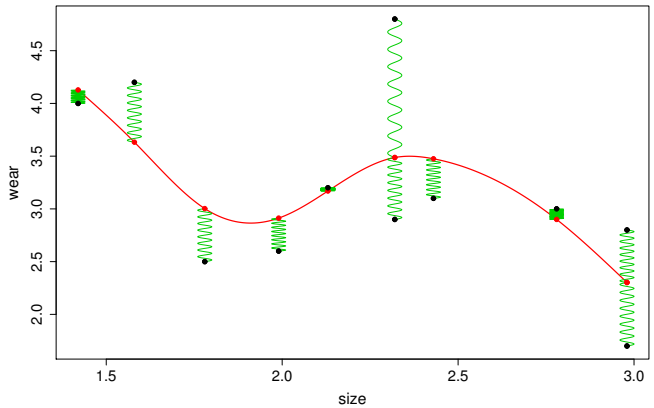
Representing smooth functions: splines

- To motivate how to represent several smooth terms in a model, first consider a simpler smoothing problem.
- Consider estimating the smooth function f in the model $y_i = f(x_i) + \epsilon_i$ from x_i, y_i data using *smoothing splines*...

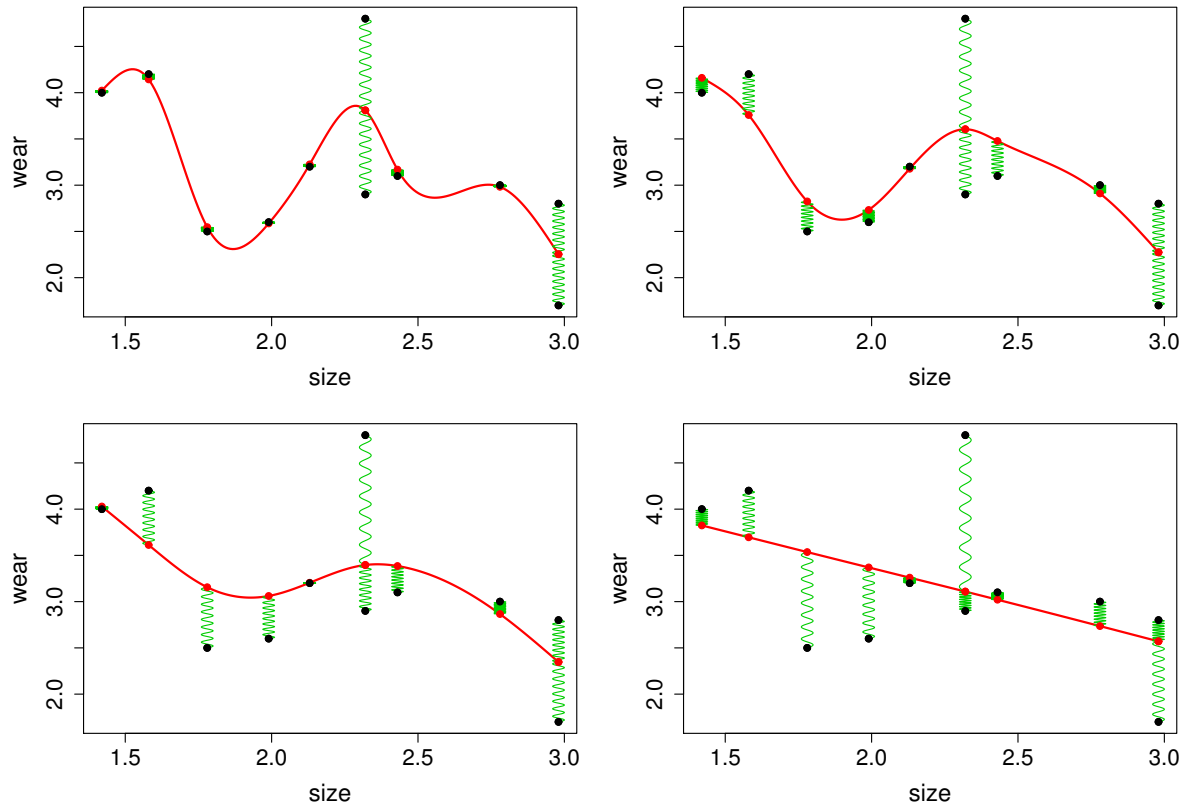


- The red curve is the *function* minimizing

$$\sum_i (y_i - f(x_i))^2 + \lambda \int f''(x)^2 dx.$$

Splines and the smoothing parameter

- Smoothing parameter λ controls the stiffness of the spline.



- But the spline can be written $\hat{f}(x) = \sum_i \beta_i b_i(x)$, where the basis functions $b_i(x)$ do not depend on λ.

P-splines: B-spline basis & approx penalty