Particle Swarm Simulation

Leader–Follower Model

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Introduction

The core idea of the Vicsek model is that each particle updates its direction based on the average direction of the neighborhood plus random noise, and then moves forward at a constant speed. The improvement idea of the Leader-Follower model is to introduce a few "leaders" to guide some "followers"

Table 1: Key Simulation Parameters

Parameter	Symbol
Domain Size	L_{domain}
Number of Followers	N_F
Number of Leaders	N_L
Leader Capacity	`leader_capacity`
Particle Speed	v_0
Interaction Radius	$R_{\text{interaction}}$
Noise Amplitude	η
Time Step	Δt
Communication Interval	$T_{\text{communication}}$
Leader Repulsion Strength	`leader_repulsion_strength`
Crowd Radius Factor	`R_crowd_factor`
Local Repel Radius Factor	`leader_local_repel_radius_factor`

Mathematical Formulation

2.1 Particle Types

- **Leaders (N_L particles):** These particles have the ability to directly influence a limited number of followers. They also interact among themselves and periodically synchronize their overall direction.

- **Followers (N_F particles):** These particles are influenced by leaders (if assigned) or by their local neighborhood following Vicsek rules.

Mathematical Formulation

2.2 Follower Behavior

At each simulation step t, follower behavior is determined as follows:

1. **Leader Assignment:** Each follower f identifies the set of leaders L_{near} within a certain visual range (or all leaders). From L_{near}, it selects the leader L^* that is closest and currently has available capacity (i.e., number of currently assigned followers < `leader_capacity`). If multiple such leaders exist, the closest one is chosen. If no leader can accept the follower, it remains unassigned for this step.

2. **Direction Update (for step $t + dt$):**

   - **Directly Assigned Followers:** If follower f was assigned to leader L^* at step t, its heading $\theta_f(t+dt)$ becomes the heading of L^* at step t, $\theta_{L^*}(t)$, plus a small random noise $\Delta_f(t) \sim \text{Unif}[-\eta/2, \eta/2]$.
   $$\theta_f(t + dt) = \theta_{L^*}(t) + \Delta_f(t) \tag{1}$$

   - **Unassigned/Free Followers:** If follower f was not assigned to any leader at step t, its heading is updated according to the standard Vicsek model rule, by aligning with the average heading of all particles (leaders and other followers) within its interaction radius $R_{\text{interaction}}$, plus noise:
   $$\theta_f(t + dt) = \text{Arg}\left\{ \sum_{j \in \mathcal{N}_f(t)} e^{i\theta_j(t)} \right\} + \Delta_f(t) \tag{2}$$
   where $\mathcal{N}_f(t)$ is the set of all particles (leaders or followers) within distance $R_{\text{interaction}}$ of follower f at time t.

Mathematical Formulation

2.3 Leader Behavior

Leaders aim to coordinate their movement while also influencing followers.

1. **Non-Communication Steps:** In steps where no global communication occurs, leaders update their headings based on Vicsek-like interactions with *all other particles (leaders and followers)* within their interaction radius $R_{\text{interaction}}$:

$$\theta_L(t+dt) = \text{Arg}\left\{ \sum_{j \in \mathcal{N}_L^{all}(t)} e^{i\theta_j(t)} \right\} + \Delta_L(t) \tag{3}$$

   where $\mathcal{N}_L^{all}(t)$ is the set of all particles (any type) within distance $R_{\text{interaction}}$ of leader L at time t (excluding L itself).

2. **Communication Steps (Periodic Synchronization):** Every $T_{\text{communication}}$ steps, all leaders instantaneously synchronize their direction. The new heading for all leaders becomes the current average heading of all leaders, plus noise:

$$\bar{\theta}_{leaders}(t) = \text{Arg}\left\{ \sum_{k=1}^{N_L} e^{i\theta_{L_k}(t)} \right\} \tag{4}$$

$$\theta_{L_j}(t+dt) = \bar{\theta}_{leaders}(t) + \Delta_{L_j}(t) \quad \forall j = 1, \ldots, N_L \tag{5}$$

Mathematical Formulation

2.4 Position Update

All particles (leaders and followers) update their positions $\mathbf{x}_i \in \mathbb{R}^2$ using their new heading $\theta_i(t+dt)$ and a constant speed v_0:

$$\mathbf{x}_i(t+dt) = \mathbf{x}_i(t) + v_0 \begin{pmatrix} \cos\theta_i(t+dt) \\ \sin\theta_i(t+dt) \end{pmatrix} \Delta t \tag{6}$$

Periodic boundary conditions are applied to keep particles within the domain $[0, L_{\text{domain}}] \times [0, L_{\text{domain}}]$.

Experimental Configuration

Parameter	Pure Vicsek Run	Leader-Follower Run
Total Particles (N_{total})	1000	1000
- Followers (N_F)	1000 (all are type follower)	990
- Leaders (N_L)	0	10
Leader Capacity (per leader)	N/A	10
Total Leader Capacity	N/A	100 (covers 10.1% of N_F)
Leader Communication Interval ($T_{\text{communication}}$)	N/A	10 steps
Max Simulation Steps	300	300
Convergence Criteria Parameters		
Polarization Threshold (for stopping)	0.95	0.95
Convergence Window (checks)	40	40
Check Interval (steps)	5	5
Physical Parameters (Defaults Used)		
Domain Size (L_{domain})	20.0	20.0
Particle Speed (v_0)	0.5	0.5
Interaction Radius ($R_{\text{interaction}}$)	2.0	2.0
Noise Amplitude (η)	0.2	0.2
Time Step (Δt)	0.5	0.5
Leader Initial Distribution	N/A	Random

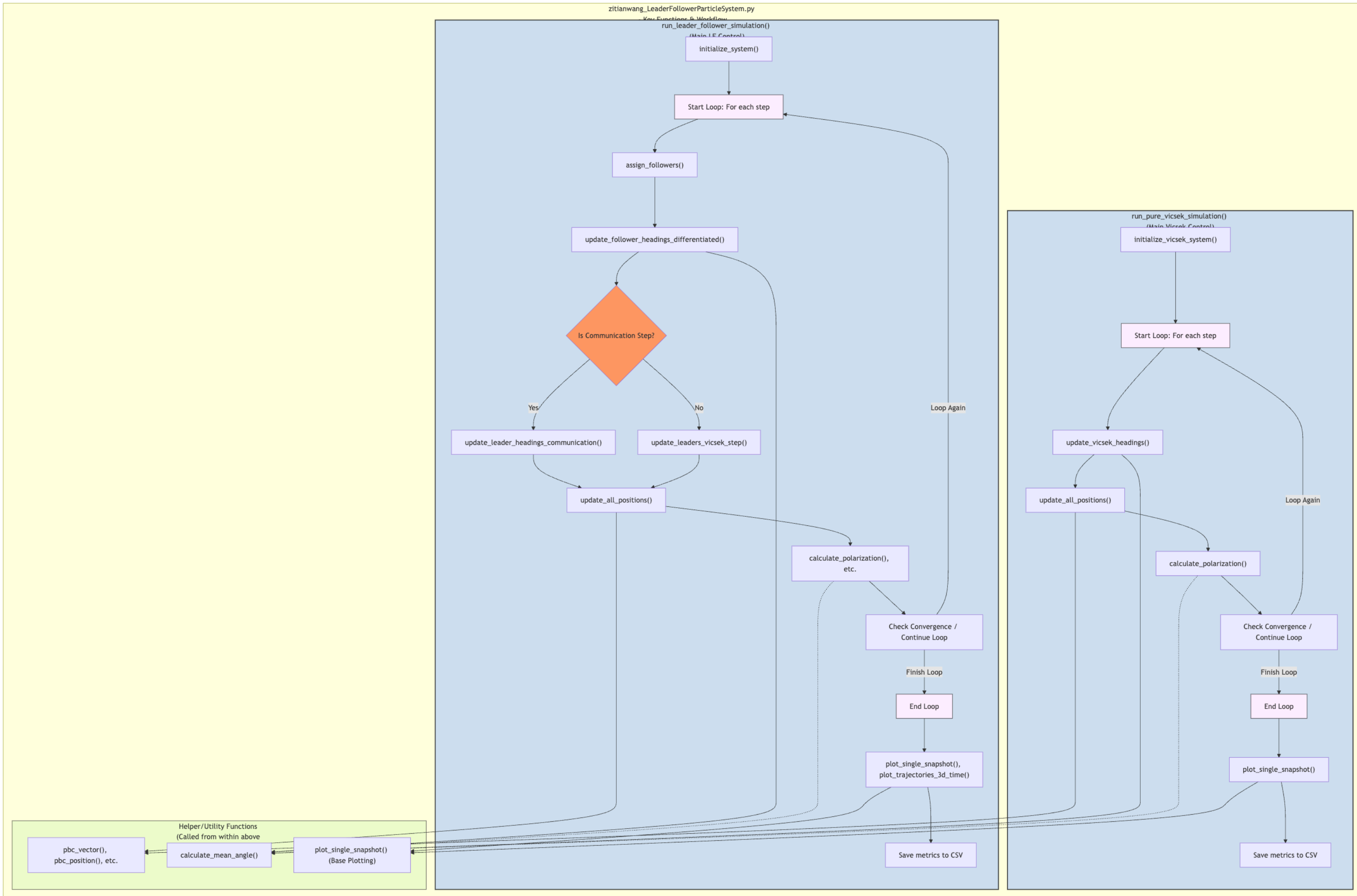
Note: N/A indicates the parameter is not applicable to that model type. The Leader-Follower run uses the default leader distribution mode (random). Total leader capacity coverage is calculated as ($N_L \times$ Leader Capacity)/N_F.

Model Implementation Details

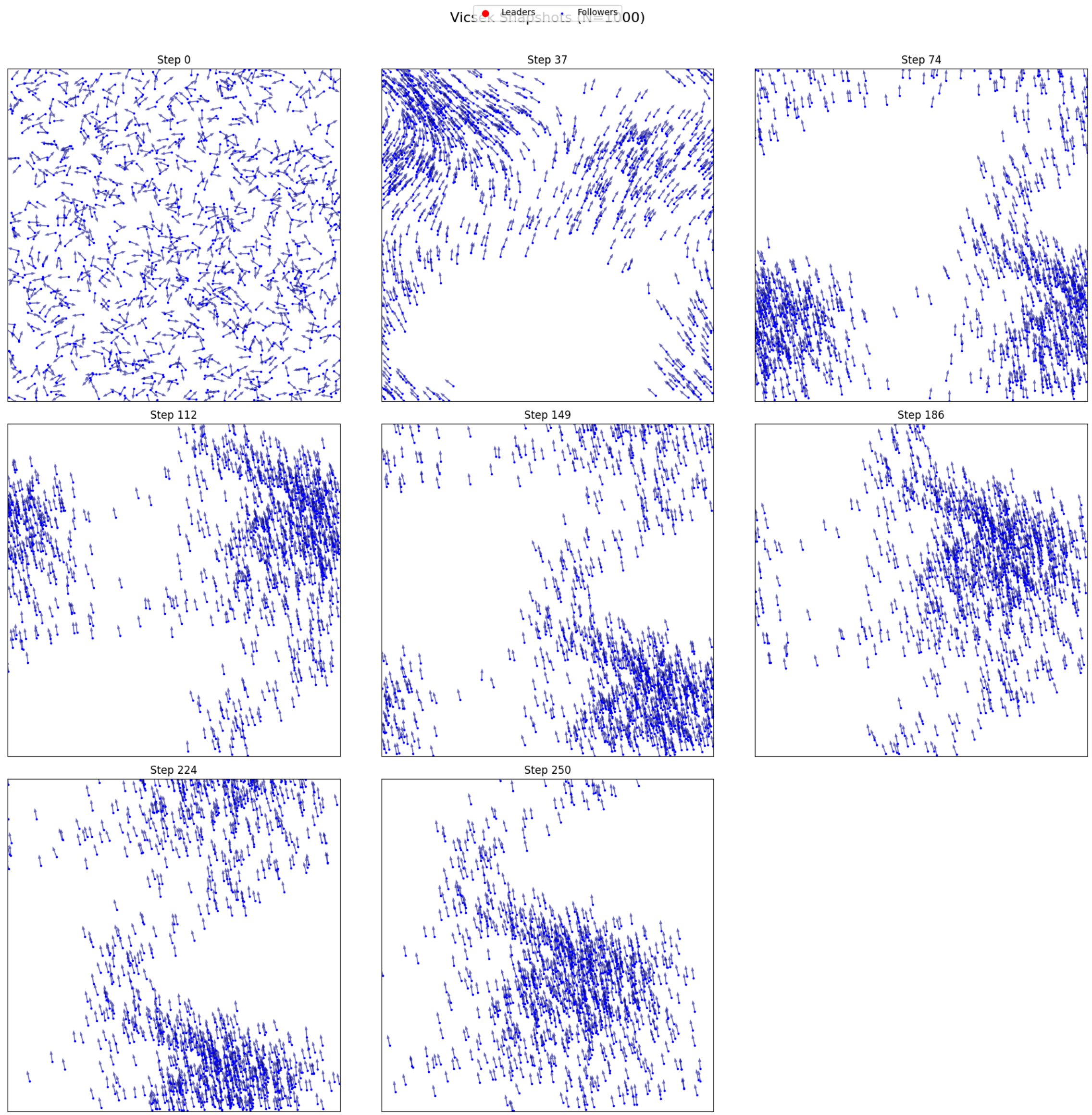
Algorithm 1 Single Simulation Step Update Sequence

Input: Current system state at step t (positions $\mathbf{x}(t)$, headings $\theta(t)$, types, etc.)
Output: Updated system state for step $t + dt$
procedure UPDATESTEP(t)
    Calculate distances and identify neighbors for all particles based on $R_{\text{interaction}}$ and $\mathbf{x}(t)$.
    if model is Leader-Follower (LF) **then**
        Assign Followers to Leaders based on proximity and `leader_capacity`.
        Update Follower headings $\theta_F(t + dt)$ (differentiated for assigned/unassigned).
        if current step t is a Leader Communication step ($t \pmod{T_{\text{communication}}} == 0$) **then**
            Leaders perform Global Synchronization of headings $\theta_L(t + dt)$.
        else
            Leaders perform Local Vicsek Update of headings $\theta_L(t + dt)$ among all particles.
    else (model is Pure Vicsek)
        Calculate average neighbor heading for all particles.
        Update all particle headings $\theta(t + dt)$ (add noise η).
    Update all particle positions $\mathbf{x}(t+dt) = \mathbf{x}(t) + v_0 \cdot (\cos\theta(t+dt), \sin\theta(t+dt)) \cdot \Delta t$.
    Apply Periodic Boundary Conditions (PBC) to $\mathbf{x}(t + dt)$.
    Calculate and record metrics for step $t + dt$ (e.g., Polarization Φ).
    Check convergence criteria.

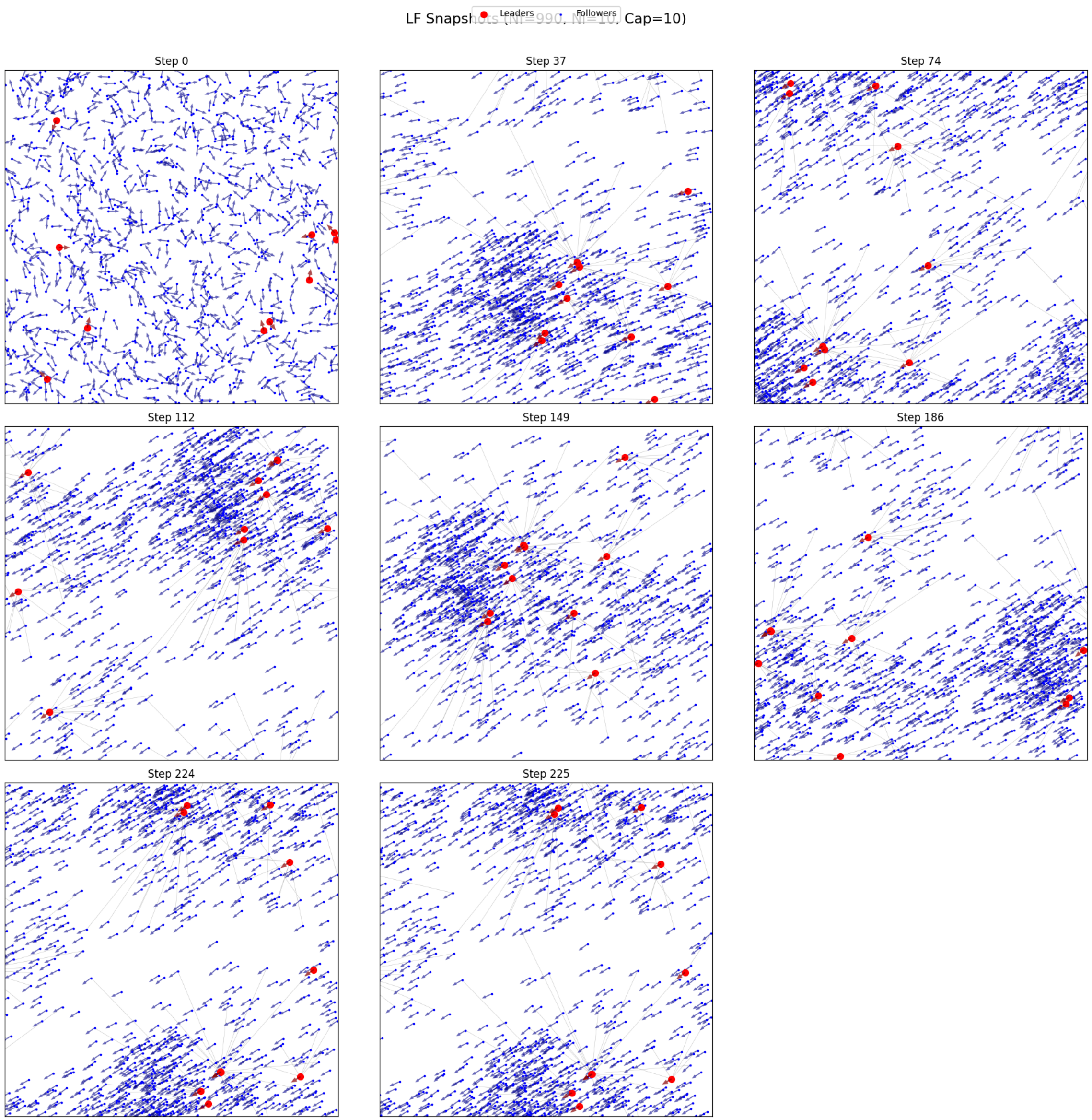
Model Implementation Details



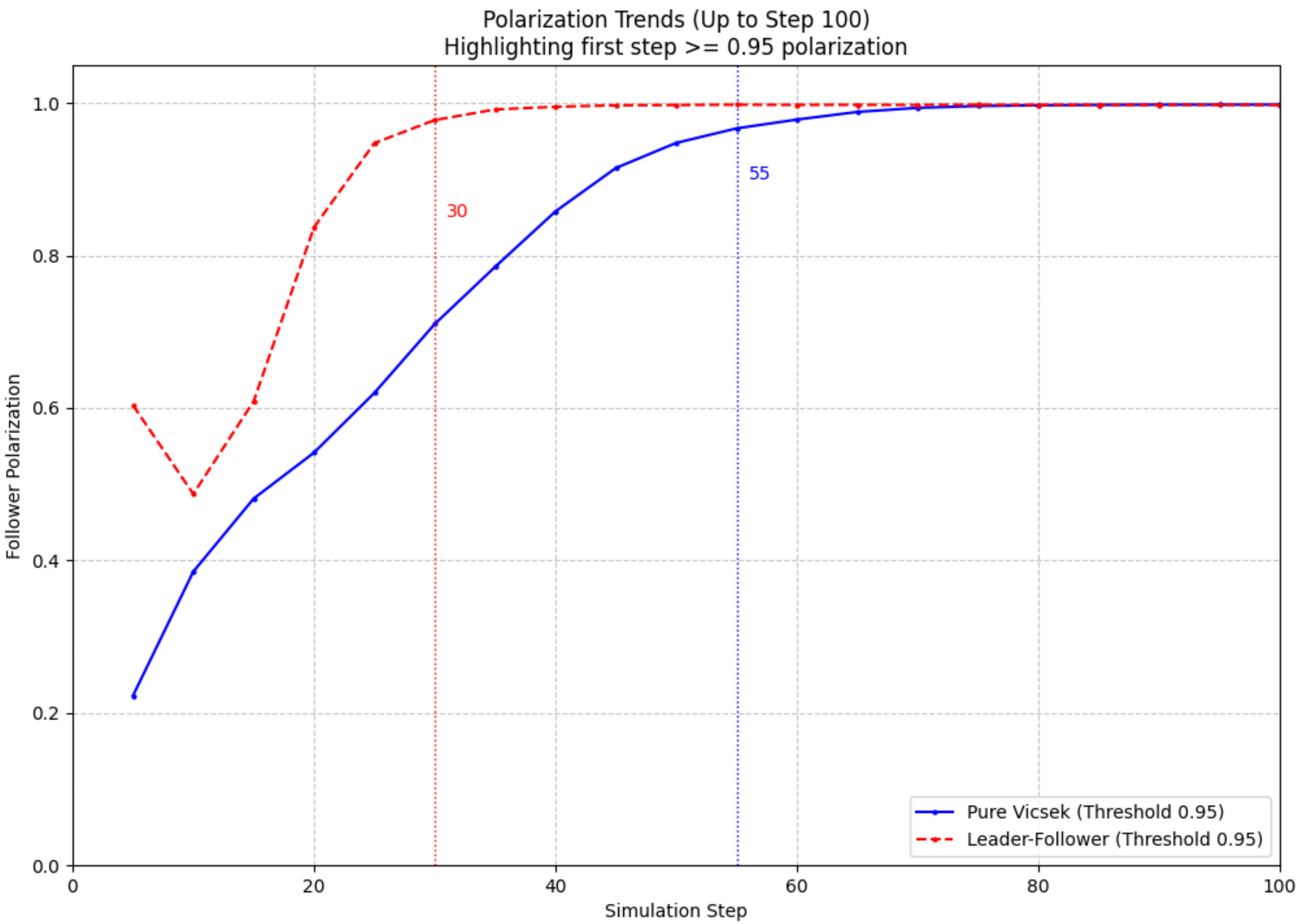
Pure Vicsek: From Chaos to Coherence



Leader–Follower: A Guided March



Experimental Results



Extended Experiment

Exp. ID	Core Objective	Key Varied Parameter(s)	Key Controlled Conditions	
Exp. 1:	Investigate impact of total particle number on collective behavior, convergence, and performance.	Total particles N_{total} (e.g., $\sim$100, $\sim$800, $\sim$10k).	$N_L/N_F \approx 1/25$;	Leader coverage $\sim$30%.
Exp. 2:	Study how the proportion of followers directly influenced by leaders alters system dynamics.	`leader_capacity` (e.g., to achieve 10%, 30%, 50% coverage of N_F).	N_F (e.g., 500);	N_L (e.g., 20).
Exp. 3:	Compare single high-capacity vs. multiple low-capacity leaders, given same total follower coverage.	N_L and `leader_capacity` (e.g., 1×150; 10×15; 30×5).	N_F (e.g., 500);	Total leader capacity (e.g., 150).
Exp. 4:	Examine if initial spatial placement of leaders affects system evolution and emergent structures.	Initial Leader Configuration (Random, Center, Grid, Periphery).	Other params (N_F, N_L, coverage, physical params).	
Exp. 5:	Determine how leader communication frequency impacts system convergence and dynamics.	$T_{communication}$ (e.g., 5, 15, 30, 100, 200, ∞).	Other params (N_F, N_L, coverage, physical params).	
Exp. 6:	Evaluate system robustness and behavior under imperfect leader communication.	Leader Communication Success Rate (or Failure Rate) per communication event (e.g., 100%, 90%, 75%, 50%).	Fixed $T_{communication}$ (e.g., 30);	Other params (N_F, N_L, coverage, physical params).

Note: This table summarizes the experimental design. Specific values for "e.g." (exempli gratia, for example) are illustrative and would be precisely defined for actual runs. "∞" for $T_{communication}$ in Exp. 5 implies leaders only perform Vicsek interactions post-initialization without global synchronization. For Exp. 6, success rate implies the probability each leader successfully participates in a given synchronization event.

Addition: Use reinforcement learning to dynamically adjust the leader's behavior strategy or optimize its initial deployment position.

Thank You!

https://github.com/ZenWang00/Particle-Methods-SP-2025/tree/main/homework5