Repeated Systematic Sampling

- Digunakan bila tidak bisa mengestimasi $V(\overline{y}_{sy})$ karena sampel sistematik memberikan sedikit informasi untuk biaya perunit daripada sampel acak sederhana.
- Membutuhkan pemilihan lebih dari satu sampel sistematik.
- Contoh :

  Suatu populasi dengan N=960 diberi nomor secara berurutan. Untuk mendapatkan systematic sample sebesar n=60. Berapa nilai sistematik sistematik berulang $n_s = 10$?

  Penyelesaian:

  1. $k = \frac{N}{n} = \frac{960}{60} = 16$
  2. $k` = n_s . k = 10(16) = 160$
  3. Ambil 10 sampel acak antara 1-160 dari populasi N=960
     Misal didapat 73, 42, 81, 145, 6 ,21, 86, 17, 112, 102
  4. Angka tersebut diurutkan dan dijadikan titik awal (starting point), kemudian setiap titik awal ditambah 1 yang kemudian akan menjadi samoel unsur kedua dan selanjutnya sampai 6 kali.
  5. Jadi, untuk mendapatkan unsur ke-n, yaitu dari ukuran sampel bisa langsung dicari dengan cara: Starting point+(ukuran sistematik berulang-1)k`

  Misal:

Random Starting Point	Starting point + 5k`
6	6+5(160) = 806
17	17+5(160) =817
21	21+5(160) = 821
42	42+5(160) = 842
73	73+5(160) = 873
81	81+5(160) = 881
86	86+5(160) = 886
102	102+5(160) = 902
112	112+5(160) = 912
145	145+5(160) = 945

Selection of repeated systematic samples

Random starting point	Second element in sample	Third element in sample	...	Sixth element in sample
6	166	326	...	806
17	177	337	...	817
21	181	341	...	821
42	202	362	...	842
73	233	393	...	873
81	241	401	...	881
86	246	406	...	886
102	262	422	...	902
112	272	432	...	912
145	305	465	...	945

Estimator of the population mean μ, using n_s 1-in-k' systematic samples:

$$\hat{\mu} = \sum_{i=1}^{n_s} \frac{\overline{y}_i}{n_s} \qquad (7.12)$$

where $\overline{y}_i$ represents the mean of the ith systematic sample.

Estimated variance of $\hat{\mu}$:

$$\hat{V}(\hat{\mu}) = \left(1 - \frac{n}{N}\right)\frac{s_{\overline{y}}^2}{n_s}$$

where

$$s_{\overline{y}}^2 = \frac{\sum_{i=1}^{n_s} (\overline{y}_i - \hat{\mu})^2}{n_s - 1} \qquad (7.13)$$

Estimator of the population total τ using n_s 1-in-k' systematic samples:

$$\hat{\tau} = N\hat{\mu} = N\sum_{i=1}^{n_s} \frac{\overline{y}_i}{n_s} \qquad (7.14)$$

Estimated variance of $\hat{\tau}$:

$$\hat{V}(\hat{\tau}) = N^2\hat{V}(\hat{\mu}) = N^2\left(1 - \frac{n}{N}\right)\frac{s_{\overline{y}}^2}{n_s} \qquad (7.15)$$

Contoh :

Data on number of persons per car (the responses y_i are in parentheses)

Random starting point	Second element	Third element	Fourth element	Fifth element	Sixth element	Seventh element	Eighth element	$\overline{y}_i$
2(3)	52(4)	102(5)	152(3)	202(6)	252(1)	302(4)	352(4)	3.75
5(5)	55(3)	105(4)	155(2)	205(4)	255(2)	305(3)	355(4)	3.38
7(2)	57(4)	107(6)	157(2)	207(3)	257(2)	307(1)	357(3)	2.88
13(6)	63(4)	113(6)	163(7)	213(2)	263(3)	313(2)	363(7)	4.62
26(4)	76(5)	126(7)	176(4)	226(2)	276(6)	326(2)	376(6)	4.50
31(7)	81(6)	131(4)	181(4)	231(3)	281(6)	331(7)	381(5)	5.25
35(3)	85(3)	135(2)	185(3)	235(6)	285(5)	335(6)	385(8)	4.50
40(2)	90(6)	140(2)	190(5)	240(5)	290(4)	340(4)	390(5)	4.12
45(2)	95(6)	145(3)	195(6)	245(4)	295(4)	345(5)	395(4)	4.25
46(6)	96(5)	146(4)	196(6)	246(3)	296(3)	346(5)	396(3)	4.38

EXAMPLE 7.6 A state park charges admission by carload rather than by person, and a park official wants to estimate the average number of people per car for a particular summer holiday. She knows from past experience that there should be approximately 400 cars entering the park, and she wants to sample 80 cars. To obtain an estimate of the variance, she uses repeated systematic sampling with ten samples of eight cars each. Using the data given in Table 7.3, estimate the average number of people per car and place a bound on the error of estimation.

SOLUTION For one systematic sample,

$$k = \frac{N}{n} = \frac{400}{80} = 5$$

Hence, for $n_s = 10$ samples,

$$k' = 10k = 10(5) = 50$$

Cars with these numbers form the random starting points for the systematic samples. For Table 7.3, the quantity $\overline{y}_1$ is the average for the first row, $\overline{y}_2$ is the average for the second row, and so forth. The estimate of μ is

$$\hat{\mu} = \frac{1}{n_s}\sum_{i=1}^{n_s}\overline{y}_i = \frac{1}{10}(3.75 + 3.38 + \cdots + 4.38) = 4.16$$

with $s_{\overline{y}} = 0.675$. Thus, the estimated standard error of $\hat{\mu}$ is

$$\sqrt{\hat{V}(\hat{\mu})} = \sqrt{\left(1 - \frac{n}{N}\right)\frac{1}{n_s}s_{\overline{y}}} = \sqrt{\left(1 - \frac{80}{400}\right)\frac{1}{10}(0.675)} = 0.19$$

Therefore, our best estimate of the mean number of people per car is 4.16 plus or minus approximately 0.38. ■