02: Modulus & Inner Product

Chapter Goal: To formally define and understand two essential vector concepts: the **length** of a vector (modulus) and the computational definition of the **Dot Product** (inner product).

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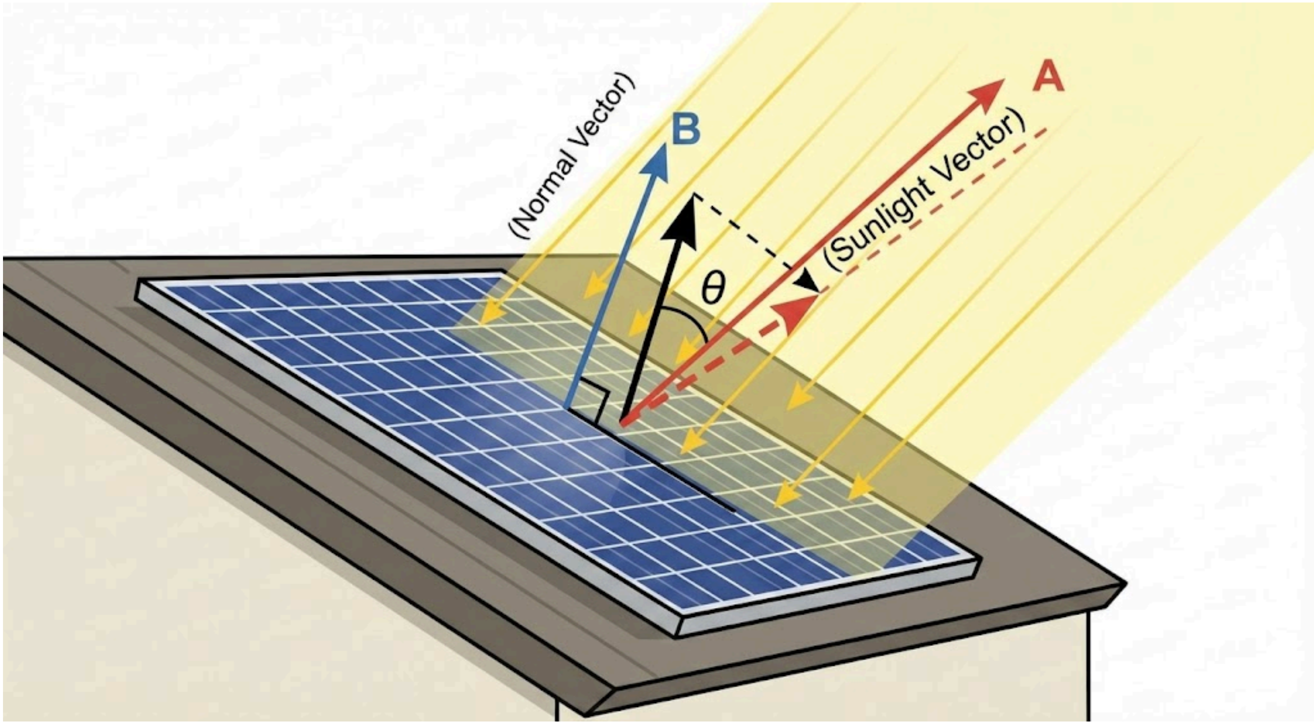
1. Background & Goal

We already know the two basic vector operations: Vector Addition and Scalar Multiplication.

Now, we will define two other essential concepts:

- The **length/size** of a vector (also called **modulus** or **magnitude**).
- The **Dot Product** (also called **inner/scalar product**), a way of "multiplying" two vectors that results in a single number (a scalar).

implementation :



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2. Calculating Length (Modulus)

- **Core Idea:** A geometric vector has a length and a direction, regardless of any coordinate system.
- **How to Calculate (in an Orthogonal Coordinate System):**
  - If a vector `r` in 2D is written as `a*î + b*ĵ` (where `î` and `ĵ` are orthogonal unit basis vectors), we can use the **Pythagorean Theorem**.

- r becomes the hypotenuse of a right-angled triangle with sides a and b.
- **Length (Modulus) Formula:**

> $|r| = \sqrt{(a^2 + b^2)}$
> (The notation $|r|$ or $||r||$ means "the length of r")

- **Generalization:** This definition is extended to any n-dimensional space, even if the "axes" are not spatial dimensions.

> The size of a vector v is defined as the square root of the sum of the squares of its components.
> $|v| = \sqrt{(v_1^2 + v_2^2 + ... + v_n^2)}$

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3. The Definition of the Dot Product (Inner Product)

- **How to Calculate (Computational Definition):**
  - Take two vectors $r = [r_1, r_2]$ and $s = [s_1, s_2]$.
  - Their dot product, $r \cdot s$, is calculated by:
    1. Multiplying the corresponding components ($r_1 * s_1$, $r_2 * s_2$).
    2. Summing up all those products.
- **Formula:**

> $r \cdot s = r_1 s_1 + r_2 s_2 + ... + r_n s_n$

- **Key Point:** The result of a dot product is a single **NUMBER (scalar)**, not a new vector.

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4. Key Properties of the Dot Product

- **Commutative** (Order doesn't matter):
  $r \cdot s = s \cdot r$
  - **Why?** Because the multiplication of regular numbers is commutative ($r_1 s_1 = s_1 r_1$).
- **Distributive** over Addition:
  $r \cdot (s + t) = (r \cdot s) + (r \cdot t)$
  - **Why?** This can be proven by expanding the algebra. It means we can "distribute" the dot product into a sum.
- **Associative** with Scalar Multiplication:
  $r \cdot (a*s) = a * (r \cdot s)$
  - **Why?** Because a (a scalar) can be factored out of each term in the sum.

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5. The Magical Connection: Dot Product and Length

- **What happens if a vector is dotted with itself?**
  $r \cdot r = r_1 * r_1 + r_2 * r_2 + \ldots = r_1^2 + r_2^2 + \ldots$
- **The "Aha!" Moment:** This expression $r_1^2 + r_2^2 + \ldots$ is exactly the **square of the length** of r!

  > $r \cdot r = |r|^2$

- **Practical Conclusion:**

  > The length of a vector can be calculated by dotting the vector with itself and then taking the square root.
  > $|r| = \sqrt{(r \cdot r)}$

This is a very neat and fundamental relationship.

Tags: #mml-specialization #linear-algebra #vectors #modulus #inner-product