Weekend Training programme on FEA for electromagnetic design

Presented on 14 -10 -2011

Vectors - Coordinate systems and Maxwell's Equation

By

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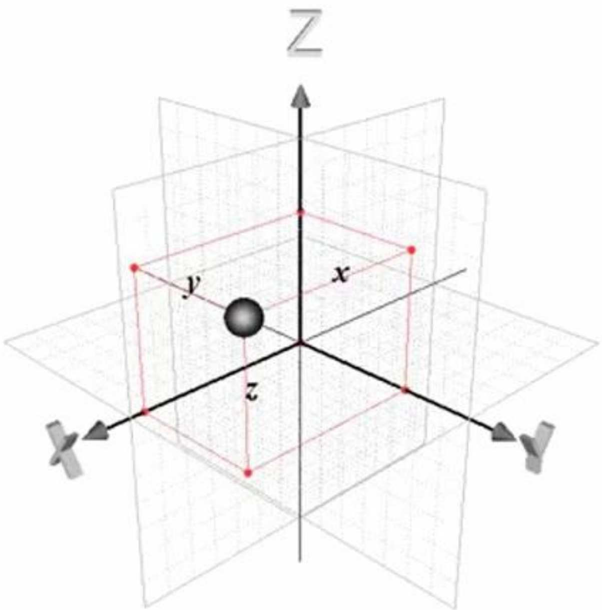
"Design is a funny word. Some people think design means how it looks. But of course, if you dig deeper, it's really how it works."

- Steve Jobs



"All that can be counted doesn't count ,
All that Counts cannot be counted"

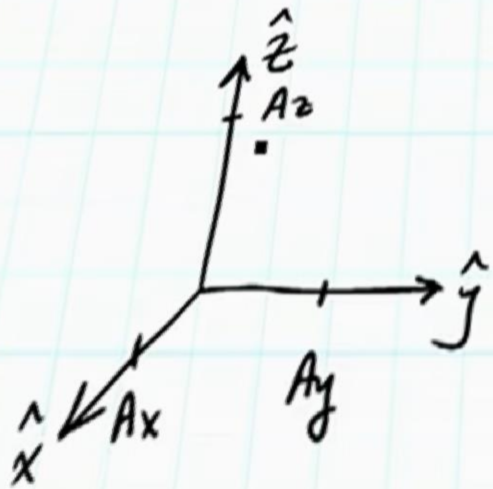
--Albert Einstein

Vectors and Coordinate SystemS

- Vector, Unit Vector, Direction and Distance between two points, Perpendicular & Parallel Vectors, Vector Math (+,-,•,x)
- Coordinate Systems (Cartesian, Cylindrical, Spherical)
- Integrals (Line, Surface, Volume)

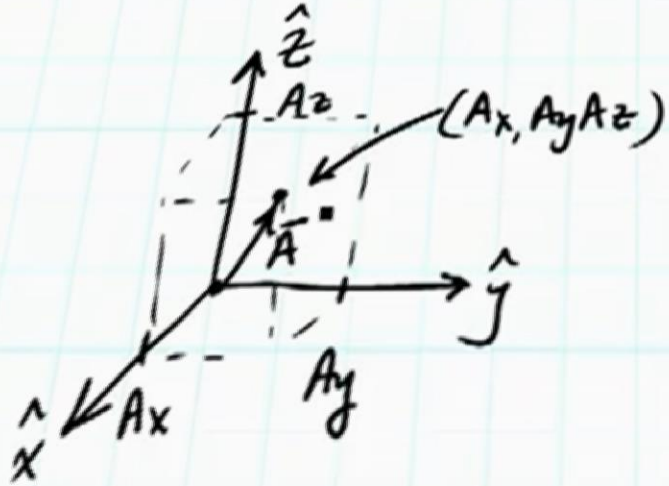
Vector, Unit Vector

$$\overline{A} = \left(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z\right)$$



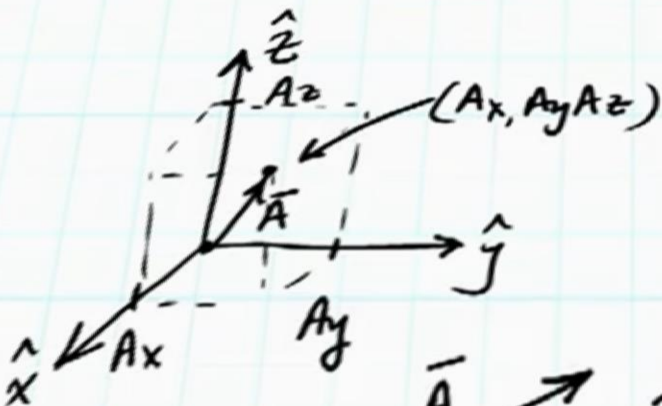
Vector, Unit Vector

$$\overline{A} = \left(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z\right)$$



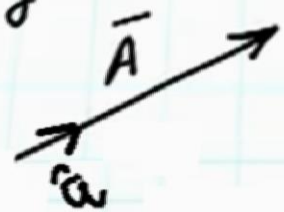
Vector, Unit Vector

$$\overline{A} = \left(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z\right) = \hat{a}\ |\overline{A}|$$



$$|\overline{A}| = \sqrt{A_x^2 + A_y^2 + A_z}$$

$$\hat{a} = \frac{\overline{A}}{|\overline{A}|}$$



$$\hat{a} = \hat{x}\frac{A_x}{|A|} + \hat{j}\frac{A_y}{|A|} + \hat{z}\frac{A_z}{|A|}$$

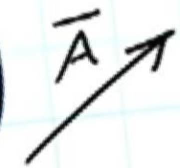
Adding and Subtracting VectorS

$$\overline{A} = \left(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z\right) \quad \bar{A}$$

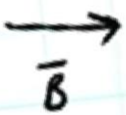
$$\overline{B} = \left(\hat{x}B_x + \hat{y}B_y + \hat{z}B_z\right) \quad \vec{B}$$

Adding and Subtracting VectorS

$$\overline{A} = \left(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z\right)$$

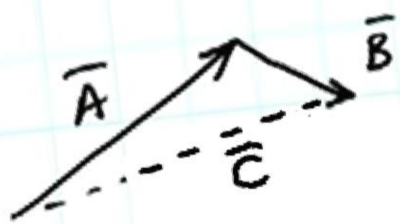


$$\overline{B} = \left(\hat{x}B_x + \hat{y}B_y + \hat{z}B_z\right)$$

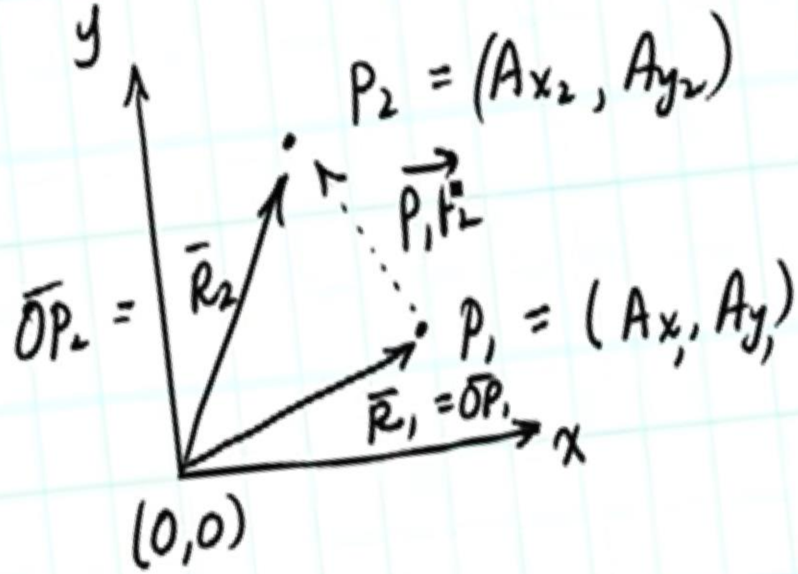


$$+ \ \bar{C} = \hat{x}\left(A_x + B_x\right) + \hat{y}\left(A_y + B_y\right) + \hat{z}\left(A_z + B_z\right)$$

$$\bar{A} + \bar{B} = \bar{B} + A$$



Direction and Distance between two points



$$\overrightarrow{P_1P_2} = \overline{R}_2 - \overline{R}_1$$

$$= \hat{x}\,(A_{x_2} - A_{x_1}) + \hat{y}\,(A_{y_2} - A_{y_1})$$

$$\overline{R}_1 = \hat{x}\,A_{x_1} + \hat{y}\,A_{y_1}$$

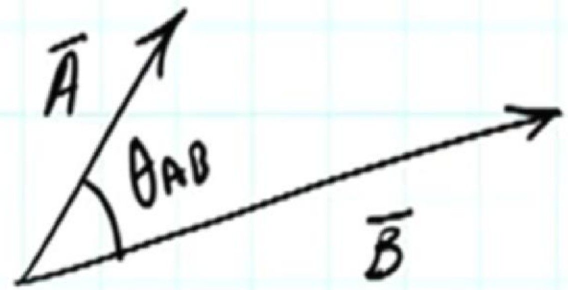
$$\overline{R}_2 = \hat{x}\,A_{x_2} + \hat{y}\,A_{y_2}$$

$$d = |\overrightarrow{P_1P_2}| = \sqrt{(A_{x_2} - A_{x_1})^2 + (A_{y_2} - A_{y_1})^2}$$

Dot Product

$$\bar{A} = \hat{x} A_x + \hat{y} A_y$$

$$\bar{B} = \hat{x} B_x + \hat{y} B_y$$



$$\bar{A} \cdot \bar{B} = C$$

Scalar

$$C = A_x B_x + A_y B_y$$

$$= |\bar{A}||\bar{B}| \cos \theta_{AB}$$

$$\bar{A} \underset{?}{\perp} \bar{B} = |\bar{A}||B| \cos \theta_{AB} = 0$$

90° Yes! $\perp$

0

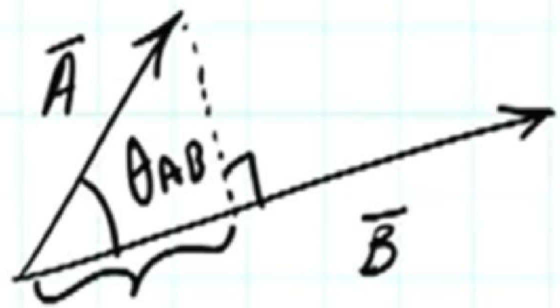
Dot Product

$$\bar{A} = \hat{x} A_x + \hat{y} A_y$$

$$\bar{B} = \hat{x} B_x + \hat{y} B_y$$

$$\bar{A} \cdot \bar{B} = C \quad \swarrow \text{Scalar}$$

$$C = A_x B_x + A_y B_y$$



Projection of $\bar{A}$ on $\bar{B}$

How much (mag) of A is in the dir$^{\underline{n}}$ as $\bar{B}$

Cross Product

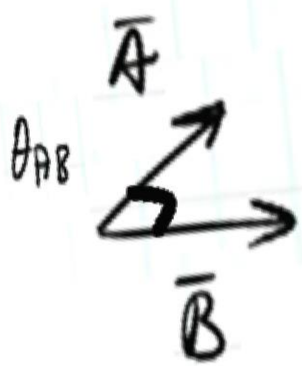
$\bar{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z$

$\bar{B} = \hat{x} B_x + \hat{y} B_y + \hat{z} B_z$

$\bar{A} \times \bar{B} = \bar{C} = |A||B| \sin\theta_{AB} \; \hat{n}$

vector

normal to both $\bar{A}\,\bar{B}$



Cross Product

$$\bar{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z$$

$$\bar{B} = \hat{x} B_x + \hat{y} B_y + \hat{z} B_z$$

$$\bar{A} \times \bar{B} = \bar{C}$$

$$\begin{vmatrix} \hat{x} & -\hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}\left(A_y B_z - B_y A_z\right)$$

$$- \hat{y}\left(A_x B_z - A_z B_x\right)$$

$$+ \hat{z}\left(A_x B_y - A_y B_x\right)$$

Cross Product

$$\bar{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z$$

$$\bar{B} = \hat{x} B_x + \hat{y} B_y + \hat{z} B_z$$

vector

$$\bar{A} \times \bar{B} = \bar{C} = |A||B| \sin\theta_{AB} \, \hat{n}$$

normal to both $\bar{A}\ \bar{B}$

$$\begin{vmatrix} \hat{x} & -\hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x} \cdot (A_y B_z - B_y A_z)$$

$$- \hat{y} (A_x B_z - A_z B_x)$$

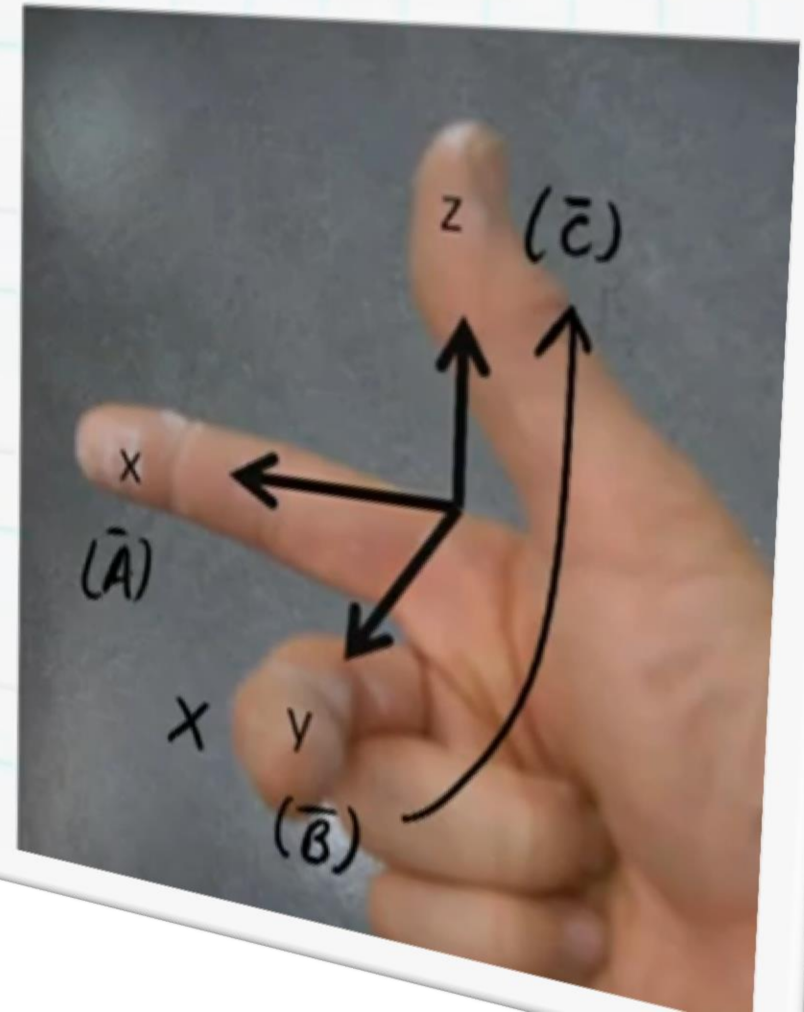
$$+ \hat{z} (A_x B_y - A_y B_x)$$

Right Hand Rule

$$\hat{x} \times \hat{y} = \hat{z}$$

$$\bar{A} \times \bar{B} = \bar{C}$$

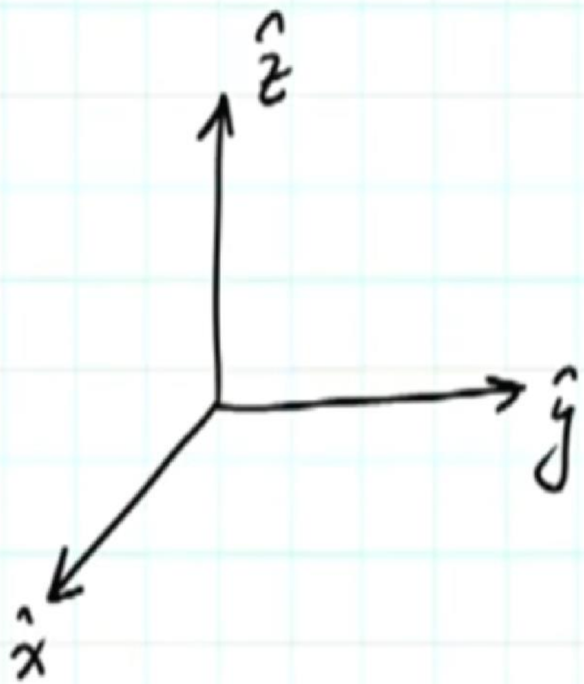
$$\bar{B} \times \bar{A} = -\bar{C}$$



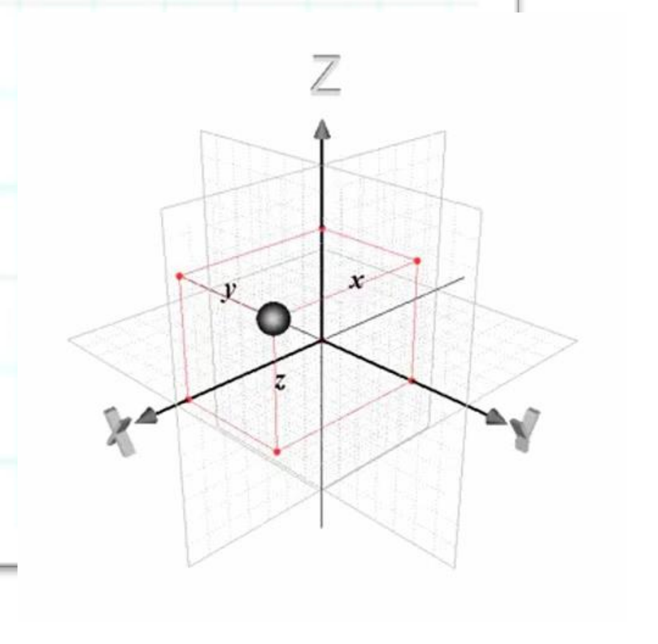
Another Form of R Hand Rule

here. B is around this way and C is pointing up.

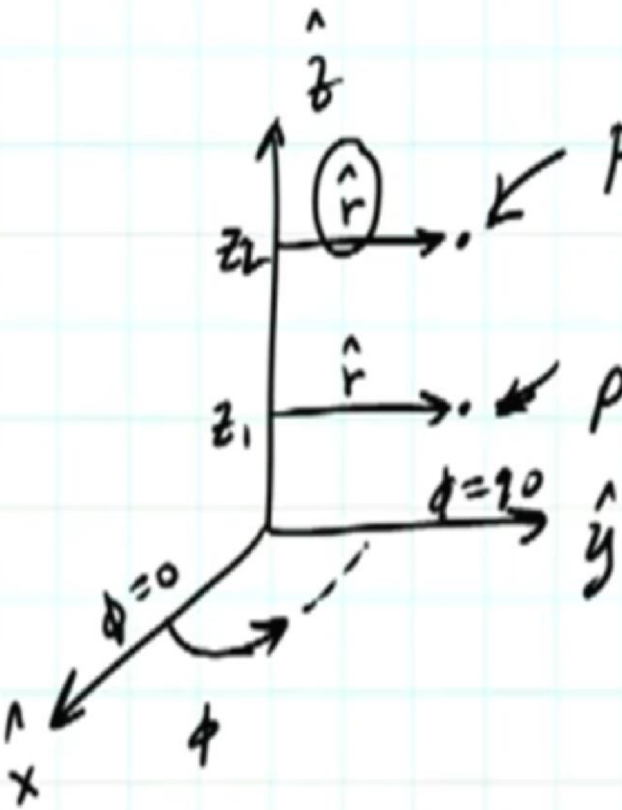
Coordinate Systems
Rectangular (Cartesian)



$$\hat{x} \times \hat{y} = \hat{z}$$



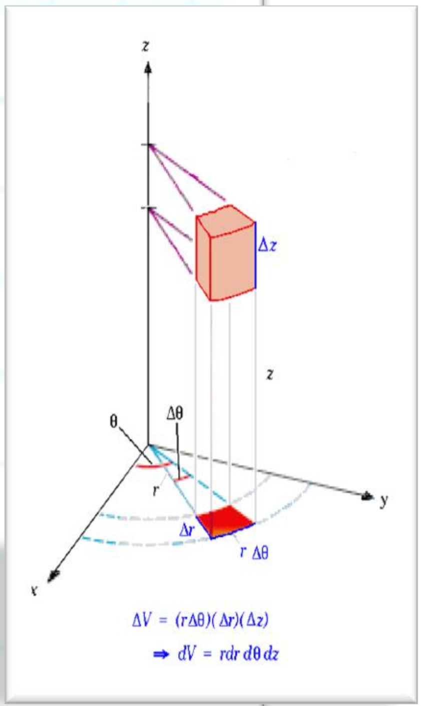
Cylindrical Coordinate System



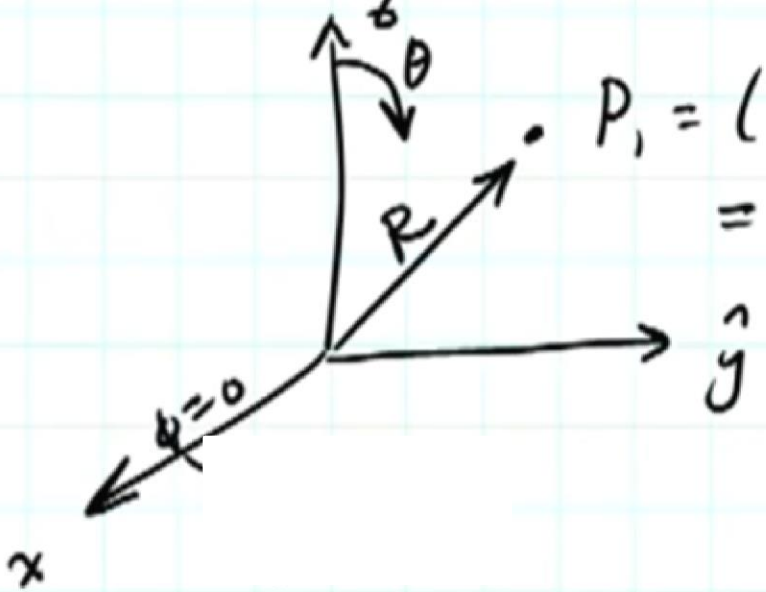
$P_2 = (r, \phi, z)$

$(1m, \phi = 90°, 3m)$

$P_1 = (1m, 90°, 1m)$

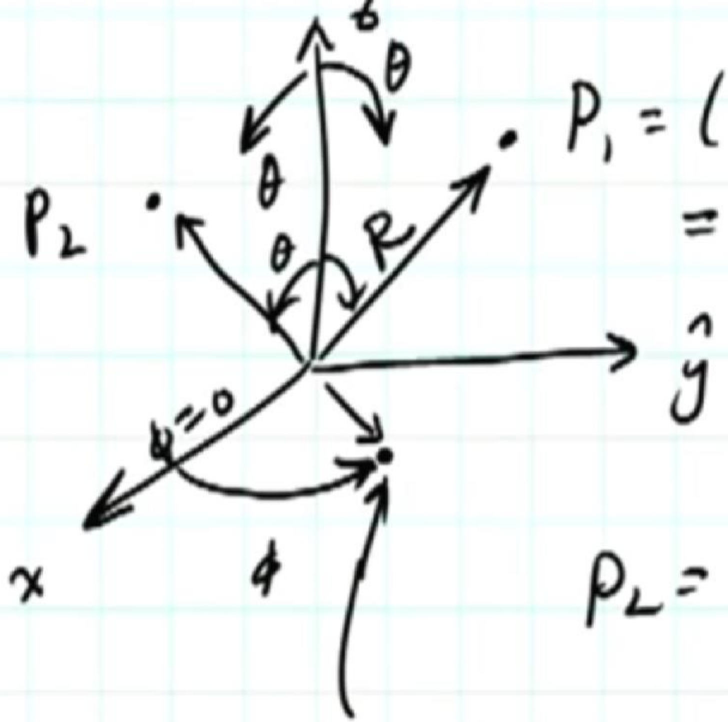


Spherical Coordinate System



$$P_1 = (R, \phi, \theta)$$
$$= (1\,m, 90^\circ, 45^\circ)$$

Spherical Coordinate System



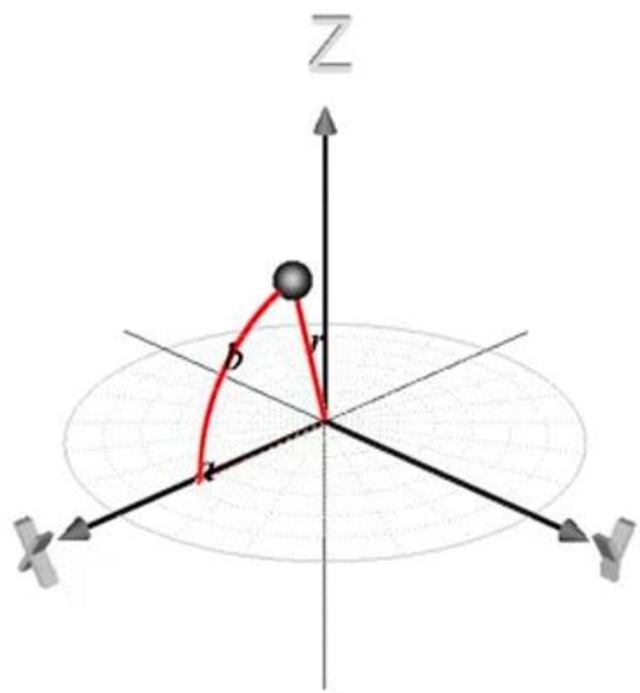
$P_1 = (R, \phi, \theta)$

$= (1m, 90°, 45°)$

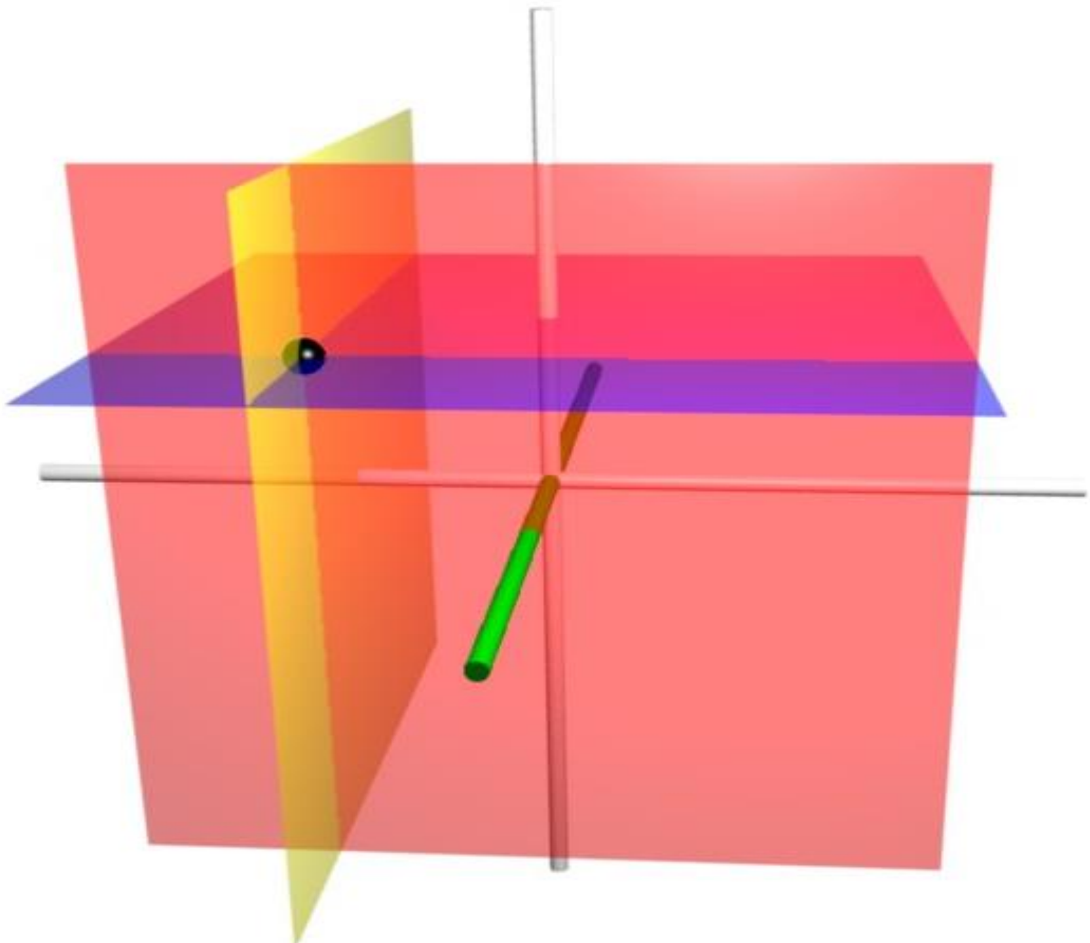
$P_2 = (1m, 0, 45°)$

$P_3 = (1m, 45°, 90°)$

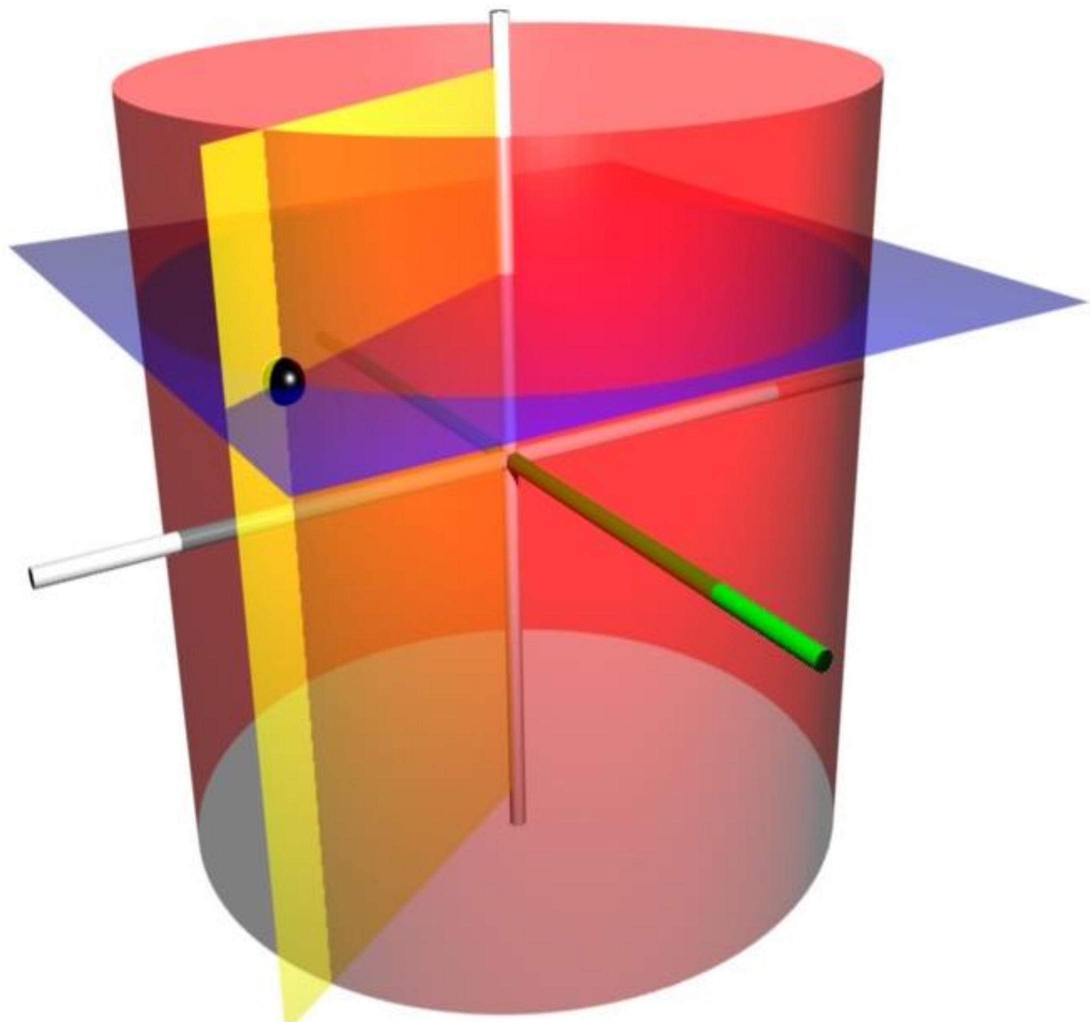
Spherical Coordinate System



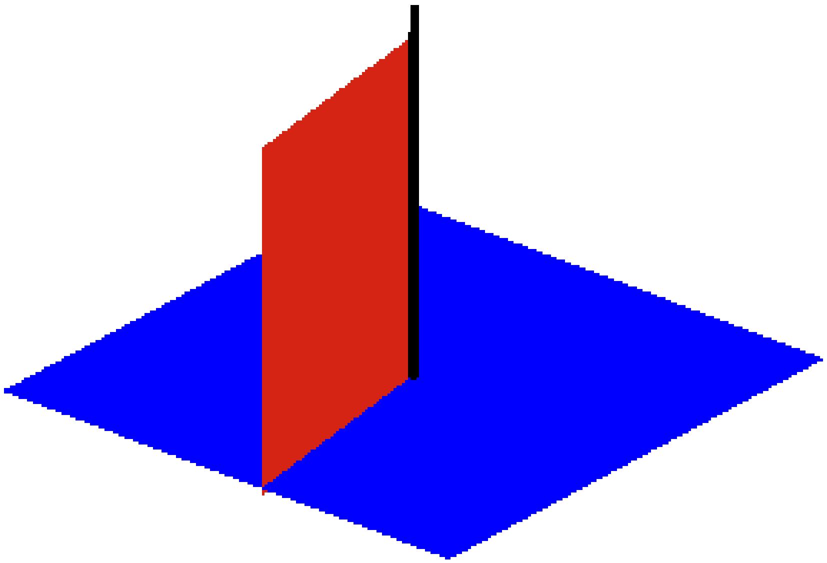
Rectangular Coordinate System



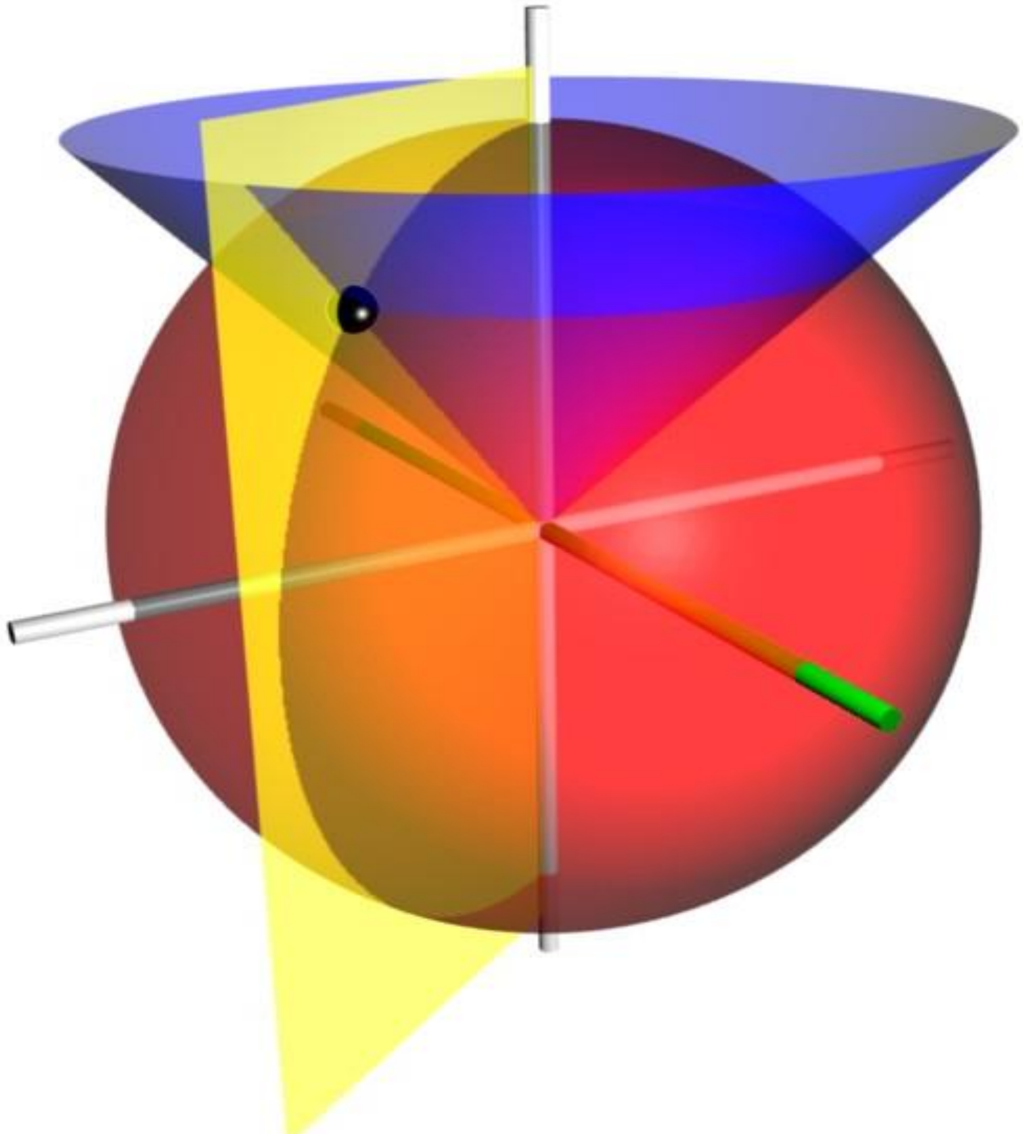
Cylindrical Coordinate System



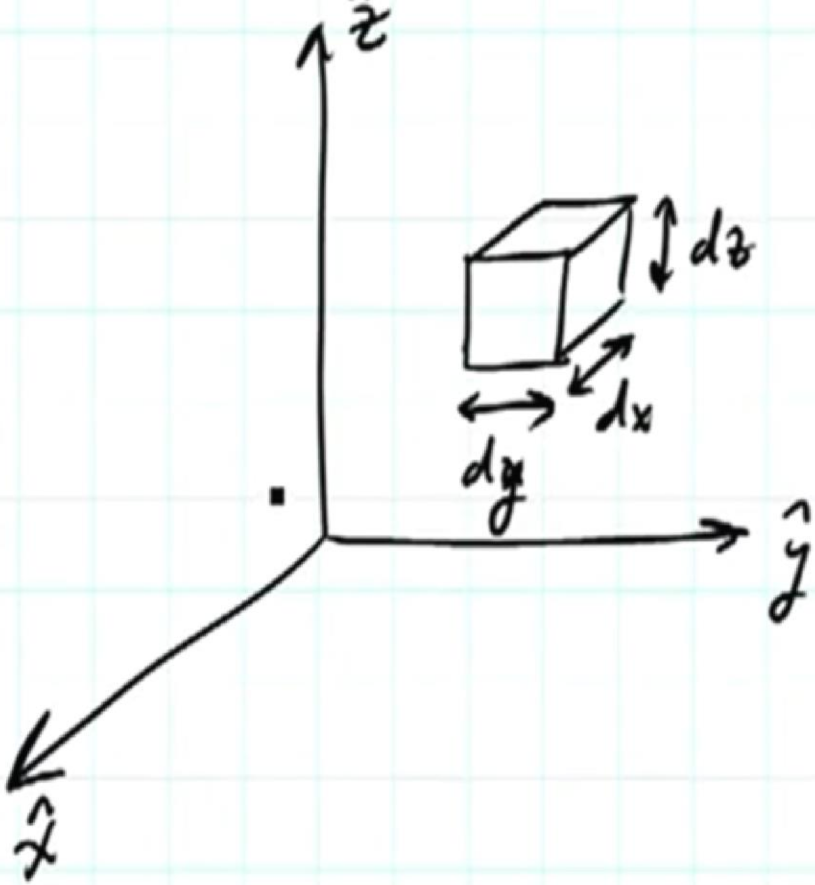
Cylindrical Coordinate System



Cylindrical Coordinate System



Differential Elements of Line, Surface, Volume -- Rectangular

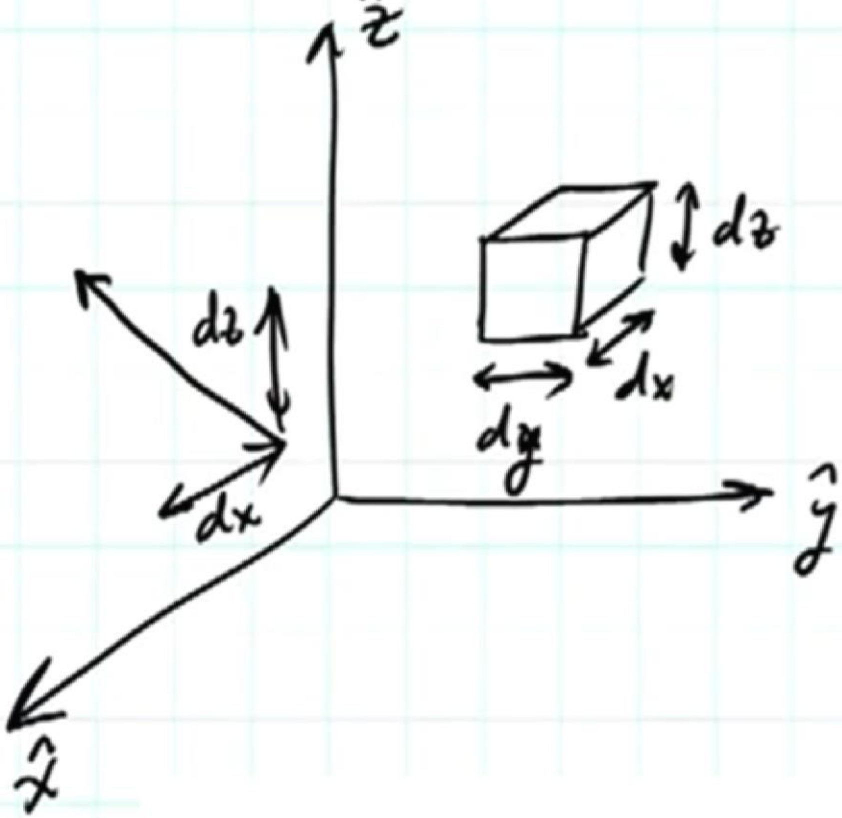


From Table 3-1:

$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$
$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$

$dv = dx\,dy\,dz$ scalar

Differential Elements of Line, Surface, Volume -- Rectangular



From Table 3-1:

$$d\vec{l} = \hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$$

$$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$$

$$d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$$

$$d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$$

$$dv = dx\,dy\,dz \quad \text{Scalar}$$

Differential Elements of Line, Surface, Volume -- Rectangular

From Table 3-1:

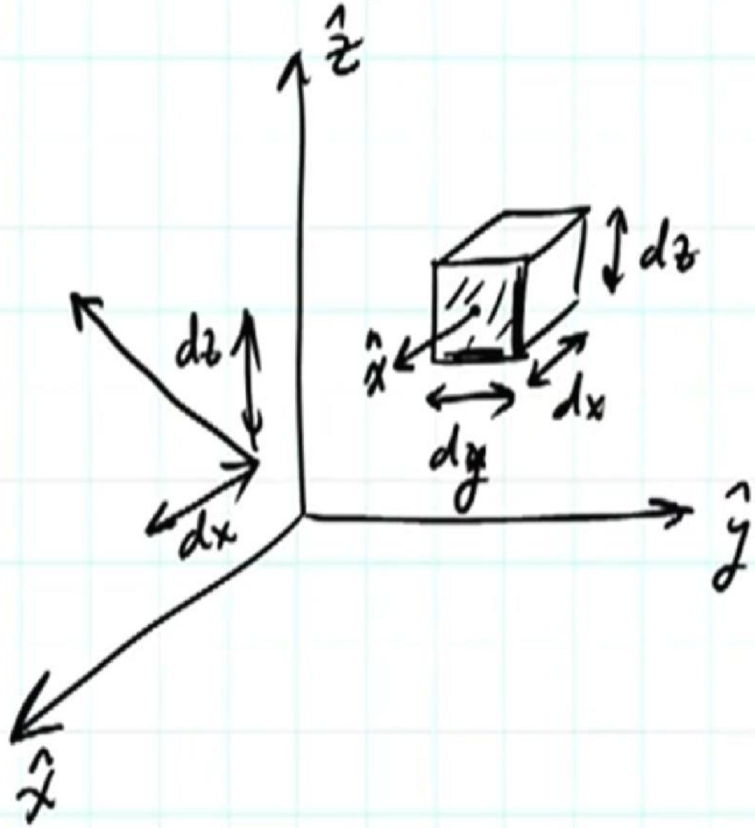
$$d\bar{l} = \hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$$

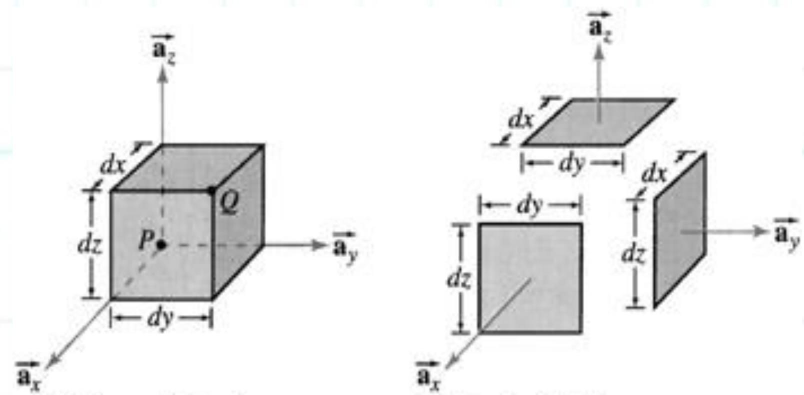
$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ Front

$d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$

$d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$

$dv = dx\,dy\,dz$ scalar





a) Differential volume b) Exploded view

Differential Elements of Line, Surface, Volume -- Rectangular

From Table 3-1:

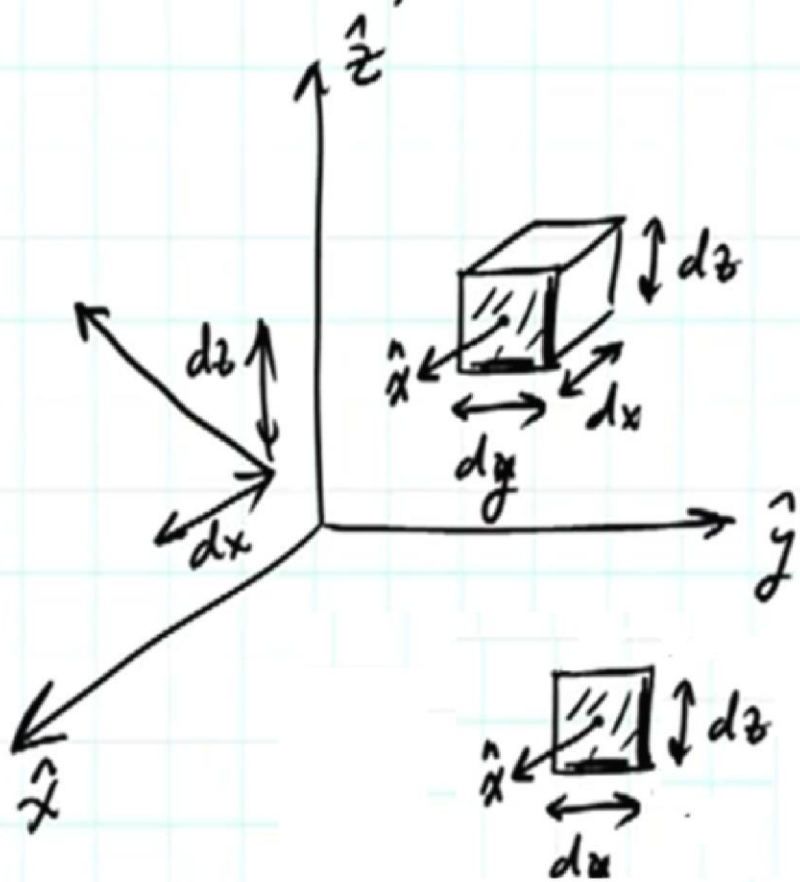
$d\bar{l} = \hat{\mathbf{x}}dx + \hat{\mathbf{y}}dy + \hat{\mathbf{z}}dz$

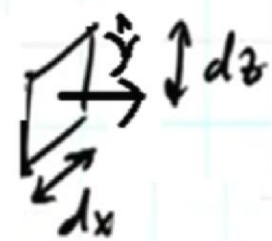
$ds_x = \hat{\mathbf{x}}dydz$ Front

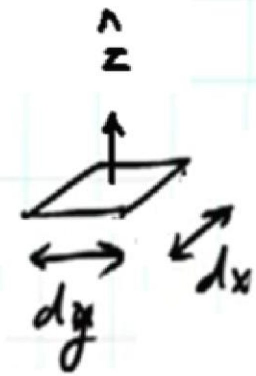
$ds_y = \hat{\mathbf{y}}dxdz$

$ds_z = \hat{\mathbf{z}}dxdy$

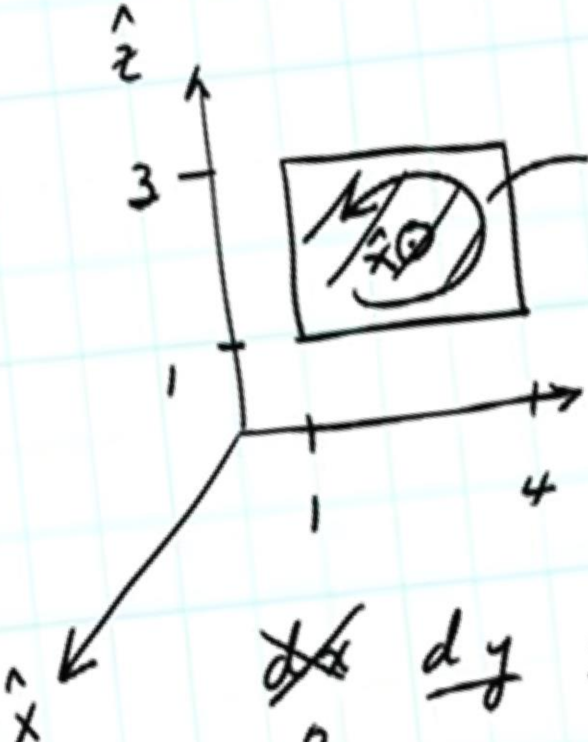
$dv = dxdydz$ Scala







Surface Integral



$$Q = \left(4\frac{C}{m^2}\right)(2\times 3) = 24\,C$$

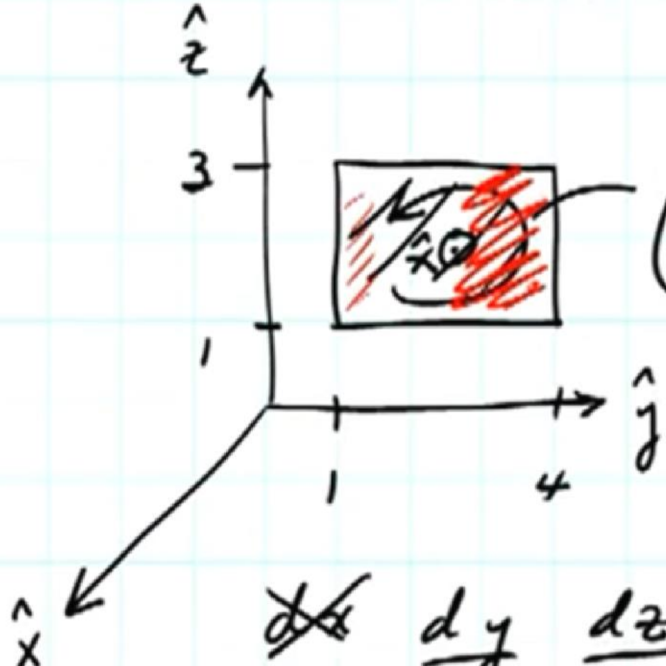
From Table 3-1:

$\hat{x}dx+\hat{y}dy+\hat{z}dz$
$ds_x = \hat{x}\,dy\,dz$ ✓ $ds_y = \hat{y}\,\cancel{dx}\,dz$ $ds_z = \hat{z}\,\cancel{dx}\,dy$
$dx\,dy\,dz$

$\underset{0}{\cancel{dx}}\ \ \underline{dy}\ \ \underline{dz}$

$$\int_{z=1}^{3}\int_{y=1}^{4}\left(4\frac{C}{m^2}\right)dy\,dz = 4\frac{C}{m^2}\; y\Big|_{1}^{4}\; z\Big|_{1}^{3} = 4\cdot 3\cdot 2 = 24\,C$$

Surface Integral



$Q =$

$\left(4y\frac{C}{m^2}\right)(2\times 3)$

$= 24\,C$

From Table 3-1:

$\hat{\mathbf{x}}\,dx+\hat{\mathbf{y}}\,dy+\hat{\mathbf{z}}\,dz$
$ds_x = \hat{\mathbf{x}}\,dy\,dz$ ✓ $ds_y = \hat{\mathbf{y}}\,\cancel{dx}\,dz$ $ds_z = \hat{\mathbf{z}}\,\cancel{dx}\,dy$
$dx\,dy\,dz$

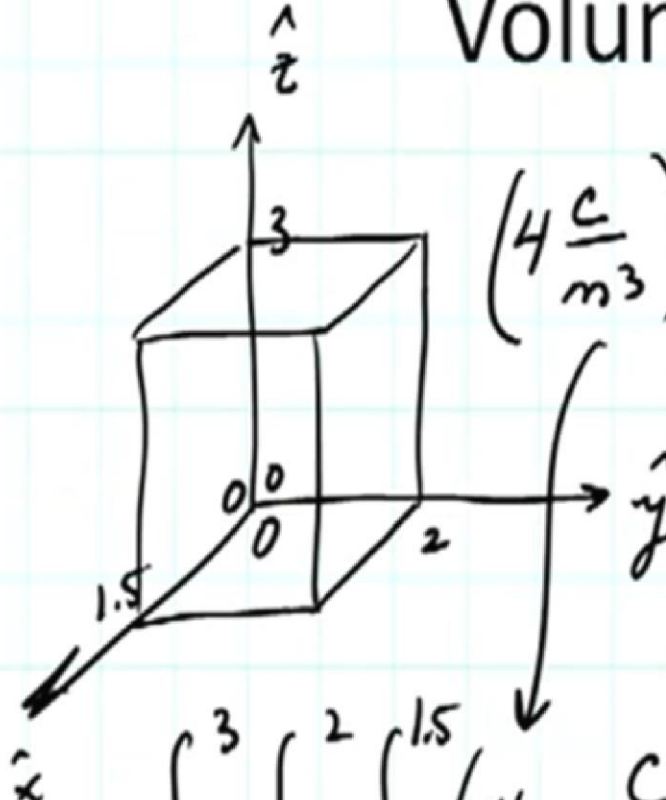
$\cancel{dx}$ $\underline{dy}$ $\underline{dz}$

0

$$\int_{z=1}^{3}\int_{y=1}^{4}\left(4y\frac{C}{m^2}\right)dy\,dz = 4\frac{C}{m^2}\,\frac{y^2}{2}\Big|_1^4\; z\Big|_1^3$$

$$= 4\cdot\frac{3^2}{2}\cdot 2 = \cancel{24}\;C$$

Volume Integral



$$\left(4\frac{C}{m^3}\right)(1.5)(2)(3) = Q_{TOT}$$

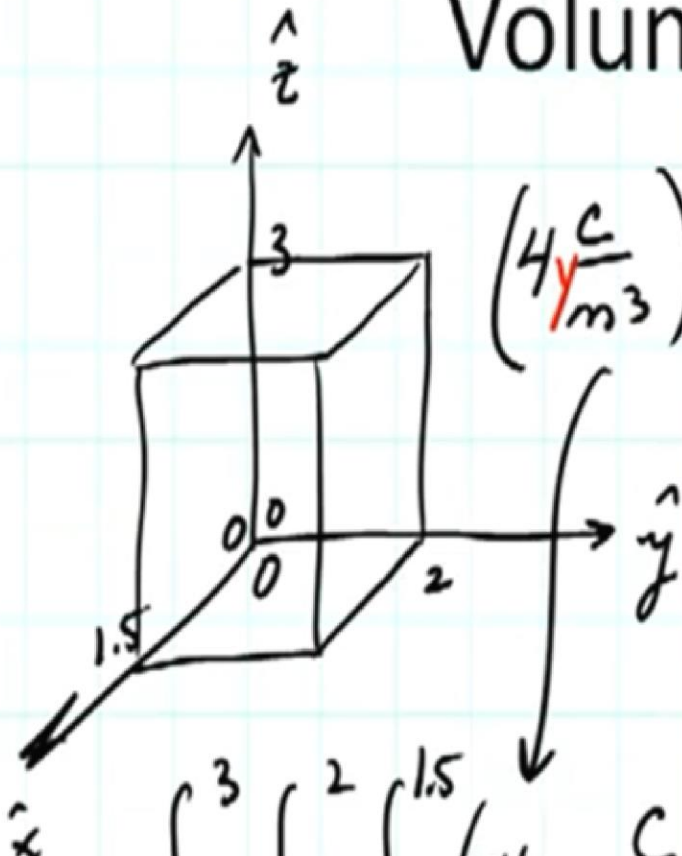
From Table 3-1:

$\hat{\mathbf{x}}\,dx+\hat{\mathbf{y}}\,dy+\hat{\mathbf{z}}\,dz$
$d\mathbf{s}_x=\hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y=\hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z=\hat{\mathbf{z}}\,dx\,dy$
$dv =$ $dx\,dy\,dz$

$$\int_{z=0}^{3}\int_{y=0}^{2}\int_{x=0}^{1.5}\left(4\ \frac{C}{m^3}\right)dx\,dy\,dz$$

$$4\ x\Big|_0^{1.5}\ y\Big|_0^{2}\ z\Big|_0^{3} = Q_{TOT}$$

Volume Integral



$$\left(4 y \frac{C}{m^3}\right)(1.5)(2)(3) \cancel{=} Q_{TOT}$$

From Table 3-1:

$\hat{\mathbf{x}}\,dx+\hat{\mathbf{y}}\,dy+\hat{\mathbf{z}}\,dz$
$d\mathbf{s}_x=\hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y=\hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z=\hat{\mathbf{z}}\,dx\,dy$
$dx\,dy\,dz$

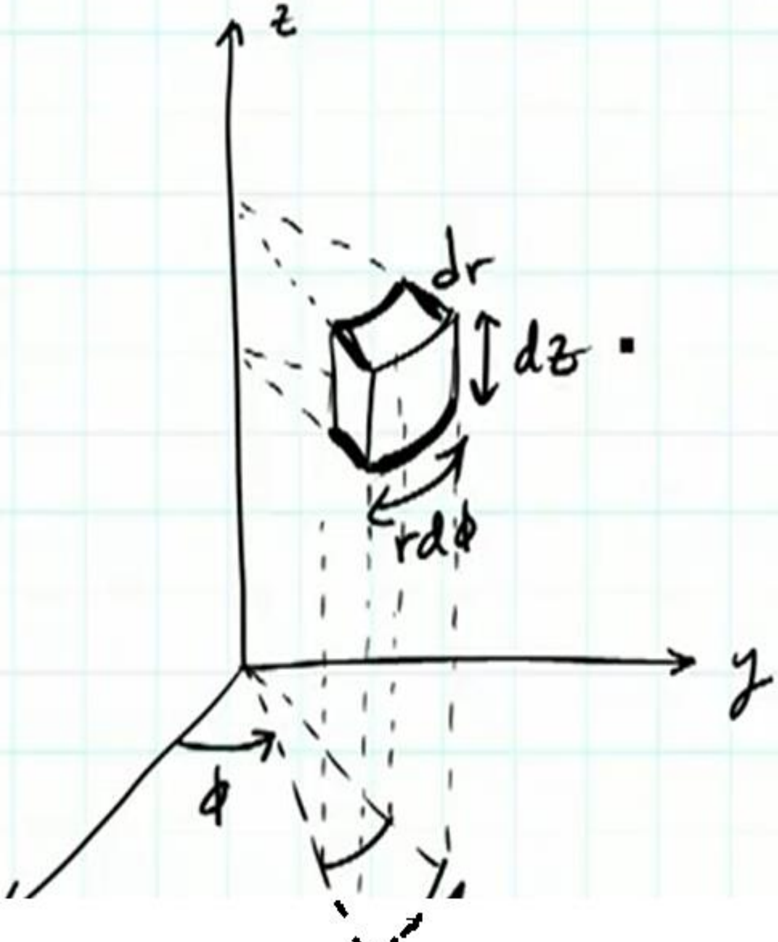
$dv =$

$$\int_{z=0}^{3}\int_{y=0}^{2}\int_{x=0}^{1.5}\left(4\, y\, \frac{C}{m^3}\right) dx\, dy\, dz$$

nonuniform.

$$4\, x\Big|_0^{1.5}\ \frac{y^2}{2}\Big|_0^{2}\ z\Big|_0^{3}$$

Differential Elements of Line, Surface, Volume -- Cylindrical



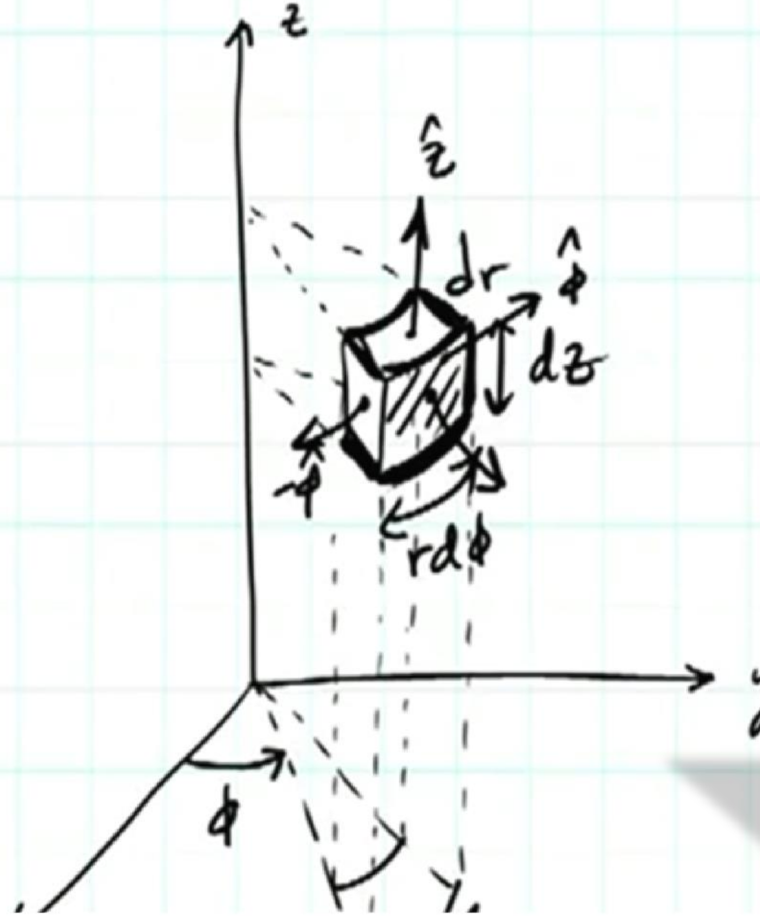
From Table 3-1:

$$d\bar{l} = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}\,r\,d\phi + \hat{\mathbf{z}}\,dz$$

$$d\bar{s} = \begin{cases} d\mathbf{s}_r = \hat{\mathbf{r}}\,r\,d\phi\,dz \\ d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz \\ d\mathbf{s}_z = \hat{\mathbf{z}}\,r\,dr\,d\phi \end{cases}$$

$$dV = r\,dr\,d\phi\,dz$$

Differential Elements of Line, Surface, Volume -- Cylindrical

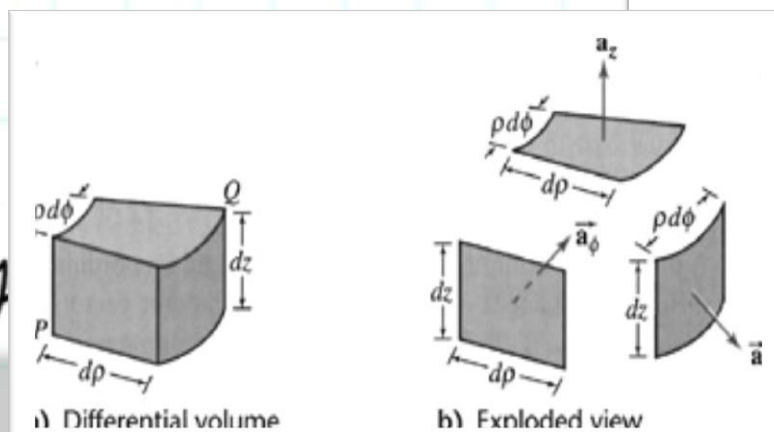


From Table 3-1:

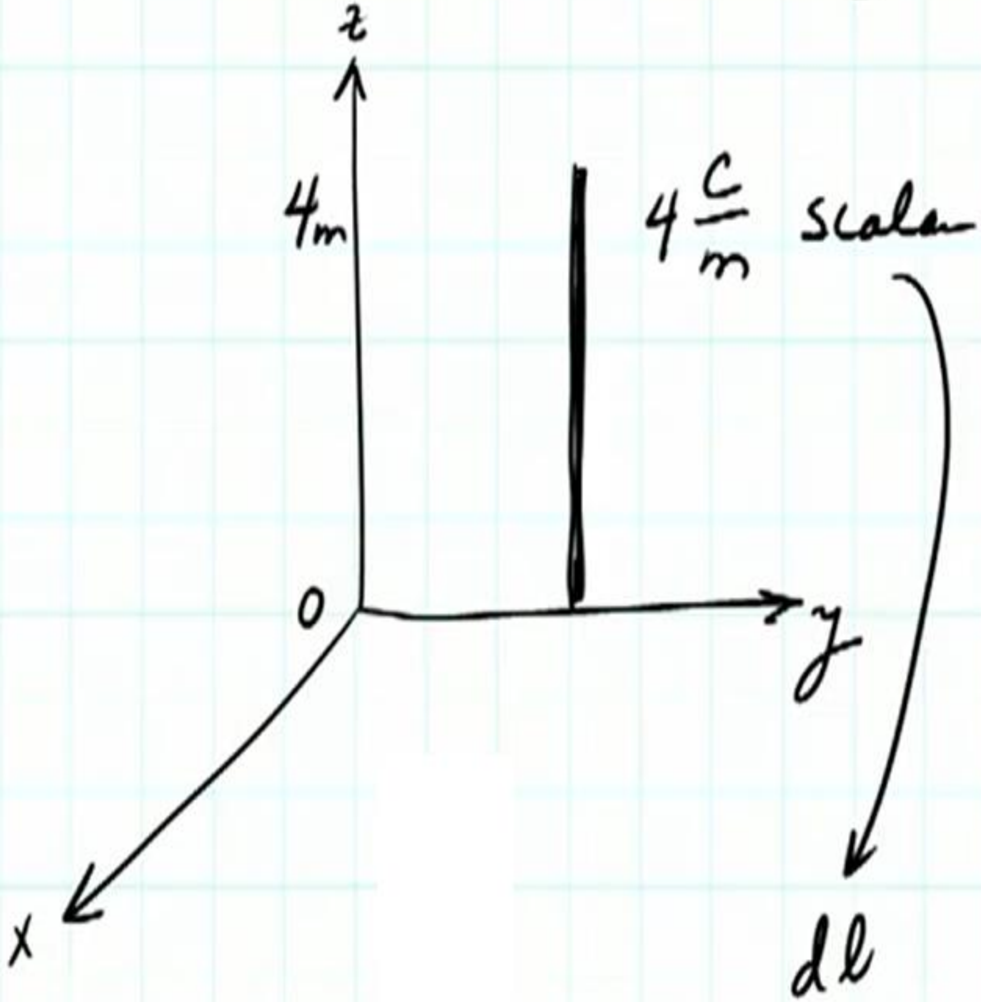
$$d\bar{l} = \hat{r}\,dr + \hat{\phi}\,r\,d\phi + \hat{z}\,dz$$

$$d\bar{s} = \begin{cases} ds_r = \hat{r}\,r\,d\phi\,dz & \text{FRONT} \\ ds_\phi = \hat{\phi}\,dr\,dz & \text{BACK} \\ ds_z = \hat{z}\,r\,dr\,d\phi & \text{TOP} \end{cases}$$

$$dV = r\,dr\,d\phi\,dz$$



Line Integral



From Table 3-1:

$d\bar{\ell} = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}\,r\,d\phi + \hat{\mathbf{z}}\,dz$

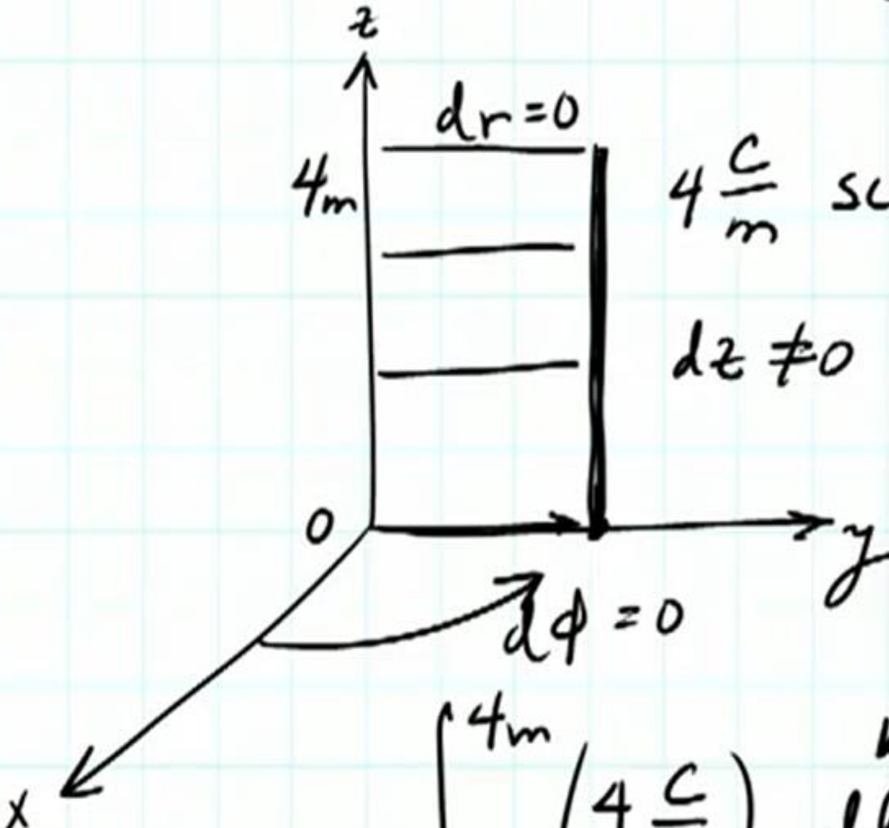
$ds_r = \hat{\mathbf{r}}\,r\,d\phi\,dz$

$ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$

$ds_z = \hat{\mathbf{z}}\,r\,dr\,d\phi$

$r\,dr\,d\phi\,dz$

Line Integral



$4\frac{C}{m}$ scalar

$dz \neq 0$

From Table 3-1:

$d\bar{\ell} = \cancel{\hat{\mathbf{r}}\,dr} + \cancel{\hat{\boldsymbol{\phi}}\,r\,d\phi} + \hat{\mathbf{z}}\,dz$

$ds_r = \hat{\mathbf{r}}\,r\,d\phi\,dz$

$ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$

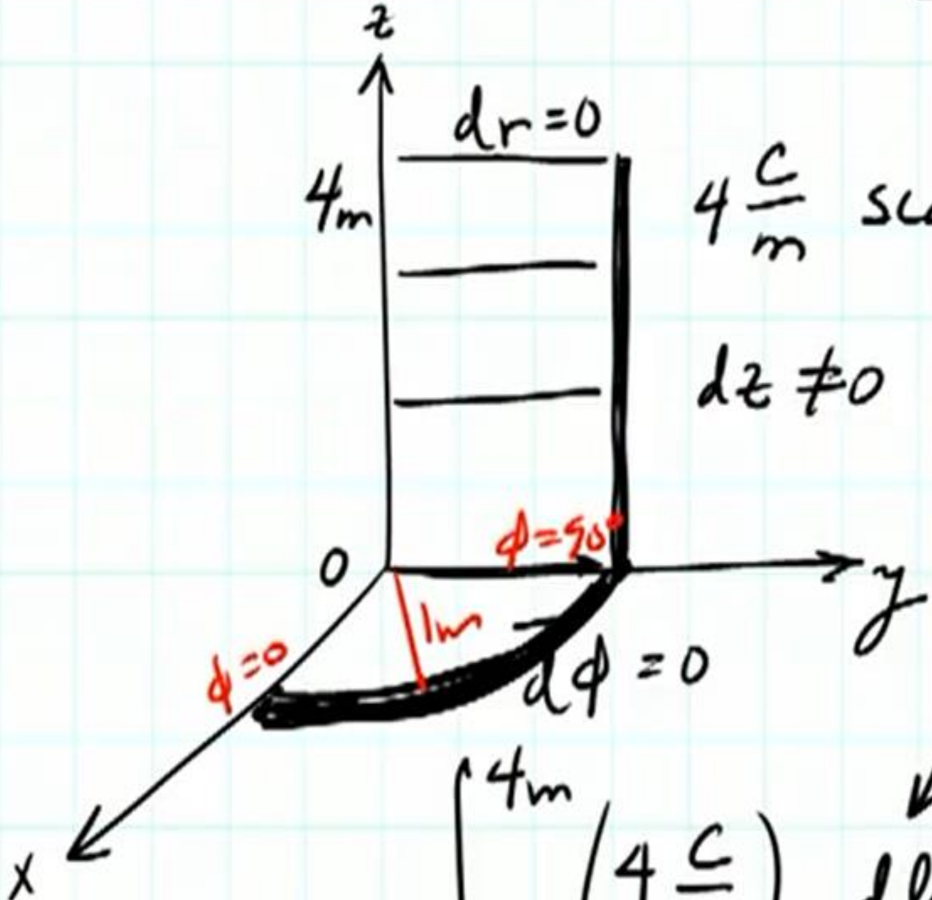
$ds_z = \hat{\mathbf{z}}\,r\,dr\,d\phi$

$r\,dr\,d\phi\,dz$

$$\int_{z=0}^{4m} \left(4\frac{C}{m}\right) \underset{dz}{d\ell} = \left(4\frac{C}{m}\right) z\Big|_0^4$$

$$= 16\ C/m$$

Line Integral



$4\frac{c}{m}$ scalar

$dz \neq 0$

From Table 3-1:

$d\bar{\ell} = \cancel{\hat{\mathbf{r}}\,dr} + \hat{\boldsymbol{\phi}}\, r\, d\phi + \hat{\mathbf{z}}\, dz$

$ds_r = \hat{\mathbf{r}}\, r\, d\phi\, dz$

$ds_\phi = \hat{\boldsymbol{\phi}}\, dr\, dz$

$ds_z = \hat{\mathbf{z}}\, r\, dr\, d\phi$

$r\, dr\, d\phi\, dz$

$$\int_{\phi=0}^{\pi/4} \left(4\frac{c}{m}\right) \overset{1m}{r}\, d\phi$$

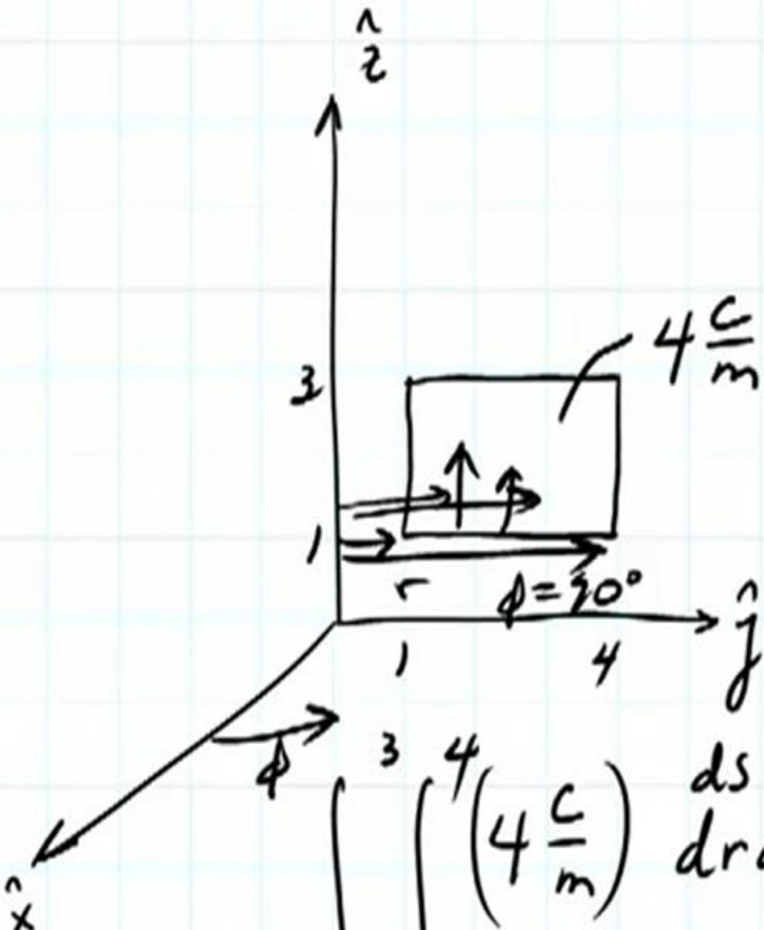
$$\left(4\frac{c}{m}\right) \overset{1m}{r}\, \phi\Big|_0^{\pi/4}$$

$$4\frac{\pi}{4}\, C$$

$$\int_z^{4m} \left(4\frac{c}{m}\right) \underset{dz}{d\ell} = \left(4\frac{c}{m}\right) z\Big|_0^4$$

$$= 16\ C/m$$

Surface Integral



$dr \neq 0$ ✓

$d\phi = 0$

$dz \neq 0$ ✓

From Table 3-1:

$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}\,r\,d\phi + \hat{\mathbf{z}}\,dz$

$ds_r = \hat{\mathbf{r}}\,r\,\cancel{d\phi}\,dz$

$ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$

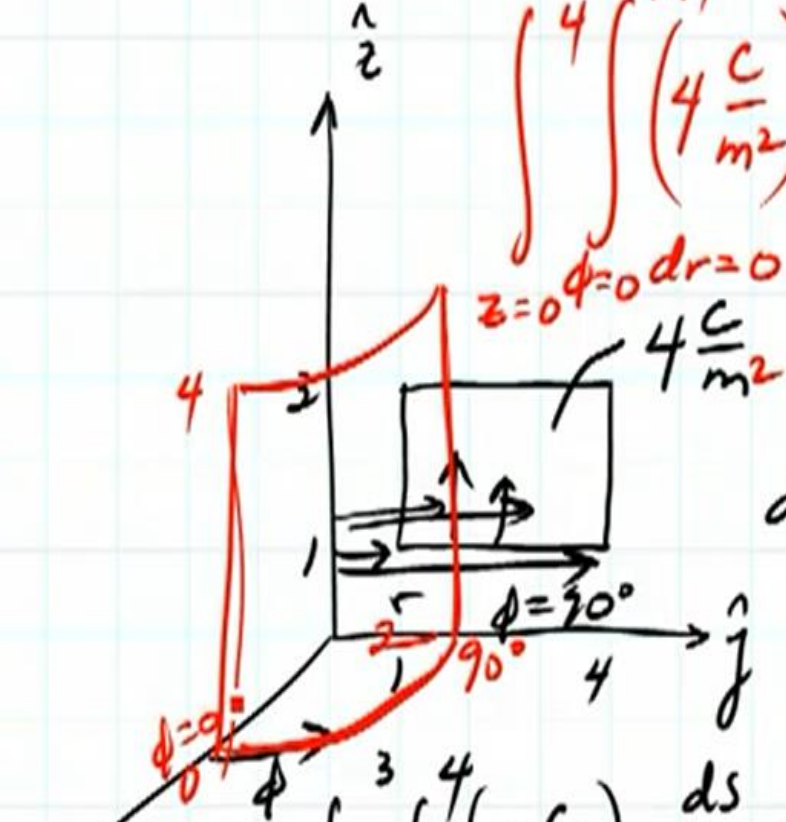
$ds_z = \hat{\mathbf{z}}\,r\,dr\,\cancel{d\phi}$

$r\,dr\,d\phi\,dz$

$$\int_{z=1}^{3}\int_{r=1}^{4}\left(4\frac{C}{m}\right)\overbrace{dr\,dz}^{ds} = \left(4\frac{C}{m}\right) r\Big|_1^4 \; z\Big|_1^3$$

$$= 4\cdot 4\cdot 3 \; ✓$$

Surface Integral



$$\int_{z=0}^{4}\int_{\phi=0}^{\pi/4}\left(4\frac{C}{m^2}\right) r\,d\phi\,dz \qquad dr=0$$

2m

$$\left(4\frac{C}{m^2}\right)(2)\,\phi\Big|_0^{\pi/4}$$

$$z\Big|_0^4$$

From Table 3-1:

$\hat{r}\,dr + \hat{\phi}\,r\,d\phi + \hat{z}\,dz$

$ds_r = \hat{r}\,r\,d\phi\,dz$

$ds_\phi = \hat{\phi}\,dr\,dz$

$ds_z = \hat{z}\,r\,dr\,d\phi$

$r\,dr\,d\phi\,dz$

$dr \neq 0$ ✓

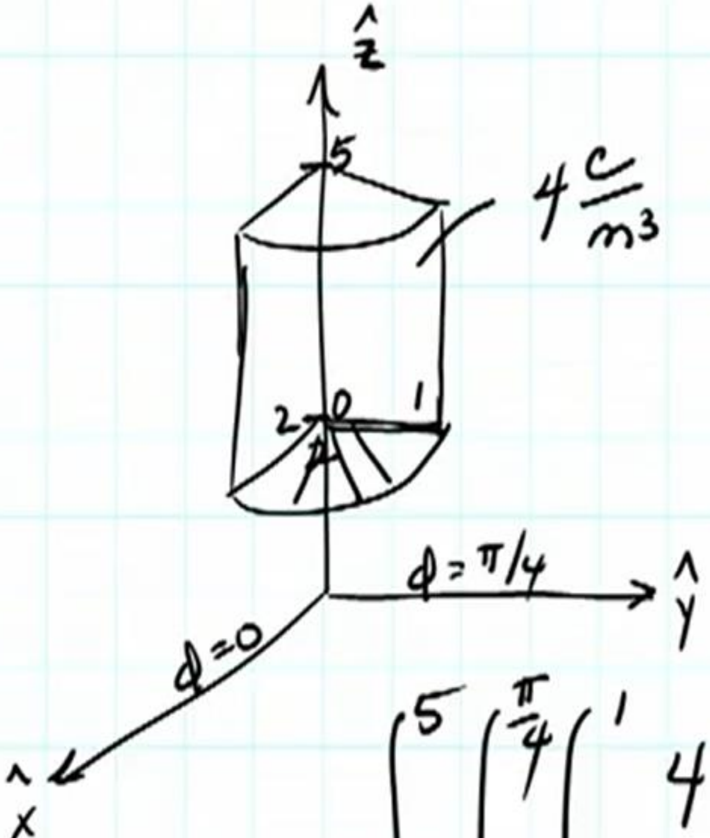
$d\phi = 0$

$dz \neq 0$ ✓

$$\int_{z=1}^{3}\int_{r=1}^{4}\left(4\frac{C}{m^2}\right) \overset{ds}{dr\,dz} = \left(4\frac{C}{m^2}\right) r\Big|_1^4 \; z\Big|_1^3$$

$$= 4 \cdot 4 \cdot 3$$ ✓

Volume Integral



From Table 3-1:

$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}\,r\,d\phi + \hat{\mathbf{z}}\,dz$

$ds_r = \hat{\mathbf{r}}\,r\,d\phi\,dz$

$ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$

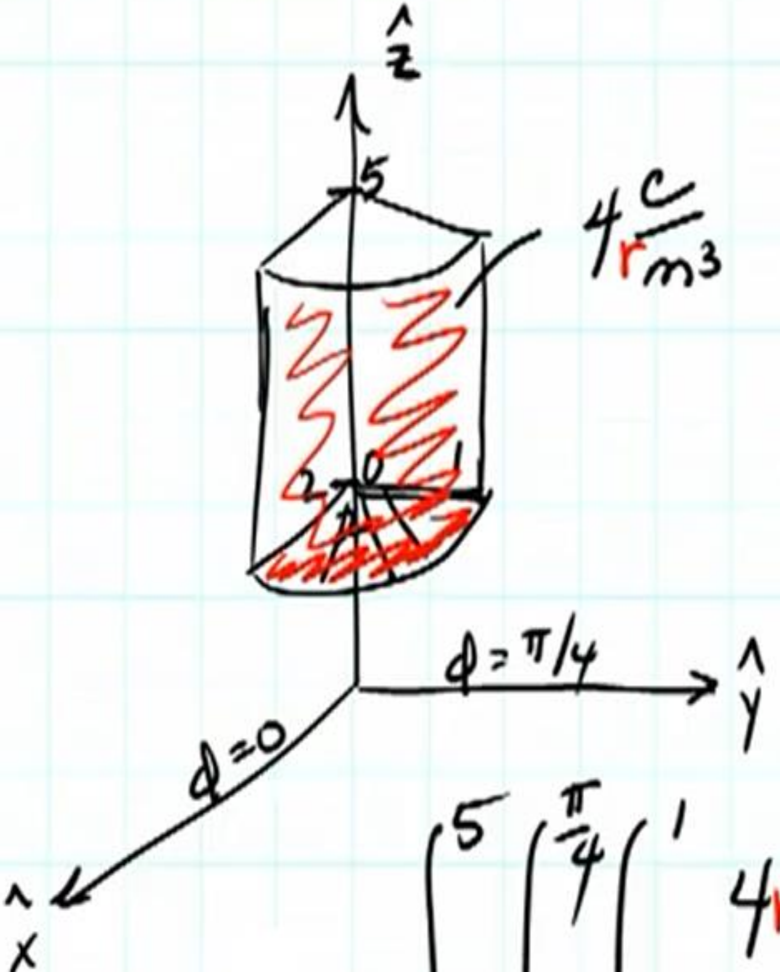
$ds_z = \hat{\mathbf{z}}\,r\,dr\,d\phi$

$dv = r\,dr\,d\phi\,dz$

$$\int_{z=2}^{5}\int_{\phi=0}^{\frac{\pi}{4}}\int_{r=0}^{1} 4\frac{C}{m^3}\, \underline{r}\,dr\,d\phi\,dz$$

$$\left(4\frac{C}{m^3}\right)\frac{r^2}{2}\Big|_0^1\ \phi\Big|_0^{\pi/4}\ z\Big|_2^5$$

Volume Integral



From Table 3-1:

$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}\,r\,d\phi + \hat{\mathbf{z}}\,dz$

$d\mathbf{s}_r = \hat{\mathbf{r}}\,r\,d\phi\,dz$

$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$

$d\mathbf{s}_z = \hat{\mathbf{z}}\,r\,dr\,d\phi$

$dv = r\,dr\,d\phi\,dz$

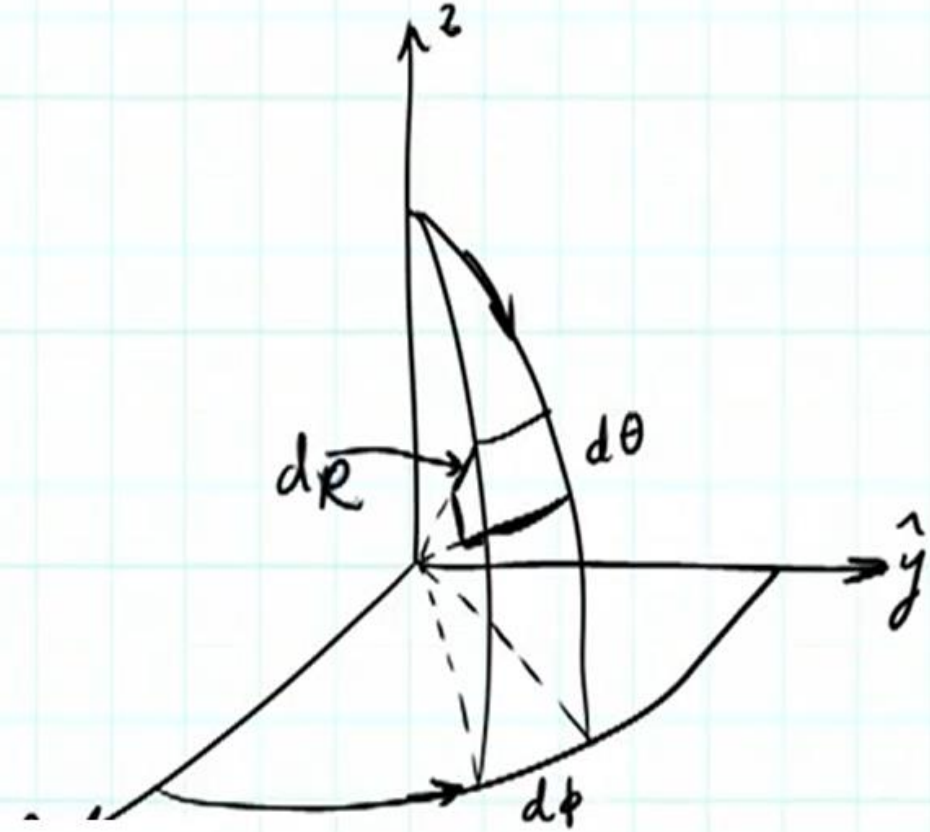
$$\int_{z=2}^{5}\int_{\phi=0}^{\frac{\pi}{4}}\int_{r=0}^{1} 4r\frac{C}{m^3}\; r\, dr\, d\phi\, dz$$

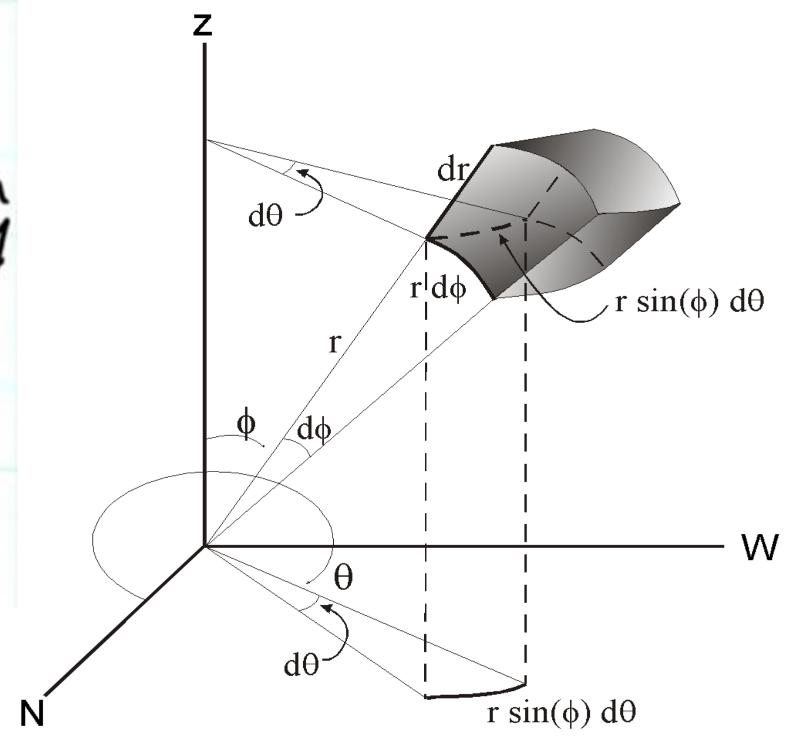
$$\left(4\frac{C}{m^3}\right)\frac{r^3}{3}\Big|_0^1 \;\phi\Big|_0^{\pi/4}\; z\Big|_2^5$$

Differential Elements of Line, Surface, Volume -- Spherical

From Table 3-1:

$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
$R^2\sin\theta\,dR\,d\theta\,d\phi$





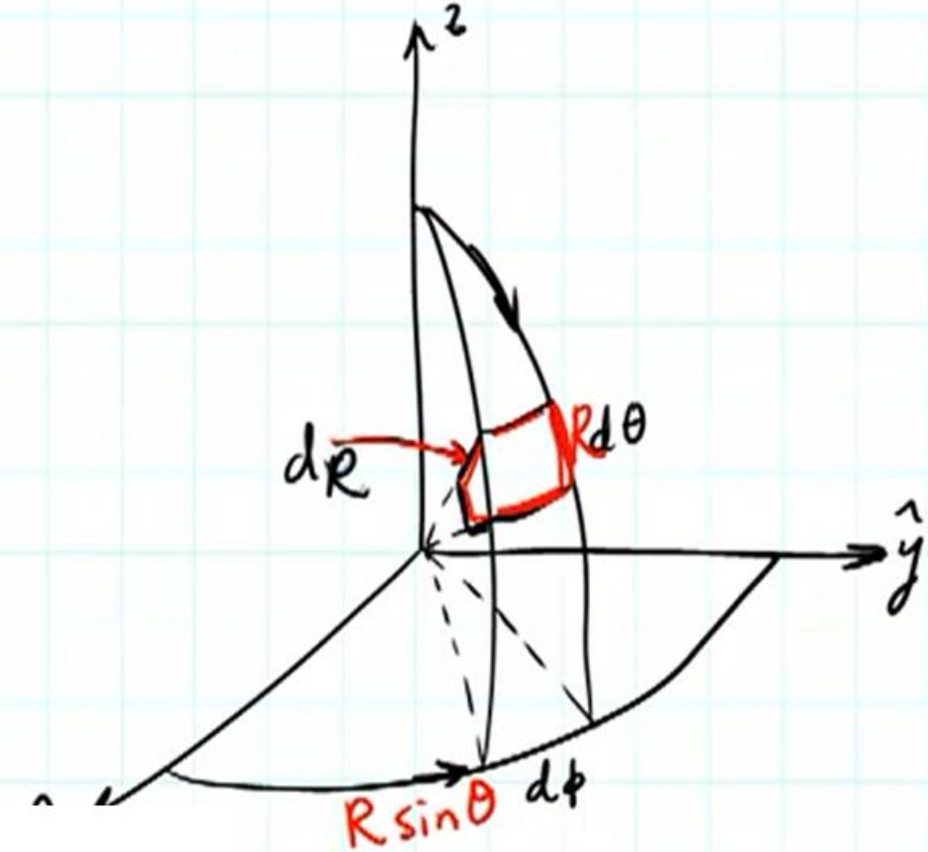
Differential Elements of Line, Surface, Volume -- Spherical

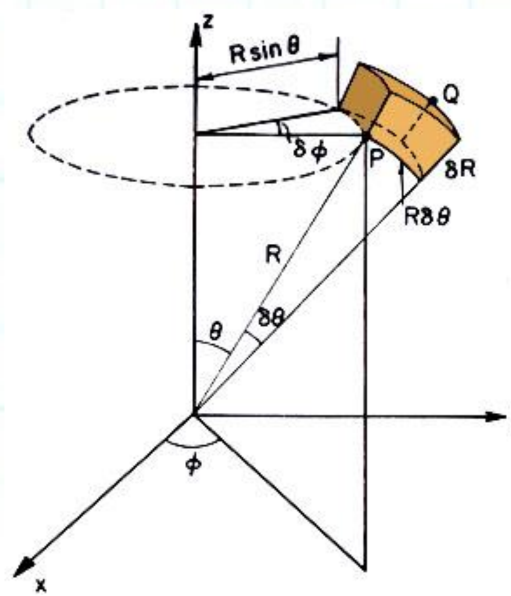
From Table 3-1:

$$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$$

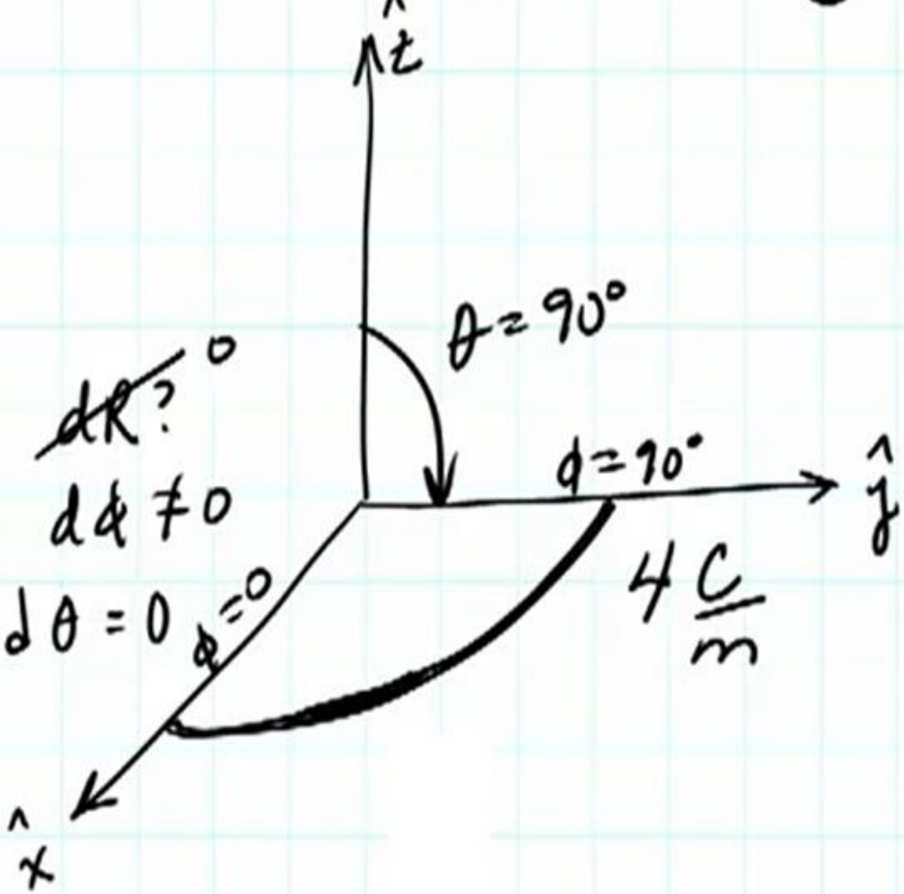
$$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$$
$$d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$$
$$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$$

$$R^2\sin\theta\,dR\,d\theta\,d\phi$$





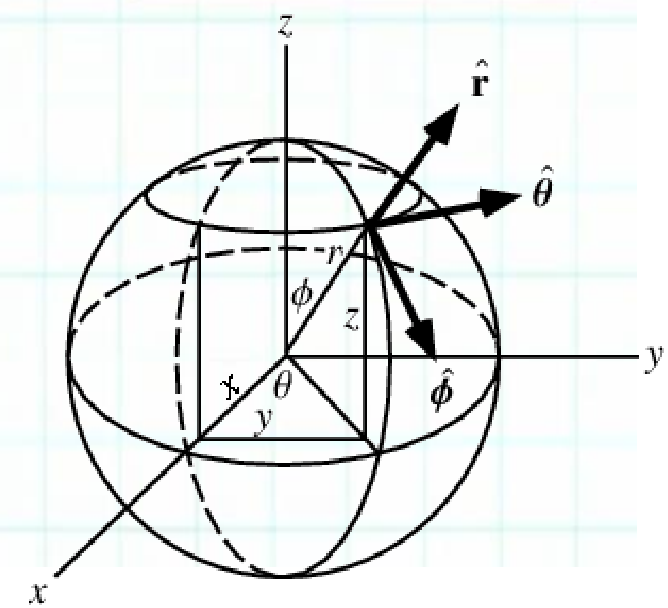
Line Integral



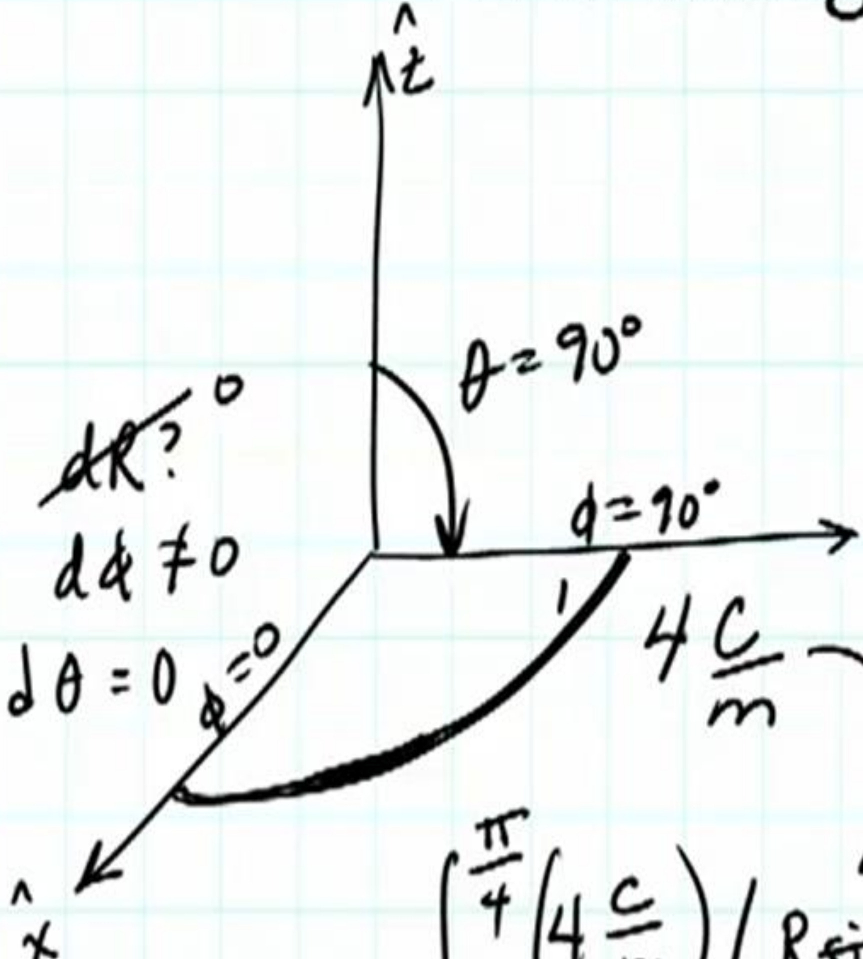
From Table 3-1:

$d\bar{l} =$

$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
$R^2\sin\theta\,dR\,d\theta\,d\phi$



Line Integral



$4\,\frac{C}{m}$ scalar

dl

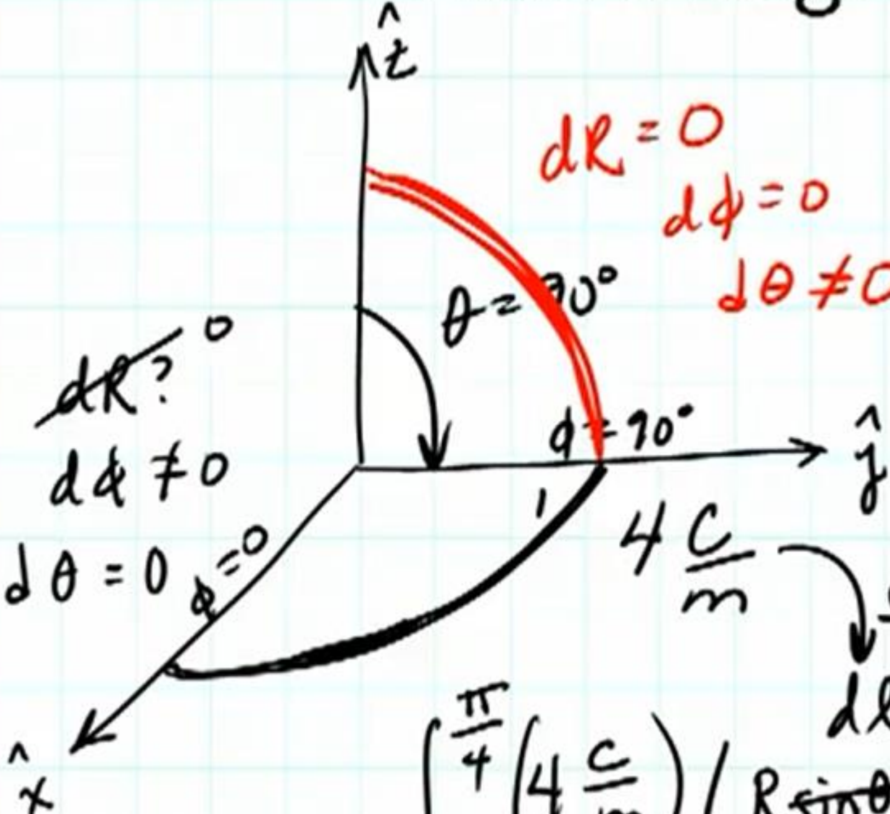
From Table 3-1:

$$d\bar{l} = \hat{R}\,\cancel{dR}^{\,0} + \hat{\theta}R\,\cancel{d\theta}^{\,0} + \hat{\phi}R\sin\theta\,d\phi$$

$d\mathbf{s}_R = \hat{R}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\theta}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\phi}R\,dR\,d\theta$
$R^2\sin\theta\,dR\,d\theta\,d\phi$

$$\int_{\phi=0}^{\frac{\pi}{4}} \left(4\frac{C}{m}\right)\overbrace{\left(\underset{1m}{R}\,\underset{90°}{\cancel{\sin\theta}}\,d\phi\right)}^{dl} \qquad \left(4\frac{C}{m}\right)(1)(1)\,\phi\Big|_0^{\pi/4}$$

Line Integral



$d\theta \neq 0 \quad d\bar{l} =$

From Table 3-1:

$$\cancel{\hat{\mathbf{R}}\,dR} + \hat{\boldsymbol{\theta}} R\, d\theta + \hat{\boldsymbol{\phi}} R \sin\theta\, \cancel{d\phi}$$

$$d\mathbf{s}_R = \hat{\mathbf{R}} R^2 \sin\theta\, d\theta\, d\phi$$
$$d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}} R \sin\theta\, dR\, d\phi$$
$$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}} R\, dR\, d\theta$$

$$R^2 \sin\theta\, dR\, d\theta\, d\phi$$

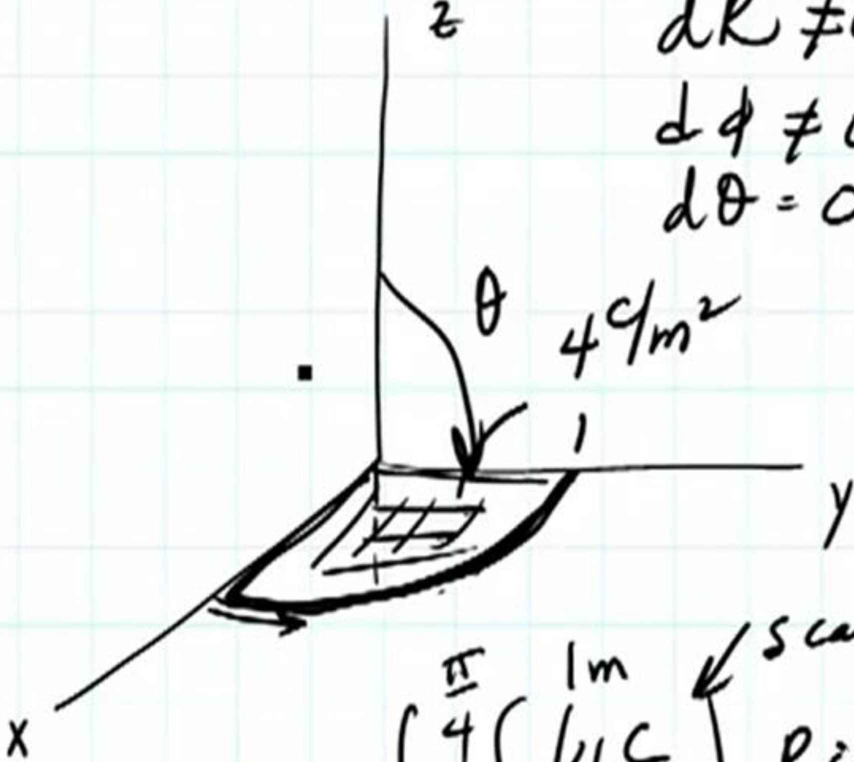
$4\,\frac{C}{m}$ scalar

dl

$$\int_{\phi=0}^{\frac{\pi}{4}} \left(4\,\frac{C}{m}\right)\left(\underset{1\,m}{R}\,\underset{90^\circ}{\cancel{\sin\theta}}\, d\phi\right) \qquad \left(4\,\frac{C}{m}\right)(1)(1)\,\phi\Big|_0^{\pi/4}$$

$$\int_{\theta=0}^{\pi/4} 4\,C/m \qquad R\,d\theta \quad \leftarrow 1\,m$$

Surface Integral



$d R \neq 0$

$d\phi \neq 0$

$d\theta = 0$

$4\,C/m^2$

From Table 3-1:

$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
$ds_R = \hat{\mathbf{R}}R^2\sin\theta\,\cancel{d\theta}\,d\phi$ 0
$ds_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$
$ds_\phi = \hat{\boldsymbol{\phi}}R\,dR\,\cancel{d\theta}$ 0
$R^2\sin\theta\,dR\,d\theta\,d\phi$

scalar

$$\int_{\phi=0}^{\frac{\pi}{4}} \int_{R=0}^{1m} \left(4\frac{C}{m^2}\right) R\sin\theta\, dR\, d\phi$$

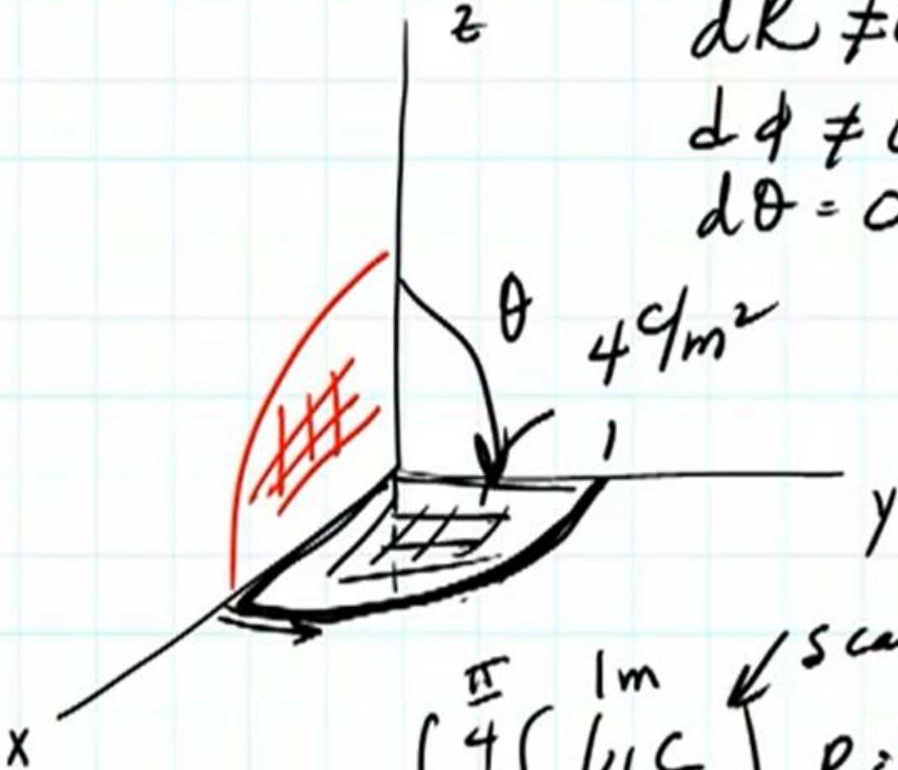
$$\left(4\,C/m^2\right) \frac{R^2}{2}\Big|_0^1 \cos\theta \Big|_0^{\pi/4}$$

Surface Integral

$dR \neq 0$ ✓

$d\phi \neq 0$ $\quad \phi = 0$

$d\theta = 0$ $\quad \theta \neq 0$



From Table 3-1:

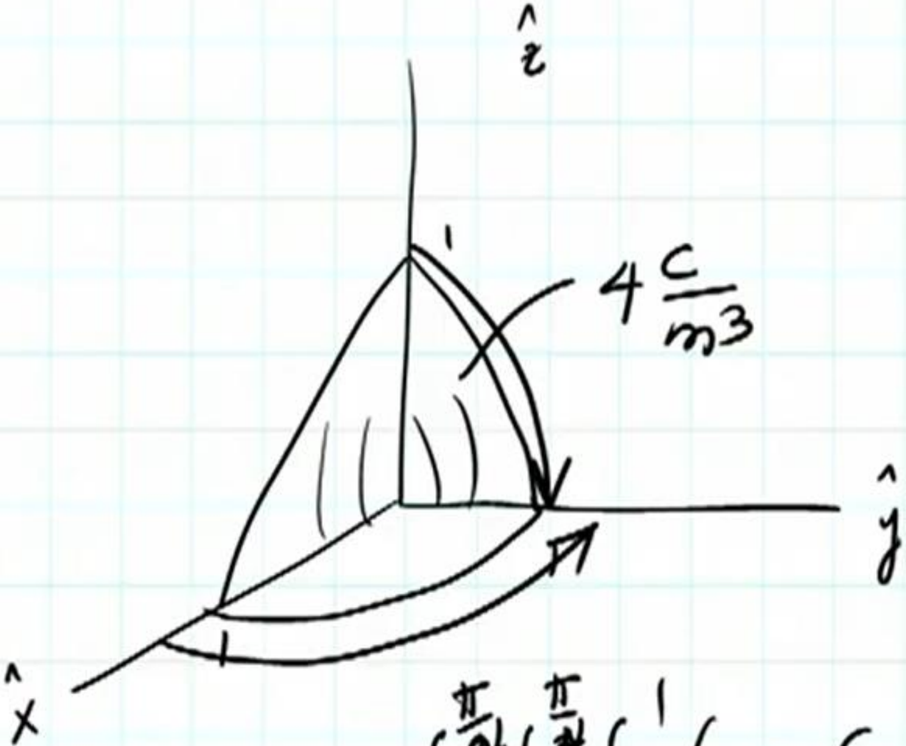
$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
$ds_R = \hat{\mathbf{R}}R^2\sin\theta\,\cancel{d\theta}\,\cancel{d\phi}$ 0
$ds_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,\cancel{d\phi}$ 0
$ds_\phi = \hat{\boldsymbol{\phi}}R\,dR\,\cancel{d\theta}$ 0
$R^2\sin\theta\,dR\,d\theta\,d\phi$

scalar

$$\int_{\phi=0}^{\frac{\pi}{4}}\int_{R=0}^{1\,m}\left(4\frac{C}{m^2}\right) R\sin\theta\, dR\, d\phi$$

$$\left(4\,C/m^2\right)\frac{R^2}{2}\Big|_0^1 \cos\theta\Big|_0^{\pi/4}$$

Volume Integral



From Table 3-1:

$\hat{\mathbf{R}}\,dR+\hat{\boldsymbol{\theta}}R\,d\theta+\hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
$ds_R=\hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $ds_\theta=\hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $ds_\phi=\hat{\boldsymbol{\phi}}R\,dR\,d\theta$
$R^2\sin\theta\,dR\,d\theta\,d\phi$

$$\int_{0}^{\frac{\pi}{2}}\int_{\theta=0}^{\frac{\pi}{2}}\int_{R=0}^{1}\left(4\cdot\frac{C}{m^3}\right)R^2\sin\theta\,dR\,d\theta\,d\phi$$

$$4\quad\frac{R^3}{3}\Big|_0^1\ \cos\theta\Big|_0^{\pi/2}\ \phi\Big|_0^{\pi/2}$$

The operator Del.......

$$\left(\hat{\boldsymbol{\imath}}\,\frac{\partial}{\partial x} + \hat{\boldsymbol{\jmath}}\,\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\,\frac{\partial}{\partial z}\right)$$

Operator ∇ – {Vector Differentiation Operator}

→ *Cartesian Coordinate*

$$\nabla = (\partial/\partial x)U_x + (\partial/\partial y)U_y + (\partial/\partial z)U_z$$

→ *Cylindrical Coordinate*

$$\nabla = (\partial/\partial r)U_r + 1/r\,(\partial/\partial \Phi)\,U_\Phi + (\partial/\partial z)\,U_z$$

→ *Spherical Coordinate*

$$\nabla = (\partial/\partial r)U_r + 1/r(\partial/\partial \vartheta)U_\vartheta + 1/r\sin\vartheta(\partial/\partial \Phi)U_\Phi$$

Using ∇

Gradient of a scalar V is ∇V

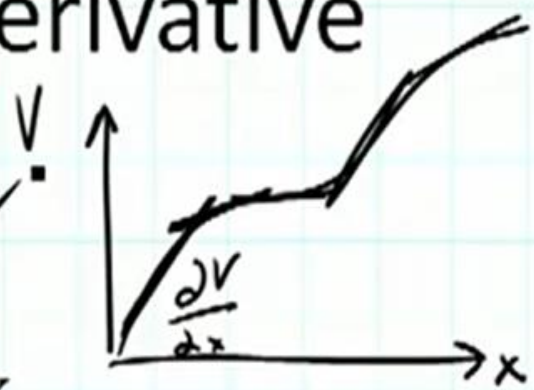
Divergence of Vector A $\nabla \bullet A$

Curl of a Vector A $\nabla X A$

Laplacian of a scalar V $\nabla^2 V$

Major Concepts: Gradient
= Directional Derivative

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

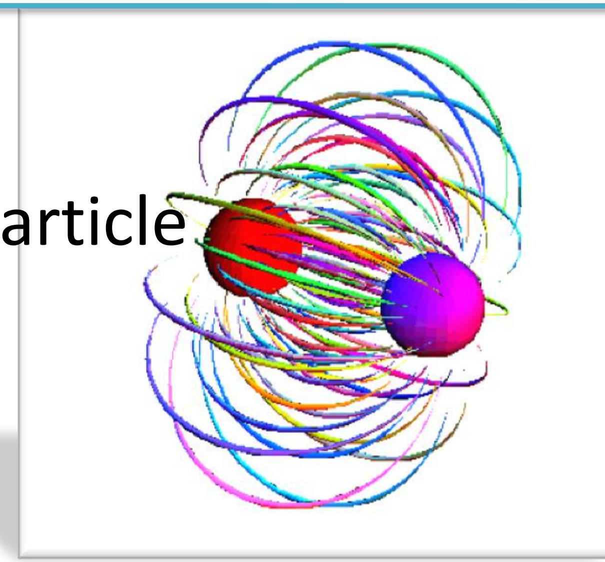


$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

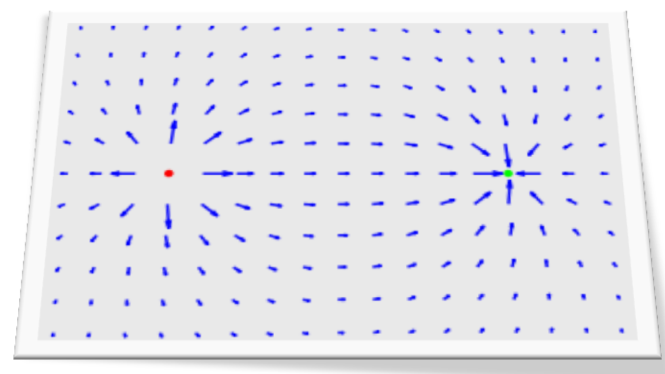
$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

What's an Electric Field

A field of force surrounding a charged particle



The change in the region around a charge that makes other charges attract or repel it

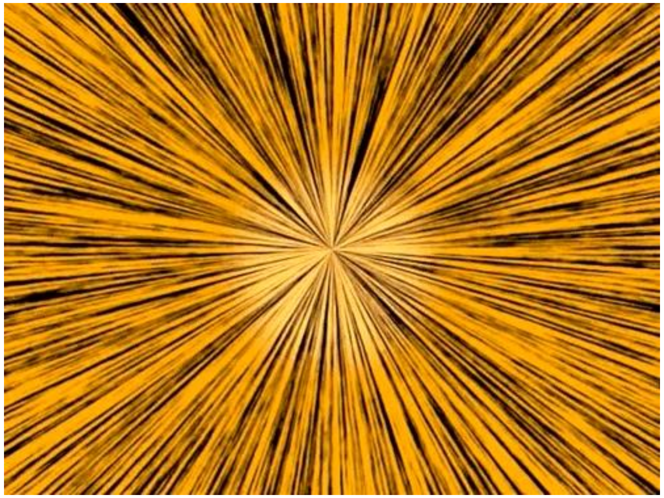


Translational Effects of Fields - Divergence

In physical terms, the divergence of a three dimensional vector field is the extent to which the vector field flow behaves like a source or a sink at a given point.

Flux of E = the charge enclosed (Gauss Law)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$



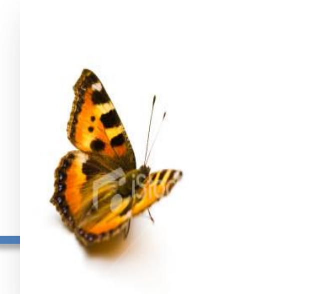
$$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} - \overrightarrow{M}$$

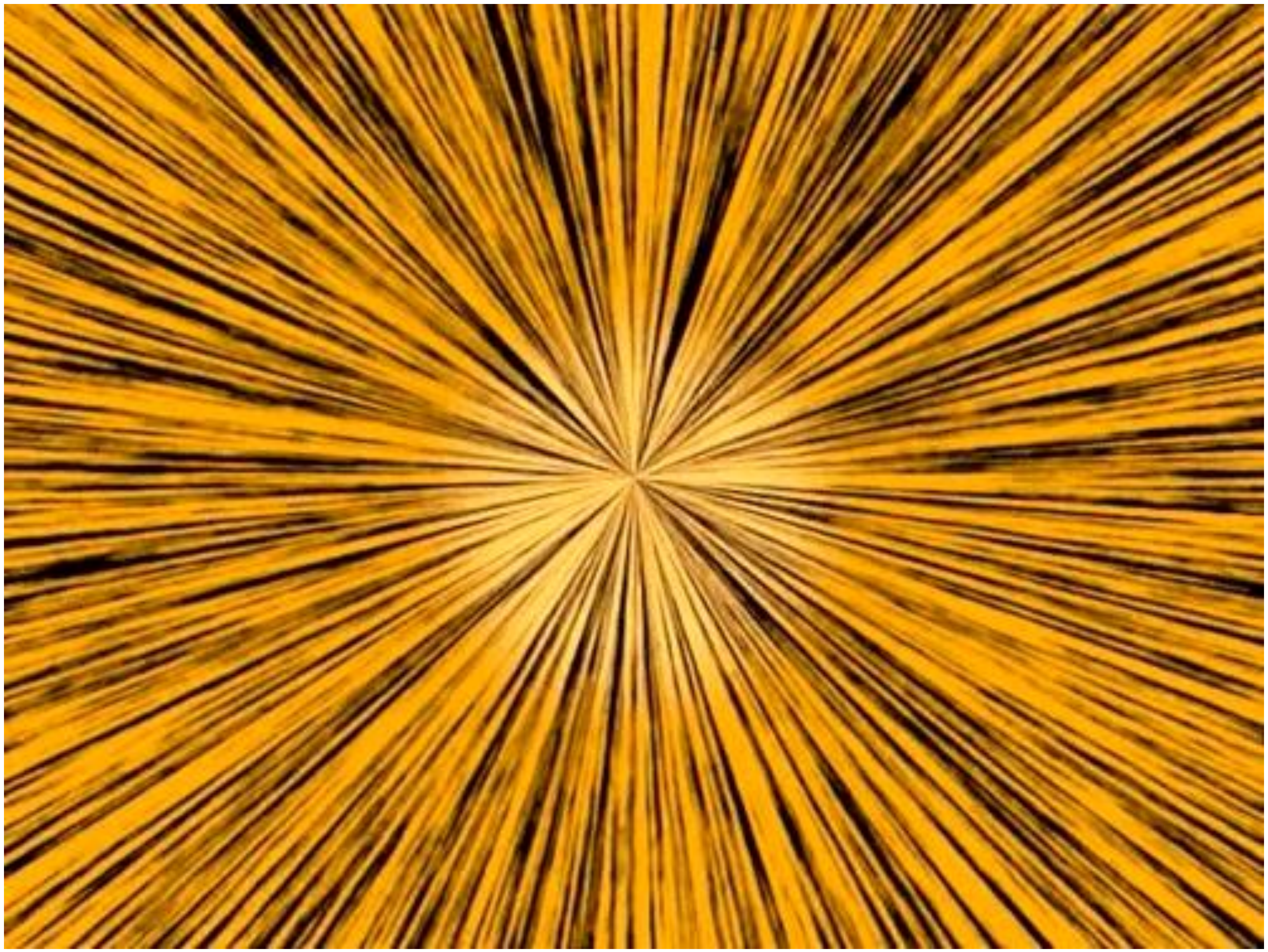
$$\nabla \times \overrightarrow{H} = -\frac{\partial \overrightarrow{D}}{\partial t} + \overrightarrow{J}$$

$$\nabla \cdot \overrightarrow{D} = \rho$$

$$\nabla \cdot \overrightarrow{B} = 0$$

Divergent or convergent Fields





$$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} - \overrightarrow{M}$$

$$\nabla \times \overrightarrow{H} = -\frac{\partial \overrightarrow{D}}{\partial t} + \overrightarrow{J}$$

$$\nabla \cdot \overrightarrow{D} = \rho$$

$$\nabla \cdot \overrightarrow{B} = 0$$

5.3 The divergence of a vector field

The divergence computes a scalar quantity from a vector field by differentiation.

If $\mathbf{a}(x, y, z)$ is a vector function of position in 3 dimensions, that is $\mathbf{a} = a_1\hat{\boldsymbol{\imath}} + a_2\hat{\boldsymbol{\jmath}} + a_3\hat{\boldsymbol{k}}$, then its divergence at any point is defined in Cartesian co-ordinates by

$$\mathrm{div}\mathbf{a} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}$$

We can write this in a simplified notation using a scalar product with the $\boldsymbol{\nabla}$ vector differential operator:

$$\mathrm{div}\mathbf{a} = \left(\hat{\boldsymbol{\imath}}\,\frac{\partial}{\partial x} + \hat{\boldsymbol{\jmath}}\,\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\,\frac{\partial}{\partial z}\right) \cdot \mathbf{a} = \boldsymbol{\nabla} \cdot \mathbf{a}$$

Notice that the divergence of a vector field is a scalar field.

Gauss ,Divergence Theorem:

$$\iiint_V (\nabla \cdot \mathbf{F})\, dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$$

Intuition

Recall that if a vector field **F** represents the flow of a fluid, then the divergence of **F** represents the expansion or compression of the fluid. The divergence theorem says that the total expansion of the fluid inside some solid V equals the total flux of the fluid out of the boundary of S. In math terms, this means the triple integral of div **F** over the solid V is equal to the flux integral of **F** over the surface S that is the boundary of S (with outward pointing normal):

Divergence is a vector operator operating on vector field to give a vector field

Rotational Effects of Fields -- Curl

Curliness in the field pushes things around in circles

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

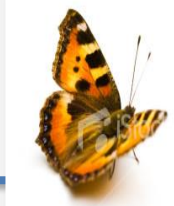
$$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} - \overrightarrow{M}$$

$$\nabla \times \overrightarrow{H} = -\frac{\partial \overrightarrow{D}}{\partial t} + \overrightarrow{J}$$

$$\nabla \cdot \overrightarrow{D} = \rho$$

$$\nabla \cdot \overrightarrow{B} = 0$$

Curling Fields



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

5.6 The curl of a vector field

- cross it with a vector field $\boldsymbol{\nabla} \times \mathbf{a}$

This gives the **curl of a vector field**

$$\boldsymbol{\nabla} \times \mathbf{a} \equiv \mathrm{curl}(\mathbf{a})$$

We can follow the pseudo-determinant recipe for vector products, so that

$$\begin{aligned}
\boldsymbol{\nabla} \times \mathbf{a} &= \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix} \quad \text{(remember it this way)} \\
&= \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z} \right) \hat{\boldsymbol{\imath}} + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial y} \right) \hat{\boldsymbol{\jmath}} + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \right) \hat{\boldsymbol{k}}
\end{aligned}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} - \vec{M}$$

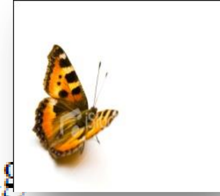
$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

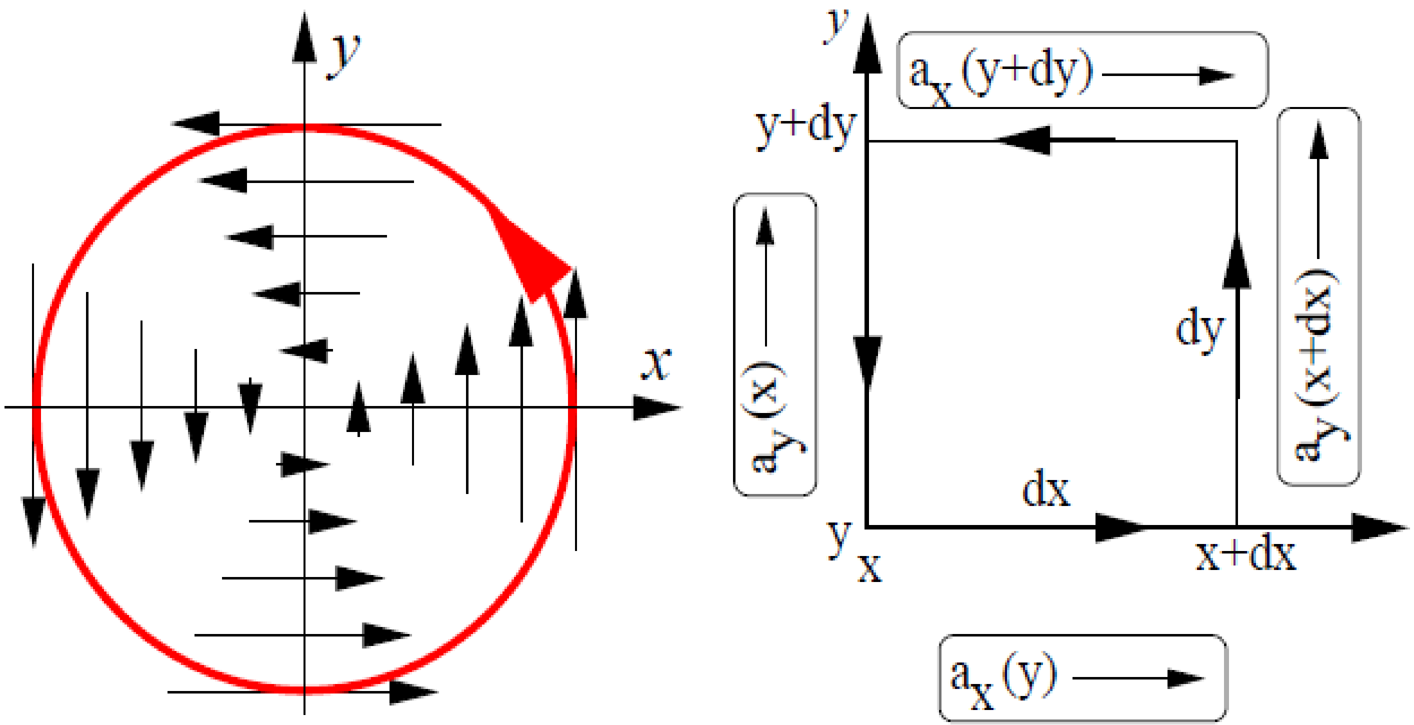
$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

The signficance of curl

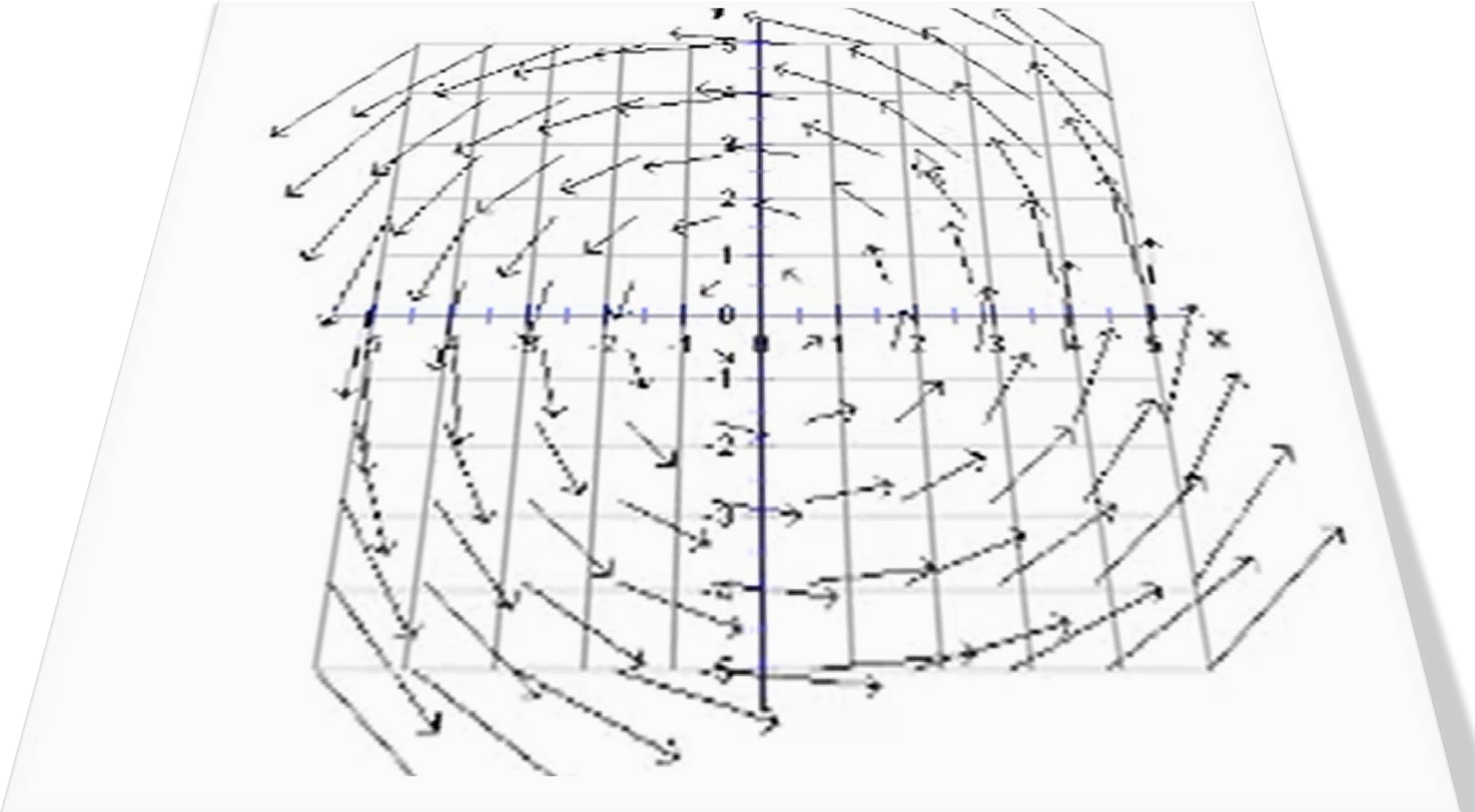
Perhaps the first example gives a clue. The field $\mathbf{a} = -y\hat{\imath} + x\hat{\jmath}$ is sketched in Fig 5.3(a). (It is the field you would calculate as the velocity field of an object rotating with $\boldsymbol{\omega} = [0, 0, 1]$.) This field has a curl of $2\hat{\boldsymbol{k}}$, which is in the r-h screw sense out of the page. You can also see that a field like this must give a finite value to the line integral around the complete loop $\oint_C \mathbf{a} \cdot d\mathbf{r}$.



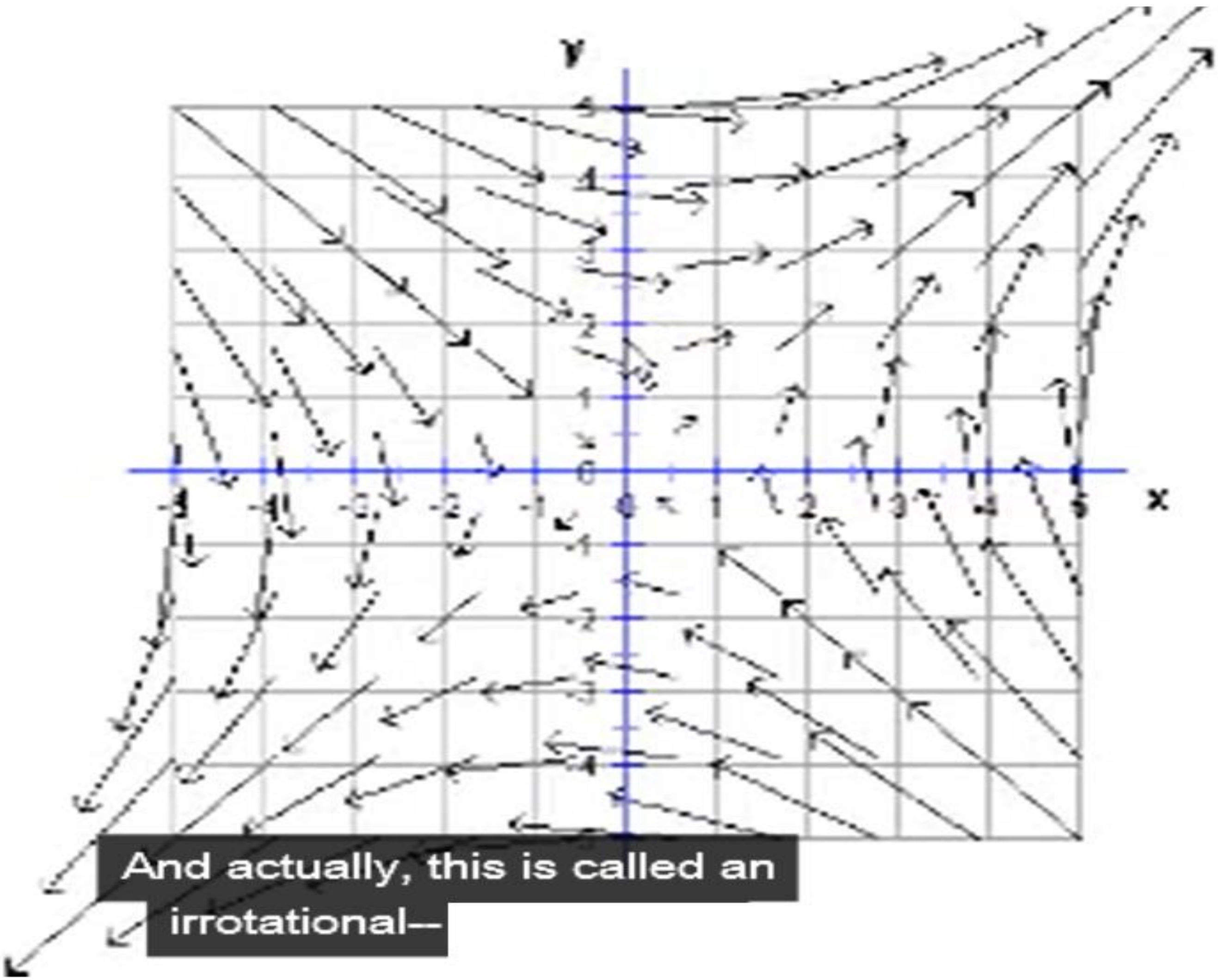


A rough sketch of the vector field $-y\hat{\imath} + x\hat{\jmath}$.

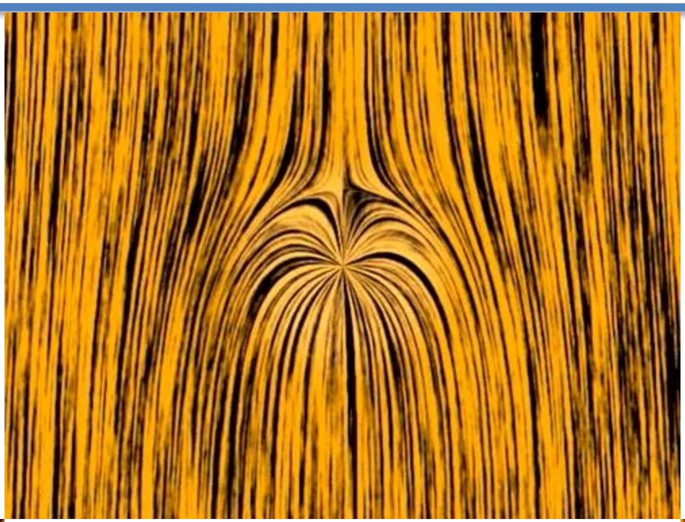
a = -yi + xj

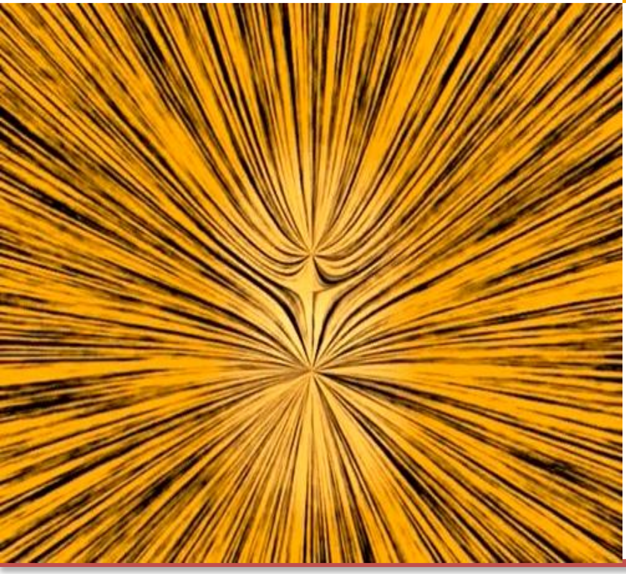


$a = y\mathbf{i} + x\mathbf{j}$

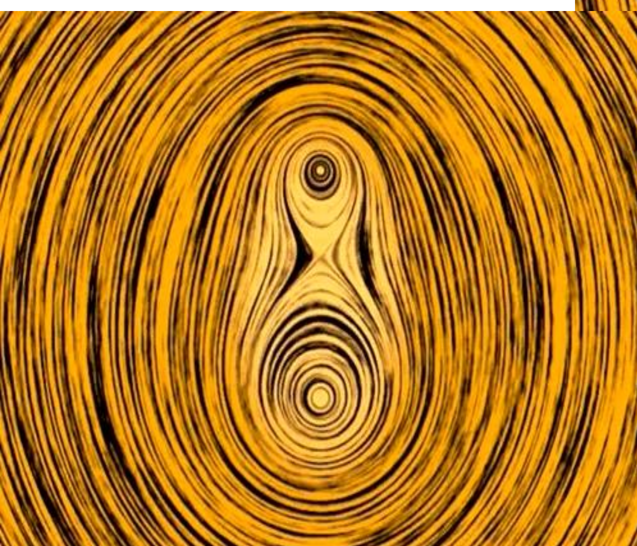


Different combination of fields





Different combination of fields



Different combination of fields

Major Concepts: Laplacian
Rate of Change

$$\nabla^2 V = \nabla \bullet \nabla V$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

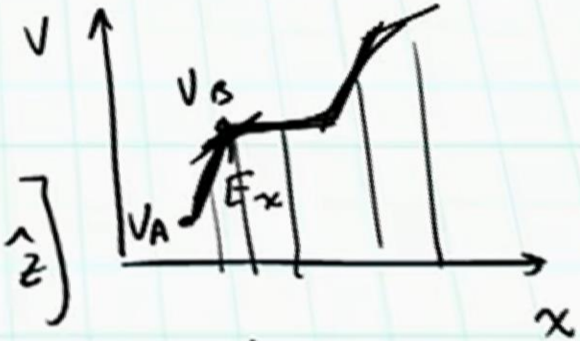
$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

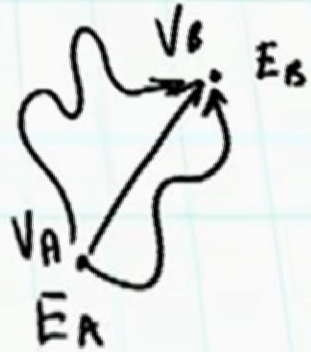
Major Concepts:
E and V

$$\overline{E} = -\nabla V$$

$$-\left[\underbrace{\left(\frac{\partial V}{\partial x}\right)}_{E_x}\hat{x} + \overbrace{\left(\frac{\partial V}{\partial y}\right)}^{E_y}\hat{y} + \overbrace{\left(\frac{\partial V}{\partial z}\right)}^{E_z}\hat{z}\right]$$



$$V_{BA} = V_B - V_A = -\int_A^B \overline{E} \bullet d\overline{\ell}$$



Major Concepts:

- Poisson's Equation

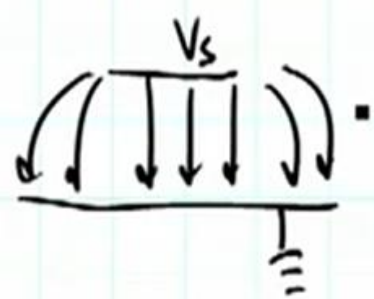
C/m^3

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

- Laplace's Equation

Charge Free Region $\rho_v = 0$
Dielectric $\sigma = 0$

$$\nabla^2 V = 0$$



Maxwell's the butterfly…..

Fields that escape into free space from material space are called Electromagnetic fields or waves

James Clerke Maxwell (1831-1879) was an interesting character - Scottish physicist

Maxwell formalised and extended the work of Ampère and Faraday

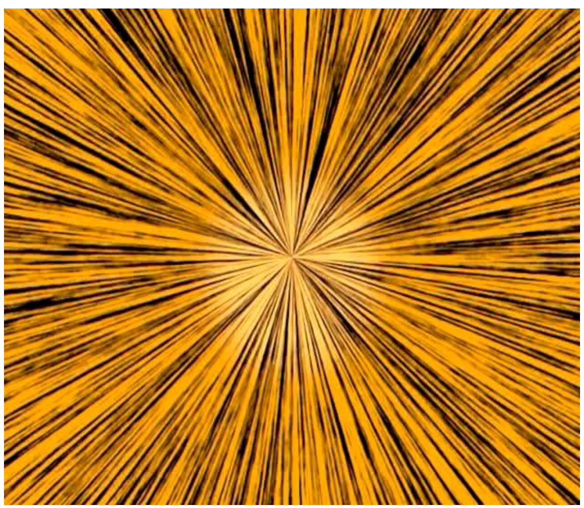
Besides his famous work on Electromagnetic theory, he was a leading contributor to the kinetic theory in gases and to the theory of colour vision.

The nature of charge…

In physical terms, the divergence of a three dimensional vector field is the extent to which the vector field flow behaves like a source or a sink at a given point.

Flux of E = the charge enclosed (Gauss Law)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

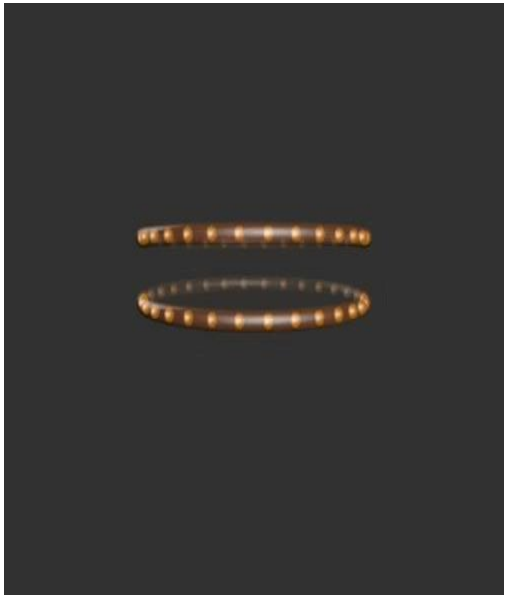
$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Magnets don't have a monopole….

In physical terms, if the divergence of a vector field is zero then it tells us lines of Magnetic field never diverge from anything, and so must form closed loops

Non existence of the Magnetic monopole



$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

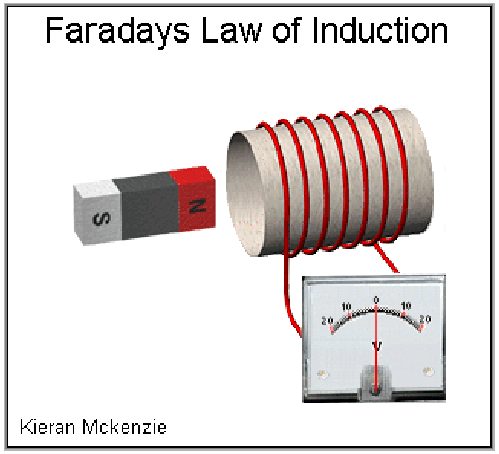
$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Maxwell's the butterfly.....

Faradays Law of electromagnetic Induction

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Maxwell's the butterfly …..

The Last Equation of Electromagnetism was found to be a problem then……….

States that magnetic fields can be generated by electrical current

States that magnetic fields can be generated by electrical current and by changing electric fields

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Maxwell's the butterfly .....

The Last Equation of Electromagnetism was found to be a problem……….

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

DEL (Ampere Equation)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$\cancel{\Delta . J = 0}$

DEL (Ampere Equation) = 0

Fails to explain the conservation of Charge

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

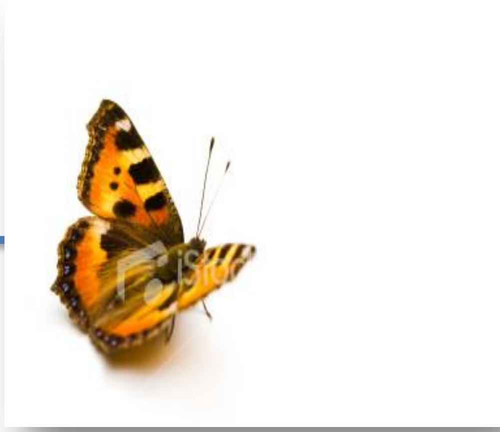
$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Charge cannot be produced……



It has to come from some were else….

Current Continuity Equation states this mathematically …

$$\Delta . J = - \partial \rho / \partial t$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

DEL (Maxwell's Equation)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

explain the conservation of Charge

$\Delta . \partial E / \partial t = - \Delta . J / \varepsilon_o$ - From Gauss Law .... (1)

$\Delta . J = - \partial \rho / \partial t$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Thank you...

-ZnTill [ ]

I said nothing it was all said earlier …..

"Cheerfulness Keeps the sprit of one who possesses it and brings a smile to the lips of others "

Natural ability without education has more often attained to glory and virtue than education without natural ability

---- Cicero