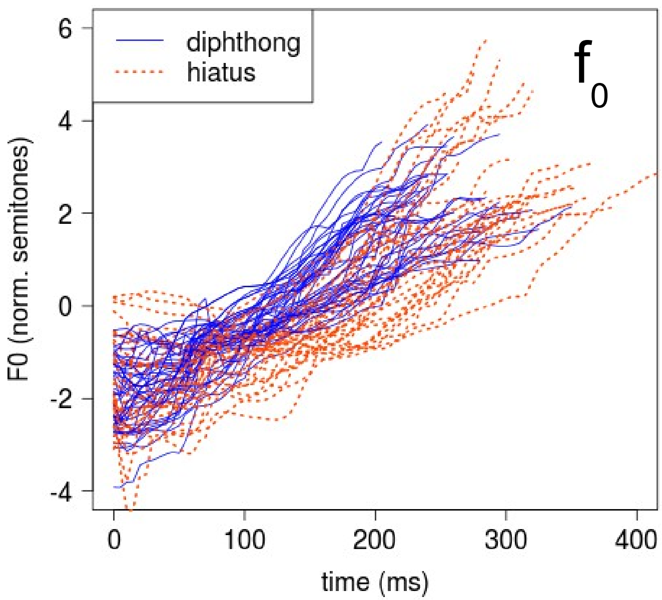
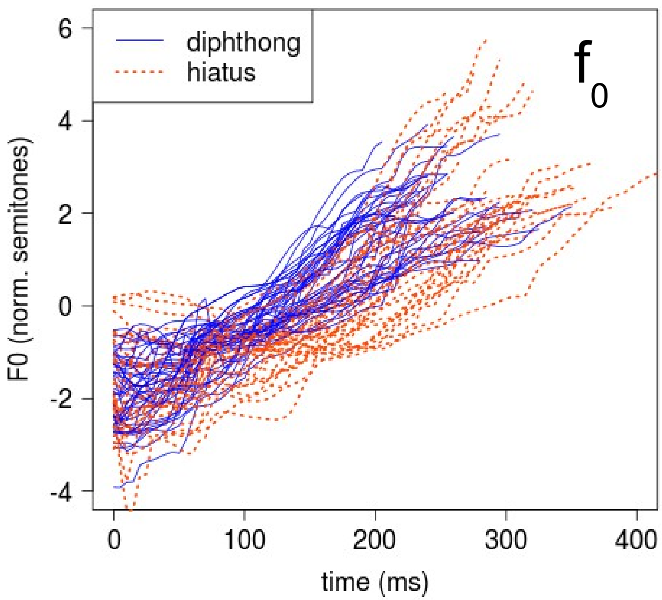
Functional Data Analysis (FDA) for phonetic research

Michele Gubian
University of Bristol, UK



- European Spanish
- **Diphthong**: /ja/
- **Hiatus** /i.a/
- Rising pitch accent
- Tonal alignment?



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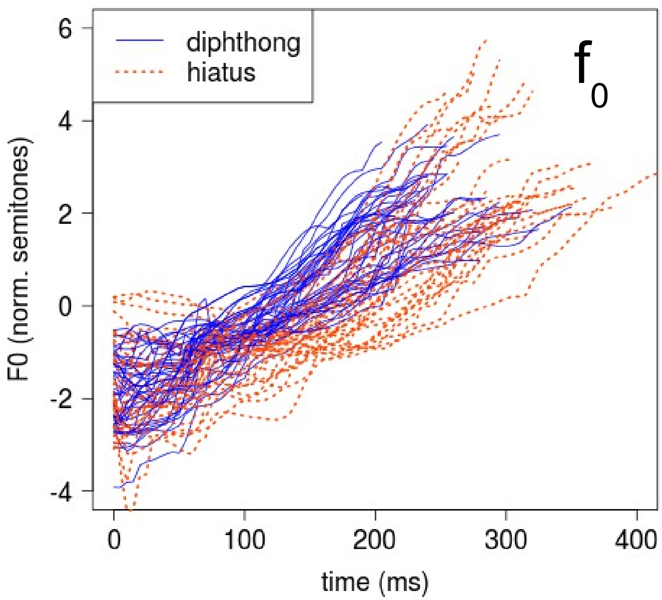
ANOVA

LR

LMER

CURVES



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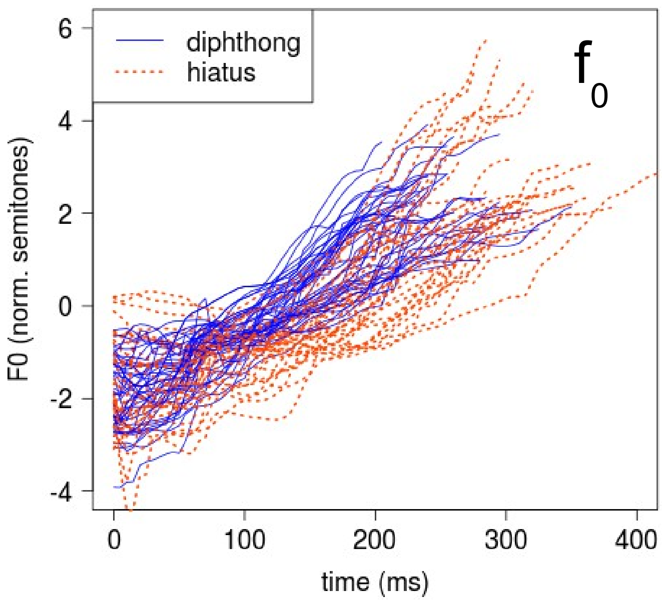
MIND THE GAP

NUMBERS

- ANOVA
- LR
- LMER

CURVES

NUMBERS



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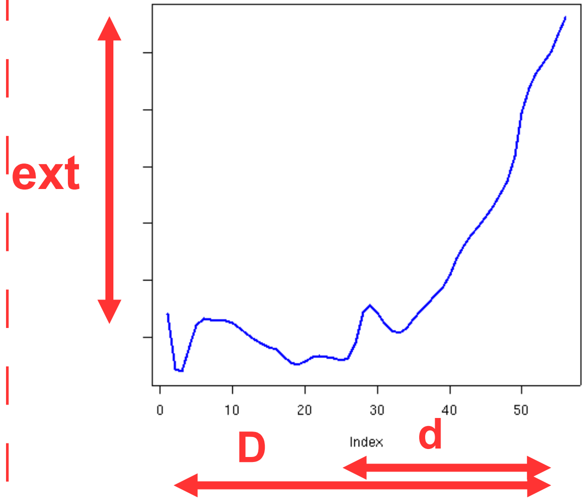


ext (st)	d/D	Cat.
5.3	0.9	D
4.6	0.7	H
...	...	...

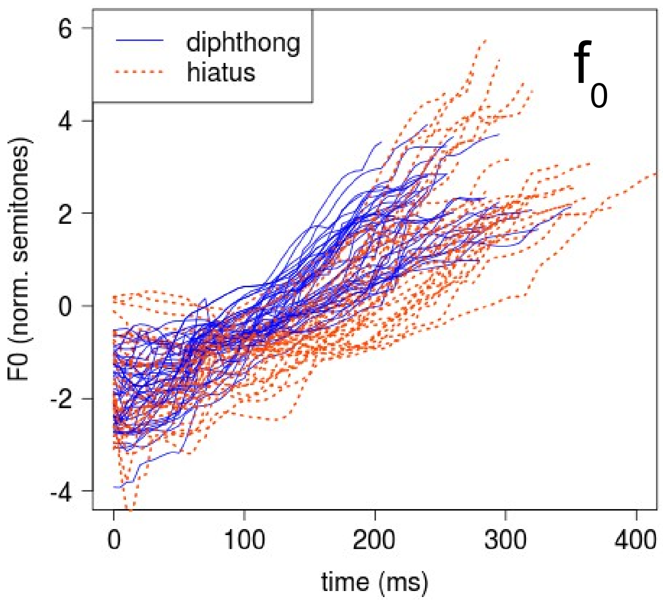
ANOVA

LR

LMER

CURVES

NUMBERS



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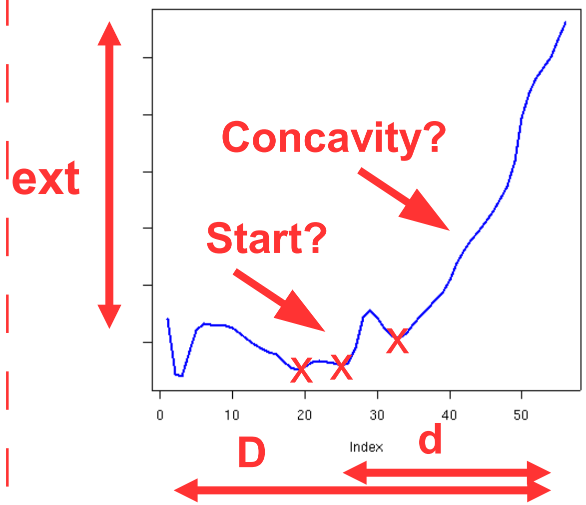


ext (st)	d/D	Cat.
5.3	0.9	D
4.6	0.7	H
...	...	...

ANOVA

LR

LMER

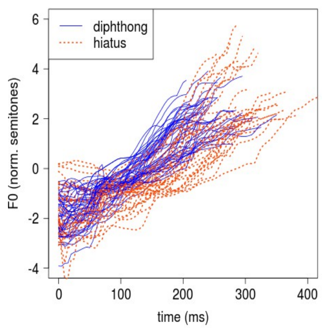
MISSION

automate curve parametrisation

- Data driven
- Few parameters
- Interpretable

Road map

CURVES



Interpolate using a function basis

- Data driven

Dimensionality reduction tool

- Few parameters
- Interpretable

NUMBERS

ANOVA

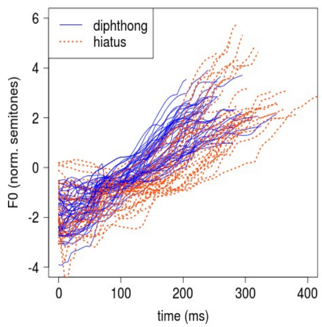
LM

LMER

Road map

CURVES

NUMBERS



Interpolate using a function basis

Dimensionality reduction tool

ANOVA

LM

LMER

- Data driven

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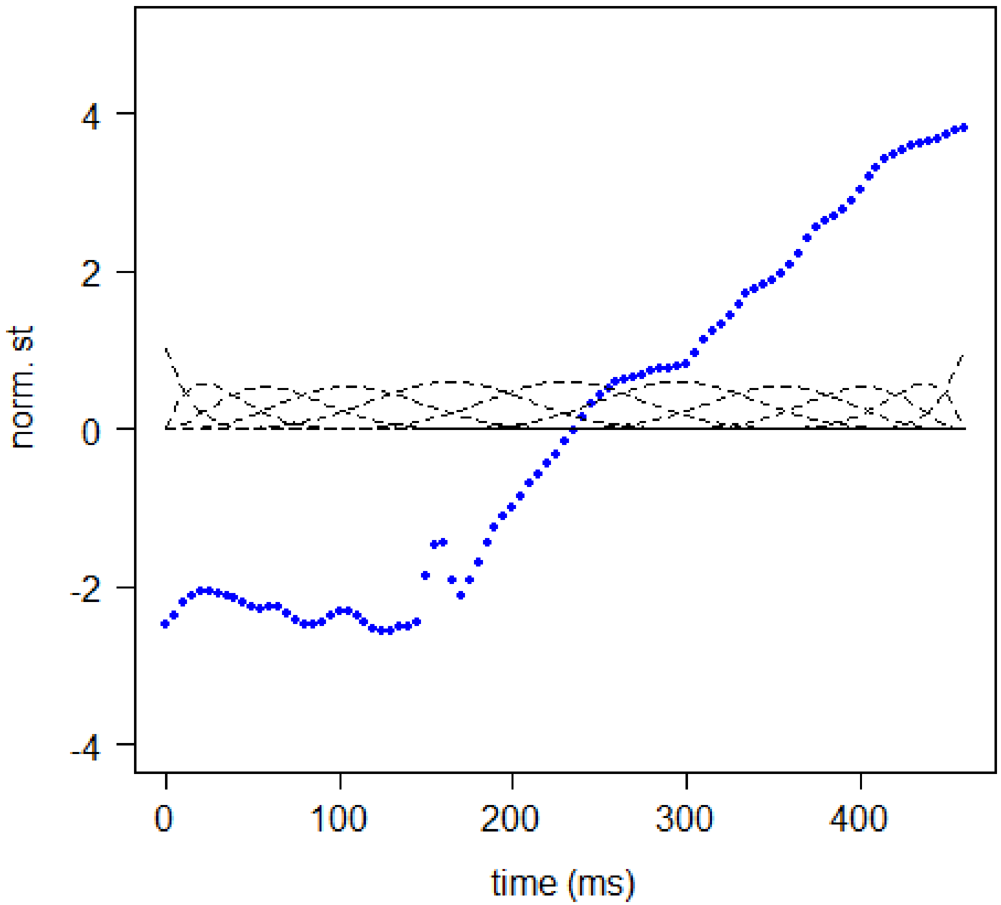
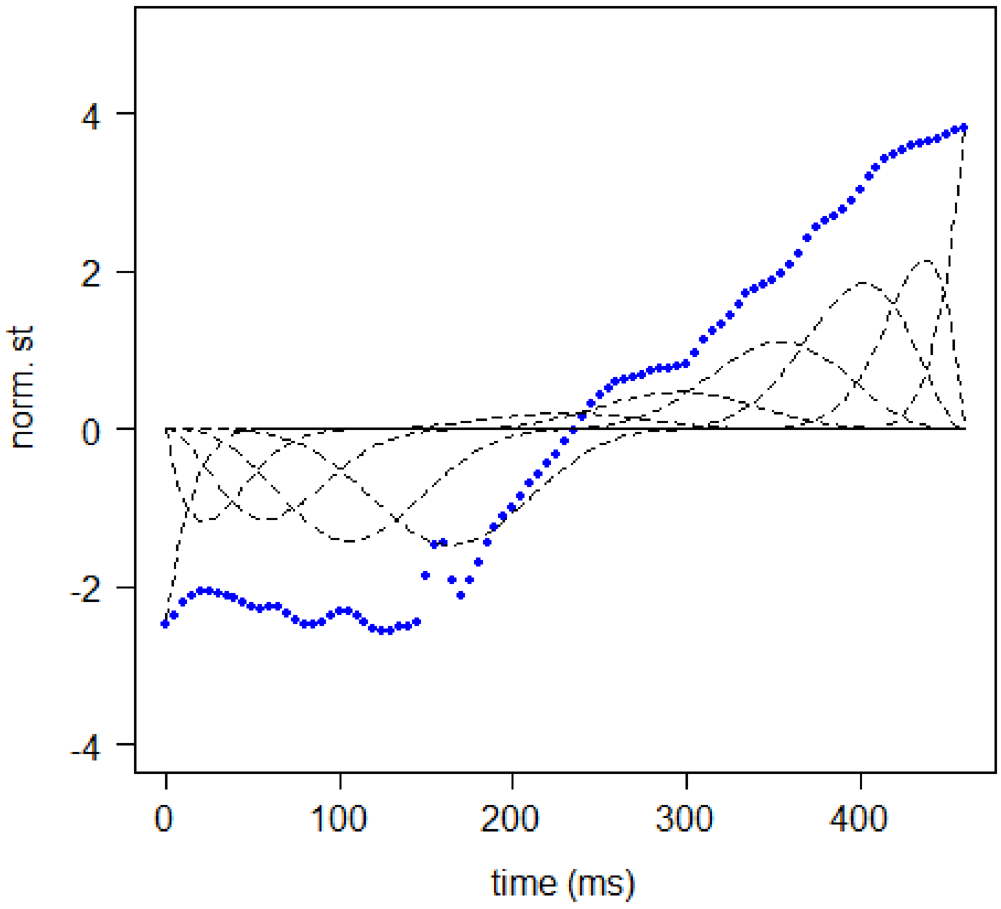
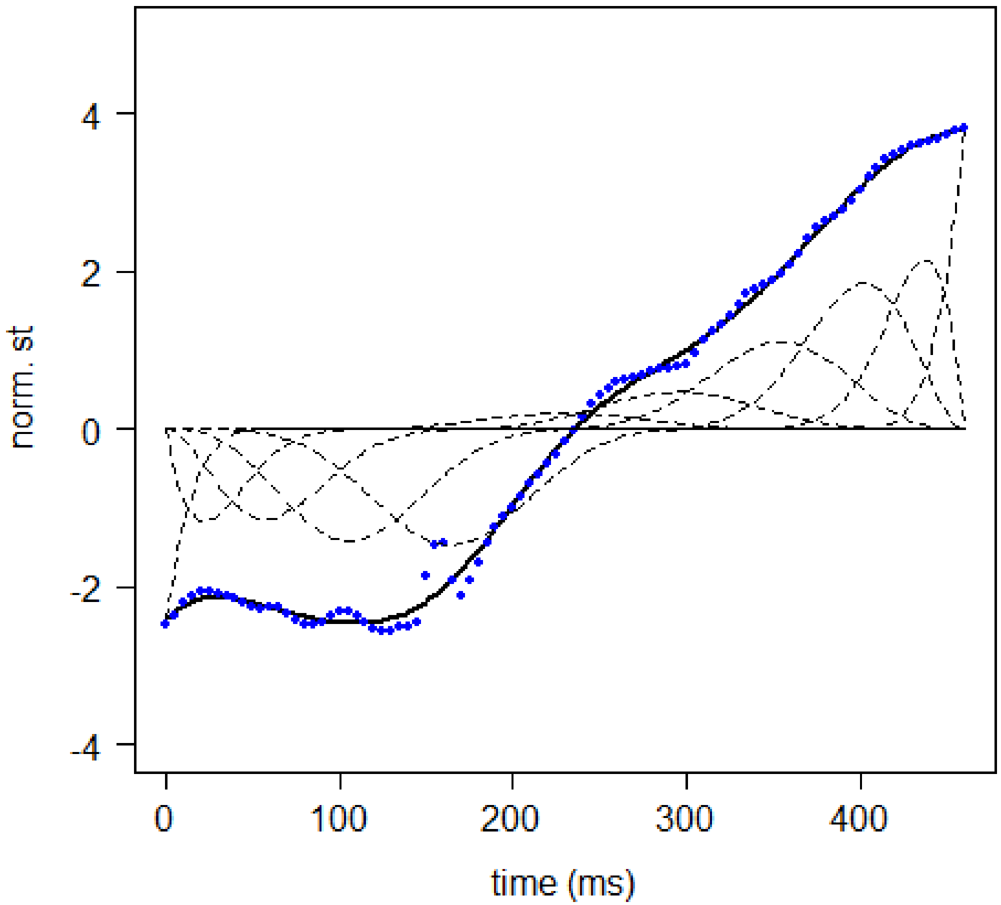
Interpolation with B-splines



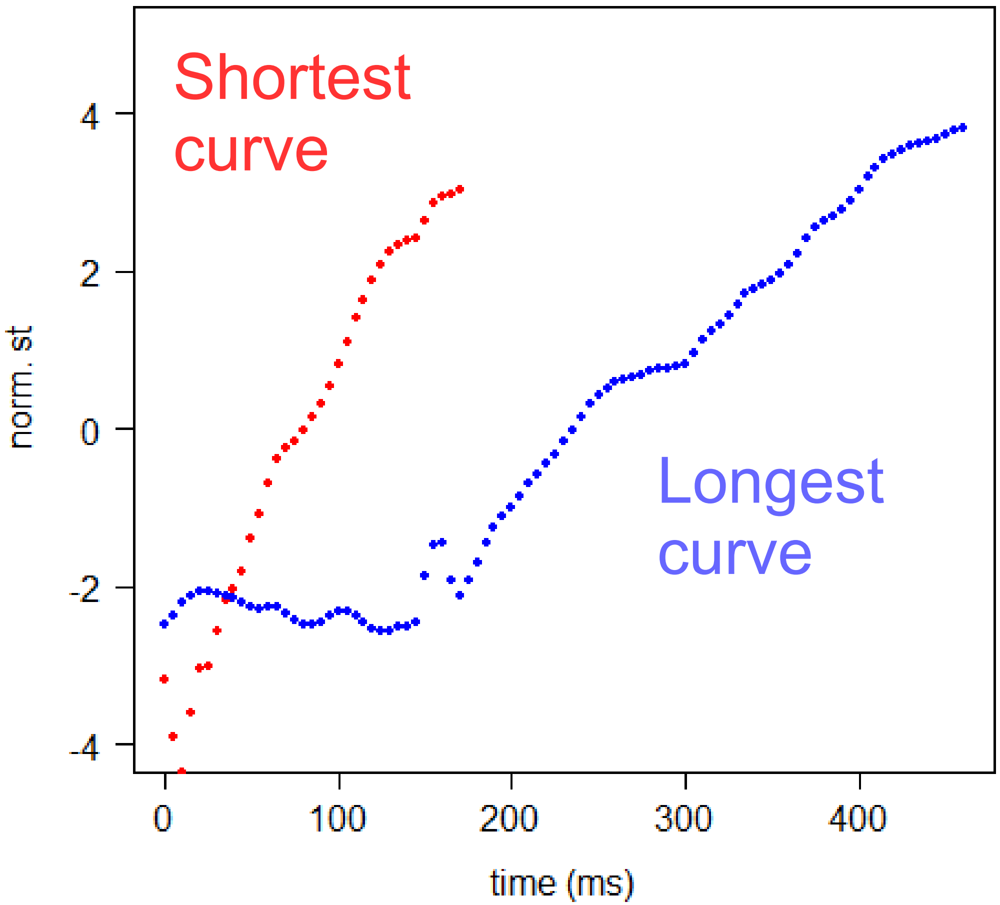
Interpolation with B-splines



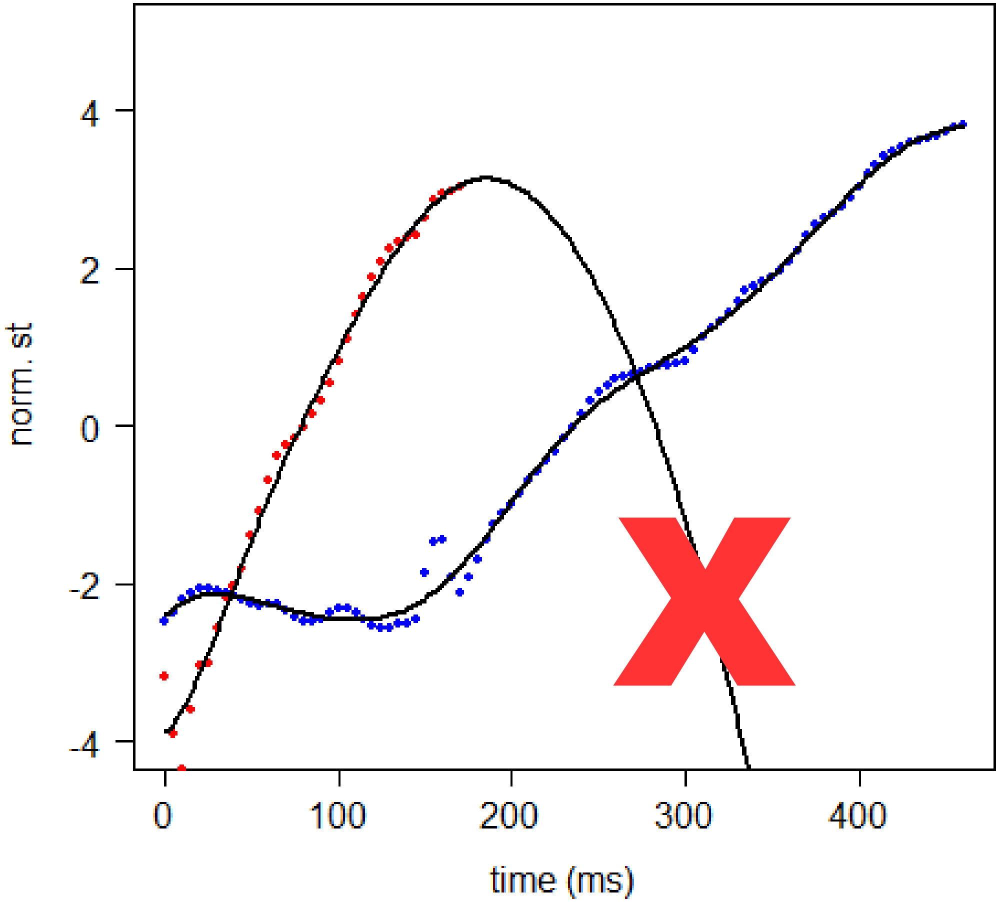
Interpolation with B-splines



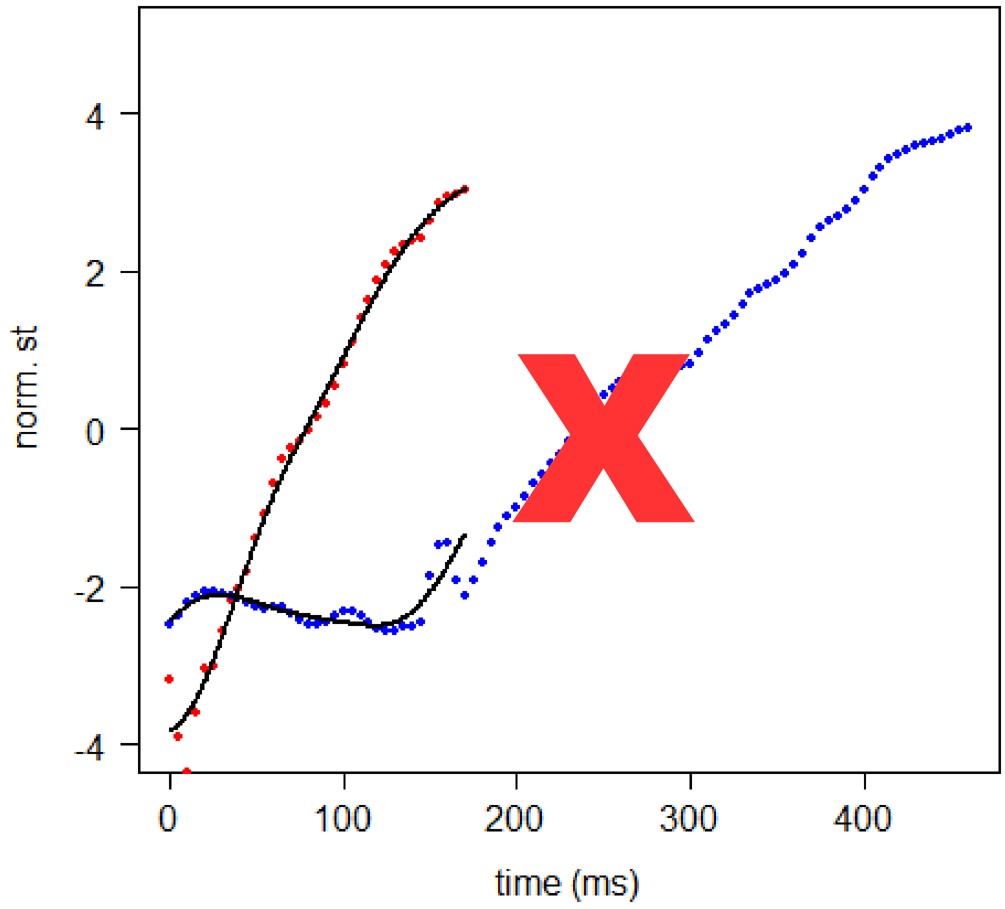
Different durations



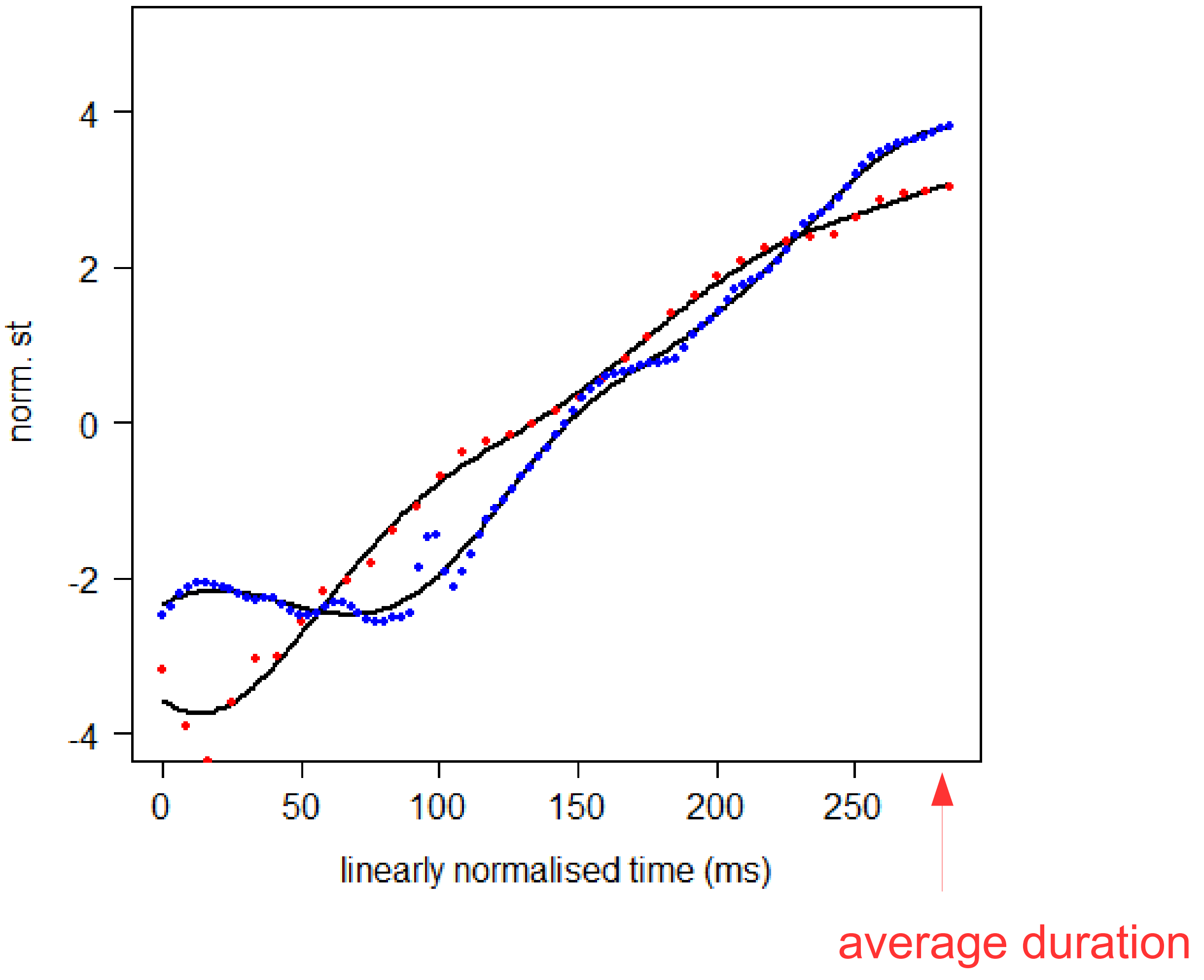
Take longest duration



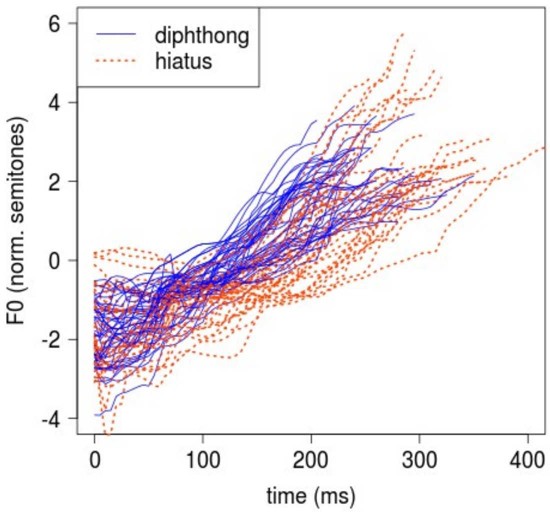
Take shortest duration



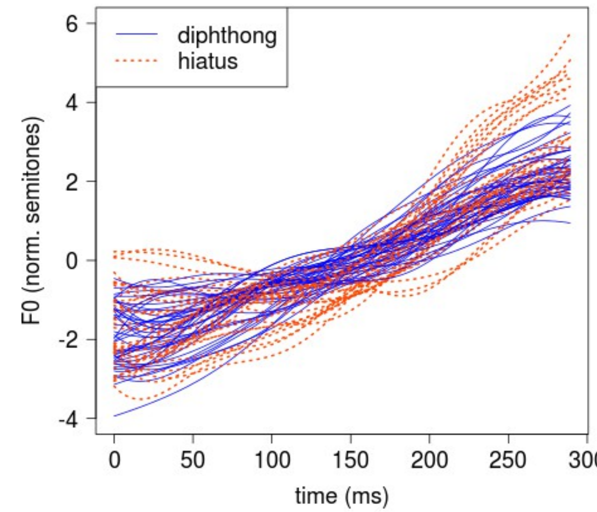
Linear time normalisation



Linear time normalisation



Interpolate to
the same
time interval

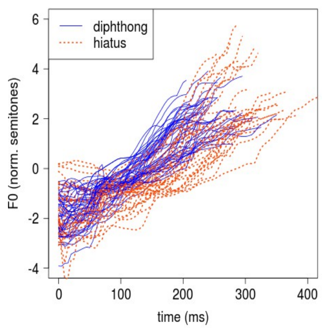


- We must use the same time interval
- This implies linear time normalisation
- Durations have to be reintroduced at the end of the analysis

Road map

CURVES

NUMBERS



Interpolate to <u>the same</u> time interval

Dimensionality reduction tool

ANOVA

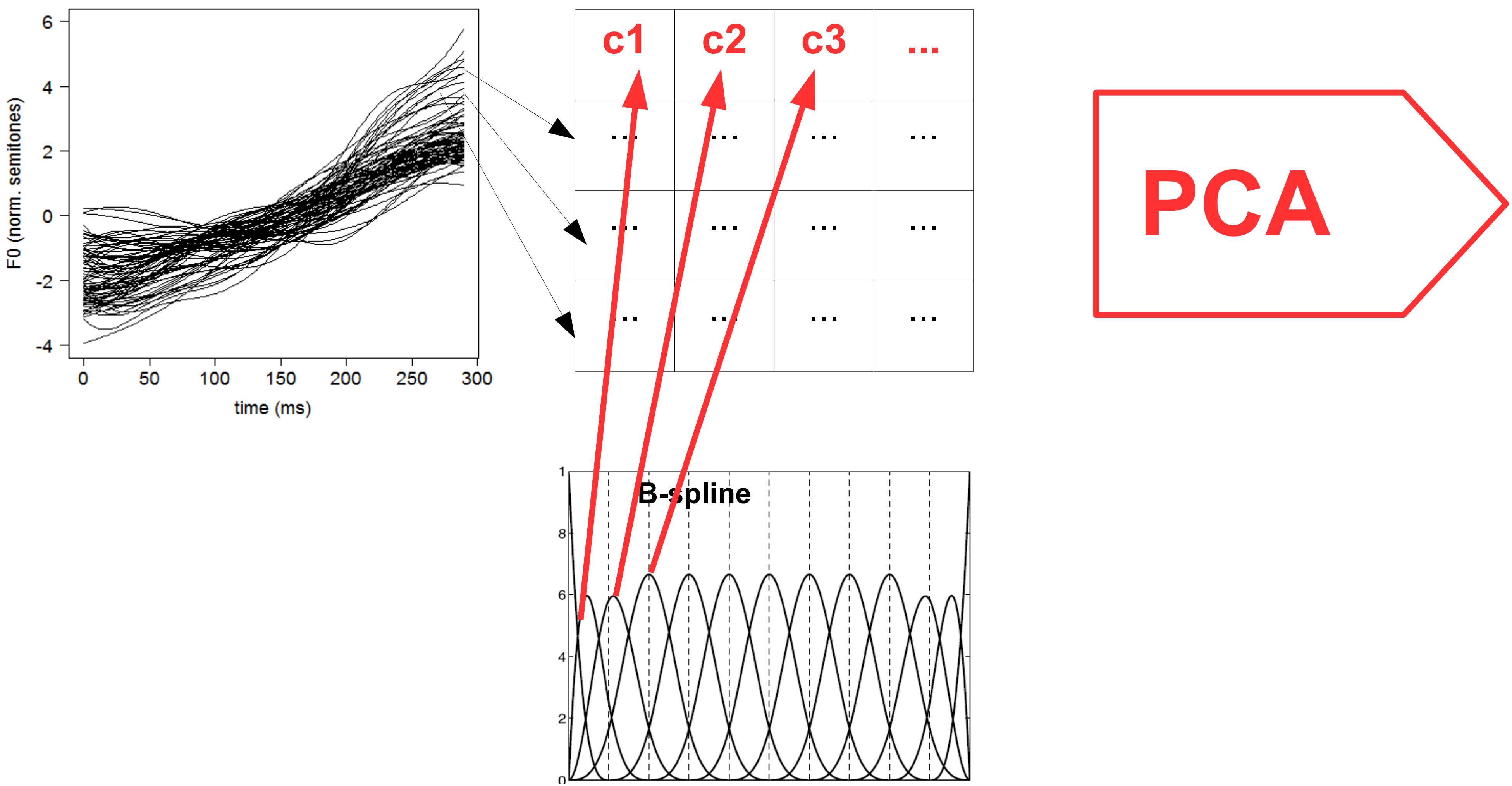
LM

LMER

- Data driven

- Few parameters
- Interpretable

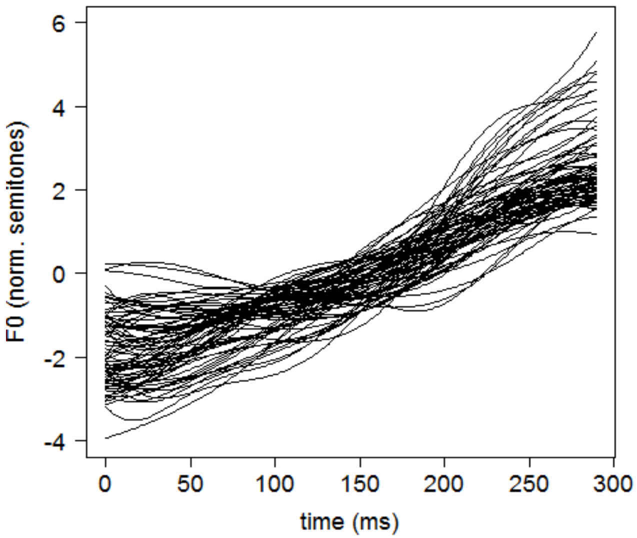
Principal Component Analysis



PCA limitations

- PCA does not use any explicit information related to the curve shapes or the B-splines shapes
- e.g. the sequence of coefficients c1, c2,.. reflects time adjacency of polynomial components, i.e. overlapping 'hills'

Discrete Cosine Transform



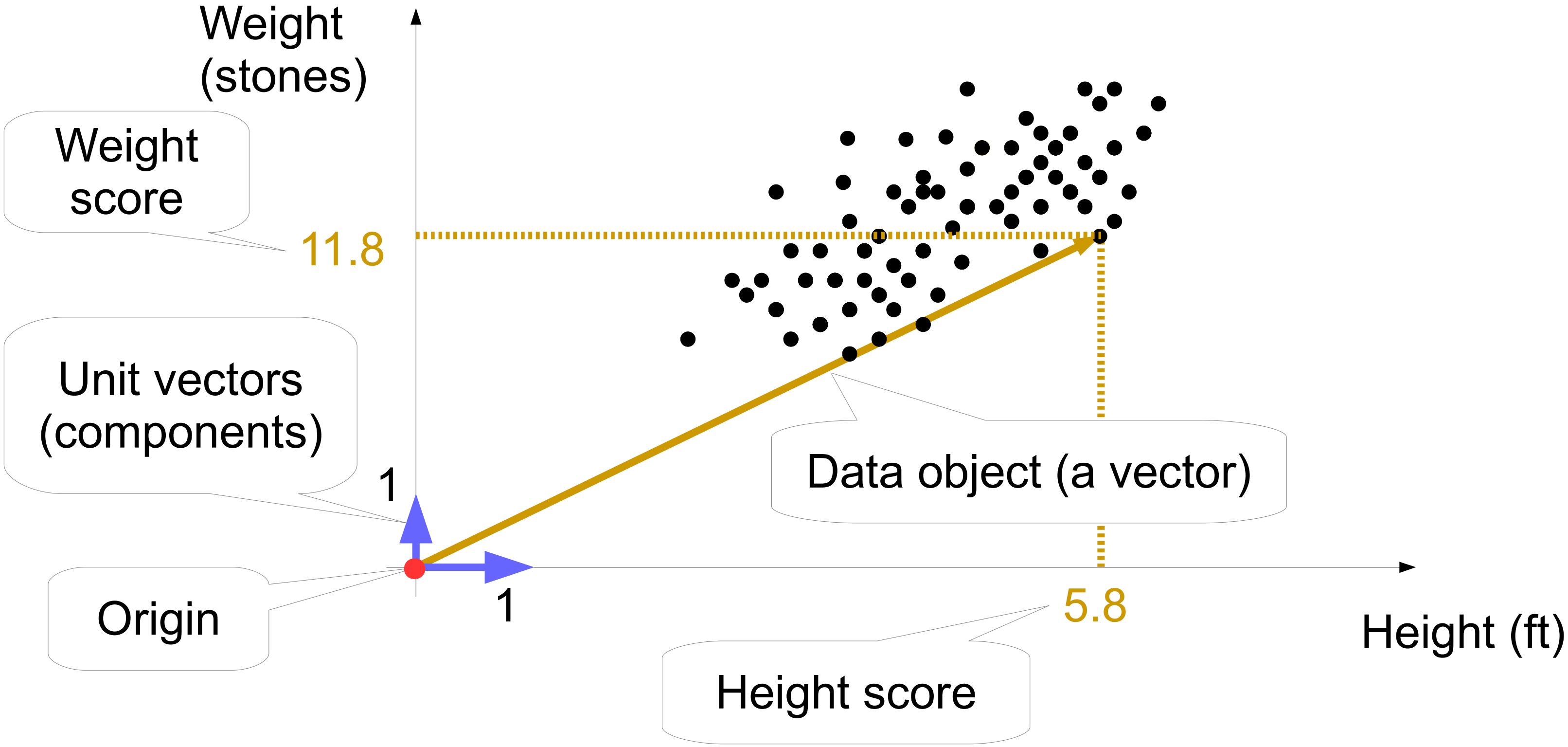
k0	k1	k2	...
...	...	...	...
...	...	...	...
...	...	...	...

DCT limitations

- DCT does not (easily) encode time-localised information, e.g. a small hump in the same (time-normalised) position
- Typically only k0, k1 and k2 are used, which have a geometric interpretation
- Extracting several k's brings up the need of PCA
- In general, not effective to encode long signals

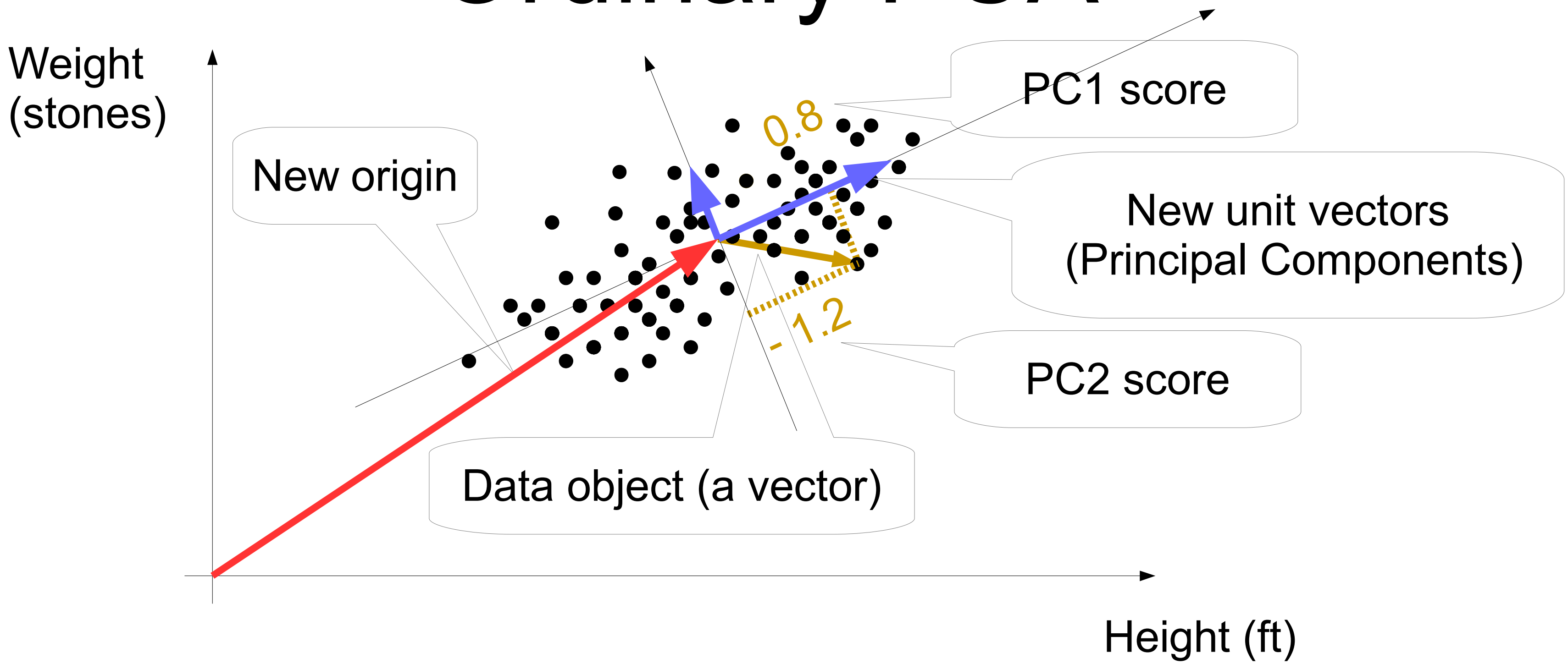
Introducing
Functional PCA

Vectors



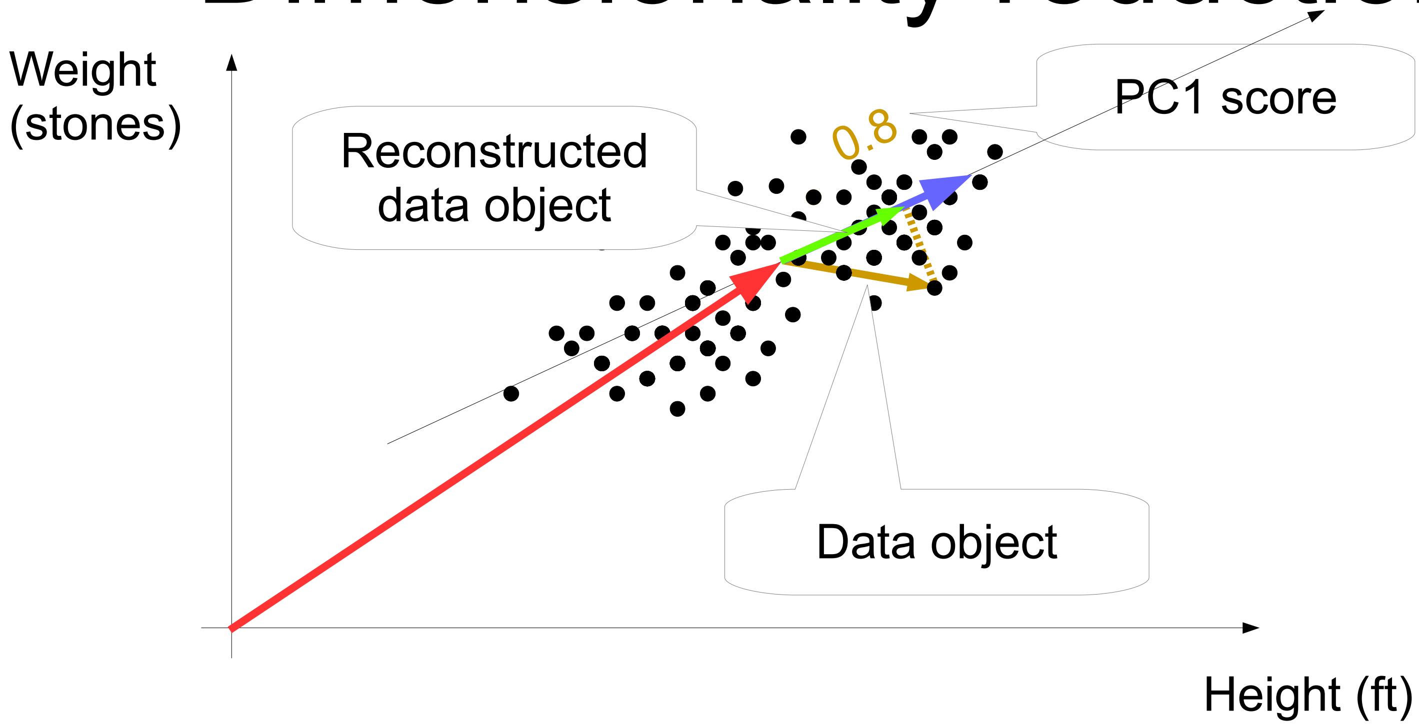
- Data objects and components are vectors
- From scores (numbers) we can reconstruct data objects (vectors)

Ordinary PCA

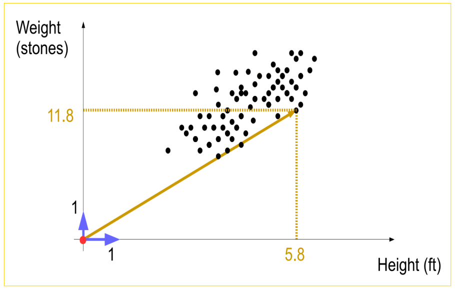


- PCA computes new origin and unit vectors which best suit the data
- From PC scores we can reconstruct data objects

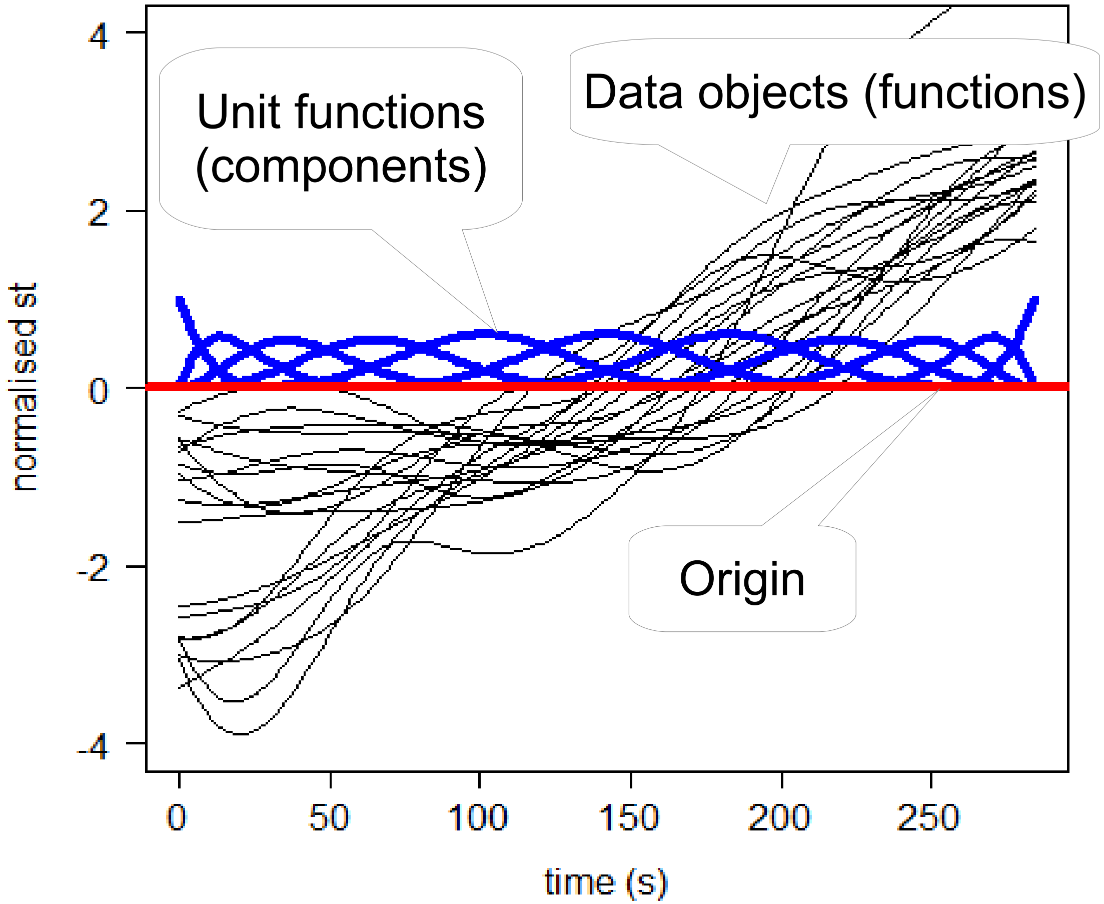
Dimensionality reduction



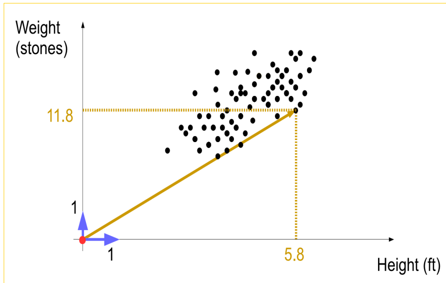
- We can use only part of the PCs
- This reduces the data dimensionality
- But introduces reconstruction errors too



Functions (curves)



- Origin, components and data objects are functions
- Origin is a flat line
- Components are 11 B-spline curves



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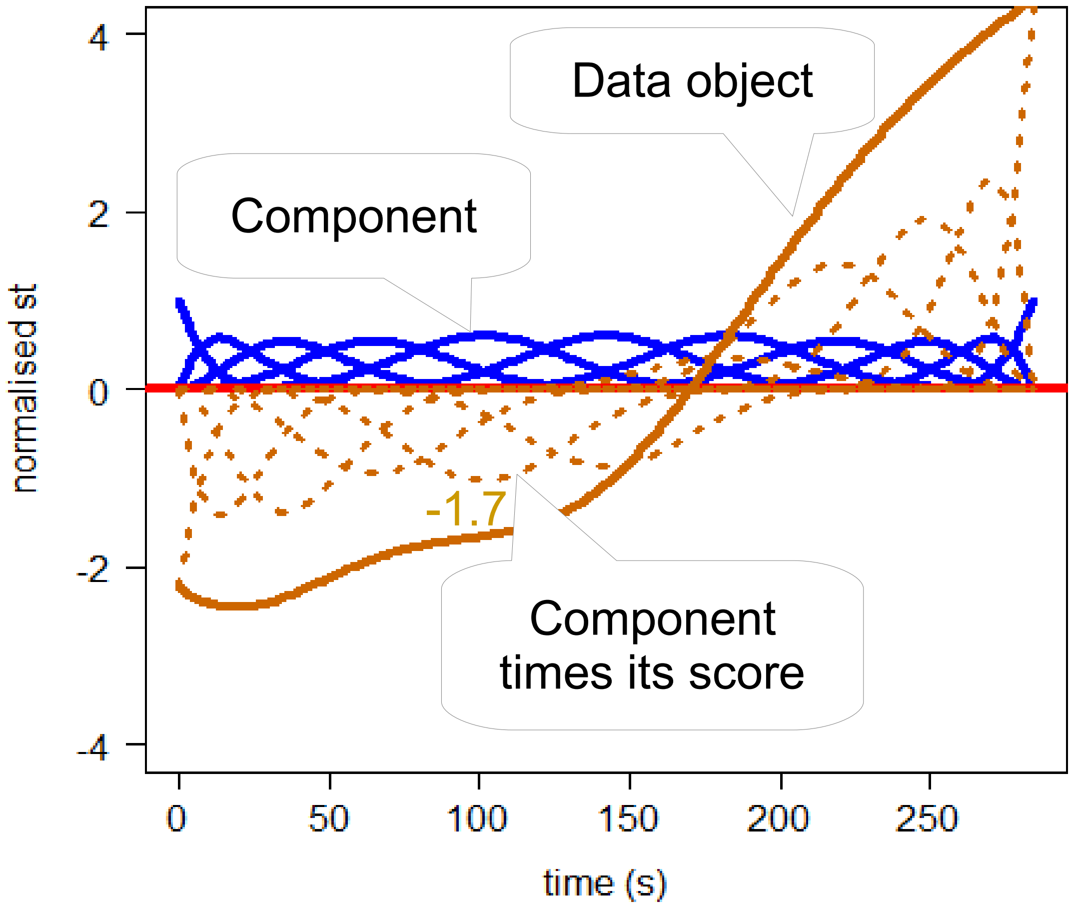
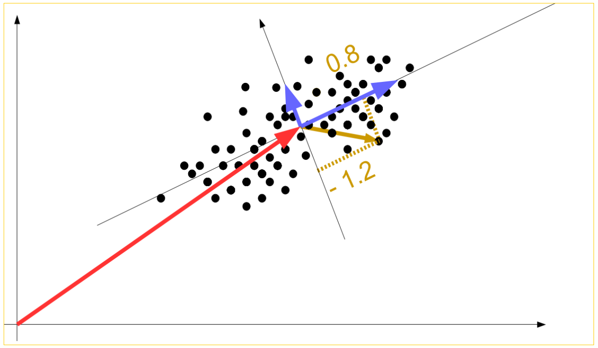
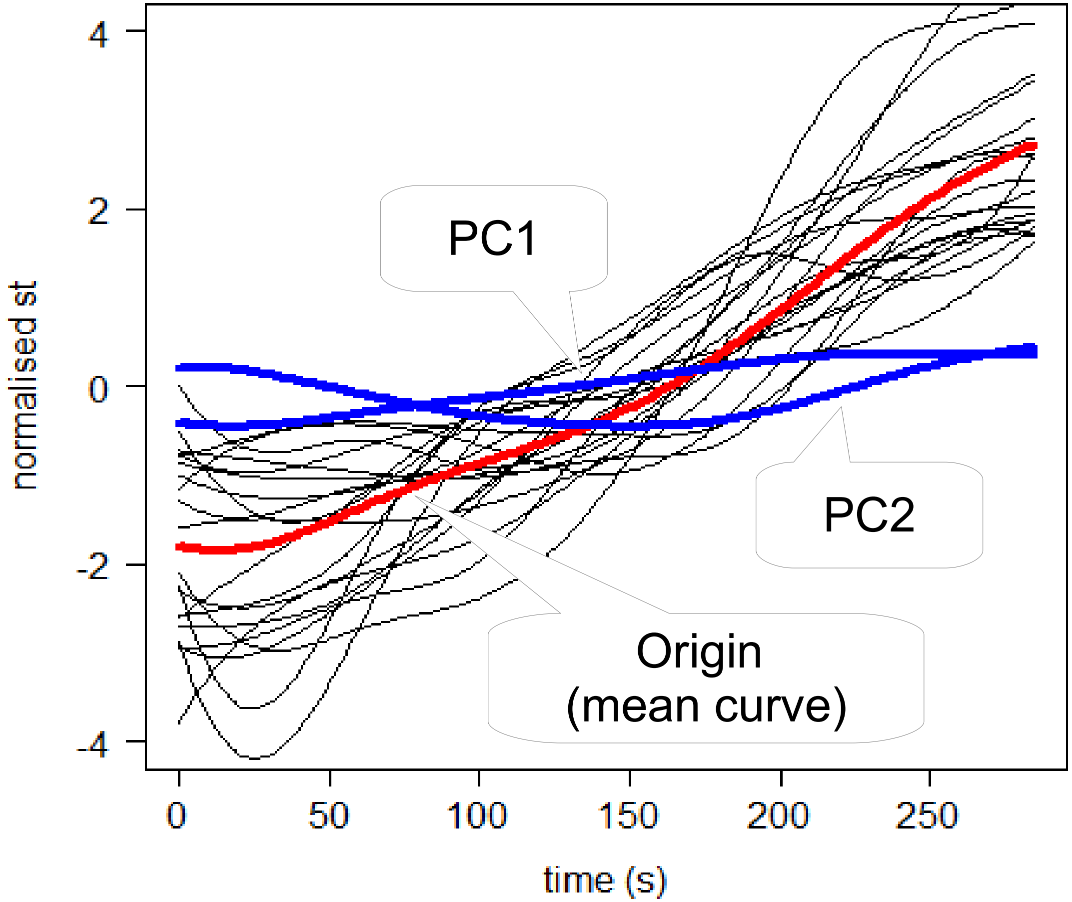


- Each of the 11 components is multiplied by a score
- These are summed together to obtain a data object

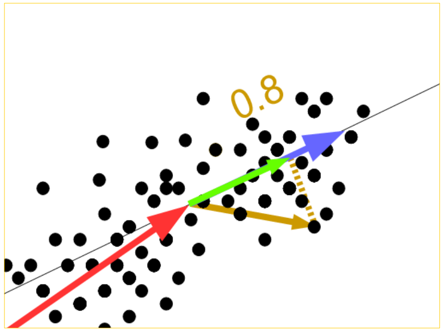


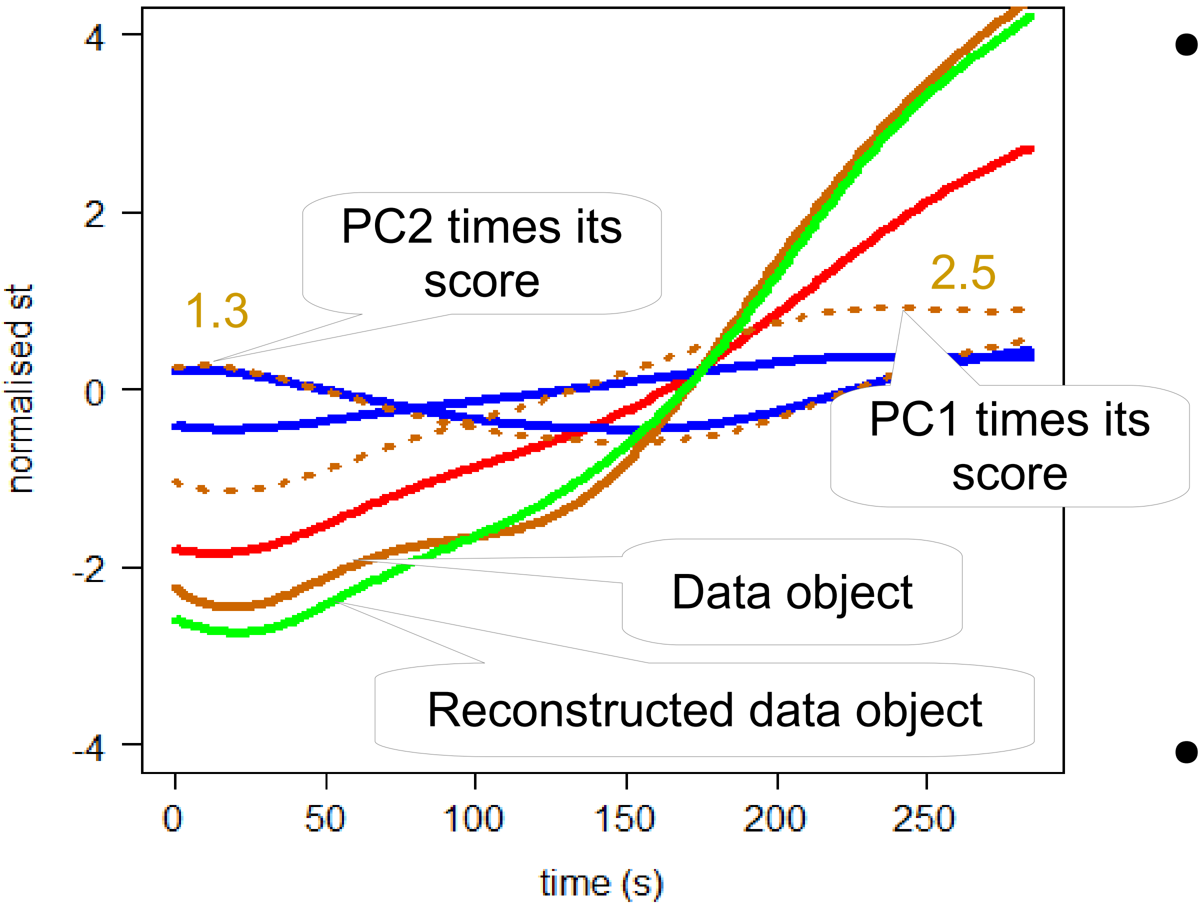
Functional PCA



- FPCA computes new origin and component functions which best suit the data

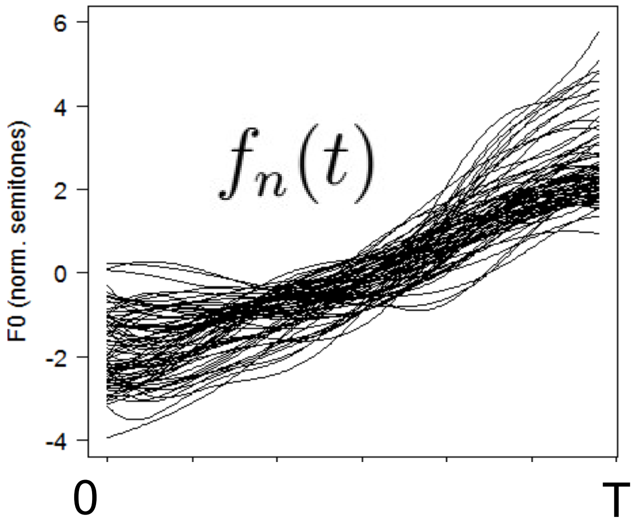
Functional PCA





- The sum of origin (mean) curve + PCs times their scores gives an approx reconstruction of the original curve
- Dimensions from 11 (B-splines) down to 2 (PCs)

Functional PCA



$$\max\left\{ var_n \left(\int_0^T PC1(t) f_n(t)\,dt \right) \right\}$$

$$\text{subject to} \int_0^T PC1^2(t) = 1$$

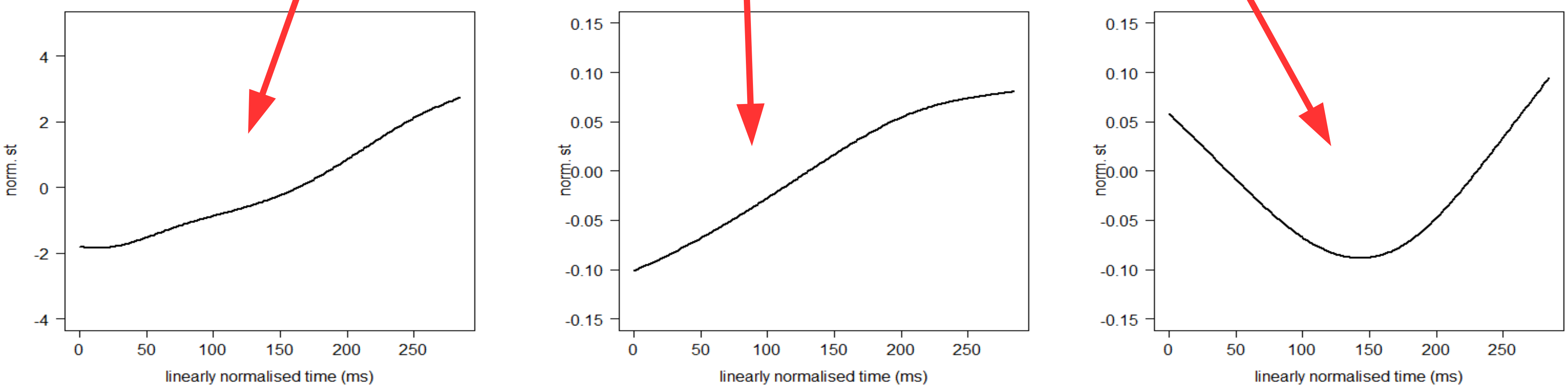
- FPCA definition uses the input curves $f_n(t)$
- FPCA is independent of the B-splines used to smooth $f_n(t)$

Functional PCs

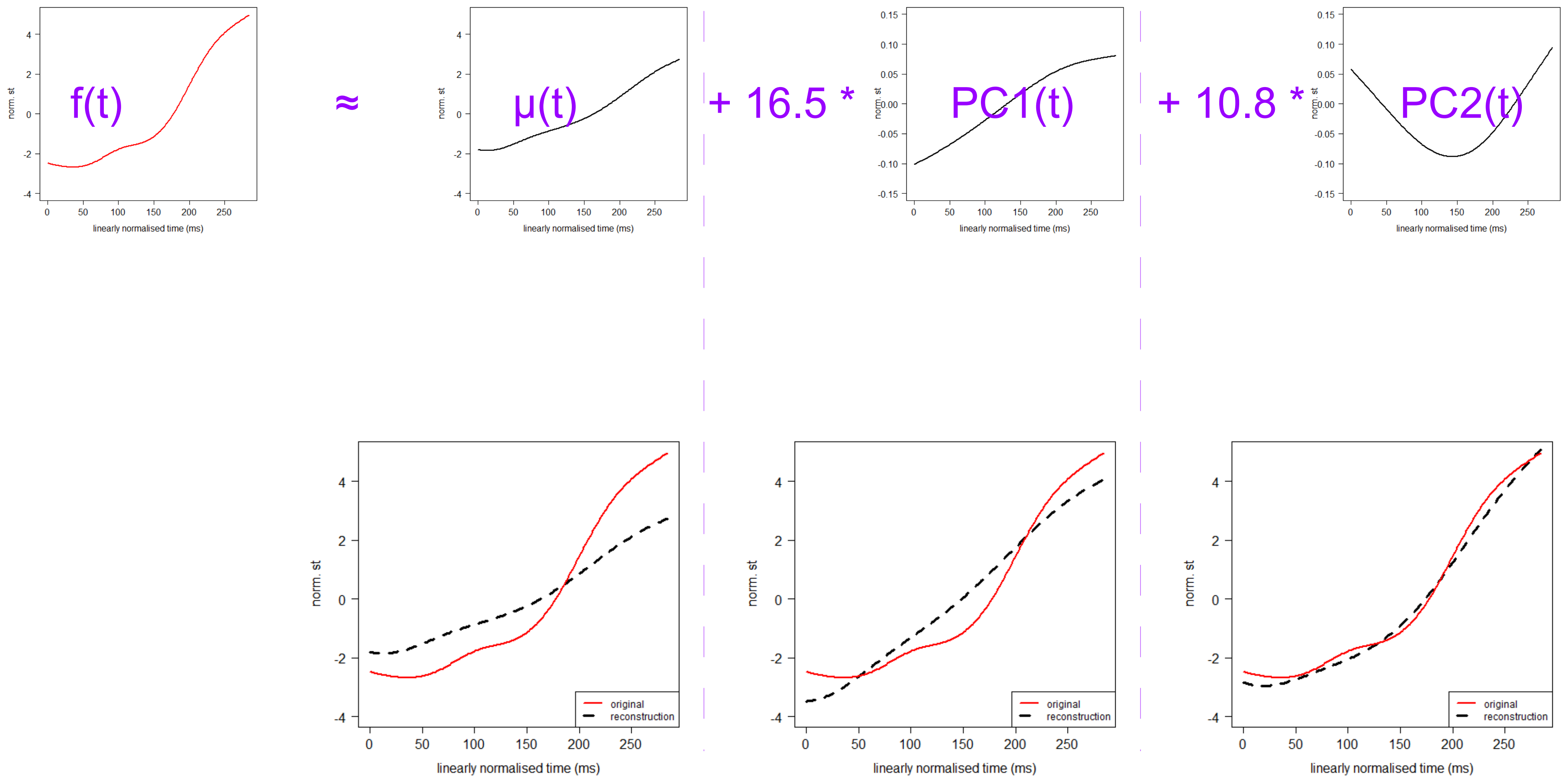
$$\boxed{f(t) \approx \mu(t) + s_1 \cdot PC1(t) + s_2 \cdot PC2(t) + \cdots}$$

PC1 score

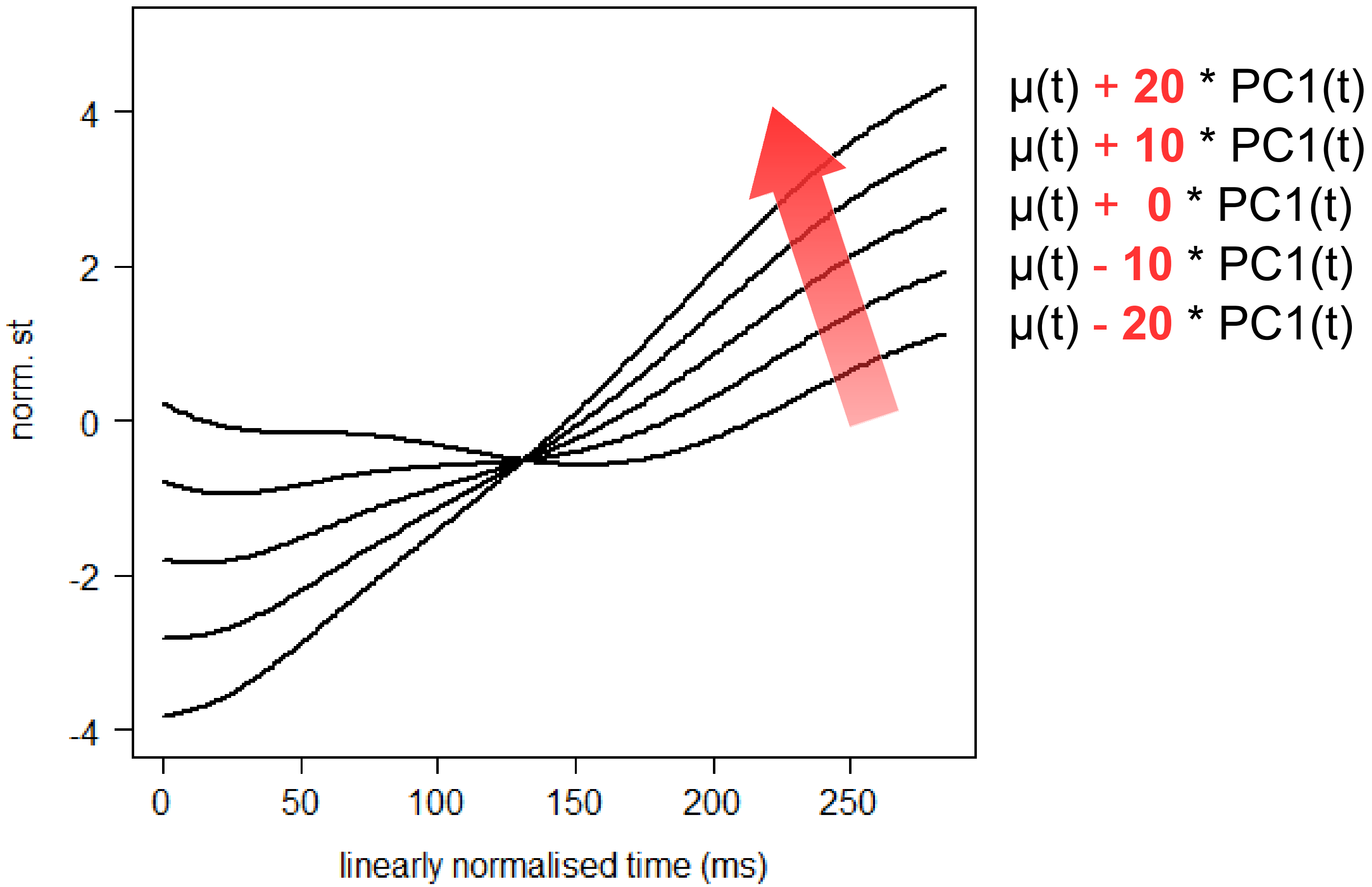
PC2 score



Curve reconstruction

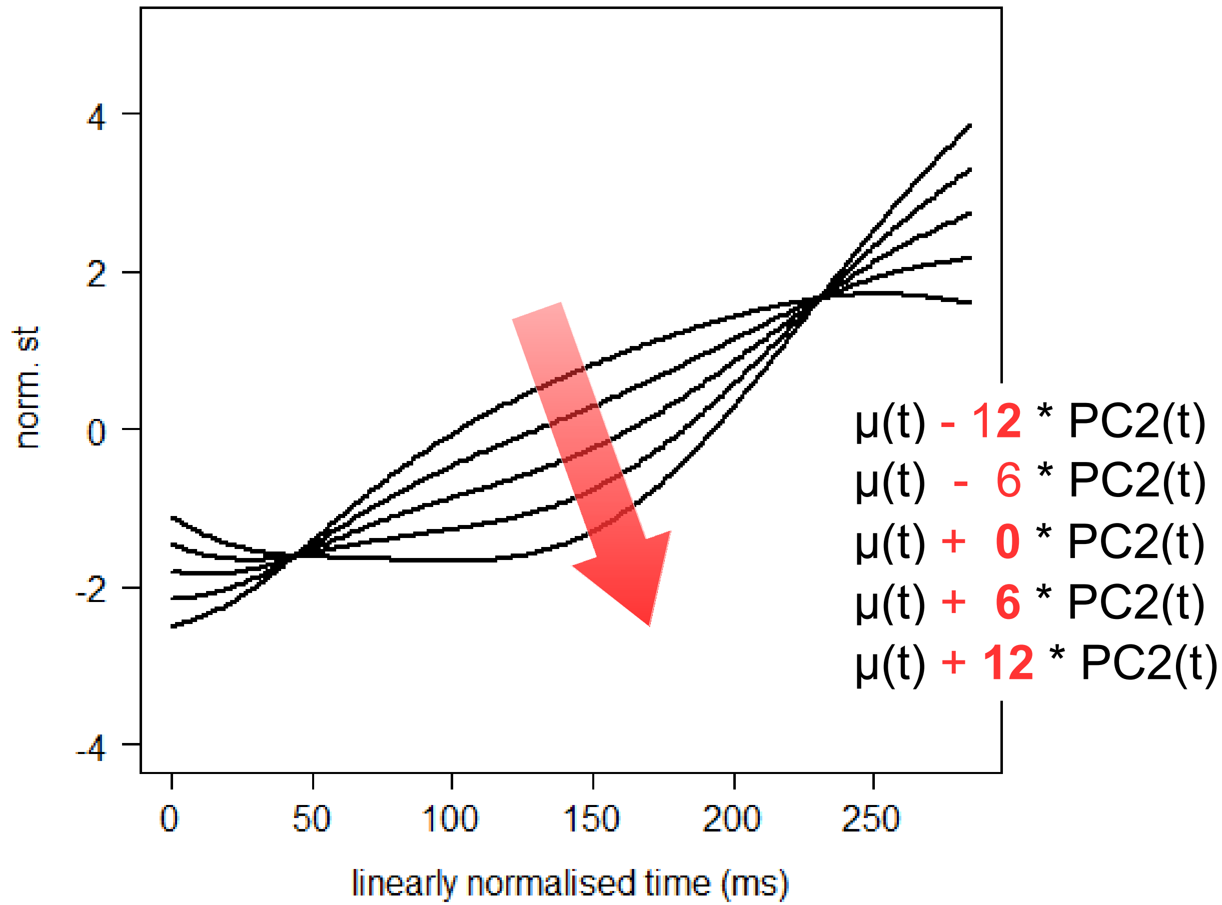


PC1 scores



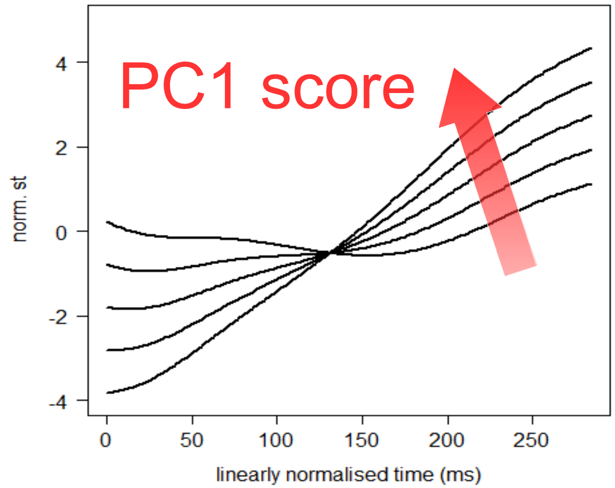
4
2
0
-2
-4
norm. st
0
50
100
150
200
250
linearly normalised time (ms)
μ(t) + 20 * PC1(t)
μ(t) + 10 * PC1(t)
μ(t) + 0 * PC1(t)
μ(t) - 10 * PC1(t)
μ(t) - 20 * PC1(t)

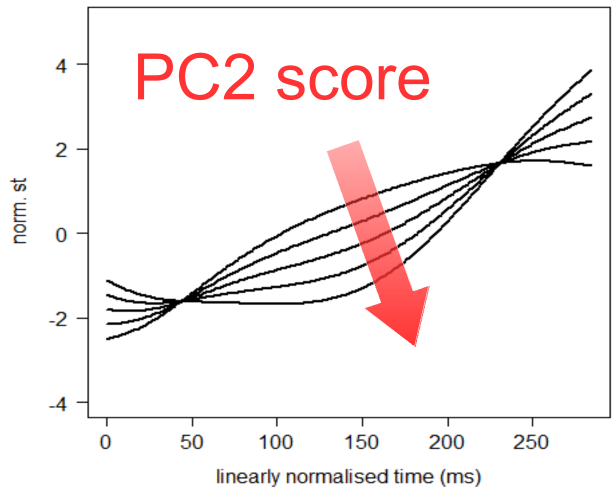
PC2 scores

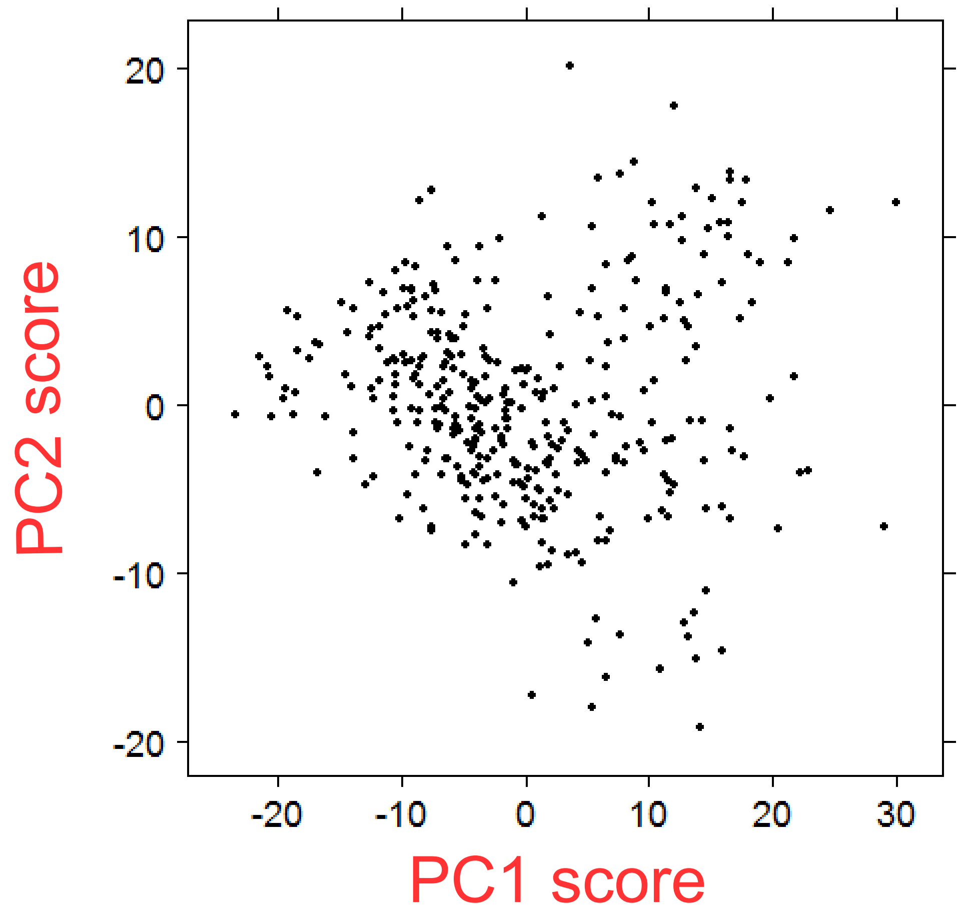


4
2
0
-2
-4
norm. st
0
50
100
150
200
250
linearly normalised time (ms)
μ(t) - 12 * PC2(t)
μ(t) - 6 * PC2(t)
μ(t) + 0 * PC2(t)
μ(t) + 6 * PC2(t)
μ(t) + 12 * PC2(t)

Curve parametrisation







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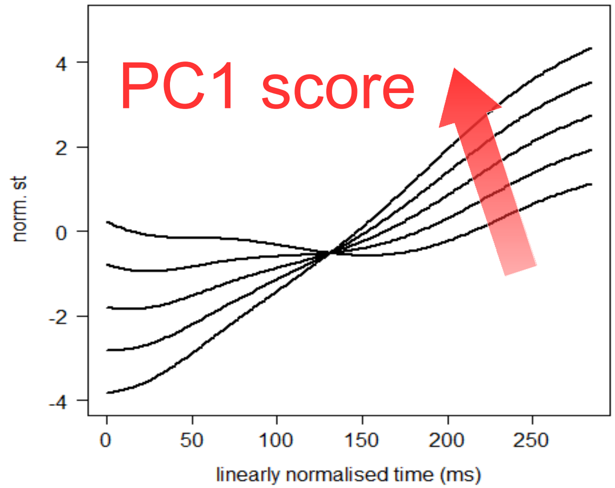
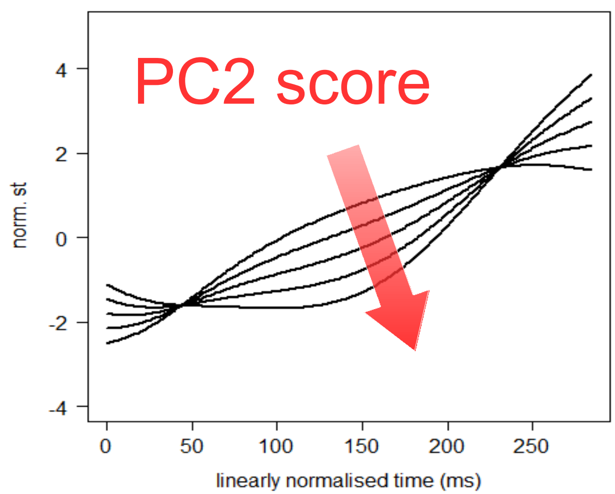
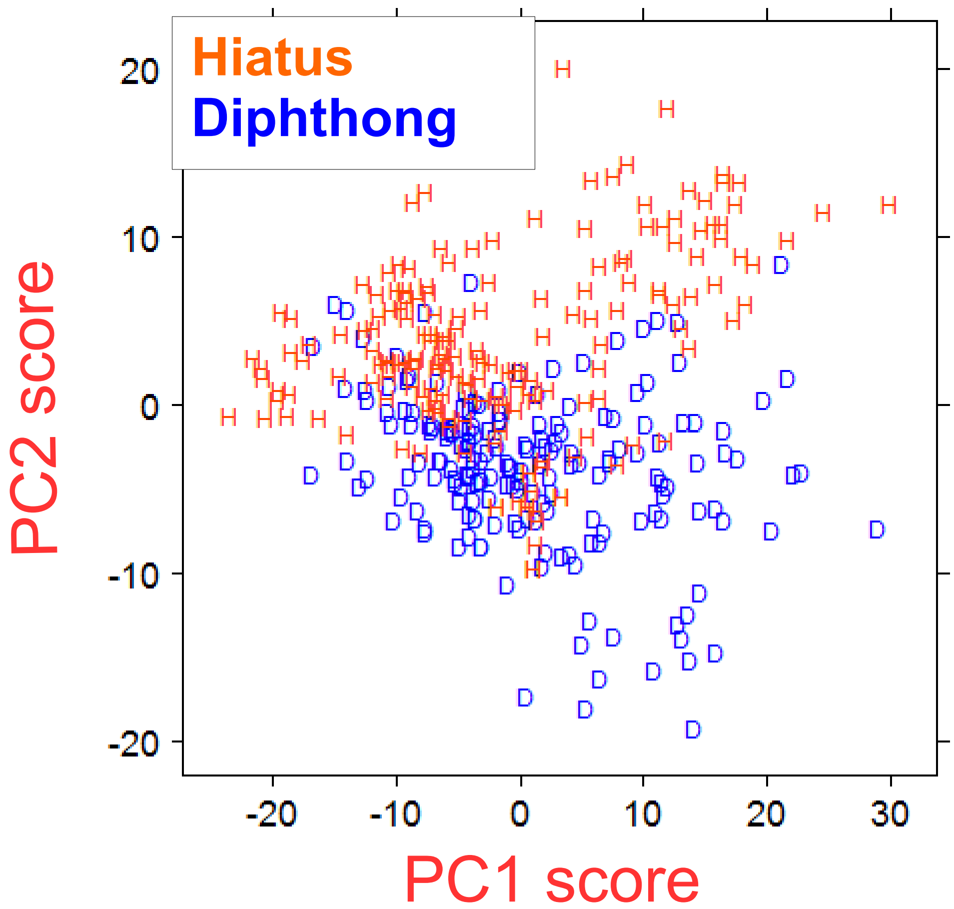


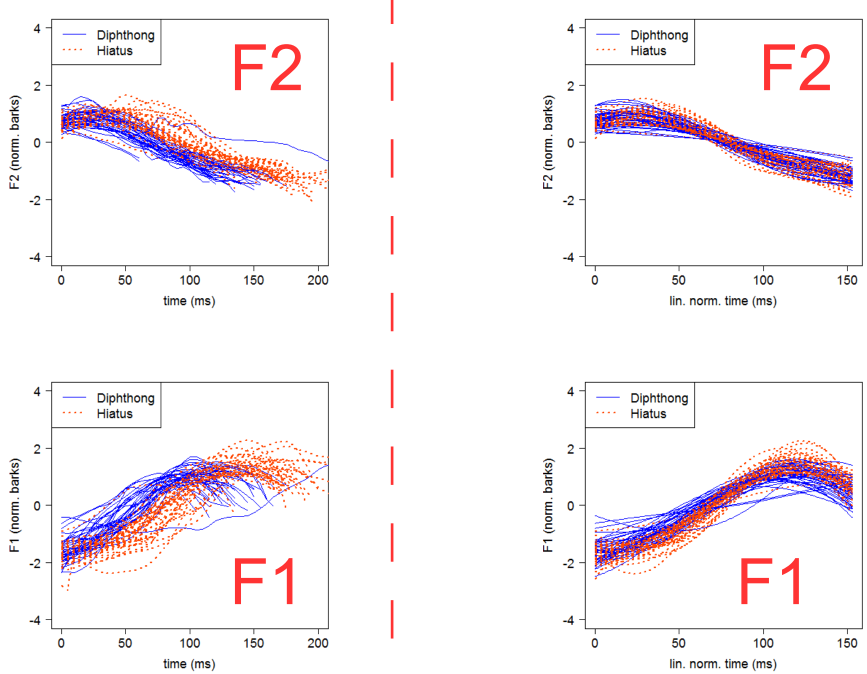




Formants

2D CURVES

NUMBERS



FPCA

FPCA

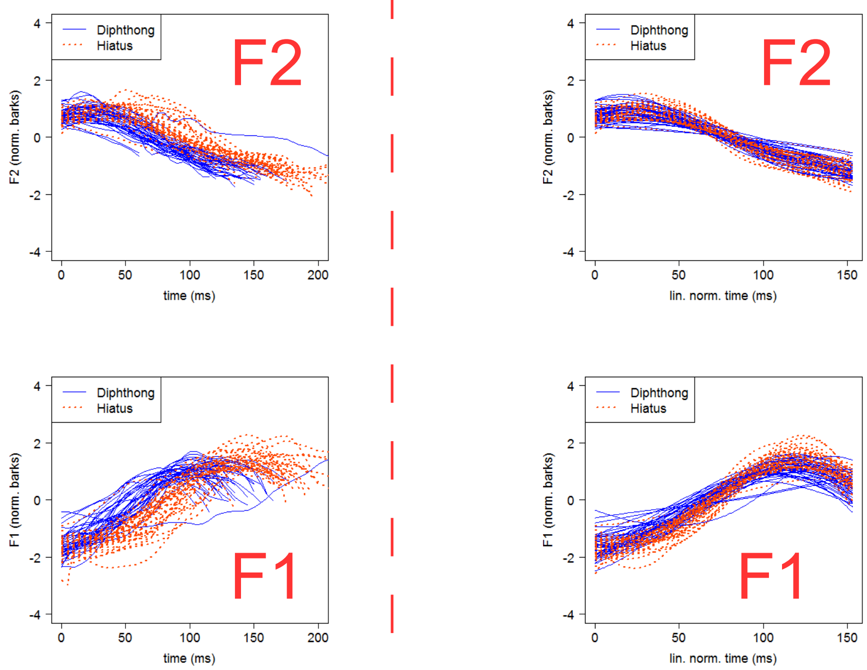
ANOVA

LM

LMER

Formants

2D CURVES



2D FPCA

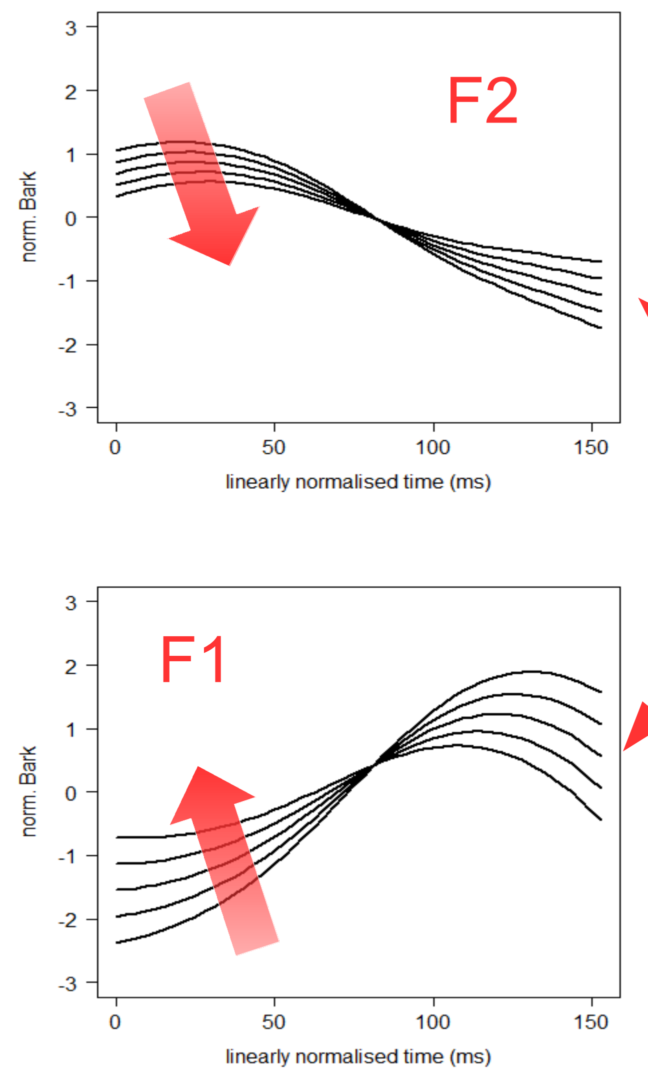
NUMBERS

ANOVA

LM

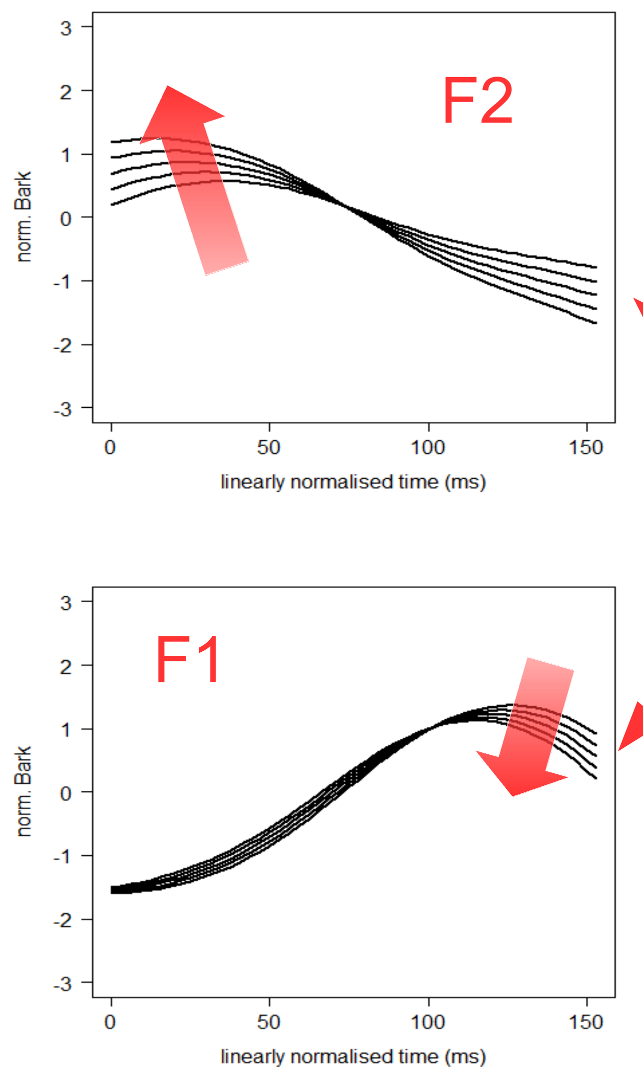
LMER

PC1 scores



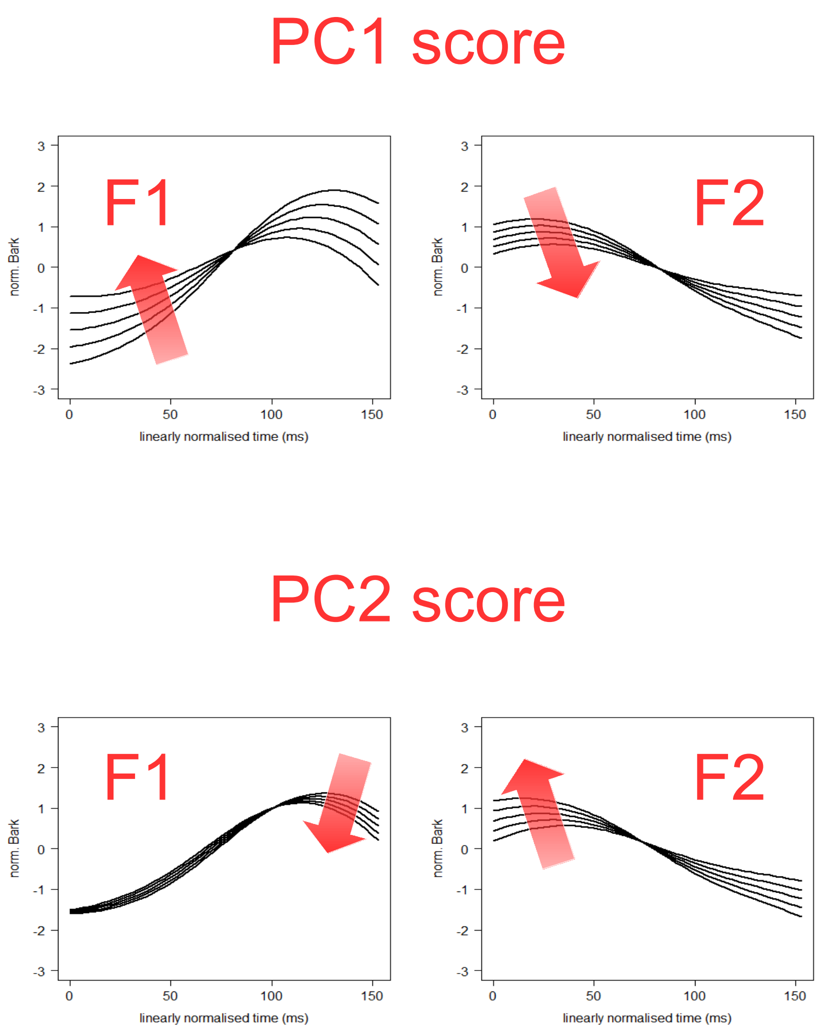
$\mu(t)$ **+ 8** * PC1(t)
$\mu(t)$ **+ 4** * PC1(t)
$\mu(t)$ **+ 0** * PC1(t)
$\mu(t)$ **- 4** * PC1(t)
$\mu(t)$ **- 8** * PC1(t)

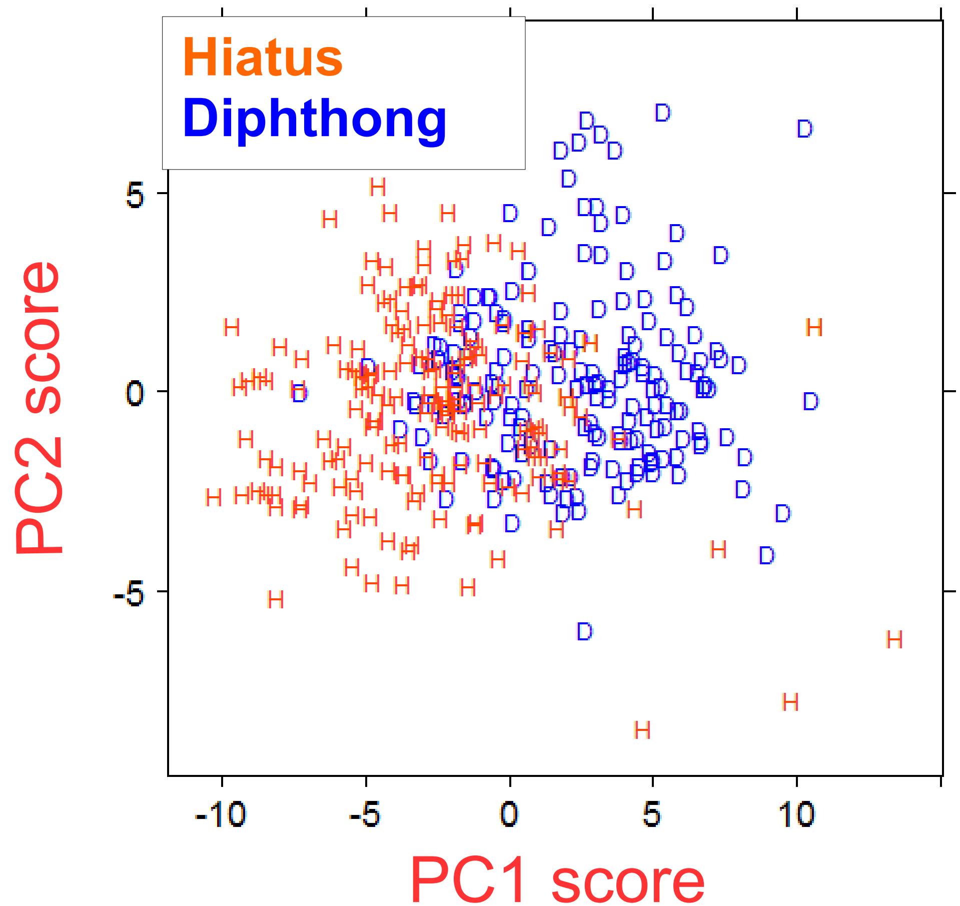
PC2 scores



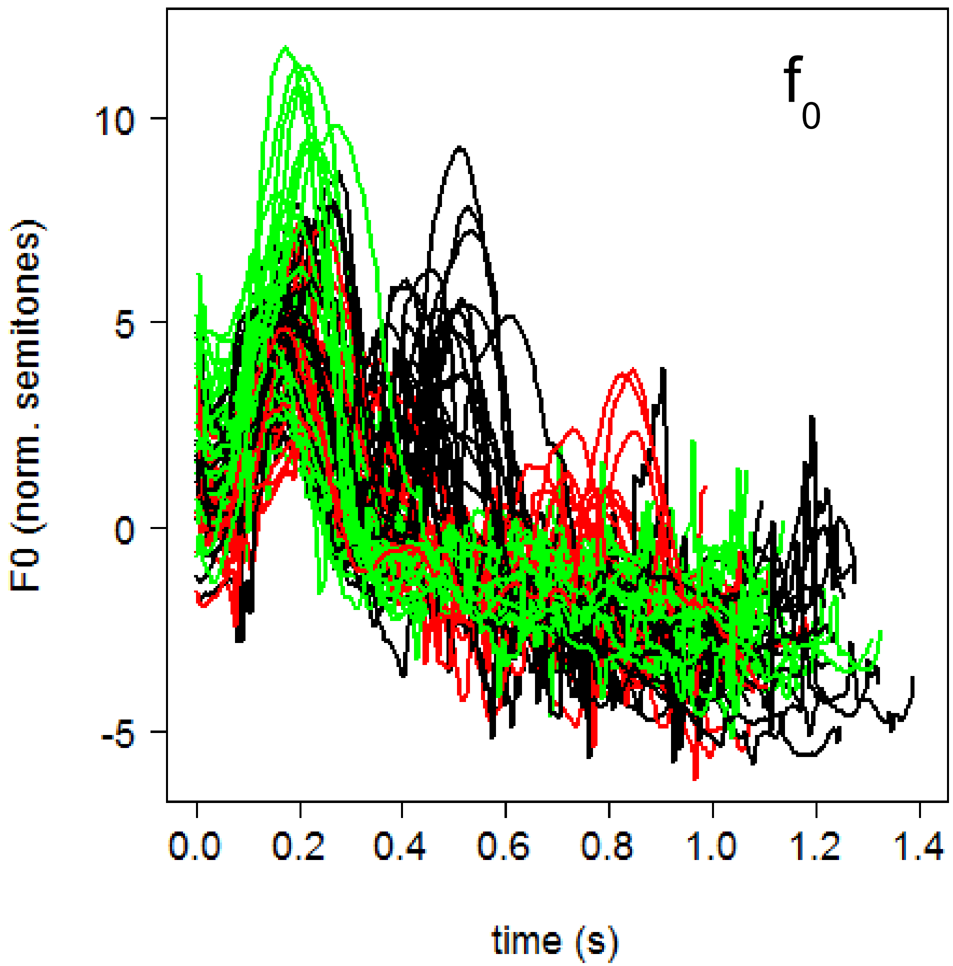
$\mu(t) + 4 * PC1(t)$
$\mu(t) + 2 * PC1(t)$
$\mu(t) + 0 * PC1(t)$
$\mu(t) - 2 * PC1(t)$
$\mu(t) - 4 * PC1(t)$

2D curve parametrisation





Many segments



- Narrow focus in Neapolitan Italian
- Focus on

  Subject, **Verb** or **Prop. Phrase**

  Danilo vola da Roma

  (*Danilo flies from Rome*)
- 8 CV syllables

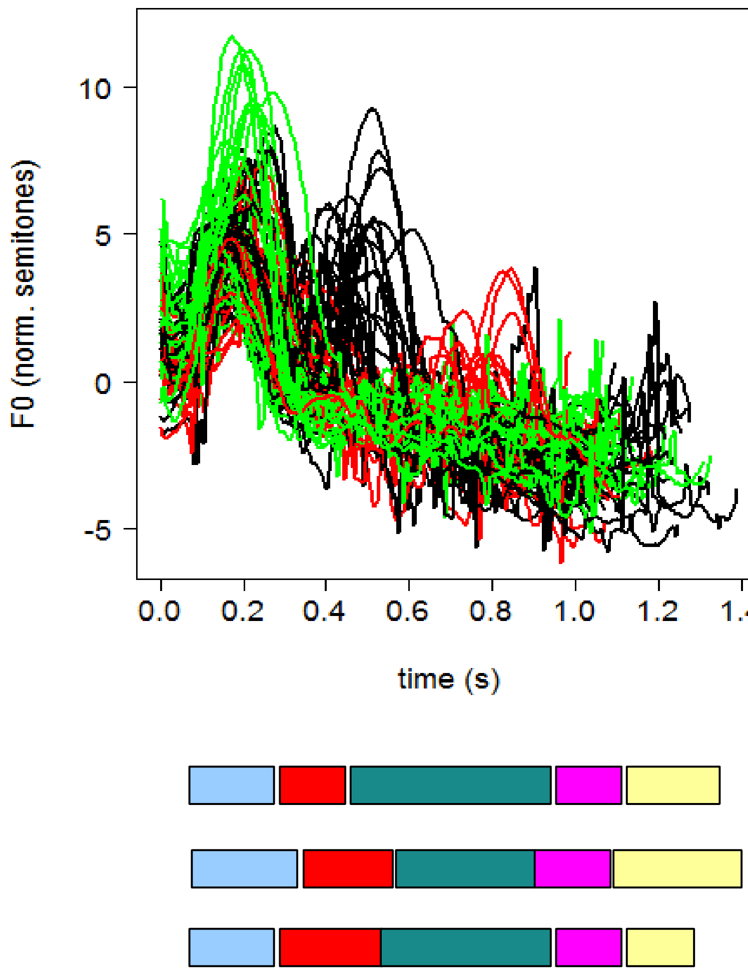
  first C was excluded (too short)

  VCVCV CVCV CV CVCV

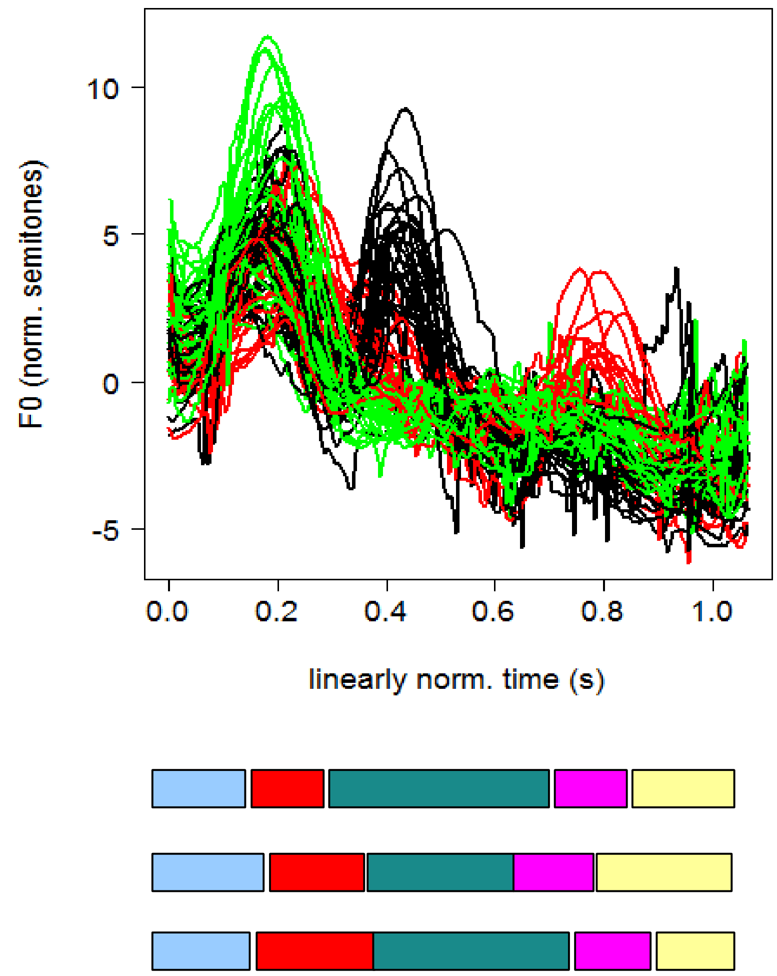
… 15 segments!

Linear time normalisation

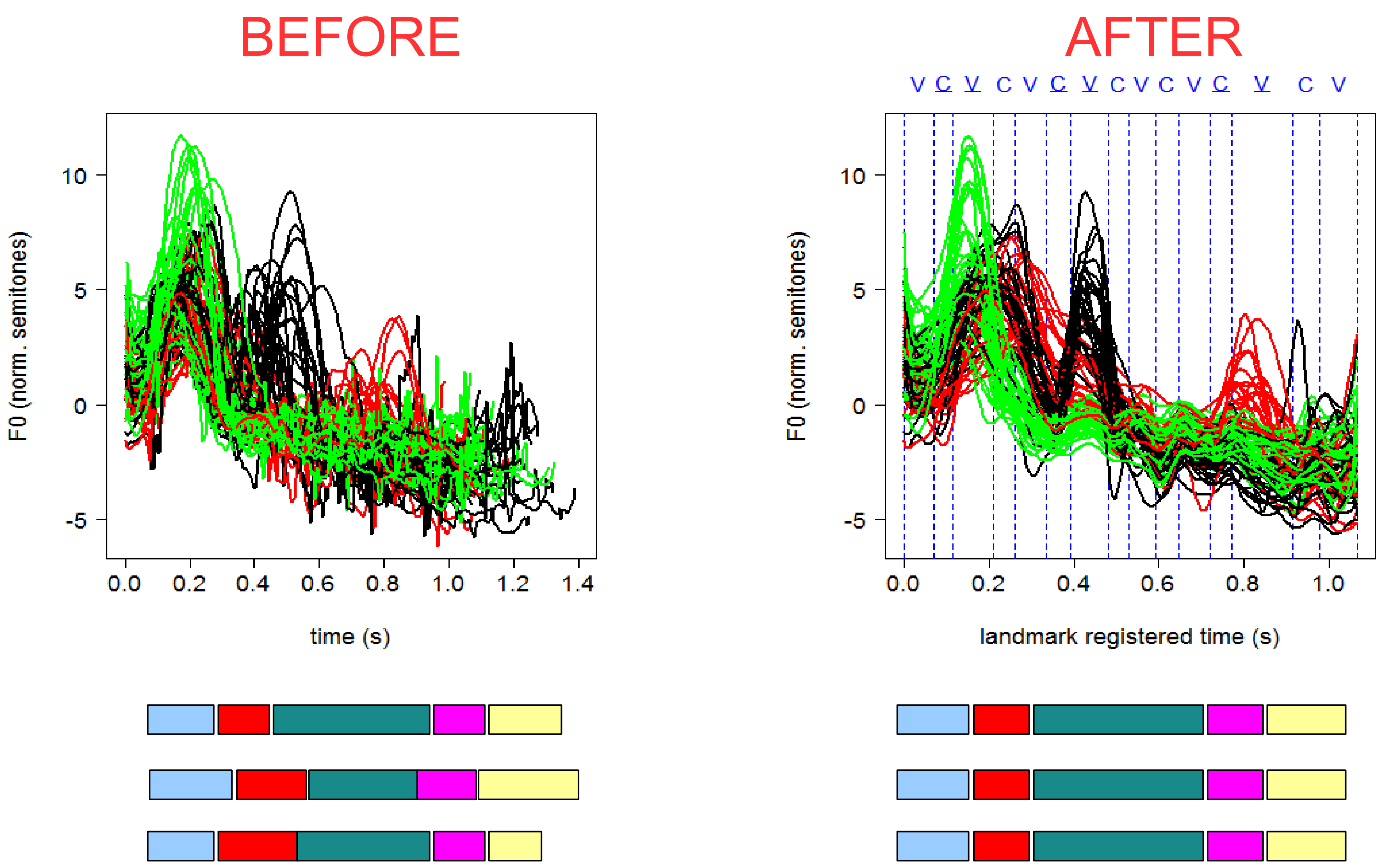
BEFORE



AFTER

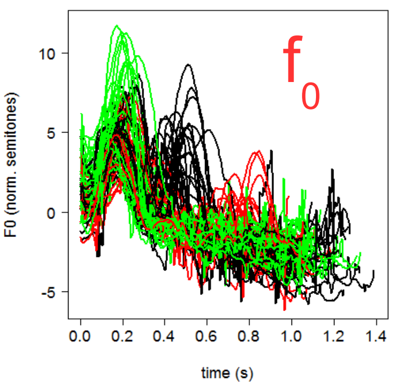


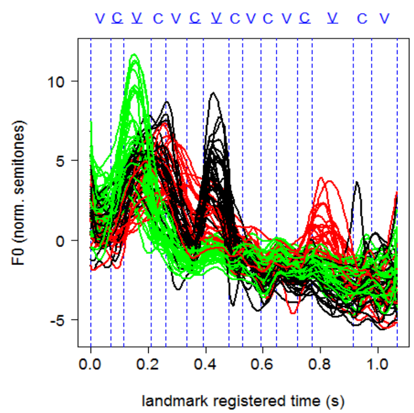
Landmark registration



Using landmark registration

CURVES





segment durations

d1	d2	...	d15
...	...	...	...
...	...	...	...
...	...	...	...

FPCA

PCA

NUMBERS

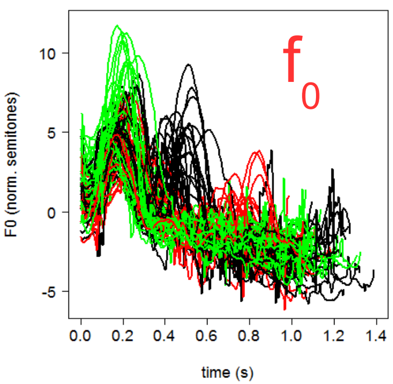
ANOVA

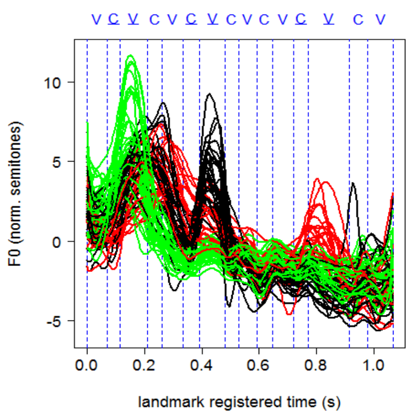
LM

LMER

Using landmark registration

CURVES





segment durations

d1	d2	...	d15
...	...	...	...
...	...	...	...
...	...	...	...

MIND THE GAP

FPCA

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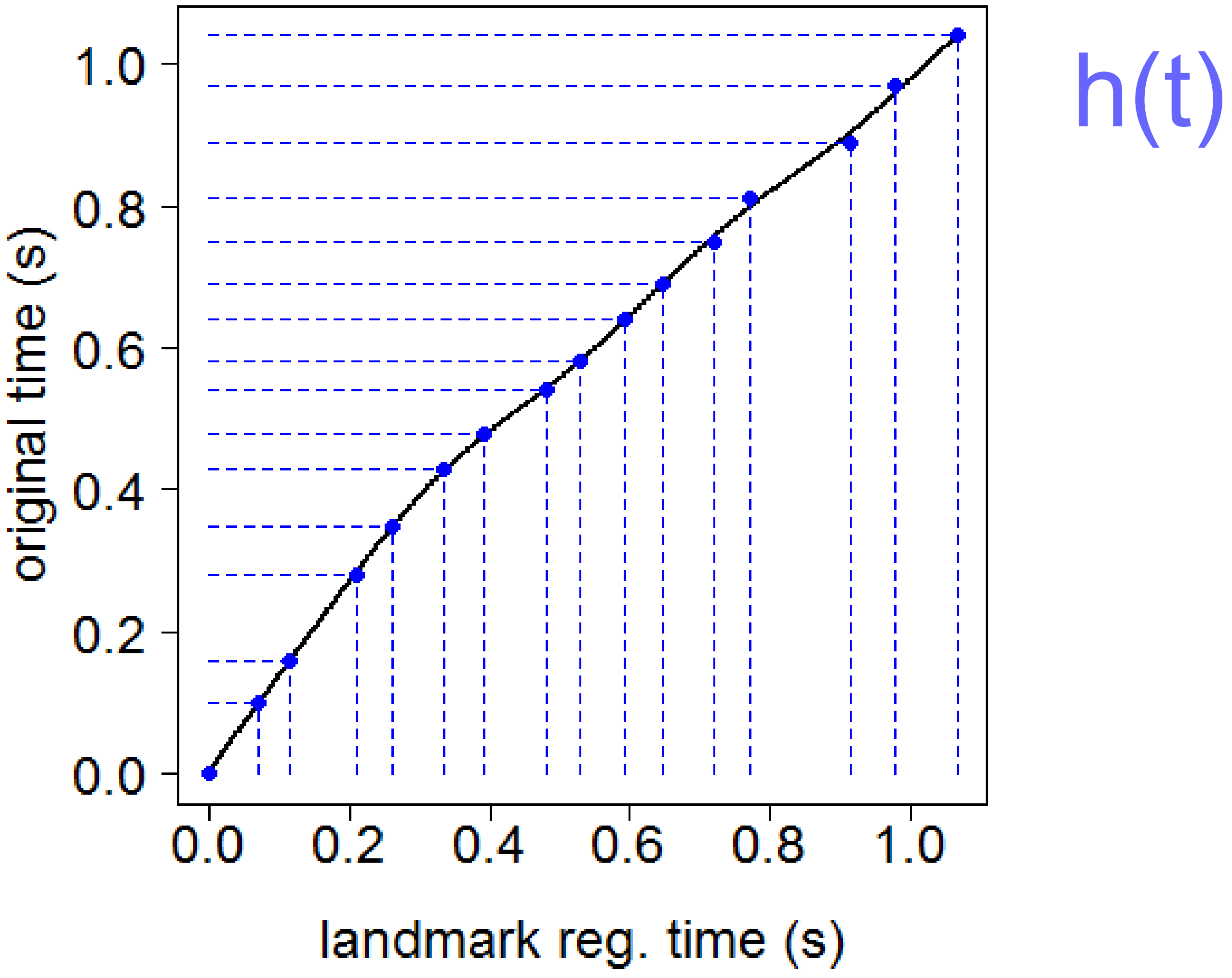
LM

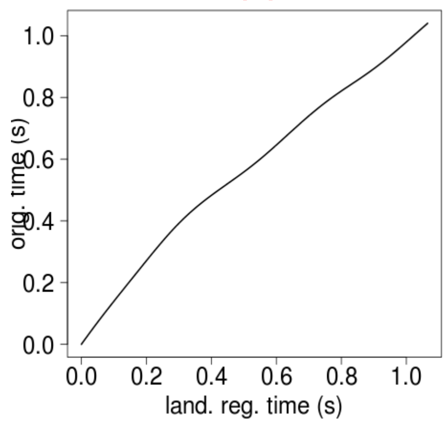
LMER

Inside landmark registration

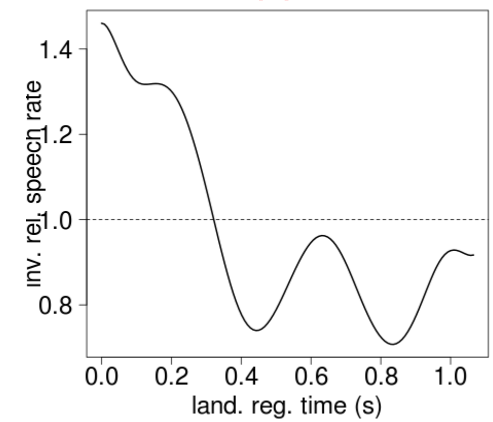


Relative log rate

1 h(t)



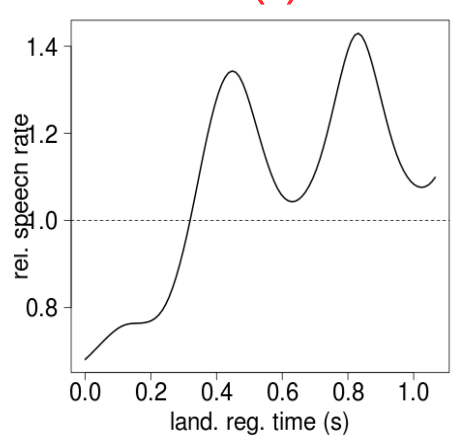
2 dh(t)/dt



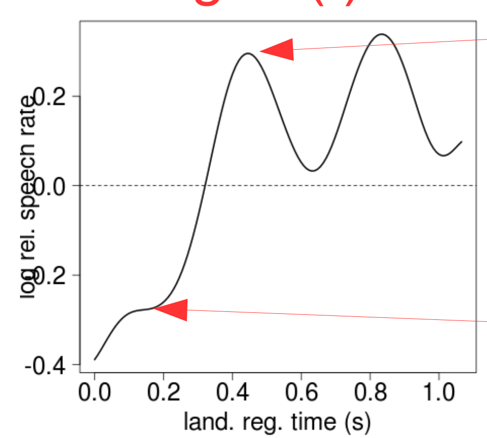
REVERSIBLE!

1 ⟷ 4

3 - dh(t)/dt



4 - log dh(t)/dt

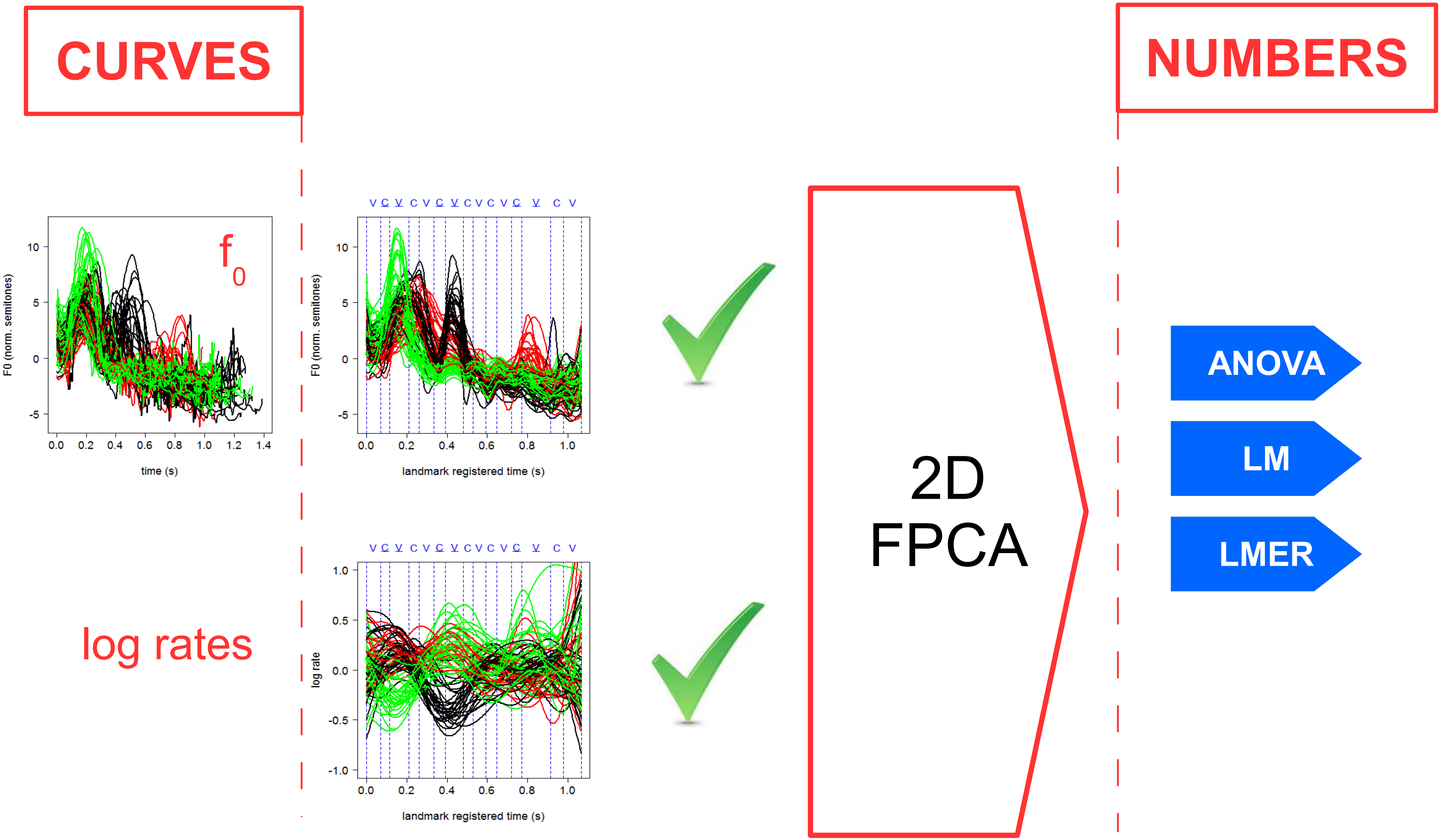


+ 0.25 → duration / 1.28

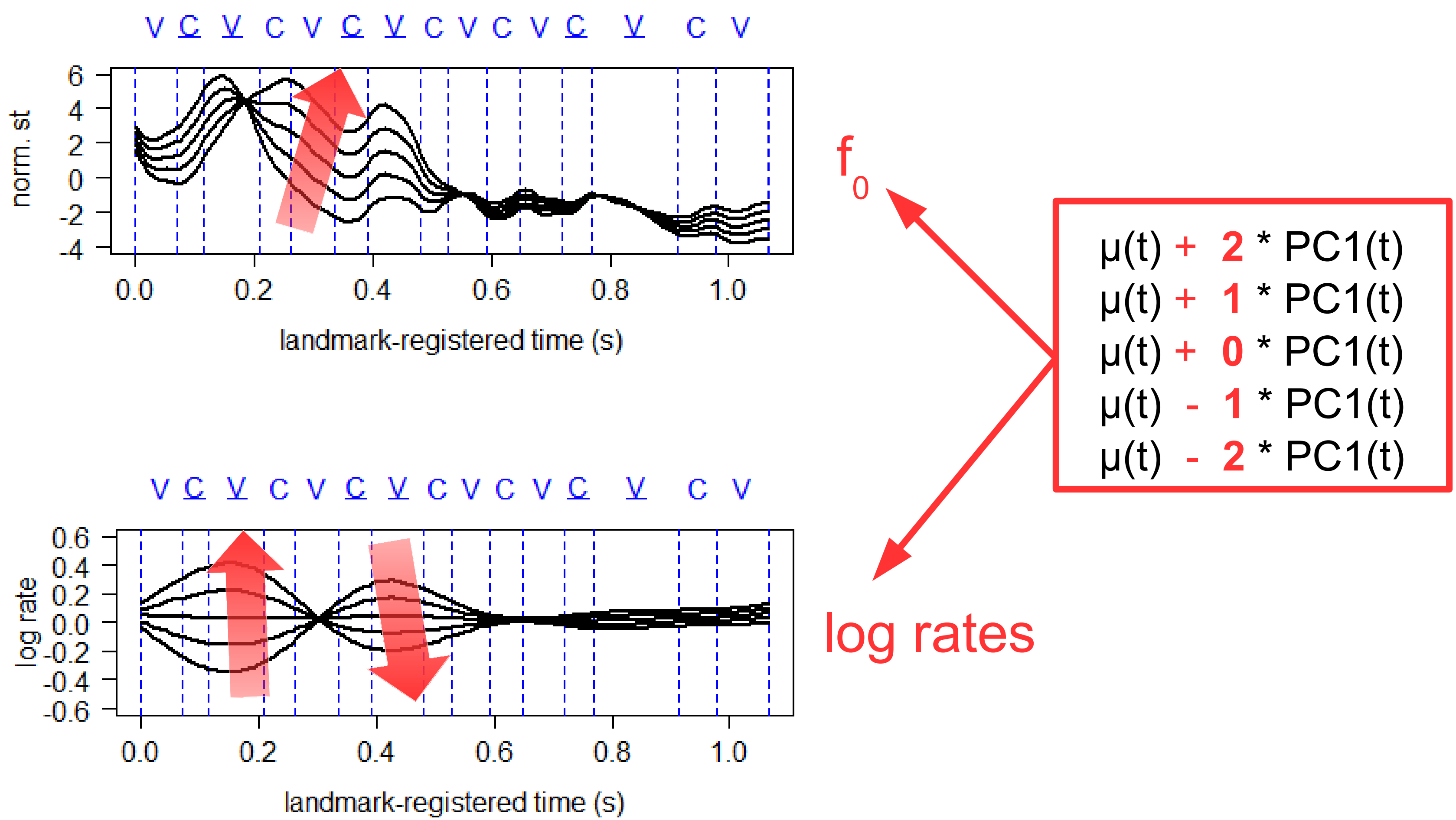
0 → same duration

- 0.25 → duration * 1.28

Using log rates

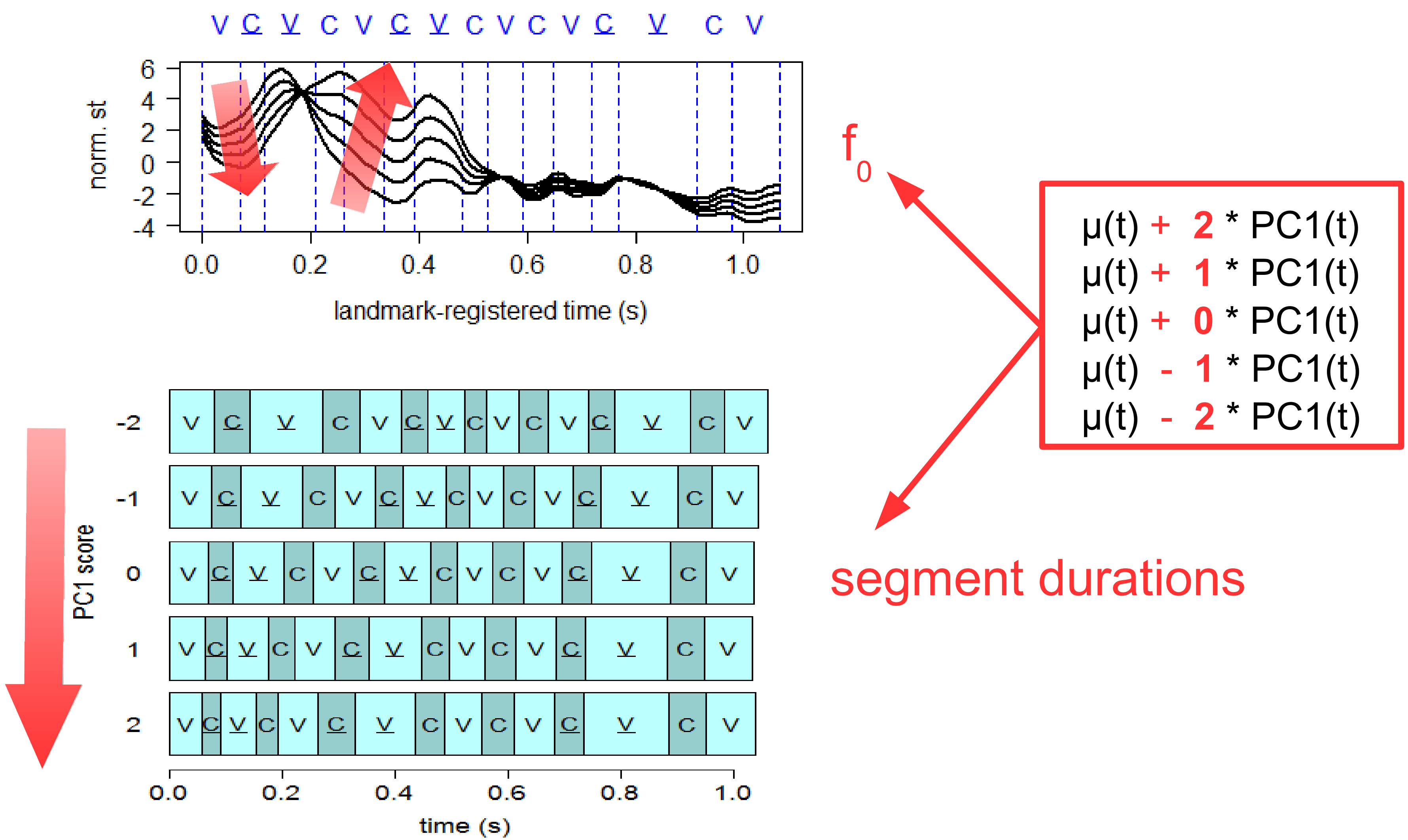


PC1 scores

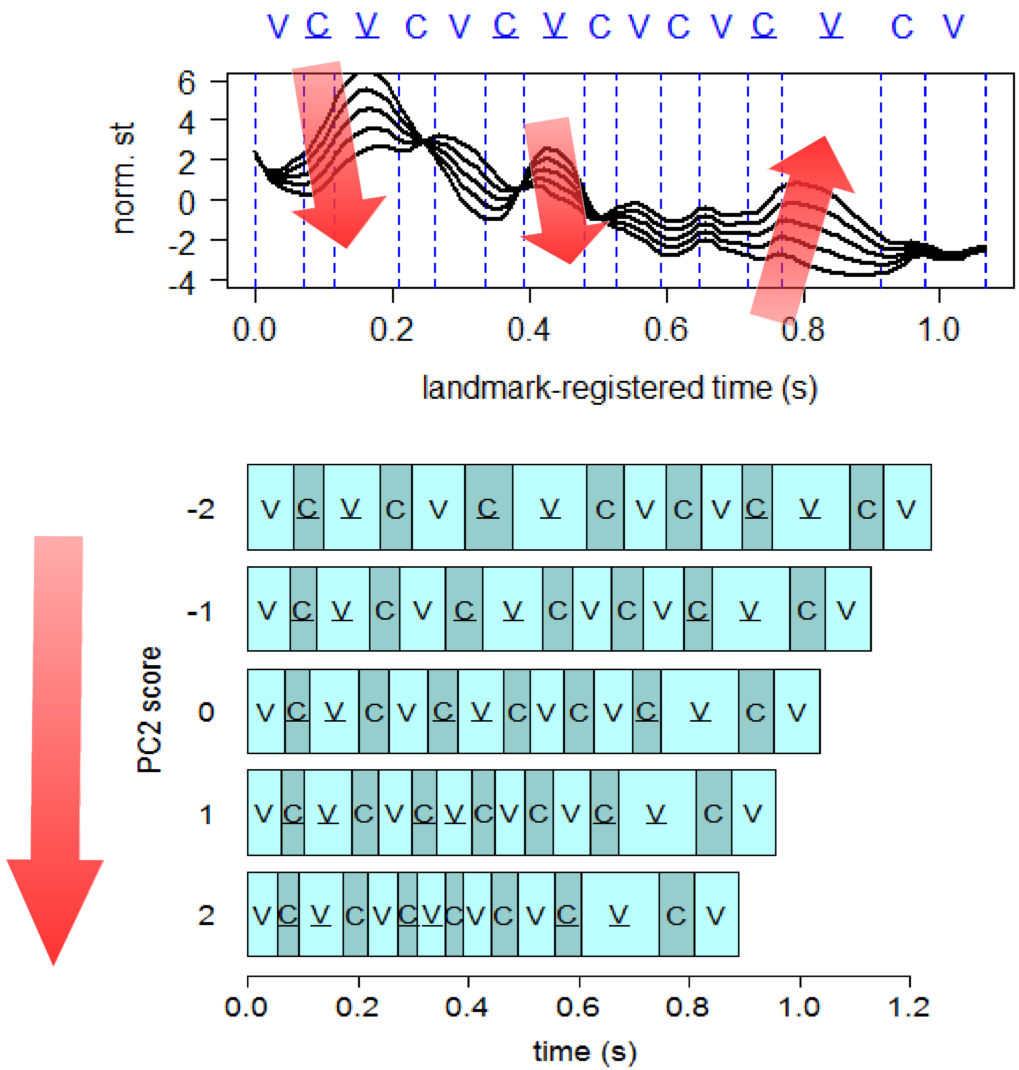


V C V C V C V C V C V C V C V
norm. st
landmark-registered time (s)
log rate
f0
log rates
μ(t) + 2 * PC1(t)
μ(t) + 1 * PC1(t)
μ(t) + 0 * PC1(t)
μ(t) - 1 * PC1(t)
μ(t) - 2 * PC1(t)

PC1 scores


V C V C V C V C V C V C V C V
norm. st
landmark-registered time (s)
f0
μ(t) + 2 * PC1(t)
μ(t) + 1 * PC1(t)
μ(t) + 0 * PC1(t)
μ(t) - 1 * PC1(t)
μ(t) - 2 * PC1(t)
segment durations
PC1 score
time (s)

PC2 scores



f_0

$\mu(t) + 2 * PC1(t)$

$\mu(t) + 1 * PC1(t)$

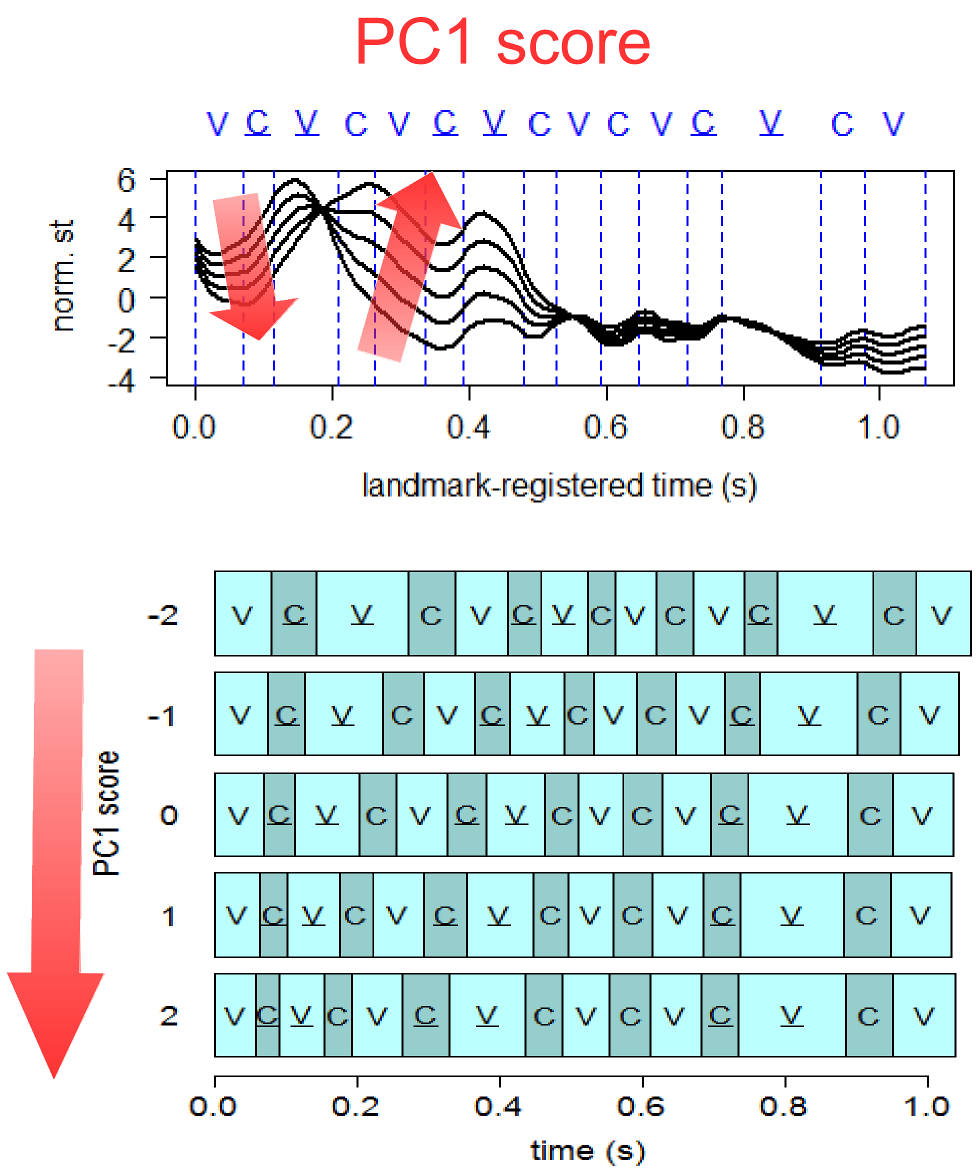
$\mu(t) + 0 * PC1(t)$

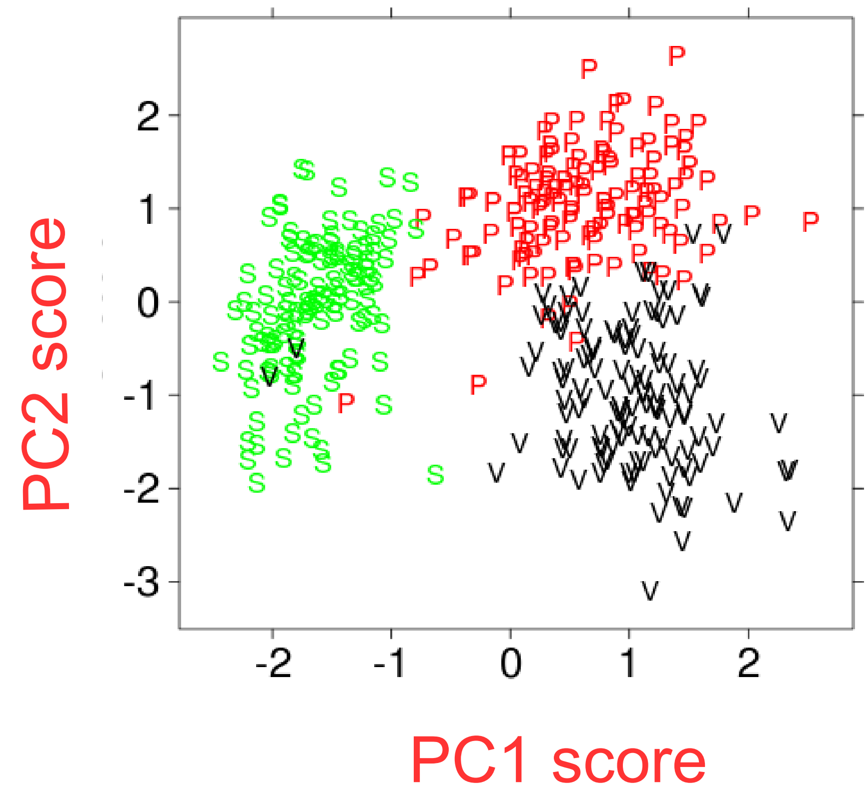
$\mu(t) - 1 * PC1(t)$

$\mu(t) - 2 * PC1(t)$

segment durations

multi-segment curve parametrisation





multi-segment curve parametrisation