Discretization Methods

Spring Semester 2025

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Exercise Sheet 3

Date of submission: 28.04.2025

Exercise 1

Consider the variable coefficient problem

$$\frac{\partial u}{\partial t} + \sin(x)\frac{\partial u}{\partial x} = 0,$$

subject to periodic boundary conditions. Derive a Fourier-Galerkin approximation. Is $P_N u = u_N$?

Exercise 2

Consider the variable coefficient problem

$$\frac{\partial u}{\partial t} + \sin(x)\frac{\partial u}{\partial x} = 0,$$

subject to the boundary conditions

$$u(0,t) = u(\pi,t) = 0.$$

Derive a Fourier-Galerkin approximation.

Exercise 3

Consider the variable coefficient problem as in exercise 2, with the same boundary conditions
Assume that the solution is approximated by

$$u_N(x,t) = \sum_{n=0}^{N+N_b} \hat{u}_n(t)\cos(nx).$$

Derive a tau approximation.

Exercise 4

Consider Burgers equation

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = \varepsilon\frac{\partial^2 u}{\partial x^2},$$

subject to periodic boundary conditions. Derive a Fourier-Collocation approximation.