Functional Principal Component Analysis (FPCA) for Phonetic Research

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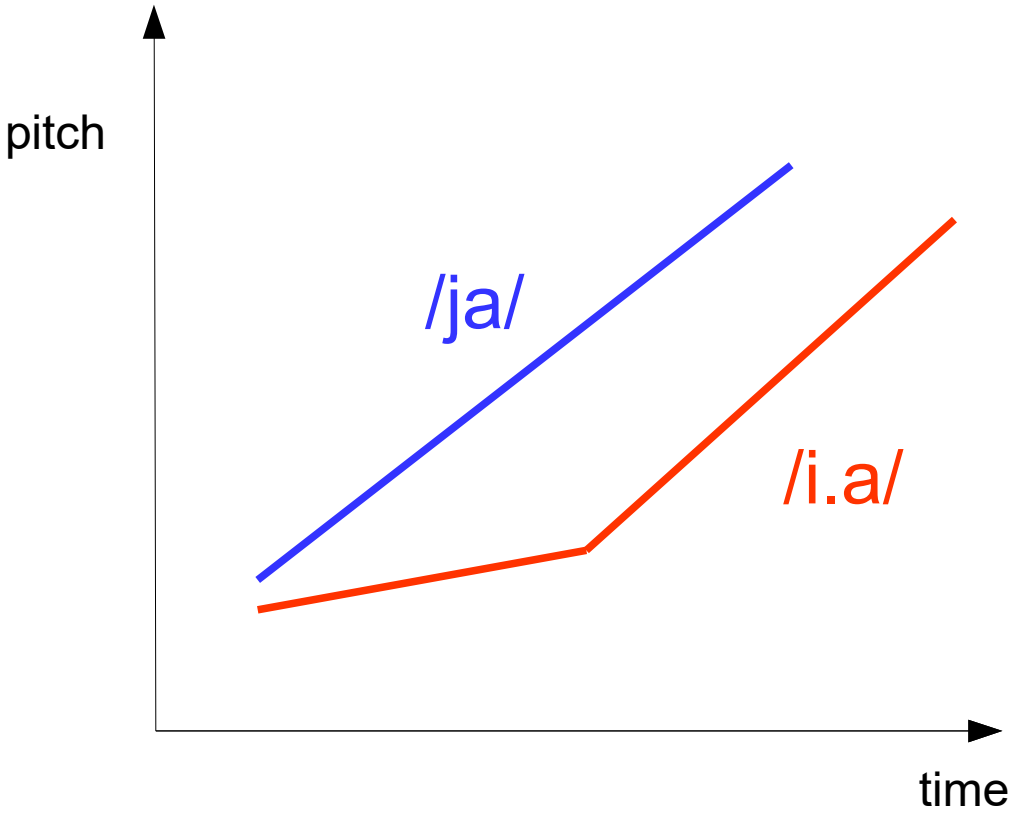
now at LMU Munich, Germany

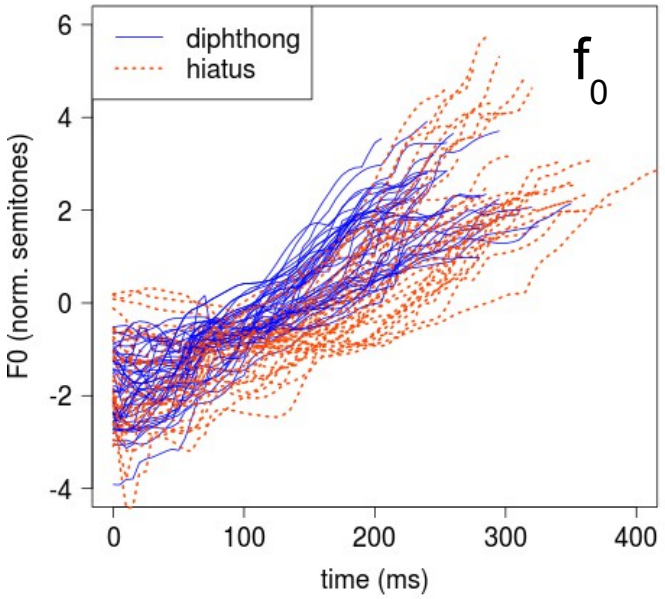
Alignment of rising pitch accents in Spanish



- European Spanish
- **Diphthong**: /ja/

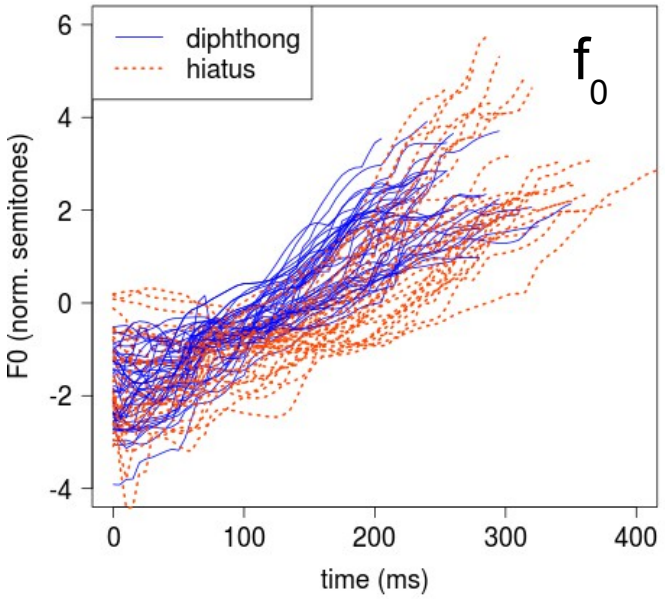
  e.g. *Emili<u>a</u>na*
- **Hiatus** /i.a/

  e.g. *pi<u>a</u>no*
- Rising pitch accent should align to syllabic structure



- Read speech
- 9 participants
- 20 Diphthongs +

  20 Hiatuses each



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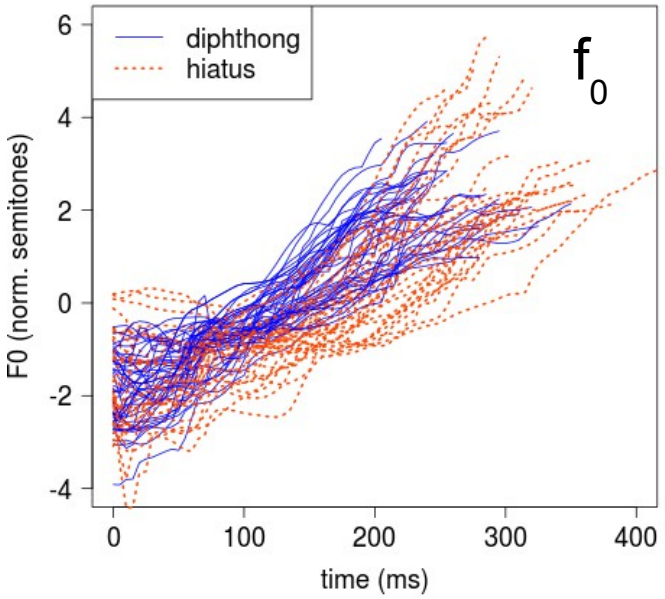
ANOVA

LR

LMER

CURVES



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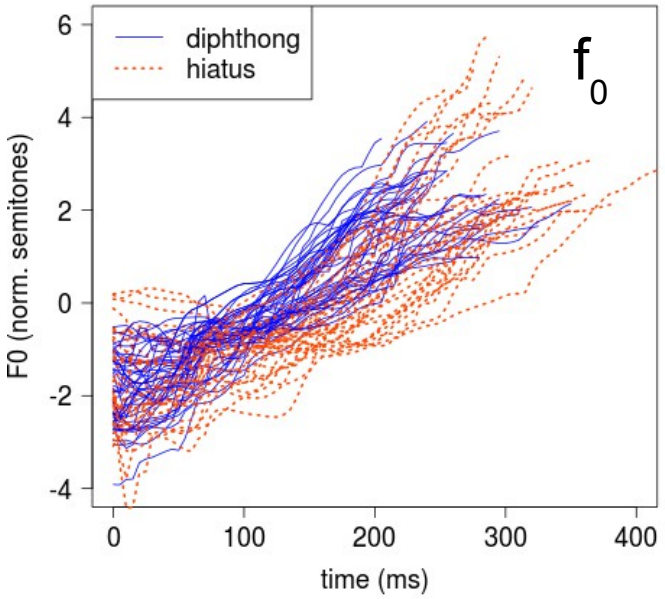

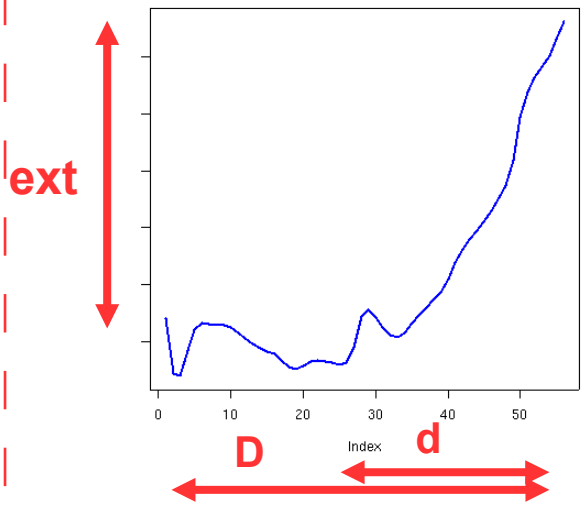

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ext (st)	d/D	Cat.
5.3	0.9	D
4.6	0.7	H
...	...	...

NUMBERS

ANOVA

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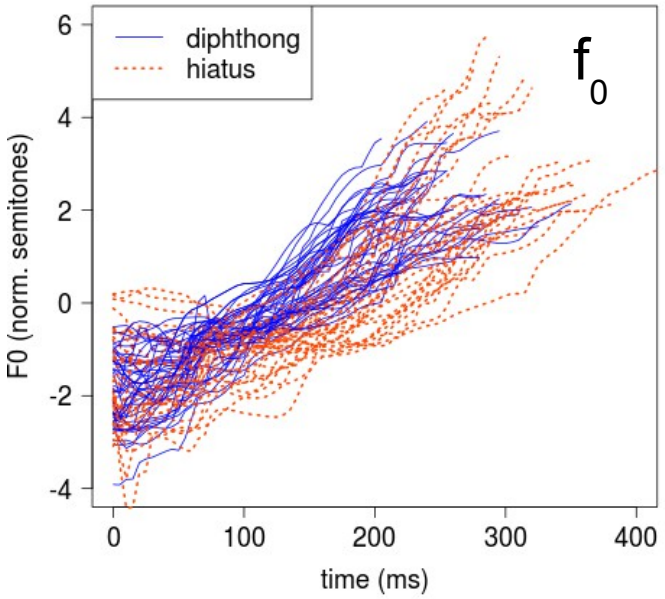
LMER

CURVES



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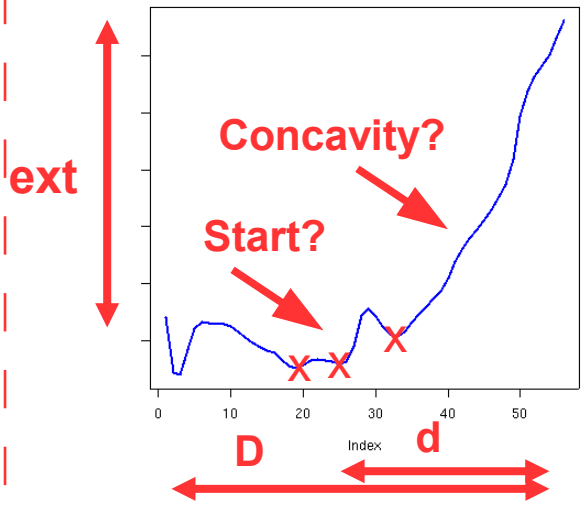

ext (st)	d/D	Cat.
5.3	0.9	D
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...	...	...

NUMBERS

ANOVA

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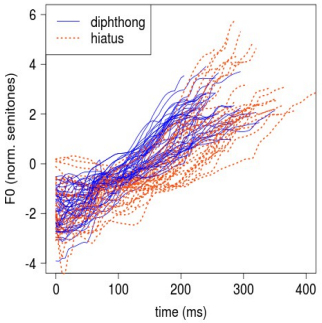
MISSION

automate curve parametrisation

- Data driven
- Few parameters
- Interpretable

Road map

CURVES



Interpolate using a function basis

- Data driven

Dimensionality reduction tool

- Few parameters
- Interpretable

NUMBERS

- ANOVA
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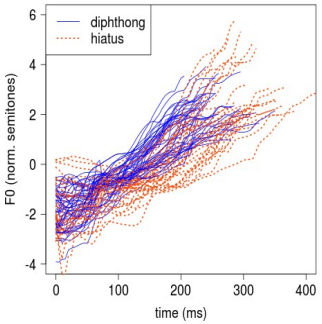
Road map

CURVES



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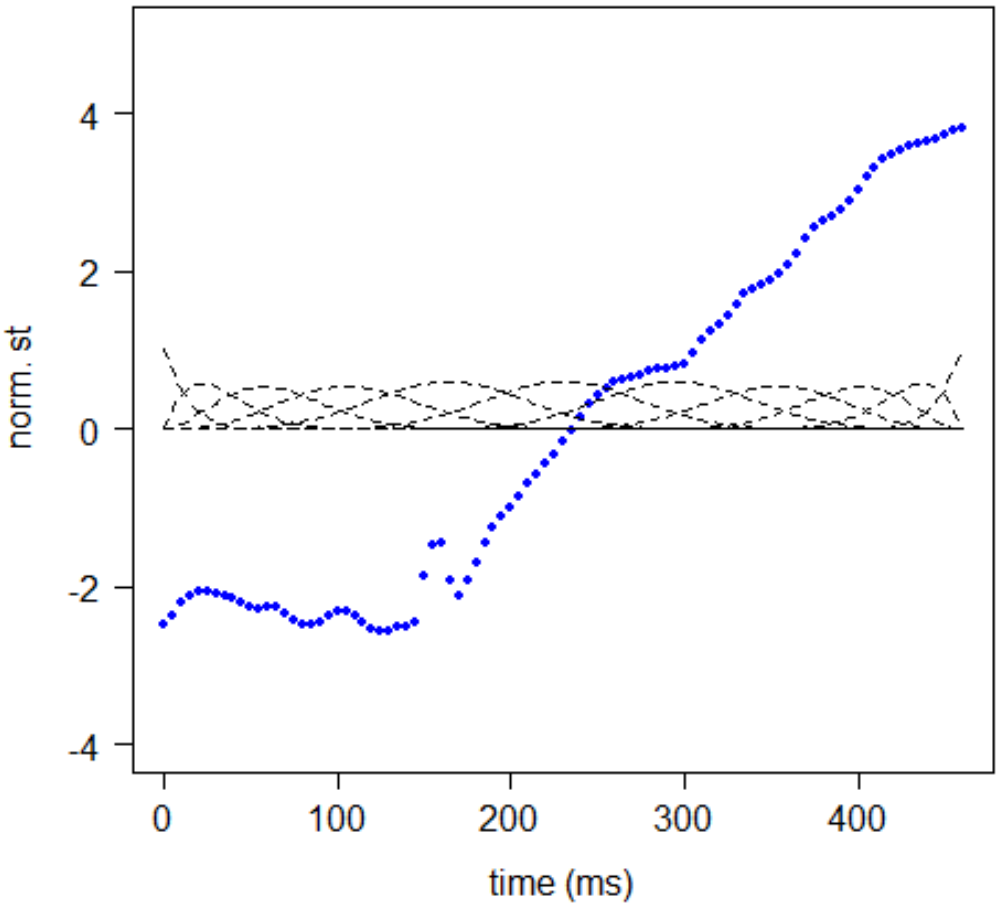
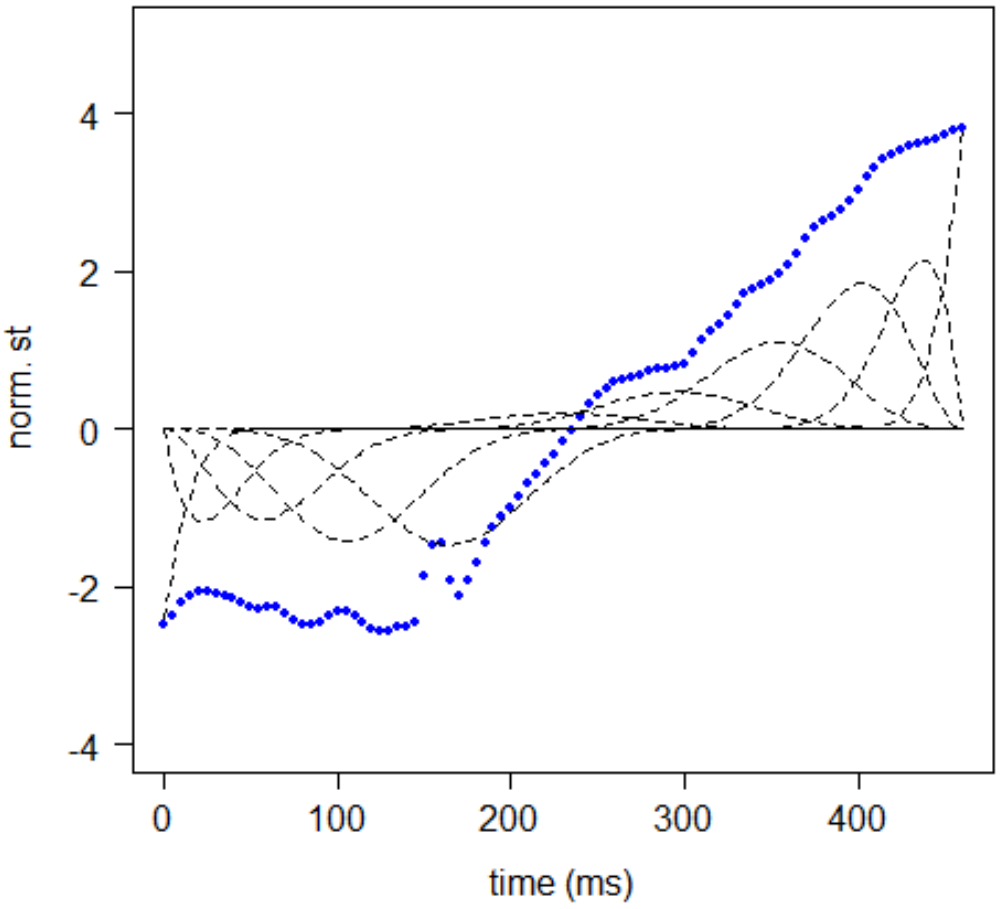
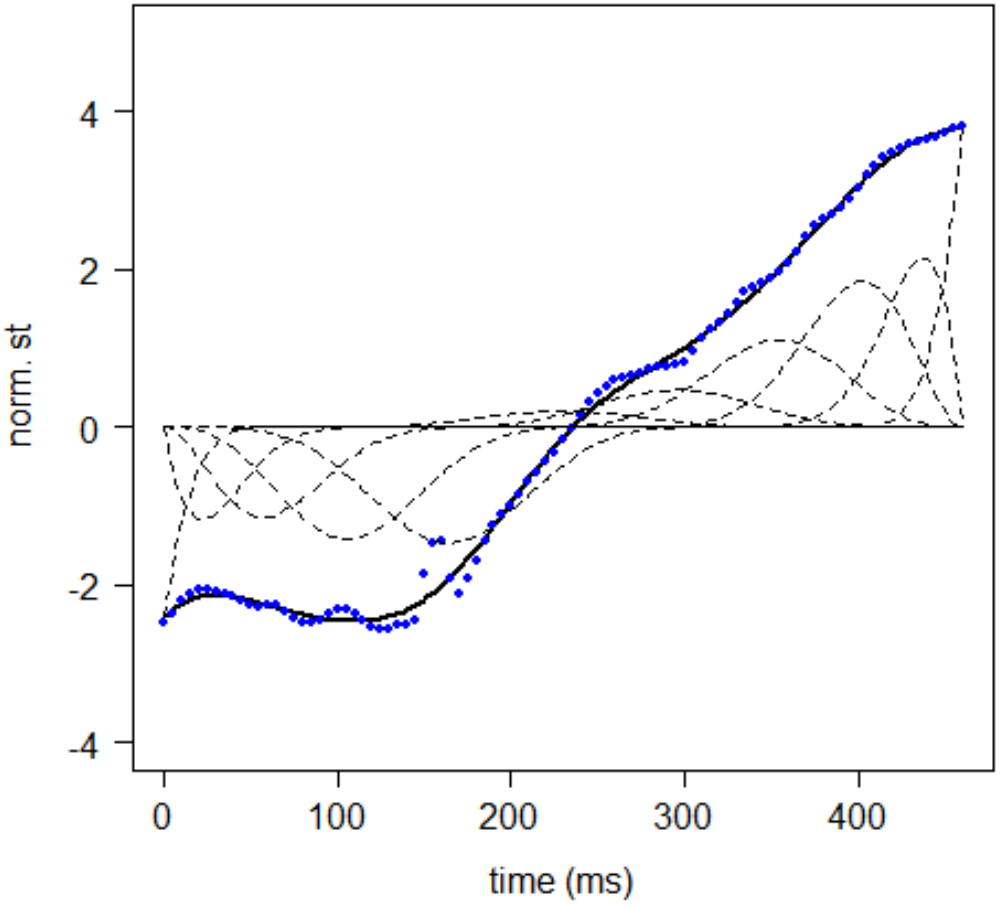
Interpolation with B-splines



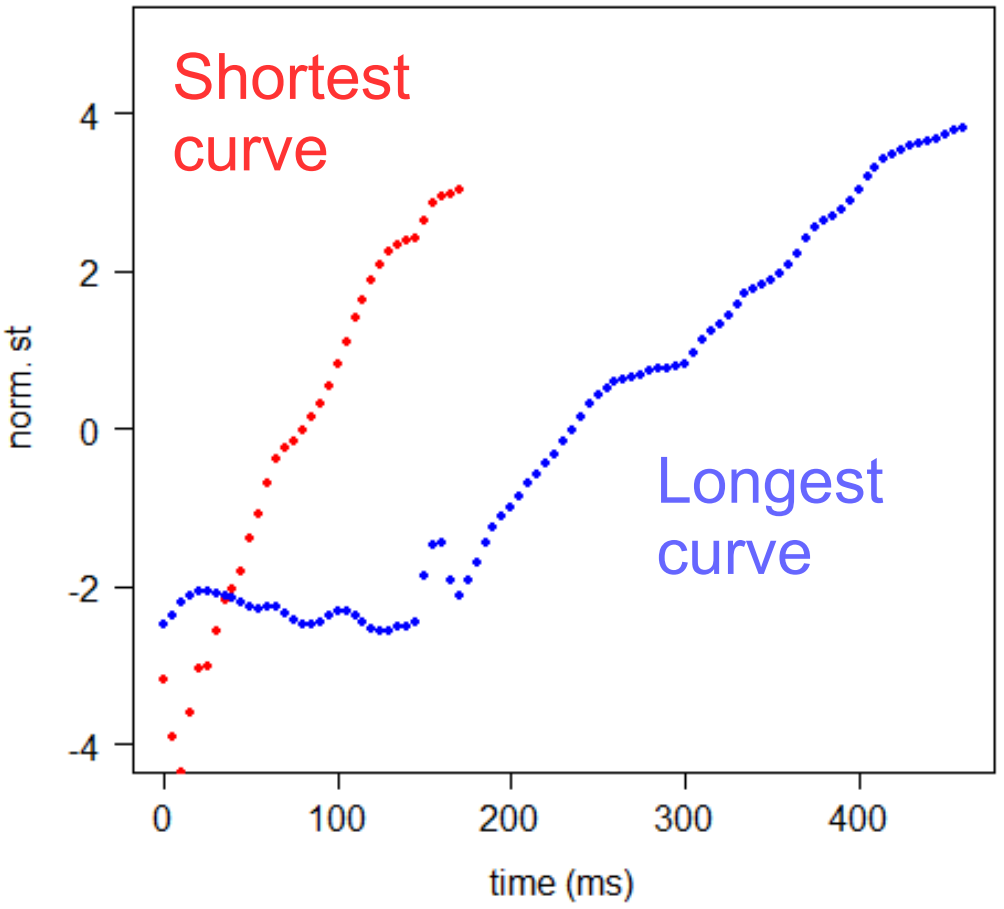
Interpolation with B-splines



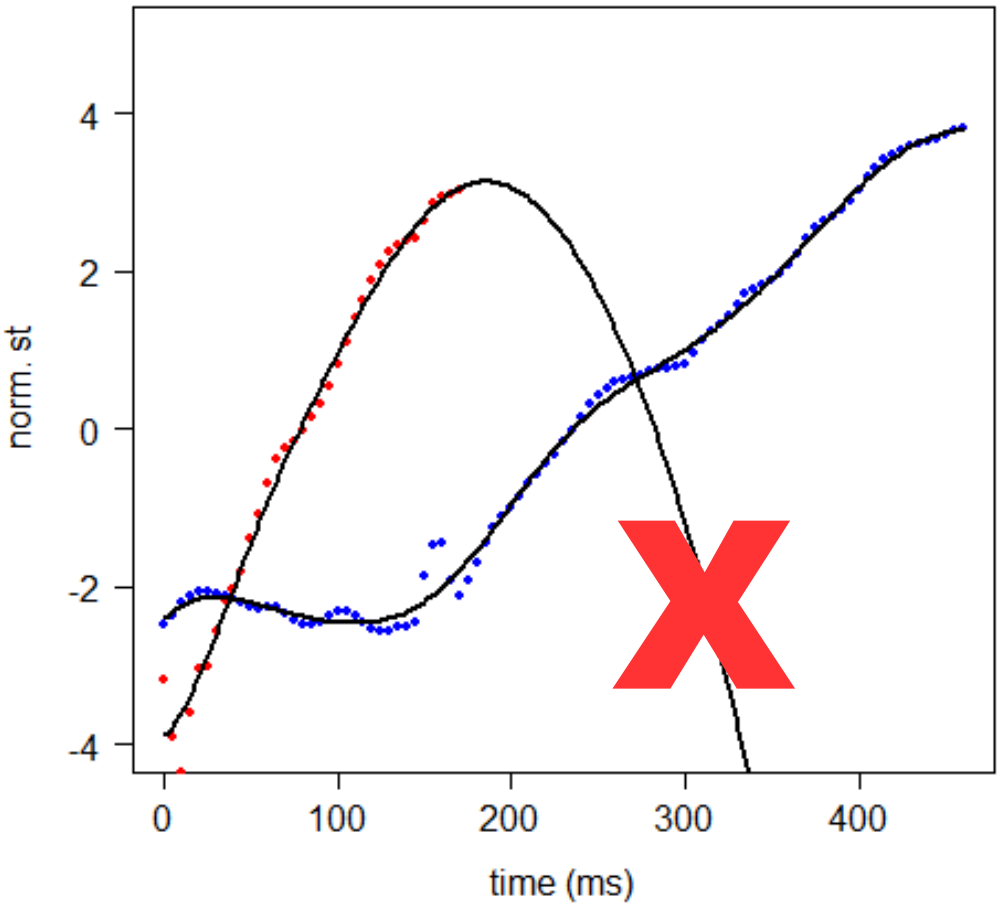
Interpolation with B-splines



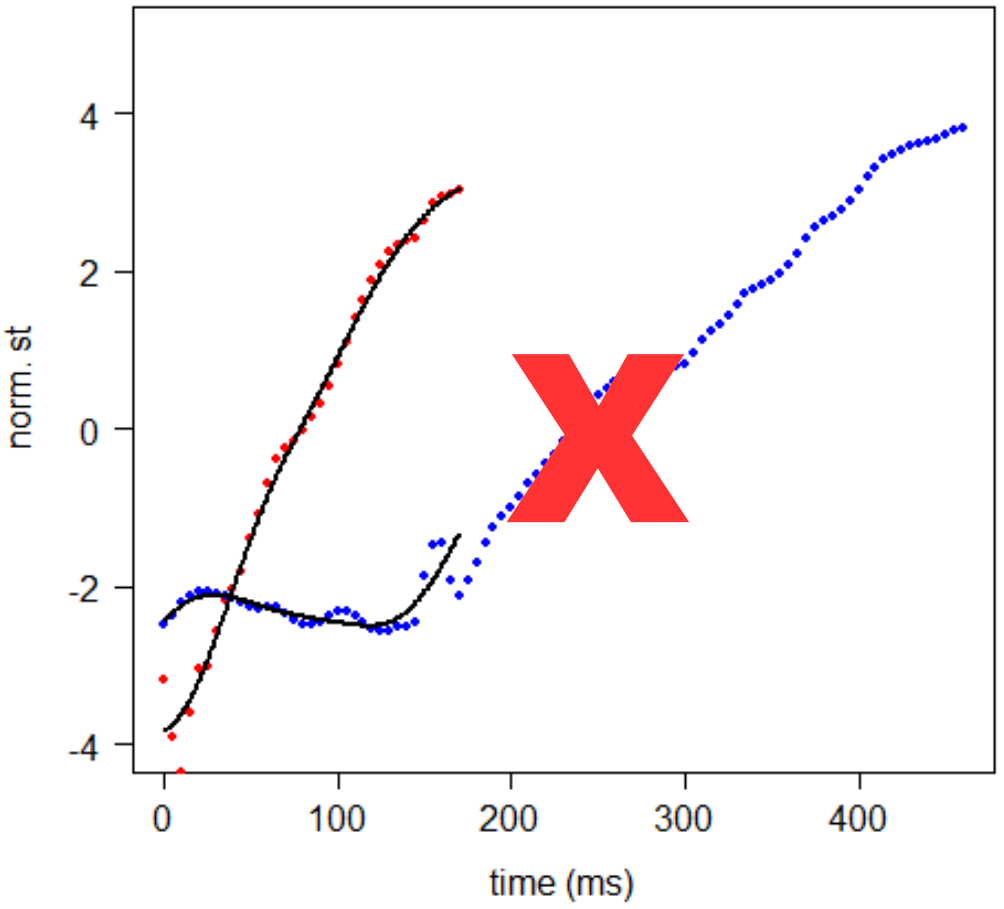
Different durations



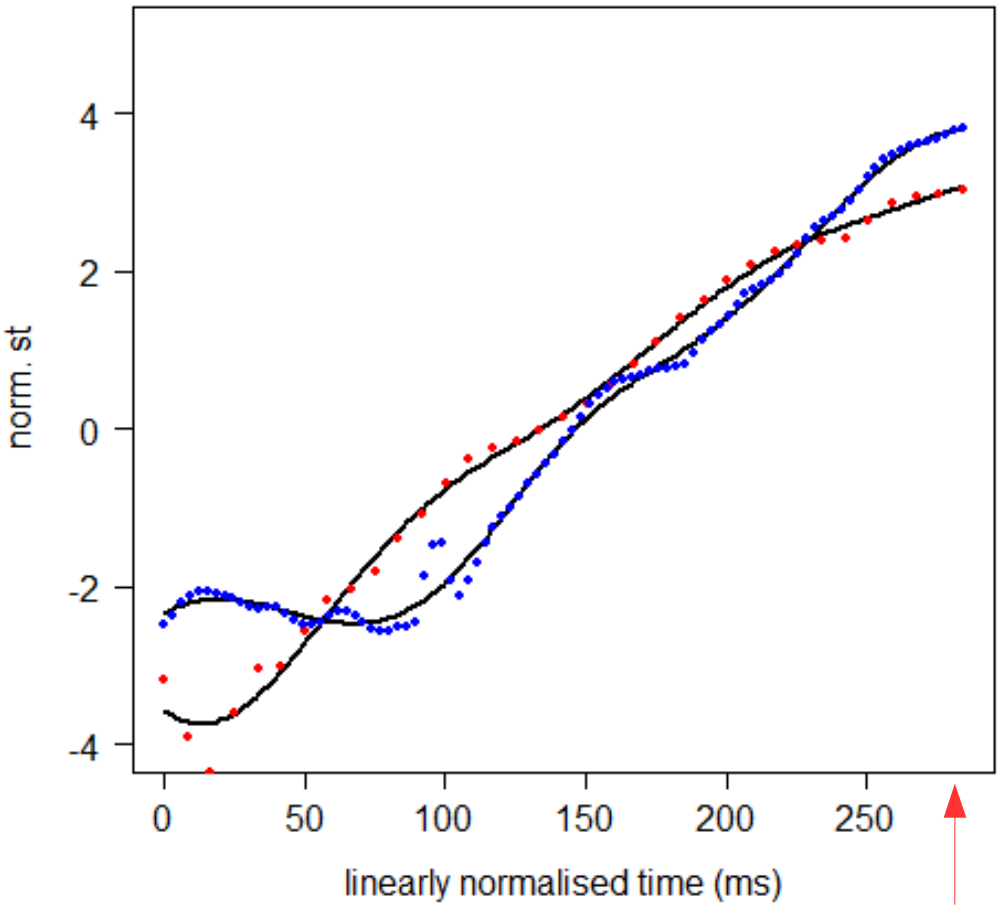
Take longest duration



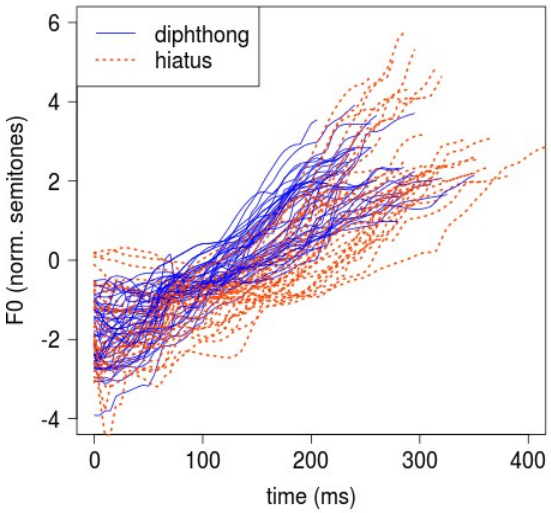
Take shortest duration



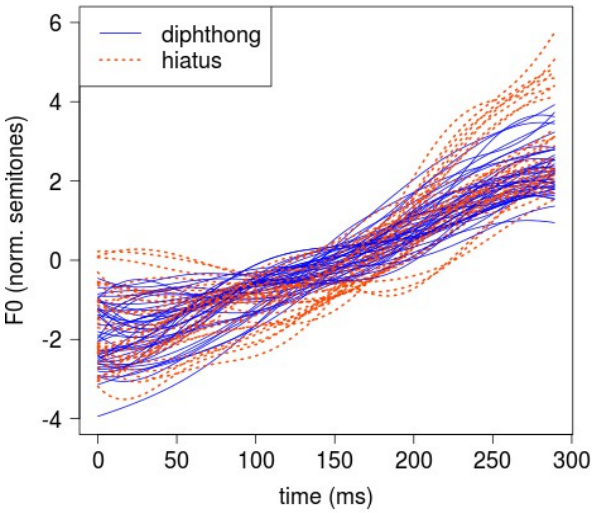
Linear time normalisation



Linear time normalisation



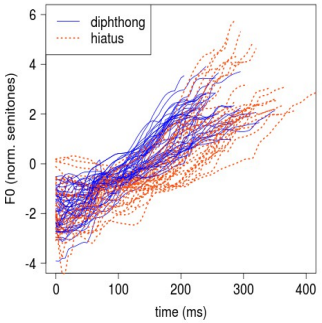
Interpolate to
the same
time interval



- We must use the same time interval
- This implies linear time normalisation
- Durations have to be reintroduced at the end of the analysis

Road map

CURVES



Interpolate to
<u>the same</u>
time interval

- Data driven

Dimensionality
reduction tool

- Few parameters
- Interpretable

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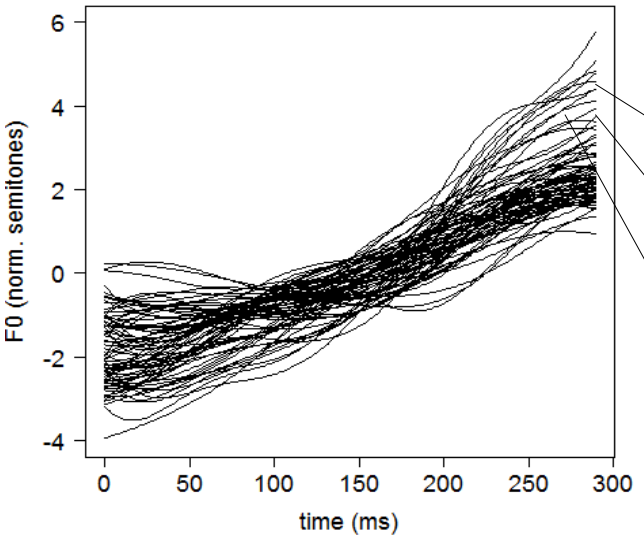
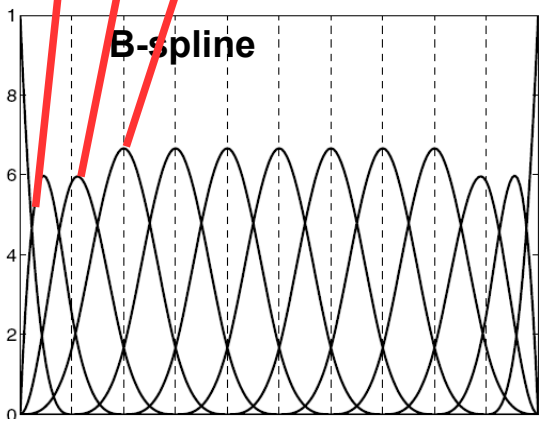
Principal Component Analysis



c1	c2	c3	...
...	...	...	...
...	...	...	...
...	...	...	...

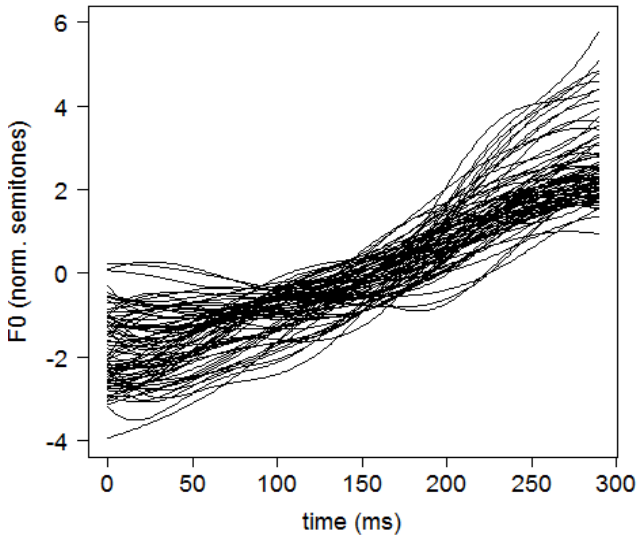


PCA

PCA limitations

- PCA does not use any explicit information related to the curve shapes or the B-splines shapes
- e.g. the sequence of coefficients c1, c2,.. reflects time adjacency of polynomial components, i.e. overlapping 'hills'

Discrete Cosine Transform



DCT

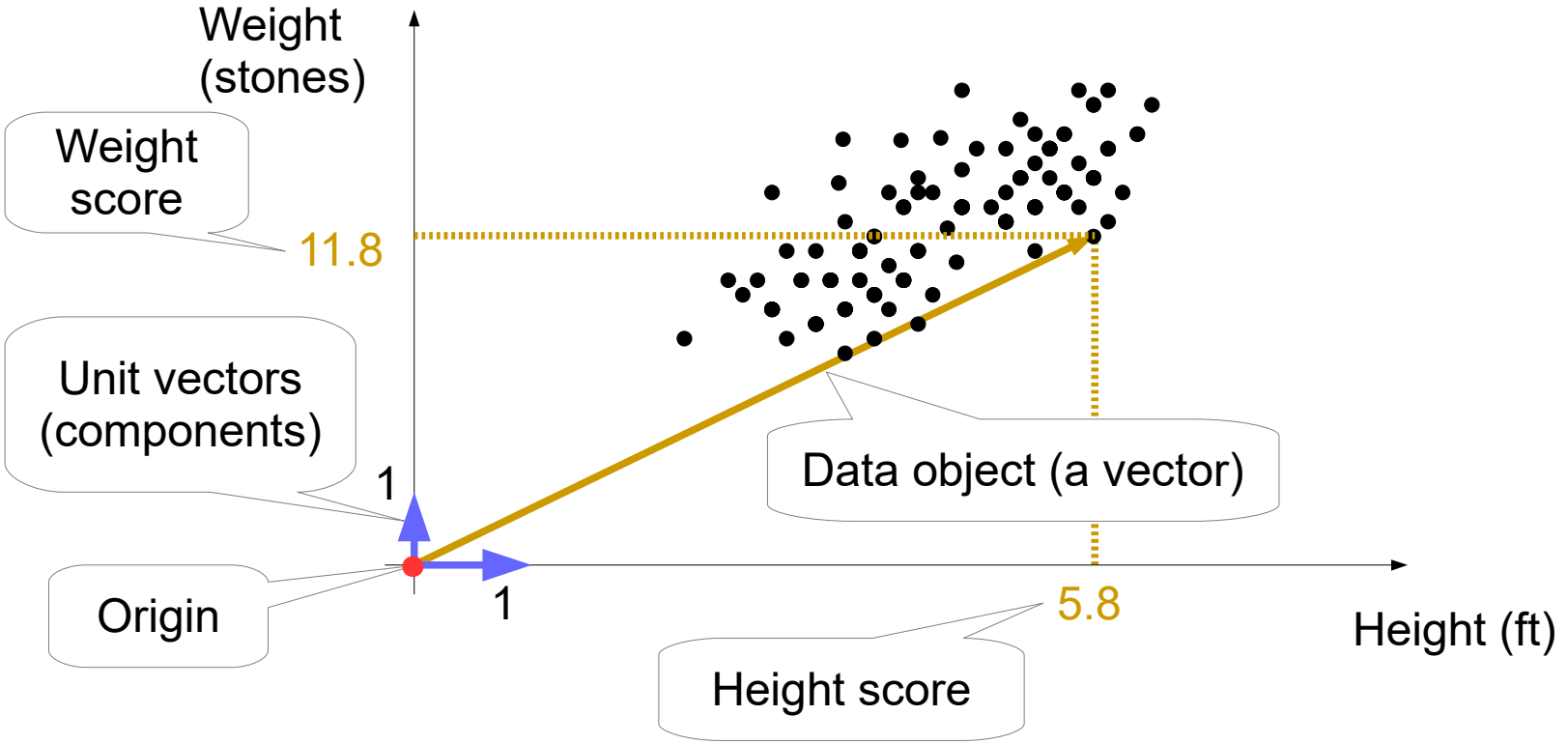
k0	k1	k2	...
...	...	...	...
...	...	...	...
...	...	...	...

DCT limitations

- DCT does not (easily) encode time-localised information, e.g. a small hump in the same (time-normalised) position
- Typically only k0, k1 and k2 are used, which have a geometric interpretation
- Extracting several k's brings up the need of PCA
- In general, not effective to encode long signals

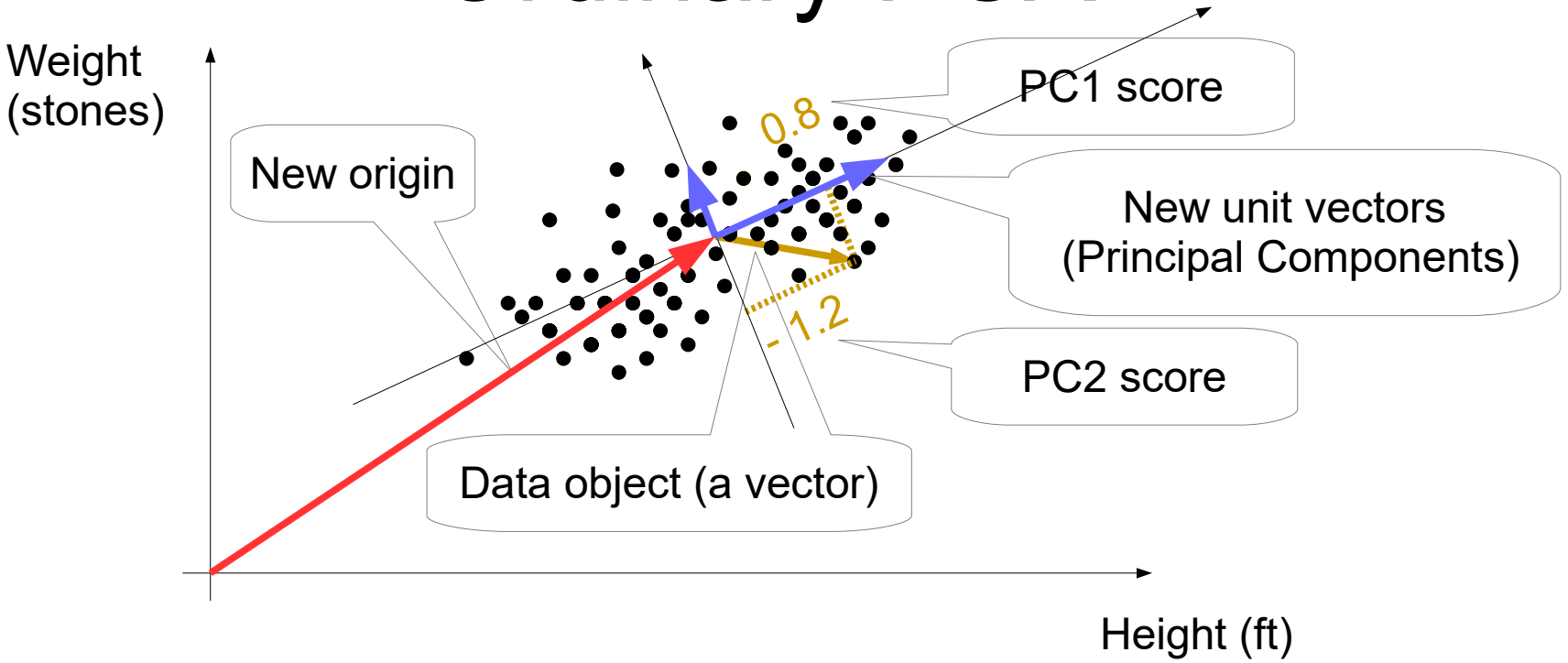
Introducing Functional PCA

Vectors



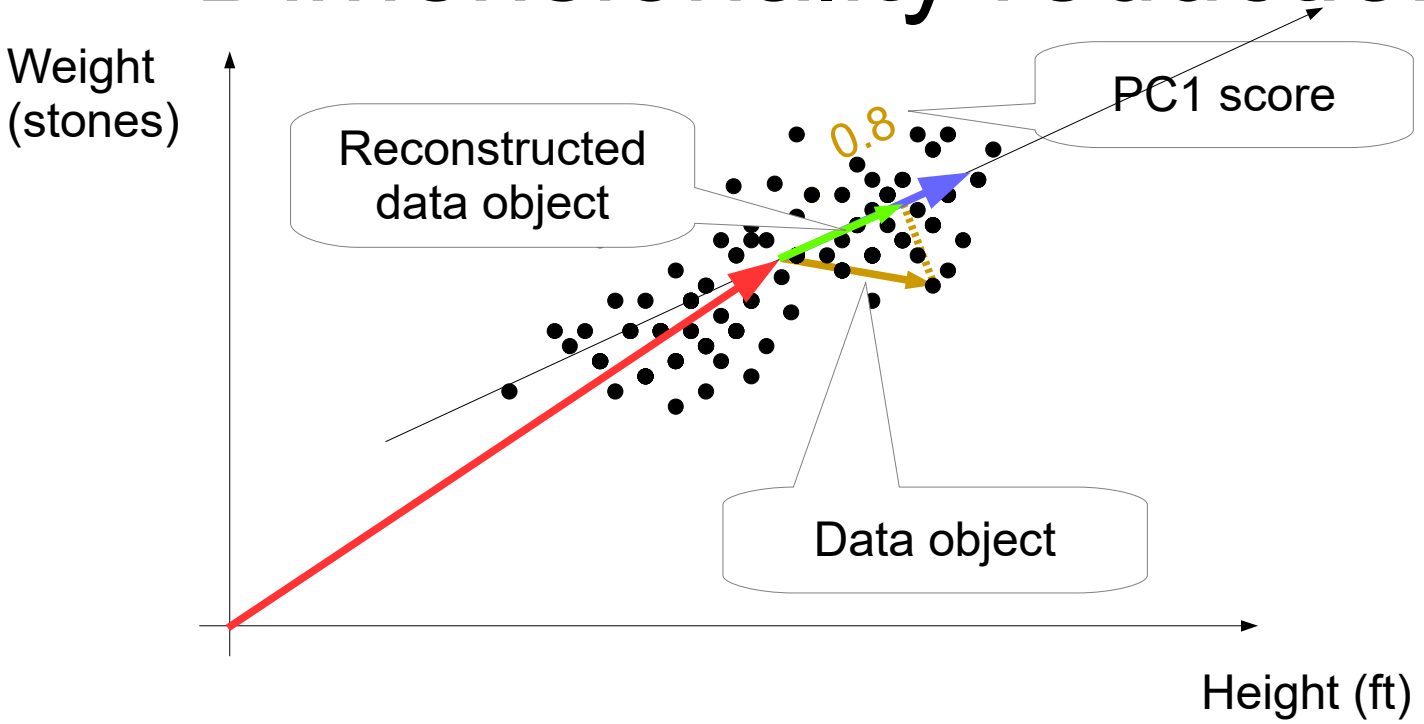
- Data objects and components are vectors
- From scores (numbers) we can reconstruct data objects (vectors)

Ordinary PCA

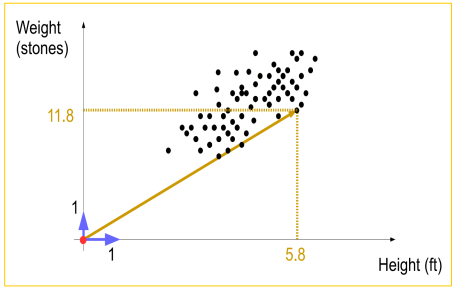


- PCA computes new origin and unit vectors which best suit the data
- From PC scores we can reconstruct data objects

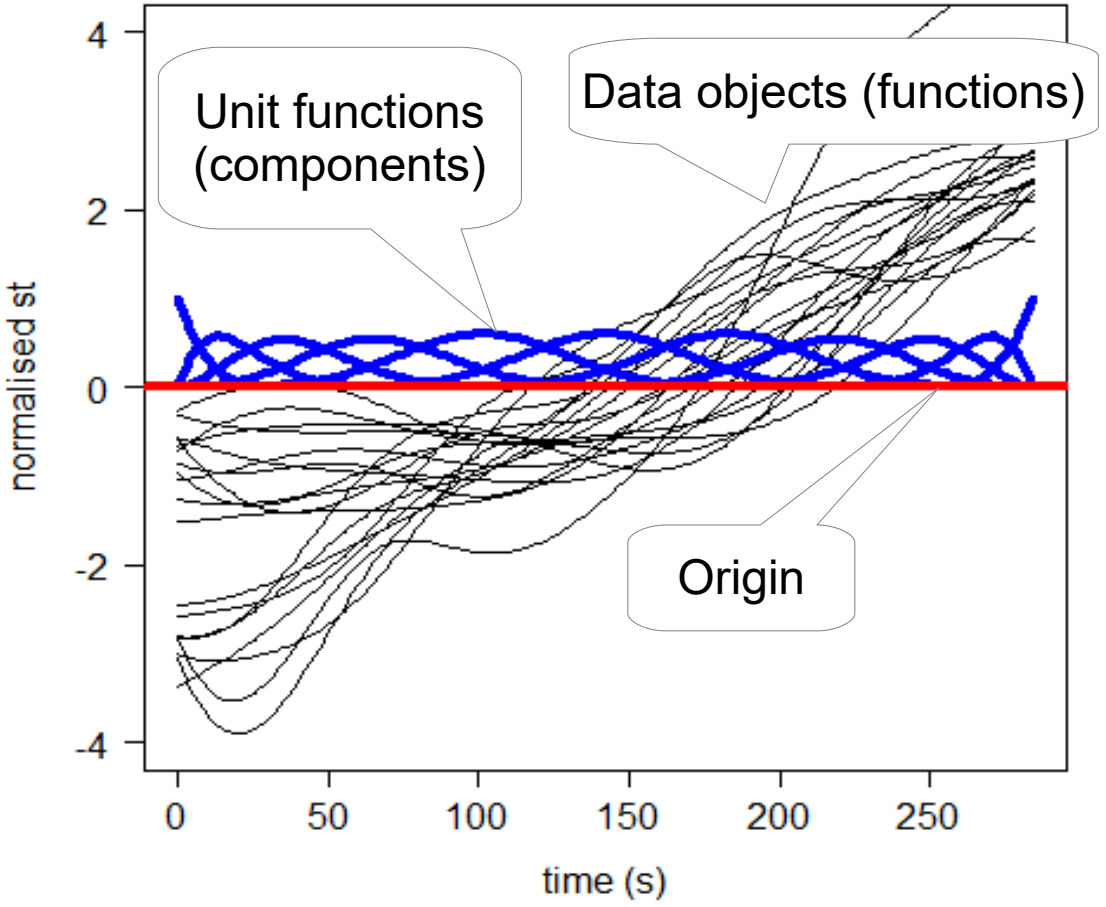
Dimensionality reduction



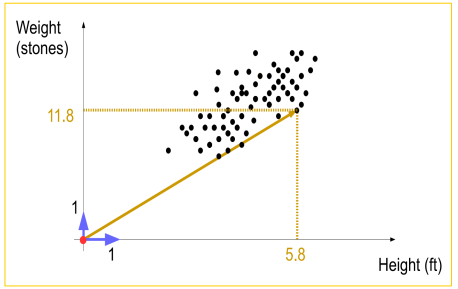
- We can use only part of the PCs
- This reduces the data dimensionality
- But introduces reconstruction errors too



Functions (curves)



- Origin, components and data objects are functions
- Origin is a flat line
- Components are 11 B-spline curves



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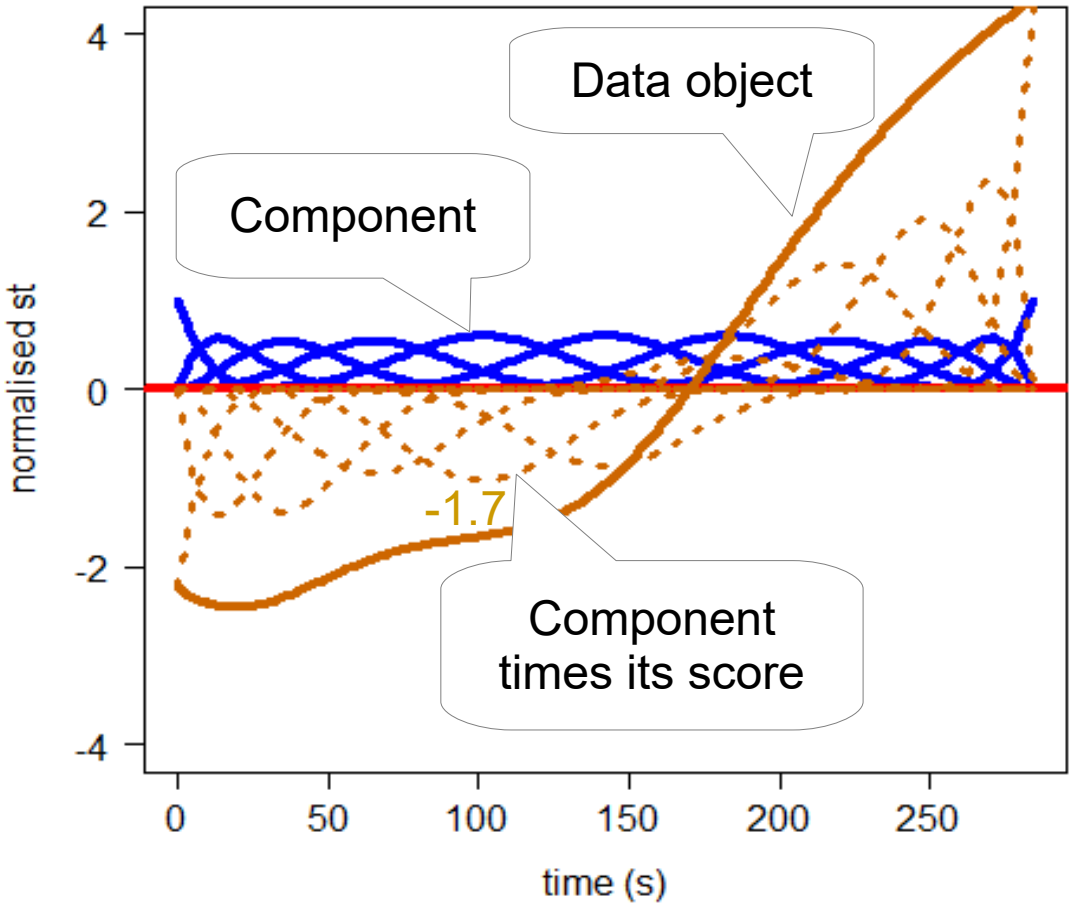
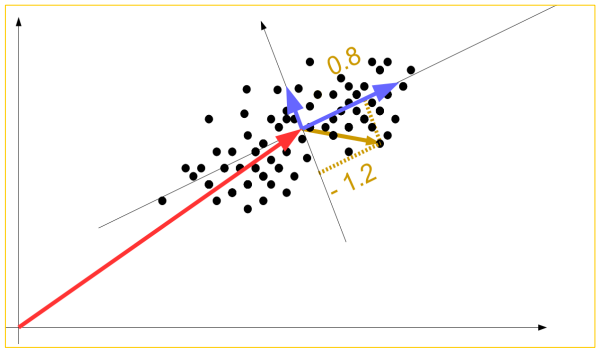
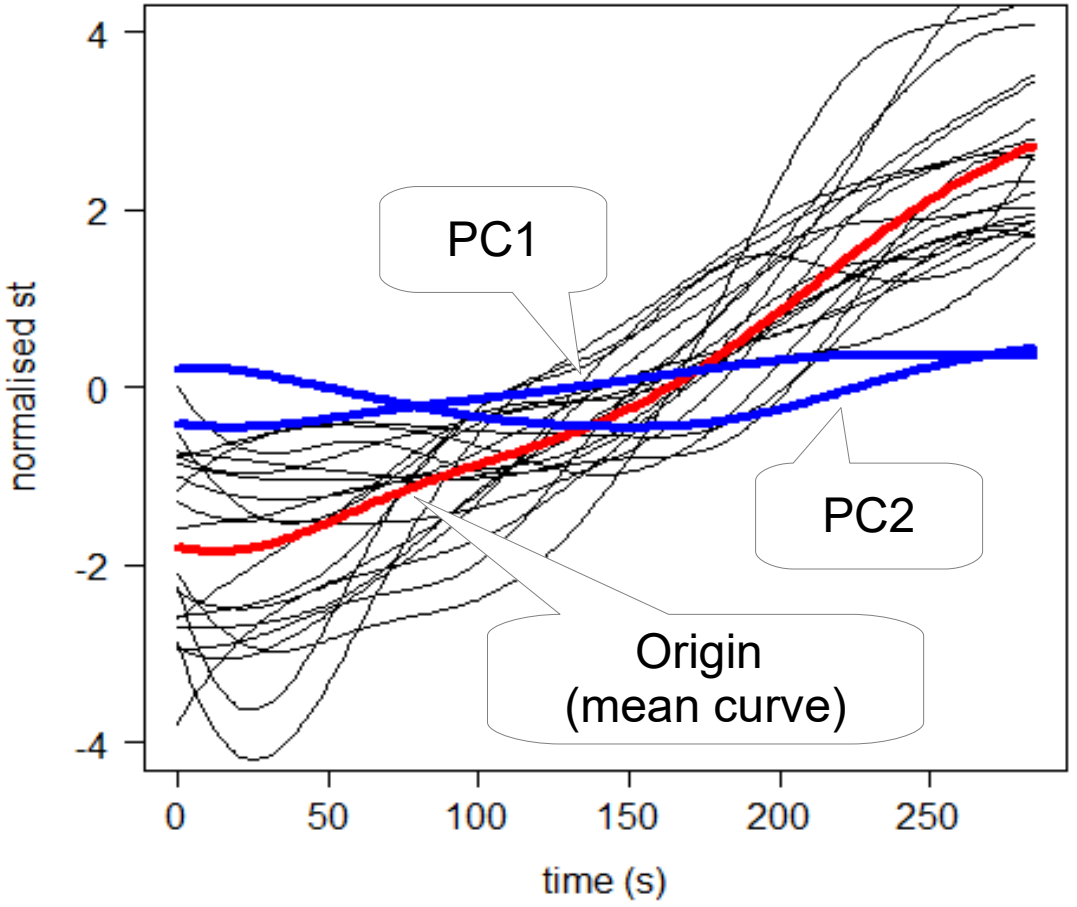


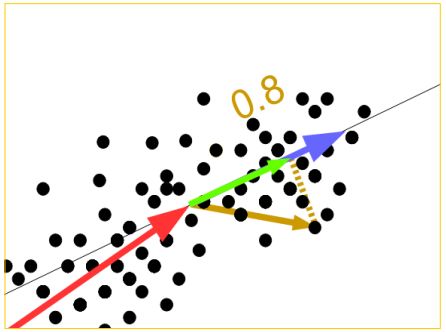
- Each of the 11 components is multiplied by a score
- These are summed together to obtain a data object



Functional PCA



- FPCA computes new origin and component functions which best suit the data



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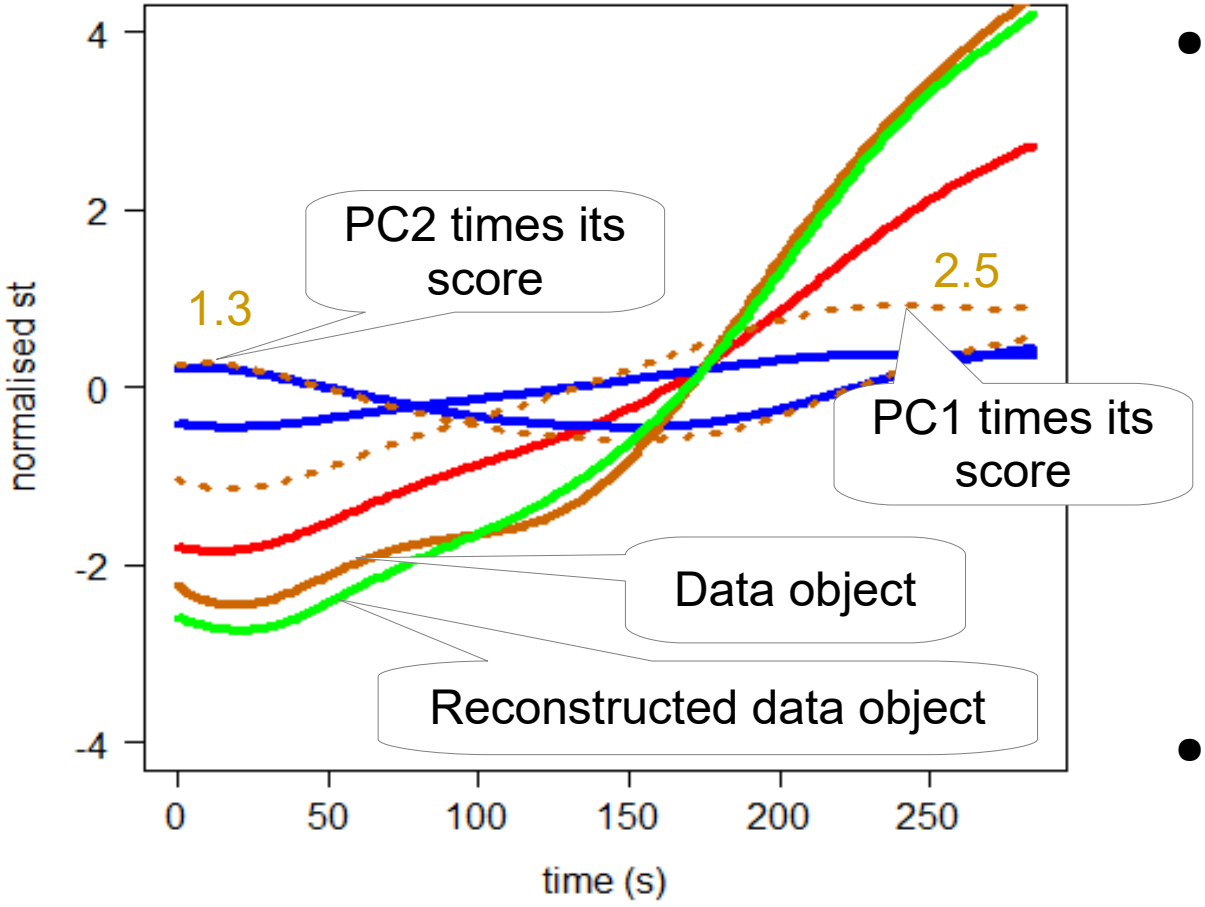
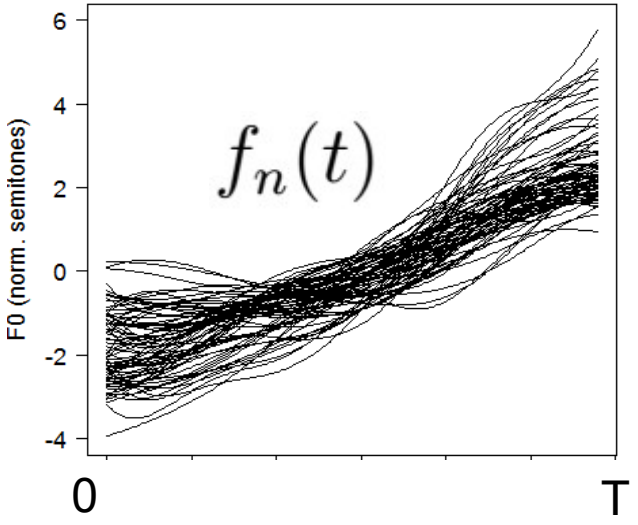


- The sum of origin (mean) curve + PCs times their scores gives an approx reconstruction of the original curve
- Dimensions from 11 (B-splines) down to 2 (PCs)

Functional PCA



$$\max\left\{var_n\left(\int_0^T PC1(t)f_n(t)dt\right)\right\}$$

$$\text{subject to}\int_0^T PC1^2(t)=1$$

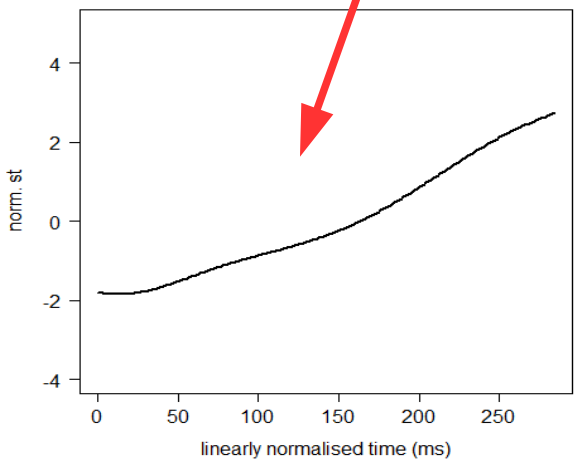
- FPCA definition uses the input curves $f_n(t)$
- FPCA is independent of the B-splines used to smooth $f_n(t)$

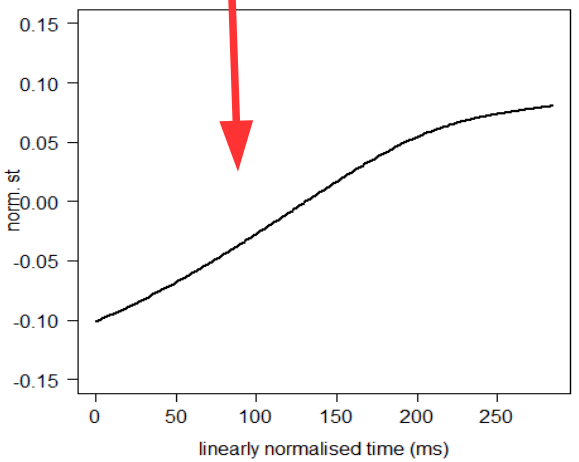
Functional PCs

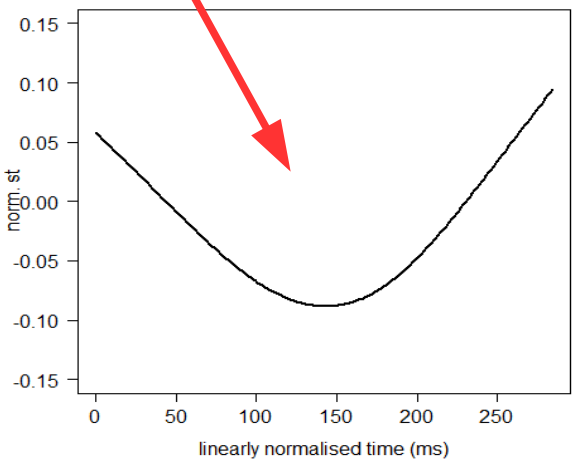
$$f(t) \approx \mu(t) + s_1 \cdot PC1(t) + s_2 \cdot PC2(t) + \cdots$$

PC1 score

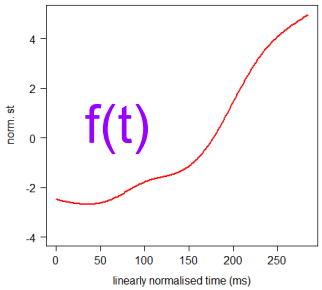
PC2 score







Curve reconstruction



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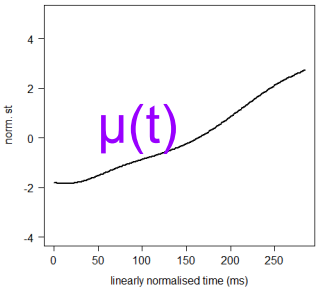
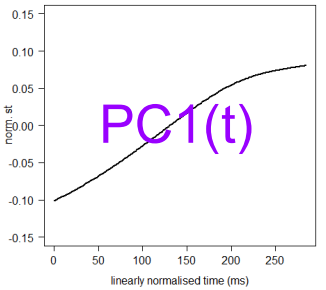
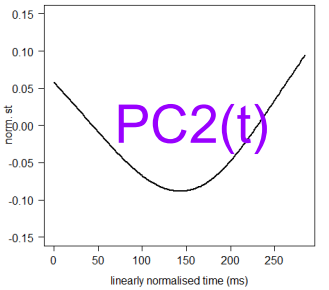
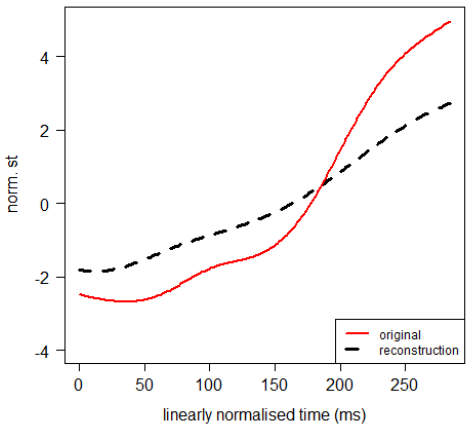


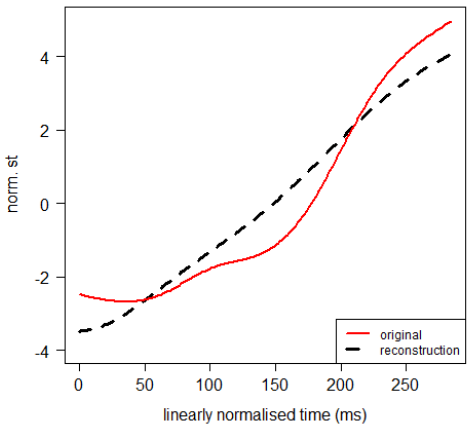
+ 16.5 *

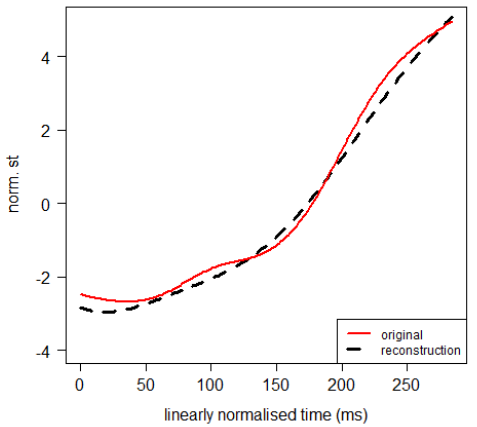


+ 10.8 *

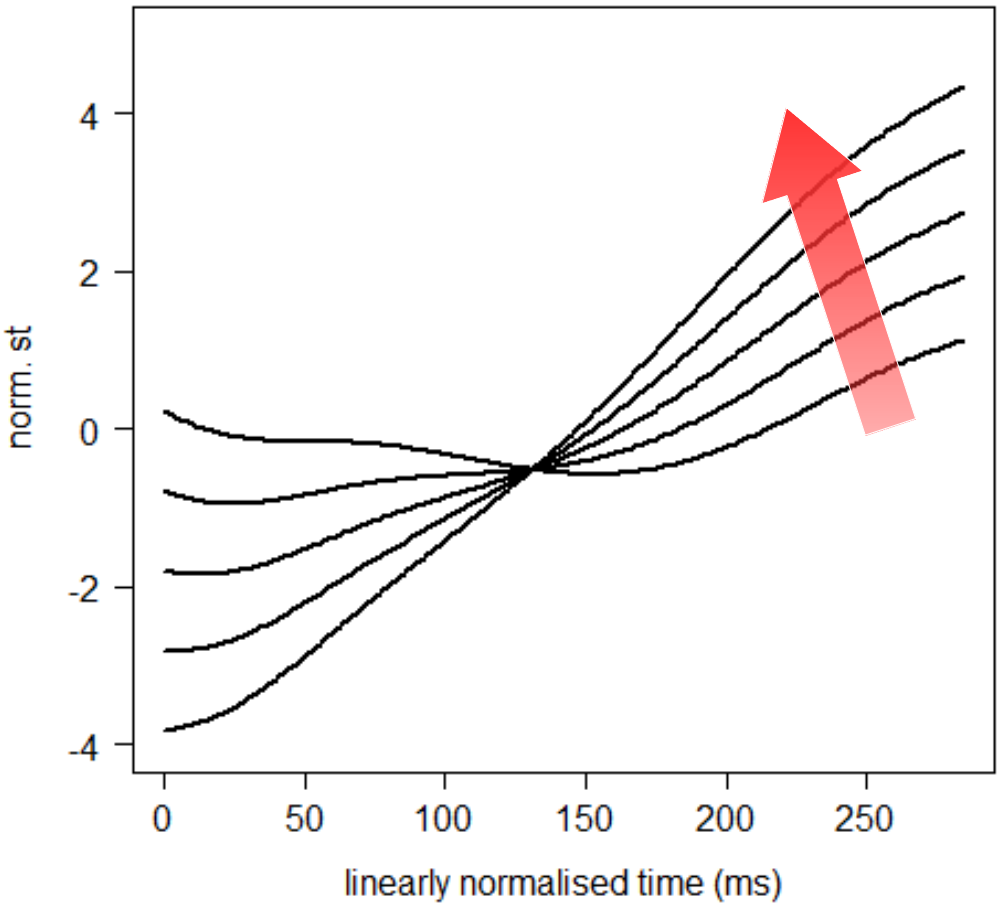






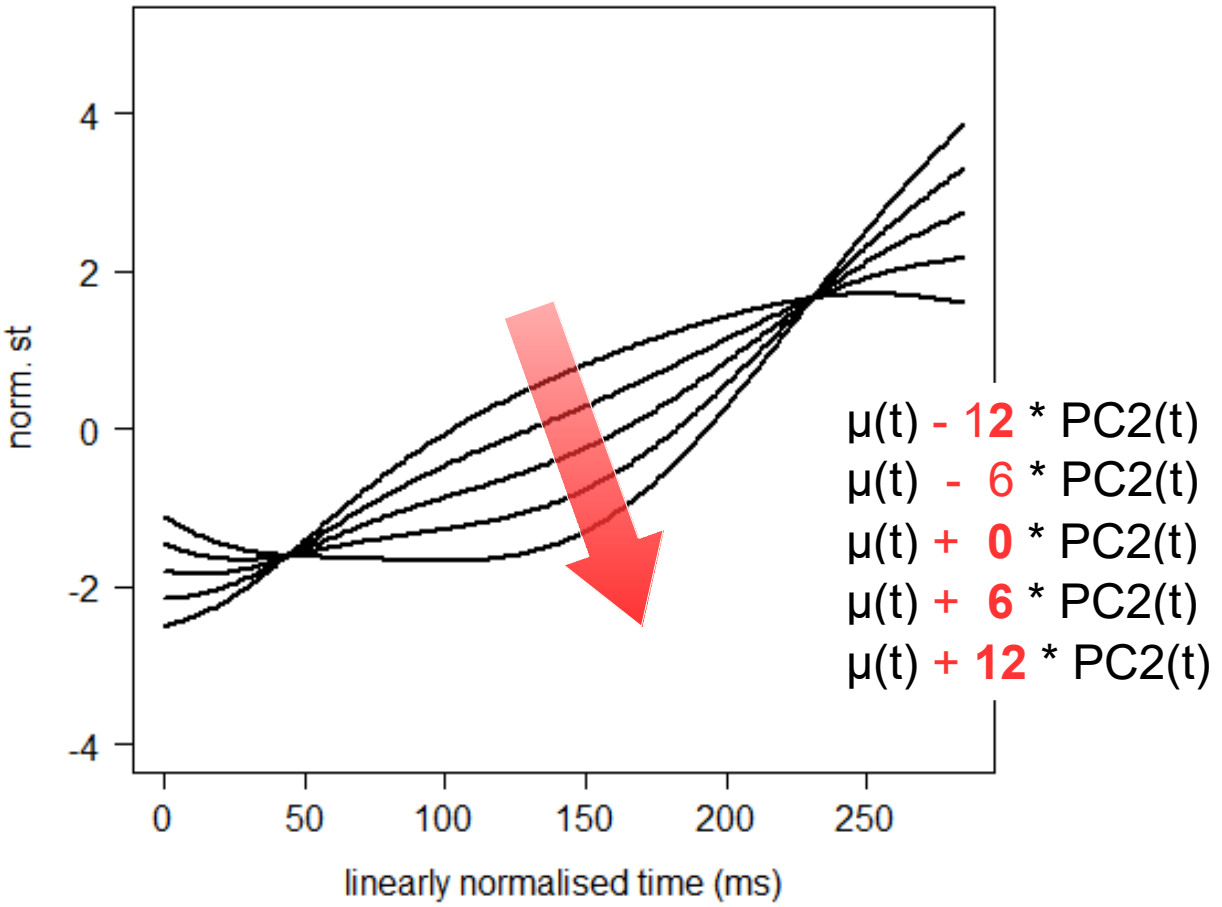


PC1 scores

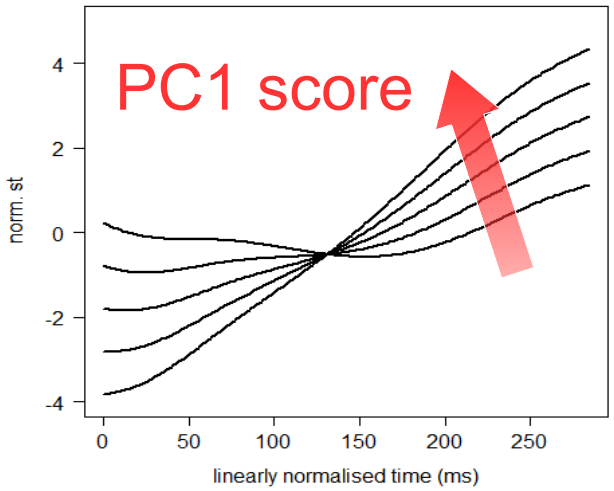


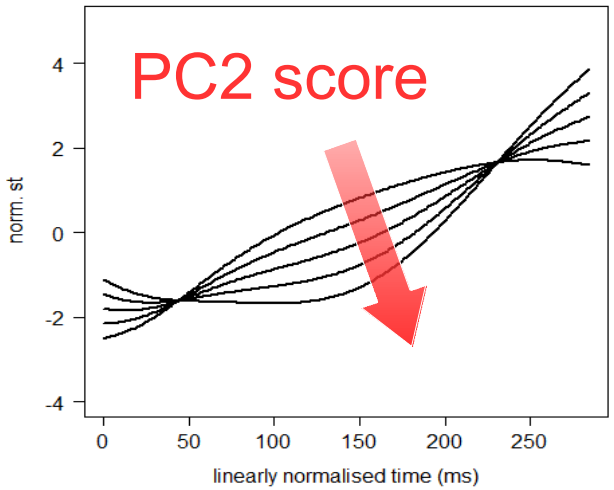
μ(t) + **20** * PC1(t)
μ(t) + **10** * PC1(t)
μ(t) + **0** * PC1(t)
μ(t) - **10** * PC1(t)
μ(t) - **20** * PC1(t)

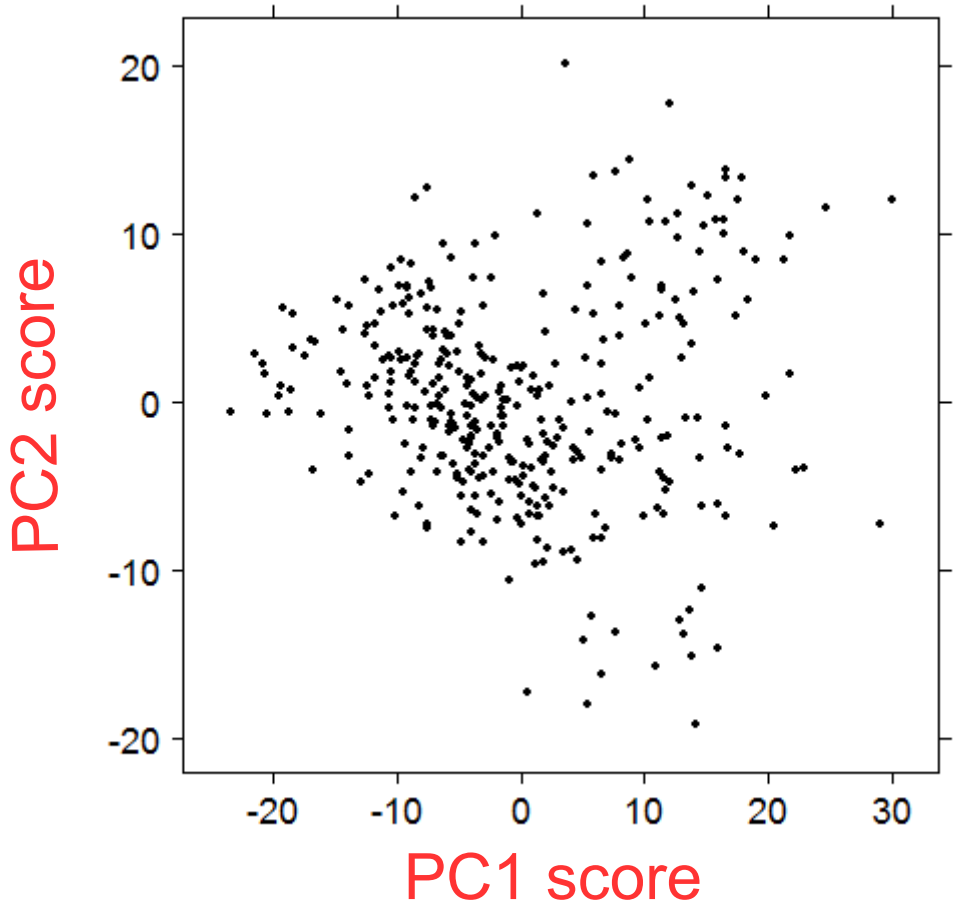
PC2 scores


4
2
0
-2
-4
norm. st
0
50
100
150
200
250
linearly normalised time (ms)
μ(t) - 12 * PC2(t)
μ(t) - 6 * PC2(t)
μ(t) + 0 * PC2(t)
μ(t) + 6 * PC2(t)
μ(t) + 12 * PC2(t)

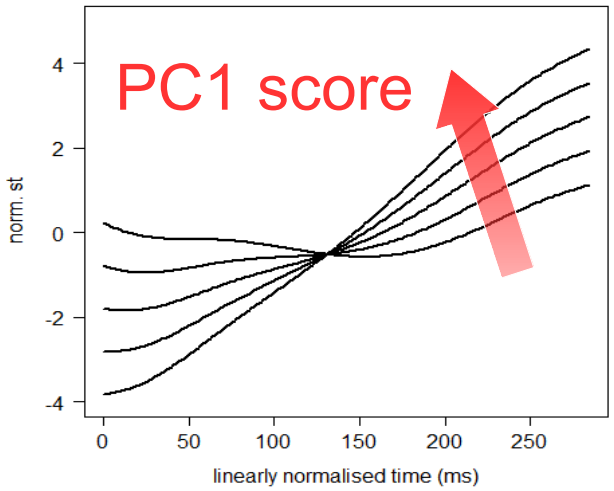
Curve parametrisation

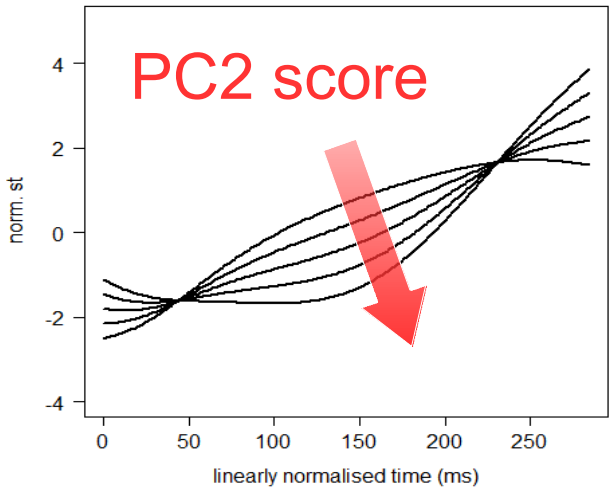


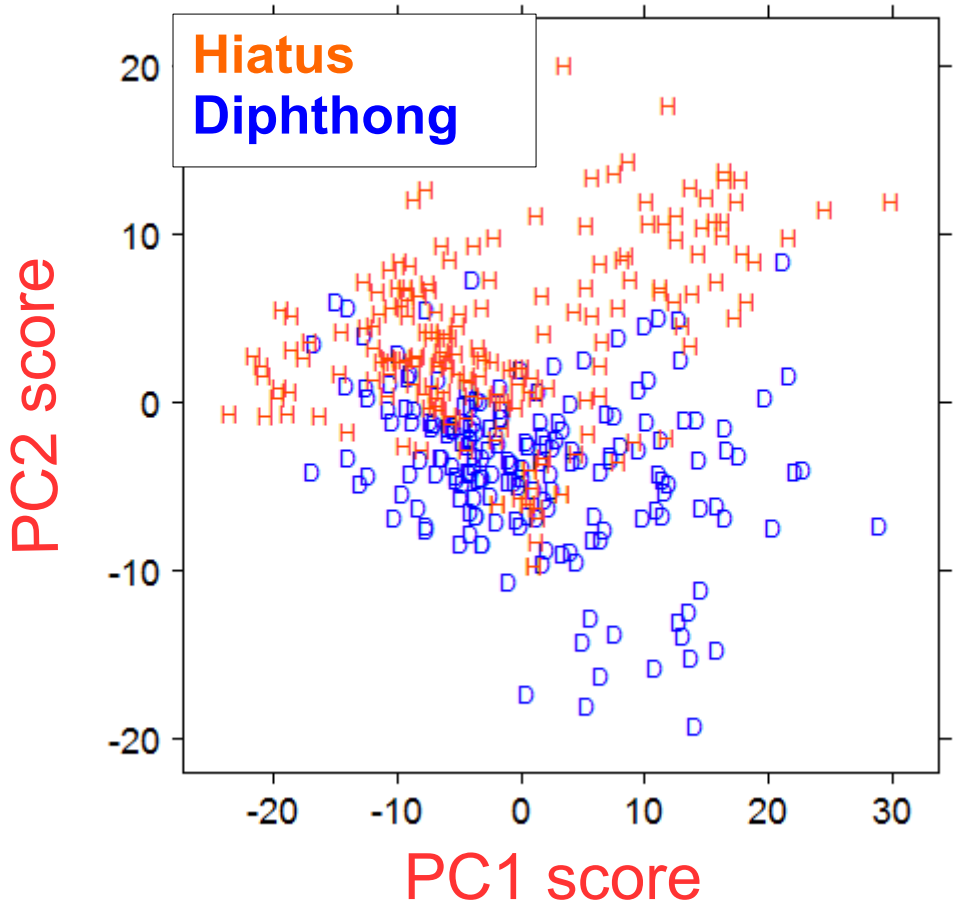




Curve parametrisation

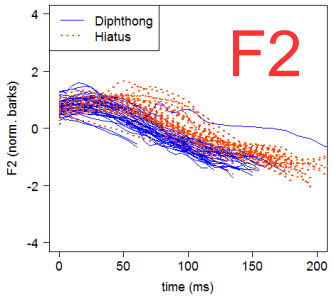


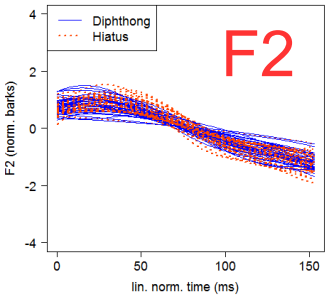


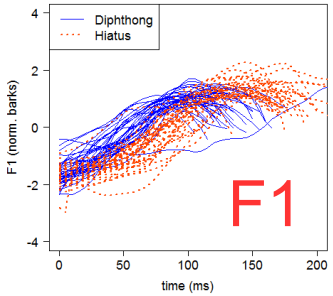


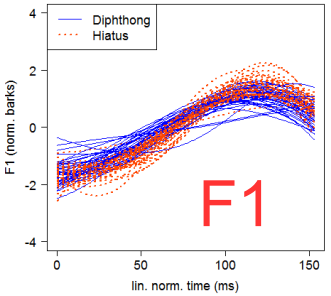
Formants

2D CURVES









FPCA

FPCA

NUMBERS

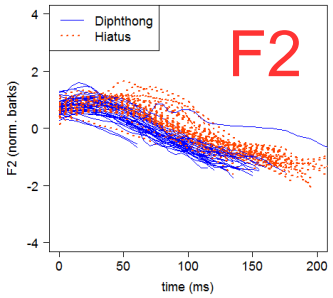
ANOVA

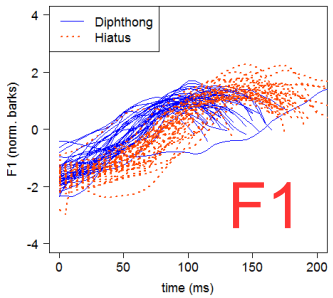
LM

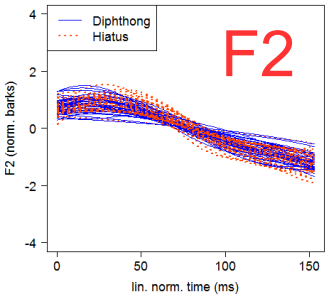
LMER

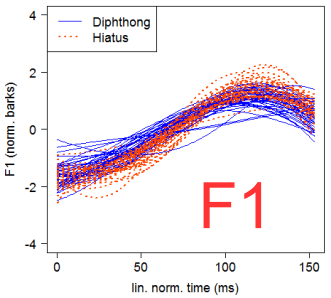
Formants

2D CURVES







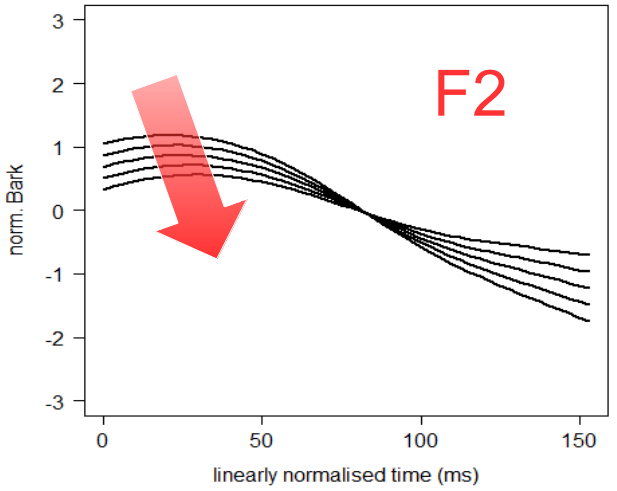


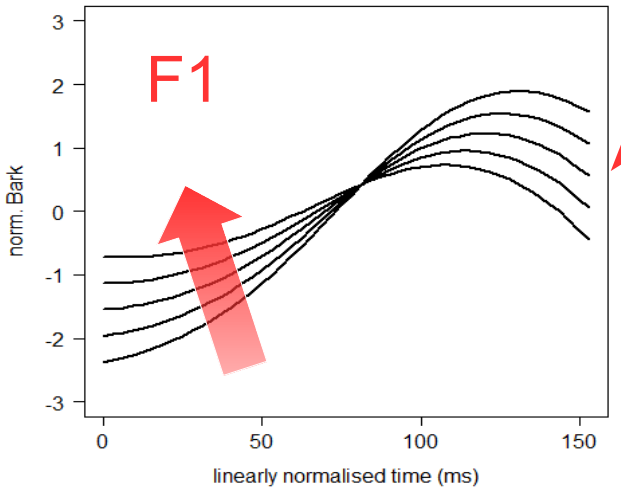
2D
FPCA

NUMBERS

- ANOVA
- LM
- LMER

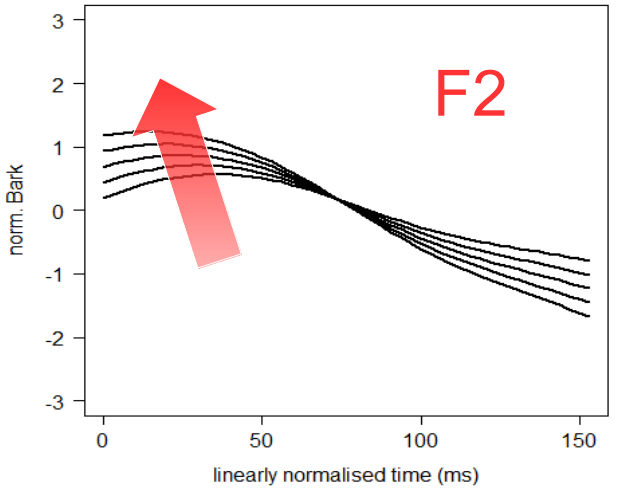
PC1 scores

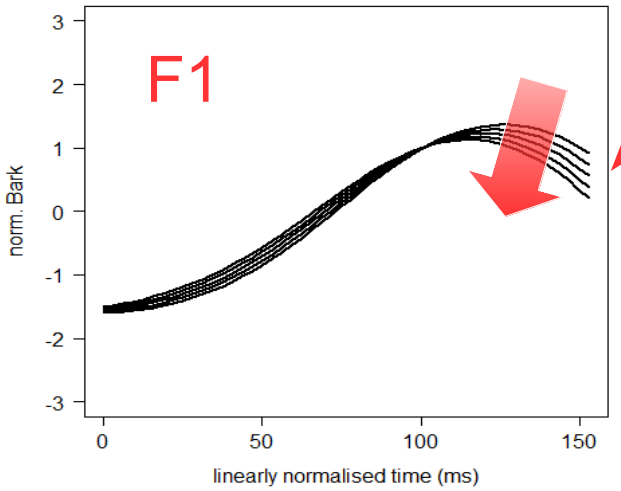




μ(t) + **8** * PC1(t)
μ(t) + **4** * PC1(t)
μ(t) + **0** * PC1(t)
μ(t) - **4** * PC1(t)
μ(t) - **8** * PC1(t)

PC2 scores

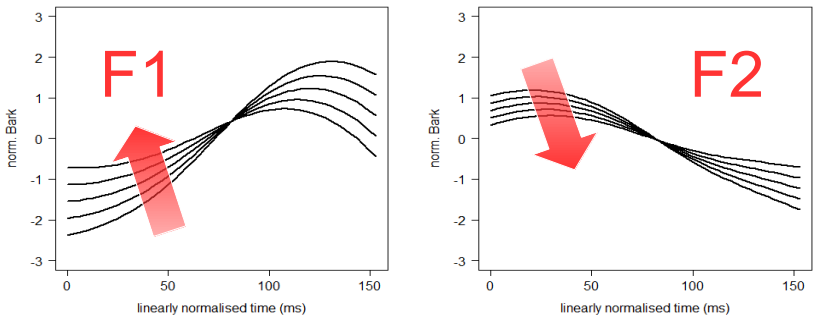




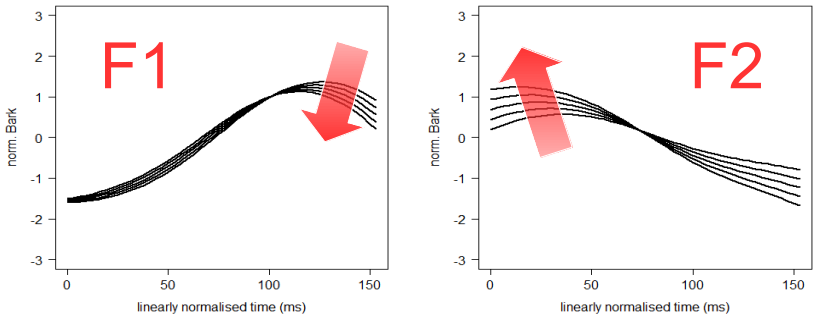
μ(t) + **4** * PC1(t)
μ(t) + **2** * PC1(t)
μ(t) + **0** * PC1(t)
μ(t) - **2** * PC1(t)
μ(t) - **4** * PC1(t)

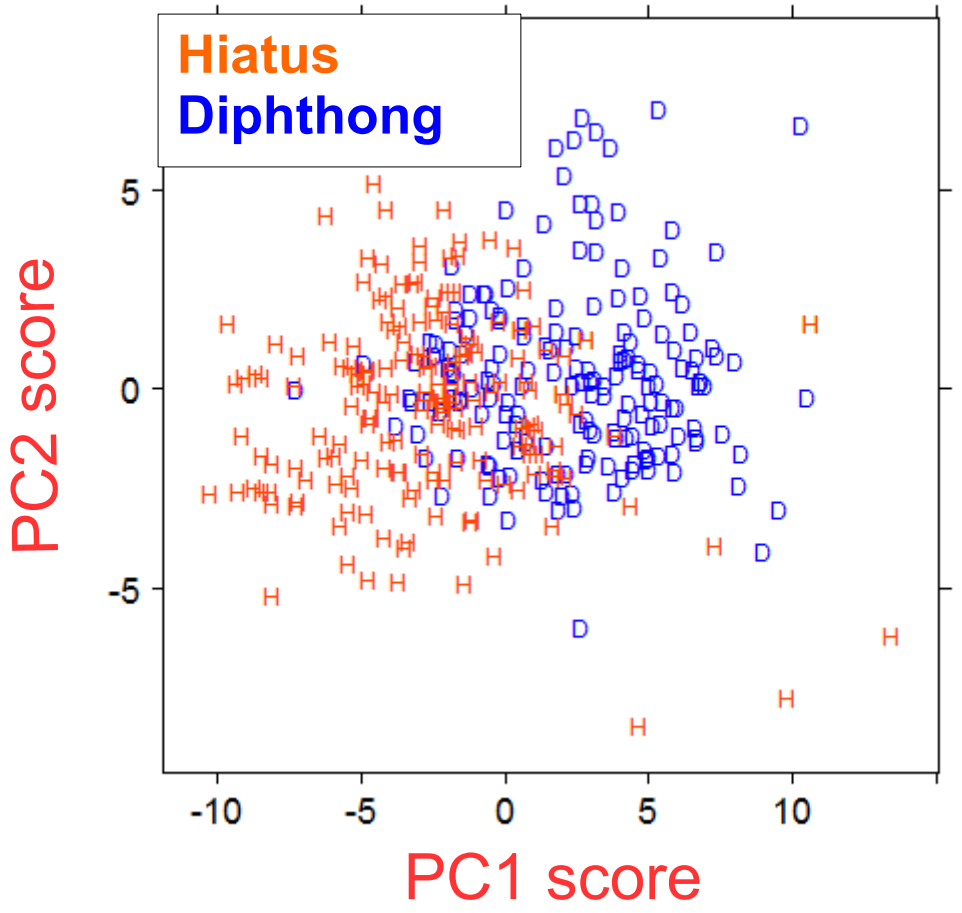
2D curve parametrisation

PC1 score

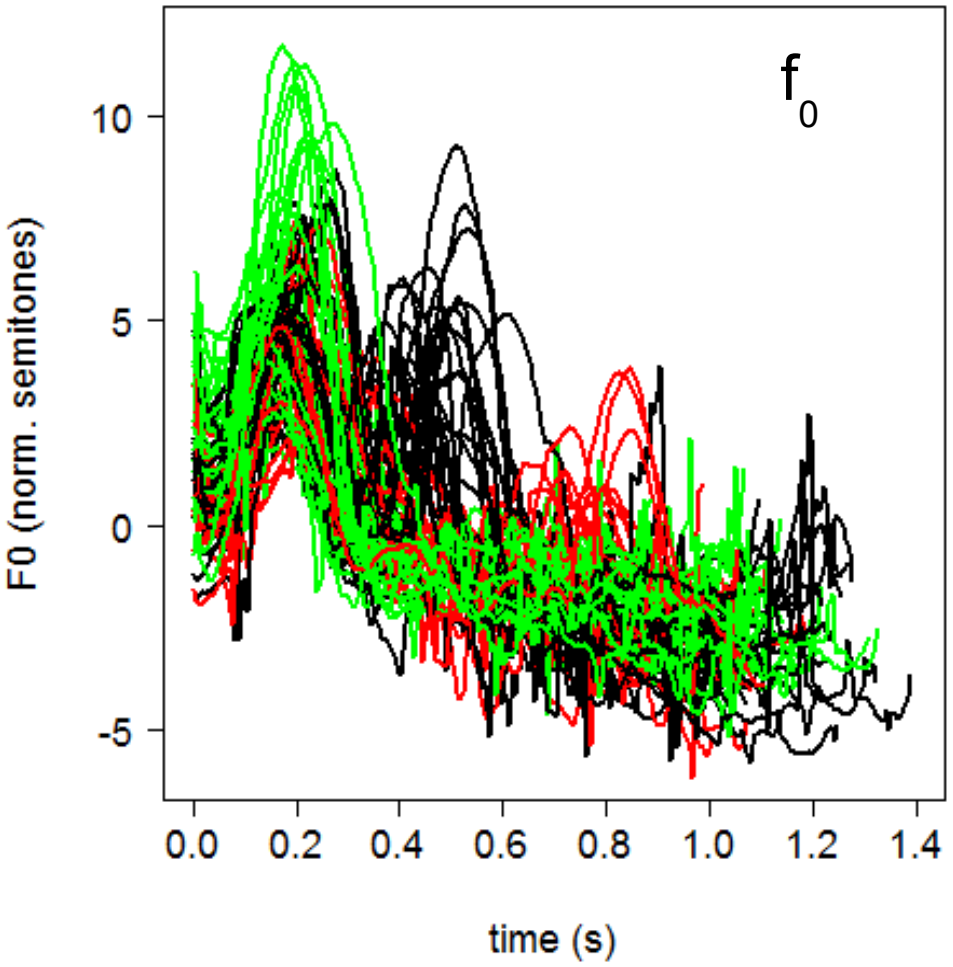


PC2 score





Many segments



- Narrow focus in Neapolitan Italian
- Focus on

  Subject, **Verb** or **Prop. Phrase**

  Danilo vola da Roma

  (*Danilo flies from Rome*)
- 8 CV syllables

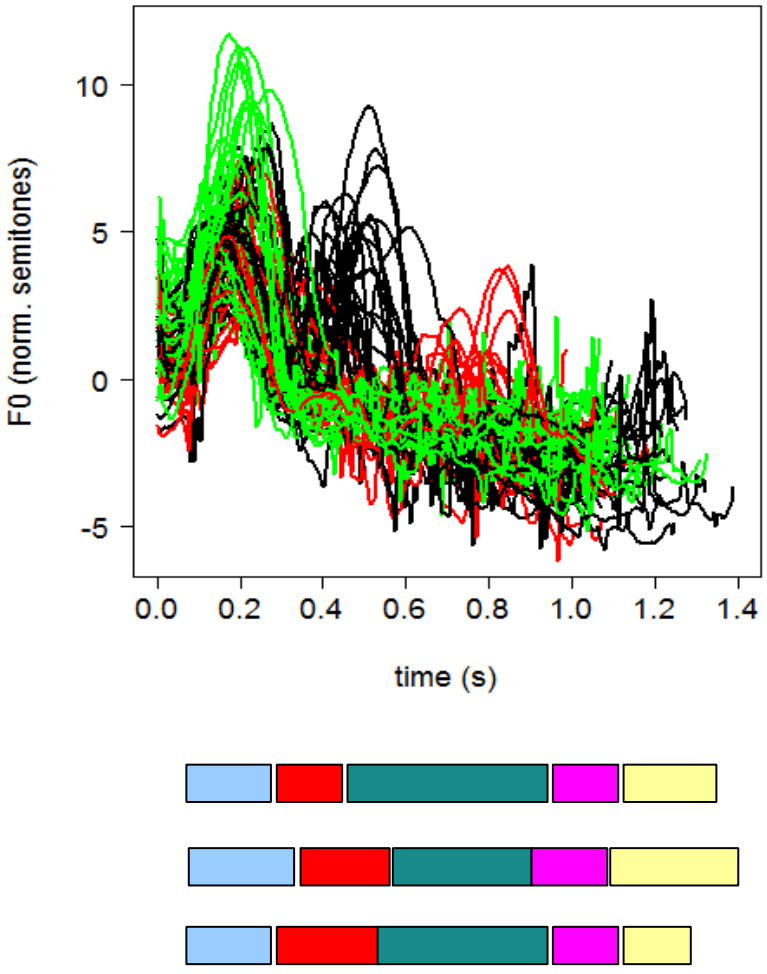
  first C was excluded (too short)

  VCVCV CVCV CV CVCV

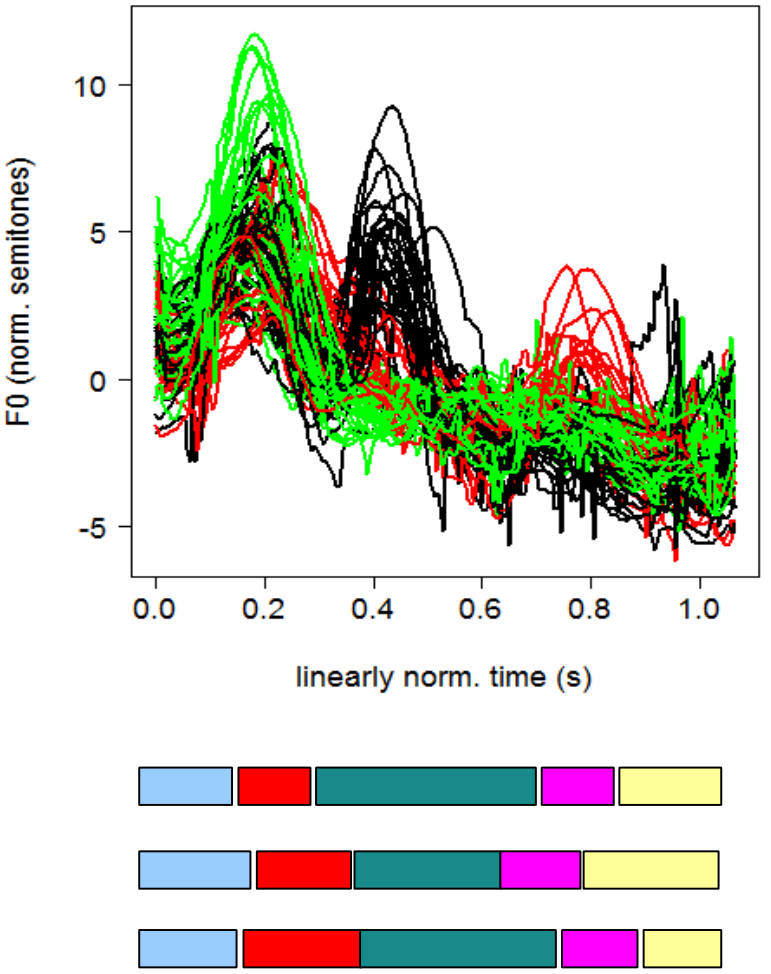
… 15 segments!

Linear time normalisation

BEFORE

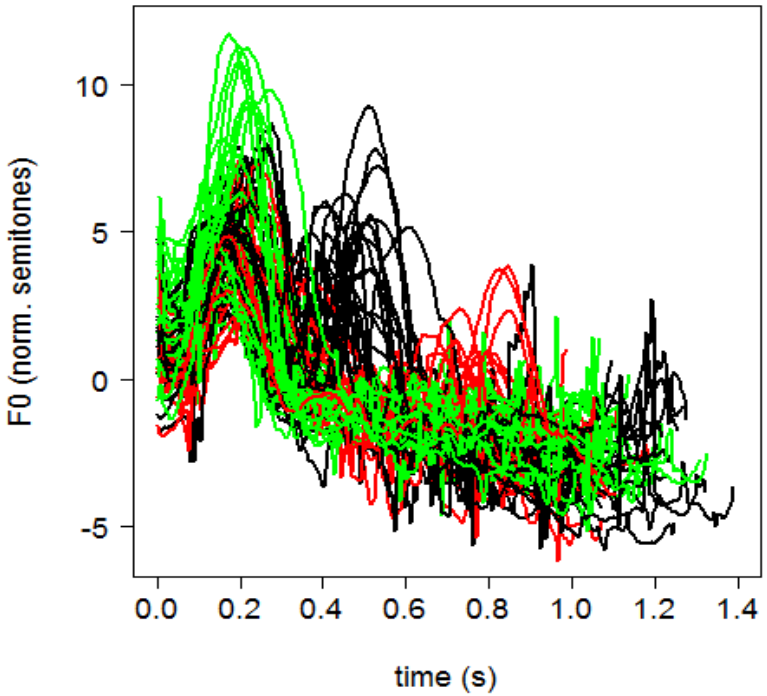


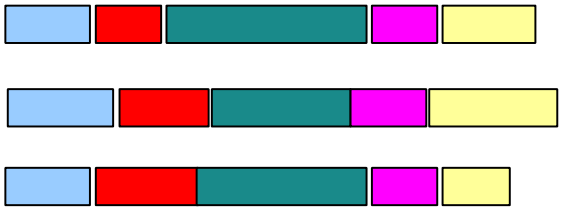
AFTER



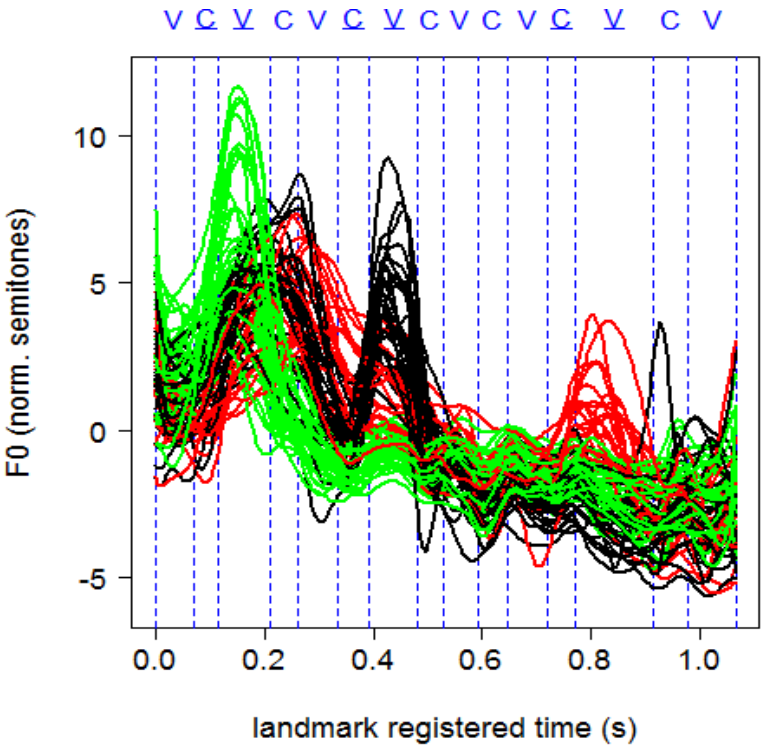
Landmark registration

BEFORE



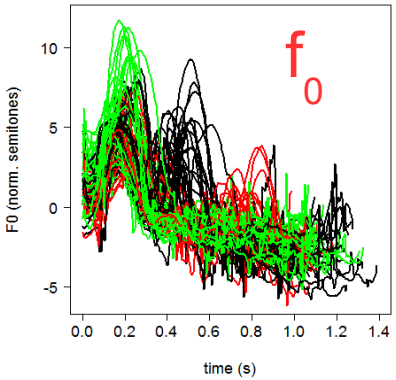


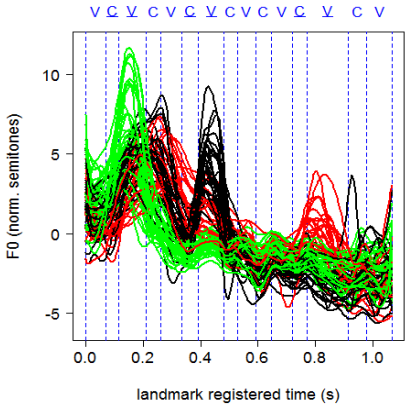
AFTER



Using landmark registration

CURVES





segment durations

d1	d2	...	d15
...	...	...	...
...	...	...	...
...	...	...	...

FPCA

PCA

NUMBERS

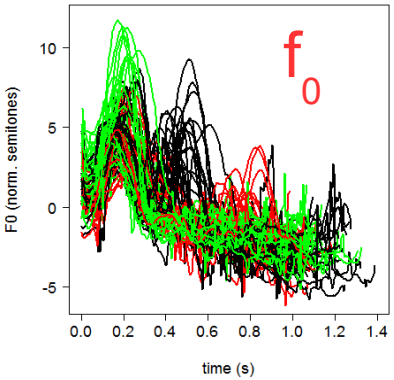
ANOVA

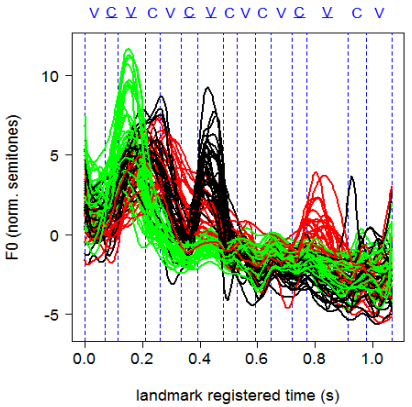
LM

LMER

Using landmark registration

CURVES





segment durations

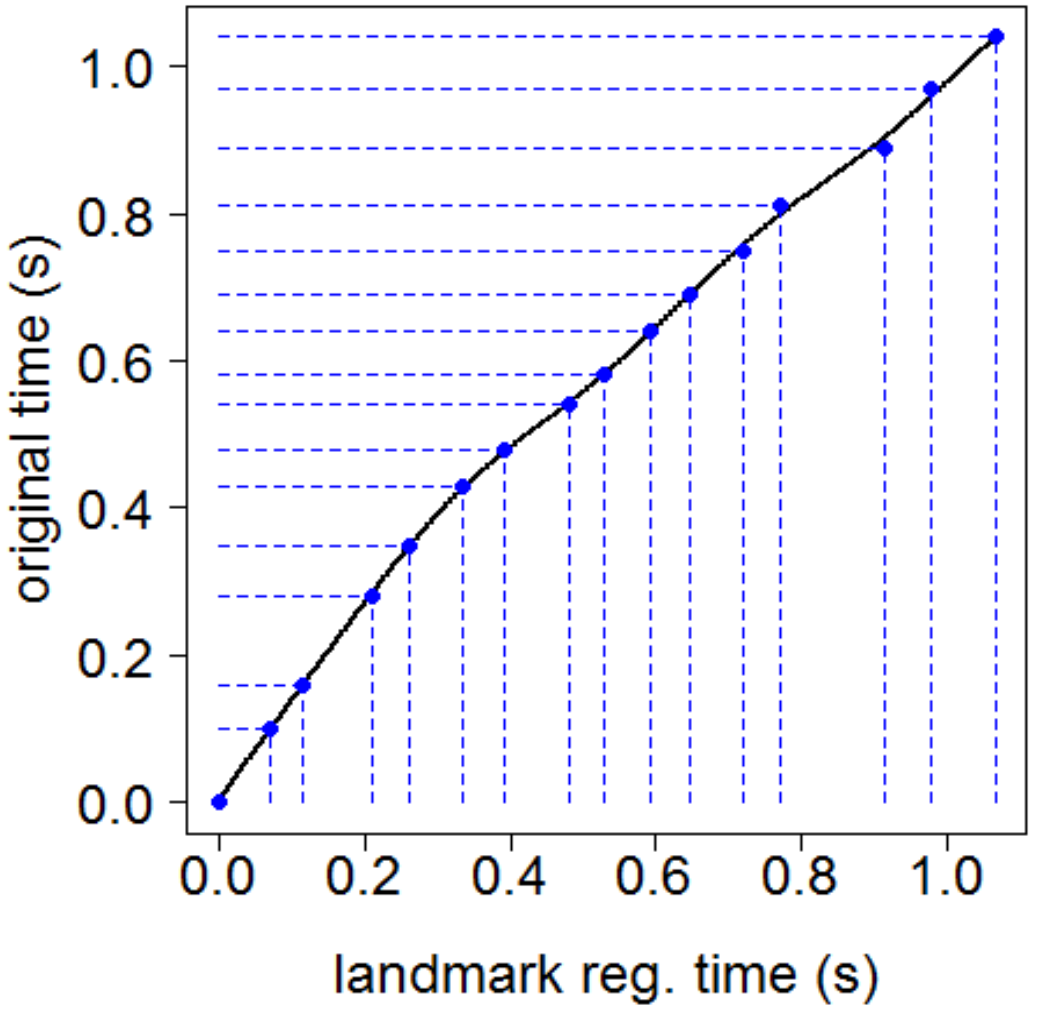
d1	d2	...	d15
...	...	...	...
...	...	...	...
...	...	...	...

FPCA

NUMBERS

- ANOVA
- LM
- LMER

Inside landmark registration

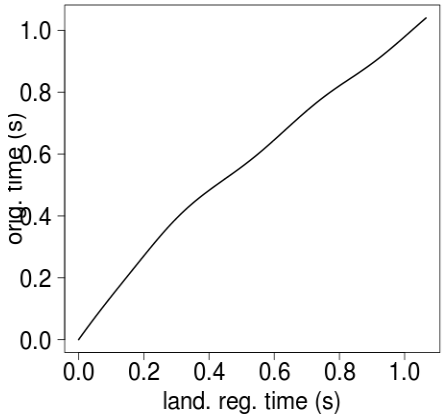


h(t)

Relative log rate

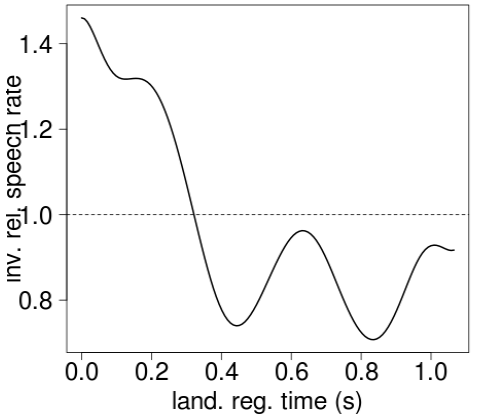
1

h(t)



2

dh(t)/dt

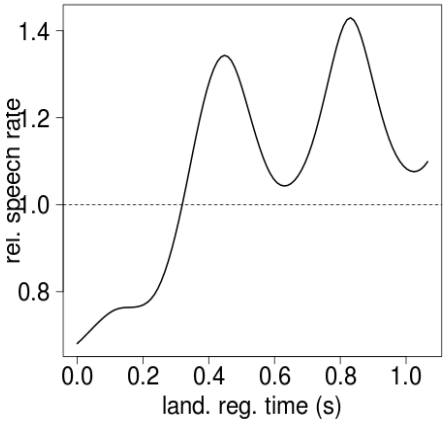


REVERSIBLE!

1 ⟷ 4

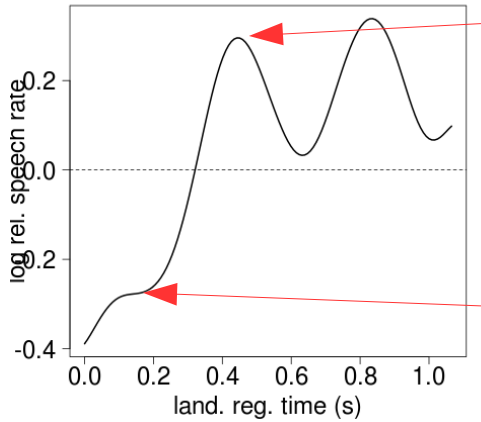
3

- dh(t)/dt



4

- log dh(t)/dt



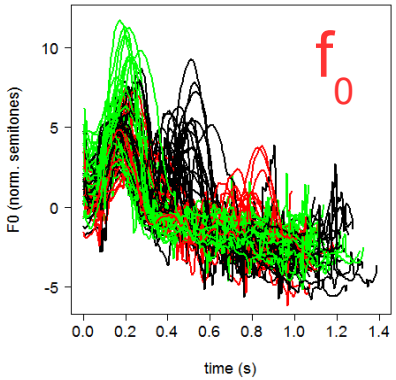
+ 0.25 → duration / 1.28

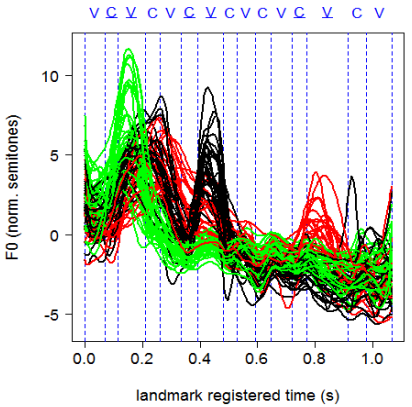
0 → same duration

- 0.25 → duration * 1.28

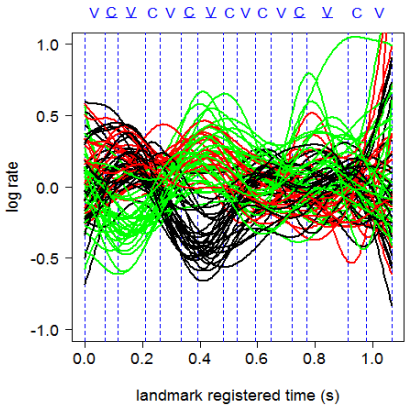
Using log rates

CURVES





log rates

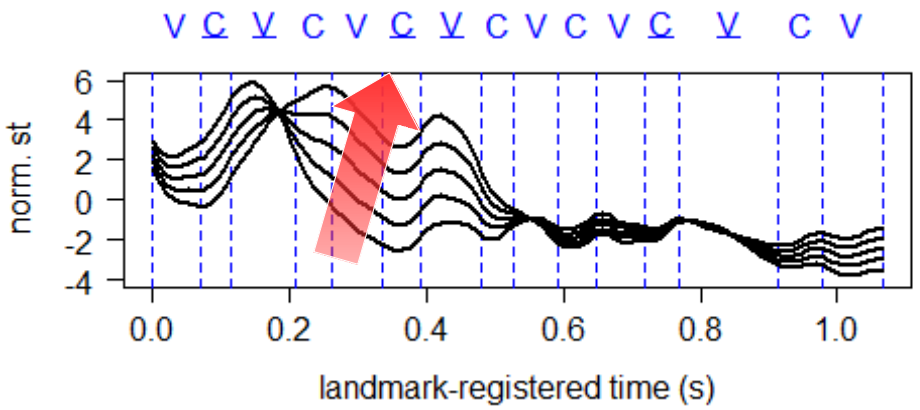


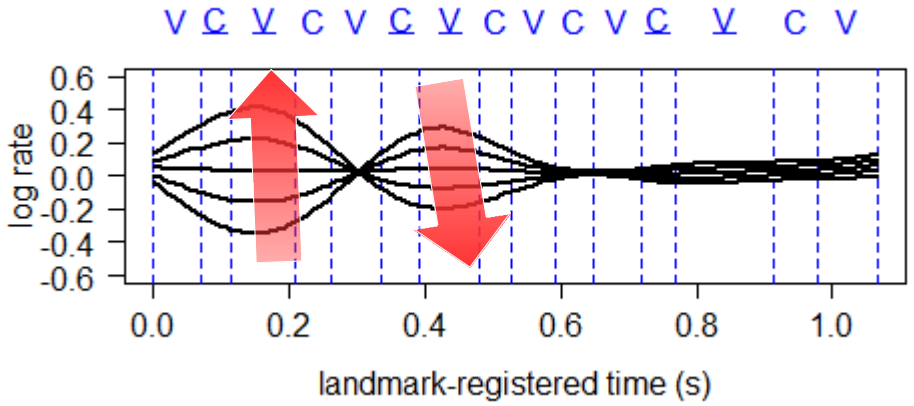
2D FPCA

NUMBERS

- ANOVA
- LM
- LMER

PC1 scores



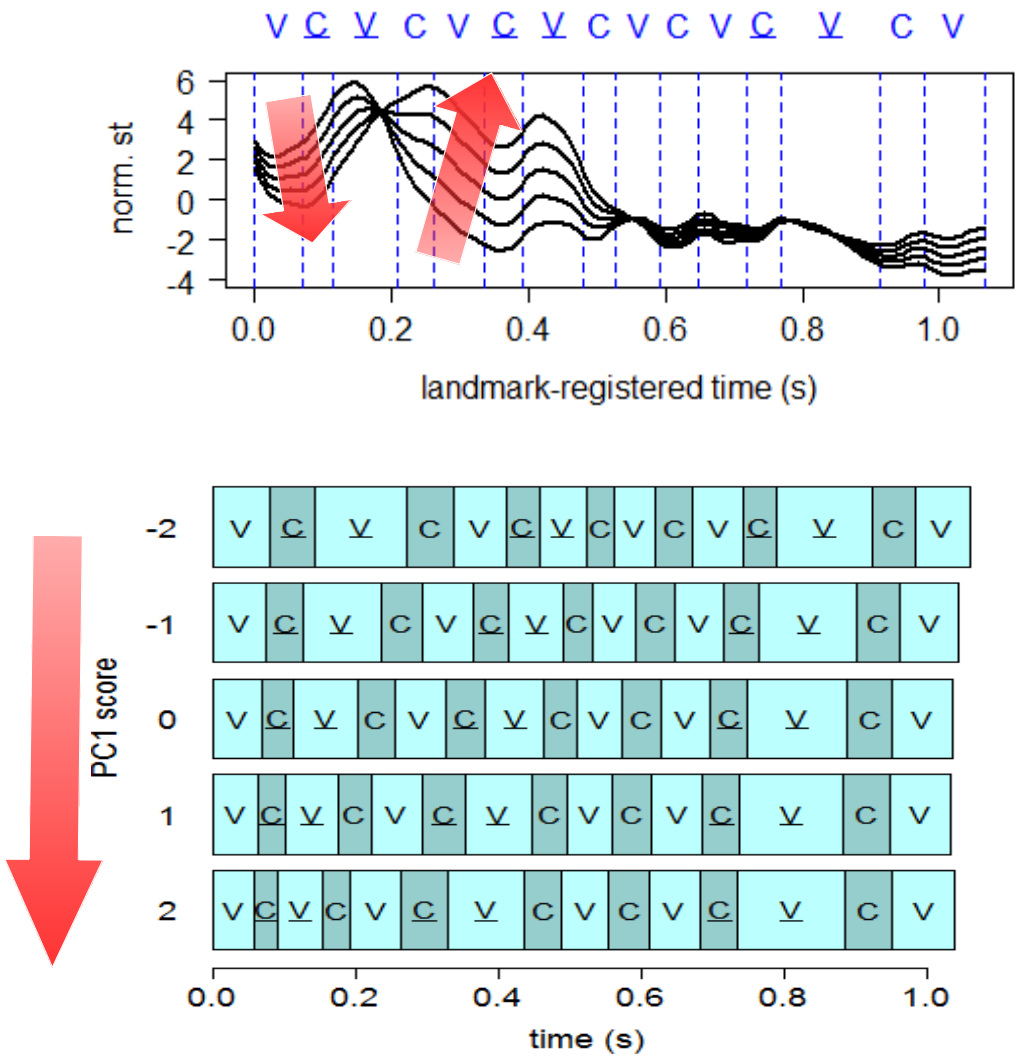


f_0

log rates

$\mu(t) + 2 * PC1(t)$
$\mu(t) + 1 * PC1(t)$
$\mu(t) + 0 * PC1(t)$
$\mu(t) - 1 * PC1(t)$
$\mu(t) - 2 * PC1(t)$

PC1 scores

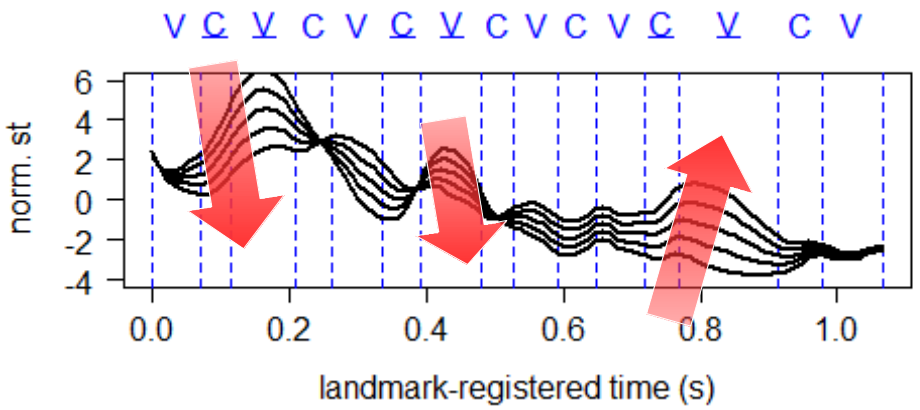


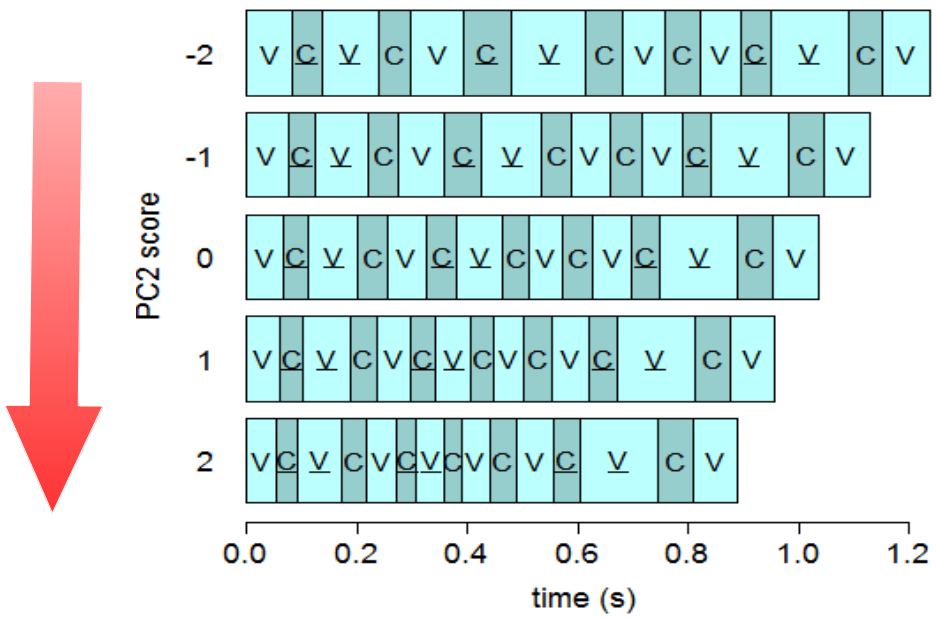
f_0

$\mu(t) + 2 * PC1(t)$
$\mu(t) + 1 * PC1(t)$
$\mu(t) + 0 * PC1(t)$
$\mu(t) - 1 * PC1(t)$
$\mu(t) - 2 * PC1(t)$

segment durations

PC2 scores





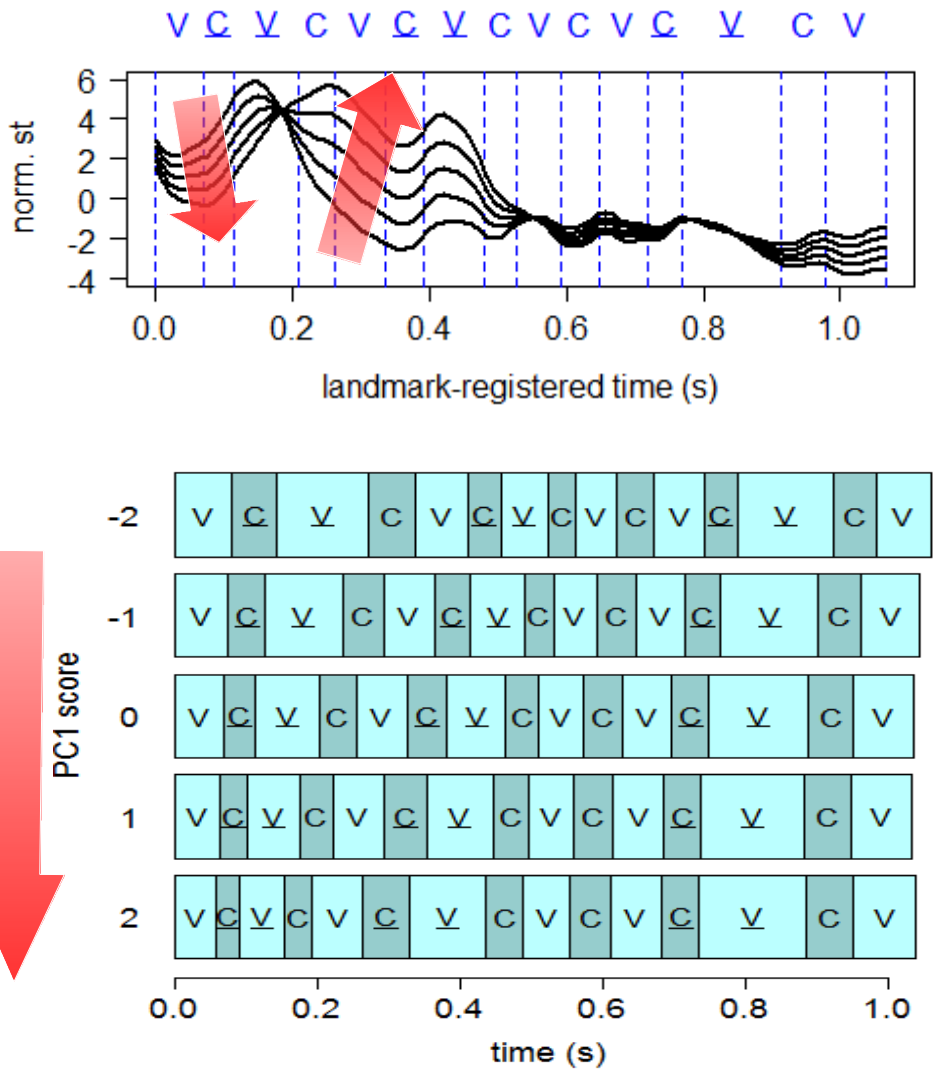
f_0

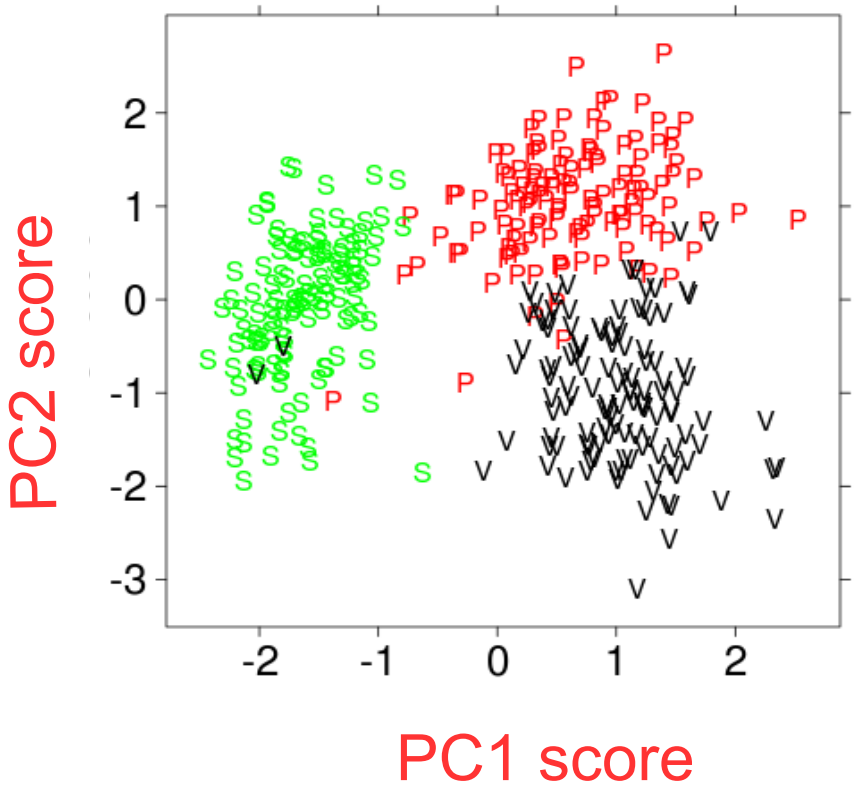
$\mu(t) + 2 * PC1(t)$
$\mu(t) + 1 * PC1(t)$
$\mu(t) + 0 * PC1(t)$
$\mu(t) - 1 * PC1(t)$
$\mu(t) - 2 * PC1(t)$

segment durations

multi-segment curve parametrisation

PC1 score





multi-segment curve parametrisation