

1. Vektor acak $X^t = [X_1, X_2, X_3, X_4]$
 Vektor nilai tengah $\mu^t = [4, 3, 2, 1]$
 matriks ragam kovarian

$$\Sigma_X = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 4 \end{bmatrix} \quad \left| \begin{array}{l} A = \begin{bmatrix} 1 & -1 \end{bmatrix} \\ B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \end{array} \right.$$

$$\text{partisi } X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix}$$

a) $E(X^{(1)}) = E \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

b) $E(AX^{(1)}) = (1 \ -1) \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 1$

c) $\text{cov}(X^{(1)}) = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

d) $\text{cov}(AX^{(1)}) = (1 \ -1) \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = (3 \ -1) \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 4$

e) $E(X^{(2)}) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

f) $E(BX^{(2)}) = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

g) $\text{cov}(X^{(2)}) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

h) $\text{cov}(BX^{(2)}) = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 8 & 4 \end{bmatrix}$

i) $\text{cov}(X^{(1)}, X^{(2)}) = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

j) $\text{cov}(AX^{(1)}, BX^{(2)}) = (1 \ -1) \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} = (-1 \ 2) \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} = (4 \ 2)$

2. $X \sim N_3(\mu, \Sigma)$ dengan $\mu^t = [-3, 1, 4]$ dan $\Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 9 & 0 \\ -1 & 0 & 2 \end{bmatrix}$
 Meninjau saling bebas?

a) X_1 dan X_2

$\rightarrow \rho_{12} = \frac{\sigma_{12}}{\sqrt{\sigma_{11} \sigma_{22}}} = \frac{0}{\sqrt{4 \cdot 9}} = 0$ karena $\rho_{12} = 0$ maka X_1 & X_2 saling bebas

b) X_1 dan X_3

$\rightarrow \rho_{13} = \frac{\sigma_{13}}{\sqrt{\sigma_{11} \sigma_{33}}} = \frac{-1}{\sqrt{4 \cdot 2}} \neq 0$ karena $\rho_{13} \neq 0$ maka X_1 & X_3 tidak saling bebas

c) X_2 dan X_3

$\rightarrow \rho_{23} = \frac{\sigma_{23}}{\sqrt{\sigma_{22} \sigma_{33}}} = \frac{0}{\sqrt{9 \cdot 2}} = 0$ karena $\rho_{23} = 0$ maka X_2 dan X_3 saling bebas

d) (X_1, X_3) dan X_2

$\rightarrow (X_1, X_3) = \begin{bmatrix} X_1 \\ X_3 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 9 & 0 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix}$

$\text{cov}(X_1, X_3) \text{ dan } X_2 = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 9 & 0 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ karena sama dengan vektor 0, maka (X_1, X_3) dan X_2 saling bebas

e) X_1 dan $X_1 + 3X_2 - 2X_3$

$\text{cov}(X_1, X_1 + 3X_2 - 2X_3) = \text{cov}(X_1, X_1) + 3\text{cov}(X_1, X_2) - 2\text{cov}(X_1, X_3)$
 $= \sigma_{11} + 3\sigma_{12} - 2\sigma_{13} = 4 + 0 - 2(-1) \neq 0$

karena $\neq 0$, maka tidak saling bebas