



Department of Statistics

Study Program in Statistics and Data Science

Sebaran Kepekatan Peluang Normal (Normal Distribution)

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Program Studi Statistika dan Sains Data

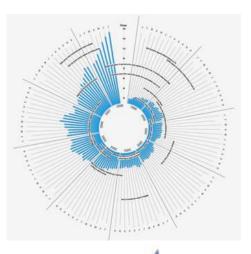






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Outline

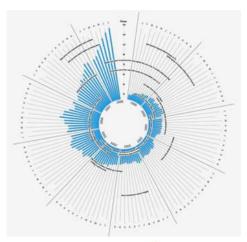
- 1. Peubah Acak Kontinu
- 2. Sebaran Peluang Normal
- 3. Sebaran Peluang Normal Baku
- 4. Penghitungan Peluang Normal







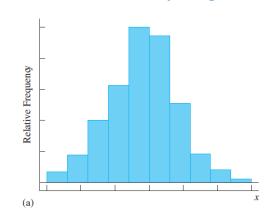


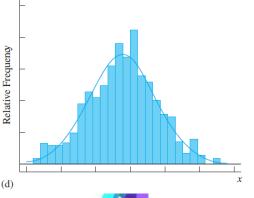


- Peubah acak kontinu. Dari contoh acak diukur tinggi dan bobot badan, keduanya merupakan peubah acak kontinu. Untuk peubah acak kontinu maka maka peluangnya pada satu titik selalu sama dengan nol.
- Mengapa? Karena P(a < X < b) adalah integral fungsi kepekatan f(X) dari titik a samapai ke titik b.



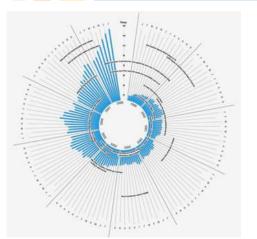
- Sebaran peluang peubah acak diskret dapat didekati (aproksimasi) dengan sebaran kontinu.
- Seperti nampak pada gambar, histogram frekuensi relatif yang menggambarkan sebaran data dapat didekati dengan fungsi kontinu.
- Jika jumlah pengukurannya besar sekali maka selang kelas menjadi semakin sempit dan histogram semakin mulus.
- Kurva yang mulus dalam histogram itu (lihat histogram yang di bawah) mencerminkan sebaran peluang dari peubah acak kontinu.



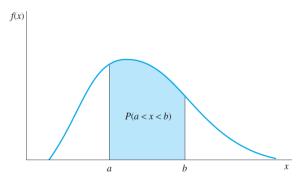






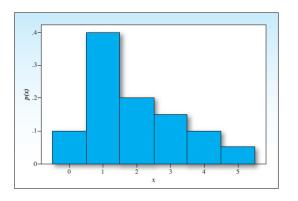


- Bagaimana model dari sebaran peluang tersebut?
- Nilai peubah acak kontinu adalah semua bilangan nyata.
- Sebaran peluangnya dibangun sedemikian sehingga membentuk fungsi seperti nampak dalam gambar.
- Kepekatan peluang (*probability density*), dituliskan dengan rumus matematika f(x) yang dikenal sebagai sebaran peluang (*probability distribution*) atau fungsi kepekatan peluang untuk peubah acak x.



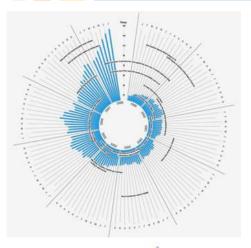


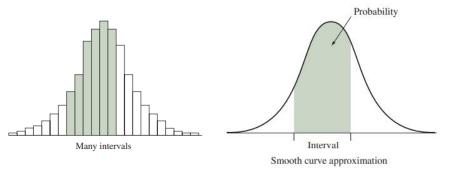
- Ingat bahwa untuk peubah acak diskret berlaku:
 - 1. Jumlah seluruh peluangnya yaitu p(x) sama dengan 1
 - 2. Peluang x ada di dalam suatu selang merupakan jumlah dari peluang yang ada dalam selang tersebut.

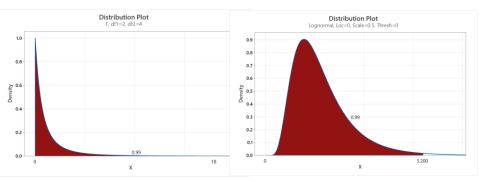












- Setiap interval memiliki peluang diantara 0 s.d 1
- Peluang dari suatu interval diberikan dari luas di bawah kurva
- Total peluang semua interval = total luas di bawah kurva = 1



- ✓ X = Lamanya waktu tunggu dalam suatu antrian
- ✓ Y = Lamanya waktu menelpon seseorang
- ✓ Z = nilai UTS Metode Statistika mahasiswa FKH

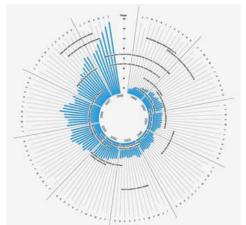








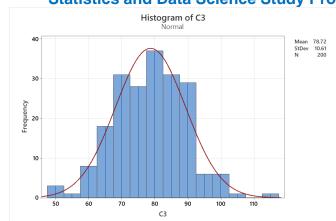
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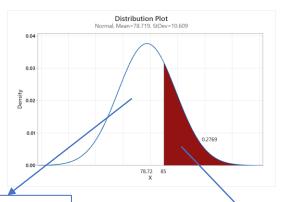


Nilai UTS untuk 200 mahasiswa

Stem-and-leaf of UTS N = 200

1	4	8	uency
4	5	014	Fred
7	5	799	
18	6	0022222344	
42	6	5566666777788889999999	
73	7	000000000111111222233444444444	
(34)	7	555556666666777778888888899999999	
93	8	00000111111111112222233333333344	
61	8	555555666666666777777788888888889999999	
20	9	011222334	
11	9	566779	
5	10	011	
2	10	6	





Luas area di bawah kurva = 1

- Misalkan peubah acak X = nilai UTS mahasiswa
- Berapa P(Xsama dengan 85

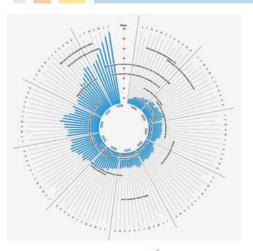
Leaf Unit = 1

Peluang nilai UTS \geq 85 \rightarrow P(X \geq 85) dapat diperoleh dengan menghitung luas di bawah kurva pada selang \geq 85





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• Untuk suatu fungsi kepekatan peluang kontinu f(x) berlaku:

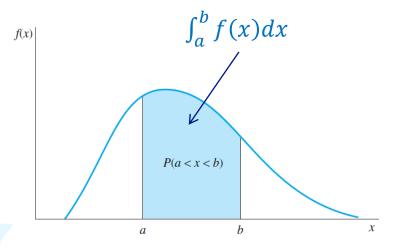
• Luas area di bawah
$$f(x)$$
 sama dengan 1. \longrightarrow $\int_{-\infty}^{\infty} f(x) dx = 1$

 Peluang x ada di dalam suatu selang dari a sampai b, sama dengan luas area di bawah f(x) dari a sampai b.

■
$$P(x = a) = 0$$
 untuk peubah acak x yang kontinu. $\longrightarrow \int_a^a f(x) dx = 0$

- Akibatnya $P(x \ge a) = P(x > a) \operatorname{dan} P(x \neq a) = P(x < a)$.
- Dua hal ini tidak berlaku untuk peubah acak x yang diskret.

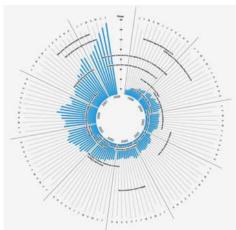






Sebaran Peluang Seragam (*Uniform*)







- Ilustrasi. Perhatikan peubah acak X yaitu waktu penerbangan pesawat terbang dari Jakarta ke Bali.
- Waktu penerbangan bersifat kontinu dengan nilai dalam selang dari 120 menit hingga 140 menit.
- Misalkan dari data penerbangan aktual yang cukup tersedia diketahui peluang waktu penerbangan dalam interval 1 menit manapun sama dengan peluang waktu penerbangan dalam interval 1 menit lainnya yang terkandung dalam interval yang lebih besar dari 120 hingga 140 menit.

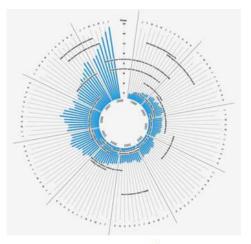






Sebaran Peluang Seragam (*Uniform*)

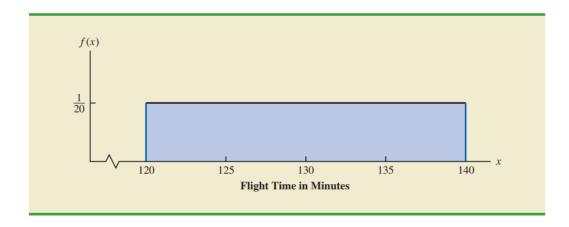




- Dengan setiap interval 1 menit memiliki kemungkinan yang sama, peubah acak X dikatakan memiliki **sebaran peluang seragam**.
- Fungsi kepekatan peluang, yang mendefinisikan sebaran seragam untuk peubah acak waktu terbang, adalah

$$f(x) = \begin{cases} 1/20 & \text{for } 120 \le x \le 140 \\ 0 & \text{elsewhere} \end{cases}$$

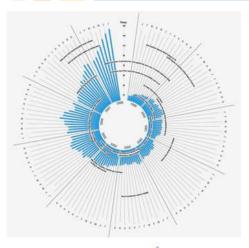






Sebaran Peluang Seragam (*Uniform*)







- Berapa peluang bahwa waktu penerbangan antara 120 dan 130 menit? Artinya, berapa P(120 < X < 130)?
- Karena waktu penerbangan harus antara 120 dan 140 menit dan karena peluang digambarkan seragam selama interval ini, kita bisa memperoleh P(120 < x < 130) = .50.

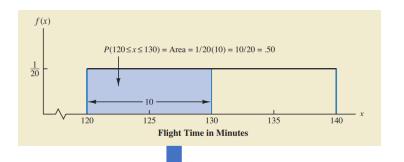


Misalkan $X \sim \operatorname{seragam}(a, b)$

$$E(X) = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$





Perhatikan gambar di atas: $X \sim \text{seragam}(120,140)$

$$E(X) = \frac{120 + 140}{2} = 130$$

$$Var(X) = \frac{(140 - 120)^2}{12} = 33.33$$

Sehingga $\sigma = \sqrt{Var(X)} = 5.77$

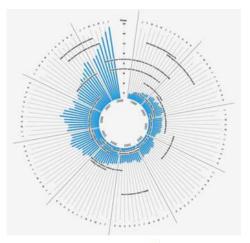


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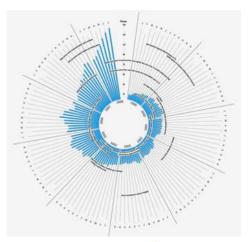




Sebaran Peluang Normal



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Sebaran Peluang Normal

- Sebaran peluang kontinu dapat bermacam-macam bentuknya.
- Salah satu yang paling banyak ditemui dalam proses alam adalah peubah acak dengan sebaran frekuensi yang berbentuk genta, yang dikenal dengan sebaran peluang normal.
- Rumus fungsi kepekatan peluangnya seperti di bawah ini.

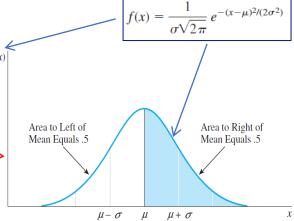
TI

NORMAL PROBABILITY DISTRIBUTION

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty$$

The symbols e and π are mathematical constants given approximately by 2.7183 and 3.1416, respectively; μ and σ (σ > 0) are parameters that represent the population mean and standard deviation, respectively.

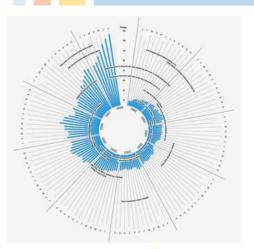
- Dalam sebaran peluang normal nilaitengah μ merupakan pusat dari sebaran, dan bentuknya simetrik terhadap μ .
- Karena simetrik maka luas areal di sebelah kiri μ sama dengan luas area di sebelah kanan μ , masing-masing sama dengan 0.5.



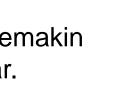
Sebaran Peluang Normal



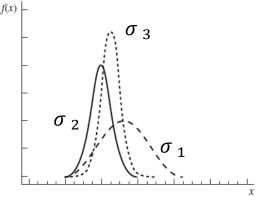




Bentuk dari sebaran normal ditentukan oleh parameter σ, yaitu simpangan baku populasi.

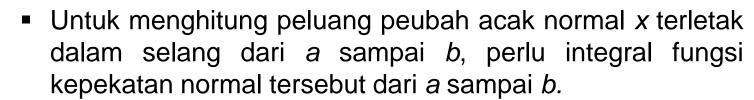


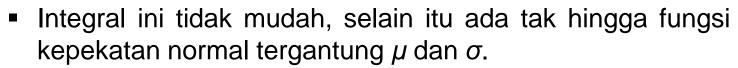
Lihat gambar di samping ini, semakin besar σ , semakin pendek kurva, dan keragamannya semakin besar.



 Sebaliknya, jika σ makin kecil maka semakin tinggi kurva normal, dan semakin kecil keragamannya.

 $\sigma_1 > \sigma_2 > \sigma_3$





• Untungnya, setiap peubah acak normal $x(\mu,\sigma^2)$ selalu dapat dibakukan menjadi peubah acak normal z(0,1).

Peubah acak z normal baku (standard normal)

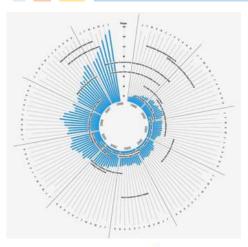




Sebaran Peluang Normal







Cara membakukan sebaran normal

■ Suatu peubah acak normal $x(\mu,\sigma^2)$ dapat dibakukan (**standardized**) menjadi peubah acak z(0,1) sbb:

$$z = \frac{x - \mu}{\sigma}$$

- Jika x lebih kecil dari μ, maka z bernilai negatif.
- Jika x lebih besar dari μ, maka z bernilai positif.
- Jika $x = \mu$, maka z = 0.

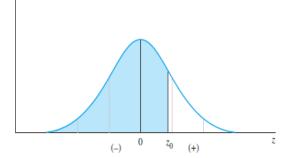
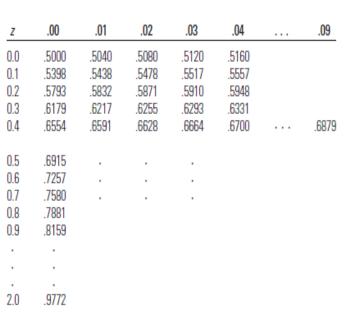


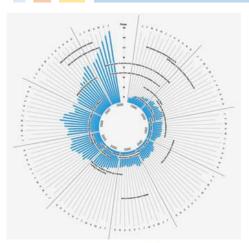
Table 3. Areas Under the Normal Curve						
Z	.00	.01	.02	.03		.09
-3.4	.0003	.0003	.0003	.0003		
-3.3	.0005	.0005	.0005	.0004		
-3.2	.0007	.0007	.0006	.0006		
-3.1	.0010	.0009	.0009	.0009		
-3.0	.0013	.0013	.0013	.0012		.0010
-2.9	.0019					
-2.8	.0026					
-2.7	.0035					
-2.6	.0047					
-2.5	.0062					
-2.0	.0228					





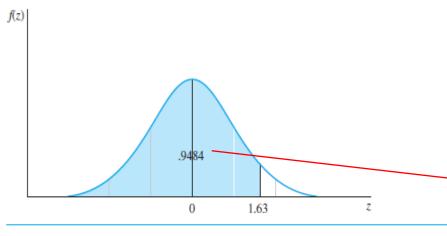
Menentukan peluang normal





Find $P(z \le 1.63)$. This probability corresponds to the area to the left of a point z = 1.63 standard deviations to the right of the mean (see Figure 6.8).

Solution The area is shaded in Figure 6.8. Since Table 3 in Appendix I gives areas under the normal curve to the left of a specified value of z, you simply need to find the tabled value for z = 1.63. Proceed down the left-hand column of the table to z = 1.6 and across the top of the table to the column marked .03. The intersection of this row and column combination gives the area .9484, which is $P(z \le 1.63)$.



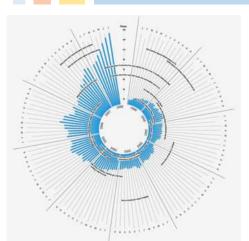
Areas to the left of z = 0 are found using negative values of z.

z	.00	.01	.02	.03	.04		
0.0	.5000	.5040	.5080	.5120	.5160		
0.1	.5398	.5438	.5478	.5517	.5557		
0.2	.5793	.5832	.5 871	.5910	.5948		
0.3	.6179	.6217	.6255	.6293	.6331		
0.4	.6554	.6591	.6628	.6664	.6700		
0.5	.6915	.6950	.6985	.7019	.7054		
0.6	.7257	.7291	.7324	.7357	.7389		
0.7	.7580	.7611	.7642	.7673	.7704		
8.0	.7881	.7910	.7939	.7967	.7995		
0.9	.8159	.8186	.8212	.8238	.8264		
1.0	.8413	.8438	.8461	.8485	.8508		
1.1	.8643	.8665	.8686	.8708	.8729		
1.2	.8849	.8869	.8888.	.8907	.8925		
1.3	.9032	.9049	.9066	.9082	.9099		
1.4	.9192	.9207	.9222	.9236	.9251		
1.5	.9332	.9345	.9357	.9370	.9382		
1.6	.9452	.9463	.9474	9484	.9495		
1.7	.9554	.9564	.9573	.9582	.9591		
1.8	.9641	.9649	.9656	.9664	.9671		
1.9	.9713	.9719	.9726	.9732	.9738		



Menentukan peluang normal



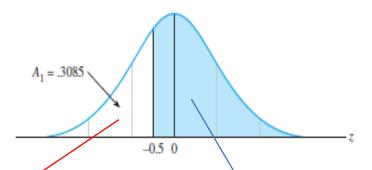




z	.00	.01	.02	.03
-3.4	.0003	.0003	.0003	.0003
-3.3	.0005	.0005	.0005	.0004
-3.2	.0007	.0007	.0006	.0006
-3.1	.0010	.0009	.0009	.0009
-3.0	.0013	.0013	.0013	.0012
-2.9	.0019	.0018	.0017	.0017
-2.8	.0026	.0025	.0024	.0023
-2.7	.0035	.0034	.0033	.0032
-2.6	.0047	.0045	.0044	.0043
-2.5	.0062	.0060	.0059	.0057
-2.4	.0082	.0080	.0078	.0075
-2.3	.0107	.0104	.0102	.0099
-2.2	.0139	.0136	.0132	.0129
-2.1	.0179	.0174	.0170	.0166
-2.0	.0228	.0222	.0217	.0212
-1.9	.0287	.0281	.0274	.0268
-1.8	.0359	.0351	.0344	.0336
-1.7	.0446	.0436	.0427	.0418
-1.6	.0548	.0537	.0526	.0516
-1.5	.0668	.0655	.0643	.0630
-1.4	.0808	.0793	.0778	.0764
-1.3	.0968	.0951	.0934	.0916
-1.2	.1151	.1131	.1112	.1093
-1.1	.1357	.1335	.1314	.1292
-1.0	.1587	.1562	.1529	.1515
-0.9	.1841	.1814	.1788	.1762
-0.8	.2119	.2020	.2061	.2033
-0.7	.2420	.2389	.2358	.2327
-0.6	2743	.2709	.2676	.2643
-0.5	.3085	.3050	.3015	.2981



Find $P(z \ge -.5)$. This probability corresponds to the area to the *right* of a point z = -.5 standard deviation to the left of the mean (see Figure 6.9).



Solution The area given in Table 3 in Appendix I is the area to the left of a specified value of z. Indexing z = -.5 in Table 3, we can find the area A_1 to the *left* of -.5 to be .3085.

Since the area under the curve is 1, we find

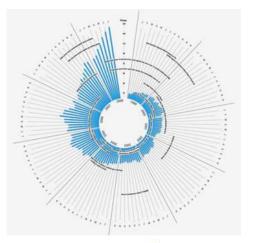
$$P(z \ge -.5) = 1 - A_1 = 1 -.3085 = (.6915)$$





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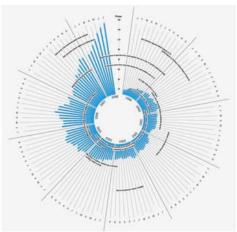


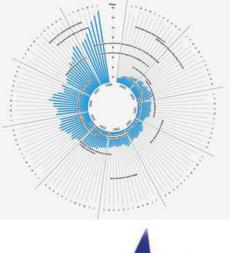




Menentukan peluang normal





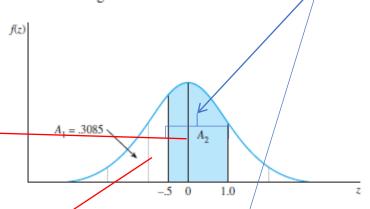




z	.00	.01	.02	.01	.02	.03
0.0	.5000	.5040	.5080	.0003	.0003	.0003
0.1	.5398	.5438	.5478	.0005	.0005	.0004
0.2	.5793	.5832	.5 871	.0007	.0006	.0006
0.3	.6179	.6217	.6255	.0009	.0009	.0009
0.4	.6554	.6591	.6628	.0013	.0013	.0012
0.5	.6915	.6950	.6985	.0018	.0017	.0017
0.6	.7257	.7291	.7324	.0025	.0024	.0023
0.7	.7580	.7611	.7642	.0034	.0033	.0032
8.0	.7881	.7910	.7939	.0045	.0044	.0043
0.9	.8159	.8186	.8212	.0060	.0059	.0057
1.0	.8413)	.8438	.8461	.0080	.0078	.0075
1.1	.8643	.8665	.8686	.0104	.0102	.0099
1.2	.8849	.8869	.8888	.0136	.0132	.0129
1.3	.9032	.9049	.9066	.0174	.0170	.0166
1.4	.9192	.9207	.9222	.0222	.0217	.0212
		-1.9	.0287	.0281	.0274	.0268
		-1.8	.0359	.0351	.0344	.0336
		-1.7	.0446	.0436	.0427	.0418
		-1.6	.0548	.0537	.0526	.0516
		-1.5	.0668	.0655	.0643	.0630
		-1.4	.0808	.0793	.0778	.0764
		-1.3	.0968	.0951	.0934	.0918
		-1.2	.1151	.1131	.1112	.1093
		-1.1	.1357	.1335	.1314	.1292
		-1.0	.1587	.1562	.1533	.1515
		-0.9	.1841	.1814	.1788	.1762
		-0.8	.2119	.2090	.2061	.2033
		-0.7	.2420	.2389	.2358	.2327
		-0.6	2743	.2709	.2676	.2643
		-0.5	.3085	.3050	.3015	.2981

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Find $P(-.5 \le z \le 1.0)$. This probability is the area between z = -.5 and z = 1.0, as shown in Figure 6.10.



Solution The area required is the shaded area A_2 in Figure 6.10. From Table 3 in Appendix I, you can find the area to the left of z = -.5 ($A_1 = .3085$) and the area to the left of z = 1.0 ($A_1 + A_2 = .8413$). To find the area marked A_2 , we subtract the two entries:

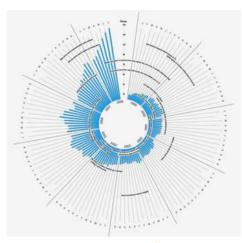
$$A_2 = (A_1 + A_2) - A_1 = .8413 - 3085 = .5328$$

That is, $P(-.5 \le z \le 1.0) = (5328.$



Menentukan peluang normal





Let x be a normally distributed random variable with a mean of 10 and a standard deviation of 2. Find the probability that x lies between 11 and 13.6.

Solution The interval from x = 11 to x = 13.6 must be standardized using the formula for z. When x = 11,

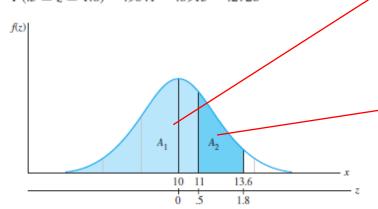
$$z = \frac{x - \mu}{\sigma} = \frac{11 - 10}{2} = .5$$

and when x = 13.6,

$$z = \frac{x - \mu}{\sigma} = \frac{13.6 - 10}{2} = 1.8$$

The desired probability is therefore $P(.5 \le z \le 1.8)$, the area lying between z = .5 and z = 1.8, as shown in Figure 6.12. From Table 3 in Appendix I, you find that the area to the left of z = .5 is .6915, and the area to the left of z = 1.8 is .9641. The desired probability is the difference between these two probabilities, or

$$P(.5 \le z \le 1.8) = .9641 - .6915 = .2726$$



Z	.00	.01	.02	.03	.04	
0.0	.5000	.5040	.5080	.5120	.5160	
0.1	.5398	.5438	.5478	.5517	.5557	
0.2	.5793	.5832	.5 871	.5910	.5948	
0.3	.6179	.6217	.6255	.6293	.6331	
0.4	.6554	.6591	.6628	.6664	.6700	
0.5	6915	.6950	.6985	.7019	.7054	
0.6	.7257	.7291	.7324	.7357	.7389	
0.7	.7580	.7611	.7642	.7673	.7704	
0.8	.7881	.7910	.7939	.7967	.7995	
0.9	.8159	.8186	.8212	.8238	.8264	
1.0	.8413	.8438	.8461	.8485	.8508	
1.1	.8643	.8665	.8686	.8708	.8729	
1.2	.8849	.8869	.8888	.8907	.8925	
1.3	.9032	.9049	.9066	.9082	.9099	
1.4	.9192	.9207	.9222	.9236	.9251	
1.5	.9332	.9345	.9357	.9370	.9382	
1.6	.9452	.9463	.9474	.9484	.9495	
1.7	.9554	.9564	.9573	.9582	.9591	
1.8	9641	.9649	.9656	.9664	.9671	
1.9	.9713	.9719	.9726	.9732	.9738	

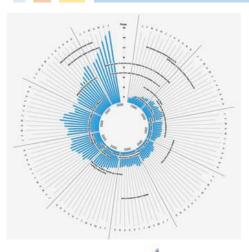




Ilustrasi

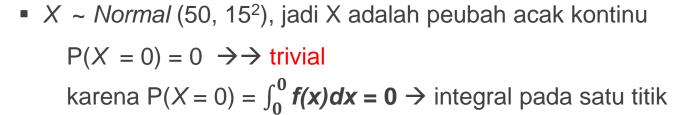


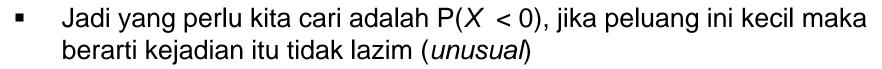




Dari Mendenhall:

6.16 A normal random variable x has mean 50 and standard deviation 15. Would it be unusual to observe the value x = 0? Explain your answer. Suatu peubah acak *X* menyebar normal dengan nilaitengah 50 dan simpangan baku 15. Apakah tidak lazim (unusual) jika kita memperoleh X = 0? Jelaskan..!





■
$$P(X < 0) = P(Z < (0-50)/15) = P(Z < -3.33) = 0.0004 \leftarrow lihat di tabel.$$

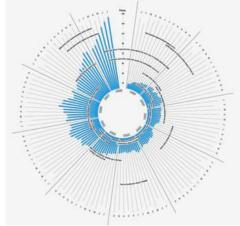
- Karena peluangnya sangat kecil, jauh lebih kecil dari 5%, maka kita simpulkan bahwa kejadian X = 0 adalah kejadian yang langka atau tidak lazim (unsual).
- Apakah juga tidak lazim untuk memperoleh X = 20?

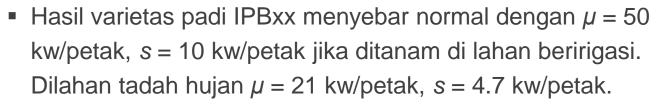


Ilustrasi









- Percobaan di lahan irigasi menghasilkan 65 kw/petak sedangkan di lahan tadah hujan hasilnya 30 kw/petak.
- Pertanyaan:
 - Brp peluang memperoleh hasil ≥ 65 kw di lahan irigasi?
 - Brp peluang memperoleh hasil ≥ 30 kw di tadah hujan?
- Misalkan X_{ir} = hasil varietas IPBxx di lahan irigasi, dan X_{th} = hasil varietas IPBxx di lahan tadah hujan.
- $X_{ir} \sim N(50, 10^2) \text{ dan } X_{th} \sim N(21, 4.7^2)$
 - $P[X_{ir} \ge 65] = P[(X_{ir} \mu)/s \ge (65-50)/10]$ = $P[Z \ge 1.50] = 0.0668 \leftarrow dari tabel^2$
 - $P[X_{th} \ge 30]$ = $P[(X_{th} \mu)/s \ge (30-21)/4.7]$

 $= P[Z \ge 1.91] = 0.0281 \leftarrow dari tabel \leftarrow$

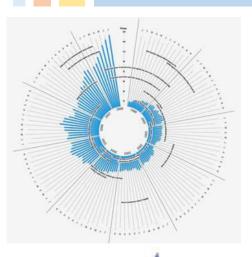
9	M M	
MÍ		

z	.00	.01	.02	.03
-3.4	.0003	.0003	.0003	.0003
-3.3	.0005	.0005	.0005	.0004
-3.2	.0007	.0007	.0006	.0006
-3.1	.0010	.0009	.0009	.0009
-3.0	.0013	.0013	.0013	.0012
-2.9	.0019	.0018	.0017	.0017
-2.8	.0026	.0025	.0024	.0023
-2.7	.0035	.0034	.0033	.0032
-2.6	.0047	.0045	.0044	.0043
-2.5	.0062	.0060	.0059	.0057
-2.4	.0082	.0080	.0078	.0075
-2.3	.0107	.0104	.0102	.0099
-2.2	.0139	.0136	.0132	.0129
-2.1	.0179	.0174	.0170	.0166
-2.0	.0228	.0222	.0217	.0212
-1.9	.0287	.0281	.0274	.0268
-1.8	.0359	0351	.0344	.0336
-1.7	.0446	.0436	.0427	.0418
-1.6	.0548	.0537	.0526	.0516
-1.5	.0668	.0655	.0643	.0630
1.4	.0808	.0793	.0778	.0764
-1.3	.0968	.0951	.0934	.0918
-1.2	.1151	.1131	.1112	.1093
-1.1	.1357	.1335	.1314	.1292
-1.0	.1587	.1562	.1539	.1515
-0.9	.1841	.1814	.1788	.1762
-0.8	.2/119	.2090	.2061	.2033
-0.7	.7420	.2389	.2358	.2327
-0.6	2743	.2709	.2676	.2643
-0.5	.3085	.3050	.3015	.2981

Pendekatan Normal thd Binom





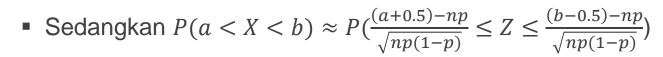


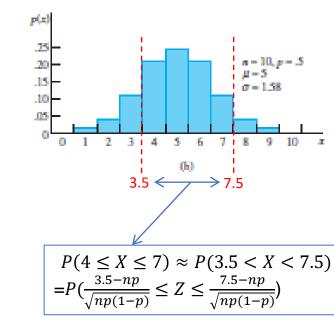
- Menghitung peluang binom (diskret) bisa lebih mudah jika didekati dengan sebaran normal (kontinu) asalkan n cukup besar ($np \ge 5$).
- Jika $X \sim Binom(n, p)$ maka p.a X diaproksimasi dengan sebaran normal, artinya $X \sim Normal(np, np(1-p))$.
- Dengan pendekatan normal

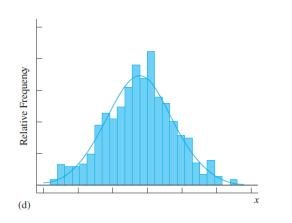
$$P(a \le X \le b) \approx P((a - 0.5) \le X \le (b + 0.5)).$$

- Sedangkan $P(a < X < b) \approx P((a + 0.5) \le X \le (b 0.5))$.
- Trasformasi ke normal baku (Z):

$$P(a \le X \le b) \approx P(\frac{(a-0.5)-np}{\sqrt{np(1-p)}} \le Z \le \frac{(b+0.5)-np}{\sqrt{np(1-p)}})$$





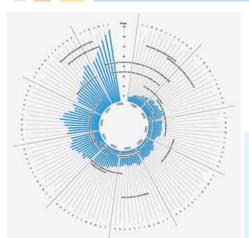




Pendekatan Normal thd Binom





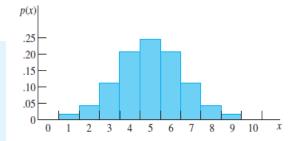


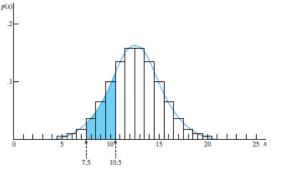
 Berikut ini ilustrasi sebaran normal untuk menghitung peluang sebaran binom.

For large n and π not too near 0 or 1, the distribution of a binomial random variable y may be approximated by a normal distribution with $\mu = n\pi$ and $\sigma = \sqrt{n\pi} (1 - \pi)$. This approximation should be used only if $n\pi \ge 5$ and $n(1 - \pi) \ge 5$. A continuity correction will improve the quality of the approximation in cases in which n is not overwhelmingly large.



A large drug company has 100 potential new prescription drugs under clinical test. About 20% of all drugs that reach this stage are eventually licensed for sale. What is the probability that at least 15 of the 100 drugs are eventually licensed? Assume that the binomial assumptions are satisfied, and use a normal approximation with continuity correction.





$$P(y \ge 14.5) = P\left(z \ge \frac{14.5 - 20}{4.0}\right) = P(z \ge -1.38) = 1 - P(z < -1.38)$$
$$= 1 - .0838 = .9162$$

Perhatikan bahwa angka \geq 15 (binom) didekati dengan angka \geq 14.5 (normal).

Mengapa? Karena angka 15 (diskret) tidak lain adalah dari 14.5 sampai 15.5.(kontinu).

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Study Programs in Statistics and Data Science