Assignment 4: Queuing Theory

Due date: *Sunday, April 13, 2025 at 22:00*

This is an individual assignment. You may discuss it with others, but your code and documentation must be written on your own. In particular, do not use automated code-generation tools such as ChatGPT or GitHub copilot.

In this assignment, you are asked to solve some problems related to queuing theory.

Maximum Rate Possible

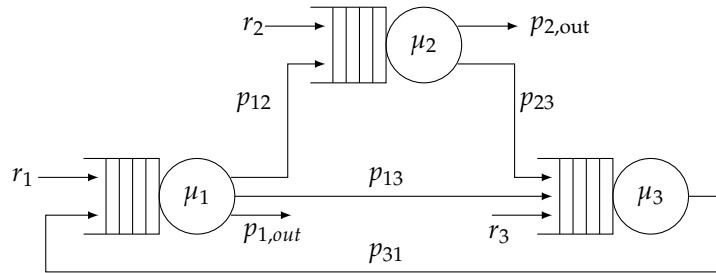Consider the network of queues shown in Figure 1.

Figure 1: A network of queues with probabilistic routing.

Each server serves at an average rate of 10 jobs/sec. Suppose that $r_2 = r_3 = 1$ and that $p_{12} = p_{2,\text{out}} = 0.8$, $p_{23} = p_{13} = 0.2$, $p_{1,\text{out}} = 0$, and $p_{31} = 1$. What is the bound on r_1 to keep this system stable?
Hint: A system is stable when the average arrival rate is less than the average service rate, $\lambda < \mu$. In the given network, we must ensure that $\lambda_i < \mu_i, \forall i \in \{1, 2, 3\}$ where $\lambda_i = r_i + \sum_j p_{ji}\lambda_j$.

Little's Theorem

Patients arriving at the Grand River Hospital Emergency Room have a mean waiting time of 3 hours. It has been found that, averaged over the period of a day, patients arrive at the rate of one every 5 minutes.

   a. How many patients are awaiting treatment on average at any given point in time?

   b. What should the size of the waiting room be so that it can always accommodate arrivals?

M/M/1 Queue Length

For an M/M/1 queue with utilization ρ, prove that

$$E[N_Q] = \frac{\rho^2}{1 - \rho}$$

M/M/1 Queue

Consider a link to which packets arrive as a Poisson process at a rate of 450 packets/sec such that the time taken to service a packet is exponentially distributed. Suppose that the mean packet length is 250 bytes and that the link capacity is 1 Mbps.

a. What is the probability that the link's queue has one, two, and ten packets respectively,

b. What is the mean number of packets in the system? What is the mean number in the queue?

c. What is the mean waiting time?

Submission Instructions

Submit a small report containing your solutions to the proposed problems through the iCorsi system. Note that the report must be in pdf format. The report must include a section to properly acknowledge any and all external sources of information you may have used, including suggestions, and comments from other students.