M02: Vector Operations - Formula Summary

This document contains a summary of the fundamental formulas discussed in Module 2, focusing on vector operations.

---

1. Vector Representation

- A vector $\vec{v}$ with `n` components.

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

---

2. Vector Length (Modulus / Magnitude)

- Calculates the length of a vector $\vec{v}$.

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

---

3. The Dot Product (Inner Product)

3.1. Computational Definition

- Calculated by summing the products of corresponding components.

$$\vec{r} \cdot \vec{s} = r_1 s_1 + r_2 s_2 + \cdots + r_n s_n$$

3.2. Geometric Definition

- Relates the dot product to the vector lengths and the angle (θ) between them.

$$\vec{r} \cdot \vec{s} = |\vec{r}||\vec{s}| \cos(\theta)$$

---

4. Key Relationships & Applications

4.1. Length from Dot Product

- The square of a vector's length is the dot product of the vector with itself.

$$|\vec{v}|^2 = \vec{v} \cdot \vec{v} \implies |\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$$

4.2. Angle Between Vectors

- Used to find the angle between two vectors.

$$\cos(\theta) = \frac{\vec{r} \cdot \vec{s}}{|\vec{r}||\vec{s}|}$$

4.3. Orthogonality Check

- Two non-zero vectors are mutually orthogonal if and only if their dot product is zero.

$$\vec{r} \cdot \vec{s} = 0 \iff \vec{r} \perp \vec{s}$$

---

5. Projections

5.1. Scalar Projection

- The **length** of the "shadow" of vector $\vec{s}$ onto vector $\vec{r}$.

$$\text{Scalar Projection} = \frac{\vec{r} \cdot \vec{s}}{|\vec{r}|}$$

5.2. Vector Projection

- The **vector** that represents the "shadow" of $\vec{s}$ onto $\vec{r}$.

$$\text{proj}_{\vec{r}}(\vec{s}) = \left(\frac{\vec{r} \cdot \vec{s}}{\vec{r} \cdot \vec{r}} \right) \vec{r}$$

Note

Perhatikan soal dibawah !

Contoh soal sederhana :

3. The lectures introduced the idea of projecting one vector onto another. The following diagram shows the projection of $\mathbf{s}$ onto $\mathbf{r}$ when the vectors are in two dimensions:

1 point

Remember that the scalar projection is the *size* of the green vector. If the angle between $\mathbf{s}$ and $\mathbf{r}$ is greater than $\pi/2$, the projection will also have a minus sign.

We can do projection in any number of dimensions. Consider two vectors with three components, $\mathbf{r} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$.

What is the scalar projection of $\mathbf{s}$ onto $\mathbf{r}$?

- ○ $\frac{1}{2}$
- ◉ 2
- ○ $-\frac{1}{2}$
- ○ -2

Gambar: Scalar Projection (Ukuran Bayangan)

Di soal ini, pertanyaannya simpel: **"Berapa PANJANG bayangan hijau itu?"** (Ingat, hasilnya harus angka/skalar).

Mari kita hitung pakai logika "Seberapa Ngefek":

1. Hitung Total Interaksi (Dot Product):

   Kita kalikan komponen yang bersesuaian lalu jumlahkan.

$$= \ 0$$

$$= 10 \ 6$$

$$\cdot = ()(10) + ()() + (0)(6)$$

$$\cdot = 0 \ \ 20 + 0 =$$

   (Ternyata interaksi totalnya positif, berarti searah/mendukung).

2. Cari Panjang Landasan (Panjang Vektor r):

   Kita butuh tahu panjang si sebagai pembagi.

$$|| = \sqrt{^2 + ()^2 + 0^2} = \sqrt{+16} = \sqrt{2} = \mathbf{5}$$

3. Scalar Projection (Bayangan):

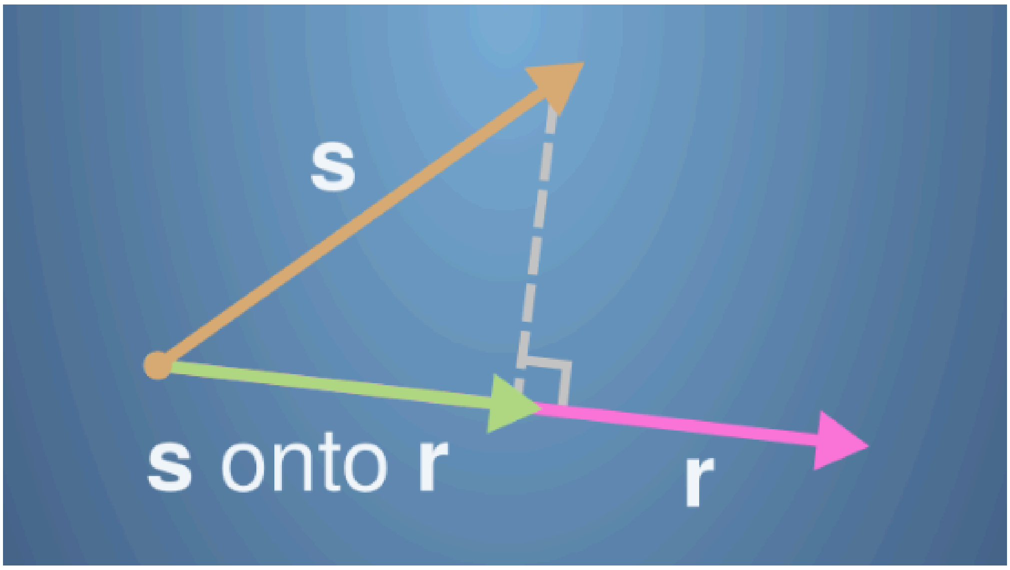
"Total interaksi dibagi panjang landasan."

$$\frac{10}{} =$$

Artinya: Panjang bayangan hijau di gambar itu adalah **2 satuan**.

---

4. Remember that in the projection diagram, the vector projection *is* the green vector:



Let $\mathbf{r} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and let $\mathbf{s} = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$.

What is the vector projection of $\mathbf{s}$ onto $\mathbf{r}$?

- $\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 30 \\ -20 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 6/5 \\ -8/5 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 6 \\ -8 \\ 0 \end{bmatrix}$

Vector Projection (Bentuk Bayangan)

Sekarang soalnya nanya: **"Tuliskan bayangan hijau itu sebagai VEKTOR lengkap (ada arahnya)."**

Logikanya begini:

Kita sudah tahu panjang bayangannya adalah 2 (dari soal sebelumnya). Kita tinggal "meminjam" arah dari vektor .

1. Ambil Arah (Unit Vector):

   Vektor aslinya 0 panjangnya 5.

   Supaya kita dapat "murni arah" tanpa panjang, kita bagi 5.

   $$\text{Arah } = - - \frac{0}{}$$

2. Kalikan dengan Panjang Bayangan (Scalar tadi):

   Kita mau bikin panah baru yang arahnya sama kayak , tapi panjangnya cuma 2.

   $$\text{Vector Proj} = 2 \times - - 0$$

   $$\text{Vector Proj} = \frac{6}{} - 0$$

Artinya: Panah hijau itu secara koordinat ada di posisi . (Jawabanmu yang dilingkari biru itu benar!).

Tags: #mml-specialization #linear-algebra #formula-sheet #cheatsheet