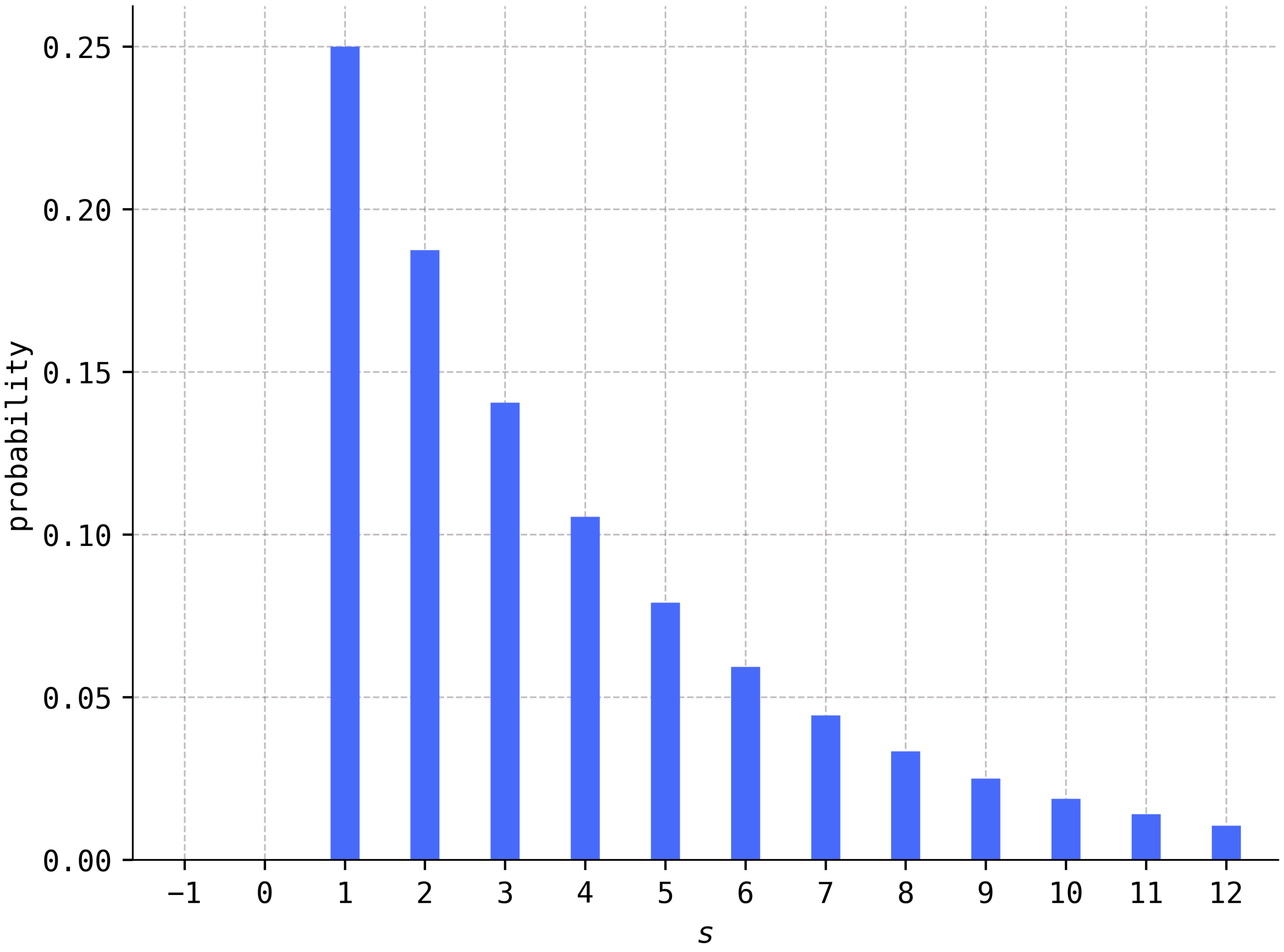
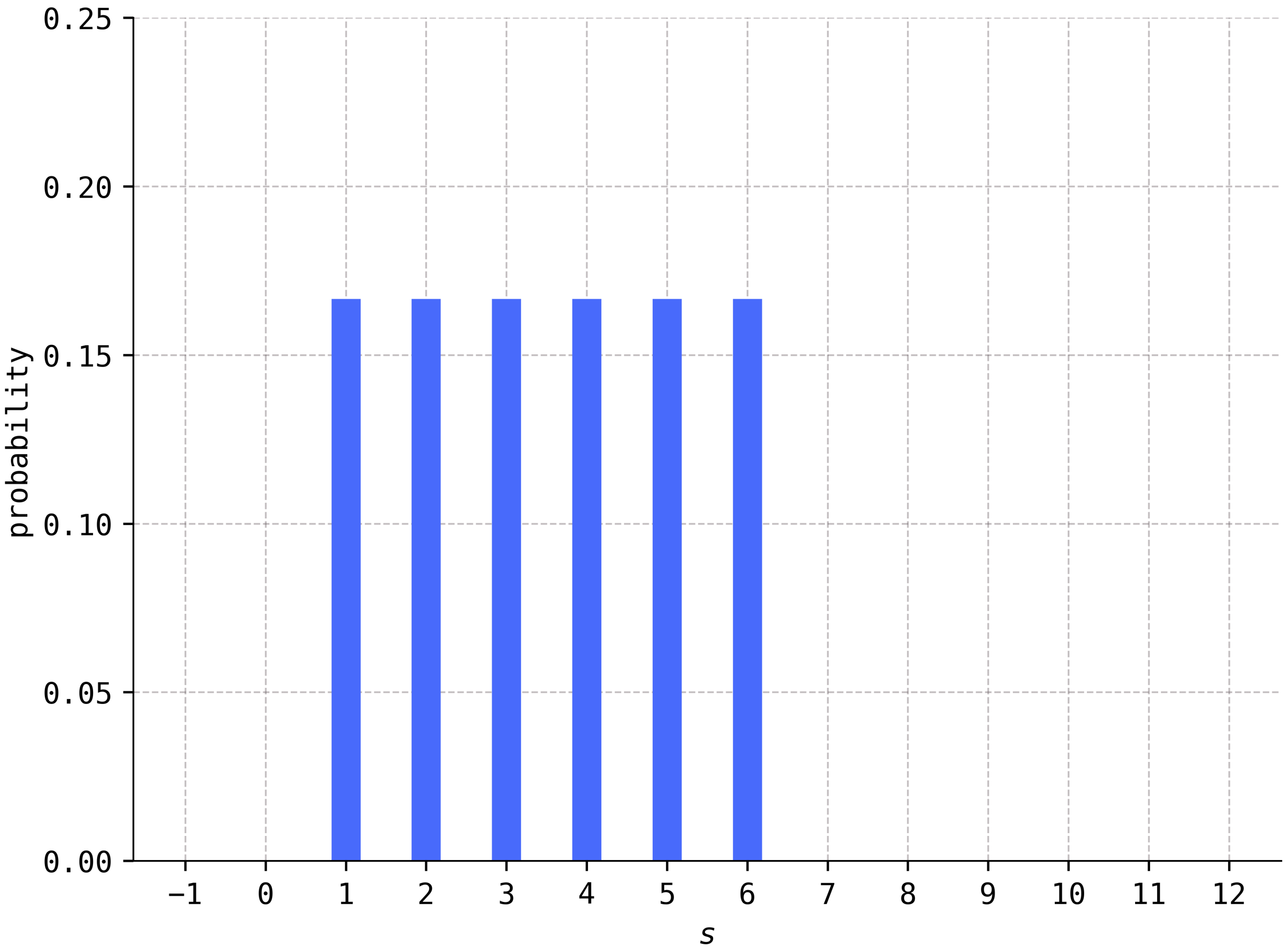
2.7. Probability histograms

$$p(s) = \begin{cases} (0.25)(0.75)^{s-1} & : s = 1, 2, \ldots, \\ 0 & : \text{otherwise.} \end{cases}$$



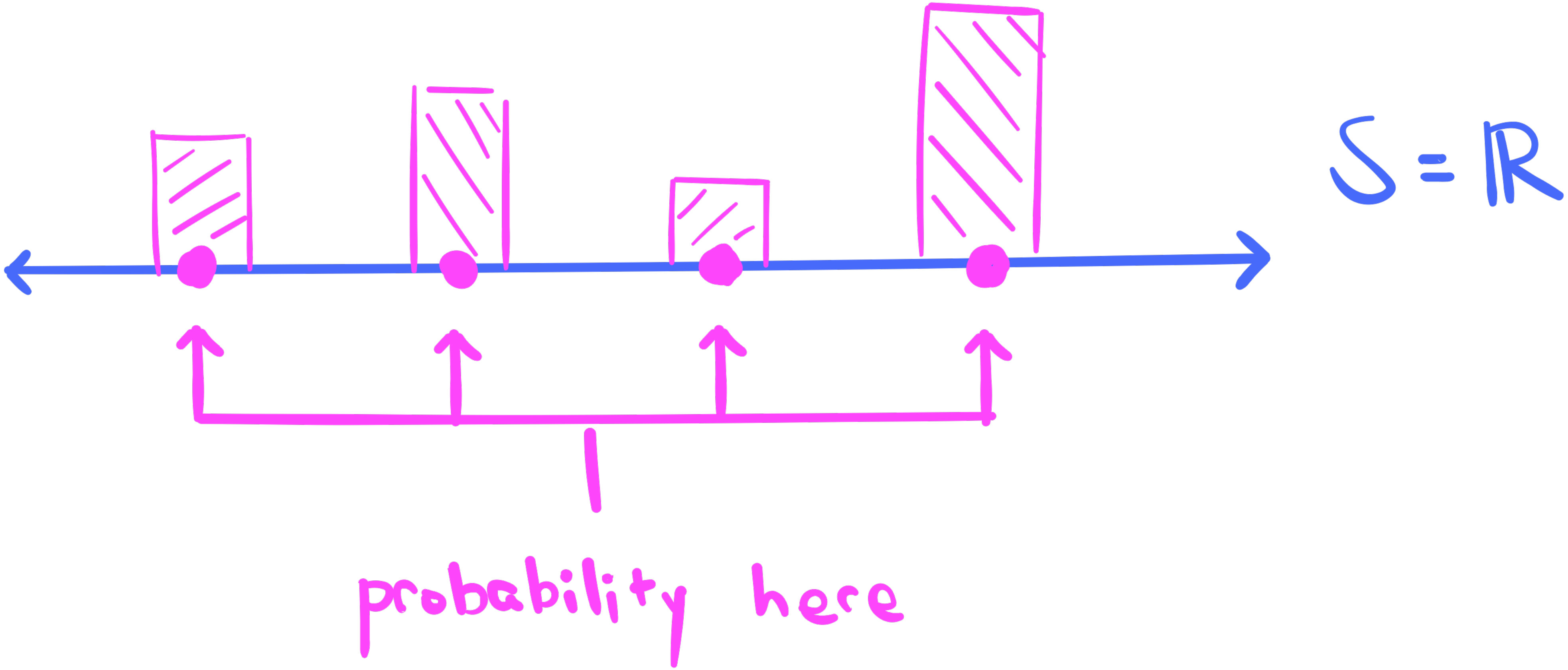
$$p(s) = \begin{cases} (0.25)(0.75)^{s-1} & : s = 1, 2, \ldots, \\ 0 & : \text{otherwise.} \end{cases}$$



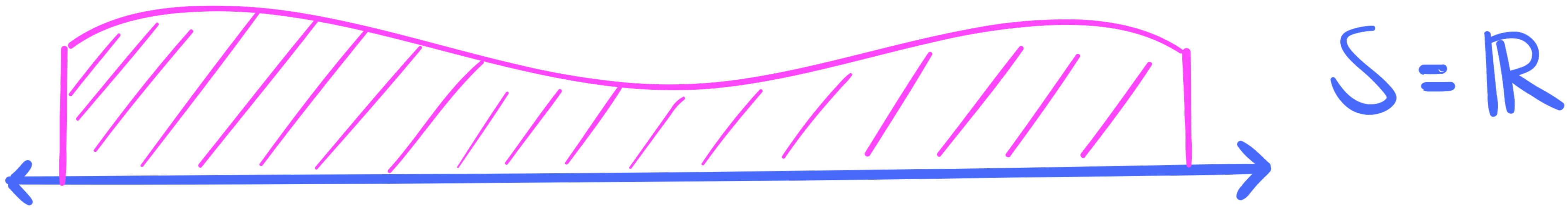
$$\text{support} = \{1, 2, 3, 4, 5, 6\} \quad \text{and} \quad p(s) = \frac{1}{6}$$

2.8. Continuous probability measures

Discrete probability measure



continuous probability measure



probability everywhere!

Definition 2.9

Let P be a probability measure on $\mathbb{R}$. We shall say P is *continuous* if there is a function $f : \mathbb{R} \to \mathbb{R}$ such that

$$P(A) = \int_A f(s)\,\mathrm{d}s \tag{2.4}$$

for all events $A \subset \mathbb{R}$. In this case, the function $f(s)$ is called the *probability density function* (PDF) of the probability measure P, and $\mathbb{R}$ is called a *continuous probability space* (when equipped with P).

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Definition 2.7

Let P be a probability measure on a sample space S. We shall say P is *discrete* if every subset $A \subset S$ is an event and there is a function $p : S \to \mathbb{R}$ with discrete support such that

$$P(A) = \sum_{s\in A} p(s), \tag{2.2}$$

for all events A. In this case, the function p is called the *probability mass function* (PMF) of the probability measure P (or sometimes just the *probability function*), and S is called a *discrete probability space* (when equipped with P).

Problem Prompt

Do problems 12 and 13 on the worksheet.

Theorem 2.3 (Properties of Probability Mass Functions)

Let $p(s)$ be the probability mass function of a discrete probability measure P. Then:

1. $p(s) \geq 0$ for all $s \in S$, and
2. $\sum_{s \in S} p(s) = 1$.

Theorem 2.4 (Discrete Probability Construction Lemma)

Let S be a set and $p : S \to \mathbb{R}$ a function with discrete support. If

1. $p(s) \geq 0$ for all $s \in S$, and
2. $\sum_{s \in S} p(s) = 1$,

then there is a unique discrete probability measure P on S such that

$$P(A) = \sum_{s \in A} p(s)$$

for all $A \subset S$.

Theorem 2.5 (Properties of Probability Density Functions (univariate version))

Let $f(s)$ be the probability density function of a continuous probability measure P on $\mathbb{R}$. Then:

1. $f(s) \geq 0$ for all $s \in \mathbb{R}$, and
2. $\int_{\mathbb{R}} f(s)\, \mathrm{d}s = 1$.

Theorem 2.6 (Continuous Probability Construction Lemma (univariate version))

Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that

1. $f(s) \geq 0$ for all $s \in \mathbb{R}$, and
2. $\int_{\mathbb{R}} f(s)\, \mathrm{d}s = 1$.

Then there is a unique continuous probability measure P on $\mathbb{R}$ such that

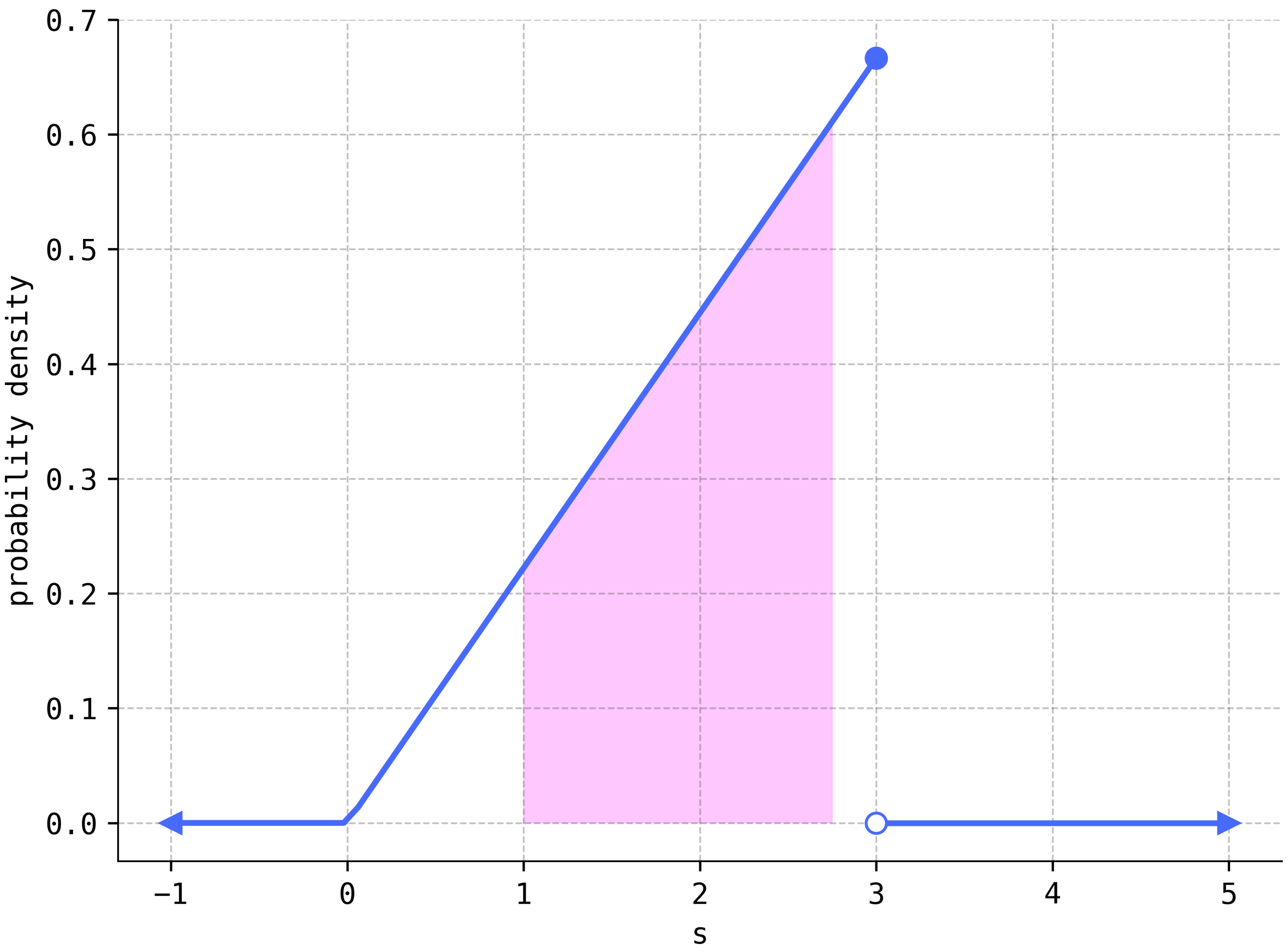
$$P(A) = \int_A f(s)\, \mathrm{d}s$$

for all events $A \subseteq \mathbb{R}$.

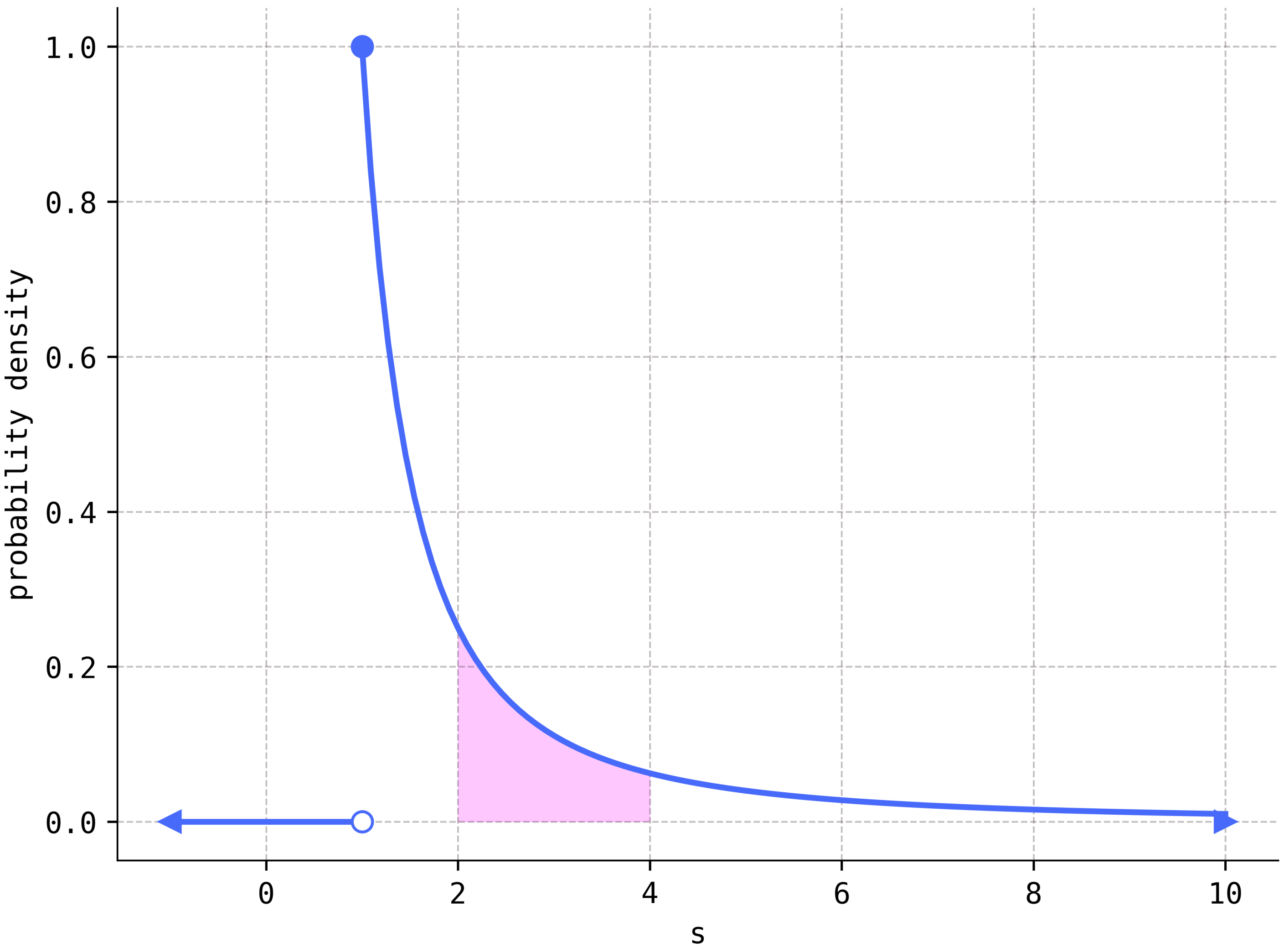
Problem Prompt

Do problem 14 on the worksheet.

2.9. Probability density graphs (univariate versions)



$$f(s) = \begin{cases} \frac{2}{9}s & : 0 \leq s \leq 3, \\ 0 & : \text{otherwise.} \end{cases} \qquad P((1, 2.75)) = \int_1^{2.75} f(s)\,\mathrm{d}s \approx 0.73.$$



$$f(s) = \begin{cases} \frac{1}{s^2} & : s \geq 1, \\ 0 & : \text{otherwise.} \end{cases} \qquad P([2,4]) = \int_2^4 f(s)\, \mathrm{d}s = \frac{1}{4}$$