01: Functions

Chapter Goal: To build a philosophical and intuitive understanding of what a **Function** is, its role in science and modeling, and how it relates to Calculus.

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1. Intuitive Definition: What is a Function?

- **Core Idea:** A function is a "machine" or a rule that defines a relationship between **inputs** and **outputs**.
- You put something in (the input), and the machine gives you one specific result back (the output).
- **Multi-Variable Example:**
    - **Function for the Temperature in a Room:** $T(x, y, z, t)$
    - **Input:** 4 numbers (the coordinates `x,y,z` and the time `t`).
    - **Output:** 1 number (the temperature at that specific location and time).

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2. The Problem of Notation & "Learning the Language"

- **Common Problem:** Mathematical notation can often be confusing and unintuitive at first.
- **Analogy:** Learning mathematics is like learning a new language (e.g., French).
    - You cannot immediately enjoy the "poetry" (advanced applications like Machine Learning) without first learning the "vocabulary and grammar" (basic notation and rules), which can sometimes feel tedious.
- **Examples of Confusion:**
    - `f(x)`: Why does this mean "f is a function of x" and not "f multiplied by x"?
        - **Answer:** Convention. We just have to accept it as part of the "grammar".
    - `f(x) = ... g(x) ... h ... a`: Is `g` a function? Are `h` and `a` constants?
        - **Answer:** Context is crucial. Just like in a normal language, we often need the surrounding sentences to understand the meaning of a word.
- **Key Message:**
    - Feeling confused by notation is **normal**, even for professionals.
    - With time and practice, you will get better at "guessing" the meaning from the context, just like when learning a new language.

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3. The Creative Role of Functions: The "Poetry" of Science

- **The Heart of Science:** Choosing or "inventing" a function is the core of the scientific and creative process.
- **The Scientific Process:**
  1. **Observation:** You see a phenomenon in the real world (e.g., the height data).
  2. **Hypothesis (The Creative Step):** You say, "I believe this phenomenon can be modeled by function X." (e.g., the Normal Distribution function). Choosing this candidate function is the "poetry".
  3. **Verification (The Hard Work):** After that comes the long process of testing whether your hypothesis is correct.
- The geniuses of science (Newton, Einstein) are remembered for the creative step #2: finding the right function to describe nature.

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4. The Relationship to Calculus

- **The Role of Calculus:**
  - If choosing a function is the art, then calculus is the **technical toolkit**.
  - Calculus is the study of **how functions change** with respect to their inputs.
  - It gives you the tools to investigate, manipulate, and **optimize** the functions that you have creatively chosen.
- **Final Conclusion:**
  - **Functions** are the language for describing the world.
  - **Choosing** the right function is an art.
  - **Calculus** is the toolkit for analyzing and working with that language.

This course will teach you how to use that toolkit to model real-world data.

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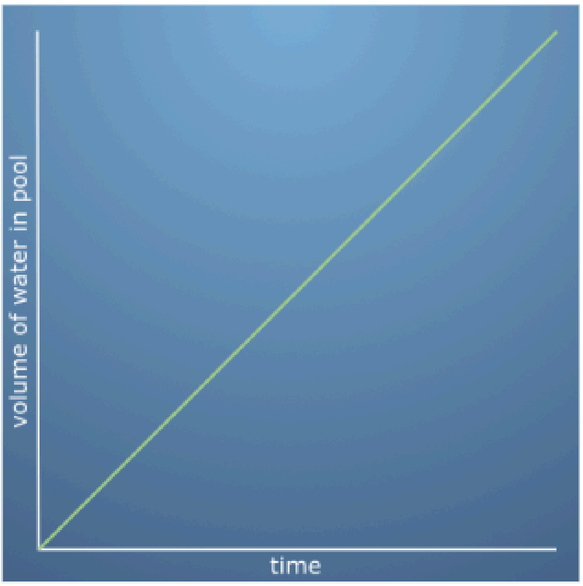
5. Worked Example

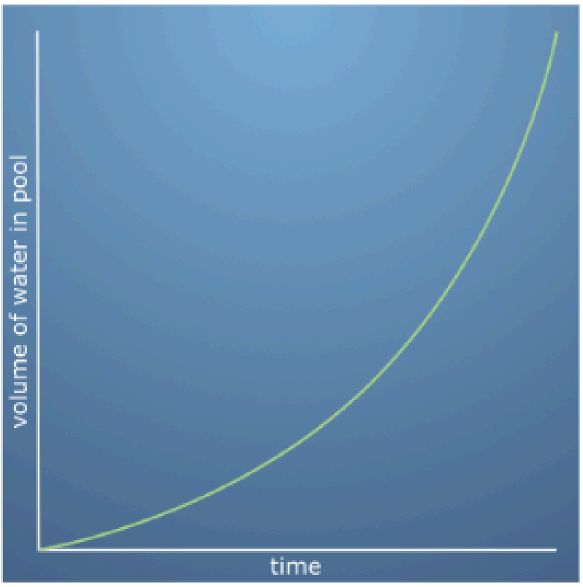
1. In this quiz you will get a refresher in functions - in particular, matching a description of a function to the graph of the function.

1 / 1 point

Imagine that you place one end of a water hose into a swimming pool and turn the tap on at the other end. Water then pours into the pool *at a constant rate*, causing the volume of water in the pool to increase at a constant rate.

While the swimming pool is still filling up with water, what would we expect the plot of the function of volume of water in the pool with respect to time to look like?

- [x] 

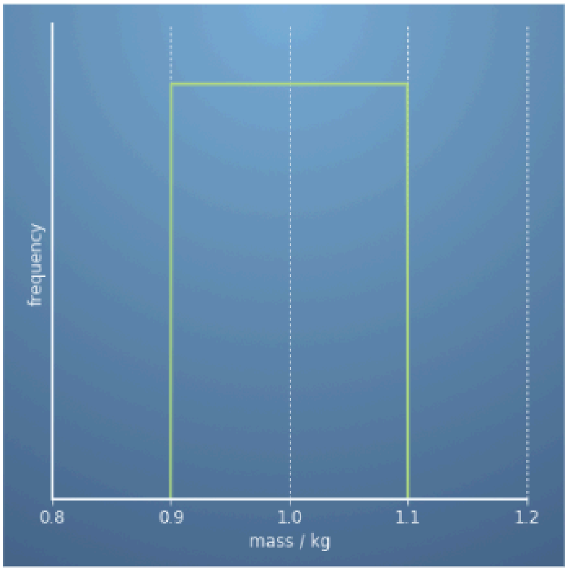
- [ ] 

4. Bags of flour labelled 1 kg from a supermarket are weighed. Most of the weights measured are very close to 1 kg, with some a little more and others a little less. Those which are further away from 1 kg are found less and less often, with almost no bags more than 100 g out.

1 / 1 point

What might we expect the plot of frequency (i.e. how often a type of bag is found) against mass to look like?

- [ ] $$$$



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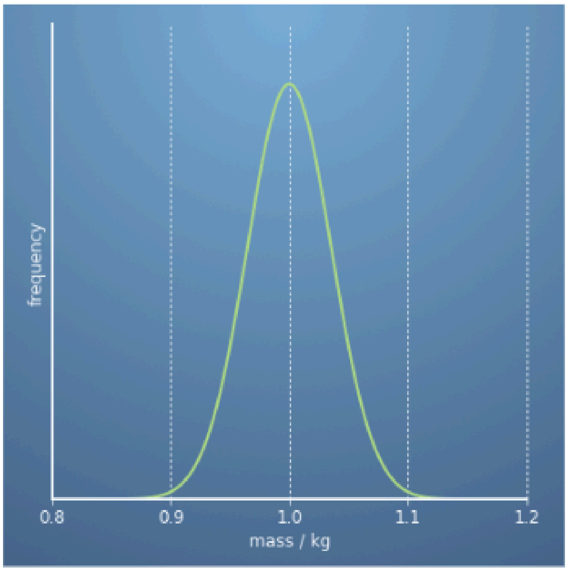

5. A mass is attached to a string and hung from the ceiling. It is then pulled away from its natural hanging position (called equilibrium) and released, so that it swings backwards and forwards. Let's assume there is no air resistance, so that when the mass swings back it returns all the way back to where it was originally released. It completes a full swing, away and back, every 2 seconds.

1 / 1 point

What is a reasonable plot for the displacement of the mass from equilibrium with respect to time?

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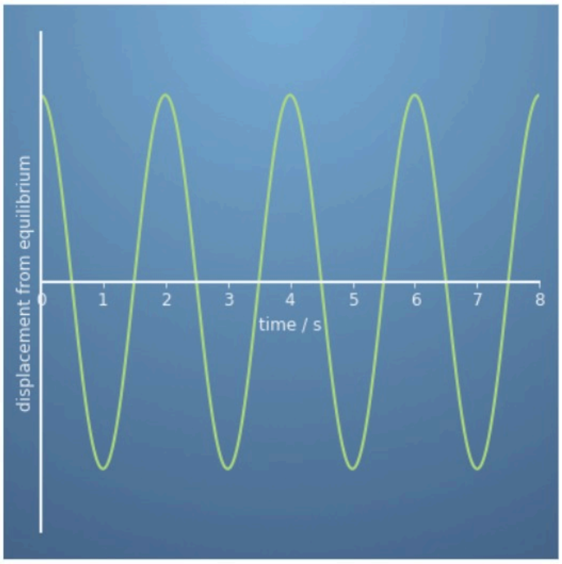

Tags: #mml-specialization #multivariate-calculus #functions #notation #modeling