Responsi Metode Statistika (STA-1211)

PERTEMUAN 5

Sebaran Peubah Acak (Kontinu)

Asisten: Laily Nissa Atul Mualifah

Sebaran Seragam

$$X \sim Seragam\ (a, b)$$

maksimal
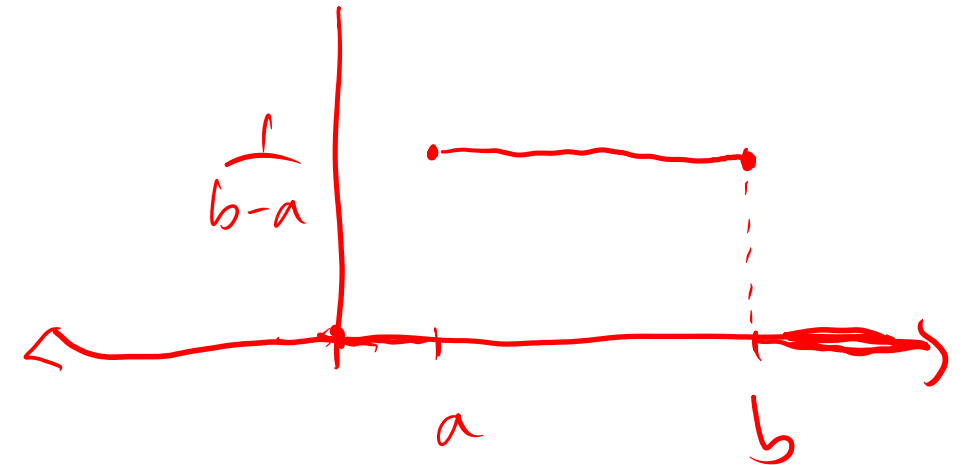
minimal

Fkp dari peubah acak X:

$$f(x) = \frac{1}{b-a} \qquad, a \leq x \leq b$$

$$f_X = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & , x \text{ y lain} \end{cases}$$

Nilai harapan $X : E(X) = \frac{a+b}{2}$

Ragam $X : Var\ (X) = \frac{(b-a)^2}{12}$

$$E(x^2) - E(x)^2$$

Wackerly, *et al.*, 2008

$$\frac{1}{5m}, \quad 2000 \le x < 25000$$

4.45 Upon studying low bids for shipping contracts, a microcomputer manufacturing company finds that intrastate contracts have low bids that are uniformly distributed between 20 and 25, in units of thousands of dollars. Find the probability that the low bid on the next intrastate shipping contract

a is below $22,000. $P(X < 22000)$

b is in excess of $24,000.

$P(X < 24000)$

$$P(X \geq 24.000) = \int_{24000}^{25000} f(x)\, dx = 1 - \int_{20m}^{24000} f(x)\, dx$$

$$\text{a) } P(X < 22000) = \int_{20m}^{22} f(x)\, dx = \int_{20}^{22} \frac{1}{5000}\, dx = \frac{1}{5000} x \Big]_{20m}^{22000} = \frac{2}{5}$$

Wackerly, *et al.*, 2008

4.51 The cycle time for trucks hauling concrete to a highway construction site is uniformly distributed over the interval 50 to 70 minutes. What is the probability that the cycle time exceeds 65 minutes if it is known that the cycle time exceeds 55 minutes?
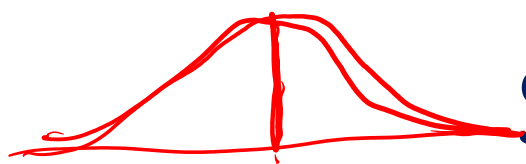
$$a = 50, \quad b = 70$$

$$f_x = \frac{1}{20}, \quad 50 \le x \le 70$$

$$\frac{\frac{1}{20} x \big]_{65}^{70}}{\frac{1}{20} x \big]_{55}^{70}} = \frac{5}{15} = \frac{1}{3}$$

$$P(x > 65 \mid x > 55) = \frac{P(x > 65) \cap P(x > 55)}{P(x > 55)} = \frac{P(x > 65)}{P(x > 55)} = \frac{\int_{65}^{70} \frac{1}{20} dx}{\int_{55}^{70} \frac{1}{20} dx} =$$

Sebaran Normal

nilai tengah

$$X \sim Normal\;(\mu, \sigma^2)$$

ragam

Fkp dari peubah acak X:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (x^2 - 2x\mu + \mu^2) \quad , -\infty < x < \infty$$

Nilai harapan $X : E(X) = \mu$

Ragam $X : Var\;(X) = \sigma^2$

Distribusi Normal Baku

$$Z \sim Normal(0,1)$$

Fkp:

$$f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}} \quad , -\infty < x < \infty$$

Hubungan dengan sebaran lain:

$$z = \frac{x - \mu}{\sigma} \sim Normal\;(0,1)$$

Transformasi $Z^2 \sim \chi^2_{(1)}$ → *transformasi*

$$Y = Z^2 \sim \chi^2_{(1)}$$

Latihan

$$P(a < X < b) = P(X < b) - P(X < a)$$

$$Z = \frac{x - \mu}{\sigma}$$

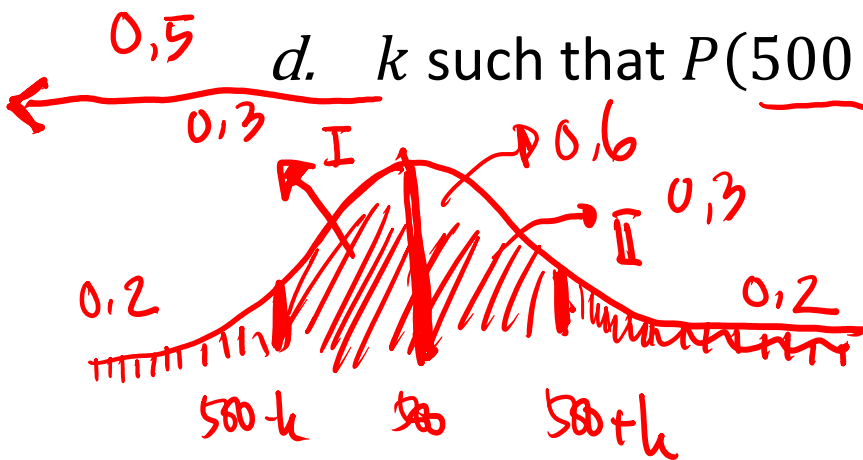Let y be a normal random variable with $\mu = 500$ and $\sigma = 100$. Find the following probabilities:

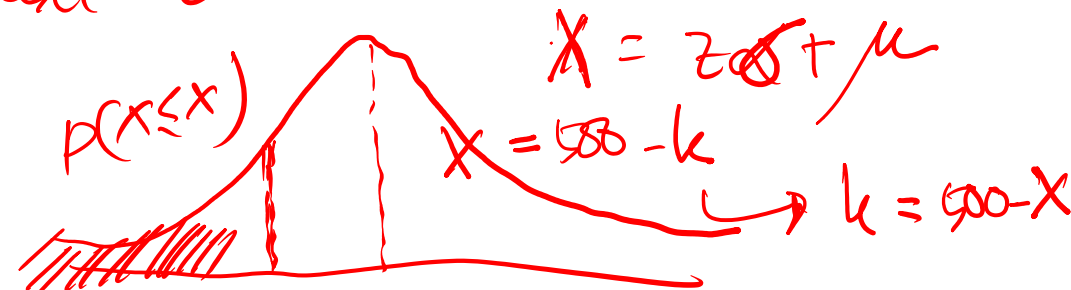$$z_2 = \int z^b$$

$$z_1 = z_2 \quad = \int_{665} f(x)\, dx = \text{Tabel } Z$$

a. $P(500 < y < 665)$

$$= 1 - \int_{-n}$$

b. $P(y > 665) = \int_{665}$

$$P(X \leq x)$$

$$X = z\sigma + \mu$$

$$X = 500 - k$$

c. $P(304 < y < 665)$?

$$k = 500 - X$$

?

d. k such that $P(500 - k < y < 500 + k) = 0.60$

0,5

0,3   I   0,6

0,3

0,2   II   0,2

500-k   500   500+k

$$P(X < 500 - k) = 0,2$$

$$P(X > 500 + k) = 0,2$$

0,8

0,2
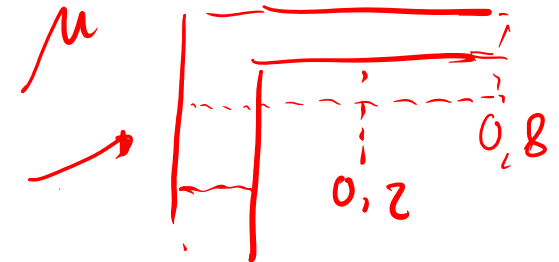
$$\boxed{Z} = 0, \ldots + 0,6 \ldots$$

$$Z = 0, \ldots + 0,6 \ldots$$

Mendenhall, *et al.,* 2013

6.18 **Hamburger Meat** The meat department at a local supermarket specifically prepares its "1-pound" packages of ground beef so that there will be a variety of weights, some slightly more and some slightly less than 1 pound. Suppose that the weights of these "1-pound" packages are normally distributed with a mean μ of 1.00 pound and a standard deviation of .15 pound. σ

by syntax

a. What proportion of the packages will weigh more than 1 pound? $P(X>1)$

b. What proportion of the packages will weigh between .95 and 1.05 pounds? $P(0.95 < X < 1.05)$

c. What is the probability that a randomly selected package of ground beef will weigh less than .80 pound? $P(X<0.80)$

d. Would it be unusual to find a package of ground beef that weighs 1.45 pounds? How would you explain such a large package? $P(X=1.45)=0$

$P(X \geq 1.45)$? $\quad P(X \leq 1.45)$

Sebaran bagi contohan

$X \sim Normal(\mu, \sigma^2)$

$\bar{X} \sim Normal\left(\mu, \dfrac{\sigma^2}{n}\right)$, $\dfrac{s^2}{n}$

Statistik contoh

Agresti, *et al.*, 2018

6.63 **Normal heart rate** The normal resting heart rate for adults is 60 to 100 beats per minute. The heartbeat rate in a sample of 400 patients was tested. It was found that the distribution of the number of beats per minute is roughly normally distributed with an average of 80 and a standard deviation of 12. s

$P(60 < X < 100)$

nilai rata = X

a. Find the proportion of individuals in the sample whose heartbeat rate is in the normal range.

b. Tachycardia refers to any heartbeat rate greater than 100 beats per minute. Estimate the number of individuals in the sample who could have tachycardia.

X

$z = \dfrac{X - \mu}{\sigma}$ Populasi

$z = \dfrac{X - \bar{X}}{s/\sqrt{n}}$ Sampel

$\sigma^2 = \sqrt{\sigma^2} = \sigma \leftarrow \sigma^2 = \dfrac{s^2}{n} \rightarrow \sigma = \sqrt{\dfrac{s^2}{n}} = \dfrac{s}{\sqrt{n}}$

b) $P(X > 100) = \ldots ?$

proporsi $\rightarrow \hat{p} = \dfrac{X}{n}$

Latihan

Agresti, *et al.*, 2018

6.67   **Used car prices**   Data from the Web site carmax.com compiled in July 2014 show that prices for used Audi A4 cars advertised on the Web site have a mean of $23,800 and a standard deviation of $4,380. Assume a normal distribution for the price.

    a.   What percent of used Audi A4s cost more than $25,000?

    b.   What percent of used Audi A4s cost between $18,000 and $22,000?

    c.   The least expensive 10% of used Audi A4s offered on the Web site cost at most how much?

Mendenhall, *et al.*, 2013

6.50 **The Rh Factor** In a certain population, 15% of the people have Rh-negative blood. A blood bank serving this population receives 92 blood donors on a particular day.

a. What is the probability that 10 or fewer are Rh-negative?

b. What is the probability that 15 to 20 (inclusive) of the donors are Rh-negative?

c. What is the probability that more than 80 of the donors are Rh-positive?

Latihan

Mendenhall, *et al.,* 2013

6.82 **Long Distance** It is known that 30% of all calls coming into a telephone exchange are long-distance calls. If 200 calls come into the exchange, what is the probability that at least 50 will be long-distance calls?

TERIMAKASIH
'Semoga Sukses'

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