04: Projection

Chapter Goal: To use the geometric definition of the Dot Product to formally understand and calculate the **projection** of one vector onto another.

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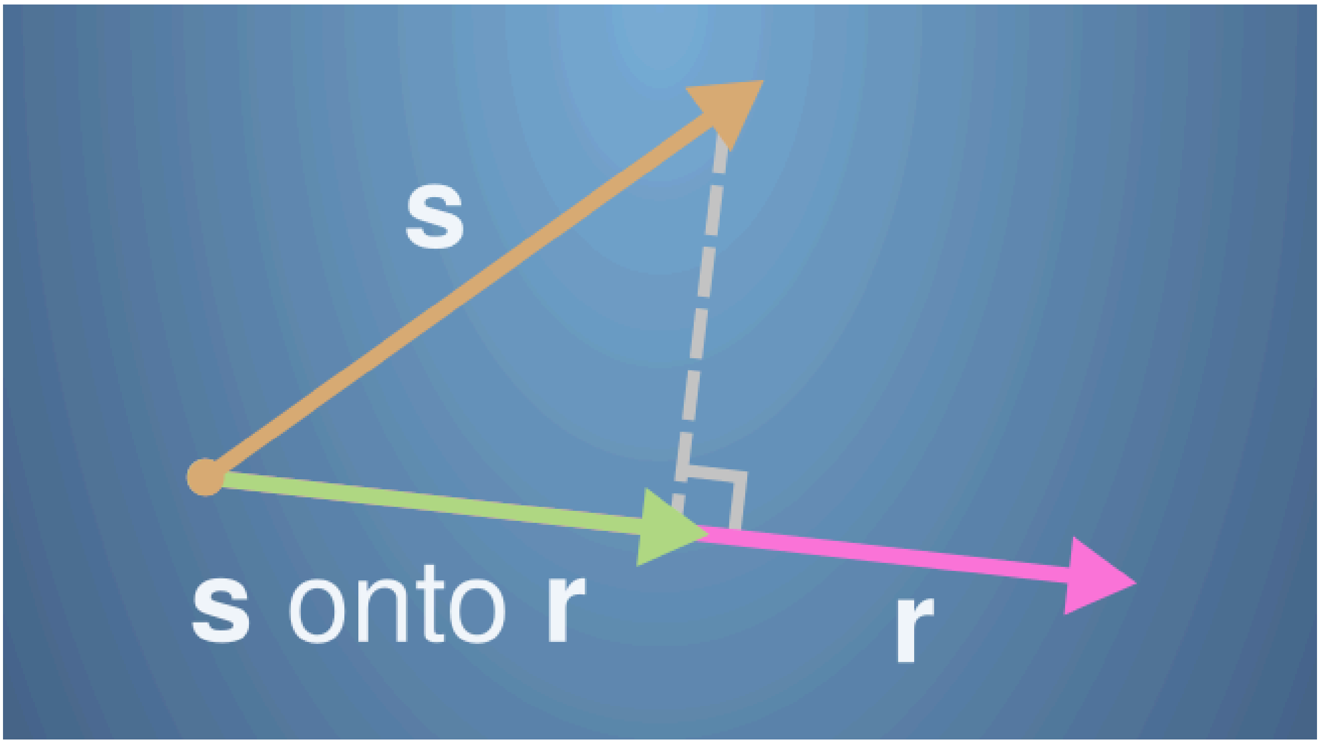
1. Core Idea: The "Shadow" of One Vector onto Another

- **Setup:** We have two vectors, $\vec{r}$ and $\vec{s}$, with an angle θ between them.
- **Projection:** Imagine a light source directly above, casting a shadow from vector $\vec{s}$ perpendicularly onto the line of vector $\vec{r}$. This shadow is the projection.
- **Geometrically:** This is the **adjacent** side of a right-angled triangle where $\vec{s}$ is the **hypotenuse**.
- **Visualization:**
  Picture vectors $\vec{r}$ and $\vec{s}$ starting from the same origin. From the tip of $\vec{s}$, draw a dashed line that is perpendicular to the line of $\vec{r}$. The "shadow" that forms along the line of $\vec{r}$ is the "Projection".



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2. Connecting the Dot Product with Projection

- **Geometric Definition of the Dot Product:**

$$\vec{r} \cdot \vec{s} = |\vec{r}||\vec{s}| \cos(\theta)$$

- **Focus on the $|\vec{s}|\cos(\theta)$ part:**
  - From basic trigonometry (SOH CAH TOA), we know $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$.
  - So, $\text{Adjacent} = \text{Hypotenuse} \cdot \cos(\theta)$.
  - In our triangle, $\text{Hypotenuse} = |\vec{s}|$, and the `Adjacent` side is the length of the "shadow" or projection.
  - Therefore, $|\vec{s}|\cos(\theta)$ **is the length of the shadow of $\vec{s}$ onto $\vec{r}$.**
- **"Aha!" Moment:**
  If we substitute this back into the dot product formula:

  > $\vec{r} \cdot \vec{s} = |\vec{r}| \cdot (\text{Length of the Projection of } \vec{s} \text{ onto } \vec{r})$

- **New Interpretation:** The dot product $\vec{r} \cdot \vec{s}$ tells us the length of $\vec{r}$ multiplied by the length of $\vec{s}$'s shadow on $\vec{r}$. This is why the dot product is sometimes called a "Projection Product".

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3. Two Types of Projection

We can rearrange the equation above to formally define two types of projection.

A. Scalar Projection

- **Goal:** To find the **LENGTH** of the shadow (a single number/scalar).
- **How to Calculate:** From the equation above, we just divide by $|\vec{r}|$.

$$\text{Scalar Projection of } \vec{s} \text{ onto } \vec{r} = \frac{\vec{r} \cdot \vec{s}}{|\vec{r}|}$$

- This is a number that tells us "how far $\vec{s}$ goes in the direction of $\vec{r}$".

B. Vector Projection

- **Goal:** To find the **VECTOR** of the shadow itself (an arrow with both length AND direction).
- **Logic:**
  1. We already have the **length**: $\frac{\vec{r} \cdot \vec{s}}{|\vec{r}|}$.
  2. We need its **direction**: The direction must be the same as $\vec{r}$'s direction.
  3. The unit vector in the direction of $\vec{r}$ is $\frac{\vec{r}}{|\vec{r}|}$.
- **Combine the two:** `Vector Projection = (Length) * (Unit Direction Vector)`
- **Formula:**

$$\text{proj}_{\vec{r}}(\vec{s}) = \left(\frac{\vec{r} \cdot \vec{s}}{|\vec{r}|}\right)\frac{\vec{r}}{|\vec{r}|} = \left(\frac{\vec{r} \cdot \vec{s}}{|\vec{r}|^2}\right)\vec{r} = \left(\frac{\vec{r} \cdot \vec{s}}{\vec{r} \cdot \vec{r}}\right)\vec{r}$$

- This is a new vector that represents the "shadow" of $\vec{s}$ onto $\vec{r}$.

4. Summary of the Dot Product's Uses

The Dot Product is a very versatile tool. It can tell us:

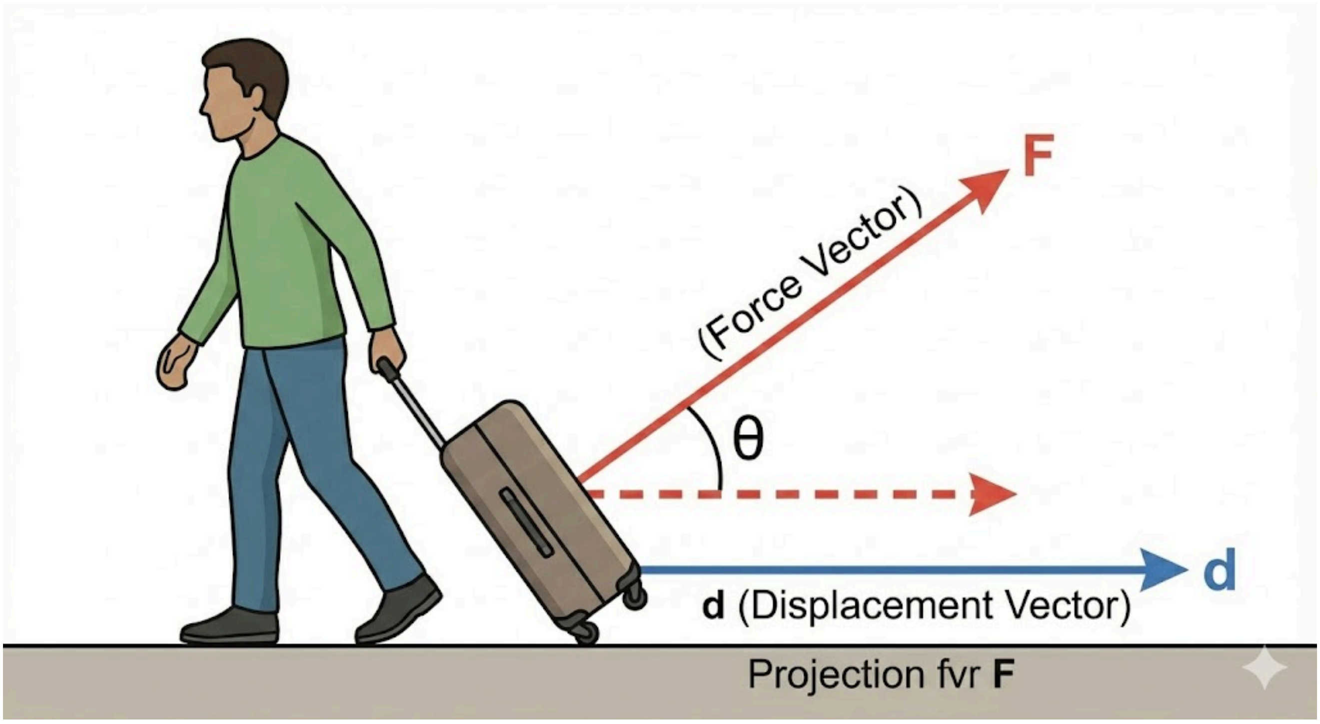
- The **length** of a vector: $|\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}}$.
- The **angle** between two vectors (through $\cos(\theta)$).
- If two vectors are **perpendicular** ($\vec{r} \cdot \vec{s} = 0$).
- The **projection** of one vector onto another (both its length and the vector itself).

Catatan Tambahan :

⚠ **SECARA INTUITIF**

Menggunakan bahasa, agar lebih mudah dipahami

1. The Big Picture: Apa Bedanya?



Bayangkan lagi contoh **Menarik Koper** tadi.

1. **Dot Product:** Sebuah **Angka Skor**. (Misal: 50 Joule).
   - *Masalahnya:* Angka ini tercampur antara "Kuatnya tarikanmu" dan "Panjangnya jarak tempuh". Susah dibedakan mana yang mana.
2. **Scalar Projection:** Sebuah **Angka Panjang**. (Misal: 5 Newton).
   - *Artinya:* Ini murni "Kuatnya tarikanmu yang searah gerak". Kita membuang faktor jarak tempuh.

3. **Vector Projection:** Sebuah **Panah Baru**. (Misal: Panah 5 Newton ke arah kanan).
    - *Artinya:* Ini adalah "Bayangan"-nya itu sendiri yang digambar ulang sebagai panah.

2. Scalar Projection (Panjang Bayangan)

Istilah kerennya: *"Component of $\mathbf{a}$ along $\mathbf{b}$"*.

Intuisinya:

Kita ingin tahu panjang bayangan $\mathbf{a}$ yang jatuh ke $\mathbf{b}$, TAPI kita tidak peduli seberapa panjang $\mathbf{b}$ itu sendiri. Kita cuma mau tahu seberapa efektif si $\mathbf{a}$.

Dari mana rumusnya?

Ingat rumus Dot Product:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \underbrace{|\mathbf{b}|}_{\text{Pengganggu}} \cos\theta$$

Kita mau mencari panjang bayangannya saja, yaitu $|\mathbf{a}|\cos\theta$.

Jadi, kita harus "membuang" si $|\mathbf{b}|$ (panjang vektor B). Caranya? Bagi saja dengan $|\mathbf{b}|$.

Rumus Scalar Projection:

$$\text{Scalar Proj} = |\mathbf{a}|\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$$

> Contoh Koper:
>
> Kamu tarik koper (Vektor A) sekuat 10 Newton dengan sudut 60°.
>
> - Dot Product mungkin angkanya besar karena dikali jarak.
> - Scalar Projection cuma peduli tenagamu: $10 \cdot \cos(60^\circ) = \mathbf{5}\ \text{Newton}$.
> - Hasilnya cuma angka **5**.

3. Vector Projection (Wujud Bayangan)

Istilah kerennya: *"Projecting $\mathbf{a}$ onto $\mathbf{b}$"*.

Intuisinya:

Kalau Scalar Projection itu cuma ngasih tau "Panjang bayangannya 5 cm", Vector Projection itu bilang: "Nih, aku gambarin panah baru yang panjangnya 5 cm dan arahnya sama persis dengan B."

Jadi hasilnya bukan angka, tapi **Vektor (Panah)**.

Cara Meraciknya:

Untuk membuat vektor, kita butuh dua bahan:

1. **Ukuran (Size):** Kita ambil dari *Scalar Projection* tadi.
2. **Arah (Direction):** Kita pinjam arahnya si $\mathbf{b}$.

Tapi tunggu, kita tidak boleh ambil panjangnya $\mathbf{b}$, kita cuma butuh arahnya. Di matematika, "Arah Murni" itu disebut **Unit Vector** ($\frac{\mathbf{b}}{|\mathbf{b}|}$).

Rumus Vector Projection:

$$\text{ector Proj} = \underbrace{(\text{Scalar Proj})}_{\text{uran}} \; (\underbrace{\frac{\mathbf{b}}{|\mathbf{b}|}}_{\text{Arah}})$$

Kalau disederhanakan rumusnya jadi terlihat rumit (padahal konsepnya simpel):

$$\text{ector Proj} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2} \right) \mathbf{b}$$

> **Contoh Koper:**
>
> - Scalar Projection bilang: "Kekuatannya 5 Newton."
> - Vector Projection bilang: "Kekuatannya **5 Newton**, arahnya **Mendatar ke Kanan**." (Dia menggambar panah hantu di lantai).

Ringkasan Visual

Biar gampang diingat:

1. **Dot Product ($\mathbf{a} \cdot \mathbf{b}$):**
   - *Output:* Angka Besar (Skalar).
   - *Arti:* Total interaksi A dan B (panjang A panjang B kemiringan).
2. **Scalar Projection ($\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$):**
   - *Output:* Angka Kecil (Skalar).
   - *Arti:* Panjangnya bayangan A doang (panjang A kemiringan). Panjang B dibuang.
3. **Vector Projection:**
   - *Output:* Gambar Panah (Vektor).
   - *Arti:* Panah bayangan A yang ditempel di atas garis B.

Jadi, urutannya biasanya:

Hitung Dot Product dulu Bagi panjang B buat dapet Scalar Proj Kasih arah B buat dapet Vector Proj.

Tags: #mml-specialization #linear-algebra #dot-product #projection #scalar-projection #vector-projection