Pendahuluan

STK473 – Praktikum 1

MRJ

Warm Up

Saya punya 27 buah bola besi yang masing-masing memiliki berat 1 kg. Ternyata salah satu bola besi *fake* memiliki berat 1.1 kg. Bagaimana caranya agar saya bisa menemukan bola besi yang berbeda tersebut jika saya hanya punya kesempatan menimbang menggunakan timbangan neraca **maksimal 3 kali**?

Peubah Acak

- Bernoulli
- Binomial
- Seragam Diskret
- Poisson
- Seragam
- Normal
- Eksponensial

dan lain-lain...

Peubah Acak Diskret

Bernoulli(p)

pmf	$P(X = x \mid p) = p^x(1-p)^{1-x}; \quad x = 0, 1; \quad 0 \le p \le 1$
mean and variance	$\mathrm{E}X = p, \quad \mathrm{Var}\, X = p(1-p)$
mgf	$M_X(t) = (1-p) + pe^t$

Binomial(n, p)

pmf	$P(X = x \mid n, p) = \binom{n}{x} p^x (1-p)^{n-x}; \quad x = 0, 1, 2, \ldots, n; \quad 0 \le p \le 1$
mean and variance	$\mathrm{E}X = np, \quad \mathrm{Var}\, X = np(1-p)$
mgf	$M_X(t) = [pe^t + (1-p)]^n$

Peubah Acak Diskret

Discrete uniform

pmf	$P(X = x\|N) = \frac{1}{N}; \quad x = 1, 2, \ldots, N; \quad N = 1, 2, \ldots$
mean and variance	$\mathrm{E}X = \frac{N+1}{2}, \quad \mathrm{Var}\, X = \frac{(N+1)(N-1)}{12}$
mgf	$M_X(t) = \frac{1}{N} \sum_{i=1}^{N} e^{it}$

Poisson(λ)

pmf	$P(X = x\|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; \quad x = 0, 1, \ldots; \quad 0 \le \lambda < \infty$
mean and variance	$\mathrm{E}X = \lambda, \quad \mathrm{Var}\, X = \lambda$
mgf	$M_X(t) = e^{\lambda(e^t - 1)}$

Peubah Acak Kontinu

Uniform(a, b)

pdf	$f(x\|a,b) = \frac{1}{b-a}, \quad a \leq x \leq b$
mean and variance	$\mathrm{E}X = \frac{b+a}{2}, \quad \mathrm{Var}\, X = \frac{(b-a)^2}{12}$
mgf	$M_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$

Exponential(β)

pdf	$f(x\|\beta) = \frac{1}{\beta} e^{-x/\beta}, \quad 0 \leq x < \infty, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \beta, \quad \mathrm{Var}\, X = \beta^2$
mgf	$M_X(t) = \frac{1}{1-\beta t}, \quad t < \frac{1}{\beta}$

Peubah Acak Kontinu

$\boldsymbol{Normal}(\mu, \sigma^2)$

pdf	$f(x\|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$
mean and variance	$\mathrm{E}X = \mu, \quad \mathrm{Var}\, X = \sigma^2$
mgf	$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$

Review R

```
#membuat matriks
a1<-matrix(1:9,3,3)
a1

#mengakses isi matriks
a1[3,c(1,3)]
a1[1:2,c(1,3)]

#menggunakan looping
sum<-0
for (i in 1:10) sum<-sum+i
sum
```

Review R

```
#membangkitkan bilangan yang menyebar seragam
seragam<-runif(1000,3,14)
hist(seragam)
plot(density(seragam))
plot.ecdf(seragam)

#membangkitkan bilangan yang menyebar binomial
binomial<-rbinom(1000,20,0.3)
barplot(table(binomial))
plot.ecdf(binomial)
```

Contoh Soal

Misalkan terdapat 12 nasabah asuransi di suatu tempat. Diketahui bahwa proporsi nasabah telat bayar polis ialah 1/6. Jika antar nasabah saling bebas, tentukanlah peluang bahwa terdapat 7 sampai 9 nasabah yang telat bayar polis!

- X = banyaknya nasabah asuransi yang telat membayar
- $X \sim \text{binomial}(n = 12, p = 1/6)$
- $P(7 \leq X \leq 9)$

```
> pbinom(9,size=12,p=1/6) - pbinom(6,size=12,p=1/6)
[1] 0.001291758
> dbinom(7,12,1/6) + dbinom(8,12,1/6) + dbinom(9,12,1/6)
[1] 0.001291758
```

Mengambil Contoh Acak

```
seragam1<-sample(seragam,100,replace=T)
hist(seragam1)

binomial1<-sample(binomial,100,replace=T)
barplot(table(binomial1))
```

Uniform Random Number

STK473 – Praktikum 2

Uniform Random Variable

PDF random variables `X ~ Uniform (a,b)`

$$f(x) = \begin{cases} \dfrac{1}{b-a} & , a \le x \le b \\ 0 & , x \; lainnya \end{cases}$$

We need to generate n random number $x_1, x_2, \dots, x_n$ that uniformly distributed.

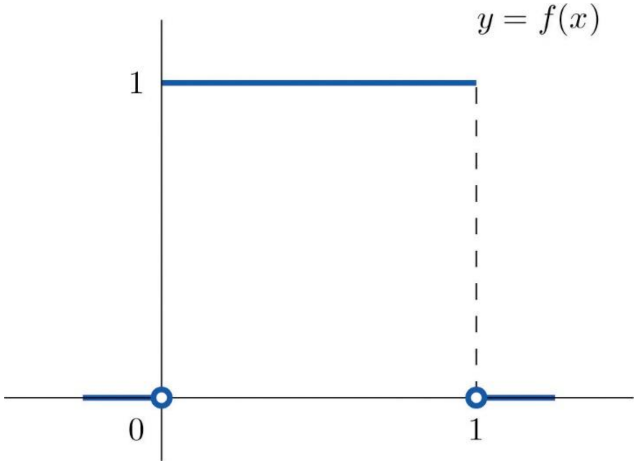
Why Uniform First?

1. The simplest of continuous distribution that described distribution of some number intervals
2. From Uniform, can be transformed to be others complicated distributions

So, how we generate random numbers that uniformly distributed ?

Start from the simplest Uniform distribution:

X ~ Uniform (0,1)



Congruential Generator

X ~ Uniform (0,1)

So we have intervals of x_i, $0 \leq x_i \leq 1$

Congruential Generator :

$$X_{n+1} = aX_n + b \ (\mathrm{mod}\ m), n \geq 0$$

$$U_i = \frac{X_i}{m} \sim U(0,1)$$

Congruential Generator (2)

Maximum cycle

- b & m don't have same factors
- (a-1) (mod prime factor of m) = 0
- (a-1) (mod 4) = 0, if m (mod 4) = 0

Independence of Observations

$$cov(X_i, X_j) \approx 0$$

$$\rho = \left[\frac{1}{a} - \frac{6b}{am}\left(1 - \frac{b}{m}\right)\right] \pm \frac{a}{m}$$

So we got :

$$\boxed{m = 2^k, k \geq 2}$$

$$\boxed{a = 4c + 1}$$

$$\boxed{b > 0, \text{odd numbers}}$$

$$X_{n+1} = aX_n + b \pmod{m}, n \geq 0$$

$$U_i = \frac{X_i}{m} \sim U(0,1)$$

$a = 1598$

$b = 17$

$m = 1000$

$X_0 = 78$

i	$aX_{i-1} + b$	X_i	U_i
0		78	
1	124661	661	0.661
2	1056295	295	0.295
3	471427	427	0.427
4	682363	363	0.363
5	580091	91	0.091
6	145435	435	0.435
7	695147	147	0.147
8	234923	923	0.923
9	1474971	971	0.971
10	1551675	675	0.675
11	1078667	667	0.667
12	1065883	883	0.883
13	1411051	51	0.051

Application in R

```
x0<-78
n<-250
xi<-matrix(NA,n,3)
colnames(xi)<-c("aX(i-1)+b","Xi","Ui")
for (i in 1:n)
{
  xi[i,1]<-(1598*x0+17)
  xi[i,2]<-xi[i,1]%%1000
  xi[i,3]<-xi[i,2]/1000
  x0<-xi[i,2]
}
hist(xi[,3])
```

Other Generator Function

$$\boxed{U_{n+1} = (\pi + U_n)^5 (\mathrm{mod}\ 1), n \geq 0}$$

```
n<-1000
x1<-0.9
for (i in 2:n) x1[i]<-(pi+x1[i-1])^5%%1
hist(x1)
```

```
> cbind("i"=1:20, "Xi"=xi[1:20,2], "i"=101:120,
  "Xi"=xi[101:120,2], "i"=201:220,
  "Xi"=xi[201:220,2])

         i  Xi   i  Xi   i  Xi
 [1,]    1 661 101 411 201 411
 [2,]    2 295 102 795 202 795
 [3,]    3 427 103 427 203 427
 [4,]    4 363 104 363 204 363
 [5,]    5  91 105  91 205  91
 [6,]    6 435 106 435 206 435
 [7,]    7 147 107 147 207 147
 [8,]    8 923 108 923 208 923
 [9,]    9 971 109 971 209 971
[10,]   10 675 110 675 210 675
[11,]   11 667 111 667 211 667
[12,]   12 883 112 883 212 883
[13,]   13  51 113  51 213  51
[14,]   14 515 114 515 214 515
[15,]   15 987 115 987 215 987
[16,]   16 243 116 243 216 243
[17,]   17 331 117 331 217 331
[18,]   18 955 118 955 218 955
[19,]   19 107 119 107 219 107
[20,]   20   3 120   3 220   3
```

Repeated Series

If we have $X \sim U(0,1)$, how to generate $Y \sim U(a,b)$?

$$\texttt{X ~ Uniform (0,1)} \xrightarrow{?} \texttt{Y ~ Uniform (a,b)}$$

X ~ Uniform (0,1)

$$f(x) = \begin{cases} 1, 0 \le x \le 0 \\ 0 \ , x \ lainnya \end{cases}$$

Y ~ Uniform (a,b)

$$f(x) = \begin{cases} \dfrac{1}{b-a}, a \le x \le b \\ 0 \ , x \ lainnya \end{cases}$$

Y = (b-a)X + a

Application in R

Y ~ Uniform (3,20)

```
Y<-(20-3)*x1+3
hist(Y)
```

thank you!