Ilustrasi

Suatu perusahaan periklanan ingin menentukan berapa lama waktu yang akan dialokasikan untuk beriklan melalui televisi di suatu wilayah. Untuk tujuan tersebut perusahaan melakukan survei untuk menduga rata-rata lama waktu (jam) menonton televisi per rumahtangga per minggu di wilayah tersebut.

Informasi yang tersedia mengenai wilayah tersebut sbb: Wilayah terdiri dari dua kota, yaitu kota A dan kota B, serta daerah pedesaan yang mengitarinya. Kota A dibangun di sekitar pabrik, dan kebanyakan rumahtangga terdiri atas pekerja pabrik dengan anak-anak usia sekolah. Kota B adalah sub-urban eksklusif dari suatu kota di wilayah tetangga dan terdiri dari orang-orang yang lebih tua dengan sedikit anak yang tinggal di rumah.

Buatlah rancangan penarikan contoh untuk masalah di atas.

Rancangan Survei

Lapisan 1 (Kota A)	Lapisan 2 (Kota B)	Lapisan 3 (Pedesaan)
$N_1 = 155$	$N_2 = 62$	$N_3 = 93$

Data sampel: Waktu menonton TV (jam/minggu) di ketiga lapisan

Kota A

35	28	26	41
43	29	32	37
36	25	29	31
39	38	40	45
28	27	35	34

Kota B

27	4	49	10
15	41	25	30

Pedesaan

8	15	21	7
14	30	20	11
12	32	34	24

Ringkasan Perhitungan Setiap Lapisan

Lapisan	N_i	n_i	Rata-rata $(\bar{y})$	Ragam (s^2)
Kota A	155	20	33.900	35.358
Kota B	62	8	25.125	232.411
Pedesaan	93	12	19.000	87.636
Total	310	40		

Nilai dugaan rata-rata lama waktu menonton TV per keluarga (jam/minggu)

Di seluruh wilayah

$$
\begin{aligned}
\bar{y}_{st} &= \frac{1}{N}\left[N_1\bar{y}_1 + N_2\bar{y}_2 + N_3\bar{y}_3\right] \\
&= \frac{1}{310}\left[(155)(33.900) + (62)(25.125) + (93)(19.000)\right] \\
&= 27.675
\end{aligned}
$$

Nilai dugaan ragam rataan:

$$
\begin{aligned}
\hat{V}(\bar{y}_{st}) &= \frac{1}{N^2}\left[\sum_{i=1}^{3} N_i\left(N_i - n_i\right)\frac{s_i^2}{n_i}\right] \\
&= \frac{1}{310^2}\left[(155)(155-20)\left(\frac{35.358}{20}\right) + (62)(62-8)\left(\frac{232.411}{8}\right) + (93)(93-12)\left(\frac{87.636}{12}\right)\right] \\
&= 1.97
\end{aligned}
$$

Nilai dugaan rata-rata lama waktu menonton per keluarga TV (jam/minggu)

Selang kepercayaan kira-kira 95% bagi μ

$$\begin{aligned}\bar{\mathrm{y}}_{\mathrm{st}} \pm 2\sqrt{\hat{V}(\bar{y}_{st})} &\Leftrightarrow 27.675 \pm 2\sqrt{1.97} \\ &\Leftrightarrow 27.675 \pm 2.807 \\ &\Leftrightarrow 24.868\ s/d\ 30.482\ \ jam/mg\end{aligned}$$

- Selang kepercayaan kira-kira 95% bagi μ_2 (Rata-rata lama waktu menonton TV per keluarga di kota B:

$$\begin{aligned}\bar{\mathrm{y}}_2 \pm 2\sqrt{\hat{V}(\bar{y}_2)} &\Leftrightarrow \bar{\mathrm{y}}_2 \pm 2\sqrt{\left(\frac{N_2 - n_2}{N_2}\right)\frac{s_2^2}{n_2}} \\ &\Leftrightarrow 25.125 \pm 2\sqrt{\left(\frac{62-8}{62}\right)\frac{232.411}{8}} \\ &\Leftrightarrow 25.125 \pm 10.060 \Leftrightarrow 15.065\ s/d\ 35.185\ \ jam/mg\end{aligned}$$

Nilai dugaan total lama waktu menonton TV seluruh keluarga (jam)

Penduga titik bagi τ

$$\hat{\tau} = N\bar{y}_{st} = 310(27.675) = 8579.250\ jam$$

- Nilai dugaam ragam total

$$\hat{V}(\hat{\tau}) = \hat{V}(N\bar{y}_{st}) = N^2\hat{V}(\bar{y}_{st}) = 310^2(1.97) = 189278.560\ jam$$

- Selang kepercayaan kira-kira 95% bagi τ (Total lama waktu menonton TV semua keluarga di wilayah tersebut):

$$\begin{aligned}\hat{\tau} \pm 2\sqrt{\hat{V}(\hat{\tau})} &\Leftrightarrow 8579.25 \pm 2\sqrt{189278.560} \\ &\Leftrightarrow 8579.25 \pm 870.12 \\ &\Leftrightarrow 7709.13\ s/d\ 9449.37\ \ jam/mg\end{aligned}$$

Contoh kasus

EXAMPLE 5.5 A prior survey suggests that the stratum variances for Example 5.1 are approximately $\sigma_1^2 \approx 25$, $\sigma_2^2 \approx 225$, and $\sigma_3^2 \approx 100$. We wish to estimate the population mean by using $\bar{y}_{st}$. Choose the sample size to obtain a bound on the error of estimation equal to 2 hours if the allocation fractions are given by $a_1 = 1/3$, $a_2 = 1/3$, and $a_3 = 1/3$. In other words, you are to take an equal number of observations from each stratum.

EXAMPLE 5.6 As in Example 5.5, suppose the variances of Example 5.1 are approximated by $\sigma_1^2 \approx 25$, $\sigma_2^2 \approx 225$, and $\sigma_3^2 \approx 100$. We wish to estimate the population total τ with a bound of 400 hours on the error of estimation. Choose the appropriate sample size if an equal number of observations is to be taken from each stratum.

Penyelesaian Example 5.5

$$n = \frac{\sum_{i=1}^{L} N_i^2 \sigma_i^2 / w_i}{N^2 D + \sum_{i=1}^{L} N_i \sigma_i^2}$$

$$D = \frac{B^2}{4} = \frac{2^2}{4} = 1$$

$$\sum_{i=1}^{L} N_i^2 \sigma_i^2 / w_i = \frac{(155)^2 25}{1/3} + \frac{(62)^2 225}{1/3} + \frac{(93)^2 100}{1/3} = 6{,}991{,}275$$

$$\sum_{i=1}^{L} N_i \sigma_i^2 = (155)25 + (62)225 + (93)100 = 27{,}125$$

$$N^2 D = (310)^2 (1) = 96{,}100$$

$$n = \frac{6{,}991{,}275}{96{,}100 + 27{,}125} = 56.7 \approx 57$$

$$n_1 = (1/3)57 = 19$$

$$n_2 = (1/3)57 = 19$$

$$n_3 = (1/3)57 = 19$$

Penyelesaian Example 5.6

$$n = \frac{\sum_{i=1}^{L} N_i^2 \sigma_i^2 / w_i}{N^2 D + \sum_{i=1}^{L} N_i \sigma_i^2}$$

$$D = \frac{B^2}{4N^2} = \frac{400^2}{4N^2} = \frac{40{,}000}{4N^2}$$

$$\sum_{i=1}^{L} N_i^2 \sigma_i^2 / w_i = \frac{(155)^2 25}{1/3} + \frac{(62)^2 225}{1/3} + \frac{(93)^2 100}{1/3} = 6{,}991{,}275$$

$$\sum_{i=1}^{L} N_i \sigma_i^2 = (155)25 + (62)225 + (93)100 = 27{,}125$$

$$N^2 D = N^2 \frac{40{,}000}{N^2} = 40{,}000$$

$$n = \frac{6{,}991{,}275}{40{,}000 + 27{,}125} = 104.2 \approx 105$$

$$n_1 = (1/3)105 = 35$$
$$n_2 = (1/3)105 = 35$$
$$n_3 = (1/3)105 = 35$$

Contoh kasus

EXAMPLE 5.7 The advertising firm in Example 5.1 finds that obtaining an observation from a rural household costs more than obtaining a response in town A or B. The increase is due to the costs of traveling from one rural household to another. The cost per observation in each town is estimated to be \$9 (that is, $c_1 = c_2 = 9$), and the costs per observation in the rural area to be \$16 (that is, $c_3 = 16$). The stratum standard deviations (approximated by the strata sample variances from a prior survey) are $\sigma_1 \approx 5$, $\sigma_2 \approx 15$, and $\sigma_3 \approx 10$. Find the overall sample size n and the stratum sample sizes, n_1, n_2, and n_3, that allow the firm to estimate, at minimum cost, the average television-viewing time with a bound on the error of estimation equal to 2 hours.

Penyelesaian Example 5.7

Penentuan ukuran sampel keseluruhan (n)

$$n = \frac{\sum_{k=1}^{L} N_i \sigma_i / \sqrt{c_i} \sum_{i=1}^{L} N_i \sigma_i \sqrt{c_i}}{N^2 D + \sum_{i=1}^{L} N_i \sigma_i^2}$$

$$\sum_{k=1}^{L} N_i \sigma_i / \sqrt{c_i} = \frac{(155)(5)}{\sqrt{9}} + \frac{(62)(15)}{\sqrt{9}} + \frac{(93)(10)}{\sqrt{16}} = 800{,}83$$

$$\sum_{i=1}^{L} N_i \sigma_i \sqrt{c_i} = (155)(5)\sqrt{9} + (62)(15)\sqrt{9} + (93)(10)\sqrt{16} = 8835$$

$$N^2 D = (310)^2 (1) = 96{,}100$$

$$\sum_{i=1}^{L} N_i \sigma_i^2 = (155)25 + (62)225 + (93)100 = 27{,}125$$

$$n = \frac{(800{,}83)(8835)}{96{,}100 + 27{,}125} = 57.42 \approx 58$$

Contoh kasus – Neyman allocation

EXAMPLE 5.8 The advertising firm in Example 5.1 decides to use telephone interviews rather than personal interviews because all households in the county have telephones, and this method reduces costs. The cost of obtaining an observation is then the same in all three strata. The stratum standard deviations are again approximated by $\sigma_1 \approx 5$, $\sigma_2 \approx 15$, and $\sigma_3 \approx 10$. The firm desires to estimate the population mean μ with a bound on the error of estimation equal to 2 hours. Find the appropriate sample size n and stratum sample sizes, n_1, n_2, and n_3.

Penyelesaian Example 5.8

Penentuan ukuran sampel keseluruhan (n)

$$n = \frac{\sum_{k=1}^{L} N_i \sigma_i \sum_{i=1}^{L} N_i \sigma_i}{N^2 D + \sum_{i=1}^{L} N_i \sigma_i^2} = \frac{(\sum_{i=1}^{L} N_i \sigma_i)^2}{N^2 D + \sum_{i=1}^{L} N_i \sigma_i^2}$$

$$\sum_{k=1}^{L} N_i \sigma_i = (155)(5) + (62)(15) + (93)(10) = 2635$$

$$N^2 D = (310)^2 (1) = 96{,}100$$

$$\sum_{i=1}^{L} N_i \sigma_i^2 = (155)25 + (62)225 + (93)100 = 27{,}125$$

$$n = \frac{(2635)^2}{96{,}100 + 27{,}125}$$

$$= 56.34 \approx 57$$

Alokasi sampel per strata

$$n_i = n\left[\frac{N_i\sigma_i}{\sum_{k=1}^{L} N_k\sigma_k}\right]$$

$$n_1 = n\left[\frac{(155)(5)}{2635}\right] = 0.30n = 0.3(57) = 17$$

$$n_2 = n\left[\frac{(62)(15)}{2635}\right] = 0.35n = 0.35(57) = 20$$

$$n_3 = n\left[\frac{(93)(10)}{2635}\right] = 0.35n = 0.35(57) = 20$$

Contoh kasus

EXAMPLE 5.10 The advertising firm in Example 5.1 thinks that the approximate variances used in Examples 5.7 and 5.8 are in error and that the stratum variances are approximately equal. The common value of σ_i was approximated by 10 in a preliminary study. Telephone interviews are to be used, and hence costs will be equal in all strata. The firm desires to estimate the average number of hours per week that households in the county watch television, with a bound on the error of estimation equal to 2 hours. Find the sample size and stratum sample sizes necessary to achieve this accuracy.

Penyelesaian Example 5.10

Penentuan ukuran sampel keseluruhan (n)

$$n = \frac{\sum_{i=1}^{L} N_i \sigma_i^2}{ND + \frac{1}{N} \sum_{i=1}^{L} N_i \sigma_i^2}$$

$$\sum_{k=1}^{L} N_i \sigma_i^2 = (155)(10)^2 + (62)(10)^2 + (93)(10)^2 = 31{,}000$$

$$ND = (310)\ (1) = 310$$

$$\sum_{i=1}^{L} N_i \sigma_i^2 = (155)25 + (62)225 + (93)100 = 27{,}125$$

$$n = \frac{(31{,}000)}{310 + \frac{1}{310}(31{,}000)}$$
$$= 75.6 \approx 76$$

Alokasi sampel per strata

$$n_i = n\left(\frac{N_i}{\sum_{i=1}^{L} N_i}\right) = n\left(\frac{N_i}{N}\right)$$

$$n_1 = n\left[\frac{(155)}{310}\right] = 0.50n = 0.3(76) = 38$$

$$n_2 = n\left[\frac{(62)}{310}\right] = 0.20n = 0.20(76) = 15$$

$$n_3 = n\left[\frac{(93)}{310}\right] = 0.30n = 0.30(76) = 23$$

Contoh kasus

EXAMPLE 5.12 The advertising firm wants to estimate the proportion of households in the county of Example 5.1 that view show X. The county is divided into three strata, town A, town B, and the rural area. The strata contain $N_1 = 155$, $N_2 = 62$, and $N_3 = 93$ households, respectively. A stratified random sample of $n = 40$ households is chosen with proportional allocation. In other words, a simple random sample is taken from each stratum; the sizes of the samples are $n_1 = 20$, $n_2 = 8$, and $n_3 = 12$. Interviews are conducted in the 40 sampled households; results are shown in Table 5.3. Estimate the proportion of households viewing show X, and place a bound on the error of estimation.

TABLE **5.3**
Data for Example 5.12

Stratum	Sample size	Number of households viewing show X	$\hat{p}_i$
1	$n_1 = 20$	16	0.80
2	$n_2 = 8$	2	0.25
3	$n_3 = 12$	6	0.50

Penyelesaian

$$\hat{p}_{st} = \frac{1}{N}\left[N_1\hat{p}_1 + N_2\hat{p}_2 + \cdots + N_L\hat{p}_L\right] = \frac{1}{N}\sum_{i=1}^{L} N_i\hat{p}_i$$

Strata	Ni	ni	pi hat	Nipi
1	155	20	0.80	124
2	62	8	0.25	15.5
3	93	12	0.50	46.5
Total	**310**	**40**		**186**

$$\widehat{p_{st}} = \frac{186}{310} = 0.6$$

$$\hat{V}(\hat{p}_{st}) = \frac{1}{N^2}\sum_{i=1}^{L}\underbrace{\left[N_i^2 \frac{N_i - n_i}{N_i}\frac{\hat{p}_i(1-\hat{p}_i)}{n_i - 1}\right]}_{\text{Ai}}$$

Strata	Ni	ni	pihat	1-pihat	Ai
1	155	20	0,80	0,20	176,21
2	62	8	0,25	0,75	89,68
3	93	12	0,50	0,50	171,20
Total	**310**	**40**			**437,09**

$$\hat{V}(\hat{p}_{st}) = \frac{1}{310^2}(437.09) = 0.0045$$

$$B = 2\sqrt{0.0045} = 0.1349$$

Penentuan Ukuran Contoh untuk Menduga Proporsi

$$n = \frac{\sum_{i=1}^{L} N_i^2 p_i(1-p_i)/w_i}{N^2 \frac{B^2}{4} + \sum_{i=1}^{L} N_i p_i (1-p_i)}$$

$$w_i = \frac{N_i \sqrt{\frac{p_i(1-p_i)}{c_i}}}{\sum_{k=1}^{L} N_k \sqrt{\frac{p_k(1-p_k)}{c_k}}}$$