

Analisis Profil

PENGERTIAN

- Analisis Profil digunakan pada saat terdapat perlakuan yang terbagi ke dalam dua atau lebih group.
- Dalam kasus repeated measurement → group = waktu
- Tujuan dari analisis profil adalah membandingkan perilaku antar perlakuan dari waktu ke waktu.
- Dalam kasus lain, grup yang dimaksud bukan hanya waktu tetapi bisa berupa tempat, atau yang lainnya
- Asumsi yang digunakan:
 - Semua respon diukur dalam unit yang sama
 - Respon dari group yang berbeda saling bebas satu sama lain

Ilustrasi

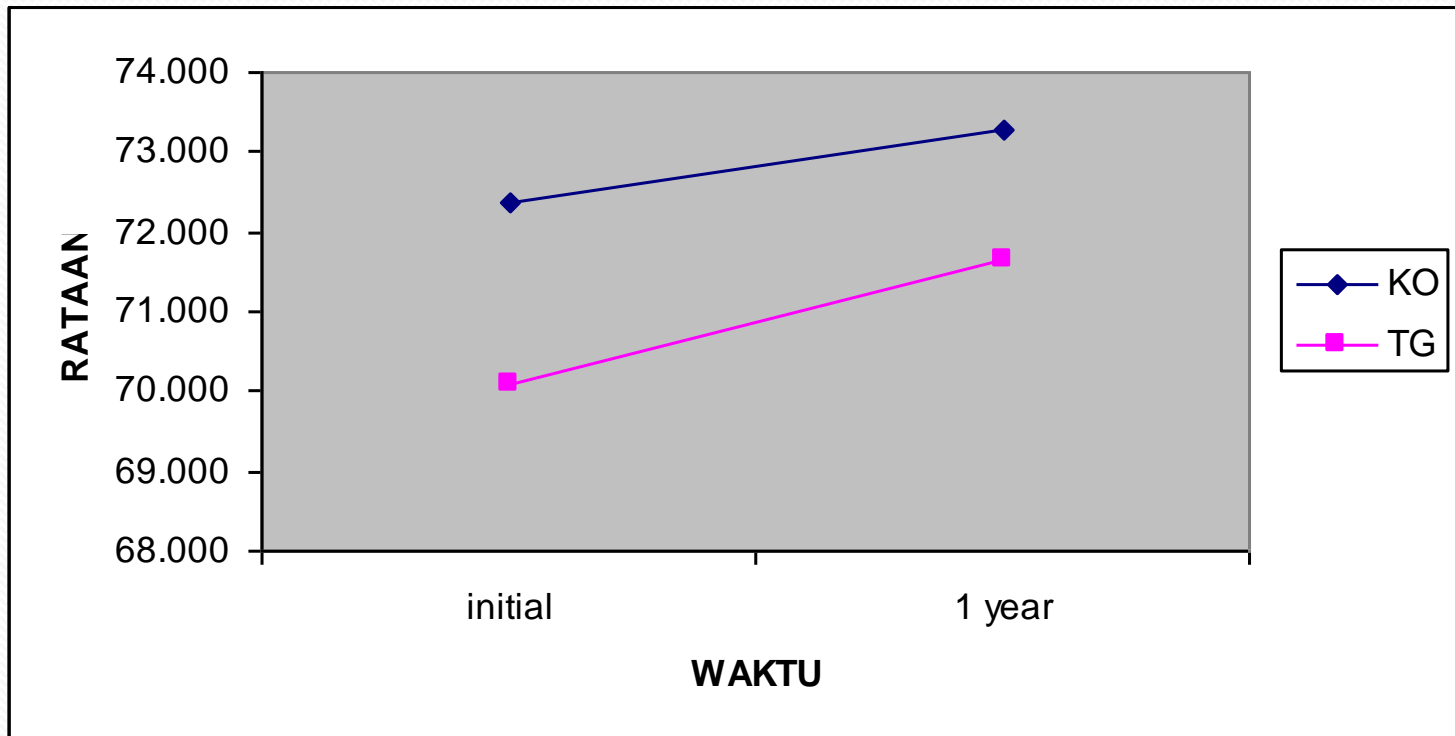
- Misal terdapat 2 perlakuan : kontrol (K_0) dan perlakuan (K_1). Masing-masing dilakukan pengukuran 4 kali (awal percobaan, 1 tahun percobaan, 2 tahun percobaan, dan 3 tahun percobaan)
- Ingin diketahui apakah rata-rata vektor antar waktu sama?

- Sebagai awal hanya akan dilihat kesamaan rata-rata vektor perlakuan $K_0 (\underline{\mu}_1)$ dibandingkan $K_1 (\underline{\mu}_2)$
- Misal $\underline{\mu}_1' = [\mu_{11}, \mu_{12}]$ dan $\underline{\mu}_2' = [\mu_{21}, \mu_{22}]$
- $H_0 : \underline{\mu}_1 = \underline{\mu}_2 \rightarrow$ perlakuan mempunyai efek yang sama (secara rata-rata) antara dua waktu tersebut

Ilustrasi Data

PERLAKUAN	INITIAL	1 YEAR	PERLAKUAN	INITIAL	1 YEAR
KO	87.30	86.90	TG	83.80	85.50
KO	59.00	60.20	TG	65.30	66.90
KO	76.70	76.50	TG	81.20	79.50
KO	70.60	76.10	TG	75.40	76.70
KO	54.90	55.10	TG	55.30	58.30
KO	78.20	75.30	TG	70.30	72.30
KO	73.70	70.80	TG	76.50	79.90
KO	61.80	68.70	TG	66.00	70.90
KO	85.30	84.40	TG	76.70	79.00
KO	82.30	86.90	TG	77.20	74.00
KO	68.60	65.40	TG	67.30	70.70
KO	67.80	69.20	TG	50.30	51.40
KO	66.20	67.00	TG	57.70	57.00
KO	81.00	82.30	TG	74.30	77.70
KO	72.30	74.60	TG	74.00	74.70
RATAAN	72.380	73.293		70.087	71.633
RAGAM	92.119	89.076		94.238	91.415
COV-KO(INITIAL,1 YEAR)	65.889				
COV-K1(INITIAL,1 YEAR)	58.115				

• $\bar{x}_1' = [72.380, 70.087]$ dan $\bar{x}_2' = [73.293, 71.633]$



Terdapat tiga hipotesis

- Apakah antar profil saling paralel?
 - $H_{01} : \mu_{12} - \mu_{11} = \mu_{22} - \mu_{21}?$
- Jika diasumsikan antar profil paralel, apakah profilnya berimpit?
 - $H_{02} : \mu_{1i} - \mu_{2i} = 0, i = 1, 2 ?$
- Jika diasumsikan saling berimpit, apakah semua rata-rata sama dengan suatu konstanta?
 - $H_{03} : \mu_{11} = \mu_{12} = \mu_{21} = \mu_{22}$

Uji Keperalelan

- H_0 dapat dituliskan:
- $H_0 : C \underline{\mu}_1 = C \underline{\mu}_2$ dimana C adalah matriks konstanta
- Statistik uji :

$$T^2 = (\bar{x}_1 - \bar{x}_2)' C' \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) C S_{pooled} C' \right]^{-1} C (\bar{x}_1 - \bar{x}_2)$$

$$S_{pooled} = \frac{n_1 - 1}{n_1 + n_2 - 2} S_1 + \frac{n_2 - 1}{n_1 + n_2 - 2} S_2$$

- Tolak H_0 jika $T^2 > c^2$, dimana $c^2 = \frac{(n_1 + n_2 - 2)(p - 1)}{n_1 + n_2 - p} F_{p-1, n_1 + n_2 - p}(\alpha)$

Back to Ilustrasi Data

- $H_{01} : \mu_{12} - \mu_{11} = \mu_{22} - \mu_{21}$

- $H_{01} : C \underline{\mu}_1 = C \underline{\mu}_2$

- $C = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad S_1 = \begin{bmatrix} 92.119 & 65.889 \\ 65.889 & 94.238 \end{bmatrix} \quad S_2 = \begin{bmatrix} 87.076 & 58.115 \\ 58.115 & 87.076 \end{bmatrix}$

$$S_{pooled} = \frac{29}{58} [S_1 + S_2] = 0.5 \begin{bmatrix} 179.195 & 124.004 \\ 124.004 & 185.653 \end{bmatrix} = \begin{bmatrix} 89.5975 & 62.0020 \\ 62.0020 & 92.8265 \end{bmatrix}$$

$$A = \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) C S_{pooled} C' \right] = \left[\left(\frac{1}{30} + \frac{1}{30} \right) [1 \ -1] \begin{bmatrix} 89.5975 & 62.002 \\ 62.002 & 92.8265 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right] = 3.89467$$

$$A^{-1} = 1/3.89467 = 0.256761$$

$$T^2 = \begin{bmatrix} -2.293 & -1.660 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} 0.256761 [1 \ -1] \begin{bmatrix} -2.293 \\ -1.660 \end{bmatrix} = 0.102881$$

$$c^2 = \frac{(30 + 30 - 2)(2 - 1)}{30 + 30 - 2} F_{1,58(0.05)} = F_{1,58(0.05)} = 4.0069$$

Kesimpulan : karena $T^2 < c^2$ maka Terima $H_0 \rightarrow$ dua garis tersebut paralel

Uji Keberhimpitan, jika diasumsikan paralel

- $H_{02} : \mathbf{1}' \underline{\mu}_1 = \mathbf{1}' \underline{\mu}_2$
- Statistik Uji :

$$T^2 = \mathbf{1}'(\bar{x}_1 - \bar{x}_2) \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) \mathbf{1}' S_{pooled} \mathbf{1} \right]^{-1} \mathbf{1}'(\bar{x}_1 - \bar{x}_2) = \left(\frac{\mathbf{1}'(\bar{x}_1 - \bar{x}_2)}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2} \right) \mathbf{1}' S_{pooled} \mathbf{1}}} \right)^2$$

- Tolak H_0 jika $T^2 > c^2$, dimana

$$c^2 = t_{n_1+n_2+2}^2 \left(\frac{\alpha}{2} \right) = F_{1, n_1+n_2-2}(\alpha)$$

Back to Ilustrasi Data

- $H_{02} : \mu_{11} + \mu_{12} = \mu_{21} + \mu_{22}$

- $H_{02} : 1' \underline{\mu}_1 = 1' \underline{\mu}_2$

- $1' = \begin{bmatrix} 1 & 1 \end{bmatrix}$ $S_1 = \begin{bmatrix} 92.119 & 65.889 \\ 65.889 & 94.238 \end{bmatrix}$ $S_2 = \begin{bmatrix} 87.076 & 58.115 \\ 58.115 & 87.076 \end{bmatrix}$

$$S_{pooled} = \frac{29}{58} [S_1 + S_2] = 0.5 \begin{bmatrix} 179.195 & 124.004 \\ 124.004 & 185.653 \end{bmatrix} = \begin{bmatrix} 89.5975 & 62.0020 \\ 62.0020 & 92.8265 \end{bmatrix}$$

$$A = \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) 1' S_{pooled} 1 \right] = \left[\left(\frac{1}{30} + \frac{1}{30} \right) \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 89.5975 & 62.002 \\ 62.002 & 92.8265 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right] = 20.4285$$

$$T^2 = \left[\frac{1'(\bar{x}_1 - \bar{x}_2)}{\sqrt{A}} \right]^2 = \left[\frac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -2.293 \\ -1.660 \end{bmatrix}}{\sqrt{20.4285}} \right]^2 = \begin{bmatrix} -0.874597 \end{bmatrix}^2 = 0.764921$$

$$c^2 = F_{1,58(0.05)} = 4.0069$$

Kesimpulan : karena $T^2 < c^2$ maka Terima $H_0 \rightarrow$ dua garis saling berhimpit

Uji Kesamaan

- $H_0 : \mu_1 = \mu_2$
- $H_0 : C \underline{\mu} = \underline{0}$
- Statistik uji :

$$T^2 = (n_1 + n_2) \bar{x}' C' [C S C']^{-1} C \bar{x}$$

- Tolak H_0 jika $T^2 > c^2$, dimana

$$c^2 = \frac{(n_1 + n_2 - 1)(p - 1)}{n_1 + n_2 - p + 1} F_{p-1, n_1 + n_2 - p}(\alpha)$$

- S = matrix variance covariance dari $n_1 + n_2$ pengamatan

Back to Ilustration Data

PERLAKUAN	INITIAL	1 YEAR	PERLAKU AN	INITIAL	1 YEAR
KO	87.3	86.9	TG	83.8	85.5
KO	59.0	60.2	TG	65.3	66.9
KO	76.7	76.5	TG	81.2	79.5
KO	70.6	76.1	TG	75.4	76.7
KO	54.9	55.1	TG	55.3	58.3
KO	78.2	75.3	TG	70.3	72.3
KO	73.7	70.8	TG	76.5	79.9
KO	61.8	68.7	TG	66.0	70.9
KO	85.3	84.4	TG	76.7	79.0
KO	82.3	86.9	TG	77.2	74.0
KO	68.6	65.4	TG	67.3	70.7
KO	67.8	69.2	TG	50.3	51.4
KO	66.2	67.0	TG	57.7	57.0
KO	81.0	82.3	TG	74.3	77.7
KO	72.3	73.2	TG	69.8	71.4
RATAAN	72.75			70.61	
RAGAM	87.58			95.95	
COV(K0,TG)	62.36				

Back to Ilustrasi Data

- $H_0 : C \underline{\mu} = \underline{0}$
- $C = [1 \ -1]$

$$S = \begin{bmatrix} 87.689 & 62.355 \\ 62.355 & 90.245 \end{bmatrix} \quad \bar{x} = \begin{bmatrix} 72.837 \\ 70.860 \end{bmatrix}$$

$$A = CSC' = [1 \ -1] \begin{bmatrix} 87.689 & 62.355 \\ 62.355 & 90.245 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 55.2240 \quad A^{-1} = 1/55.2240 = 0.0181081$$

$$T^2 = (30 + 30) \begin{bmatrix} 72.837 & 70.860 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} 0.0181081 [1 \ -1] \begin{bmatrix} 72.837 \\ 70.860 \end{bmatrix} = 4.2466$$

$$c^2 = F_{1,58(0.05)} = 4.0069$$

Kesimpulan : karena $T^2 > c^2$ maka Tolak $H_0 \rightarrow$ Rataan kedua populasi berbeda



Sekian dan Terima Kasih