Konsep Matriks dalam APG

Teras Matriks

$$tr(\mathbf{A}) = \sum_{i=1}^{n} a_{ii}$$

Contoh

$$tr\left(\begin{bmatrix} 5 & 3 & 5 \\ 4 & -1 & 2 \\ -3 & 8 & 7 \end{bmatrix}\right) = 5 - 1 + 7 = 11.$$

Transpose Matriks

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 4 & 1 & 3 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 4 \\ 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Inverse matriks

- ✓ Matriks invers dari suatu matriks A adalah matriks B yang apabila dikalikan dengan matriks A memberikan satuan I
- ✓ Notasi matriks invers : A^{-1}

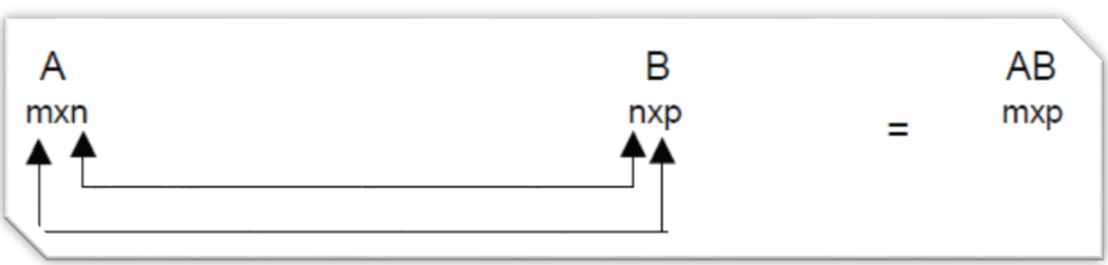
Contoh

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Perkalian matriks

- ✓ Syarat perkalian adalah jumlah banyaknya kolom pertama matriks sama dengan jumlah banyaknya baris matriks kedua.



Contoh

$$A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$$A * B = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (3*3) + (2*1) + (1*0) \end{bmatrix} = \begin{bmatrix} 11 \end{bmatrix}$$

$$B * A = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} * \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3*3 & 3*2 & 3*1 \\ 1*3 & 1*2 & 1*1 \\ 0*3 & 0*2 & 0*1 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 3 \\ 3 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

<u>Mencari matriks korelasi dari matriks kovarian</u>

- Misal **X** memiliki matriks

kovarian yakni $\boldsymbol{\Sigma} = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$

- Maka tentukan $\boldsymbol{\rho}$

- $= \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$ maka $\boldsymbol{\rho} = \begin{bmatrix} \frac{\sigma_{11}}{\sqrt{\sigma_{11}}\sqrt{\sigma_{11}}} & \frac{\sigma_{12}}{\sqrt{\sigma_{11}}\sqrt{\sigma_{22}}} & \frac{\sigma_{13}}{\sqrt{\sigma_{11}}\sqrt{\sigma_{33}}} \\ \frac{\sigma_{12}}{\sqrt{\sigma_{11}}\sqrt{\sigma_{22}}} & \frac{\sigma_{22}}{\sqrt{\sigma_{22}}\sqrt{\sigma_{22}}} & \frac{\sigma_{23}}{\sqrt{\sigma_{22}}\sqrt{\sigma_{33}}} \\ \frac{\sigma_{13}}{\sqrt{\sigma_{11}}\sqrt{\sigma_{33}}} & \frac{\sigma_{23}}{\sqrt{\sigma_{22}}\sqrt{\sigma_{33}}} & \frac{\sigma_{33}}{\sqrt{\sigma_{33}}\sqrt{\sigma_{33}}} \end{bmatrix}$ (i)

Diketahui $\boldsymbol{\Sigma} = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$ maka untuk mencari matriks $\boldsymbol{\rho}$, digunakan persamaan (i) di atas sehingga :

$$\boldsymbol{\rho} = \begin{bmatrix} \frac{25}{25} & \frac{-2}{10} & \frac{4}{15} \\ \frac{-2}{10} & \frac{4}{4} & \frac{1}{6} \\ \frac{4}{15} & \frac{1}{6} & \frac{9}{9} \end{bmatrix} = \begin{bmatrix} 1 & \frac{-1}{5} & \frac{4}{15} \\ \frac{-1}{5} & 1 & \frac{1}{6} \\ \frac{4}{15} & \frac{1}{6} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -0.2 & 0.267 \\ -0.2 & 1 & 0.167 \\ 0.267 & 0.167 & 1 \end{bmatrix}$$

$$\rho_{ij} = \frac{\operatorname{cov}(x_i, x_j)}{\sqrt{\operatorname{var}(x_i)\operatorname{var}(x_j)}} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}}$$

Mencari Simpangan Baku dari Matriks Kovarian

- Misal **X** memiliki matriks kovarian yakni

- $\boldsymbol{\Sigma} = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$

  - Maka tentukan $\boldsymbol{V}^{1/2}$

- Untuk mencari matriks $\boldsymbol{V}^{1/2}$ digunakan persamaan sebagai

berikut : $\boldsymbol{V}^{1/2} = \begin{bmatrix} \sqrt{\sigma_{11}} & 0 & 0 \\ 0 & \sqrt{\sigma_{22}} & 0 \\ 0 & 0 & \sqrt{\sigma_{33}} \end{bmatrix}$

sehingga

$$\boldsymbol{V}^{1/2} = \begin{bmatrix} \sqrt{\sigma_{11}} & 0 & 0 \\ 0 & \sqrt{\sigma_{22}} & 0 \\ 0 & 0 & \sqrt{\sigma_{33}} \end{bmatrix} = \begin{bmatrix} \sqrt{25} & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{9} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Mencari Akar Ciri dan Vektor Ciri

$$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$$

A positif definit jika $\boldsymbol{x}'A\boldsymbol{x} > 0$ untuk sembarang vektor x ≠ 0

Karena vektor **x** tidak diketahui, maka untuk menunjukkan A adalah definit positif dengan cara mencari vektor cirinya terlebih dahulu.

|A – λI|=0

$$\left|\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix} 9-\lambda & -2 \\ -2 & 6-\lambda \end{bmatrix}\right| = 0$$

$$[(9-\lambda)(6-\lambda)] - [(-2)(-2)] = 0$$

$$[(54 - 9\lambda - 6\lambda + \lambda^2) - 4] = 0$$

$$50 - 15\lambda + \lambda^2 = 0$$

$$(\lambda - 10)(\lambda - 5) = 0$$

$\lambda_1 = 10$ atau $\lambda_2 = 5$

sehingga akar ciri dari matriks A adalah $\lambda_1 = 10$ atau $\lambda_2 = 5$

1. Mencari vektor ciri

- Untuk $\lambda = 10$

(A – λI)x=0

$$\left(\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} - 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{array}{cc|c} -1 & -2 & 0 \\ -2 & -4 & 0 \end{array}\right) \underset{\sim}{B_{2+1(-2)}} \left(\begin{array}{cc|c} -1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$-x_1 - 2x_2 = 0$$

$$-x_1 = 2x_2$$

$$x_1 = -2x_2$$

Misal $x_2 = s$

$$x = \begin{bmatrix} -2s \\ s \end{bmatrix}$$

Misal s=1, maka

$$x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Jadi vektor ciri dari akar ciri $\lambda_1 = 10$ adalah $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

A definit positif jika $\boldsymbol{x'Ax} > 0$

$x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ dengan $[-2 \quad 1]$

$$\boldsymbol{x'Ax} = [-2 \quad 1] \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$[-20 \quad 10] \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \mathbf{50} > 0$$

Karena $\boldsymbol{x'Ax} = \mathbf{50} > 0$ **maka** terbukti bahwa A definit positif

- Untuk $\lambda \;= 5$

(A – λI)x=0

$$\left(\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{array}{cc|c} 4 & -2 & 0 \\ -2 & 1 & 0 \end{array}\right) \underset{\sim}{B_{2+1(\frac{1}{2})}} \left(\begin{array}{cc|c} 4 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$4x_1 - 2x_2 = 0$$

$$4x_1 = 2x_2$$

$$x_1 = \frac{1}{2}x_2$$

Misal $x_1 = t$

$$x = \begin{bmatrix} t \\ 2t \end{bmatrix}$$

Misal t=1, maka

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Jadi vektor ciri dari akar ciri $\lambda_2 = 5$ adalah $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

A definit positif jika $\boldsymbol{x'Ax} > 0$

$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ dengan $[1 \quad 2]$

$$\boldsymbol{x'Ax} = [1 \quad 2] \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$[5 \quad 10] \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \mathbf{25} > 0$$

Karena $\boldsymbol{x'Ax} = \mathbf{25} > 0$ **maka** terbukti bahwa A definit positif

Latihan Soal

1. Diketahui matriks A, B, dan C berikut.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a. Tentukan akar ciri dan vektor ciri dari matriks **A**, **B**, dan **C**.

✓ Untuk matriks $\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

akar ciri akan diperoleh dengan menyelesaikan $|\boldsymbol{A} - \lambda\mathbf{I}| = 0$

$$\left|\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 2-\lambda & 0 & 0 \\ 0 & 4-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{pmatrix}\right| = 0$$

$(2-\lambda)(4-\lambda)(3-\lambda) = 0$

Maka, akar ciri untuk matriks A:

$2-\lambda = 0$ $\quad$ $4-\lambda = 0$ $\quad$ $3-\lambda = 0$

$\lambda_1 = 2$ $\quad$ $\lambda_2 = 4$ $\quad$ $\lambda_3 = 3$

Vektor ciri akan diperoleh dengan menyelesaikan persamaan $\boldsymbol{Ax} = \lambda\boldsymbol{x}$

➢ $\lambda_1 = 2$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 2\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$2x_1 = 2x_1$
$4x_2 = 2x_2$ → diperoleh $x_1 = t, t \in R$, $x_2 = 0\ dan\ x_3 = 0$ sehingga
$3x_3 = 2x_3$

$$\mathbf{x} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} t\ , \mathrm{t} \in R$$

vektor ciri untuk $\lambda_1 = 2$ adalah :

$$\boldsymbol{e} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}\cdot\boldsymbol{x}}}$$

$$\boldsymbol{e} = \frac{\begin{pmatrix}1\\0\\0\end{pmatrix}}{\sqrt{\begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}}} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

- $\lambda_2 = 4$

  $$\begin{pmatrix}2 & 0 & 0\\0 & 4 & 0\\0 & 0 & 3\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = 4\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

  $2x_1 = 4x_1$
  $4x_2 = 4x_2$ → diperoleh $x_1 = 0$, $x_2 = t$, $t \in R$ dan $x_3 = 0$ sehingga
  $3x_3 = 4x_3$

  $$\mathbf{x} = \begin{pmatrix}0\\t\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix}t \text{ , t} \in R$$

  vektor ciri untuk $\lambda_2 = 4$ adalah :

  $$\boldsymbol{e} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}\cdot\boldsymbol{x}}}$$

  $$\boldsymbol{e} = \frac{\begin{pmatrix}0\\1\\0\end{pmatrix}}{\sqrt{\begin{pmatrix}0 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}}} = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

- $\lambda_3 = 3$

  $$\begin{pmatrix}2 & 0 & 0\\0 & 4 & 0\\0 & 0 & 3\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = 3\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

  $2x_1 = 3x_1$
  $4x_2 = 3x_2$ → diperoleh $x_1 = 0$, $x_2 = 0 \; dan \; x_3 = t, t \in R$ sehingga
  $3x_3 = 3x_3$

  $$\mathbf{x} = \begin{pmatrix}0\\0\\t\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}t \text{ , t} \in R$$

  vektor ciri untuk $\lambda_3 = 3$ adalah :

  $$e = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}\cdot\boldsymbol{x}}}$$

  $$e = \frac{\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

✓ Untuk matriks $\mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Akar ciri akan diperoleh dengan menyelesaikan $|\text{?} - \lambda\mathbf{I}| = 0$

$$\left|\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 2-\lambda & 0 & 0 \\ 0 & 4-\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix}\right| = 0$$

$(2-\lambda)(4-\lambda)(-\lambda) = 0$

Maka, nilai ciri untuk matriks **B**:

$2 - \lambda = 0$ $\quad 4 - \lambda = 0$ $\quad -\lambda = 0$

$\lambda_1 = 2$ $\quad \lambda_2 = 4$ $\quad \lambda_3 = 0$

Vektor ciri akan diperoleh dengan menyelesaikan persamaan $\mathbf{B}\boldsymbol{x} = \lambda\boldsymbol{x}$

- $\lambda_1 = 2$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 2\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$2x_1 = 2x_1$
$4x_2 = 2x_2$ → diperoleh $x_1 = t, t \in R$, $x_2 = 0$ dan $x_3 = 0$ sehingga
$0 = 2x_3$

$$\mathbf{x} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} t\ , \mathrm{t} \in R$$

vector ciri untuk $\lambda_2 = 2$ adalah :

$$\boldsymbol{e} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}{\cdot}\boldsymbol{x}}}$$

$$\boldsymbol{e} = \frac{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\sqrt{(1 \quad 0 \quad 0)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- $\lambda_2 = 4$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 4\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$2x_1 = 4x_1$
$4x_2 = 4x_2$ → diperoleh $x_1 = 0$, $x_2 = t$, $t \in R$ dan $x_3 = 0$ sehingga
$0 = 4x_3$

$$\mathbf{x} = \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} t \text{ , t} \in R$$

vector ciri untuk $\lambda_2 = 4$ adalah :

$$\boldsymbol{e} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}'\boldsymbol{x}}}$$

$$\boldsymbol{e} = \frac{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- $\lambda_3 = 0$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$2x_1 = 0$
$4x_2 = 0$ → diperoleh $x_1 = 0$, $x_2 = 0\ dan\ x_3 = t, t \in R$ sehingga
$0 \quad = 0$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t \text{ , t} \in R$$

vector ciri untuk $\lambda_3 = 3$ adalah :

$$\boldsymbol{e} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}'\boldsymbol{x}}}$$

$$\boldsymbol{e} = \frac{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

✓ Untuk matriks $\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Nilai ciri akan diperoleh dengan menyelesaikan $|\boldsymbol{C} - \lambda\mathbf{I}| = 0$

$$\left|\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{pmatrix}\right| = 0$$

$(2-\lambda)(4-\lambda)(-\lambda) = 0$

Maka, nilai ciri untuk matriks A:

$1-\lambda = 0 \quad 1-\lambda = 0 \qquad 1-\lambda = 0$

$\lambda_1 = 1 \qquad \lambda_2 = 1 \qquad \lambda_3 = 1$

Vektor ciri akan diperoleh dengan menyelesaikan persamaan $\boldsymbol{Cx} = \lambda\boldsymbol{x}$

- $\lambda_1 = 1$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 1\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$x_1 = x_1$
$x_2 = x_2$ → diperoleh $x_1 = t, x_2 = t\ dan\ x_3 = t, t \in R$ sehingga
$x_3 = x_3$

$$\mathbf{x} = \begin{pmatrix} t \\ t \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} t\text{ , t} \in R$$

vector ciri untuk $\lambda_1 = \lambda_2 = \lambda_3 = 1$ adalah :

$$e = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x}'\boldsymbol{x}}}$$

$$e = \frac{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

b. Apakah matriks A, B, dan C merupakan matriks definit positif?
   - ✓ untuk matriks ***A*** : $\lambda_1.\lambda_2.\lambda_3 = 2.4.3 \ > 0$, maka matriks **A** definit positif
   - ✓ untuk matriks ***B*** : $\lambda_1.\lambda_2.\lambda_3 = 2.4.0 \ \geq 0$, maka matriks **B** semi definit positif
   - ✓ untuk matriks ***C*** : $\lambda_1.\lambda_2.\lambda_3 = 1.1.1 \ > 0$, maka matriks **C** definit positif

2. Diketahui, $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ adalah matriks simetris. Tunjukkan bahwa determinan A adalah sama dengan akar ciri pertama kali akar ciri kedua, atau $|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \lambda_1\lambda_2$

➢ Mencari akar ciri dari matriks **A**:

$$|\mathbf{A} - \boldsymbol{\lambda}\mathbf{I}| = 0$$

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$$

$$(a_{11} - \lambda)(a_{22} - \lambda) - a_{12}a_{21} = 0$$

$$a_{11}a_{22} - a_{11}\lambda - a_{22}\lambda + \lambda^2 - a_{12}a_{21} = 0$$

$$\lambda^2 - (a_{11} + a_{22})\lambda + a_{11}a_{22} - a_{12}a_{21} = 0$$

Bentuk ini analog dengan rumus abc untuk persamaan kuadrat $a\lambda^2 + b\lambda + c = 0$, *dimana* $\lambda_1\lambda_2 = \frac{c}{a}$

Sehingga didapat:

$$\lambda_1\lambda_2 = \frac{a_{11}a_{22} - a_{12}a_{21}}{1} = a_{11}a_{22} - a_{12}a_{21}$$

➢ Mencari determinan dari matriks **A**:

$$|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$\therefore$ terbukti bahwa $|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \lambda_1\lambda_2$.