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Jawaban Bagian A
Ujian Tengah Semester
STA1342 – Teknik Peubah Ganda

1. Vektor acak $\mathbf{X}^t = [X_1, X_2, X_3, X_4]$ memiliki vektor nilai tengah $\boldsymbol{\mu}_{\mathbf{X}}^t = [4,3,2,1]$ dan matriks ragam peragam

$$\mathbf{\Sigma}_{\mathbf{X}} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

Misalkan partisi dilakukan pada **X** sebagai berikut

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \hline X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} \mathbf{X}^{(1)} \\ \hline \mathbf{X}^{(2)} \end{bmatrix}$$

dan didefinisikan

$\mathbf{A} = [1 \quad 2]$ dan $\mathbf{B} = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$

Hitunglah:

a. $E(\mathbf{X}^{(1)})$

$E(\mathbf{X}^{(1)}) = E\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

$E(\mathbf{X}^{(1)}) = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$

$E(\mathbf{X}^{(1)}) = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

b. $E(\boldsymbol{A}\mathbf{X}^{(1)})$

$E(\boldsymbol{A}\mathbf{X}^{(1)}) = \boldsymbol{A} \times E(\mathbf{X}^{(1)})$

$E(\boldsymbol{A}\mathbf{X}^{(1)}) = \boldsymbol{A} \times \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$

$E(\boldsymbol{A}\mathbf{X}^{(1)}) = [1 \quad 2][\begin{smallmatrix} 4 \\ 3 \end{smallmatrix}]$

$E(\boldsymbol{A}\mathbf{X}^{(1)}) = 10$

c. $Cov(\mathbf{X}^{(1)})$

$Cov(\mathbf{X}^{(1)}) = \mathbf{\Sigma_{11}}$

$Cov(\mathbf{X}^{(1)}) = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

d. $Cov(\boldsymbol{A}\mathbf{X}^{(1)})$

$Cov(\boldsymbol{A}\mathbf{X}^{(1)}) = \boldsymbol{A} \times Cov(\mathbf{X}^{(1)}) \times \boldsymbol{A}'$

$Cov(\boldsymbol{A}\mathbf{X}^{(1)}) = [1 \quad 2]\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$Cov(\boldsymbol{A}\mathbf{X}^{(1)}) = 7$

e. $E(\mathbf{X}^{(2)})$

$E(\mathbf{X}^{(2)}) = E\begin{bmatrix} X_3 \\ X_4 \end{bmatrix}$

$E(\mathbf{X}^{(2)}) = \begin{bmatrix} \mu_3 \\ \mu_4 \end{bmatrix}$

$E(\mathbf{X}^{(2)}) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

f. $E(\boldsymbol{B}\mathbf{X}^{(2)})$

$E(\boldsymbol{B}\mathbf{X}^{(2)}) = \boldsymbol{B} \times E(\mathbf{X}^{(2)})$

$E(\boldsymbol{B}\mathbf{X}^{(2)}) = \boldsymbol{B} \times \begin{bmatrix} \mu_3 \\ \mu_4 \end{bmatrix}$

$E(\boldsymbol{B}\mathbf{X}^{(2)}) = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$E(\boldsymbol{B}\mathbf{X}^{(2)}) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

g. $Cov(\mathbf{X}^{(2)})$

$Cov(\mathbf{X}^{(2)}) = \mathbf{\Sigma_{22}}$

$Cov(\mathbf{X}^{(2)}) = \begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}$

h. $Cov(\boldsymbol{B}\mathbf{X}^{(2)})$

$Cov(\boldsymbol{B}\mathbf{X}^{(2)}) = \boldsymbol{B} \times Cov(\mathbf{X}^{(2)}) \times \boldsymbol{B}'$

$Cov(\boldsymbol{B}\mathbf{X}^{(2)}) = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$

$Cov(\boldsymbol{B}\mathbf{X}^{(2)}) = \begin{bmatrix} 33 & 36 \\ 36 & 48 \end{bmatrix}$

i. $Cov(\mathbf{X}^{(1)}, \mathbf{X}^{(2)})$

$Cov(\mathbf{X}^{(1)}, \mathbf{X}^{(2)}) = \mathbf{\Sigma_{12}}$

$Cov(\mathbf{X}^{(1)}, \mathbf{X}^{(2)}) = \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

j. $Cov(\boldsymbol{A}\mathbf{X}^{(1)}, \boldsymbol{B}\mathbf{X}^{(2)})$

$Cov(\boldsymbol{A}\mathbf{X}^{(1)}, \boldsymbol{B}\mathbf{X}^{(2)}) = \boldsymbol{A} \times Cov(\mathbf{X}^{(1)}, \mathbf{X}^{(2)}) \times \boldsymbol{B}'$

$Cov(\boldsymbol{A}\mathbf{X}^{(1)}, \boldsymbol{B}\mathbf{X}^{(2)}) = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$

$Cov(\boldsymbol{A}\mathbf{X}^{(1)}, \boldsymbol{B}\mathbf{X}^{(2)}) = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

2. Misalkan $\mathbf{X} \sim N_3(\boldsymbol{\mu}, \mathbf{\Sigma})$ dengan $\boldsymbol{\mu}^t = [-3,1,4]$ dan

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Berikan penjelasan manakah di antara peubah acak berikut yang saling bebas

a. X_1 dan X_2

Saling bebas jika $\text{Cov}(X_1, X_2) = 0$

$\text{Cov}(X_1, X_2) = \mathbf{\Sigma_{11}}$

$\text{Cov}(X_1, X_2) = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

$\text{Cov}(X_1, X_2) \neq 0$ maka tidak X_1 dan X_2 saling bebas

b. X_2 dan X_3

Saling bebas jika $\text{Cov}(X_2, X_3) = 0$

$\text{Cov}(X_2, X_3) = \boldsymbol{\sigma_{23}}$

$\text{Cov}(X_2, X_3) = 0$ maka saling bebas

c. (X_1, X_2) dan X_3

$X_1, X_2 = \mathbf{X}^{(1)}$

$Cov\left(\mathbf{X}^{(1)}, X_3\right) = \boldsymbol{\Sigma_{12}}$

$Cov\left(\mathbf{X}^{(1)}, X_3\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\text{Cov}\left(\mathbf{X}^{(1)}, X_3\right) = 0$ maka saling bebas

d. $\frac{X_1+X_2}{2}$ dan X_3

$$AX = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$A\mathbf{\Sigma}\text{A}' = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A\mathbf{\Sigma}\text{A}' = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}$$

$\boldsymbol{\Sigma_{12}} = 0$ maka saling bebas