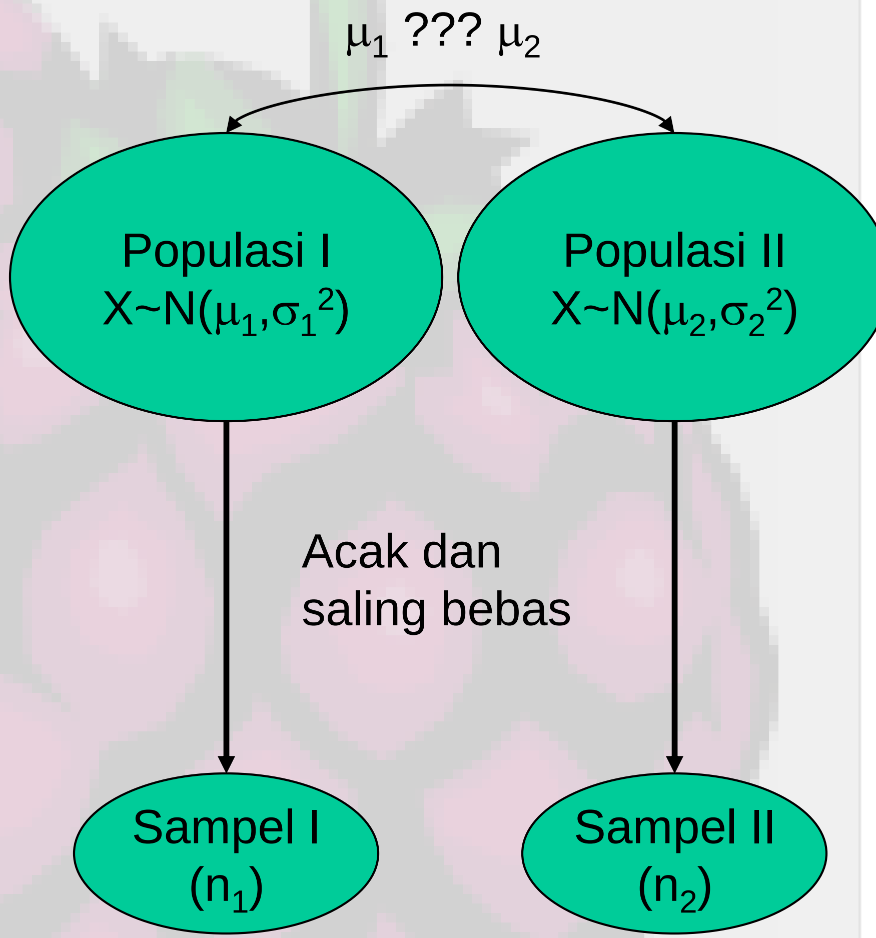
Inferensia Vektor Nilai Tengah (Lanjutan)

Perbandingan Dua Vektor Nilai Tengah

Kasus Dua Sample Saling Bebas

- Setiap populasi diambil sampel acak berukuran tertentu (bisa sama, bisa juga tidak sama)
- Pengambilan kedua sampel saling bebas
- Tujuannya adalah menguji apakah parameter μ_1 sama dengan parameter μ_2



Deskripsi masing-masing sampel

Multivariate:

Ukuran Pemusatan dan Penyebaran

Misal:

vektor peubah acak untuk sampel 1 adalah $\underline{x}_1' = (x_{11}, x_{12}, \ldots, x_{1p})$ dan vektor peubah acak sampel 2 adalah $\underline{x}_2' = (x_{21}, x_{22}, \ldots, x_{2p})$

$$\underline{\bar{\mathbf{x}}}_1 = \frac{1}{n_1} \sum_{j=1}^{n_1} \underline{\mathbf{x}}_{1j} \qquad \mathbf{S}_1 = \frac{1}{n_1 - 1} \sum_{j=1}^{n_1} \left(\mathbf{x}_{1j} - \bar{\mathbf{x}}_1\right)\left(\mathbf{x}_{1j} - \bar{\mathbf{x}}_1\right)'$$

$$\underline{\bar{\mathbf{x}}}_2 = \frac{1}{n_2} \sum_{j=1}^{n_1} \underline{\mathbf{x}}_{2j} \qquad \mathbf{S}_2 = \frac{1}{n_2 - 1} \sum_{j=1}^{n_1} \left(\mathbf{x}_{2j} - \bar{\mathbf{x}}_2\right)\left(\mathbf{x}_{2j} - \bar{\mathbf{x}}_2\right)'$$

Langkah Pengujiannya

Bentuk Hipotesis: H_0: $\underline{\mu}_1 = \underline{\mu}_2$ vs H_1: $\underline{\mu}_1 \neq \underline{\mu}_2$.

Statistik uji:

a. Ragam sama

$$T^2 = (\underline{\bar{\mathbf{x}}}_1 - \underline{\bar{\mathbf{x}}}_2)' \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) S_{gab} \right]^{-1} (\underline{\bar{\mathbf{x}}}_1 - \underline{\bar{\mathbf{x}}}_2)$$

$$S_{gab} = \frac{(n_1 - 1)S_1 + (n_2 - 1)S_2}{n_1 + n_2 - 2}$$

Daerah penolakan H0: $T^2 \geq c^2 = \dfrac{(n_1 + n_2 - 2)p}{(n_1 + n_2 - p - 1)} F_{p,n1+n2-p-1}(\alpha)$

Statistik uji:

a. Ragam tidak sama (Gunakan matriks kovarian masing-masing sample

$$T^2 = \left(\underline{\bar{\mathbf{x}}}_1 - \underline{\bar{\mathbf{x}}}_2\right)' \left[\left(\frac{S_1}{n_1} + \frac{S_2}{n_2}\right)\right]^{-1} \left(\underline{\bar{\mathbf{x}}}_1 - \underline{\bar{\mathbf{x}}}_2\right)$$

Daerah penolakan H0:

$$T^2 \geq c^2 = \chi^2_{(\alpha,p)}$$

Ilustrasi

Misal:

x_1=lebar badan kura-kura; x_2=panjang badan kura-kura

Sampel 1:

(n_1=24)

$$\underline{\bar{x}}_J = \begin{bmatrix} 102.583 \\ 52.042 \end{bmatrix} \qquad S_J = \begin{bmatrix} 171.732 & 101.844 \\ 101.844 & 64.737 \end{bmatrix}$$

Sampel 2:

(n_2=24)

$$\underline{\bar{x}}_B = \begin{bmatrix} 88.292 \\ 40.708 \end{bmatrix} \qquad S_B = \begin{bmatrix} 50.042 & 21.654 \\ 21.654 & 11.259 \end{bmatrix}$$

Hipotesis :

$$\mathrm{H}_0 : \underline{\mu}_J = \underline{\mu}_B \quad \mathrm{H}_1 : \underline{\mu}_J \neq \underline{\mu}_B$$

- Kasus ragam sama

$$T^2 = \left(\underline{\bar{x}}_1 - \underline{\bar{x}}_2\right)' \left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right) S_{gab}\right]^{-1} \left(\underline{\bar{x}}_1 - \underline{\bar{x}}_2\right)$$

$$S_{gab} = \frac{(n_1 - 1)S_1 + (n_2 - 1)S_2}{n_1 + n_2 - 2} \qquad S_{gab} = \begin{bmatrix} 110.887 & 61.749 \\ 61.749 & 37.998 \end{bmatrix}$$

$$\underline{\bar{x}}_1 - \underline{\bar{x}}_2 = \begin{bmatrix} 14.292 \\ 11.333 \end{bmatrix} \qquad S_{gab}^{-1} = \begin{bmatrix} 0.095 & -0.154 \\ -0.154 & 0.277 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 14.292 & 11.333 \end{bmatrix} \left(\frac{1}{24} + \frac{1}{24}\right) \begin{bmatrix} 0.0945 & -0.154 \\ -0.154 & 0.277 \end{bmatrix} \begin{bmatrix} 14.292 \\ 11.333 \end{bmatrix} = 4.995x\frac{24}{2}$$

- Tolak Ho, jika

$$T^2 > c^2 = \frac{(n_1 + n_2 - 2)p}{(n_1 + n_2 - p - 1)} F_{p, n_1 + n_2 - p - 1}(.01) = \frac{(46)2}{45}(2.44) = 4.988$$

- Kasus ragam tidak sama

$$T^2 = \left(\underline{\bar{x}}_1 - \underline{\bar{x}}_2\right)' \left[\frac{S_1}{n_1} + \frac{S_2}{n_2}\right]^{-1} \left(\underline{\bar{x}}_1 - \underline{\bar{x}}_2\right)$$

$$T^2 = \begin{bmatrix} 14.292 & 11.333 \end{bmatrix} \begin{bmatrix} 57.197 & 25.898 \\ 25.898 & 13.956 \end{bmatrix}^{-1} \begin{bmatrix} 14.292 \\ 11.333 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 14.292 & 11.333 \end{bmatrix} \begin{bmatrix} 0.109 & -0.203 \\ -0.203 & 0.448 \end{bmatrix} \begin{bmatrix} 14.292 \\ 11.333 \end{bmatrix} = 14.170$$

- Tolak Ho, jika

$$T^2 > \chi^2_{(0.05;2)} = 5.99$$

Kasus dua sampel berpasangan

- X_{j1} ⇨ respon untuk perlakuan 1(atau respon sebelum perlakuan) dan

  X_{j2} ⇨ respon perlakuan 2 (atau respon setelah perlakuan) untuk pengamatan ke j.

  Maka (X_{j1} , X_{j2}) adalah pasangan pengamatan ke j. Beda ke n dari pangan tersebut

  $D_j = X_{j1} - X_{j2}$, j=1,2,...,n

  Hipotesis :

$$H_0 : \delta = 0$$

$$H_1 : \delta \neq 0$$

Statistik uji

$$T^2 = n(\overline{D} - \delta)' S_d^{-1} (\overline{D} - \delta)$$

dimana,

$$\overline{D} = \frac{1}{n} \sum_{j=1}^{n} D_j$$

$$s_d^2 = \frac{1}{n-1} \sum_{j=1}^{n} (D_j - \overline{D})\ (D_j - \overline{D})$$

Ilustrasi

- Data mengenai kandungan BOD dan SS pada limbah sungai. Limbah berasal dari satu program pantuan. Setengah limbah dibawa ke Lab swasta dan setengahnya dibawa ke lab pemerintah. Apakah ada perbedaan hasil uji yang dilakukan oleh kedua lab tersebut? (diambil dari Johnson and Wichern, 1998 : 294)

Contoh ke-j	Lab swasta X1 (BOD)	Lab Swasta X2 (SS)	Lab pemerintah X1 (BOD)	Lab pemerintah X2 (SS)
1	6	27	25	15
2	6	23	28	13
3	18	64	36	22
4	8	44	35	29
. .				
11	20	14	39	21

$$H_0 : \delta' = [\delta_1 \ \ \delta_2] = [0 \ \ \ 0]$$

d_1	-19	-22	-18	-27	-4	-10	-14	17	9	4	-19
d_2	12	10	42	15	-1	11	-4	60	-2	10	7

$$\bar{d} = \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \end{bmatrix} = \begin{bmatrix} -9.36 \\ 13.27 \end{bmatrix}$$

$$s_d = \begin{bmatrix} 199.26 & 88.38 \\ 88.38 & 418.61 \end{bmatrix}$$

Perhitungan statistik uji

$$T^2 = 11\begin{bmatrix}-9.36 & 13.27\end{bmatrix}\begin{bmatrix}0.0055 & -0.0012 \\ -0.0012 & 0.0026\end{bmatrix}\begin{bmatrix}-9.36 \\ 13.27\end{bmatrix} = 13.6$$

- Tolak H_0 jika

  $T^2 > c$

  $$c = \frac{p(n-1)}{n-p} F_{p,n-p}(0.05) = \frac{2(10)}{9} F_{2,9}(0.05) = 9.47$$

  keputusan : Tolak H_0

TERIMA KASIH