

Exam Part 1

Part 1

Consider the advection diffusion equation given as

$$\frac{\partial u(x, t)}{\partial t} + U_0(x) \frac{\partial u(x, t)}{\partial x} = \nu \frac{\partial^2 u(x, t)}{\partial x^2}, \quad (1)$$

where $U_0(x)$ is periodic and bounded and ν is assumed to be constant. Also $u(x, t)$ is assumed to be smooth and periodic as is the initial condition.

(a)

State sufficient conditions on $U_0(x)$ and ν that ensures Eq. 1 to be well-posed.

(b)

Assume that Eq. 1 is approximated using Fourier Collocation method. Is the approximation consistent and what is the expected convergence rate when increasing N , the number of modes used in the approximation.

(c)

Assume now that $U_0(x)$ is constant and Eq. 1 is approximated using a Fourier Collocation method with odd number of modes. Prove that the semi-discrete approximation, i.e. continuous time and approximated space, is stable.