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Contoh 1 **PERTEMUAN 1**

Ada bola di dalam tas. Diketahui bahwa setidaknya salah satu dr bola berwarna hitam, tetapi ragu apakah keduanya hitam atau yang satu hitam dan yang satu putih. Ada 2 kemungkinan:

BB: Kedua bola berwarna hitam

BW: satu hitam dan satu putih

Kita tahu bahwa hanya satu dari pernyataan/hipotesis berikut ini yg benar. Dilakukan mengeluarkan salah satu bola, & mengamati warna-nya. (D: bola yang dikeluarkan dari tas berwarna hitam).

Jawab:

Asumsikan P(BB) = 0.5 dan P(BW) = 0.5 → nilai prior

Dari kejadian D, kita hitung kemungkinan nilai likelihood

Hypotheses	Possible Data	Probability
BB	Black Ball	1
	White Ball	0
BW	Black Ball	0.5
	White Ball	0.5

Lalu diperoleh Bayes's Box berikut secara lengkap

Hypotheses	prior	likelihood	h = prior × likelihood	posterior
BB	0.5	1	0.5	0.667
BW	0.5	0.5	0.25	0.333
Totals:	1		0.75	1

Latihan 1

1. Jika dilihat dari sudut pandang Bayesian, data diasumsikan bersifat acak yang memiliki sebaran. Salah
2. Keunggulan dari metode Bayesian adalah dapat memperbaharui informasi yang diperoleh. Benar
3. Posterior dalam konsep Bayesian merupakan sebaran awal sebelum data diperoleh. Salah
4. Parameter memiliki sebaran adalah konsep dari sudut pandang Frequentist. Salah

Latihan 2

Dari 200 orang pelamar terdapat 150 berasal dari SMA dan sisanya berasal dari SMK. Dari 150 pelamar SMA tersebut hanya 80 yg mempunyai sertifikat bahasa Inggris. Pelamar yg berasal dari SMK, hanya 20% yang mempunyai sertifikat bahasa Inggris. Jika seorang pelamar ditarik secara acak, berapa peluang pelamar tsb ber-pendidikan SMK jika diketahui ybs mempunyai sertifikat b.Inggris?

Jawab:

K1 = sekolah	K2 = sertifikat
SMA (150) -> P(A) = ¾	S (80) -> P(S|A) = 80/150 = 8/15
	T (70) -> P(T|A) = 70/150 = 7/15
SMK (50) -> P(K) = ¼	S (50*20%= 10) -> P(S|K) = 1/5
	T (40) -> P(T|K) = 4/5

$$P(K|S) = \frac{P(K\cap S)}{P(S)} = \frac{P(K)P(S|K)}{P(S|A)P(A) + P(S|K)P(K)}$$

$$P(K|S) = \frac{\frac{1}{4}\left(\frac{1}{5}\right)}{\frac{8}{15}\left(\frac{3}{4}\right) + \frac{1}{5}\left(\frac{1}{4}\right)} = \frac{\frac{1}{20}}{\frac{9}{20}} = \frac{1}{9}$$

Latihan 3

You move into a new house which has a phone installed. You can't remember the phone number, but you suspect it might be 555-3226 (some of you may recognise this as being the phone number for Homer Simpson's "Mr Plow" business). To test this hypothesis, you carry out an experiment by picking up the phone and dialing 555-3226.

If you are correct about the phone number, you will definitely hear a busy signal because you are calling yourself. If you are incorrect, the probability of hearing a busy signal is 1/100. However, all of that is only true if you assume the phone is working, and it might be broken! If the phone is broken, it will always give a busy signal.

When you do the experiment, the outcome (the data) is that you do actually get the busy signal. The question asked us to consider the following four hypotheses, and to calculate their posterior probabilities:

Hypothesis	Description	Prior Probability
H_1	Phone is working and 555-3226 is correct	0.4
H_2	Phone is working and 555-3226 is incorrect	0.4
H_3	Phone is broken and 555-3226 is correct	0.1
H_4	Phone is broken and 555-3226 is incorrect	0.1

Jawab:

^Ada 3 kejadiann

K1 = Telepon	K2 = nomor	K3 = sinyal
Berfungsi (F)	Benar (B)	Sibuk (K) = 1
		Tidak (T) = 0
	Salah (S)	Sibuk (K) = 1/100 = 0.01
		Tidak (T) = 99/100 = 0.99
Rusak ®	Benar (B)	Sibuk (K) = 1
		Tidak (T) = 0
	Salah (S)	Sibuk (K) = 1
		Tidak (T) = 0

Hipt.	Desk.	Prior	Likelihood	h	Posterior (h/T)
H1	FB	0.4	1	0.4	0.662
H2	FS	0.4	0.01	0.004	0.006
H3	RB	0.1	1	0.1	0.116
H4	RS	0.1	1	0.1	0.116
Total				0.604	

PERTEMUAAN 2

Studi Kasus 1

kemungkinan tes medis tertentu menjadi positif adalah 90%, apakah seorang pasien memiliki ini adalah D. Diket:i 1% populasi menderita penyakit tersebut, dan teks mencatat positif palsu 5% dari waktu. jika Anda menerima tes positif, berapa peluang Anda terkena D.

Jawab:

$P(+|D) = 0.90$; $P(D) = 0.01$, $P(+|\widetilde{D}) = 0.05$

Bayes' Theorem: $P(D|+) = \frac{P(+|D)\,P(D)}{P(+)} = \frac{P(+|D)\,P(D)}{[P(+|D)\,P(D)]+[P(+|\widetilde{D})\,P(\widetilde{D})]}$

Substituting in the data: $P(D|+) = \frac{0.9\times0.01}{(0.9\times0.01)+(0.05\times0.99)} = 0.15$

Studi kasus 2 (**Teorema Bayes u/ Binomial dg Prior Diskret**)

Let $Y|\pi$ be $binomial(n=4,\pi)$. Suppose we consider that there are only three possible values for π, .4,.5, and .6. We will assume they are equally likely. The joint probability distribution $f(\pi_i, y_j)$ is found by multiplying the conditional observation distribution $f(y_j|\pi_i)$ times the prior distribution $g(\pi_i)$. In this case, the conditional observation probabilities come from the $binomial(n=4,\pi)$ distribution.

Suppose $Y = 3$ was observed.

Bayes' Box

π	prior	likelihood	prior × likelihood	posterior
.4	$\frac{1}{3}$	.1536	.0512	$\frac{.0512}{.2497}$ = .205
.5	$\frac{1}{3}$	.2500	.0833	$\frac{.0833}{.2497}$ = .334
.6	$\frac{1}{3}$	.3456	.1152	$\frac{.1152}{.2497}$ = .461
marginal $P(Y=3)$			.2497	1.000

Studi kasus 3 (**Teorema Bayes u/ Poisson dg Prior Diskret**)

Let $Y|\mu$ be Poisson(μ). Suppose that we believe there are only four possible values for μ, 1,1.5,2, and 2.5. Suppose we consider that the two middle values, 1.5 and 2, are twice as likely as the two end values 1 and 2.5. Suppose $y = 2$ was observed. Plug the value $y = 2$ into formula

$$f(y|\mu) = \frac{\mu^y e^{-\mu}}{y!}$$

To give the likelihood

Bayes' Box

μ	prior	likelihood	prior × likelihood	posterior
1.0	$\frac{1}{6}$	$\frac{1.0^2e^{-1.0}}{2!}$ = .1839	.0307	$\frac{.0307}{.2473}$ = .124
1.5	$\frac{1}{3}$	$\frac{1.5^2e^{-1.5}}{2!}$ = .2510	.0837	$\frac{.0837}{.2473}$ = .338
2.0	$\frac{1}{3}$	$\frac{2.0^2e^{-2.0}}{2!}$ = .2707	.0902	$\frac{.0902}{.2473}$ = .365
2.5	$\frac{1}{6}$	$\frac{2.5^2e^{-2.5}}{2!}$ = .2565	.0428	$\frac{.0428}{.2473}$ = .173
marginal $P(Y=2)$			.2473	1.000

Latihan 1

Suppose there is a medical diagnostic test for a disease. The *sensitivity* of the test is .95. This means that if a person has the disease, the probability that the test gives a positive response is .95. The *specificity* of the test is .90. This means that if a person does not have the disease, the probability that the test gives a negative response is .90, or that the *false positive* rate of the test is .10. In the population, 1% of the people have the disease. What is the probability that a person tested has the disease, given the results of the test is positive? Let D be the event "the person has the disease" and let T be the event "the test gives a positive result."

Jawab:

Diket: D = mengidap penyakit; T = hasil positif; $P(T|\widetilde{D})$=0.1

Sensitivitas = P(T|D) = 0.95; Spesivisitas = $P(\widetilde{T}|\widetilde{D})$ = 0.90;

KI KII

0,01 D → T 0,95 ; $\widetilde{T}$ 0,05

0,99 $\widetilde{D}$ → T 0,1 ; $\widetilde{T}$ 0,9

$\rightarrow P(D|T) = \frac{P(D).P(T|D)}{P(D).P(T|D) + P(\widetilde{D}).P(T|\widetilde{D})}$

$= 8,75\%$

Jadi, peluang seseorang yg dites mengidap penyakit tsb jika hasil tesnya positif a/ 0.0875 atau 8.75%.

Latihan 2

Suppose there is a medical screening procedure for a specific cancer that has *sensitivity* = .90, and *specificity* = .95. Suppose the underlying rate of the cancer in the population is .001. Let B be the event "the person has that specific cancer," and let A be the event "the screening procedure gives a positive result."

a) What is the probability that a person has the disease given the results of the screening is positive?

b) Does this show that screening is effective in detecting this cancer?

Jawab:

KI KII

0,001 B → A 0,9 ; $\bar{A}$ 0,1

0,999 $\widetilde{B}$ → A 0,05 ; $\bar{A}$ 0,95

$P(B|A) = \frac{P(B).P(A|B)}{P(B).P(A|B) + P(\bar{B}).P(A|\bar{B})}$

$= \frac{0,001 \cdot 0,9}{0,001 \cdot 0,9 + 0,999 \cdot 0,05} = 0,01769$

Latihan 3

There is an urn containing 9 balls, which can be either green or red. The number of red balls in the urn is not known.

Let X be the number of red balls in the urn.

X = bnyk bola merah dlm guci

X = {0,1,2,...9}

Suppose we look at the two draws from the urn (without replacement) as a single experiment. The results were first draw red, second draw green. Find the posterior distribution of X by filling in the simplified table.

1M 8H

X	prior	likelihood (MH)	prior × likelihood	posterior
0	P(X=0) = 1/10	0		
1	P(X=1) = 1/10	$\frac{\binom{1}{1}\binom{8}{1}}{\binom{9}{2}} = \frac{8}{36}$		
⋮				
9	P(X=9) = 1/10			

X	Prior	Likelihood → MH	h = P.L	Posterior
0	1/10	0 ?	0	0
1	1/10	$\binom{1}{1}\binom{8}{1}/\binom{9}{2} = 8/36$	8/360	8/120
2	1/10	$\binom{2}{1}\binom{7}{1}/\binom{9}{2} = 14/36$	14/360	14/120
3	1/10	$\binom{3}{1}\binom{6}{1}/\binom{9}{2} = 18/36$	18/360	18/120
4	1/10	$\binom{4}{1}\binom{5}{1}/\binom{9}{2} = 20/36$	20/360	20/120
5	1/10	$\binom{5}{1}\binom{4}{1}/\binom{9}{2} = 20/36$	20/360	20/120
6	1/10	$\binom{6}{1}\binom{3}{1}/\binom{9}{2} = 18/36$	18/360	18/120
7	1/10	$\binom{7}{1}\binom{2}{1}/\binom{9}{2} = 14/36$	14/360	14/120
8	1/10	$\binom{8}{1}\binom{1}{1}/\binom{9}{2} = 8/36$	8/360	8/120
9	1/10	0 ?	0	0

Total = $\frac{120}{360}$

Latihan 4

$Y_1 \sim binomial(n=10, \pi)$

Let Y_1 be the number of successes in $n = 10$ independent trials where each trial results in a success or failure, and π, the probability of success, remains constant over all trials. Suppose the 4 possible values of π are .20, .40, .60, and .80. We do not wish to favor any value over the others so we make them equally likely. We observe $Y_1 = 7$. Find the posterior distribution by filling in the simplified table.

$Y_1 = 7 \rightarrow P(Y_1=7) = \binom{10}{7}\pi^7(1-\pi)^3$

π	prior	likelihood	prior × likelihood	posterior
0,2	1/4			
0,4	1/4			
0,6	1/4			
0,8	1/4			

π	Prior	Likelihood → $P(Y_1=7)=\binom{10}{7}\pi^7(1-\pi)^3$	h = P.L	Posterior
0,2	1/4	$\binom{10}{7}0,2^7(0,8)^3$ = 0,000786	0,0001965	0,00171
0,4	1/4	$\binom{10}{7}0,4^7(0,6)^3$ = 0,042467	0,010617	0,09241
$\boxed{0,6}$	1/4	$\binom{10}{7}0,6^7(0,4)^3$ = 0,214991	0,053748	$\boxed{0,46781}$
0,8	1/4	$\binom{10}{7}0,8^7(0,2)^3$ = 0,201326	0,050332	0,438075

Total 0,1148935

$Y_1 \rightarrow$ Sukses $\rightarrow$ pendapat $Y_1 = 7$

$\pi = 0,6$

Latihan 5

Let n be the unknown number of customers that visit a store on the day of a sale. The number of customers that make a purchase is $Y|n \sim \text{Binomial}(n, \theta)$ where θ is the known probability of making a purchase given the customer visited the store. The prior is $n \sim \text{Poisson}(5)$. Assuming θ is known and n is the unknown parameter, plot the posterior distribution of n for all combinations of $Y \in \{0, 5, 10\}$ and $\theta \in \{0.2, 0.5\}$ and comment on the effect of Y and θ on the posterior.

Latihan 1 **PERTEMUAN 3**

Sophie, the editor of the student newspaper, is going to conduct a survey of students to determine the level of support for the current president of the students' association. She needs to determine her prior distribution for π, the proportion of students who support the president. She decides her prior mean is .5, and her prior standard deviation is .15.

(a) Determine the $beta(a, b)$ prior that matches her prior belief.

(b) What is the equivalent sample size of her prior?

(c) Out of the 68 students that she polls, $y = 21$ support the current president. Determine her posterior distribution.

Jawab:

(a) $\pi \sim beta(a,b)$; $E(\pi) = 0.5$; $\sqrt{Var(\pi)} = 0.15$

→ $X \sim beta(\alpha, \beta)$

$$f_x(x) \begin{cases} \frac{\Gamma(\alpha)+\Gamma(\beta)}{\Gamma(\alpha).\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}; & 0 < x < 1 \\ 0; & x \text{ lainnya} \end{cases}$$

$$E(X) = \frac{\alpha}{\alpha+\beta} \qquad Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$E(\pi) = 0.5$ → $\frac{a}{a+b} = 0.5$ → $a = 0.5a + 0.5b$ → $0.5a = 0.5b$ → a=b

$Var(\pi) = (0.15)^2 = 0.0225 = \frac{9}{400}$

$\frac{ab}{(a+b)^2(a+b+1)} = \frac{9}{400}$ → $\frac{a^2}{4a^2(2a+1)} = \frac{9}{100}$ → $100 = 18a + 9$

$a = \frac{91}{18}$ → maka $b = \frac{91}{18}$ $\quad \therefore \pi \sim beta(\frac{91}{18}, \frac{91}{18})$

(b) $n = a + b = \frac{91}{18} + \frac{91}{18} = \frac{182}{18} = 10.111 \approx 11$

(c) $n = 68$ $\quad y \sim$ Binomial$(68, \pi)$

$y = 21$ $\quad \pi \sim$ Beta$(\frac{91}{18}, \frac{91}{18})$

→ $\pi|y \sim Beta(\alpha + y, n - y + \beta)$

→ $\pi|y \sim Beta(\frac{91}{18} + 21, 68 - 21 + \frac{91}{18})$

→ $\pi|y \sim Beta(26.05, 52.05)$

Latihan 2

Say $Y|\lambda \sim$ Poisson(λ).

(a) Derive and plot the Jeffreys' prior for λ.

(b) Is this prior proper?

(c) Derive the posterior and give conditions on Y to ensure it is proper.

Jawab:

(a) $I(\lambda) = -E\left(\frac{d^2 log f(Y|\lambda)}{d\lambda^2}\right)$

$f(Y|\lambda) = \frac{e^{-\lambda}\lambda^Y}{Y!}$

$\log(f(Y|\lambda)) = -\lambda + Y log\lambda - \log(Y!)$

$\frac{dlogf(...)}{d\lambda} = -1 + \frac{Y}{\lambda}$

$\frac{d^2logf(...)}{d\lambda^2} = -\frac{Y}{\lambda^2}$

$I(\lambda) = -E\left(-\frac{Y}{\lambda^2}\right) = \frac{E(Y)}{\lambda^2} = \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}$

Catatan:
$Y \sim Poisson(\lambda)$
$E(Y) = \lambda$

Maka, nilai JP
$\pi(\lambda) \propto \sqrt{I(\lambda)}$
$\pi(\lambda) \propto \sqrt{1/\lambda}$
$\pi(\lambda) \propto \lambda^{-\frac{1}{2}}$

(b) Karena syarat dari λ > 0, maka nilai prior π(λ) = 1/√λ akan selalu bernilai bilangan real. Artinya, dugaan prior u/ λ sudah proper/baik.

(c) Posterior

$\pi(\lambda|y) \propto f(y|\lambda).\pi(\lambda)$ $\qquad \alpha - 1 = -\frac{1}{2} + y$ $\qquad X \sim Gamma(\alpha, \beta)$

$\pi(\lambda|y) \propto \frac{e^{-\lambda}\lambda^y}{y!}\lambda^{-\frac{1}{2}}$ $\qquad \alpha = \frac{1}{2} + y$ $\qquad f_x(x) \begin{cases} \frac{\beta^\alpha}{\Gamma\alpha} e^{-\beta x} x^{\alpha-1}; x < 0 \\ 0; \quad x \text{ lainnya} \end{cases}$

$\pi(\lambda|y) \propto e^{-\lambda}\lambda^{-\frac{1}{2}+y}$

→ $\lambda \sim Gamma\left(\alpha = \frac{1}{2} + y, \beta = 1\right); y \neq \frac{1}{2}$

Latihan 3

We will use the Minitab macro *PoisGamP*, or **poisgamp** function in R, to find the posterior distribution of the Poisson probability μ when we have a random sample of observations from a $Poisson(\mu)$ distribution and we have a $gamma(r, v)$ prior for μ. The *gamma* family of priors is the conjugate family for *Poisson* observations. That means that if we start with one member of the family as the prior distribution, we will get another member of the family as the posterior distribution. The simple updating rules are "add sum of observations to r" and "add sample size to v. When we start with a $gamma(r, v)$ prior, we get a $gamma(r', v')$ posterior where $r' = r + \sum(y)$ and $v' = v + n$.

Suppose we have a random sample of five observations from a $Poisson(\mu)$ distribution. They are:

3	4	3	0	1

(a) Suppose we start with a positive uniform prior for μ. What $gamma(r, v)$ prior will give this form?

(b) **[Minitab:]** Find the posterior distribution using the Minitab macro *PoisGamP* or the R function **poisgamp**.

[R:] Find the posterior distribution using the R function **poisgamp**.

(c) Find the posterior mean and median.

(d) Find a 95% Bayesian credible interval for μ.

Jawab:

a) $\pi(\mu|\sim Gamma(r, v)$; $\quad y|\mu \sim Poisson(\mu)$

$r = \sum_{i=1}^{5} y_i = 3 + 4 + 3 + 0 + 1 = 11$ → shape

$v = n = 5$ (banyaknya amatan) → rate

$\pi(\mu) \sim Gammma(11, 5)$

b) $\mu|y \sim Gamma(r = 22, v = 10)$ → Posterior

c) $E(\mu|y) = \frac{r}{v} = \frac{22}{10} = 2.2$ d) 95% credible interval for μ=[1.38, 3.21]

Cara R:

```
# a. gamma(r,v)
# r = hasil penjumlahan setiap observasi = 3+4+3+0+1=11
# v = banyaknya observasi = 5
# install.packages("Bolstad")
library(Bolstad)
sebaran = c(3,4,3,0,1)
posterior = poisgamp(sebaran, shape = sum(sebaran), rate =
length(sebaran))
#Summary statistics for data
#--------------------------
# Number of observations: 5
#Sum of observations: 11
# Summary statistics for posterior
# --------------------------------
# Shape parameter (r): 22
# Rate parameter (v): 10
# 95% credible interval for mu: [1.38, 3.21] (jawaban soal
d)
# b. Sehingga, didapatkan hasil posterior mu|y ~
gamma(r=22,v=10)
# c. E(mu | y) = r/v = 22/10 = 2.2
# bukti
posterior$mean
# 2.2
# Median
# quantile = 0.5
quantile(posterior, probs = 0.5)
# 2.166758
# d. Bayesian Inverval 95%
c(quantile(posterior, probs = 0.025), quantile(posterior,
probs = 0.975))
# 1.378728 3.210073
```

Latihan 4 (Lanjutan no 3)

Suppose we start with a Jeffreys' prior for the Poisson parameter μ.

$$g(\mu) = \mu^{-\frac{1}{2}}$$

(a) What $gamma(r, v)$ prior will this form?

(b) Find the posterior distribution using the macro *PoisGamP* in Minitab or or the function **poisgamp** in R.

(c) Find the posterior mean and median.

(d) Find a 95% Bayesian credible interval for μ.

Jawab:

a) Posterior

$\pi(\mu|y) \propto f(y_1, .., y_5|\mu)g(\mu)$ $\qquad X \sim Gamma(\alpha, \beta)$

$\pi(\mu|y) \propto f(y_1|\mu).f(y_2|\mu) \dots f(y_5|\mu)g(\mu)$ $\qquad f(y|\mu) = \frac{e^{-\mu}\mu^y}{y!}$

$\pi(\mu|y) \propto \frac{e^{-\mu}\mu^{y_1}}{y_1!} \dots \frac{e^{-\mu}\mu^{y_1}}{y_1!}.\mu^{-\frac{1}{2}}$

$\propto \frac{e^{-5\mu}\mu^{\Sigma y_i}}{\pi(y_i!)}\mu^{-\frac{1}{2}}$

$\boxed{\begin{aligned} \alpha - 1 &= -\tfrac{1}{2} + y \\ \alpha &= \tfrac{1}{2} + y \end{aligned}}$

$\propto e^{-5\mu}\mu^{-\frac{1}{2}+\Sigma y_i}$

$\sim gamma\left(\alpha = \frac{1}{2} + \Sigma y_i,\ \beta = 5\right)$

$\sim gamma\left(\alpha = 11\frac{1}{2},\ \beta = 5\right)$

```
#b.
posterior2 <- poisgamp(sebaran, shape = 11.5, rate = 5)
#Summary statistics for data
#--------------------------
# Number of observations: 5
#Sum of observations: 11
#Summary statistics for posterior
#--------------------------------
# Shape parameter (r): 22.5
#Rate parameter (v): 10
#95% credible interval for mu: [1.42, 3.27] (Jawaban soal
d)
# b. Sehingga, didapatkan hasil posterior mu|y ~
gamma(r=22.5,v=10)
# c. E(mu | y) = r/v = 22.5/10 = 2.25
# bukti
posterior2$mean
# 2.25
# Median
# quantile = 0.5
quantile(posterior2, probs = 0.5)
# 2.216756
# Bayesian Inverval 95%
c(quantile(posterior2, probs = 0.025),
quantile(posterior2, probs = 0.975))
# 1.418308 3.270508
```

Latihan 1 **PERTEMUAN 4&5**

Suppose that the random variable X is transmitted over a communication channel. Assume that the received signal is given by

$$Y = 2X + W,$$

where $W \sim N(0, \sigma^2)$ is independent of X. Suppose that $X = 1$ with probability p, and $X = -1$ with probability $1 - p$. The goal is to decide between $X = -1$ and $X = 1$ by observing the random variable Y. Find the MAP test for this problem.

Jawab:

$H_0 = X = 1$ vs $H_1 = X = -1$

MAP → dipilih H0 <--> $f_y(y|H_0).P(H_0) > f_y(y|H_1).P(H_1)$

*dibawah kondisi H0

$W \sim N(0, \sigma^2)$

$Y = 2 + W$ → $E(Y) = E(2+W) = 2$; $Var(Y) = Var(2+W) = Var(W) = \sigma^2$

$Y|H_0 \sim N(2, \sigma^2)$

$f_Y(y|H_0) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-2)^2}{2\sigma^2}}$ $; -\infty < y < \infty$

*di bawah kondisi H1

$Y = -2 + W$

$Y|H_1 \sim N(-2, \sigma^2)$

$f_Y(y|H_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y+2)^2}{2\sigma^2}}$ $; -\infty < y < \infty$

→Dipilih H0

$\Leftrightarrow f_Y(y|H_0)P(H_0) > f_Y(y|H_1).P(H_1)$

$\cancel{\frac{1}{\sigma\sqrt{2\pi}}} e^{-\frac{(y-2)^2}{2\sigma^2}}.p > \cancel{\frac{1}{\sigma\sqrt{2\pi}}} e^{-\frac{(y+2)^2}{2\sigma^2}}.(1-p)$

$e^{-\frac{(y-2)^2}{2\sigma^2}}.e^{\frac{(y+2)^2}{2\sigma^2}} > \frac{1-p}{p}$

$\exp\left(\frac{1}{2\sigma^2}\left(-\cancel{y^2} + 4y - \cancel{4} + \cancel{y^2} + 4y + \cancel{4}\right)\right) > \frac{1-p}{p}$

$\rightarrow \exp\left(\frac{8y}{2\sigma^2}\right) > \frac{1-p}{p}$

$\frac{4y}{\sigma^2} > \ln\left(\frac{1-p}{p}\right)$

$y > \frac{\sigma^2}{4}\ln\left(\frac{1-p}{p}\right)$

Latihan 2

In the study of water quality in New Zealand streams, documented in McBride et al. (2002), a high level of *Campylobacter* was defined as a level greater than 100 per 100 ml of stream water. $n = 116$ samples were taken from streams having a high environmental impact from birds. Out of these, $y = 11$ had a high *Campylobacter* level. Let π be the true probability that a sample of water from this type of stream has a high *Campylobacter* level.

(a) Find the frequentist estimator for π. $= \frac{y}{n} = \frac{11}{116}$

(b) Use a $beta(1, 10)$ prior for π. Calculate the posterior distribution $g(\pi|y)$.

(c) Find the posterior mean and variance. What is the Bayesian estimator for π?

(d) Find a 95% credible interval for π.

(e) Test the hypothesis

$$H_0 : \pi = .10 \quad \text{versus} \quad H_1 : \pi \neq .10$$

at the 5% level of significance.

Jawab:

→ $y|\pi \sim$ binomial(n, π) ✓

$\pi \sim$ beta$(1, 10)$

(b) Posterior :

$\pi|y \sim$ beta$(1+y, n-y+10)$

$\pi|y \sim$ beta$(12, 115)$

(c) $E(\pi|y) = \frac{\alpha}{\alpha+\beta} = \frac{12}{12+115} = \frac{12}{127}$

$Var(\pi|y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$= \frac{12.115}{127^2(128)}$

```
###Latihan 2
pi.duga <- 11/116 #[1] 0.09482759 (a)
m1 <- 12/(12+115) #[1] 0.09448819 (c)
v1 <- (12*115)/((12+115)^2*(12+115+1)) #[1]
0.0006684388 (c)
ba <- m1+1.96*sqrt(v1) #(d)
bb <- m1-1.96*sqrt(v1) #(d)
cat(paste("95% Credible Interval: [", round
(bb,5)," ", round(ba,5), "]\n", sep=""))
#(d)
#95% Credible Interval: [0.04381 0.14516]
#pi=0.1 masuk selang --> tidak tolak H0 (e)
```