STA1373-Optimisasi Statistika: Pencarian Akar Persamaan/ Penyelesaian SPL

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Contents

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Vector and Matrix

```
u <- seq(1,5)
v <- seq(6,10)
u
```

```
## [1] 1 2 3 4 5
```

```
v
```

```
## [1]  6  7  8  9 10
```

##penjumlahan vektor

```
u + v
```

```
## [1]  7  9 11 13 15
```

##pengurangan vektor

```
u - v
```

```
## [1] -5 -5 -5 -5 -5
```

Bagaimana jika panjang kedua vektor berbeda?

```
x <- seq(1,2)
x
```

```
## [1] 1 2
```

```r
u + x
```

```
## Warning in u + x: longer object length is not a multiple of shorter object
## length
```

```
## [1] 2 4 4 6 6
```

Berdasarkan contoh tersebut, R akan mengeluarkan peringatan yang menunjukkan operasi dilakukan pada vektor dengan panjang berbeda. R akan tetap melakukan perhitungan dengan menjumlahkan kembali vektor u yang belum dijumlahkan dengan vektor x sampai seluruh elemen vektor u dilakukan operasi penjumlahan Menghitung Inner Product dan Panjang Vektor

##Inner product

```r
u%*%v
```

```
##      [,1]
## [1,]  130
```

##Panjang vektor

```r
sqrt(sum(u*u))
```

```
## [1] 7.416198
```

Operasi Matrix

```r
A <- matrix(1:9,3)
B <- matrix(10:18,3)
C <- matrix(1:6,3)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    7
## [2,]    2    5    8
## [3,]    3    6    9
```

```r
B
```

```
##      [,1] [,2] [,3]
## [1,]   10   13   16
## [2,]   11   14   17
## [3,]   12   15   18
```

```r
C
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    2    5
## [3,]    3    6
```

##Penjumlahan A + B

```r
A+B
```

```
##      [,1] [,2] [,3]
## [1,]   11   17   23
## [2,]   13   19   25
## [3,]   15   21   27
```

```r
#A+C
```

##Perkalian

```
A%*%B
```

```
##      [,1] [,2] [,3]
## [1,]  138  174  210
## [2,]  171  216  261
## [3,]  204  258  312
```

Operasi Baris Elementer

##Row Scaling

```
scale_row <- function(m, row, k){
  m[row, ] <- m[row, ]*k
  return(m)
}

(A <- matrix(1:15, nrow=5))
```

```
##      [,1] [,2] [,3]
## [1,]    1    6   11
## [2,]    2    7   12
## [3,]    3    8   13
## [4,]    4    9   14
## [5,]    5   10   15
```

lakukan scaling pada row 2 dengan nilai 10

```
scale_row(m=A, row=2, 10)
```

```
##      [,1] [,2] [,3]
## [1,]    1    6   11
## [2,]   20   70  120
## [3,]    3    8   13
## [4,]    4    9   14
## [5,]    5   10   15
```

##Row Swapping

```
swap_row <- function(m, row1, row2){
  row_tmp <- m[row1, ]
  m[row1, ] <- m[row2, ]
  m[row2, ] <- row_tmp
  return(m)
}
```

Lakukan swapping baris 2 dengan baris 5

```
swap_row(m=A, row1 = 2, row2 = 5)
```

```
##      [,1] [,2] [,3]
## [1,]    1    6   11
## [2,]    5   10   15
## [3,]    3    8   13
## [4,]    4    9   14
## [5,]    2    7   12
```

##Row replacement

```
replace_row <- function(m, row1, row2, k){
  m[row2, ] <- m[row2, ] + m[row1, ]*k
  return(m)
}
```

Lakukan replacement

```
replace_row(m=A, row1=1, row2=3, k=-3)
```

```
##      [,1] [,2] [,3]
## [1,]    1    6   11
## [2,]    2    7   12
## [3,]    0  -10  -20
## [4,]    4    9   14
## [5,]    5   10   15
```

Eliminasi Gauss

##Row Echelon Form

```
ref_matrix <- function(a){
  m <- nrow(a)
  n <- ncol(a)
  piv <- 1

  # cek elemen diagonal apakah bernilai nol
  for(row_curr in 1:m){
    if(piv <= n){
      i <- row_curr
      while(a[i, piv] == 0 && i < m){
        i <- i+1
        if(i > m){
          i <- row_curr
          piv <- piv+1
          if(piv > n)
            return(a)
        }
      }

      # jika diagonal bernilai nol, lakukan row swapping
      if(i != row_curr)
        a <- swap_row(a, i, row_curr)

      # proses triangulasi untuk membentuk matriks segitiga atas
      for(j in row_curr:m)
        if(j != row_curr){
          c <- a[j, piv]/a[row_curr, piv]
          a <- replace_row(a, row_curr, j, -c)
        }
      piv <- piv+1
    }
  }
  return(a)
}
```

Diasumsikan terdapat sebuat matrix

```
am <- c(1,1,2,
        1,2,1,
        1,-1,2,
        6,2,10)
(m <- matrix(am, nrow=3))
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    6
## [2,]    1    2   -1    2
## [3,]    2    1    2   10
```

Carilah solusi dari persamaan matrix di atas

```
ref_matrix(m)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    6
## [2,]    0    1   -2   -4
## [3,]    0    0   -2   -6
```

Eliminasi Gauss - Jordan

create a matrix

```
A <- matrix(c(-3,2,-1,6,-6,7,3,-4,4),byrow = T,nrow=3,ncol=3)
A
```

```
##      [,1] [,2] [,3]
## [1,]   -3    2   -1
## [2,]    6   -6    7
## [3,]    3   -4    4
```

```
b <- matrix(c(-1,-7,-6),nrow=3,ncol=1)
b
```

```
##      [,1]
## [1,]   -1
## [2,]   -7
## [3,]   -6
```

dimension of matrix A

```
nrow <- nrow(A)
nrow
```

```
## [1] 3
```

concatenante matrix A and vector b to create Augmented Matrix Ugmt.mtx

```
Ugmt.mtx <- cbind(A,b)
Ugmt.mtx
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -3    2   -1   -1
## [2,]    6   -6    7   -7
## [3,]    3   -4    4   -6
```

```
Ugmt.mtx[1,] <- Ugmt.mtx[1,]/Ugmt.mtx[1,1]
Ugmt.mtx
```

```
##      [,1]       [,2]      [,3]       [,4]
## [1,]    1 -0.6666667 0.3333333  0.3333333
## [2,]    6 -6.0000000 7.0000000 -7.0000000
## [3,]    3 -4.0000000 4.0000000 -6.0000000
```

ILUSTRASI:

```
Ugmt.mtx[2, ] <- Ugmt.mtx[2, ] - Ugmt.mtx[2-1, ] * Ugmt.mtx[2, 2-1] #pembuat nol element matrix
Ugmt.mtx
```

```
##      [,1]       [,2]      [,3]       [,4]
## [1,]    1 -0.6666667 0.3333333  0.3333333
## [2,]    0 -2.0000000 5.0000000 -9.0000000
## [3,]    3 -4.0000000 4.0000000 -6.0000000
```

```
Ugmt.mtx[2,] <- Ugmt.mtx[2,]/Ugmt.mtx[2,2] #pembuat =1 diagonal matrix
Ugmt.mtx
```

```
##      [,1]       [,2]       [,3]       [,4]
## [1,]    1 -0.6666667  0.3333333  0.3333333
## [2,]    0  1.0000000 -2.5000000  4.5000000
## [3,]    3 -4.0000000  4.0000000 -6.0000000
```

dst, dalam bentuk loop:

```
A <- matrix(c(-3,2,-1,6,-6,7,3,-4,4),byrow = T,nrow=3,ncol=3)
A
```

```
##      [,1] [,2] [,3]
## [1,]   -3    2   -1
## [2,]    6   -6    7
## [3,]    3   -4    4
```

```
b <- matrix(c(-1,-7,-6),nrow=3,ncol=1)
b
```

```
##      [,1]
## [1,]   -1
## [2,]   -7
## [3,]   -6
```

```
nrow <- nrow(A)
Ugmt.mtx <- cbind(A,b)
Ugmt.mtx
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -3    2   -1   -1
## [2,]    6   -6    7   -7
## [3,]    3   -4    4   -6
```

```
Ugmt.mtx[1,] <- Ugmt.mtx[1,]/Ugmt.mtx[1,1]
Ugmt.mtx
```

```
##      [,1]       [,2]      [,3]       [,4]
## [1,]    1 -0.6666667 0.3333333  0.3333333
## [2,]    6 -6.0000000 7.0000000 -7.0000000
## [3,]    3 -4.0000000 4.0000000 -6.0000000
```

```
for (i in 2:nrow){ # loop over rows
  for (j in i:nrow) { # loop over columns
```

```
    Ugmt.mtx[j, ] <- Ugmt.mtx[j, ] - Ugmt.mtx[i-1, ] * Ugmt.mtx[j, i-1] # replace the row values at jth
  }
  Ugmt.mtx[i,] <- Ugmt.mtx[i,]/Ugmt.mtx[i,i]
}
# print output
Ugmt.mtx #Back Susbstitution needed
```

```
##      [,1]       [,2]       [,3]       [,4]
## [1,]    1 -0.6666667  0.3333333  0.3333333
## [2,]    0  1.0000000 -2.5000000  4.5000000
## [3,]    0  0.0000000  1.0000000 -1.0000000
```

in case we want to do it. and want to produce the solution instantly: ILUSTRASI:

```
A <- matrix(c(-3,2,-1,6,-6,7,3,-4,4),byrow = T,nrow=3,ncol=3)
A
```

```
##      [,1] [,2] [,3]
## [1,]   -3    2   -1
## [2,]    6   -6    7
## [3,]    3   -4    4
```

```
b <- matrix(c(-1,-7,-6),nrow=3,ncol=1)
b
```

```
##      [,1]
## [1,]   -1
## [2,]   -7
## [3,]   -6
```

```
# dimension of matrix A
nrow <- nrow(A)
nrow
```

```
## [1] 3
```

```
# concatenante matrix A and vector b
Ugmt.mtx <- cbind(A,b)
Ugmt.mtx
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -3    2   -1   -1
## [2,]    6   -6    7   -7
## [3,]    3   -4    4   -6
```

```
Ugmt.mtx[1,] <- Ugmt.mtx[1,]/Ugmt.mtx[1,1]
for (i in 2:nrow){ # loop over rows
  for (j in i:nrow) { # loop over columns
    Ugmt.mtx[j, ] <- Ugmt.mtx[j, ] - Ugmt.mtx[i-1, ] * Ugmt.mtx[j, i-1] # replace the row values at jth
  }
  Ugmt.mtx[i,] <- Ugmt.mtx[i,]/Ugmt.mtx[i,i]
}

Ugmt.mtx[1, ] <- Ugmt.mtx[1, ] - Ugmt.mtx[2, ] * Ugmt.mtx[1, 2]
Ugmt.mtx
```

```
##      [,1] [,2]      [,3]      [,4]
## [1,]    1    0 -1.333333  3.333333
## [2,]    0    1 -2.500000  4.500000
```

```
## [3,]    0    0  1.000000 -1.000000
```

Dengan menggunakan loop:

```
A <- matrix(c(-3,2,-1,6,-6,7,3,-4,4),byrow = T,nrow=3,ncol=3)
A
```

```
##      [,1] [,2] [,3]
## [1,]   -3    2   -1
## [2,]    6   -6    7
## [3,]    3   -4    4
```

```
b <- matrix(c(-1,-7,-6),nrow=3,ncol=1)
b
```

```
##      [,1]
## [1,]   -1
## [2,]   -7
## [3,]   -6
```

```
# dimension of matrix A
nrow <- nrow(A)
nrow
```

```
## [1] 3
```

```
# concatenante matrix A and vector b
Ugmt.mtx <- cbind(A,b)
Ugmt.mtx
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -3    2   -1   -1
## [2,]    6   -6    7   -7
## [3,]    3   -4    4   -6
```

```
Ugmt.mtx[1,] <- Ugmt.mtx[1,]/Ugmt.mtx[1,1]
for (i in 2:nrow){
  for (j in i:nrow) {
    Ugmt.mtx[j, ] <- Ugmt.mtx[j, ] - Ugmt.mtx[i-1, ] * Ugmt.mtx[j, i-1]
  }
  Ugmt.mtx[i,] <- Ugmt.mtx[i,]/Ugmt.mtx[i,i]
}
for (i in j:2){
  for (j in i:2-1) {
    Ugmt.mtx[j, ] <- Ugmt.mtx[j, ] - Ugmt.mtx[i, ] * Ugmt.mtx[j, i]
  }
}
Ugmt.mtx
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    2
## [2,]    0    1    0    2
## [3,]    0    0    1   -1
```

#Metode Iterasi

Jacobi iteration

```
jacobi <- function(A, b, x0, tol=1e-6, maxiter=1000) {
  n <- length(b)
  x <- x0
  for (iter in 1:maxiter) {
    x_new <- numeric(n)
    for (i in 1:n) {
      s <- 0
      for (j in 1:n) {
        if (j != i) {
          s <- s + A[i,j] * x[j]
        }
      }
      x_new[i] <- (b[i] - s) / A[i,i]
    }
    if (all(abs(x - x_new) < tol)) {
      break
    }
    x <- x_new
  }
  return(list(x=x, iter=iter))
}
```

Contoh penggunaan:

```
A <- matrix(c(3,1,-1,4,7,-3,2,-2,5), nrow=3, byrow=TRUE)
b <- c(5,20,10)
x0 <- c(0,0,0)
result <- jacobi(A, b, x0)
x <- result$x
iter <- result$iter
cat("The solution is x =", x, " after ", iter, " iterations\n")
```

```
## The solution is x = 1.506025 3.132531 2.650602  after  22  iterations
```

Gauss Seidel iteration

```
gaussSeidel <- function(A, b, epsilon, maxIterations) {
  # Extract the size of the system
  n <- length(b)

  # Initialize the solution vector x
  x <- numeric(n)

  # Iterate until either epsilon is reached or maxIterations is reached
  for (iteration in 1:maxIterations) {
    x_old <- x

    # Update each variable using the Gauss-Seidel formula
    for (i in 1:n) {
      x[i] <- (b[i] - sum(A[i, -i] * x[-i])) / A[i, i]
    }

    # Check if epsilon is reached
```

```
    if (max(abs(x - x_old)) < epsilon) {
      break
    }
  }

  # Return the solution and the number of iterations
  list(x = x, iterations = iteration)
}
```

Contoh aplikasi penggunaan:

```
#A <- matrix(c(4, 1, 1, 4, 1, 1, 1, 1, 4), nrow = 3)
#b <- c(6, 6, 6)
result <- gaussSeidel(A, b, 1e-6, 100)
x <- result$x
iterations <- result$iterations
cat("Number of iterations:", iterations, "\n")
```

```
## Number of iterations: 15
```

```
cat("Solution:", x, "\n")
```

```
## Solution: 1.506024 3.13253 2.650602
```

STUDI KASUS

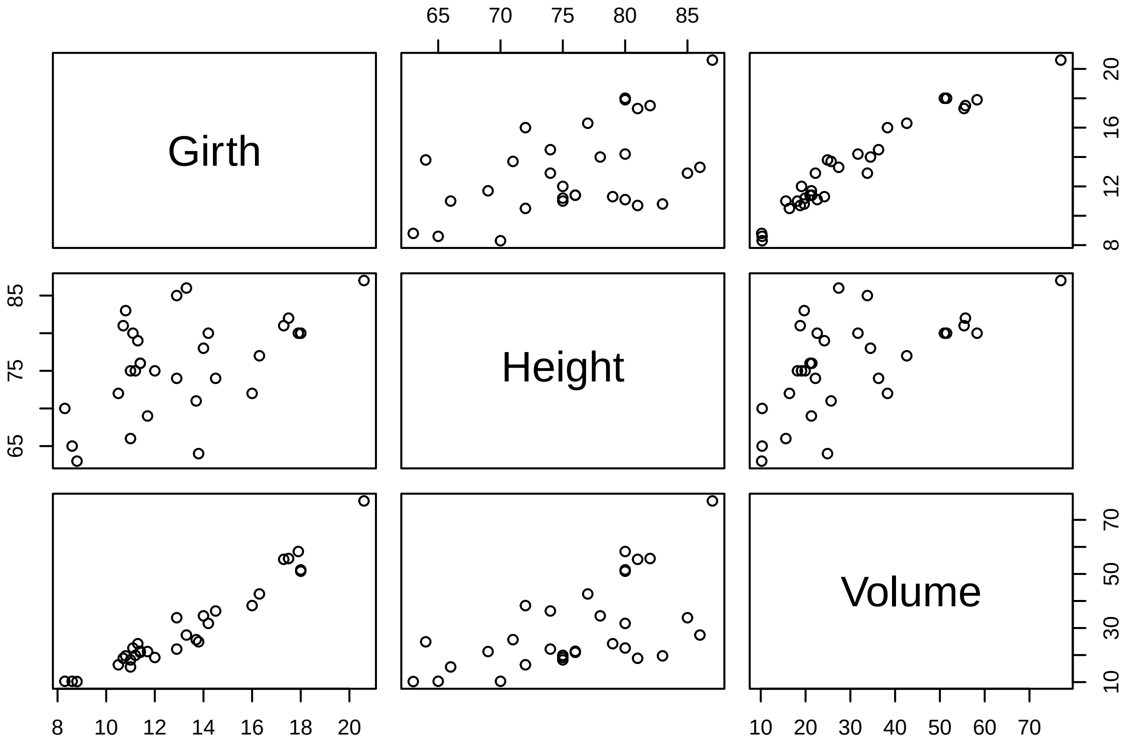
```
head(trees)
```

```
##   Girth Height Volume
## 1   8.3     70   10.3
## 2   8.6     65   10.3
## 3   8.8     63   10.2
## 4  10.5     72   16.4
## 5  10.7     81   18.8
## 6  10.8     83   19.7
```

```
str(trees)
```

```
## 'data.frame':    31 obs. of  3 variables:
##  $ Girth : num  8.3 8.6 8.8 10.5 10.7 10.8 11 11 11.1 11.2 ...
##  $ Height: num  70 65 63 72 81 83 66 75 80 75 ...
##  $ Volume: num  10.3 10.3 10.2 16.4 18.8 19.7 15.6 18.2 22.6 19.9 ...
```

```
plot(trees)
```



bentuk dalam sebuah matrix

```
pred <- cbind(intercept=1, Girth=trees$Girth, Height=trees$Height)
head(pred)
```

```
##      intercept Girth Height
## [1,]         1   8.3     70
## [2,]         1   8.6     65
## [3,]         1   8.8     63
## [4,]         1  10.5     72
## [5,]         1  10.7     81
## [6,]         1  10.8     83
```

Langkah selanjutnya adalah membentuk vektor b

```
resp<- trees$Volume
head(resp)
```

```
## [1] 10.3 10.3 10.2 16.4 18.8 19.7
```

lakukan transformasi:

```
A <- t(pred) %*% pred #(X'X)
b <- t(pred) %*% resp #(X'y)
Ab <- cbind(A,b)
```

Dengan menggunakan metode eliminasi Gauss/ Row echelon form:

```
ref_matrix(Ab)
```

```
##           intercept    Girth    Height
## intercept        31 410.7000 2356.0000  935.3000
## Girth             0 295.4374  311.5000 1496.6435
## Height            0   0.0000  889.5641  301.7857
```

Menggunakan substitusi balik didapatkan koefficient variable height: 301.7857/889.5641 = 0.3392512

```
301.7857/889.5641
```

```
## [1] 0.3392512
```

Dengan menggunakan metode eliminasi Gauss Jordan:

```
Ugmt.mtx <- cbind(A,b)
Ugmt.mtx[1,] <- Ugmt.mtx[1,]/Ugmt.mtx[1,1]
for (i in 2:nrow){
  for (j in i:nrow) {
    Ugmt.mtx[j, ] <- Ugmt.mtx[j, ] - Ugmt.mtx[i-1, ] * Ugmt.mtx[j, i-1]
  }
  Ugmt.mtx[i,] <- Ugmt.mtx[i,]/Ugmt.mtx[i,i]
}
for (i in j:2){
  for (j in i:2-1) {
    Ugmt.mtx[j, ] <- Ugmt.mtx[j, ] - Ugmt.mtx[i, ] * Ugmt.mtx[j, i]
  }
}
Ugmt.mtx
```

```
##           intercept Girth Height
## intercept         1     0      0 -57.9876589
## Girth             0     1      0   4.7081605
## Height            0     0      1   0.3392512
```

Dengan menggunakan metode iterative Jacobi ?

Dengan menggunakan metode iterative Gauss Siedel:

```
result <- gaussSeidel(A, b, 1e-6, 1000)
x <- result$x
iterations <- result$iterations
cat("Number of iterations:", iterations, "\n")
```

```
## Number of iterations: 1000
```

```
cat("Solution:", x, "\n")
```

```
## Solution: -57.77524 4.71234 0.3357443
```

Dengan menggunakan fungsi base R

```
## fungsi solve
solve(A,b)
```

```
##                  [,1]
## intercept -57.9876589
## Girth       4.7081605
## Height      0.3392512
```