UTS Bag A

No

Date

Aprilia Permata Putri

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1. Diketahui :

Vektor acak $x^t = [x_1, x_2, x_3, x_4]$ dengan $\mu_x^t = [4,3,2,1]$

$$\Sigma_x = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix} \quad \text{dan} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \hline x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x^{(1)} \\ \hline x^{(2)} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad \text{dan} \quad B = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

a.) $E(x^{(1)}) = E\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

b.) $E(Ax^{(1)}) = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = [10]$

c.) $Cov(x^{(1)}) = \Sigma_{11} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

d.) $Cov(Ax^{(1)}) = A\, cov(x^{(1)})\, A' = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = [7]$

e.) $E(x^{(2)}) = E\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \mu_3 \\ \mu_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

f.) $E(Bx^{(2)}) = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

g.) $Cov(x^{(2)}) = \Sigma_{22} = \begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}$

h.) $Cov(Bx^{(2)}) = B\, cov(x^{(2)})\, B' = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 33 & 36 \\ 36 & 48 \end{bmatrix}$

i.) $Cov(x^{(1)}, x^{(2)}) = \Sigma_{12} = \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

j.) $Cov(Ax^{(1)}, Bx^{(2)}) = A\, cov(x^{(1)}, x^{(2)})\, B'$

$= \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 6 \end{bmatrix}$

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Date

2. Misal $X \sim N_3(\mu, \Sigma)$ dg $\mu^T = [-3, 1, 4]$

$$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

a.) X_1 dan X_2

$$\rho_{12} = \frac{cov(X_1, X_2)}{\sqrt{Var(X_1)\,Var(X_2)}} = \frac{\sigma_{12}}{\sqrt{\sigma_{11} \cdot \sigma_{22}}} = \frac{-2}{\sqrt{1 \cdot 5}} = -0.894$$

$\therefore$ Karena $\rho_{12} = -0.894 \neq 0$, maka peubah acak X_1 dan X_2 tidak saling bebas (dependent)

b.) X_2 dan X_3

$$\rho_{23} = \frac{cov(X_2, X_3)}{\sqrt{Var(X_2)\,Var(X_3)}} = \frac{\sigma_{23}}{\sqrt{\sigma_{22} \cdot \sigma_{33}}} = \frac{0}{\sqrt{5 \cdot 2}} = 0$$

$\therefore$ Karena $\rho_{23} = 0$, maka p.a X_2 dan X_3 saling bebas (independent)

c.) (X_1, X_2) dan X_3

$$X = \left[\begin{array}{c} X_1 \\ X_2 \\ \hline X_3 \end{array}\right] \qquad \Sigma = \left[\begin{array}{cc|c} 1 & -2 & 0 \\ -2 & 5 & 0 \\ \hline 0 & 0 & 2 \end{array}\right] = \left[\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right]$$

$\therefore$ Karena cov dari (X_1, X_2) dan X_3 merupakan vektor nol $\Sigma_{12} = \Sigma_{21}^{T} = 0$, maka (X_1, X_2) dan X_3 saling bebas (dependent)

d.) $\frac{X_1 + X_2}{2}$ dan X_3

$$Cov\left(\tfrac{1}{2}X_1 + \tfrac{1}{2}X_2, X_3\right) = \tfrac{1}{2}Cov(X_1, X_3) + \tfrac{1}{2}cov(X_2, X_3)$$
$$= \tfrac{1}{2}(0) + \tfrac{1}{2}(0) = 0$$

$$Cov\left(X_3, \tfrac{1}{2}X_1 + \tfrac{1}{2}X_2\right) = \tfrac{1}{2}cov(X_3, X_1) + \tfrac{1}{2}cov(X_3, X_2)$$
$$= \tfrac{1}{2}(0) + \tfrac{1}{2}(0) = 0$$

$\therefore$ Karena $cov\left(\tfrac{1}{2}X_1 + \tfrac{1}{2}X_2, X_3\right) = 0$ dan $cov\left(X_3, \tfrac{1}{2}X_1 + \tfrac{1}{2}X_2\right) = 0$, maka p.a $\frac{X_1 + X_2}{2}$ dan X_3 saling bebas (dependent).