TPG minggu 3

1. Ada 20 wanita dianalisis tentang kadar gula, kadar garam dan kadar Potassium dalam darah mereka. Hasil menunjukkan.

$$\bar{x} = \begin{bmatrix} 4.64 \\ 45.40 \\ 9.96 \end{bmatrix} \quad S = \begin{bmatrix} 2.88 & 10.01 & -1.81 \\ 10.01 & 199.79 & -5.64 \\ -1.81 & -5.64 & 3.63 \end{bmatrix}$$

$H_0 = \mu' = (4 \quad 50 \quad 10)$

$H_1 : \mu' \neq (4 \quad 50 \quad 10)$

$\alpha = 10\%$ $\quad F_{3,17}(\alpha = 10\%) = 2.44$.

- $$(\bar{x} - \mu_0) = \begin{bmatrix} 4.64 \\ 45.40 \\ 9.96 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 0.64 \\ -4.6 \\ -0.04 \end{bmatrix}$$

$$\boxed{S^{-1} = \frac{1}{\det(s)} C^T}$$

$$\det(s) = \left|\begin{matrix} 2.88 & 10.01 & -1.81 \\ 10.01 & 199.79 & -5.64 \\ -1.81 & -5.64 & 3.63 \end{matrix}\right| \begin{matrix} 2.88 & 10.01 \\ 10.01 & 199.79 \\ -1.81 & -5.64 \end{matrix}$$

$= (2088,685 + 102,1861 + 102,1861) - (654.532 + 363.7264 + 91,61165)$

$= (2293.057 - 11.09.87) = 1183,187$.

- $$S^{-1} = \frac{1}{1183.187} \begin{pmatrix} 693.428 & -26.1279 & 305.1635 \\ -26.1279 & 7.1783 & -1.8749 \\ 305.1635 & -1.8749 & 475.1951 \end{pmatrix}$$

$$= \begin{pmatrix} 0.586068 & -0.02208 & 0.257917 \\ -0.02208 & 0.006067 & -0.00158 \\ 0.257917 & -0.00158 & 0.401623 \end{pmatrix}$$

>> $T^2 = n(\bar{x} - \mu_0)' S^{-1} (\bar{x} - \mu_0)$

$$= 20 \, (0.64 \quad -4.6 \quad -0.04) \begin{pmatrix} 0.586 & -0.022 & 0.257 \\ -0.02 & 0.006 & -0.01 \\ 0.257 & -0.001 & 0.401 \end{pmatrix} \begin{pmatrix} 0.64 \\ -4.6 \\ -0.04 \end{pmatrix}$$

$$= (12.8 \quad -92 \quad -0.8) \begin{pmatrix} 0.46596 \\ -0.04198 \\ 0.15629 \end{pmatrix} = \underline{9.706127}$$

>> Titik kritis

$$\frac{(n-1)p}{(n-p)} F_{p,n-p}(\alpha) = \frac{(20-1)3}{(20-3)} (2.44) = \frac{57}{17}(2.44) = \underline{8.18117}$$

$\therefore$ $T^2 >$ titik kritis

$9.706127 > 8.18117 \Rightarrow$ tolak H_0

Dengan tingkat kepercayaan 90%, dapat disimpulkan bahwa minimal ada salah satu dari rata-rata kadar gula, kadar garam, dan kadar potasium yang memiliki rata-rata tidak sama dengan nilai $(4 \quad 50 \quad 10)$

2.) Diketahui :

$$X = \begin{pmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{pmatrix} \quad n = 3 \quad p = 2$$

Hipotesis

$H_0 : \mu' = (9 \quad 5)$

$H_1 : \mu' \neq (9 \quad 5)$

$$\mu = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

Jawab:

karena $p = 2$, dan $\bar{X}$ berupa ~~matrix~~, maka untuk uji nilai tengah dilakukan dengan uji statistik T^2 Hotelling. $T^2 = n(\bar{X} - \mu_0)'(S)^{-1}(\bar{X} - \mu_0) \sim C^2$; $C^2 = \frac{(n-1)p}{n-p} F_{p,(n-p)}(\alpha)$

$\bar{X}$

$$\rightarrow \bar{X} = \begin{pmatrix} \frac{6+10+8}{2} \\ \frac{9+6+3}{3} \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

$$\rightarrow T^2 = 3\left(\begin{bmatrix} 8 \\ 6 \end{bmatrix} - \begin{bmatrix} 9 \\ 5 \end{bmatrix}\right)' (S)^{-1} \left(\begin{bmatrix} 8 \\ 6 \end{bmatrix} - \begin{bmatrix} 9 \\ 5 \end{bmatrix}\right) \sim C^2$$

$S \rightarrow$

$$S_{p \times p} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x})'$$

$$S_{1\times1} = \frac{1}{3-1}\left(x_1 - \bar{x}_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(x_1 - \bar{x}_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 6 \\ 10 \\ 8 \end{bmatrix} - 8\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(\begin{bmatrix} 6 \\ 10 \\ 8 \end{bmatrix} - 8\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}[-2 \ 2 \ 0]\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

$S_{1\times1} = 4$

$$S_{1\times2} = \frac{1}{3-1}\left(x_1 - \bar{x}_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(x_2 - \bar{x}_2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 6 \\ 10 \\ 8 \end{bmatrix} - 8\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(\begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix} - 6\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}[-2 \ 2 \ 0]\begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$$

$S_{1\times2} = -3$

$$S_{2\times1} = \frac{1}{2}\left(x_2 - \bar{x}_2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(x_1 - \bar{x}_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix} - 6\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(\begin{bmatrix} 6 \\ 10 \\ 8 \end{bmatrix} - 8\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}[3 \ 0 \ -3]\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

$S_{2\times1} = -3$

$$S_{2\times2} = \frac{1}{2}\left(x_2 - \bar{x}_2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(x_2 - \bar{x}_2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix} - 6\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)'\left(\begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix} - 6\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}[3 \ 0 \ -3]\begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$$

$S_{2\times2} = 9$

$$S = \begin{pmatrix} 4 & -3 \\ -3 & 9 \end{pmatrix}$$

$S^{-1} \rightarrow$

$$S^{-1} = \left(\begin{array}{cc|cc} 4 & -3 & 1 & 0 \\ -3 & 9 & 0 & 1 \end{array}\right) \begin{array}{l} S_1 - S_2(1/-3) \\ S_2 - S_1(3/-4) \end{array}$$

$$= \left(\begin{array}{cc|cc} 3 & 0 & 1 & 1/3 \\ 0 & \frac{27}{4} & 3/4 & 1 \end{array}\right) \begin{array}{l} S_1(1/3) \\ S_2(4/27) \end{array}$$

$$= \left(\begin{array}{cc|cc} 1 & 0 & 1/3 & 1/9 \\ 0 & 1 & 1/9 & 1/7 \end{array}\right)$$

$$S^{-1} = \begin{pmatrix} 1/3 & 1/9 \\ 1/9 & 1/7 \end{pmatrix}$$

$$\rightarrow T^2 = 3(-1 \quad 1)\begin{pmatrix} 1/3 & 1/9 \\ 1/9 & 1/7 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \sim C^2$$

$$= \left(-3 \cdot \tfrac{1}{3} + 3\tfrac{1}{9} \quad -3 \cdot \tfrac{1}{9} + 3 \cdot \tfrac{1}{3}\right)\begin{bmatrix} -1 \\ 1 \end{bmatrix} \sim C^2$$

$$= \left(-2/3 \quad \tfrac{1}{9}\right)\begin{bmatrix} -1 \\ 1 \end{bmatrix} \sim C^2$$

$$= 2/3 + \tfrac{1}{9}$$

$$T^2 = 7/9 \sim C^2$$

$$\rightarrow C^2 = \frac{(3-1)2}{3-2} \cdot F_{2(3-2)}(10\%)$$

$$= 4 \cdot 49{,}5$$

$$C^2 = 198$$

$\rightarrow$ $T^2 < C^2 \rightarrow$ terima H_0

Statistik uji < titik kritis sehingga tidak cukup bukti untuk menolak H_0 pada taraf signifikansi 10%. //

Artinya dengan tingkat kepercayaan 90%, dapat disimpulkan bahwa sampel acak dari nilai rata² populasi normal bivariate sama dengan nilai (9 5)