01: What are Eigenvalues and Eigenvectors?

Chapter Goal: To build the core geometric intuition for **Eigenvectors** and **Eigenvalues**, understanding them as the "characteristic" properties of a linear transformation.

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1. Background: The Meaning of the Word "Eigen"

- **"Eigen"** (German): Translates to **"characteristic"** or **"own"**.
- **The Eigenproblem:** Is the problem of finding the characteristic properties of a transformation.
- **Approach:** We will understand this geometrically first, before moving to the formal mathematics.

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2. Core Idea: Special Vectors That Don't Change Direction

- **Linear Transformation:** An action that changes space (stretches, rotates, shears).
- **Observation:**
  - **Most vectors:** After being transformed, are "knocked off" their original path (span). Their direction changes.
  - **Some special vectors:** After being transformed, they remain on the **same path (span)** as before the transformation.
- **Definition of an Eigenvector:**

  > An **Eigenvector** is a non-zero vector whose fundamental **direction is unchanged** by a transformation. The transformation only stretches or squishes it.

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3. Definition of an Eigenvalue

- **Eigenvalue:** Is the **scalar (number)** that acts as the **stretching/squishing factor** for an eigenvector.
- It's the "score" paired with each eigenvector.
  - If an eigenvector's length doesn't change, its eigenvalue is **1**.
  - If an eigenvector becomes twice as long, its eigenvalue is **2**.
- **Conclusion:**
  - **Eigenvector:** The "obedient" vector that stays on its path.

        - **Eigenvalue:** How much of a "push" that obedient vector receives.

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4. Visual Examples

A. Vertical Scaling

- **Transformation:** The y-direction is stretched by a factor of 2; the x-direction is unchanged.
- **Eigenvectors:**
    - **Horizontal vectors** (e.g., `[1,0]`): Their direction and length are unchanged.
        - **Eigenvalue = 1**.
    - **Vertical vectors** (e.g., `[0,1]`): Their direction is unchanged, but their length is doubled.
        - **Eigenvalue = 2**.
    - **Other vectors** (e.g., diagonals): Their direction changes (they become steeper). They are **not** eigenvectors.

B. Pure Shear

- **Transformation:** The y-direction stays fixed, the x-direction is shifted proportionally to y. (In this video's example, x stays fixed, y is shifted).
- **Eigenvectors:**
    - **Only horizontal vectors**: Their direction and length are unchanged.
        - **Eigenvalue = 1**.
    - **Other vectors:** All other vectors (including vertical ones) are "pushed sideways" and their direction changes. They are **not** eigenvectors.

C. Rotation

- **Transformation:** All vectors are rotated by the same angle.
- **Eigenvectors (in $\mathbb{R}^2$): NONE**.
- Every non-zero vector is "knocked off" its original span. There are no "magic paths" that stay fixed.

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5. Key Message

- The concept of eigenvectors and eigenvalues applies to **any number of dimensions** (3D, 4D, etc.), not just 2D.
- Finding these eigenvector-eigenvalue pairs is the way to understand the **fundamental characteristics** of a transformation.

Note

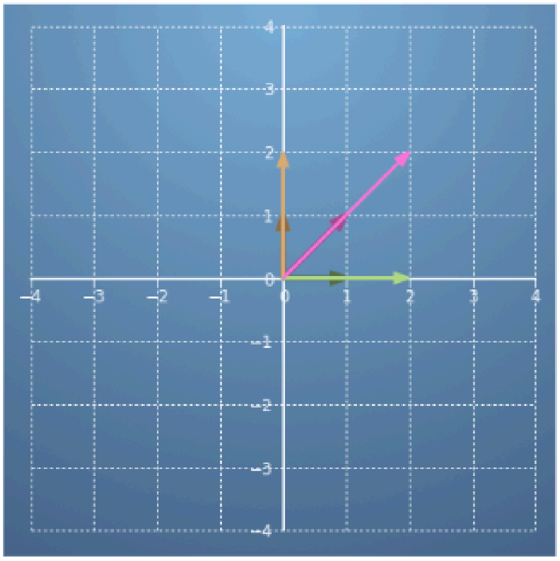
Contoh Soal dan Intuisinya

1. Recall that for a linear transformation, an eigenvector is a vector which, after applying the transformation, stays in the same span. In the following questions, you will try to geometrically see which vectors of a linear transformation are eigenvectors.

1 point

In the following diagram, the dark green vector is given by $\begin{bmatrix}1\\0\end{bmatrix}$, the purple vector by $\begin{bmatrix}1\\1\end{bmatrix}$ and the brown vector by $\begin{bmatrix}0\\1\end{bmatrix}$.

The transformation $T = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$ is applied, which sends the three vectors to the light green vector $\begin{bmatrix}2\\0\end{bmatrix}$, the magenta vector $\begin{bmatrix}2\\2\end{bmatrix}$ and the orange vector $\begin{bmatrix}0\\2\end{bmatrix}$, respectively.



Which of the three original vectors are eigenvectors of the linear transformation T?

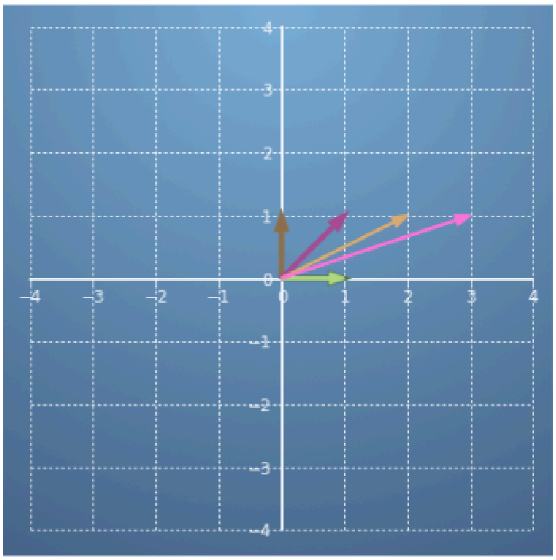
- [x] $\begin{bmatrix}1\\0\end{bmatrix}$
- [x] $\begin{bmatrix}1\\1\end{bmatrix}$
- [x] $\begin{bmatrix}0\\1\end{bmatrix}$
- [ ] None of the above.

3. Recall that for a linear transformation, an eigenvector is a vector which, after applying the transformation, stays in the same span. In the following questions, you will try to geometrically see which vectors of a linear transformation are eigenvectors.

1 point

In the following diagram, the dark green vector is given by $\begin{bmatrix}1\\0\end{bmatrix}$, the purple vector by $\begin{bmatrix}1\\1\end{bmatrix}$ and the brown vector by $\begin{bmatrix}0\\1\end{bmatrix}$.

The transformation $T = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ is applied, which sends the three vectors to the light green vector $\begin{bmatrix}1\\0\end{bmatrix}$, the magenta vector $\begin{bmatrix}3\\1\end{bmatrix}$ and the orange vector $\begin{bmatrix}2\\1\end{bmatrix}$, respectively.



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Tags: #mml-specialization #linear-algebra #eigenvectors #eigenvalues