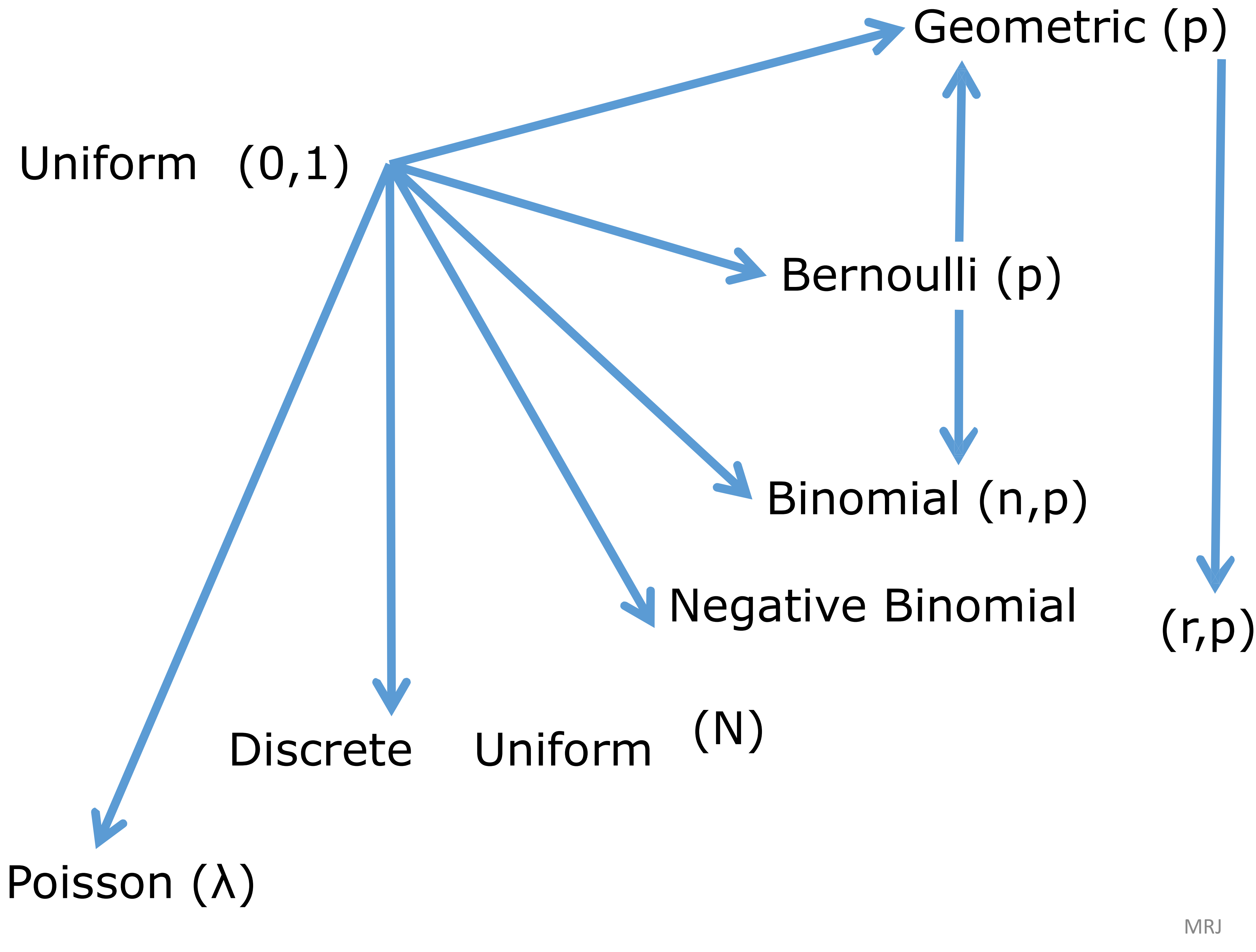
Discrete & Continous Random Number

STK473 – Praktikum 3

Discrete Random Number



Geometric (p)
Uniform (0,1)
Bernoulli (p)
Binomial (n,p)
Negative Binomial
(r,p)
Discrete Uniform (N)
Poisson (λ)
MRJ

Bernoulli (p)

Uniform (0,1)

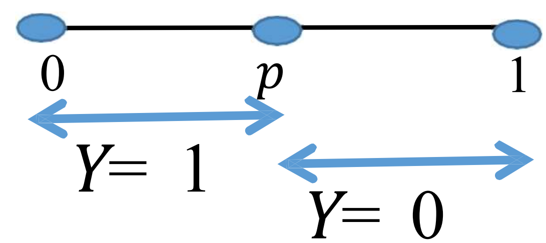
$$\Big\downarrow F_X(x)$$

Bernoulli (p)

X ~ Uniform (0,1)

$$0 \le F_X(x) \le 1$$

$$0 \le X \le 1$$



Y ~ Bernoulli (p)

Bernoulli(p)

pmf	$P(X = x \mid p) = p^x(1-p)^{1-x}; \quad x = 0, 1; \quad 0 \le p \le 1$
mean and variance	$\mathrm{E}X = p, \quad \mathrm{Var}\,X = p(1-p)$
mgf	$M_X(t) = (1-p) + pe^t$

Application in R

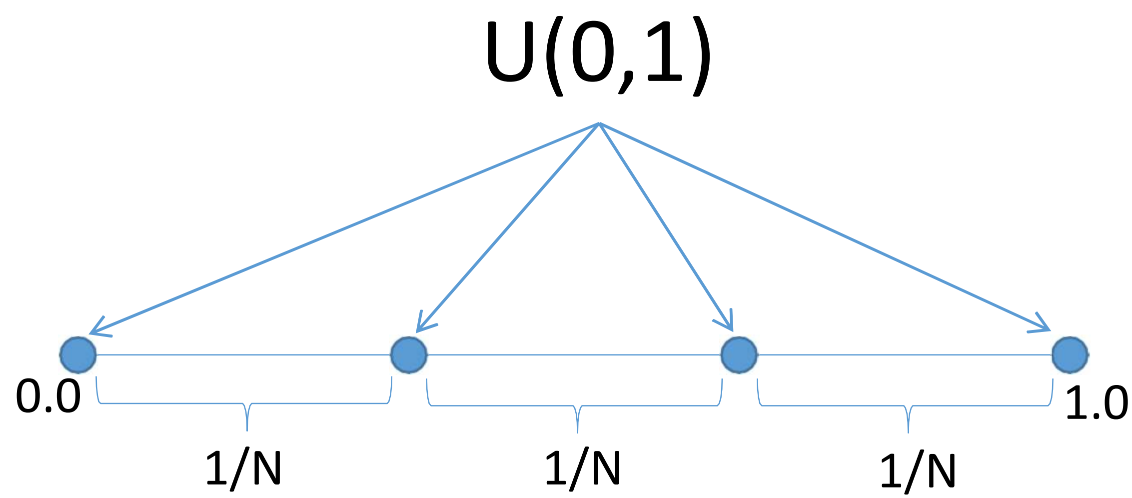
```
i<-1000
p<-.65
X<-runif(i)
Y<-NULL
for (z in 1:i) ifelse (X[z]<=p,Y[z]<-1,Y[z]<-0)
(tabel<-table(Y)/length(Y))
barplot(tabel,main="Bernoulli")

i<-1000
p<-.65
X<-runif(i)
Y<-(X<=p)+0
(tabel<-table(Y)/length(Y))
barplot(tabel,main="Bernoulli")
```

Discrete Uniform (N)

Uniform (0,1)

$\downarrow F_X(x)$

Discrete Uniform (N)



Discrete uniform

pmf	$P(X=x\|N)=\frac{1}{N};\quad x=1,2,\ldots,N;\quad N=1,2,\ldots$
mean and variance	$\mathrm{E}X=\frac{N+1}{2},\quad \mathrm{Var}\,X=\frac{(N+1)(N-1)}{12}$
mgf	$M_X(t)=\frac{1}{N}\sum_{i=1}^{N}e^{it}$

Application in R

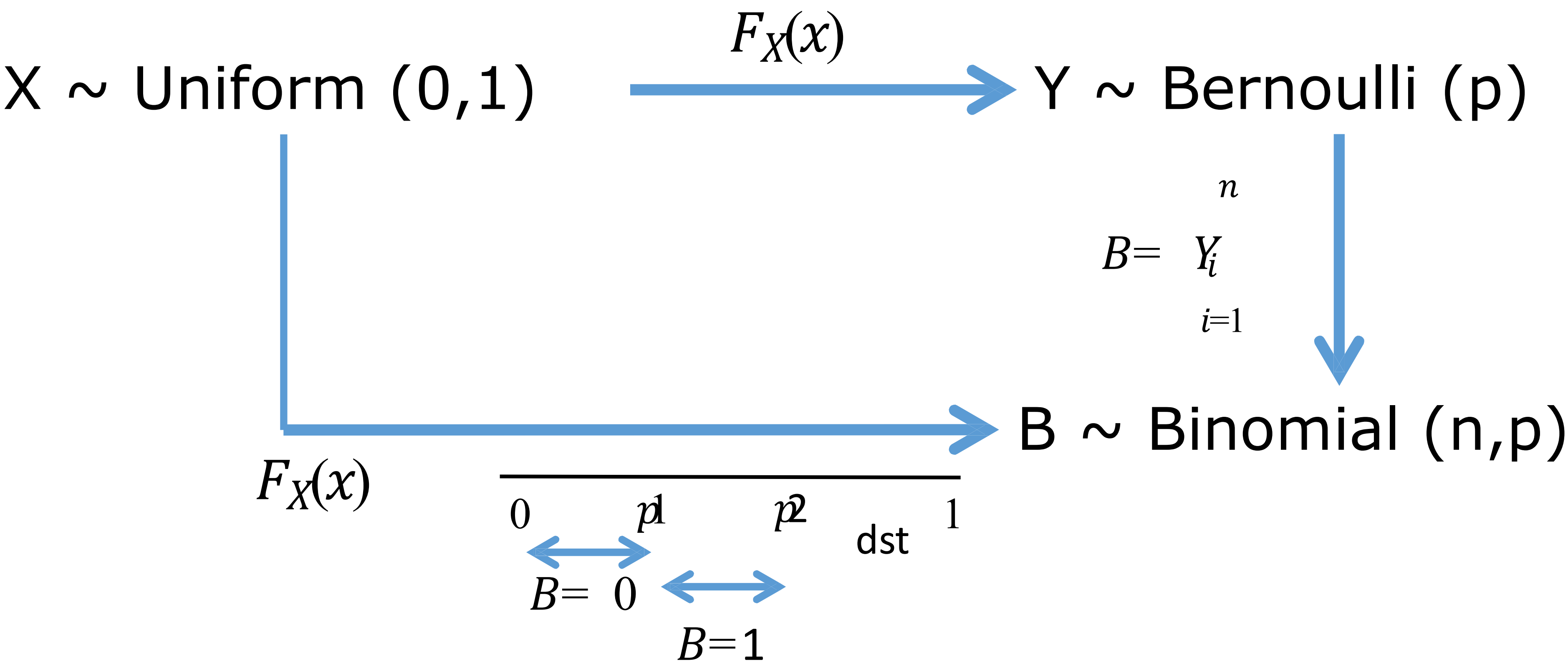
```
i<-1000
N<-4
X<-runif(i)
DU<-NULL
for (z in 1:i){
  if (X[z]<=1/N) DU[z]<-1
  else if (X[z]<=2/N) DU[z]<-2
  else if (X[z]<=3/N) DU[z]<-3
  else DU[z]<-4
}
(tabel<-table(DU)/length(DU))
barplot(tabel,main="Seragam Diskret")
```

Application in R

```
i<-1000
N<-4
X<-runif(i)
DU<-as.numeric(cut(X,breaks=c(0,1/N,2/N,3/N,1),
   include.lowest = T))
(tabel<-table(DU)/length(DU))
barplot(tabel,main="Seragam Diskret")
```

```
i<-1000
N<-4
X<-runif(i)
DU<-1+floor(N*X)
(tabel<-table(DU)/length(DU))
barplot(tabel,main="Seragam Diskret")
```

Binomial (n,p)


X ~ Uniform (0,1)
$F_X(x)$
Y ~ Bernoulli (p)
$B = \sum_{i=1}^{n} Y_i$
B ~ Binomial (n,p)
$F_X(x)$
0 p1 p2 dst 1
$B = 0$
$B = 1$

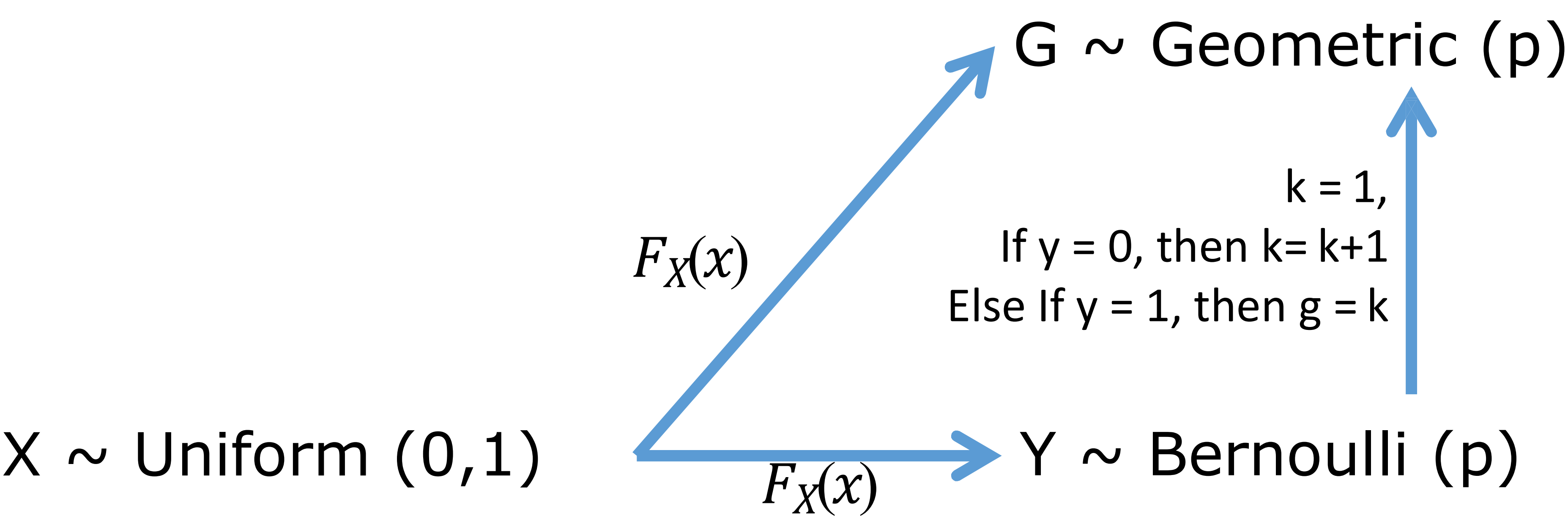
Application in R

```
#Binomial (5,0.65)
i<-1000
n<-5
p<-0.65
Binom<-NULL
for (z in 1:i){
  m<-0
  for (k in 1:n){
    y<-(runif(1)<=p)+0
    m<-m+y
  }
  Binom[z]<-m
}
(tabel<-table(Binom)/length(Binom))
barplot(tabel,main="Binomial")
```

Application in R

```
#Binomial (3,0.5)
i<-1000
X<-runif(i)
Binom<-as.numeric(cut(X,breaks=c(0,1/8,4/8,7/8,1),
   include.lowest = T))-1
(tabel<-table(Binom)/length(Binom))
barplot(tabel,main="Binomial")
```

Geometric (p)



Geometric(p)

pmf	$P(X=x\|p)=p(1-p)^{x-1};\quad x=1,2,\ldots;\quad 0\le p\le 1$
mean and variance	$\mathrm{E}X=\frac{1}{p},\quad \mathrm{Var}\,X=\frac{1-p}{p^2}$
mgf	$M_X(t)=\frac{pe^t}{1-(1-p)e^t},\quad t<-\log(1-p)$
notes	$Y=X-1$ is negative binomial$(1,p)$. The distribution is *memoryless*: $P(X>s\|X>t)=P(X>s-t)$.

Negative Binomial (r,p)

$$\begin{tikzcd}[column sep=huge, row sep=huge]
X \sim \text{Uniform }(0,1) \arrow[r, "F_X(x)"] \arrow[dr, "F_X(x)"'] & G \sim \text{Geometric }(p) \arrow[d] \\
& Y \sim \text{Neg Binomial }(r,p)
\end{tikzcd}$$

Negative binomial(r, p)

pmf $P(X = x|r, p) = \binom{r+x-1}{x} p^r (1-p)^x; \quad x = 0, 1, \ldots; \quad 0 \leq p \leq 1$

mean and variance $\mathrm{E}X = \frac{r(1-p)}{p}, \quad \mathrm{Var}\, X = \frac{r(1-p)}{p^2}$

mgf $M_X(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r, \quad t < -\log(1-p)$

notes An alternate form of the pmf is given by $P(Y = y|r, p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$, $y = r, r+1, \ldots$. The random variable $Y = X + r$. The negative binomial can be derived as a gamma mixture of Poissons. (See Exercise 4.34.)

Tugas 1

Buatlah program R untuk membangkitkan 1000 bilangan acak yang menyebar:

- **Geometrik**
- **Binomial Negatif**

Dikumpulkan paling lambat hari .....
di GCR.

Format:

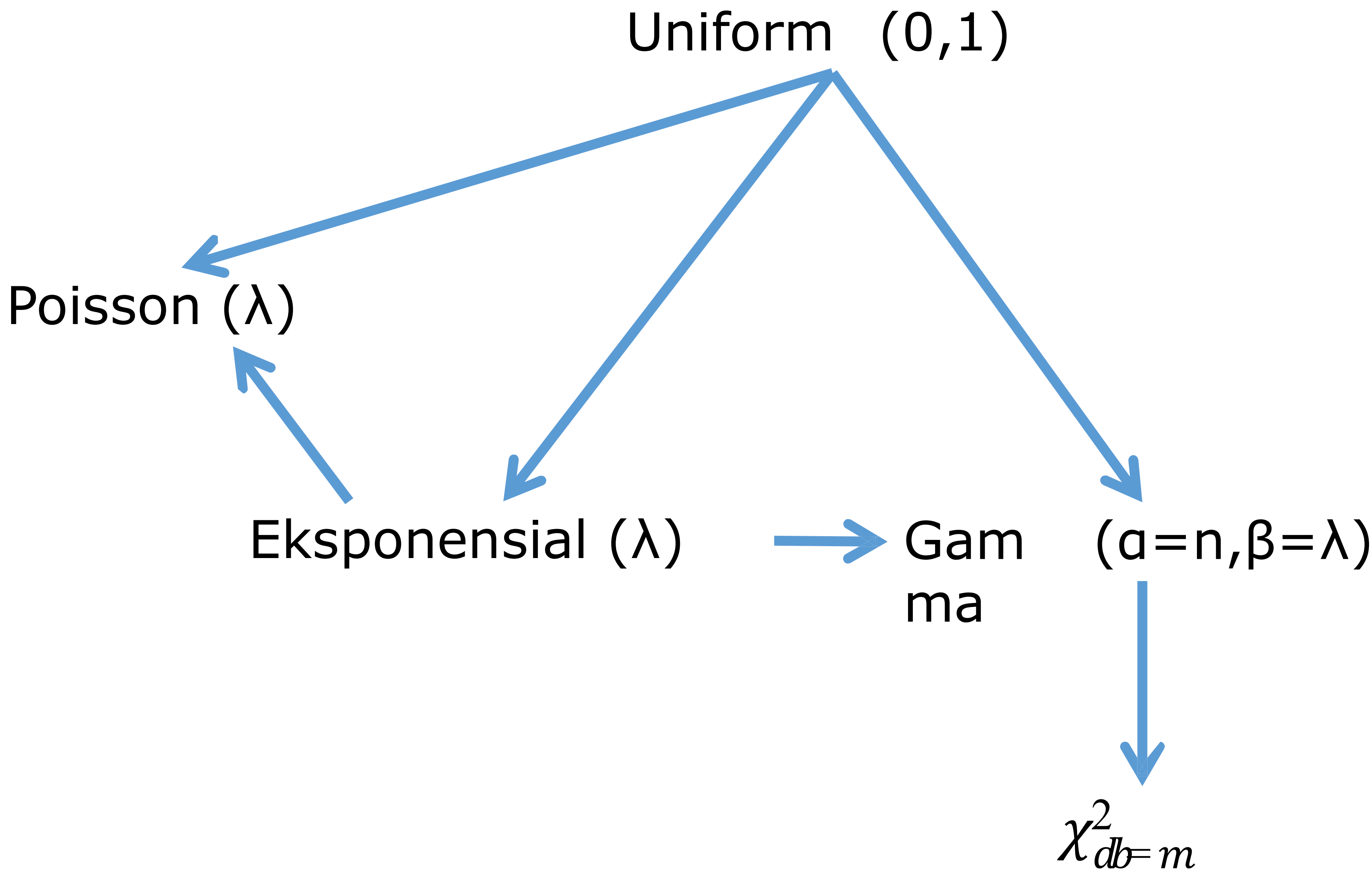
-Nama file: “Tugas 1 Kelompok [no kelompok]”

-Ekstensi file: “.r” atau “.txt”

Continous Random Numbers

Inverse Transform Method

- Metode Transformasi Kebalikan
- Dikenal juga sebagai Look-Up Table Method
- Didasari pada kenyataan bahwa
  - jika U adalah bilangan acak Seragam(0, 1)
  - dan didefinisikan $X = F^{-1}(U)$, dengan $F^{-1}(U)$ adalah fungsi kebalikan dari $F(X)$
  - maka X akan memiliki sebaran yang diinginkan
- Algoritma untuk mendapatkan bilangan acak X dengan sebaran tertentu
  - Tentukan bentuk dari fungsi sebaran kumulatif X yang diinginkan, misal $F(x)$
  - Cari fungsi kebalikan dari $F(x)$, yaitu $F^{-1}(x)$
  - Bangkitkan bilangan acak Seragam (0, 1), misal dilambangkan U
  - Hitung $X = F^{-1}(U)$



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Inverse Transform Method

Seragam (a, b)

- Ilustrasi untuk membangkitkan sebaran Seragam(a, b)

- X ~ Seragam(a, b)
  - $F(x) = (x – a) / (b – a)$
  - $U = (x – a) / (b – a)$
  - $X = a + (b – a) U$

- Algoritma:
  - Bangkitkan U, bilangan acak Seragam(0, 1)
  - Hitung $X = a + (b – a) * U$
  - Ulangi berkali-kali sesuai dengan banyaknya bilangan yang diinginkan

Inverse Transform Method

Eksponensial (λ)

- Ilustrasi untuk membangkitkan sebaran Eksponensial(λ)

- X ~ Eksponensial(λ)
  - $f(x) = \lambda e^{-\lambda x}$, untuk $x \geq 0$
  - $F(x) = 1 - e^{-\lambda x}$, untuk $x \geq 0$
  - $U = 1 - e^{-\lambda x}$, untuk $x \geq 0$
  - $X = -\ln(1 - U) / \lambda$

- Algoritma:
  - Bangkitkan U, bilangan acak Seragam(0, 1)
  - Hitung $X = -\ln(1 - U) / \lambda$
  - Ulangi berkali-kali sesuai dengan banyaknya bilangan yang diinginkan

Application in R

```
#Eksponensial(λ=3)
i<-1000
lambda<-3
U<-runif(i)
X<--log(U)/lambda
hist(X)
```

Gamma (α,β)

U~Uniform (0,1)

$$X = \frac{-\ln U}{\lambda}$$

$$G = \frac{-\sum_{i=1}^{n} \ln U_i}{\lambda}$$

X~Eksponensial (λ)

$$G = \sum_{i=1}^{n} X_i$$

G~Gamma(α=n,β=λ)

Application in R

```
#Gamma(α=5,β=3)
i<-1000
lambda<-3
alpha<-5
U<-log(runif(i*alpha))
Um<-matrix(U,i)
Y<-apply(Um,1,sum)
Gama<--Y/lambda
hist(Gama)
```

Chi-Square (m)

G~Gamma $(\alpha=n, \beta=\lambda)$

Gamma(2n/2,2)

$\chi^2_{db=m=2n}$

Gamma(v/2,2)→ $\chi^2_{db=v}$

n

Sehingga ketika v ganjil maka $n= v/2$ akan menghasilkan bilangan pecahan → tidak sesuai

Ingat:

- Jika Z~N(0,1), maka $Z^2 \sim \chi^2_{db=1}$
- Jika $X_1 \sim \chi^2_{db=m}$ dan $X_2 \sim \chi^2_{db=n}$, maka $X_3 = X_1 + X_2 \sim \chi^2_{db=(m+n)}$

Sehingga

$\chi^2_{db=v}$

v genap

$$Gamma(v/2,2)$$

v ganjil

$$Gamma\left(\frac{(v-1)}{2},2\right)+Z^2$$

$$Z^2 \sim N(0,1)$$

Application in R

```
#chi-square(10)
i<-1000
lambda<-2
alpha<-5
U<-log(runif(i*alpha))
Um<-matrix(U,i)
Y<-apply(Um,1,sum)
chi<--Y/lambda
hist(chi)
```

```
#chi-square(11)
i<-1000
lambda<-2
alpha<-5
U<-log(runif(i*alpha))
Um<-matrix(U,i)
Y<-apply(Um,1,sum)
chi<--Y/lambda
chi<-chi+(rnorm(i))^2
hist(chi)
```

Inverse Transform Method

- Kesulitan utama: memperoleh kebalikan dari fungsi sebaran kumulatif

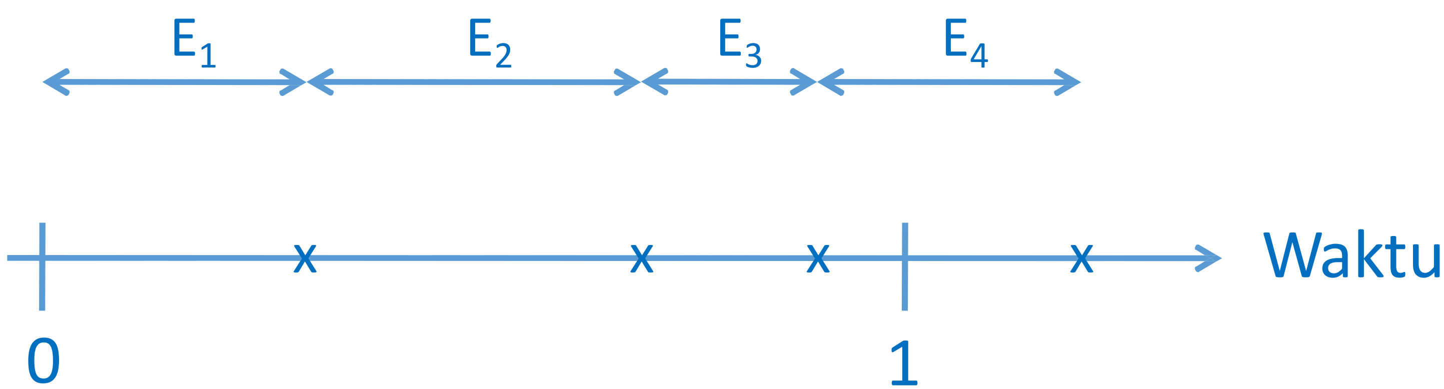
- Keunggulan: bisa digunakan untuk berbagai sebaran (termasuk sebaran diskret)

Memanfaatkan pengetahuan mengenai transformasi dan sifat sebaran peubah acak

- Y_i ~ Eksponensial(λ) $\rightarrow$ $G = \Sigma_i^n Y_i$ ~Gamma(n,λ)
- Gamma(m/2, 2) = Chi-square(m)
- Chi-square(db = 1) = Kuadrat dari Normal(0, 1)
- Jika X_1~Chi-square(m), X_2~Chi-square(n), maka $X_3=X_1+X_2$~Chi-square(m+n)
- dsb

Poisson(λ)

- Proses Poisson dengan laju sebesar λ
  - Waktu antar kejadian → saling bebas → Eksponensial (λ)
  - Banyaknya kejadian pada setiap selang waktu t menyebar Poisson(λt)



Poisson (λ)

Y~Eksponensial (λ)

$$X = \frac{-\ln U}{\lambda}$$

U~Uniform (0,1)

$$S_0 = 0$$

$$S_k = \sum_{i=1}^{k} Y_i$$

$$S_k \leq 1 \leq S_{k+1}$$

$$\prod_{i=1}^{k} U_i < e^{-\lambda}$$

K~Poisson (λ)

Application in R

```
#Poisson(1) melalui ekponensial
i<-1000
lambda<-1
K<-NULL
for (z in 1:i){
  sk<-0
  k<-0
  while (sk<=1){
    u<-runif(1)
    y<--log(u)/lambda
    sk<-y+sk
    k<-k+1
  }
  K[z]<-k-1
}
(tabel<-table(K)/length(K))
barplot(tabel)
```

Application in R

```
#Poisson(1) melalui seragam
i<-1000
lambda<-1
K<-NULL
for (z in 1:i){
  k<-0
  sk<-1
  while(sk>=exp(-lambda)){
    u<-runif(1)
    sk<-sk*u
    k<-k+1
  }
  K[z]<-k
}
(tabel<-table(K)/length(K))
barplot(tabel)
```

thank you!

MRJ