PENGANTAR MODEL LINEAR

Pertemuan 1

Overview

Introduction

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Kontrak Kerja

Perkuliahan

7 sesi UTS
7 sesi UAS

Quiz

2x quiz

pertemuan ke-7
pertemuan ke-14

Tugas

4x tugas

pertemuan ke-3
pertemuan ke-6
pertemuan ke-10
pertemuan ke-13

REVIEW MATRIX

Matriks: Matriks adalah suatu susunan bilangan berbentuk segiempat yang diatur dalam baris dan kolom Bilangan-bllangan dalam susunan itu disebut anggota/elemen/unsur dari matriks tersebut.

Notasi yang digunakan untuk menyatakan matriks bisa dengan kurung kecil : ( ), kurung siku : [ ], atau dengan garis tegak dobel : || ||

tipe-tipe matriks:

matriks persegi panjang
matriks persegi
matriks baris
matriks kolom
matriks nol
matriks diagonal
matriks skalar
matrix segitiga atas dan bawah
matrix simetri dan anti-simetri

Matrix Transpose

Let's Work Out-

Example- Find the transpose of the given matrix

$$M = \begin{bmatrix} 2 & -9 & 3 \\ 13 & 11 & -17 \\ 3 & 6 & 15 \\ 4 & 13 & 1 \end{bmatrix}$$

Solution- Given a matrix of the order 4×3.

The transpose of a matrix is given by interchanging rows and columns.

$$M^T = \begin{bmatrix} 2 & 13 & 3 & 4 \\ -9 & 11 & 6 & 13 \\ 3 & -17 & 15 & 1 \end{bmatrix}$$

Determinan Matrix

i) for a single element matrix (a scalar, $A = a_{11}$), det(A) = a_{11}.

ii) in the (2 x 2) case,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

the determinant is defined to be the difference of two terms as follows,

$$|A| = a_{11}a_{22} - a_{12}a_{21}$$

which is obtained by multiplying the two elements in the principal diagonal of A and then subtracting the product of the two off-diagonal elements.

iii) in the (3 x 3) case,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Invers Matrix

Untuk menentukan invers dari suatu matriks digunakan rumus:

$$A^{-1} = \frac{1}{\det(A)} \text{adj}(A)..$$

Tentukan invers dari matriks $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$ dengan menggunakan adjoint.

Trace Matrix

$tr(A) = a_{11} + a_{22} + a_{33} + \ldots + a_{nn}$

- $tr(A) = A$, if A is a scalar.
- $tr(A') = tr(A)$, because A is square.
- $tr(kA) = k \cdot tr(A)$, where k is a scalar.
- $tr(I_n) = n$, the trace of an identity matrix is its dimension.
- $tr(A \pm B) = tr(A) \pm tr(B)$.
- $tr(AB) = tr(BA)$.
- $tr(AA') = tr(A'A) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2$.

Orthogonal Matrix

A transpose = A invers

A.A transpose = I

Rank of Matrix

The rank of a unit matrix of order m is m.

If A matrix is of order $m \times n$, then $\rho(A) \leq \min\{m, n\}$ = minimum of m, n.

If A is of order $n \times n$ and $|A| \neq 0$, then the rank of $A = n$.

If A is of order $n \times n$ and $|A| = 0$, then the rank of A will be less than n.

Eigen Value & Eigen Vector

Jika A adalah matriks n x n, maka vektor taknol x di dalam R^n dinamakan **vektor eigen** (***eigenvector***) dari A jika Ax adalah kelipatan skalar dari x; yakni,

$$Ax = \lambda x$$

untuk suatu skalar λ. Skalar λ dinamakan **nilai eigen** (***eigenvalue***) dari A dan x dikatakan **vektor eigen** yang bersesuaian dengan λ.

Jika A adalah sebuah matriks berukuran $n \times n$, maka λ adalah nilai eigen dari A jika dan hanya jika ia memenuhi persamaan

$$det(\lambda I - A) = 0 \tag{1}$$

Persamaan tersebut disebut persamaan karakteristik dari A.

LATIHAN SOAL

No. 1

Consider the following system of equations.

$$\begin{cases} w + x + y + z = 6 \\ w \phantom{{}+x} + y + z = 4 \\ w \phantom{{}+x} + y \phantom{{}+z} = 2 \end{cases} \tag{$*$}$$

(a) List the leading variables ____________ .

(b) List the free variables ____________ .

(c) The general solution of $(*)$ (expressed in terms of the free variables) is

$$(\underline{\qquad\qquad}, \underline{\quad\;}, \underline{\quad\;}, \underline{\quad\;}).$$

(d) Suppose that a fourth equation $-2w + y = 5$ is included in the system $(*)$. What is the solution of the resulting system? Answer: (___ , ___ , ___ , ___).

(e) Suppose that instead of the equation in part (d), the equation $-2w - 2y = -3$ is included in the system $(*)$. Then what can you say about the solution(s) of the resulting system? Answer: ________________________ .

No. 2

Consider the following system of equations:

$$\begin{cases} x + \ y + \ z = 2 \\ x + 3y + 3z = 0 \\ x + \ 3y + \ 6z = 3 \end{cases} \qquad (*)$$

(a) Use Gaussian elimination to put the augmented coefficient matrix into row echelon form. The result will be $\begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 1 & b \\ 0 & 0 & 1 & c \end{bmatrix}$ where $a =$ _____ , $b =$ _____ , and $c =$ _____ .

(b) Use Gauss-Jordan reduction to put the augmented coefficient matrix in reduced row echelon form. The result will be $\begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \end{bmatrix}$ where $d =$ _____ , $e =$ _____ , and $f =$ _____ .

(c) The solutions of $(*)$ are $x =$ _____ , $y =$ _____ , and $z =$ _____ .

No. 3

Consider the following system of linear equations (where $b_1, \dots, b_5$ are constants).

$$\begin{cases} u + 2v - w - 2x + 3y \phantom{{}-4z} = b_1 \\ x - y + 2z = b_2 \\ 2u + 4v - 2w - 4x + 7y - 4z = b_3 \\ -x + y - 2z = b_4 \\ 3u + 6v - 3w - 6x + 7y + 8z = b_5 \end{cases}$$

(a) In the process of Gaussian elimination the leading variables of this system are __________ and the free variables are __________ .

(b) What condition(s) must the constants $b_1, \dots, b_5$ satisfy so that the system is consistent? Answer: ____________________________ .

(c) Do the numbers $b_1 = 1$, $b_2 = -3$, $b_3 = 2$, $b_4 = b_5 = 3$ satisfy the condition(s) you listed in (b)? ________ . If so, find the general solution to the system as a function of the free variables. Answer:

$u =$ ____________________

$v =$ __________

$w =$ __________

$x =$ __________

$y =$ __________

$z =$ __________ .

No. 4

Exercise 5.6. Let

$$M = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 0 \end{bmatrix} \in \mathcal{M}_3(\mathbb{R}).$$

(a) Determine the eigenvalues and the eigenvectors of M over the field $\mathbb{R}$.

(b) Say, justifying the answer, if there exist a matrix $V \in \mathcal{M}_3(\mathbb{R})$ and a diagonal matrix $D \in \mathcal{M}_3(\mathbb{R})$ such that $M = VDV^{-1}$.

(c) Determine a possible choice of such V and D.

No. 5

Exercise 5.1. Let M be a 2×2 matrix with real coefficients and eigenvalues 3 and 5, with eigenvectors $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$ respectively.

(a) Compute $M \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

(b) Find a diagonal matrix D and two matrices A, A^{-1} (each inverse of the other) such that $M = ADA^{-1}$.

Thank You