Project Proposal: Leader–Follower Particle System with Capacity Constraints

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1. Objectives

1. Implement from scratch the classic Vicsek alignment model as a baseline.
2. Extend it by introducing:
   - Randomly entering leader particles in a 2D domain.
   - A capacity limit $n = N_{\text{student}}/N_{\text{leader}}$ on how many followers each leader can influence.
   - Nearest-leader assignment when multiple leaders compete for the same student.
   - Periodic inter-leader communication to adjust headings for a more uniform spatial spread.
3. Define and measure convergence through:
   - Polarization of followers Φ.
   - Stable leader–follower assignments.
   - Balanced follower loads.
   - Low variance in inter-leader distances.
4. Compare the extended model against the pure Vicsek model in terms of convergence speed, final formations, and robustness.

2. Methodology

2.1 Baseline Vicsek Model

- Particles $i = 1, \ldots, N$ each have position $\mathbf{x}_i(t) \in \mathbb{R}^2$ and heading $\theta_i(t)$.
- Heading update:

$$\theta_i(t+1) = \operatorname{Arg}\Big\{\sum_{j \in \mathcal{N}_i} e^{i\theta_j(t)}\Big\} + \Delta_i(t), \quad \Delta_i \sim \operatorname{Unif}\big[-\tfrac{\eta}{2}, \tfrac{\eta}{2}\big].$$

- Position update:

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + v_0\big(\cos\theta_i(t+1), \sin\theta_i(t+1)\big).$$

2.2 Leader–Follower Extension

- **Leader spawning:** L leaders appear at random times/locations.
- **Capacity constraint:** Each leader may attract at most n students.
- **Assignment rule:** Students choose the closest leader with available capacity.
- **Follower update:** At each step, a follower aligns with the average heading of its assigned leader and Vicsek neighbors.

2.3 Inter-Leader Communication

- Every T time steps, leaders exchange positions.
- Each leader adjusts its heading to reduce local leader density (e.g. repulsion from nearest leaders).
- Between communications, leaders move according to their updated headings.

3. Convergence & Evaluation Metrics

- **Polarization:**
$$\Phi(t) = \frac{1}{N}\Big|\sum_{i=1}^{N} \mathbf{v}_i(t)\Big|, \quad \mathbf{v}_i = (\cos\theta_i, \sin\theta_i).$$
  Convergence when $\Phi \geq 0.95$ over several steps.
- **Assignment stability:** Fraction of students whose leader assignment remains unchanged over 10 steps.
- **Load balance:** Standard deviation of follower counts across leaders below a small threshold.
- **Leader spacing:** Variance of pairwise leader distances falls below a threshold.

4. Expected Outcomes

- A clean, original Python implementation of both the baseline and extended models.
- Visualizations (animations and plots) showing:
  - Pure Vicsek alignment.
  - Leader-driven clustering.

  - Effects of capacity constraints.
  - Inter-leader dispersal via communication.
- A comprehensive report detailing methods, results, and potential future extensions (e.g. dynamic capacities, obstacle avoidance).

Please let me know if this proposal aligns with your expectations or if you would suggest any modifications.

Best regards,
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