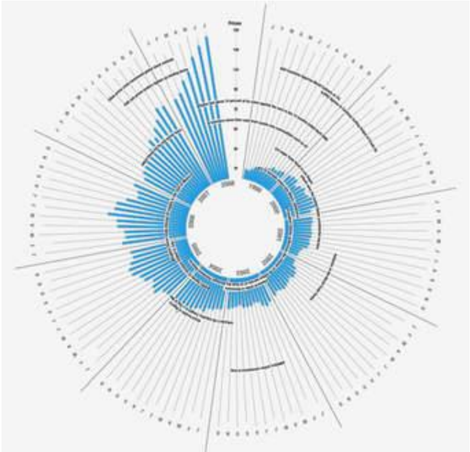


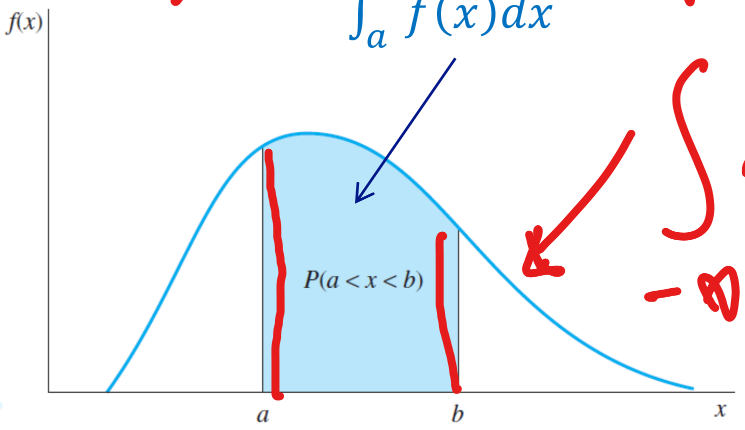
- Untuk suatu fungsi kepekatan peluang kontinu *f(x)* berlaku:
  - Luas area di bawah *f(x)* sama dengan 1. → $\int_{-\infty}^{\infty} f(x)dx = 1$
  - Peluang *x* ada di dalam suatu selang dari *a* sampai *b*, sama dengan luas area di bawah *f(x)* dari *a* sampai *b.*
- *P*(*x* = *a*) = 0 untuk peubah acak x yang kontinu. → $\int_{a}^{a} f(x)dx = 0$
- Akibatnya *P*(*x* ≥ *a*) = *P*(*x* > *a*) dan *P*(*x* q *a*) = *P*(*x* < *a*).
- Dua hal ini tidak berlaku untuk peubah acak x yang diskret.

$P(a \le X \le b) = P(X \le b) - P(X \le a)$

a b

$\int_{-\infty} f(x)dx = 1$

$\int_{a}^{b} f(x)dx$



Kontinu ← Seragam.

Sebaran Peluang Seragam (*Uniform*)

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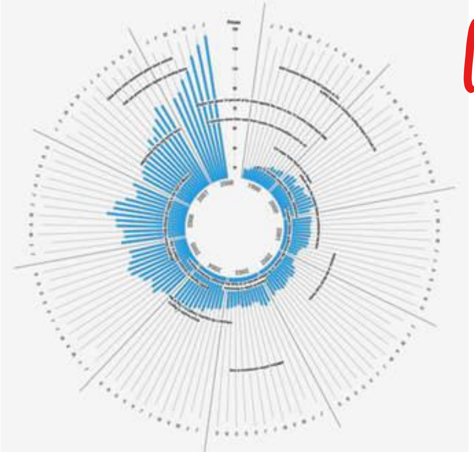
$120 \leq X \leq 140$

$P(120 < x < 121)$ $(121-120)$

120

140



- **Ilustrasi.** Perhatikan peubah acak X yaitu waktu penerbangan pesawat terbang dari Jakarta ke Bali.
- Waktu penerbangan bersifat kontinu dengan nilai dalam selang dari 120 menit hingga 140 menit.
- Misalkan dari data penerbangan aktual yang cukup tersedia diketahui peluang waktu penerbangan dalam interval 1 menit manapun sama dengan peluang waktu penerbangan dalam interval 1 menit lainnya yang terkandung dalam interval yang lebih besar dari 120 hingga 140 menit.

Diskret ← Sebaran Bernoulli.

— — Binom

— — Poisson

Sebaran Peluang Seragam (*Uniform*)

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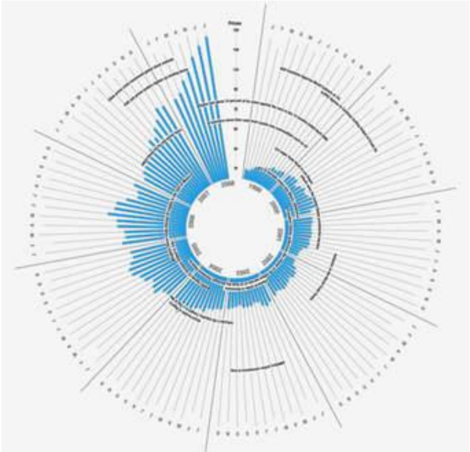


Nilai Harapan → Nilai Tengah

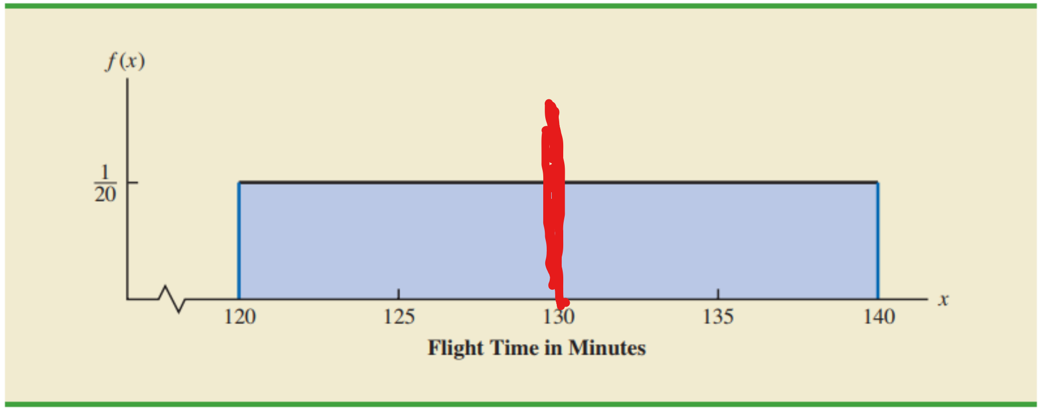
- Dengan setiap interval 1 menit memiliki kemungkinan yang sama, peubah acak X dikatakan memiliki **sebaran peluang seragam**.
- Fungsi kepekatan peluang, yang mendefinisikan sebaran seragam untuk peubah acak waktu terbang, adalah

$X \sim Unif(a,b)$

$$f(x) = \begin{cases} 1/20 & \text{for } 120 \le x \le 140 \\ 0 & \text{elsewhere} \end{cases}$$

fkp

$$\boxed{f_X = \frac{1}{b-a}}, \quad a<x<b$$



Sebaran Peluang Seragam (*Uniform*)

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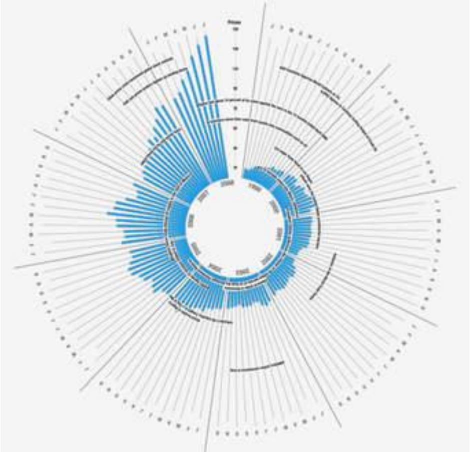


- Berdasarkan ilustrasi-2 tersebut :
- Berapa peluang bahwa waktu penerbangan antara 120 dan 130 menit? Artinya, berapa P(120 < X < 130)?
- Karena waktu penerbangan harus antara 120 dan 140 menit dan karena peluang digambarkan seragam selama interval ini, kita bisa memperoleh P(120 < x < 130) = .50.

$$\frac{120+140}{2} = \frac{260}{2} = 130$$

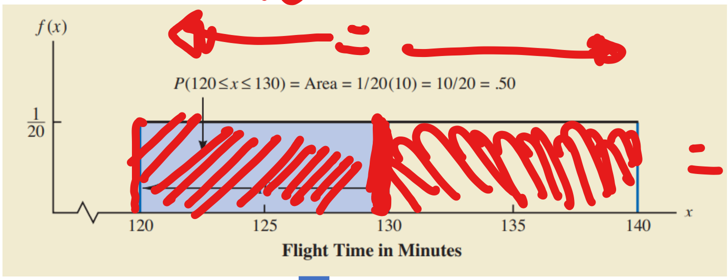
Misalkan $X \sim$seragam(a, b)

$$E(X) = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

$$Var(X) = E(X^2) - E(X)^2$$

0,5



= 1

Perhatikan gambar di atas: $P(130 < X < 140)$ = 0,5

$X \sim$seragam(120,140)

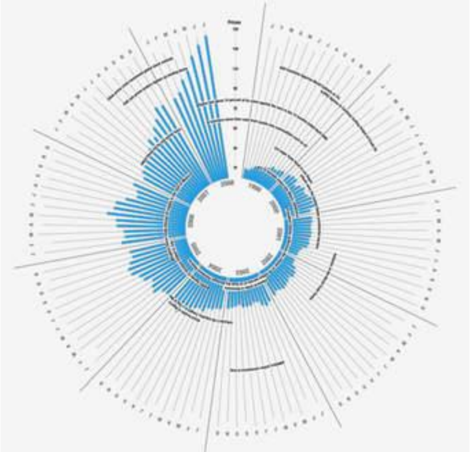
$$E(X) = \frac{120+140}{2} = 130$$

$$Var(X) = \frac{(140-120)^2}{12} = 33.33$$

Sehingga $\sigma = \sqrt{Var(X)} = 5.77$

$$\int_{120}^{130} \frac{1}{20}\,dx = \frac{1}{20}x \Big]_{120}^{130} = \frac{1}{20}\cdot 10 = \frac{1}{2} = 0,5$$

Rehat dulu ... (1)



Sebaran Peluang Normal

Seragam

Normal

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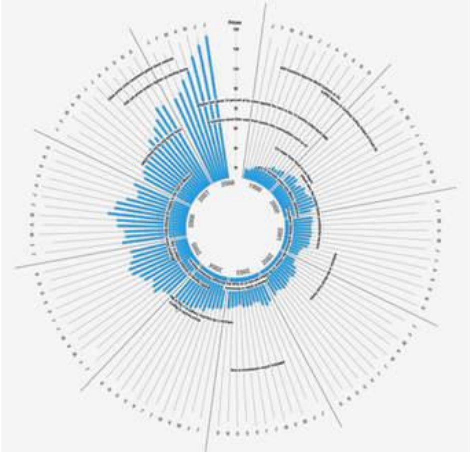
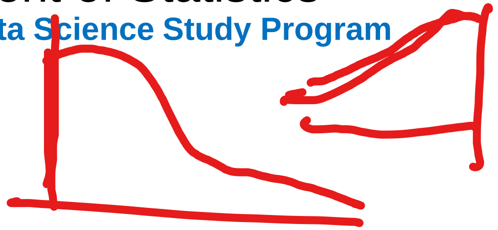




Sebaran Peluang Normal

- Sebaran peluang kontinu dapat bermacam-macam bentuknya.
- Salah satu yang paling banyak ditemui dalam proses alam adalah peubah acak dengan sebaran frekuensi yang berbentuk genta, yang dikenal dengan **sebaran peluang normal**.
- Rumus fungsi kepekatan peluangnya seperti di bawah ini.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

fkp

$$\frac{1}{b-a}$$

NORMAL PROBABILITY DISTRIBUTION

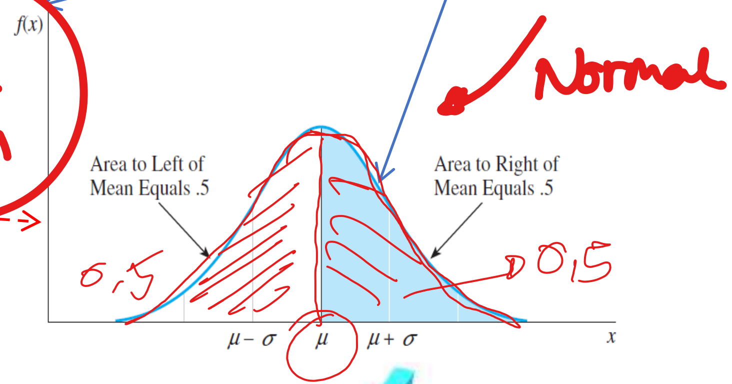
$X \sim Norm(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty$$

ragam (Varians)

nilai tengah

The symbols e and π are mathematical constants given approximately by 2.7183 and 3.1416, respectively; μ and σ ($\sigma > 0$) are parameters that represent the population mean and standard deviation, respectively.



- Dalam sebaran peluang normal nilaitengah μ merupakan pusat dari sebaran, dan bentuknya simetrik terhadap μ.
- Karena simetrik maka luas areal di sebelah kiri μ sama dengan luas area di sebelah kanan μ, masing-masing sama dengan 0.5.

Sebaran Peluang Normal

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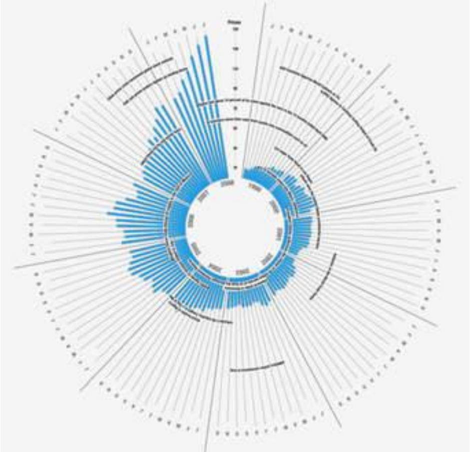
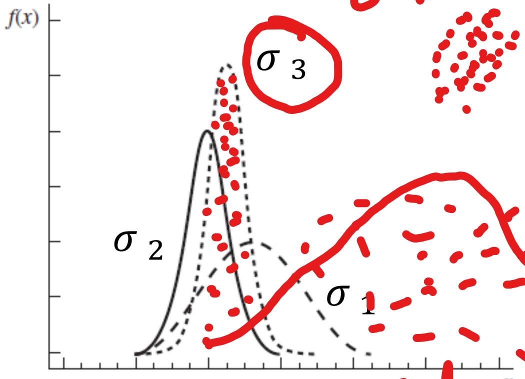


μ = Kesimetrisan kurva

σ = kurtosis (Kelangsingan)

- Bentuk dari sebaran normal ditentukan oleh parameter *σ*, yaitu simpangan baku populasi.
- Lihat gambar di samping ini, semakin besar *σ*, semakin pendek kurva, dan keragamannya semakin besar.
- Sebaliknya, jika *σ* makin kecil maka semakin tinggi kurva normal, dan semakin kecil keragamannya.



σ kecil

σ besar

$\sigma_1 > \sigma_2 > \sigma_3$

- Untuk menghitung peluang peubah acak normal *x* terletak dalam selang dari *a* sampai *b*, perlu integral fungsi kepekatan normal tersebut dari *a* sampai *b*.

luas wilayah di bawah kurva

- Integral ini tidak mudah, selain itu ada tak hingga fungsi kepekatan normal tergantung *μ* dan *σ*.
- Untungnya, setiap peubah acak normal $x(\mu,\sigma^2)$ selalu dapat dibakukan menjadi peubah acak normal $z(0,1)$.
- Peubah acak *z* normal baku (***standard normal***)

$$P(a<x<b) = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{\boxed{-(x-\mu)^2/2\sigma^2}}\,dx \;?$$

transformasi

$x \rightarrow z \sim \text{Norm}(0,1)$

Sebaran Peluang Normal

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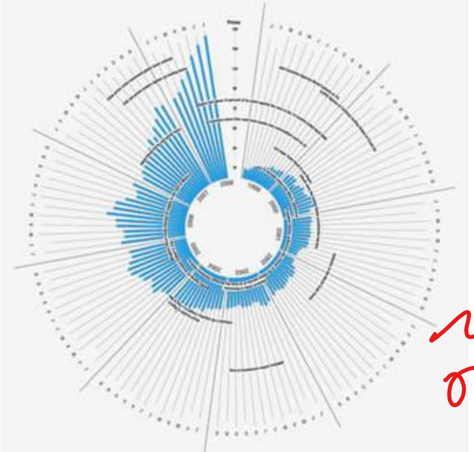


Cara membakukan sebaran normal

- Suatu peubah acak normal $x(\mu,\sigma^2)$ dapat dibakukan (***standardized***) menjadi peubah acak $z(0,1)$ sbb:

$$z = \frac{x-\mu}{\sigma}$$

$x = 29$
$\mu = 25$
$\sigma = 10$
→ Norm →

$z \sim Norm(0,1)$

- Jika x lebih kecil dari μ, maka z bernilai negatif.
- Jika x lebih besar dari μ, maka z bernilai positif.
- Jika $x = \mu$, maka $z = 0$.

$x \sim Norm(\mu, \sigma^2)$

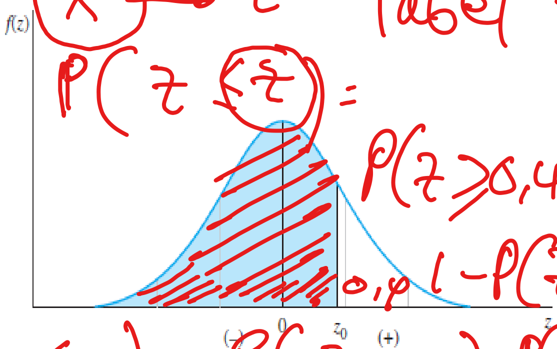
$$f_x = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$z \sim Norm(0,1)$

$$f_z = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$X \rightarrow z$ Tabel Z

$P(z \le z) =$



$P(z > 0,4) = 1 - P(z \le 0,4)$

$P(X \le x) = P(z \le z) = P(z \le 0,4)$

Table 3. Areas Under the Normal Curve

z	.00	.01	.02	.03	. . .	.09
−3.4	.0003	.0003	.0003	.0003		
−3.3	.0005	.0005	.0005	.0004		
−3.2	.0007	.0007	.0006	.0006		
−3.1	.0010	.0009	.0009	.0009		
−3.0	.0013	.0013	.0013	.0012	. . .	.0010
−2.9	.0019					
−2.8	.0026	.	.	.		
−2.7	.0035	.	.			
−2.6	.0047					
−2.5	.0062					
.	.					
.	.					
−2.0	.0228					
.	.					
.	.					
.	.					

$P(z > -3,22)$
$-3.2 + 0,02$
$1 - 0.0006 = 0.9994$

$P(z \le 0,43) = 0,4 + 0,03$

z	.00	.01	.02	.03	.04	. . .	.09
0.0	.5000	.5040	.5080	.5120	.5160		
0.1	.5398	.5438	.5478	.5517	.5557		
0.2	.5793	.5832	.5871	.5910	.5948		
0.3	.6179	.6217	.6255	.6293	.6331		
0.4	.6554	.6591	.6628	.6664	.6700	. . .	.6879
0.5	.6915	.	.	.			
0.6	.7257	.	.	.			
0.7	.7580	.	.	.			
0.8	.7881						
0.9	.8159						
.	.						
.	.						
.	.						
2.0	.9772						

→ 0,6664

$z = \frac{x-\mu}{\sigma} = \frac{29-25}{10} = 0,4$

$P(z \le 0,4) = 0,6554$

Menentukan peluang normal

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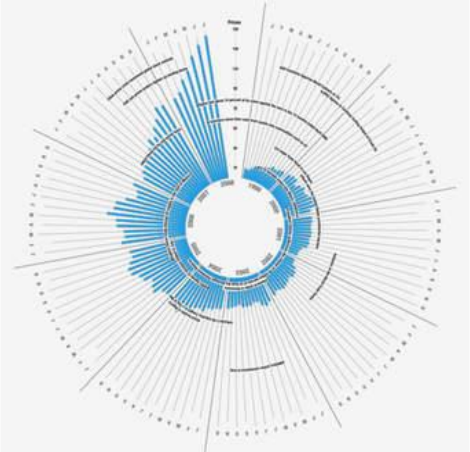


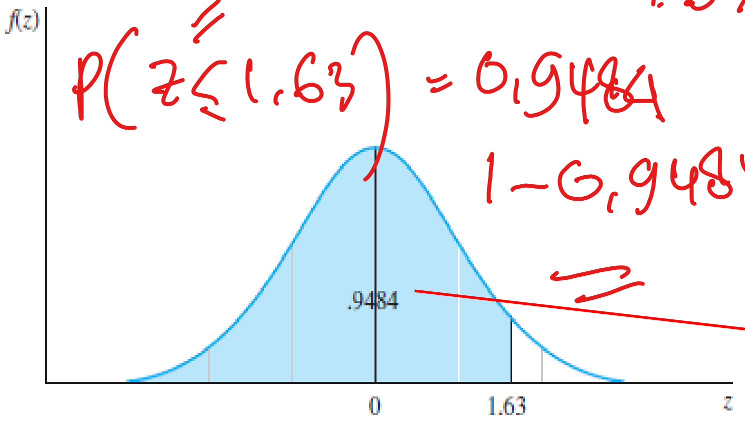
Find $P(z \leq 1.63)$. This probability corresponds to the area to the left of a point $z = 1.63$ standard deviations to the right of the mean (see Figure 6.8).

Solution The area is shaded in Figure 6.8. Since Table 3 in Appendix I gives areas under the normal curve to the left of a specified value of z, you simply need to find the tabled value for $z = 1.63$. Proceed down the left-hand column of the table to $z = 1.6$ and across the top of the table to the column marked .03. The intersection of this row and column combination gives the area .9484, which is $P(z \leq 1.63)$.

$1.63 = 1.6 + 0.03$

$P(z \leq 1.63) = 0,9484$

$1 - 0,9484$



Areas to the left of $z = 0$ are found using negative values of z.

z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5 871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700
0.5	.6915	.6950	.6985	.7019	.7054
0.6	.7257	.7291	.7324	.7357	.7389
0.7	.7580	.7611	.7642	.7673	.7704
0.8	.7881	.7910	.7939	.7967	.7995
0.9	.8159	.8186	.8212	.8238	.8264
1.0	.8413	.8438	.8461	.8485	.8508
1.1	.8643	.8665	.8686	.8708	.8729
1.2	.8849	.8869	.8888	.8907	.8925
1.3	.9032	.9049	.9066	.9082	.9099
1.4	.9192	.9207	.9222	.9236	.9251
1.5	.9332	.9345	.9357	.9370	.9382
1.6	.9452	.9463	.9474	.9484	.9495
1.7	.9554	.9564	.9573	.9582	.9591
1.8	.9641	.9649	.9656	.9664	.9671
1.9	.9713	.9719	.9726	.9732	.9738

Menentukan peluang normal

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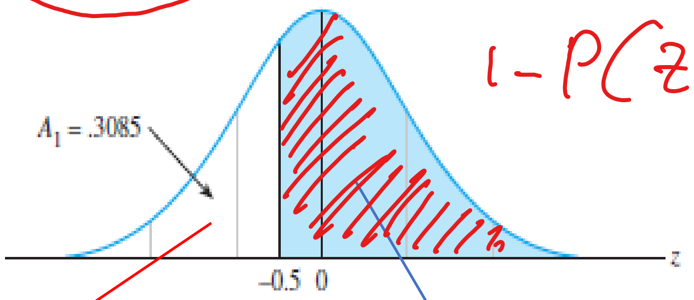
z	.00	.01	.02	.03
−3.4	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004
−3.2	.0007	.0007	.0006	.0006
−3.1	.0010	.0009	.0009	.0009
−3.0	.0013	.0013	.0013	.0012
−2.9	.0019	.0018	.0017	.0017
−2.8	.0026	.0025	.0024	.0023
−2.7	.0035	.0034	.0033	.0032
−2.6	.0047	.0045	.0044	.0043
−2.5	.0062	.0060	.0059	.0057
−2.4	.0082	.0080	.0078	.0075
−2.3	.0107	.0104	.0102	.0099
−2.2	.0139	.0136	.0132	.0129
−2.1	.0179	.0174	.0170	.0166
−2.0	.0228	.0222	.0217	.0212
−1.9	.0287	.0281	.0274	.0268
−1.8	.0359	.0351	.0344	.0336
−1.7	.0446	.0436	.0427	.0418
−1.6	.0548	.0537	.0526	.0516
−1.5	.0668	.0655	.0643	.0630
−1.4	.0808	.0793	.0778	.0764
−1.3	.0968	.0951	.0934	.0918
−1.2	.1151	.1131	.1112	.1093
−1.1	.1357	.1335	.1314	.1292
−1.0	.1587	.1562	.1539	.1515
−0.9	.1841	.1814	.1788	.1762
−0.8	.2119	.2090	.2061	.2033
−0.7	.2420	.2389	.2358	.2327
−0.6	.2743	.2709	.2676	.2643
−0.5	.3085	.3050	.3015	.2981

Find $P(z \geq -.5)$. This probability corresponds to the area to the *right* of a point $z = -.5$ standard deviation to the left of the mean (see Figure 6.9).



$1 - P(z \leq 0.5)$

Solution The area given in Table 3 in Appendix I is the area to the left of a specified value of z. Indexing $z = -.5$ in Table 3, we can find the area A_1 to the *left* of $-.5$ to be .3085.

Since the area under the curve is 1, we find

$$P(z \geq -.5) = 1 - A_1 = 1 - .3085 = .6915$$

Rehat dulu … (2)

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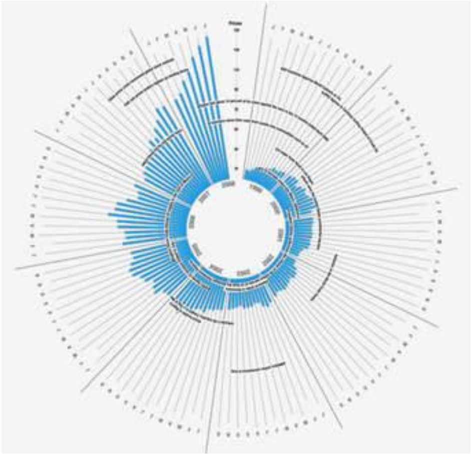

Menentukan peluang normal

$P(z \geq z)$

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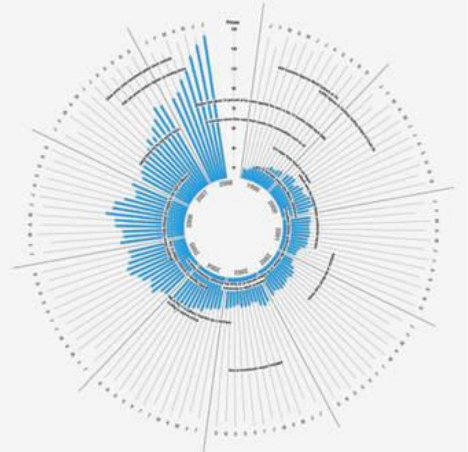
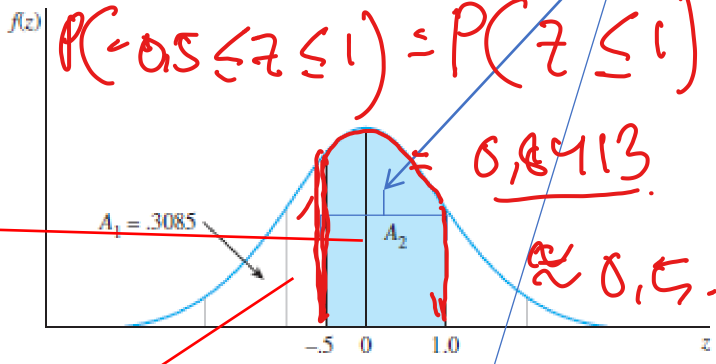


z	.00	.01	.02	.01	.02	.03
0.0	.5000	.5040	.5080	.0003	.0003	.0003
0.1	.5398	.5438	.5478	.0005	.0005	.0004
0.2	.5793	.5832	.5 871	.0007	.0006	.0006
0.3	.6179	.6217	.6255	.0009	.0009	.0009
0.4	.6554	.6591	.6628	.0013	.0013	.0012
0.5	.6915	.6950	.6985	.0018	.0017	.0017
0.6	.7257	.7291	.7324	.0025	.0024	.0023
0.7	.7580	.7611	.7642	.0034	.0033	.0032
0.8	.7881	.7910	.7939	.0045	.0044	.0043
0.9	.8159	.8186	.8212	.0060	.0059	.0057
1.0	.8413	.8438	.8461	.0080	.0078	.0075
1.1	.8643	.8665	.8686	.0104	.0102	.0099
1.2	.8849	.8869	.8888	.0136	.0132	.0129
1.3	.9032	.9049	.9066	.0174	.0170	.0166
1.4	.9192	.9207	.9222	.0222	.0217	.0212
		−1.9	.0287	.0281	.0274	.0268
		−1.8	.0359	.0351	.0344	.0336
		−1.7	.0446	.0436	.0427	.0418
		−1.6	.0548	.0537	.0526	.0516
		−1.5	.0668	.0655	.0643	.0630
		−1.4	.0808	.0793	.0778	.0764
		−1.3	.0968	.0951	.0934	.0918
		−1.2	.1151	.1131	.1112	.1093
		−1.1	.1357	.1335	.1314	.1292
		−1.0	.1587	.1562	.1539	.1515
		−0.9	.1841	.1814	.1788	.1762
		−0.8	.2119	.2090	.2061	.2033
		−0.7	.2420	.2389	.2358	.2327
		−0.6	.2743	.2709	.2676	.2643
		−0.5	.3085	.3050	.3015	.2981

$P(z \leq z)$

Find $\boxed{P(-.5 \leq z \leq 1.0)}$. This probability is the area between $z = -.5$ and $z = 1.0$, as shown in Figure 6.10.



$P(-0{,}5 \leq z \leq 1) = P(z \leq 1) - P(z \leq -0{,}5)$

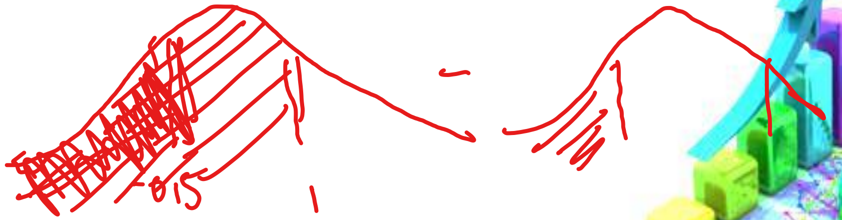
$= 0{,}8413 - 0{,}3085$

$\approx 0{,}5...$

Solution The area required is the shaded area A_2 in Figure 6.10. From Table 3 in Appendix I, you can find the area to the left of $z = -.5$ ($A_1 = .3085$) and the area to the left of $z = 1.0$ ($A_1 + A_2 = .8413$). To find the area marked A_2, we subtract the two entries:

$$A_2 = (A_1 + A_2) - A_1 = .8413 - .3085 = .5328$$

That is, $P(-.5 \leq z \leq 1.0) = .5328$.



Menentukan peluang normal

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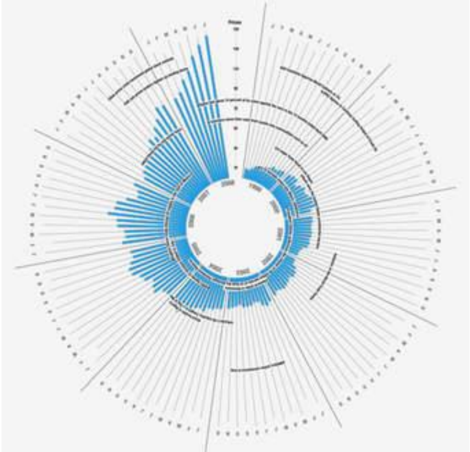


Let x be a normally distributed random variable with a mean of 10 and a standard deviation of 2. Find the probability that x lies between 11 and 13.6.

$\int_{11}^{13,6} \frac{1}{\sigma\sqrt{2\pi}}$

Solution The interval from $x = 11$ to $x = 13.6$ must be standardized using the formula for z. When $x = 11$,

$$\boxed{z = \frac{x-\mu}{\sigma} = \frac{11-10}{2} = .5}$$

$\mu = 10$

$\sigma = 2$

$\rightarrow P(11 \le X \le 13,6)$

and when $x = 13.6$,

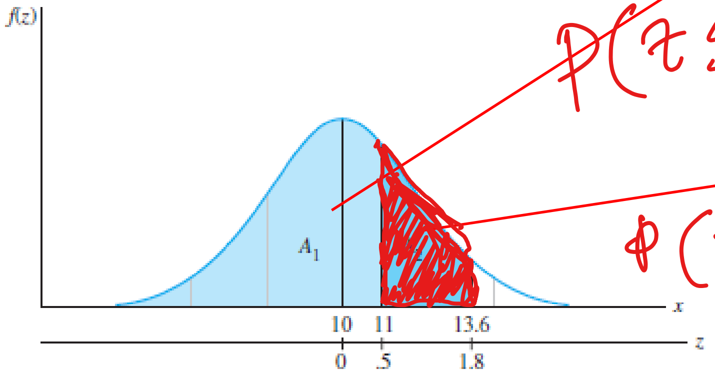
$$\boxed{z = \frac{x-\mu}{\sigma} = \frac{13.6-10}{2} = 1.8}$$

$P(0,5 < Z \le 1,8)$

The desired probability is therefore $P(.5 \le z \le 1.8)$, the area lying between $z = .5$ and $z = 1.8$, as shown in Figure 6.12. From Table 3 in Appendix I, you find that the area to the left of $z = .5$ is .6915, and the area to the left of $z = 1.8$ is .9641. The desired probability is the difference between these two probabilities, or

$$P(.5 \le z \le 1.8) = .9641 - .6915 = .2726$$

$P(Z \le 1,8) - P(Z \le 0,5)$



z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700
0.5	.6915	.6950	.6985	.7019	.7054
0.6	.7257	.7291	.7324	.7357	.7389
0.7	.7580	.7611	.7642	.7673	.7704
0.8	.7881	.7910	.7939	.7967	.7995
0.9	.8159	.8186	.8212	.8238	.8264
1.0	.8413	.8438	.8461	.8485	.8508
1.1	.8643	.8665	.8686	.8708	.8729
1.2	.8849	.8869	.8888	.8907	.8925
1.3	.9032	.9049	.9066	.9082	.9099
1.4	.9192	.9207	.9222	.9236	.9251
1.5	.9332	.9345	.9357	.9370	.9382
1.6	.9452	.9463	.9474	.9484	.9495
1.7	.9554	.9564	.9573	.9582	.9591
1.8	.9641	.9649	.9656	.9664	.9671
1.9	.9713	.9719	.9726	.9732	.9738

$P(Z < 0,5) = 0,6915$

$P(Z < -0,5) = $

$1 - 0,6915 ?$

$\boxed{0,3085}$

$P(Z < -z) = 1 - P(Z < z)$

Ilustrasi

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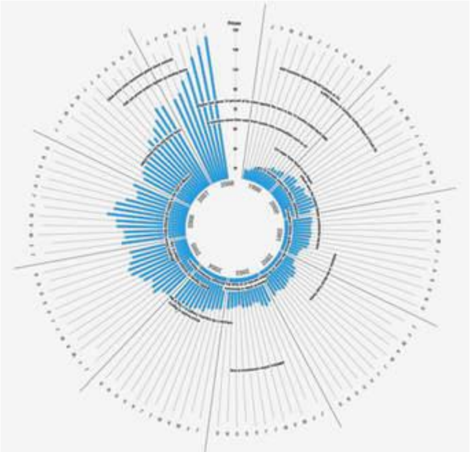

- Dari Mendenhall:

6.16 A normal random variable x has mean 50 and standard deviation 15. Would it be unusual to observe the value $x = 0$? Explain your answer.

Suatu peubah acak *X* menyebar normal dengan nilaitengah 50 dan simpangan baku 15. Apakah tidak lazim (unusual) jika kita memperoleh X = 0? Jelaskan..!

- $X \sim Normal\ (50, 15^2)$, jadi X adalah peubah acak kontinu

  $\boxed{P(X = 0)} = 0$ →→ trivial

  karena $P(X = 0) = \int_0^0 \boldsymbol{f(x)dx = 0}$ → integral pada satu titik

- Jadi yang perlu kita cari adalah $P(X < 0)$, jika peluang ini kecil maka berarti kejadian itu tidak lazim (*unusual*)
- $P(X < 0) = P(Z < (0\text{-}50)/15) = P(Z < -3.33) = 0.0004$ ← lihat di tabel.
- Karena peluangnya sangat kecil, jauh lebih kecil dari 5%, maka kita simpulkan bahwa kejadian $X = 0$ adalah kejadian yang langka atau tidak lazim (*unsual*).
- Apakah juga tidak lazim untuk memperoleh $\boxed{X = 20?}$

$\mu = 50 - 15,\ 50 + 15$

$\sigma = 15$

$z = \frac{20-50}{15} = \frac{-30}{15} = -2$

$P(X<20) = P(Z<-2)$

$= 0{,}02275$

Ilustrasi

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- Hasil varietas padi IPBxx menyebar normal dengan μ = 50 kw/petak, s = 10 kw/petak jika ditanam di lahan beririgasi. Dilahan tadah hujan μ = 21 kw/petak, s = 4.7 kw/petak.
- Percobaan di lahan irigasi menghasilkan 65 kw/petak sedangkan di lahan tadah hujan hasilnya 30 kw/petak.
- Pertanyaan:
  - Brp peluang memperoleh hasil ≥ 65 kw di lahan irigasi?
  - Brp peluang memperoleh hasil ≥ 30 kw di tadah hujan?

$P(X \geq 65$

$P(X_2 \geq 30)$

- Misalkan X_{ir} = hasil varietas IPBxx di lahan irigasi, dan X_{th}= hasil varietas IPBxx di lahan tadah hujan.

$P(Z \geq z) = P(Z \leq -z)$

- $X_{ir} \sim N(50,10^2)$ dan $X_{th} \sim N(21,4.7^2)$
  - $P[X_{ir} \geq 65] = P[(X_{ir} - \mu)/s \geq \boxed{(65-50)/10}]$
    $= P[Z \geq 1.50] = 0.0668$ ← dari tabel
  - $P[X_{th} \geq 30] = P[(X_{th} - \mu)/s \geq (30-21)/4.7]$
    $= P[Z \geq 1.91] = 0.0281$ ← dari tabel

$1 - P(Z < z) = 1 - P(Z \leq z)$

z	.00	.01	.02	.03
−3.4	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004
−3.2	.0007	.0007	.0006	.0006
−3.1	.0010	.0009	.0009	.0009
−3.0	.0013	.0013	.0013	.0012
−2.9	.0019	.0018	.0017	.0017
−2.8	.0026	.0025	.0024	.0023
−2.7	.0035	.0034	.0033	.0032
−2.6	.0047	.0045	.0044	.0043
−2.5	.0062	.0060	.0059	.0057
−2.4	.0082	.0080	.0078	.0075
−2.3	.0107	.0104	.0102	.0099
−2.2	.0139	.0136	.0132	.0129
−2.1	.0179	.0174	.0170	.0166
−2.0	.0228	.0222	.0217	.0212
−1.9	.0287	.0281	.0274	.0268
−1.8	.0359	.0351	.0344	.0336
−1.7	.0446	.0436	.0427	.0418
−1.6	.0548	.0537	.0526	.0516
−1.5	.0668	.0655	.0643	.0630
−1.4	.0808	.0793	.0778	.0764
−1.3	.0968	.0951	.0934	.0918
−1.2	.1151	.1131	.1112	.1093
−1.1	.1357	.1335	.1314	.1292
−1.0	.1587	.1562	.1539	.1515
−0.9	.1841	.1814	.1788	.1762
−0.8	.2119	.2090	.2061	.2033
−0.7	.2420	.2389	.2358	.2327
−0.6	.2743	.2709	.2676	.2643
−0.5	.3085	.3050	.3015	.2981

Pendekatan Normal thd Binom — Diskret

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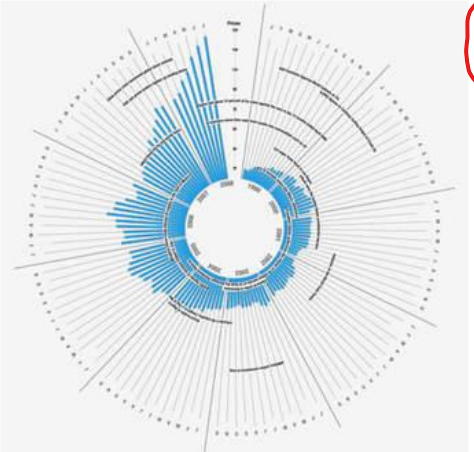


$\lambda = np$ $n \to \infty$ $p \to 0$ $X \sim Binom(n, p)$

- Menghitung peluang binom (diskret) bisa lebih mudah jika didekati dengan sebaran normal (kontinu) asalkan n cukup besar $\boxed{(np \geq 5)}$.

$E(X) = np \to$ nilai tengah $= \mu$
$Var(X) = np(1-p) \to \sigma^2$

- Jika $X \sim Binom\ (n, p)$ maka p.a X diaproksimasi dengan sebaran normal, artinya $X \sim Normal\ (np, np(1-p))$.

Kontinu

$P(X = a) = 0 \ldots ?$
$P(X = b) = 0 \ldots ?$

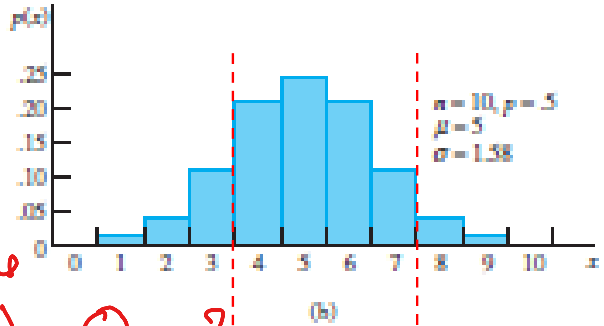
- Dengan pendekatan normal

Binom $\to$ Normal

$$P(a \leq X \leq b) \approx P\big((a - 0.5) \leq X \leq (b + 0.5)\big).$$

- Sedangkan $P(a < X < b) \approx \boxed{P\big((a + 0.5) \leq X \leq (b - 0.5)\big).}$

x_1 x_2



3.5 ↔ 7.5

$$P(4 \leq X \leq 7) \approx P(3.5 < X < 7.5)$$
$$= P\left(\frac{3.5 - np}{\sqrt{np(1-p)}} \leq Z \leq \frac{7.5 - np}{\sqrt{np(1-p)}}\right)$$

- Trasformasi ke normal baku (Z):

memuat = memperlebar

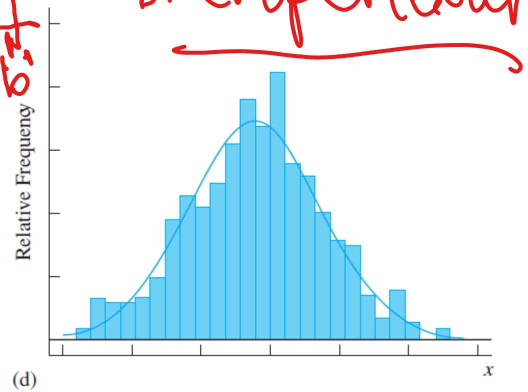
a' a b b'

$$P(a \leq X \leq b) \approx P\left(\frac{(a-0.5) - np}{\sqrt{np(1-p)}} \leq Z \leq \frac{(b+0.5) - np}{\sqrt{np(1-p)}}\right)$$

$x - \mu / \sigma$

- Sedangkan $P(a < X < b) \approx P\left(\frac{(a+0.5) - np}{\sqrt{np(1-p)}} \leq Z \leq \frac{(b-0.5) - np}{\sqrt{np(1-p)}}\right)$

σ



memuat $\neq$ mempersempit

a a' b' b

Pendekatan Normal thd Binom

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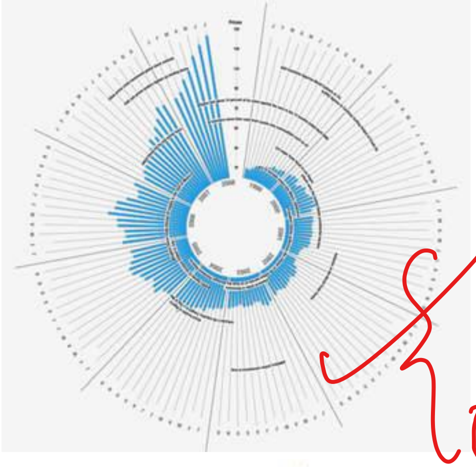
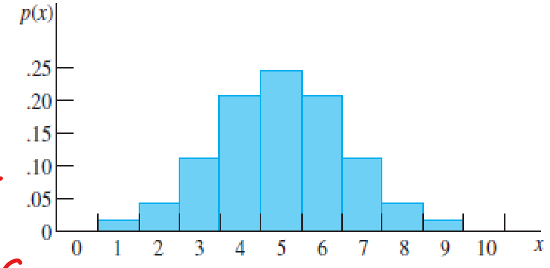
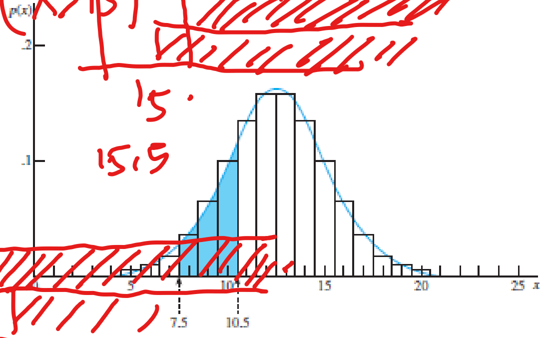


- Berikut ini ilustrasi sebaran normal untuk menghitung peluang sebaran binom.

For large n and π not too near 0 or 1, the distribution of a binomial random variable y may be approximated by a normal distribution with $\mu = n\pi$ and $\sigma = \sqrt{n\pi(1-\pi)}$. This approximation should be used only if $n\pi \ge 5$ and $n(1-\pi) \ge 5$. A continuity correction will improve the quality of the approximation in cases in which n is not overwhelmingly large.



A large drug company has 100 potential new prescription drugs under clinical test. About 20% of all drugs that reach this stage are eventually licensed for sale. What is the probability that at least 15 of the 100 drugs are eventually licensed? Assume that the binomial assumptions are satisfied, and use a normal approximation with continuity correction.



$$P(y \ge 14.5) = P\left(z \ge \frac{14.5-20}{4.0}\right) = P(z \ge -1.38) = 1 - P(z < -1.38)$$
$$= 1 - .0838 = .9162$$

Perhatikan bahwa angka ≥15 (binom) didekati dengan angka ≥ 14.5 (normal). Mengapa? Karena angka 15 (diskret) tidak lain adalah dari 14.5 sampai 15.5.(kontinu).

$\pi : p$

$\pm 0,5$

Binom

$P(X \ge 15) \rightarrow$ memperlebar

$np = 100 \cdot \frac{20}{100} = 20$

$= 91,62\%$

$z = \frac{x - \mu}{\sigma}$

$\mu = np$

$\sigma^2 = np(1-p) \rightarrow \sigma = \sqrt{\sigma^2}$

$= 100 \cdot \frac{20}{100} \cdot \frac{80}{100} = \frac{1600}{100} = 16$

$\sigma = \sqrt{16} = 4$

Syntax R :

$P(X \leq x)$

?

$P(X \geq x)$

☺ THANK YOU ☺

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