

LINEAR ALGEBRA I – HOMEWORK XII

以下為練習題。不需繳交。

- (1) Let $J \in M_{7 \times 7}(\mathbb{C})$ be an 7×7 Jordan block with eigenvalue $\lambda = 0$. Please find the Jordan canonical forms of J^3 and J^4 .

(2) Let $A = \begin{pmatrix} 2 & -1 & -1 & -3 \\ 7 & 7 & 1 & 5 \\ -4 & -2 & 3 & -4 \\ 1 & 1 & 1 & 6 \end{pmatrix} \in M_{4 \times 4}(\mathbb{C})$.

Derive the general solution to $x'(t) = A x(t)$, where $x(t)$ is a 4-dimensional column vector.

- (3) Let T be a linear transformation on an n -dimensional vector space V over \mathbb{C} . Suppose T satisfies the relation $T^2 + T + 1 = 0$. Show that T is diagonalizable.