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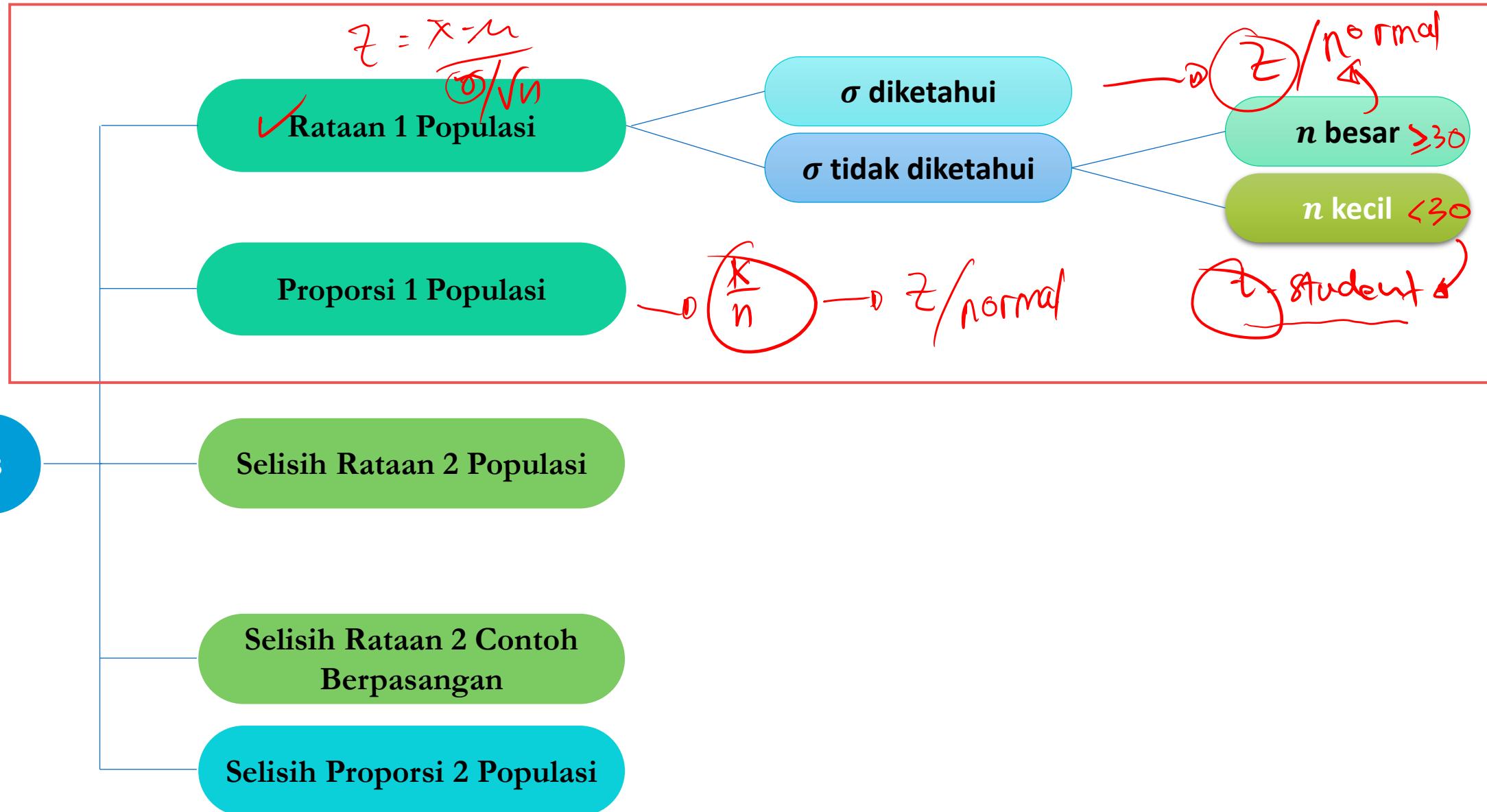
Study Program
Statistics and Data Science
Department of Statistics

Responsi Metode Statistika (STA-1211)

PERTEMUAN 8

PENGUJIAN HIPOTESIS

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nilai tengah = \bar{x}

Prosedur Pengujian Hipotesis

1. Rumuskan H_0 dan H_1 .

a. Pilih pasangan hipotesis yang relevan

dua arah

$$\begin{array}{c} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{array}$$
$$\begin{array}{c} H_0 = \\ H_1 \neq \end{array}$$
$$\begin{array}{c} H_0 \leq \\ H_1 > \end{array}$$
$$\begin{array}{c} H_0 \geq \\ H_1 < \end{array}$$

- Bisa dimulai dari H_0 atau dari H_1 . 1 arah
- Dari H_0 : jika akan menguji "asumsi", H_1 -nya menyesuaikan.
- Dari H_1 : jika akan menguji "klaim" tentang sesuatu yang baru, H_0 menyesuaikan.

b. Tentukan parameter yang diuji ($\theta = ?$).

2. Tentukan statistik dari parameter yang diuji serta Sebaran Penarikan Contoh (SPC) bagi statistik tersebut.

- Statistik untuk RATAAN atau statistik untuk PROPORSI ($\hat{\theta} = ?$).

Sebaran Penarikan Contoh:

- Jika simpangan baku populasi diketahui \rightarrow sebaran Normal
- Jika simpangan baku populasi tidak diketahui \rightarrow sebaran t

$$\begin{array}{l} H_0 : = \\ H_1 : \neq \end{array}$$

H_0 : Setelah ada tanda

$$H_1 : >$$

$H_1 :$

IP minimal 3.3

H_0

$$H_0: \mu = \mu_0$$

$$H_1: \mu \neq \mu_0$$

$$IP \geq 3.3$$

$$H_1: IP < 3.3$$

$$\bullet IP < 3.8 \Rightarrow \begin{cases} H_1: IP < 3.8 \\ H_0: IP \geq 3.8 \end{cases}$$

$$\begin{cases} H_1: IP > 3.8 \\ H_0: IP \leq 3.8 \end{cases}$$

$$z_h \leftarrow 1,25 < 1,96 \rightarrow z_{tabel}$$

Prosedur Pengujian Hipotesis

$$z = t \text{ (tabel)}$$

3. Tentukan tingkat toleransi yang akan digunakan untuk salah jenis I (α)

- Semakin kecil alpha yang dipilih \rightarrow peluang menolak H_0 semakin kecil
- Untuk percobaan-percobaan yang dilakukan di lab dengan desain yang cermat biasanya digunakan alpha 5% atau 1%

$$I \rightarrow \alpha$$

$$II \rightarrow \beta$$

$$z_{0,05} \quad z_{0,025}$$

*tabel
 H_0*

4. Hitung statistik ujinya

$$1,25$$

$$z_{th} = 2,333 \dots$$

$$z_{tabel}$$

$$z_h = \frac{\hat{\theta} - \mu_{\hat{\theta}}}{\sigma_{\hat{\theta}}}$$

atau

$$t_h = \frac{\hat{\theta} - \mu_{\hat{\theta}}}{s_{\hat{\theta}}} \quad t_{tabel}$$



5. Tentukan titik kritis, wilayah penerimaan H_0 , dan wilayah penolakan H_0 .

- Berdasarkan pasangan hipotesis yang relevan dan alpha yang digunakan.

6. Ambil keputusan menolak H_0 atau gagal menolak (menerima) H_0

- Tolak H_0 jika statistik ujinya (z_h atau t_h) berada di wilayah penolakan.
- Terima H_0 jika statistik ujinya (z_h atau t_h) berada di wilayah penerimaan.

7. Tarik kesimpulan.

Tolak H_0 : Pd farag wytu $t_h > z_{tabel}$
tolak H_0 : $p(z > z_{tabel}) < \alpha$

Tolak H_0 : $p(z > z_{tabel}) < \alpha$

$$p(z > z_{tabel}) = 0,05 < 0,05 \quad p\text{-value}$$

$$z_{tabel} \quad z_{tabel}$$

$$z_{tabel} \quad z_{tabel}$$

$$p(z > z_{tabel})$$

$$p(z > z_{tabel})$$

Rataan 1 Populasi

• (σ diketahui)

? Contoh Soal

Gaji perbulan karyawan pada sebuah perusahaan diketahui menyebar normal dengan simpangan baku Rp 600.000,-. Jika diambil sebanyak 16 contoh acak dari populasi tersebut, diperoleh rata-rata gaji Rp 1.200.000,-. Seorang atasan mengklaim bahwa rata-rata gaji karyawan di perusahaan tersebut minimal Rp 1.000.000,-. Apakah benar klaim atasan tersebut? Ujilah pada taraf nyata 5%

$$\begin{aligned}\sigma &= 600000 \\ n &= 16 \\ \bar{x} &= 1200000 \\ \alpha &= 0.05\end{aligned}$$

$$\mu \geq 1000000$$

$$z_h > z_{tabel}$$

Statistik Uji:

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Tamp Nor $\rightarrow Z$

$$Z = \frac{x - \mu / \bar{x}}{\sigma / \sqrt{n}}$$

Solusi

$$\begin{aligned}H_0: \mu &\geq 1000000 \\ H_1: \mu &< 1000000\end{aligned}$$

S

Statistik Uji:

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{1200000 - 1000000}{600000 / \sqrt{16}} = \frac{200000}{150000} = 1.33$$

$$P(Z < 1.33) = 0.908$$

$$z_\alpha \rightarrow 1.65$$

$$z_h < z_{tabel}$$

$$z_{0.05} = 1.65 \rightarrow \text{toler}$$

Kriteria keputusan: Karena $-z_{0.05} = -1.65$ dan $z_{hitung} = 1.33$ (atau berdasarkan $p-value = 0.908 > 0.05$) maka **tidak tolak H_0** .

Jadi pada taraf nyata 5% tidak cukup bukti untuk mengatakan bahwa rata-rata gaji karyawan di perusahaan tersebut kurang dari Rp 1.000.000,-.

$$16.112 < \mu < 11.89$$

Rataan 1 Populasi

(σ tidak diketahui, $n \geq 30$)

Contoh Soal

Menurut sebuah penelitian, konsumsi sereal yang telah diberi pemanis secara terus menerus menyebabkan pembusukan gigi, penyakit jantung, dan penyakit menurunnya fungsi tubuh lainnya. Dari suatu contoh acak 64 bungkus α -Bits diperoleh rata-rata kadar gula 11.3 gram dengan simpangan baku 2.45 gram. Ujilah klaim bahwa rata-rata kadar gula α -Bits kurang dari 12 gram per bungkus. Gunakan $\alpha = 5\%$.

$$\begin{aligned}s &= 2.45 \\n &= 64 \\\bar{x} &= 11.3\end{aligned}$$

$$z_h > z_{tabel} \quad z_h < -2$$

$$10.112 < \mu < 12.01$$

Statistik Uji:

$$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

1. Statistika uji $\rightarrow z/t$

2. P-value $\rightarrow P(z < z) / P(z > z)$

3. Sebag Kepercayaan

Solusi

$$\begin{aligned}H_0: \mu &\geq 12 \\H_1: \mu &< 12\end{aligned}$$

Statistik Uji:

$$z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{11.3 - 12}{2.45/\sqrt{64}} = \frac{-0.7}{0.30625} = -2.2857$$

$$P(z < -2.2857) = 0.0111 \quad \text{tolak } H_0 \quad \boxed{-2.2857 < -1.645}$$

Kriteria keputusan: Karena $-z_{0.05} = -1.645$ dan $z_{hitung} = -2.2857$ maka tolak H_0 .

Jadi pada taraf nyata 5% cukup bukti untuk mengatakan bahwa rata-rata kadar gula α -Bits kurang dari 12 gram per bungkus.

titik kritis: z_d

$$z_{0.05} = 1.645$$

$$+2.2857 > 1.645$$

$$0.0111 < 0.05$$

Norma dist

(Normalitas)

Rataan 1 Populasi

(σ tidak diketahui, $n < 30$)

Statistik Uji:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

t_α / $t_{\alpha/2}$

t_α / $t_{\alpha/2}$

derajat bebas

$= n - 1$

phon
quon
dun

pt(t, v)
 $dt(\alpha, v)$
 rt
 $dt(t, v)$

Contoh Soal

$$\begin{array}{l} 5 > |3| \\ -5 < -3 \end{array}$$

Solusi

Suatu perusahaan memproduksi bohlam yang umurnya menyebar normal. Bila dari contoh acak sebanyak 25 bohlam mencapai umur rata-rata 780 jam dengan simpangan baku 40 jam, ujilah hipotesis bahwa umur rata-rata bohlam 800 jam. Gunakan taraf nyata 4%.

$$\begin{aligned} H_0: \mu &= 800 \\ H_1: \mu &\neq 800 \end{aligned}$$

Statistik Uji:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{780 - 800}{40/\sqrt{25}} = \frac{-20}{8} = -2.5$$

$$P(t < -2.5) = 0.0098$$

$$t_h < -t_{\text{tabel}}$$

Kriteria keputusan: Karena $|t_{0.02, 24}| = 2.172$ dan $t_{\text{hitung}} = -2.5$, maka tolak H_0 .

$$-2.172 < -2.5 > 2.172$$

$$\begin{aligned} s &= 40 \\ n &= 25 \\ \bar{x} &= 780 \\ \alpha &= 0.04 \end{aligned}$$

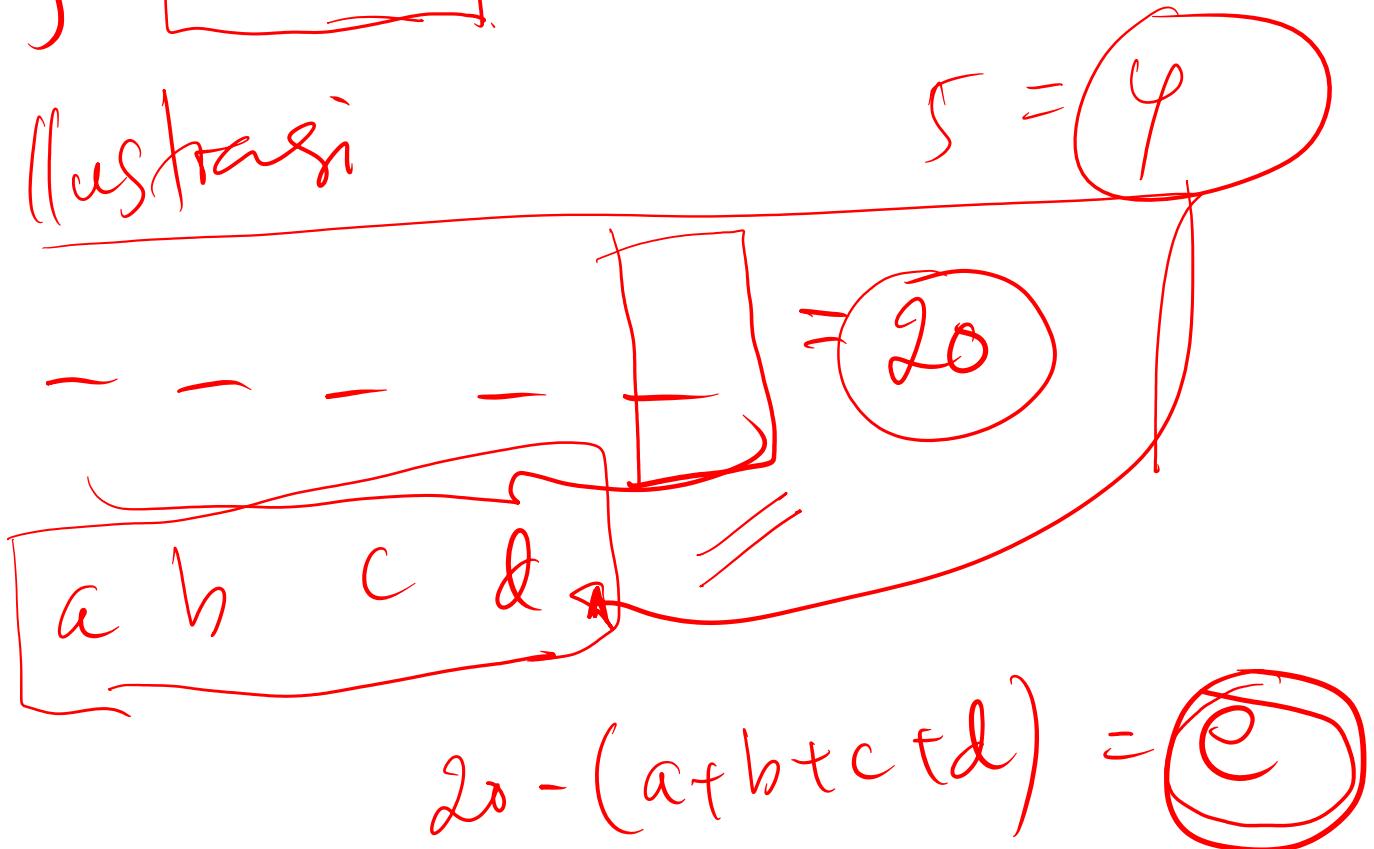
$$t_{\text{tabel}} > t_{\text{hitung}}$$



Jadi pada taraf nyata 4% cukup bukti untuk menolak klaim bahwa umur rata-rata bohlam 800 jam atau kita cukup bukti untuk mengatakan bahwa rata-rata umur bohlam tidak sama dengan 800 jam

$$g_f = \boxed{V = n - 1}$$

Illustrasi



Binomial (n , p) $C_60 = 88$

Proporsi 1 Populasi

0.8

Contoh Soal

Sebuah LSM dalam studi terbarunya menemukan bahwa sekitar 275 dari 500 penduduk di Pusat Kota A menyukai sosok calon X sebagai Walikota yang baru. LSM tersebut ingin menguji apakah benar bahwa lebih dari 50% penduduk di Pusat Kota A menyukai calon X. Gunakanlah taraf nyata 5%.

$$\hat{p} = \frac{275}{500} = 0.55$$
$$n = 500$$
$$\alpha = 0.05$$

Statistik Uji:

$$Z = \frac{X - np_0}{\sqrt{np_0(1 - p_0)}}$$

Binomial (n, p)

$$- E(X) = \mu = np$$

$$- \text{Var}(X) = [np(1-p)]$$



Solusi

$$\begin{aligned} H_0: p &\leq 0.5 \\ H_1: p &> 0.5 \end{aligned}$$

$$z = \frac{x - \mu}{\sigma / \sqrt{n}}$$

YI arah \rightarrow
titik kritik $= z_{\alpha}$

Statistik Uji:

$$Z = \frac{X - np_0}{\sqrt{np_0(1 - p_0)}} = \frac{275 - 500(0.5)}{\sqrt{500(0.5)(0.5)}} = \frac{25}{11.18} = 2.236$$

$$P(Z > 2.236) = 1 - P(Z < 2.236) = 1 - 0.9873 = 0.0127$$

~~$P(z < -2.236) =$~~ $z_{\alpha} = 1.65$ $2.236 > 1.65$

Kriteria keputusan: Karena $Z_{0.05} = 1.65$ dan $Z_{\text{hitung}} = 2.236$ (atau berdasarkan $p-value = 0.0127 < 0.05$) maka tolak H_0 .

Jadi pada taraf nyata 5% cukup bukti untuk mengatakan bahwa lebih dari 50% penduduk di Pusat Kota A menyukai calon X.

$$0.0127 < 0.05$$

Latihan Soal

Soal 1

(Mendenhall *et al.*, 2013)

9.6 A random sample of $n = 35$ observations from a quantitative population produced a mean $\bar{x} = 2.4$ and a standard deviation $s = .29$. Suppose your research objective is to show that the population mean μ exceeds 2.3.

- a. Give the null and alternative hypotheses for the test.
- b. Locate the rejection region for the test using a 5% significance level.
- c. Find the standard error of the mean.
- d. Before you conduct the test, use your intuition to decide whether the sample mean $\bar{x} = 2.4$ is likely or unlikely, assuming that $\mu = 2.3$. Now conduct the test. Do the data provide sufficient evidence to indicate that $\mu > 2.3$?

Soal 2

(Mendenhall *et al.*, 2013)

9.13 Potency of an Antibiotic A drug manufacturer claimed that the mean potency of one of its antibiotics was 80%. A random sample of $n = 100$ capsules were tested and produced a sample mean of $\bar{x} = 79.7\%$ with a standard deviation of $s = .8\%$. Do the data present sufficient evidence to refute the manufacturer's claim? Let $\alpha = .05$.

- a. State the null hypothesis to be tested.
- b. State the alternative hypothesis.
- c. Conduct a statistical test of the null hypothesis and state your conclusion.

Soal 3

(Agresti *et al.*, 2018)

9.1 H_0 or H_a ? For parts a and b, is the statement a null hypothesis, or an alternative hypothesis?

TRY

- a. In Canada, the proportion of adults who favor legalized gambling equals 0.50.
- b. The proportion of all Canadian college students who are regular smokers is less than 0.24, the value it was 10 years ago.
- c. Introducing notation for a parameter, state the hypotheses in parts a and b in terms of the parameter values.

9.2 H_0 or H_a ? State whether each of the following statements is a null hypothesis or an alternative hypothesis. Why?

TRY

- a. In 2016, the average price of solar energy in the United States was \$3.70 per watt.
- b. At least 1 out of every 8 women in the United States will develop breast cancer during her lifetime.
- c. Only 48% of all the money donated by telemarketers actually goes to charitable fundraising campaigns.

Soal 4

(Agresti *et al.*, 2018)

9.33 StatCrunch for statistics For effective learning, an instructor advised his students to practice solving statistical problems on StatCrunch, a web-based software, at least 7 hours per week. In an in-class activity, a student surveyed 15 of her 35 classmates to assess the number of hours they spend per week practicing statistics on StatCrunch. The following data was collected:

6.5, 4, 3.5, 0, 12, 5, 12, 15, 12, 0, 1, 8, 0.5, 2, 7

Is there strong evidence that the mean number of hours of StatCrunch usage in the entire class is larger than 7?

Answer by:

- a. Identifying the relevant variable and parameter.
- b. Stating null and alternative hypotheses.
- c. Finding and interpreting the test statistic value.
- d. Reporting and interpreting the P-value and stating the conclusion in context.

Soal 5

9.35 Early Detection of Breast Cancer Of those women who are diagnosed to have early-stage breast cancer, one-third eventually die of the disease. Suppose a community public health department instituted a screening program to provide for the early detection of breast cancer and to increase the survival rate p of those diagnosed to have the disease. A random sample of 200 women was selected from among those who were periodically screened by the program and who were diagnosed to have the disease. Let x represent the number of those in the sample who survive the disease.

- a. If you wish to determine whether the community screening program has been effective, state the alternative hypothesis that should be tested.
- b. State the null hypothesis.
- c. If 164 women in the sample of 200 survive the disease, can you conclude that the community screening program was effective? Test using $\alpha = .05$ and explain the practical conclusions from your test.
- d. Find the p -value for the test and interpret it.

(Mendenhall *et al.*, 2013)

Soal 6

Data pendapatan per bulan dari 30 petani (contoh acak) di desa Tegal Gundil.

Ujilah apakah mediannya sama dengan Rp 3.500.000 per bulan?

598236, 559348, 1393754, 2904429, 4172863, 1470580
3544143, 4918026, 558184, 1436698, 2598927, 2997189
6806786, 4623161, 830000, 3566447, 2707305, 3649276
2767137, 4233366, 2794188, 3116484, 4082507, 613158
7375163, 5112766, 1870627, 121805, 1330808, 863539

Hipotesis yg diuji adalah $H_0: \text{Med} = 3.500.000$

$H_1: \text{Med tdk sama } 3.500.000$

Urutan Bootstrap (umum) :

1. Ada data sebanyak n (contoh acak)
2. Ingin menguji/SK dari parameter θ , tapi statistik $\hat{\theta}$ tidak diketahui sebarannya.
3. Lakukan Bootstrap:
 - Resampling sebanyak n pengamatan terhadap **dengan pemulihan**
 - Hitung statistik $\hat{\theta}$
 - **Ulangi 100000 kali** (lakukan simulasi 100000 kali) sehingga diperoleh statistik $\hat{\theta}$ sebanyak 100000.
- Buat histogramnya dan tentukan batas kiri-kanan 95%
- Maka SK 95% diperoleh dan pengujian hipotesis bisa dilakukan.



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