Contoh 1 **PERTEMUAN 1**

Ada bola di dalam tas. Diketahui bahwa setidaknya salah satu dr bola berwarna hitam, tetapi ragu apakah keduanya hitam atau yang satu hitam dan yang satu putih. Ada 2 kemungkinan:

BB: Kedua bola berwarna hitam

BW: satu hitam dan satu putih

Kita tahu bahwa hanya satu dari pernyataan/hipotesis berikut ini yg benar. Dilakukan mengeluarkan salah satu bola, & mengamati warnanya. (D: bola yang dikeluarkan dari tas berwarna hitam).

Jawab:

Asumsikan P(BB) = 0.5 dan P(BW) = 0.5 → nilai prior

Dari kejadian D, kita hitung kemungkinan nilai likelihood

Hypotheses	Possible Data	Probability
BB	Black Ball	1
	White Ball	0
BW	Black Ball	0.5
	White Ball	0.5

Lalu diperoleh Bayes's Box berikut secara lengkap

Hypotheses	prior	likelihood	h = prior × likelihood	posterior
BB	0.5	1	0.5	0.667
BW	0.5	0.5	0.25	0.333
Totals:	1		0.75	1

Latihan 1

1. Jika dilihat dari sudut pandang Bayesian, data diasumsikan bersifat acak yang memiliki sebaran. Salah
2. Keunggulan dari metode Bayesian adalah dapat memperbaharui informasi yang diperoleh. Benar
3. Posterior dalam konsep Bayesian merupakan sebaran awal sebelum data diperoleh. Salah
4. Parameter memiliki sebaran adalah konsep dari sudut pandang Frequentist. Salah

Latihan 2

Dari 200 orang pelamar terdapat 150 berasal dari SMA dan sisanya berasal dari SMK. Dari 150 pelamar SMA tersebut hanya 80 yg mempunyai sertifkat bahasa Inggris. Pelamar yg berasal dari SMK, hanya 20% yang mempunyai sertifikat bahasa Inggris. Jika seorang pelamar ditarik secara acak, berapa peluang pelamar tsb berpendidikan SMK jika diketahui ybs mempunyai sertifikat b.Inggris?

Jawab:

K1 = sekolah	K2 = sertifikat
SMA (150) -> P(A) = ¾	S (80) -> P(S|A) = 80/150 = 8/15
	T (70) -> P(T|A) = 70/150 = 7/15
SMK (50) -> P(K) = ¼	S (50*20%= 10) -> P(S|K) = 1/5
	T (40) -> P(T|K) = 4/5

$$P(K|S) = \frac{P(K \cap S)}{P(S)} = \frac{P(K)P(S|K)}{P(S|A)P(A) + P(S|K)P(K)}$$

$$P(K|S) = \frac{\frac{1}{4}\left(\frac{1}{5}\right)}{\frac{8}{15}\left(\frac{3}{4}\right) + \frac{1}{5}\left(\frac{1}{4}\right)} = \frac{\frac{1}{20}}{\frac{9}{20}} = \frac{1}{9}$$

Latihan 3

You move into a new house which has a phone installed. You can't remember the phone number, but you suspect it might be 555-3226 (some of you may recognise this as being the phone number for Homer Simpson's "Mr Plow" business). To test this hypothesis, you carry out an experiment by picking up the phone and dialing 555-3226.

If you are correct about the phone number, you will definitely hear a busy signal because you are calling yourself. If you are incorrect, the probability of hearing a busy signal is 1/100. However, all of that is only true if you assume the phone is working, and it might be broken! If the phone is broken, it will always give a busy signal.

When you do the experiment, the outcome (the data) is that you do actually get the busy signal. The question asked us to consider the following four hypotheses, and to calculate their posterior probabilities:

Hypothesis	Description	Prior Probability
H_1	Phone is working and 555-3226 is correct	0.4
H_2	Phone is working and 555-3226 is incorrect	0.4
H_3	Phone is broken and 555-3226 is correct	0.1
H_4	Phone is broken and 555-3226 is incorrect	0.1

Jawab:

^Ada 3 kejadiann

K1 = Telepon	K2 = nomor	K3 = sinyal
Berfungsi (F)	Benar (B)	Sibuk (K) = 1
		Tidak (T) = 0
	Salah (S)	Sibuk (K) = 1/100 = 0.01
		Tidak (T) = 99/100 = 0.99
Rusak ®	Benar (B)	Sibuk (K) = 1
		Tidak (T) = 0
	Salah (S)	Sibuk (K) = 1
		Tidak (T) = 0

Hipt.	Deskripsi	Prior	Likelihood	h	Posterior (h/T)
H1	FB	0.4	1	0.4	0.662
H2	FS	0.4	0.01	0.004	0.006
H3	RB	0.1	1	0.1	0.116
H4	RS	0.1	1	0.1	0.116
Total				0.604	

PERTEMUAAN 2

Studi Kasus 1

kemungkinan tes medis tertentu menjadi positif adalah 90%, apakah seorang pasien memiliki ini adalah D. Diket:i 1% populasi menderita penyakit tersebut, dan teks mencatat positif palsu 5% dari waktu. jika Anda menerima tes positif, berapa peluang Anda terkena D.

Jawab:

P(+|D) = 0.90; P(D) = 0.01, $P(+|\widetilde{D}) = 0.05$

Bayes' Theorem: $P(D|+) = \frac{P(+|D)\,P(D)}{P(+)} = \frac{P(+|D)\,P(D)}{[P(+|D)\,P(D)]+[P(+|\widetilde{D})\,P(\widetilde{D})]}$

Substituting in the data: $P(D|+) = \frac{0.9\times0.01}{(0.9\times0.01)+(0.05\times0.99)} = 0.15$

Studi kasus 2 (**Teorema Bayes u/ Binomial dg Prior Diskret**)

Let $Y|\pi$ be $binomial(n = 4, \pi)$. Suppose that we consider that there are only three possible values for π, .4,.5, and .6. We will assume they are equally likely. The joint probability distribution $f(\pi_i, y_j)$ is found by multiplying the conditional observation distribution $f(y_j|\pi_i)$ times the prior distribution $g(\pi_i)$. In this case, the conditional observation probabilities come from the $binomial(n = 4, \pi)$ distribution.

Suppose $Y = 3$ was observed.

Bayes' Box

π	prior	likelihood	prior × likelihood	posterior
.4	$\frac{1}{3}$	.1536	.0512	$\frac{.0512}{.2497}$ = .205
.5	$\frac{1}{3}$	.2500	.0833	$\frac{.0833}{.2497}$ = .334
.6	$\frac{1}{3}$	.3456	.1152	$\frac{.1152}{.2497}$ = .461
marginal $P(Y = 3)$			.2497	1.000

Studi kasus 3 (**Teorema Bayes u/ Poisson dg Prior Diskret**)

Let $Y|\mu$ be Poisson(μ). Suppose that we believe there are only four possible values for μ, 1,1.5,2, and 2.5. Suppose we consider that the two middle values, 1.5 and 2, are twice as likely as the two end values 1 and 2.5. Suppose $y = 2$ was observed. Plug the value $y = 2$ into formula

$$f(y|\mu) = \frac{\mu^y e^{-\mu}}{y!}$$

To give the likelihood

Bayes' Box

μ	prior	likelihood	prior × likelihood	posterior
1.0	$\frac{1}{6}$	$\frac{1.0^2 e^{-1.0}}{2!}$ = .1839	.0307	$\frac{.0307}{.2473}$ = .124
1.5	$\frac{1}{3}$	$\frac{1.5^2 e^{-1.5}}{2!}$ = .2510	.0837	$\frac{.0837}{.2473}$ = .338
2.0	$\frac{1}{3}$	$\frac{2.0^2 e^{-2.0}}{2!}$ = .2707	.0902	$\frac{.0902}{.2473}$ = .365
2.5	$\frac{1}{6}$	$\frac{2.5^2 e^{-2.5}}{2!}$ = .2565	.0428	$\frac{.0428}{.2473}$ = .173
marginal $P(Y = 2)$			.2473	1.000

Latihan 1

Suppose there is a medical diagnostic test for a disease. The *sensitivity* of the test is .95. This means that if a person has the disease, the probability that the test gives a positive response is .95. The *specificity* of the test is .90. This means that if a person does not have the disease, the probability that the test gives a negative response is .90, or that the *false positive* rate of the test is .10. In the population, 1% of the people have the disease. What is the probability that a person tested has the disease, given the results of the test is positive? Let D be the event "the person has the disease" and let T be the event "the test gives a positive result."

Jawab:

Diket: D = mengidap penyakit; T = hasil positif; $P(T|\widetilde{D})$=0.1

Sensitivitas = P(T|D) = 0.95; Spesivisitas = $P(\widetilde{T}|\widetilde{D})$ = 0.90;

KI KII

0,01 D < T 0,95 ; $\bar{T}$ 0,05 ✓

0,99 $\bar{D}$ < T 0,1 ✓ ; $\bar{T}$ 0,9

$\rightarrow P(D|T) = \frac{P(D).P(T|D)}{P(D).P(T|D) + P(\bar{D}).P(T|\bar{D})}$

= 8,75%

Jadi, peluang seseorang yg dites mengidap penyakit tsb jika hasil tesnya positif a/ 0.0875 atau 8.75%.

Latihan 2

Suppose there is a medical screening procedure for a specific cancer that has *sensitivity* = .90, and *specificity* = .95. Suppose the underlying rate of the cancer in the population is .001. Let B be the event "the person has that specific cancer," and let A be the event "the screening procedure gives a positive result."

a) What is the probability that a person has the disease given the results of the screening is positive?

b) Does this show that screening is effective in detecting this cancer?

Jawab:

KI KII

0,001 B < A 0,9 ; $\bar{A}$ 0,1

0,999 $\bar{B}$ < A 0,05 ; $\bar{A}$ 0,95

$P(B|A) = \frac{P(B).P(A|B)}{P(B)\,P(A|B) + P(\bar{B}).P(A|\bar{B})}$

$= \frac{0{,}001 \cdot 0{,}9}{0{,}001 \cdot 0{,}9 + 0{,}999 \cdot 0{,}05} = 0{,}01769$

Latihan 3

There is an urn containing 9 balls, which can be either green or red. The number of red balls in the urn is not known.

Let X be the number of red balls in the urn.

X = Jmlh bola merah dlm guci

X = {0,1,2,...9}

Suppose we look at the two draws from the urn (without replacement) as a single experiment. The results were first draw red, second draw green. Find the posterior distribution of X by filling in the simplified table.

1M 8H

X	prior	likelihood (MH)	prior × likelihood	posterior
0	P(X=0) = 1/10	0		
1	P(X=1) = 1/10	$\frac{\binom{1}{1}\binom{8}{1}}{\binom{9}{2}} = \frac{8}{36}$		
⋮				
9	P(X=9) = 1/10			

X	Prior	Likelihood → MH	h = P.L	Posterior
0	1/10	0 ?	0	0
1	1/10	$\binom{1}{1}\binom{8}{1}/\binom{9}{2} = 8/36$	8/360	8/120
2	1/10	$\binom{2}{1}\binom{7}{1}/\binom{9}{2} = 14/36$	14/360	14/120
3	1/10	$\binom{3}{1}\binom{6}{1}/\binom{9}{2} = 18/36$	18/360	18/120
4	1/10	$\binom{4}{1}\binom{5}{1}/\binom{9}{2} = 20/36$	20/360	20/120
5	1/10	$\binom{5}{1}\binom{4}{1}/\binom{9}{2} = 20/36$	20/360	20/120
6	1/10	$\binom{6}{1}\binom{3}{1}/\binom{9}{2} = 18/36$	18/360	18/120
7	1/10	$\binom{7}{1}\binom{2}{1}/\binom{9}{2} = 14/36$	14/360	14/120
8	1/10	$\binom{8}{1}\binom{1}{1}/\binom{9}{2} = 8/36$	8/360	8/120
9	1/10	0 ?	0	0

Total = $\frac{120}{360}$

Latihan 4

$Y_1 \sim$ binomial $(n=10, \pi)$

Let Y_1 be the number of successes in $n = 10$ independent trials where each trial results in a success or failure, and π, the probability of success, remains constant over all trials. Suppose the 4 possible values of π are .20, .40, .60, and .80. We do not wish to favor any value over the others so we make them equally likely. We observe $Y_1 = 7$. Find the posterior distribution by filling in the simplified table.

$Y_1 = 7 \rightarrow P(Y_1 = 7) = \binom{10}{7}\pi^7(1-\pi)^3$

π	prior	likelihood	prior × likelihood	posterior
0,2	1/4			
0,4	1/4			
0,6	1/4			
0,8	1/4			

π	prior	likelihood → $P(Y_1=7) = \binom{10}{7}\pi^7(1-\pi)^3$	h = P.L	Posterior
0,2	1/4	$\binom{10}{7} 0{,}2^7 (0{,}8)^3$ = 0,000786	0,0001965	0,00171
0,4	1/4	$\binom{10}{7} 0{,}4^7 (0{,}6)^3$ = 0,042467	0,010617	0,09241
$\boxed{0{,}6}$	1/4	$\binom{10}{7} 0{,}6^7 (0{,}4)^3$ = 0,214991	0,053748	$\boxed{0{,}46787}$
0,8	1/4	$\binom{10}{7} 0{,}8^7 (0{,}2)^3$ = 0,201326	0,050332	0,938075

Total 0,114 8935

Y_1 → Sukses → pendekat $Y_1 = 7$

$\pi = 0{,}6$

Latihan 5

Let n be the unknown number of customers that visit a store on the day of a sale. The number of customers that make a purchase is $Y|n \sim$ Binomial(n, θ) where θ is the known probability of making a purchase given the customer visited the store. The prior is $n \sim$ Poisson(5). Assuming θ is known and n is the unknown parameter, plot the posterior distribution of n for all combinations of $Y \in \{0, 5, 10\}$ and $\theta \in \{0.2, 0.5\}$ and comment on the effect of Y and θ on the posterior.