What is the Best Indicators to Explain Rent?

Group member：

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In [8]:
```
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
import statsmodels.formula.api as smf
import scipy.stats as stats
from sklearn.model_selection import train_test_split
from sklearn import datasets
from sklearn import svm
from RegscorePy import mallow
import itertools
from BorutaShap import BorutaShap
import statsmodels.api as sm
from matplotlib.ticker import ScalarFormatter
from statsmodels.graphics.regressionplots import plot_ceres_residuals
```

In [2]:
```
df=pd.read_csv('train.csv', encoding = 'utf-8')
```

In [5]:
```
df.head()
```

Out[5]:

	bedroom	bathrooms	area	furnishing	avalable_for	floor_number	facing	gate_communit
0	2.0	2.0	1050.0	0.0	0.0	5.0	1.0	1.(
1	2.0	2.0	760.0	0.0	0.0	5.0	0.0	1.(
2	3.0	3.0	1122.0	0.5	0.0	1.0	7.0	1.(
3	1.0	1.0	628.0	1.0	1.0	3.0	0.0	1.(
4	2.0	2.0	668.0	0.5	2.0	6.0	4.0	1.(

5 rows × 27 columns

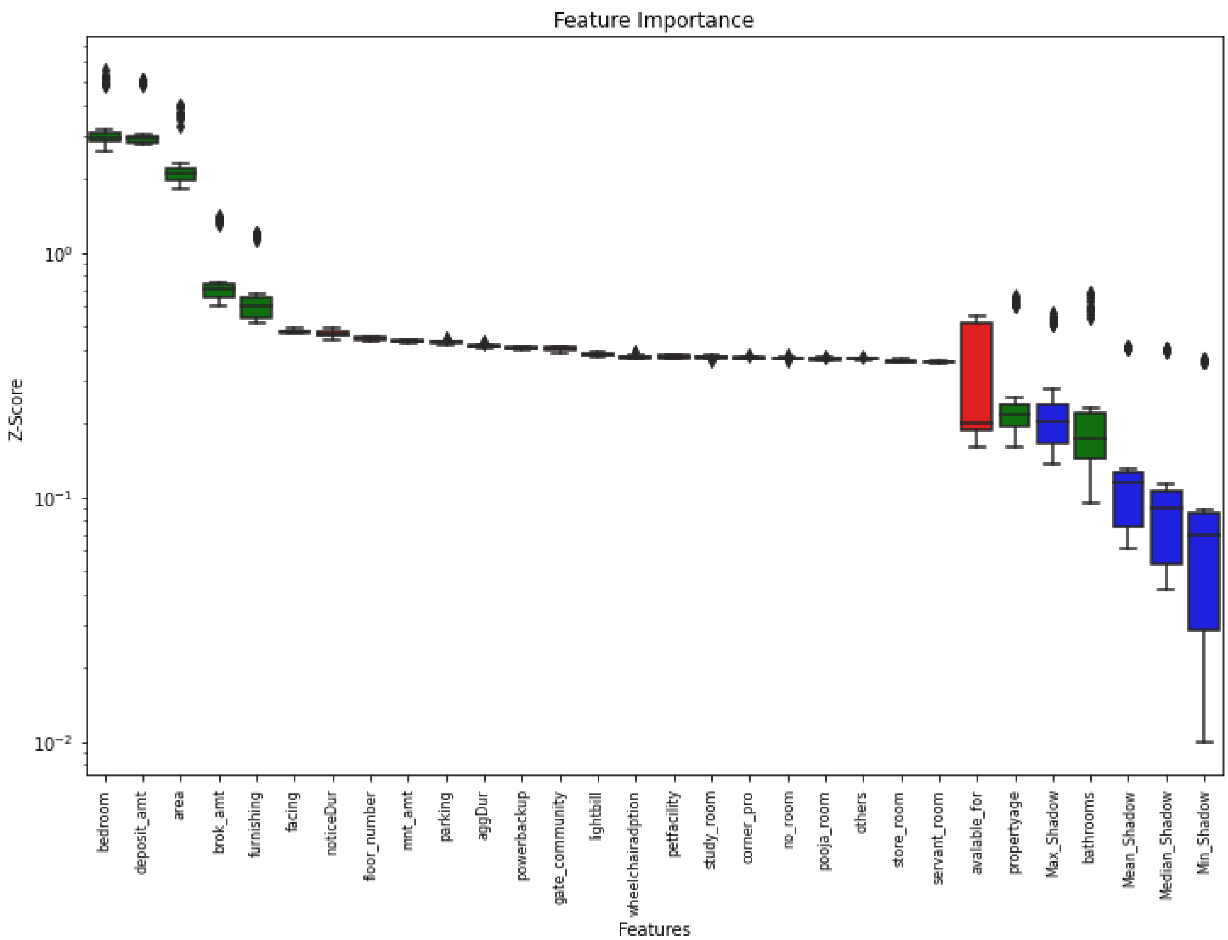
Boruta Algorithm analysis

Based on our Boruta Algorithm analysis, the top 10 predictors are bedroom, deposit_amt, area, brok_amt, furnishing, facing, noticeDur, floor_number, mnt_amt,parking. Moreover, according to the level of importance, we choose all of the above predicators for our model.

In [7]:
```
from BorutaShap import BorutaShap
boruta_data = df.copy()
df1 = df.sample(n = 2000).reset_index()
del df1['index']
```

```
df1
x = df1.iloc[:,:-1]
y = df1.iloc[:,-1]
Feature_Selector = BorutaShap(importance_measure='shap', classification=False)
Feature_Selector.fit(X=x, y=y, n_trials=50, random_state=0)
Feature_Selector.plot(which_features='all')
```

```
  0%|          | 0/50 [00:00<?, ?it/s]
7 attributes confirmed important: ['brok_amt', 'furnishing', 'bathrooms', 'pro
pertyage', 'bedroom', 'deposit_amt', 'area']
19 attributes confirmed unimportant: ['parking', 'powerbackup', 'others', 'cor
ner_pro', 'lightbill', 'petfacility', 'facing', 'no_room', 'gate_community',
'aggDur', 'avalable_for', 'noticeDur', 'floor_number', 'wheelchairadption', 's
ervant_room', 'store_room', 'study_room', 'mnt_amt', 'pooja_room']
0 tentative attributes remains: []
```



After using Boruta Algorithm shown above, we choose top 7 variables: bedroom, deposit_amt, brok_amt, area, bathrooms, furnishing, propertyage as predictors to construct the further model. The details of each predictor are showing below including Density plot, Q-Q Plot, Boxplot, and Scattered Plot respectively, also the Correlation Plots among these variables.

Descriptive Analysis

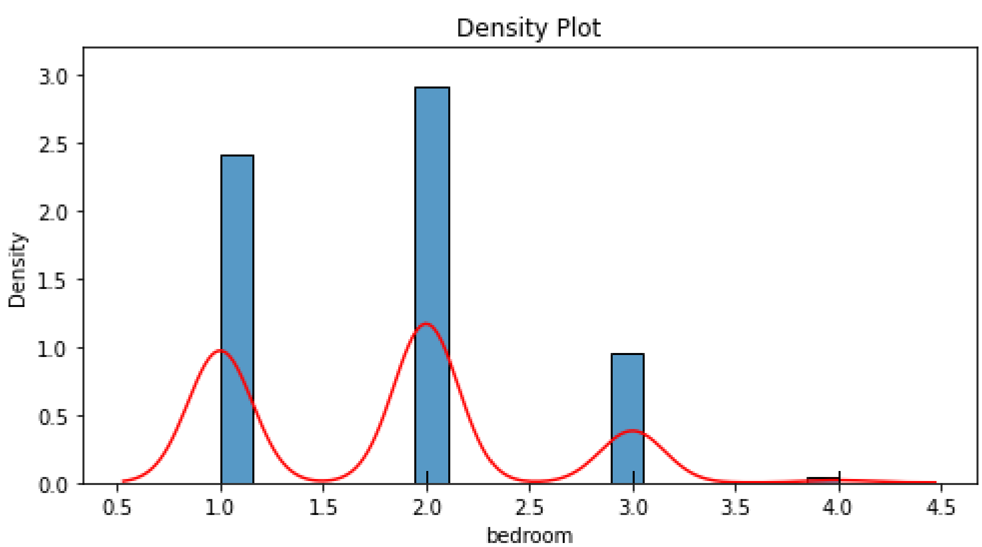
We have done the transformation to the following variables in the previous file ('project2-eda') we submitted.

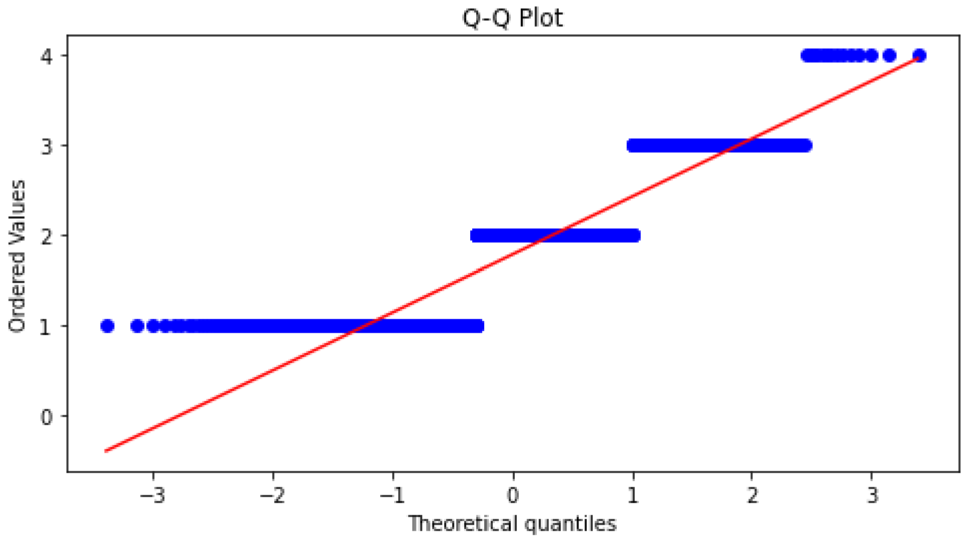
-square root transformation to 'area'

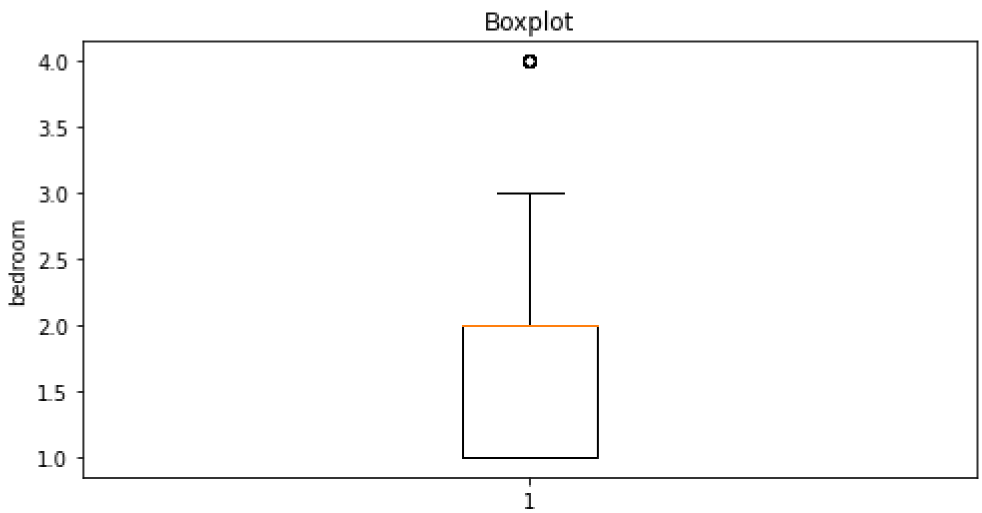
-log transformation to 'rent, deposit_amt, brok_amt'

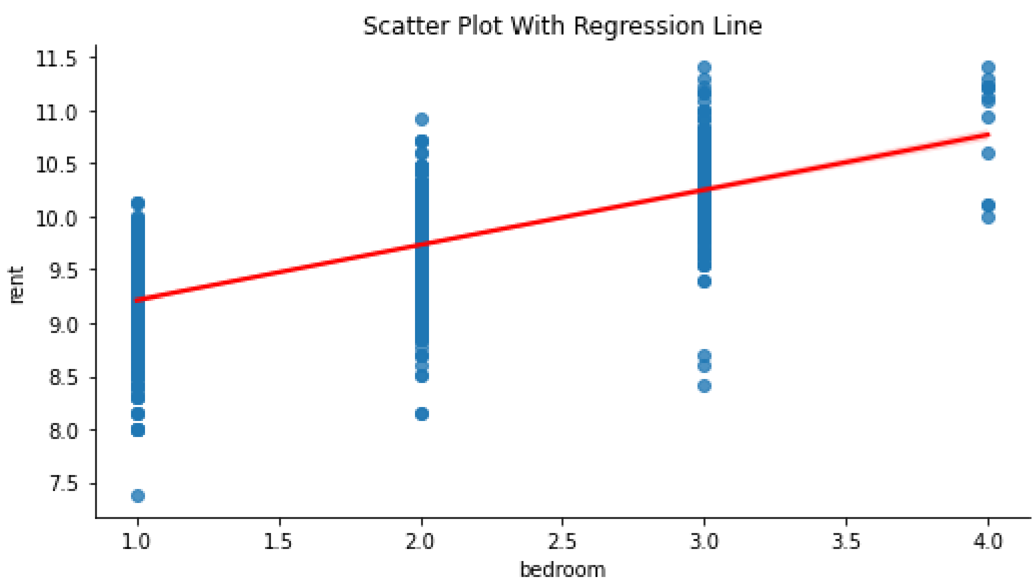
In [9]:
```
df = pd.read_csv('train5.csv')
del df['Unnamed: 0']
```

In [15]:
```
#BEDROOM

#Density Plot
plt.figure(figsize = (8,4))
sns.histplot(df['bedroom'],stat = 'density')
sns.kdeplot(df['bedroom'], color = 'red')
sns.rugplot(df['bedroom'], color = 'black')
plt.title('Density Plot')
plt.show()

#Q-Q Plot
plt.figure(figsize = (8,4))
stats.probplot(df['bedroom'], dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()

#Boxplot
plt.figure(figsize = (8,4))
plt.boxplot(df['bedroom'])
plt.ylabel('bedroom')
plt.title('Boxplot')
plt.show()

#Scatter plot with regression line
sns.lmplot(data=df,x='bedroom',y='rent',line_kws={'color':'red'},height=4,aspec
plt.title('Scatter Plot With Regression Line')
plt.show()
```





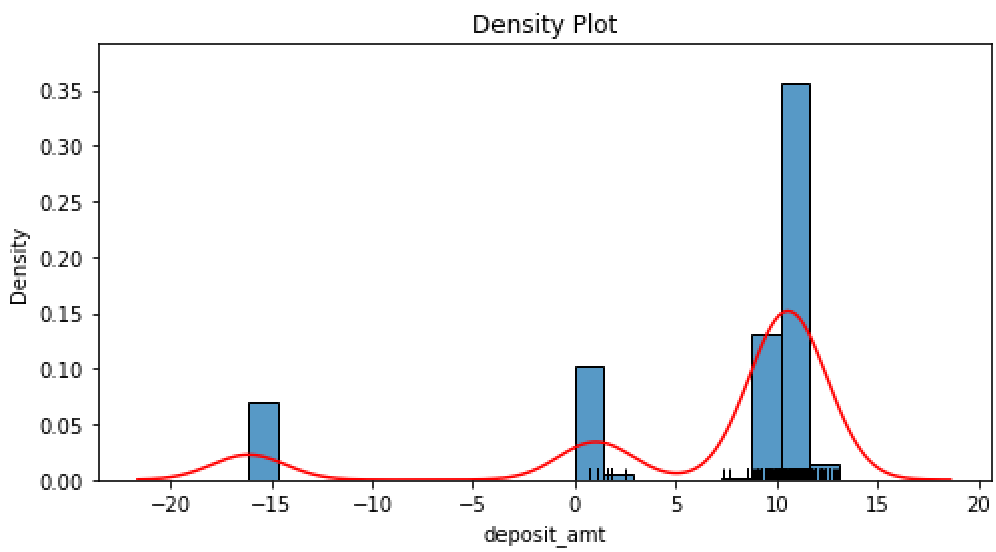


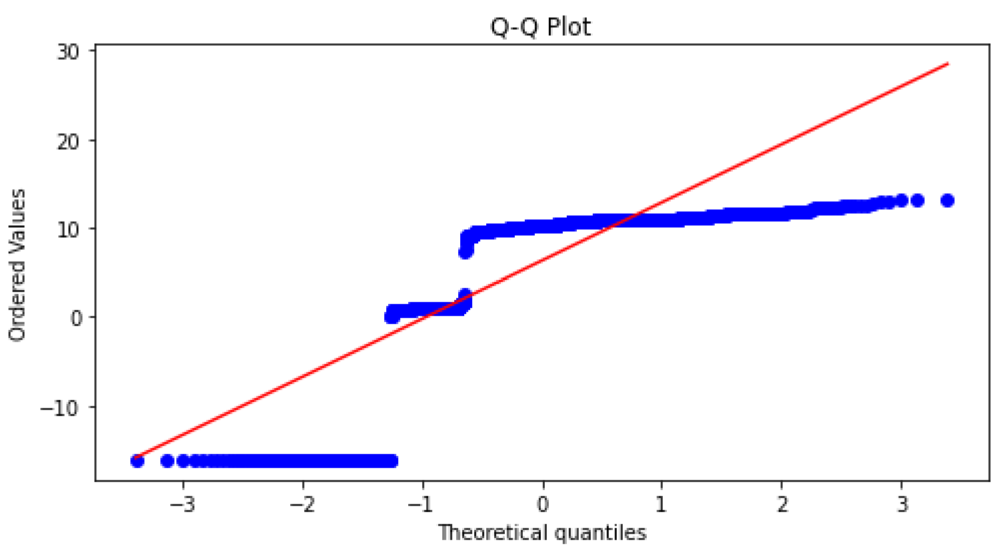


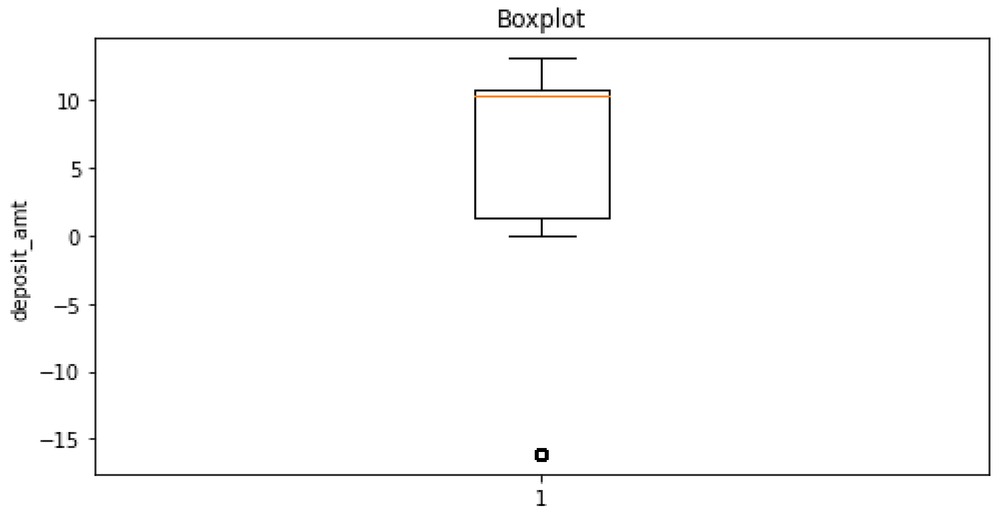
In [14]:

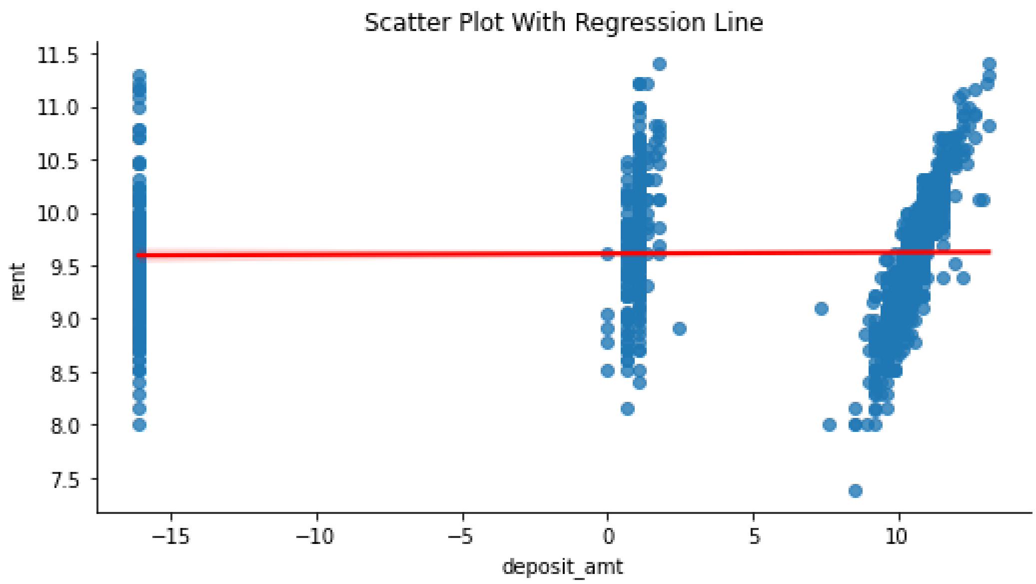
```
#DEPOSIT_AMT

#Density Plot
plt.figure(figsize = (8,4))
sns.histplot(df['deposit_amt'],stat = 'density')
sns.kdeplot(df['deposit_amt'], color = 'red')
sns.rugplot(df['deposit_amt'], color = 'black')
```

```python
plt.title('Density Plot')
plt.show()

#Q-Q Plot
plt.figure(figsize = (8,4))
stats.probplot(df['deposit_amt'], dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()

#Boxplot
plt.figure(figsize = (8,4))
plt.boxplot(df['deposit_amt'])
plt.ylabel('deposit_amt')
plt.title('Boxplot')
plt.show()

#Scatter plot with regression line
sns.lmplot(data=df,x='deposit_amt',y='rent',line_kws={'color':'red'},height=4,a
plt.title('Scatter Plot With Regression Line')
plt.show()
```



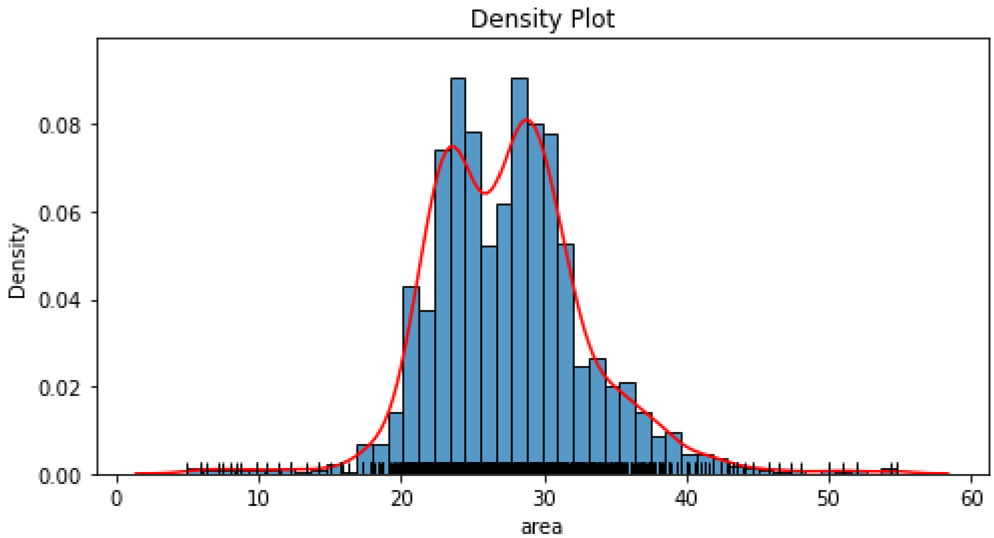


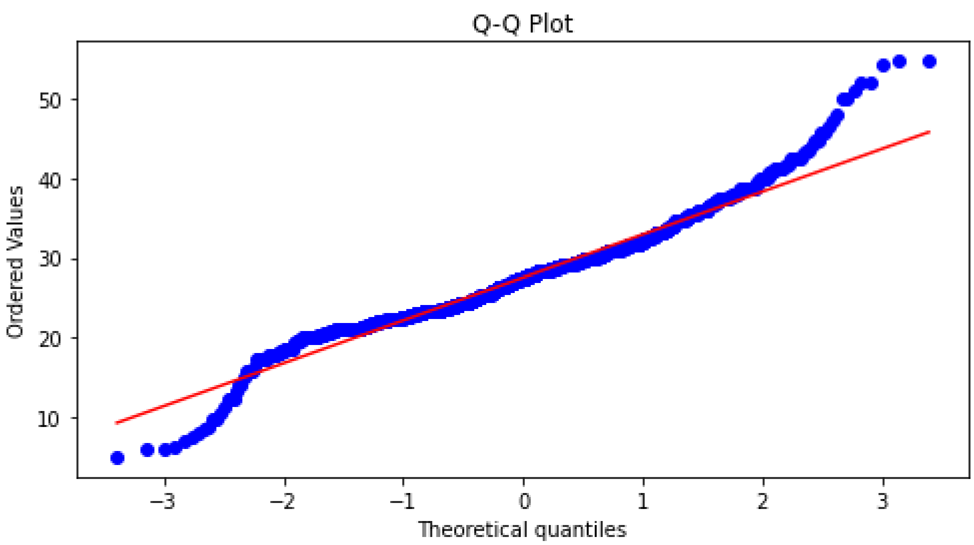


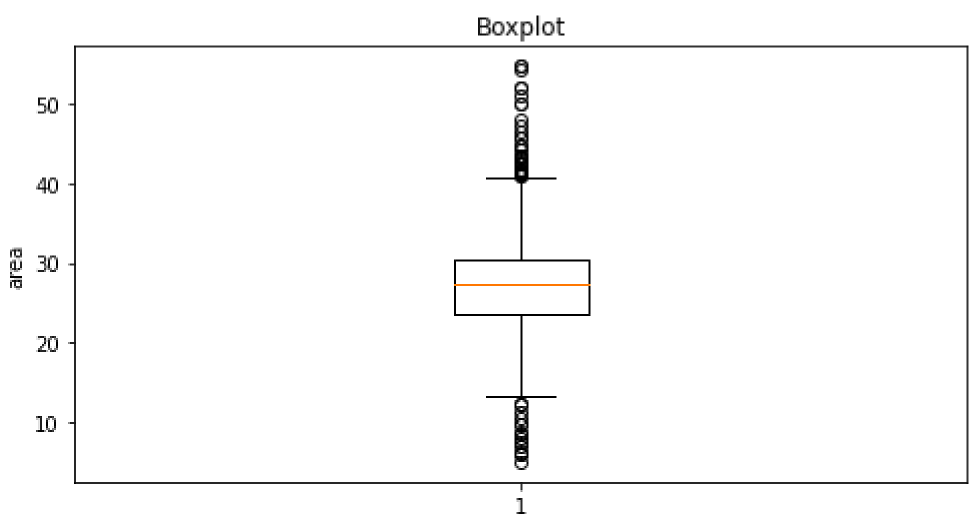


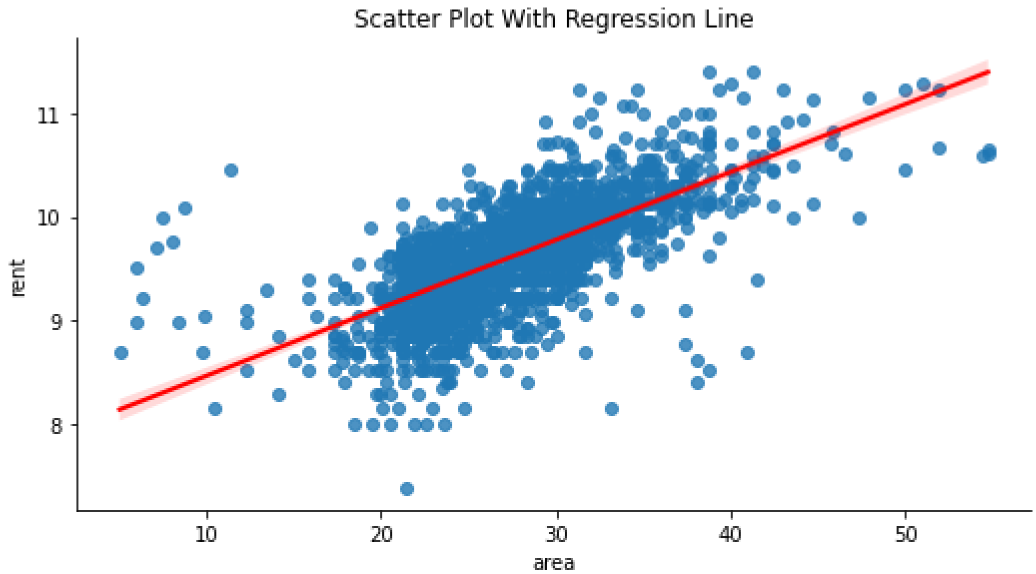
In [17]:
```
# AREA

#Density Plot
plt.figure(figsize = (8,4))
sns.histplot(df['area'],stat = 'density')
sns.kdeplot(df['area'], color = 'red')
sns.rugplot(df['area'], color = 'black')
plt.title('Density Plot')
plt.show()

#Q-Q Plot
plt.figure(figsize = (8,4))
stats.probplot(df['area'], dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()

#Boxplot
plt.figure(figsize = (8,4))
plt.boxplot(df['area'])
plt.ylabel('area')
plt.title('Boxplot')
plt.show()

#Scatter plot with regression line
```

```
sns.lmplot(data=df,x='area',y='rent',line_kws={'color':'red'},height=4,aspect=
plt.title('Scatter Plot With Regression Line')
plt.show()
```



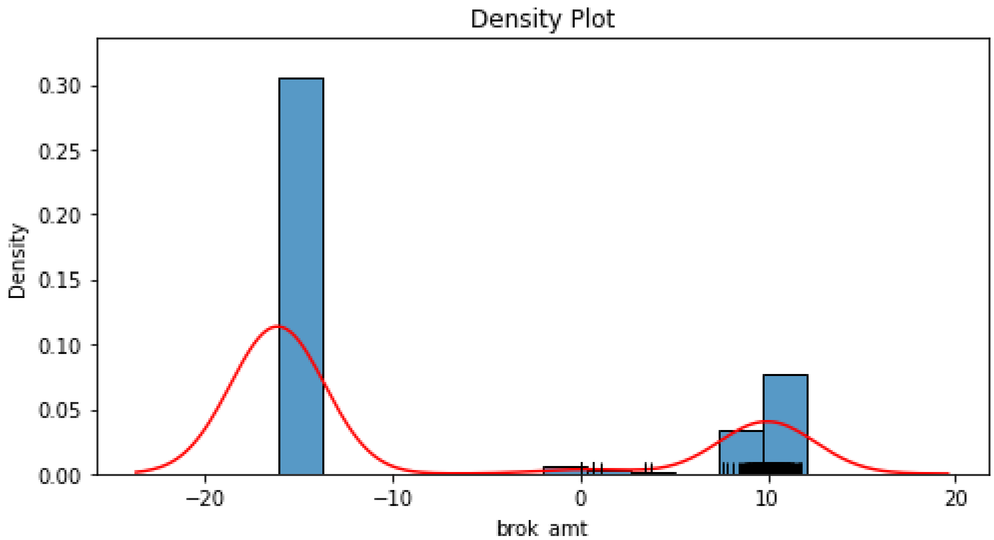


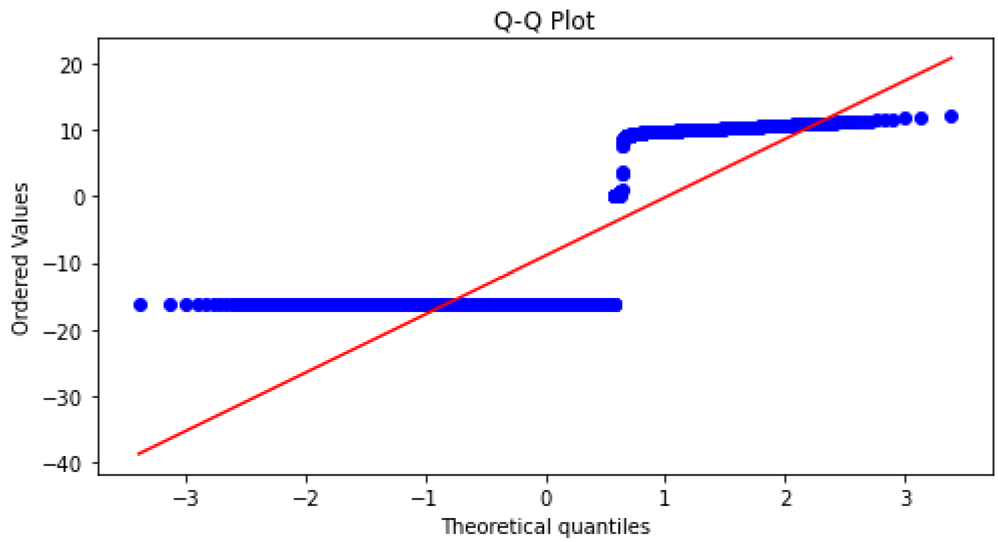


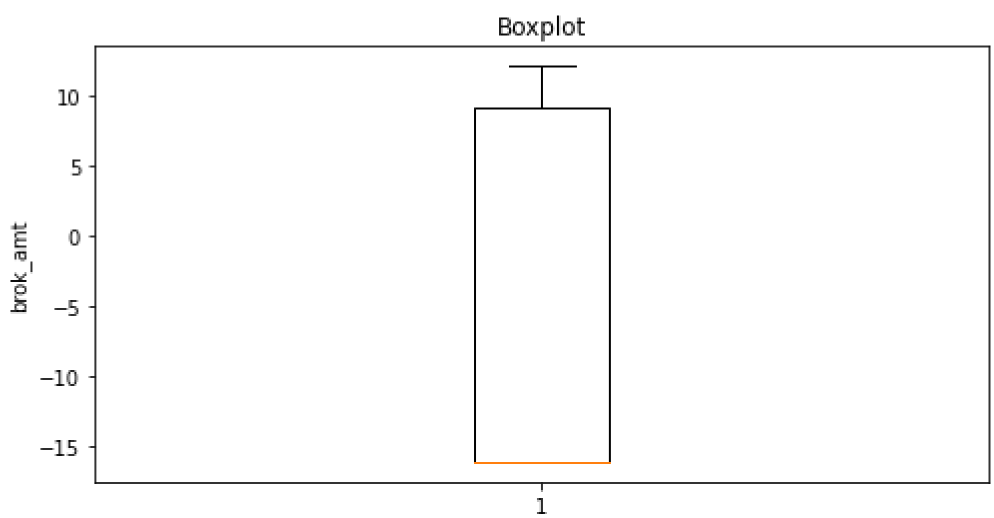


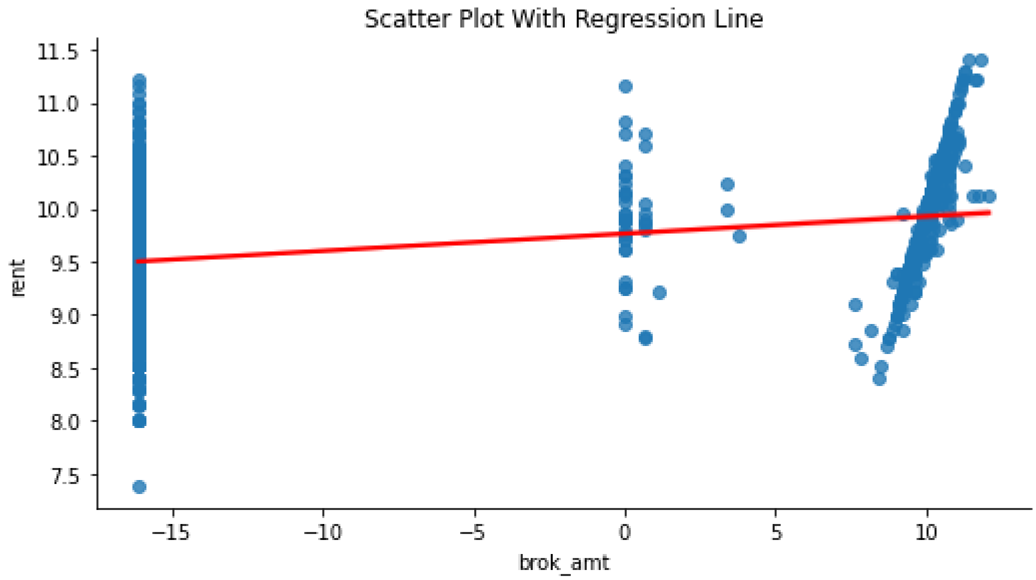
In [18]:

```
#BROK_AMT

#Density Plot
plt.figure(figsize = (8,4))
sns.histplot(df['brok_amt'],stat = 'density')
sns.kdeplot(df['brok_amt'], color = 'red')
sns.rugplot(df['brok_amt'], color = 'black')
plt.title('Density Plot')
plt.show()

#Q-Q Plot
plt.figure(figsize = (8,4))
stats.probplot(df['brok_amt'], dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()

#Boxplot
plt.figure(figsize = (8,4))
plt.boxplot(df['brok_amt'])
plt.ylabel('brok_amt')
plt.title('Boxplot')
plt.show()

#Scatter plot with regression line
sns.lmplot(data=df,x='brok_amt',y='rent',line_kws={'color':'red'},height=4,aspe
plt.title('Scatter Plot With Regression Line')
plt.show()
```


Density Plot
Density
brok_amt
0.00
0.05
0.10
0.15
0.20
0.25
0.30
−20
−10
0
10
20

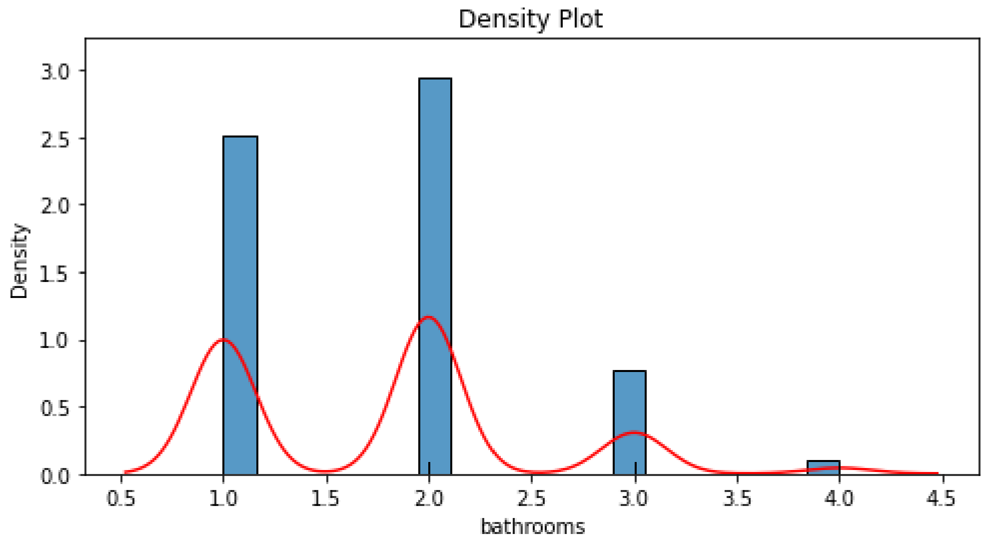

Q-Q Plot
Ordered Values
Theoretical quantiles
20
10
0
−10
−20
−30
−40
−3
−2
−1
0
1
2
3

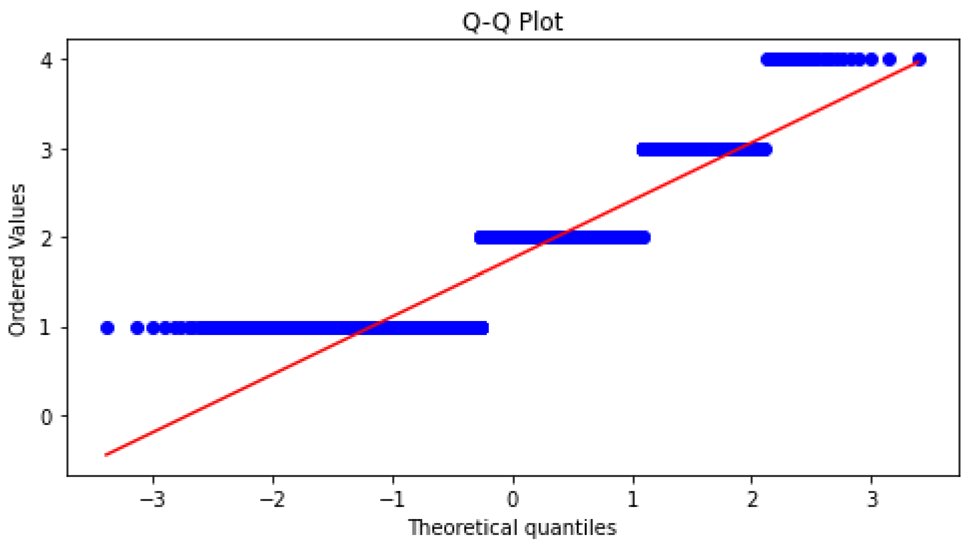

Boxplot
brok_amt
10
5
0
−5
−10
−15
1

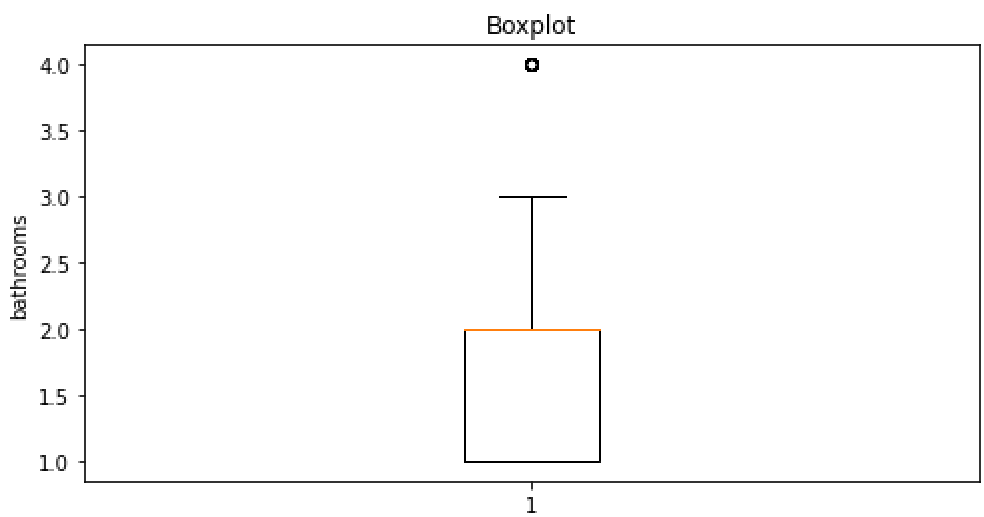


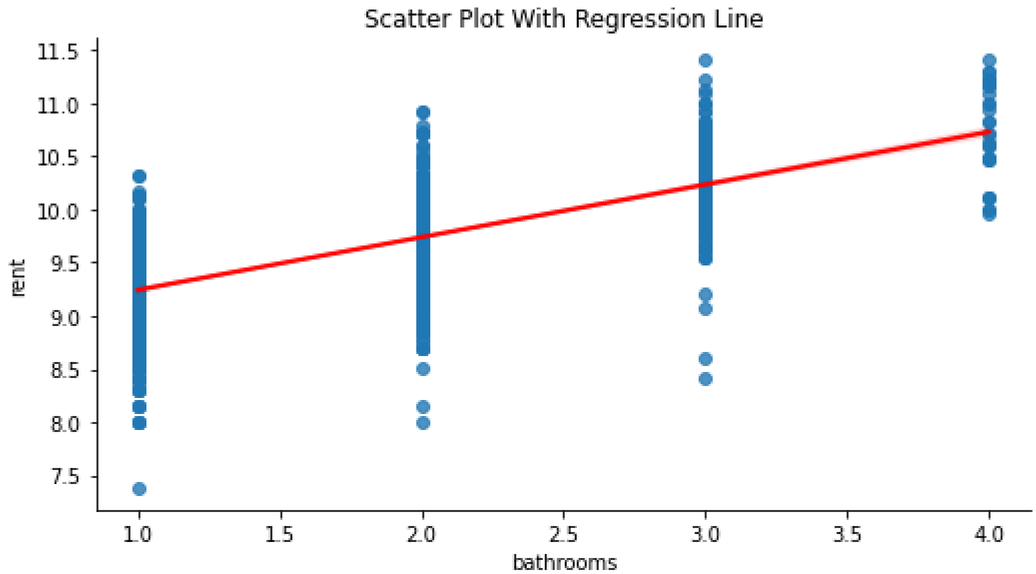
In [19]:

```
#BATHROOM

#Density Plot
plt.figure(figsize = (8,4))
sns.histplot(df['bathrooms'],stat = 'density')
sns.kdeplot(df['bathrooms'], color = 'red')
sns.rugplot(df['bathrooms'], color = 'black')
plt.title('Density Plot')
plt.show()

#Q-Q Plot
plt.figure(figsize = (8,4))
stats.probplot(df['bathrooms'], dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()

#Boxplot
plt.figure(figsize = (8,4))
plt.boxplot(df['bathrooms'])
plt.ylabel('bathrooms')
plt.title('Boxplot')
plt.show()

#Scatter plot with regression line
sns.lmplot(data=df,x='bathrooms',y='rent',line_kws={'color':'red'},height=4,asp
plt.title('Scatter Plot With Regression Line')
plt.show()
```


Density Plot
Density
bathrooms
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

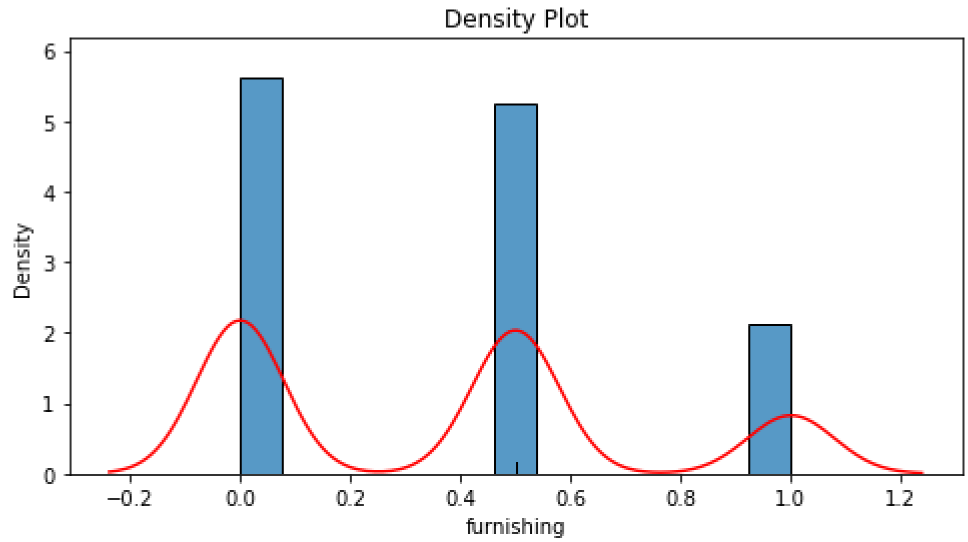

Q-Q Plot
Ordered Values
Theoretical quantiles
−3
−2
−1
0
1
2
3
4

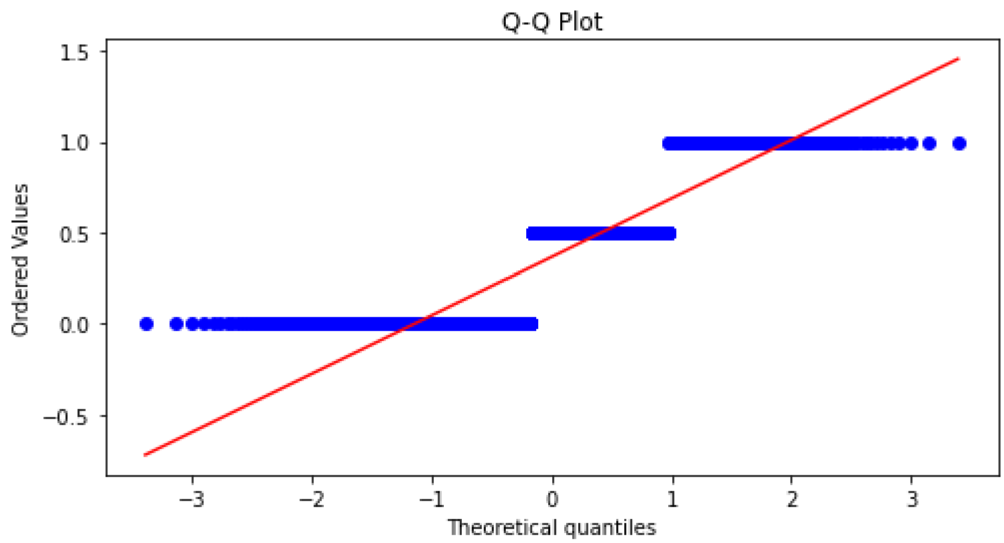

Boxplot
bathrooms
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1

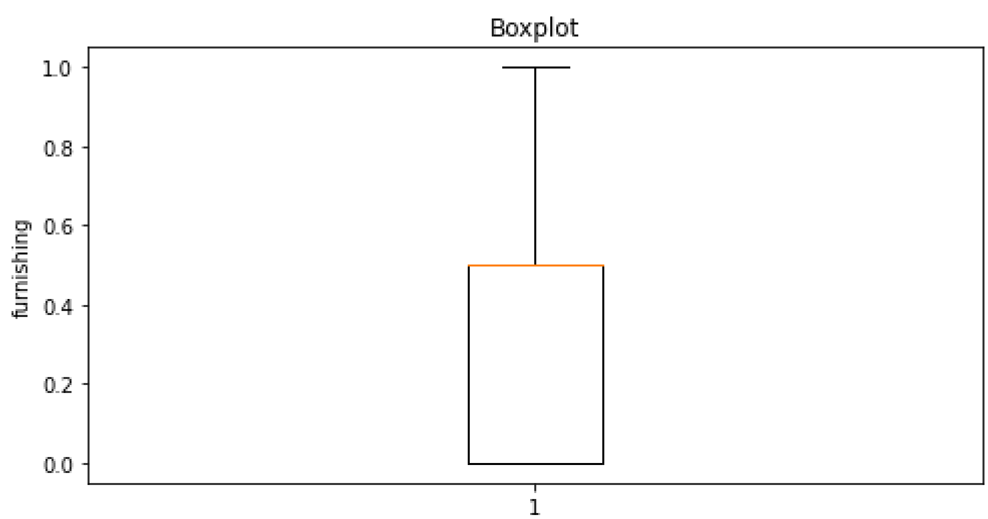


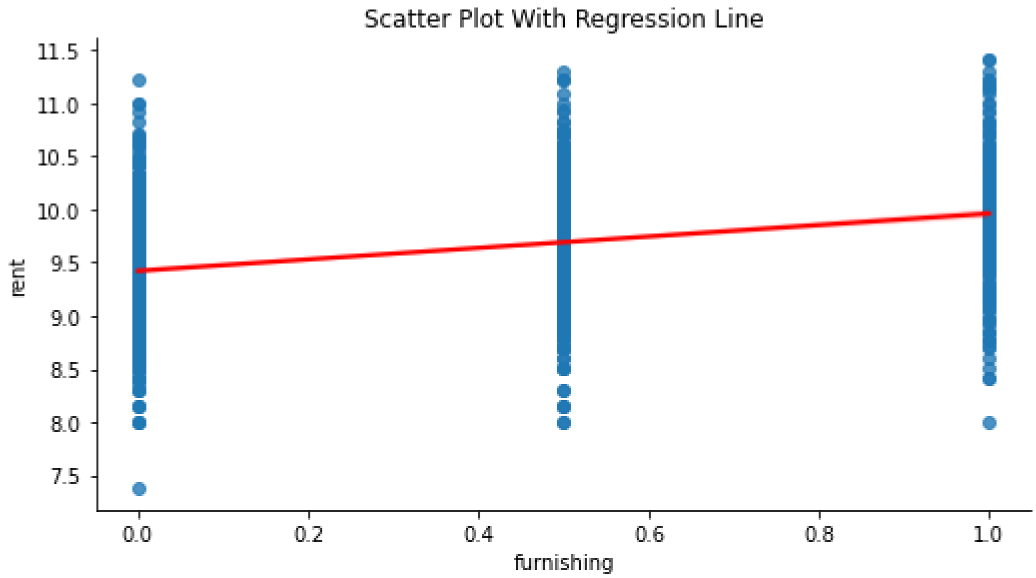
In [20]:

```
#FURNISHING

#Density Plot
plt.figure(figsize = (8,4))
sns.histplot(df['furnishing'],stat = 'density')
sns.kdeplot(df['furnishing'], color = 'red')
sns.rugplot(df['furnishing'], color = 'black')
plt.title('Density Plot')
plt.show()

#Q-Q Plot
plt.figure(figsize = (8,4))
stats.probplot(df['furnishing'], dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()

#Boxplot
plt.figure(figsize = (8,4))
plt.boxplot(df['furnishing'])
plt.ylabel('furnishing')
plt.title('Boxplot')
plt.show()

#Scatter plot with regression line
sns.lmplot(data=df,x='furnishing',y='rent',line_kws={'color':'red'},height=4,as
plt.title('Scatter Plot With Regression Line')
plt.show()
```


Density Plot
Density
furnishing
0
1
2
3
4
5
6
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2

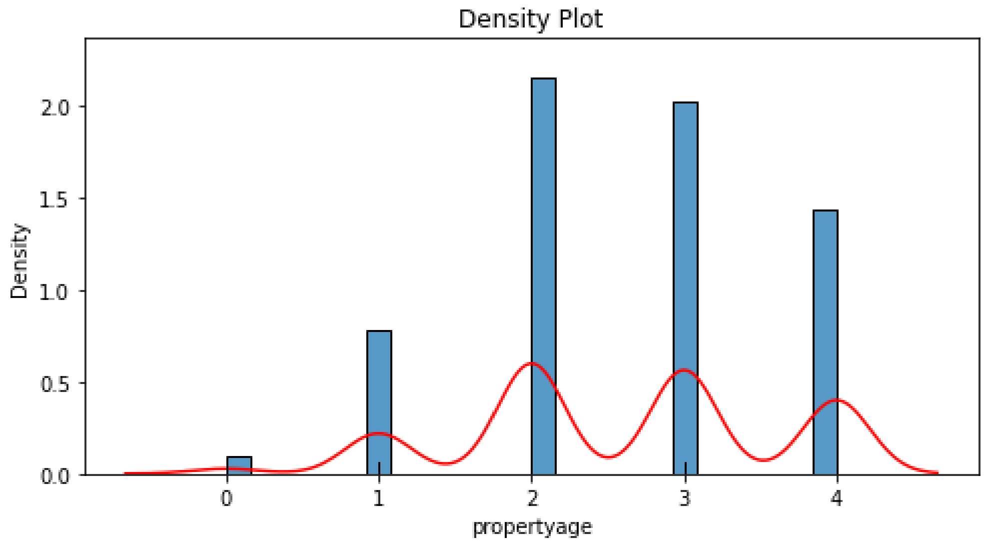

Q-Q Plot
Ordered Values
Theoretical quantiles
1.5
1.0
0.5
0.0
−0.5
−3
−2
−1
0
1
2
3

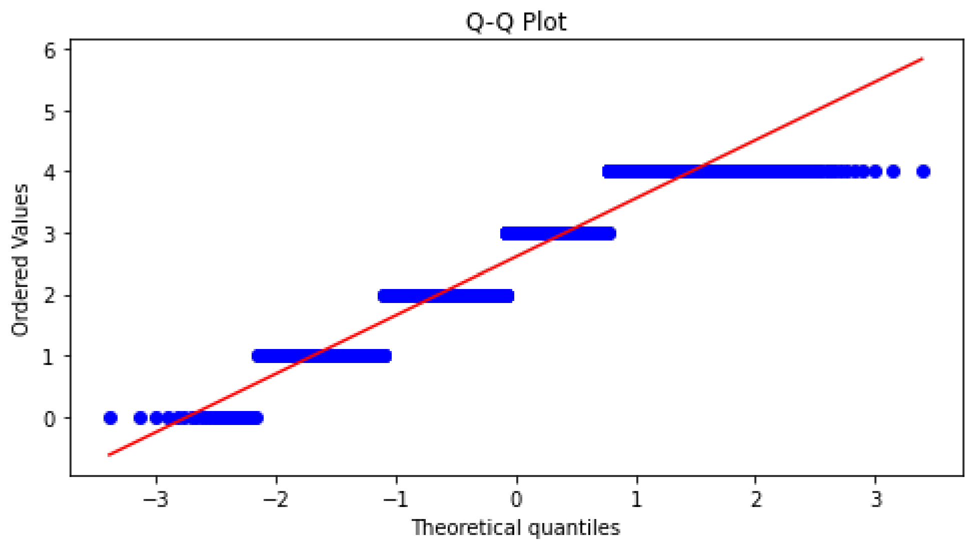

Boxplot
furnishing
1.0
0.8
0.6
0.4
0.2
0.0
1

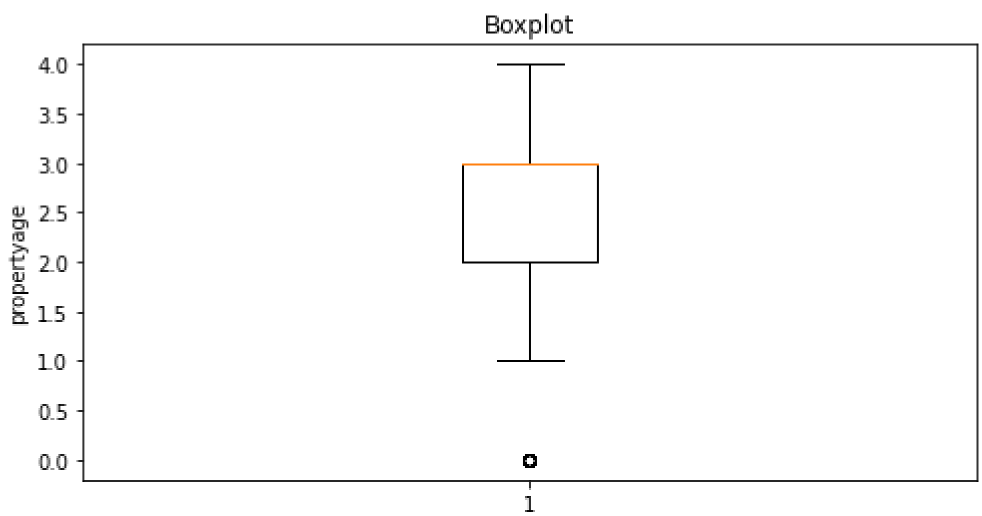


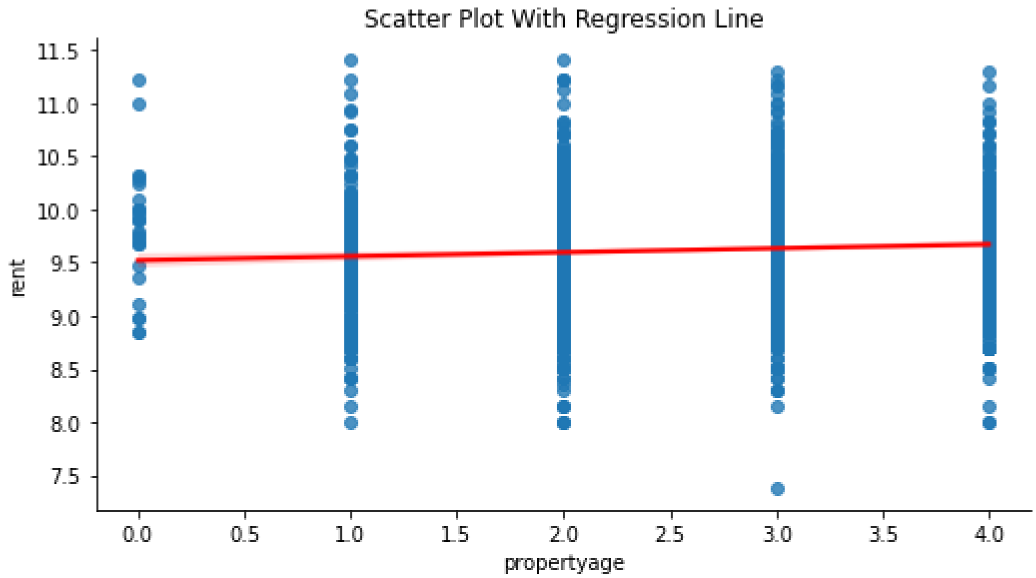
In [21]:

```
#PROPERTYAGE

#Density Plot
plt.figure(figsize = (8,4))
sns.histplot(df['propertyage'],stat = 'density')
sns.kdeplot(df['propertyage'], color = 'red')
sns.rugplot(df['propertyage'], color = 'black')
plt.title('Density Plot')
plt.show()

#Q-Q Plot
plt.figure(figsize = (8,4))
stats.probplot(df['propertyage'], dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()

#Boxplot
plt.figure(figsize = (8,4))
plt.boxplot(df['propertyage'])
plt.ylabel('propertyage')
plt.title('Boxplot')
plt.show()

#Scatter plot with regression line
sns.lmplot(data=df,x='propertyage',y='rent',line_kws={'color':'red'},height=4,a
plt.title('Scatter Plot With Regression Line')
plt.show()
```

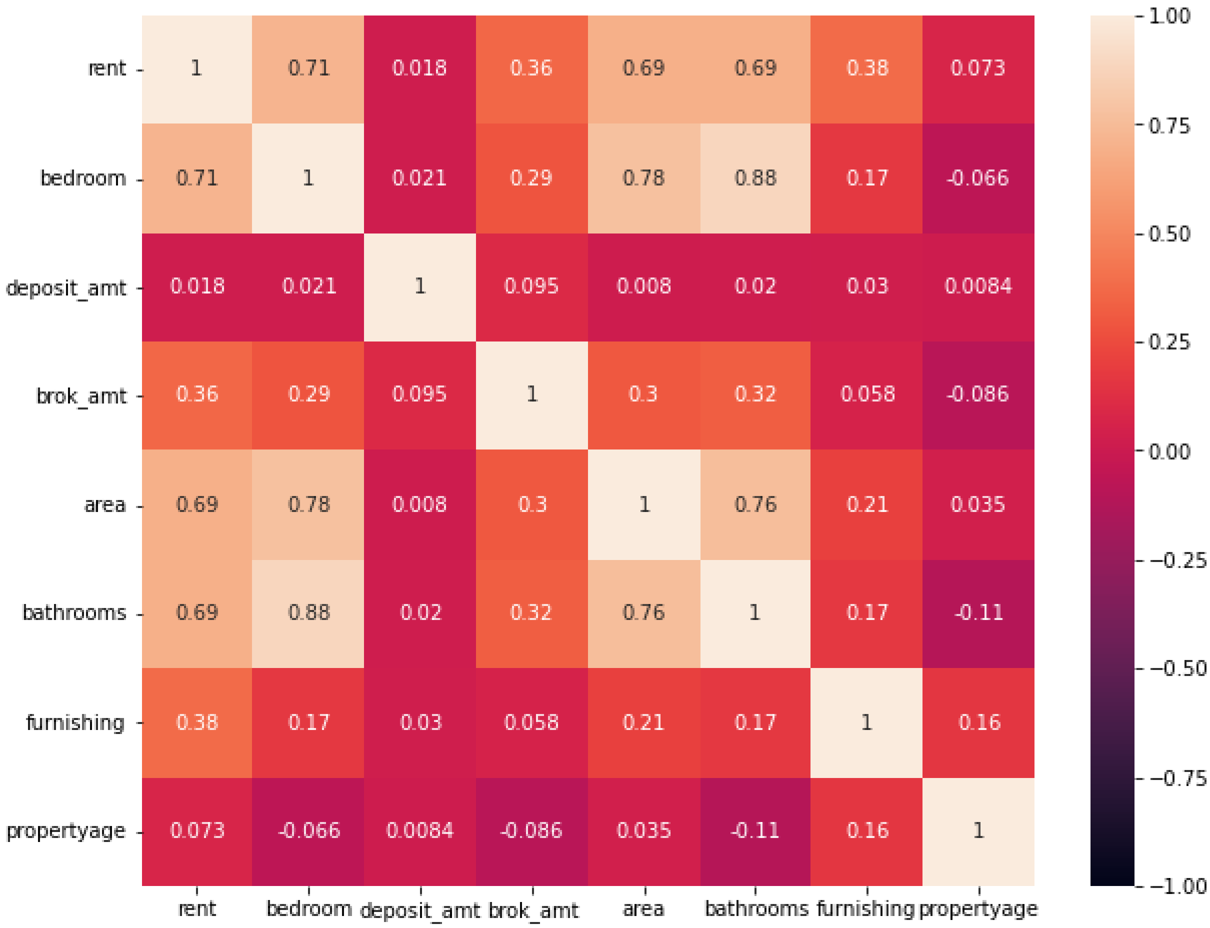

Density Plot
Density
propertyage
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4


Q-Q Plot
Ordered Values
Theoretical quantiles
0
1
2
3
4
5
6
−3
−2
−1
0
1
2
3


Boxplot
propertyage
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1



In [22]:
```
#CORRELATION PLOT

plt.figure(figsize = (10,8))
heatmap = sns.heatmap(df[['rent','bedroom','deposit_amt','brok_amt',
                          'area','bathrooms','furnishing','propertyage']].cor
                      vmin=-1, vmax=1, annot=True)
plt.show()
```



Model Building

In [23]:
```python
# test for Multicollinearity
import patsy as pt
import statsmodels.stats.outliers_influence as smo
y, X = pt.dmatrices('rent ~ bedroom+deposit_amt+brok_amt+area+bathrooms+furnish
                    return_type = 'dataframe')

k = X.shape[1]

# create an empty matrix to store results
VIF1 = np.empty(k)

# Loop for each regressor (+ intercept)
for i in range(k):
    # calculate the VIF for each
    VIF1[i] = smo.variance_inflation_factor(X.values, i)

print('VIF for model1:', VIF1)
```

```
VIF for model1: [41.11433843  5.21840762  1.01096242  1.14353907  2.83801103
5.00303716
  1.07450088  1.0794827 ]
```

In [24]:
```python
y2, X2 = pt.dmatrices('rent ~ bedroom+deposit_amt+brok_amt+area+furnishing+prop
                    return_type = 'dataframe')

k2 = X2.shape[1]

# create an empty matrix to store results
VIF2 = np.empty(k2)

# Loop for each regressor (+ intercept)
for i in range(k2):
    # calculate the VIF for each
    VIF2[i] = smo.variance_inflation_factor(X2.values, i)

print('VIF for model2:', VIF2)
```

```
VIF for model2: [41.07624503  2.61677736  1.01094743  1.12920175  2.67617032
1.07328379
  1.05923593]
```

After having 7 preferred variables, we firstly test the Multicollinearity. By observing VIF of these variables, we decided to drop 'bathrooms' with high VIF-value and low t-statistics. After droping this variable, we retest the Multicollinearity of new model and found that the VIFs becoming better.

In [25]:
```python
results8 = smf.ols('rent ~ bedroom+deposit_amt+brok_amt+area+furnishing+propert
results8.summary()
```

Out [25]:

OLS Regression Results

Dep. Variable:	rent	**R-squared:**	0.634
Model:	OLS	**Adj. R-squared:**	0.633
Method:	Least Squares	**F-statistic:**	574.6
Date:	Mon, 14 Nov 2022	**Prob (F-statistic):**	0.00
Time:	21:37:37	**Log-Likelihood:**	-526.12
No. Observations:	2000	**AIC:**	1066.
Df Residuals:	1993	**BIC:**	1105.
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	8.2449	0.045	182.448	0.000	8.156	8.334
bedroom	0.3206	0.016	20.130	0.000	0.289	0.352
deposit_amt	-0.0010	0.001	-1.133	0.257	-0.003	0.001
brok_amt	0.0069	0.001	10.586	0.000	0.006	0.008
area	0.0237	0.002	11.267	0.000	0.020	0.028
furnishing	0.3298	0.020	16.345	0.000	0.290	0.369
propertyage	0.0362	0.007	5.028	0.000	0.022	0.050

Omnibus:	337.309	**Durbin-Watson:**	2.017
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1117.279
Skew:	-0.831	**Prob(JB):**	2.43e-243
Kurtosis:	6.263	**Cond. No.**	196.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

According to the regression result of new model above, we found that 'deposit_amt' is not significant, so we dropped this variable from the model.

In [27]:

```
# Test for model misspecification
results = smf.ols('rent ~ bedroom+brok_amt+area+furnishing+propertyage', df).f:
reset_out = smo.reset_ramsey(res = results, degree = 3)
reset_out
```

Out [27]:

```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=1.4690924211297869, p=0.2303834843430593, df_denom=1.99e+03, df_num
=2>
```

According to misspecification test result, it is clearly shown that we have large p-value so that this model do not need interaction term.

In [29]:
```
# Test for heteroskedasticity

y_model, model = pt.dmatrices('rent ~ bedroom+brok_amt+area+furnishing+property
                    return_type = 'dataframe')

sm.stats.diagnostic.het_white(results.resid, model)
```

Out[29]:
```
(89.06058502000613,
 1.0815402216569976e-10,
 4.611629661645695,
 5.925495423701455e-11)
```

In [31]:
```
df['resid'] =  results.resid**2

# estimate weights
w_est = smf.ols('np.log(resid) ~ bedroom+brok_amt+area+furnishing+propertyage',

vari = np.exp(w_est.fittedvalues) #estimated variances
w = 1/vari**2

fgls =smf.wls('rent ~ bedroom+brok_amt+area+furnishing+propertyage', df, weight

print(fgls.summary())

results_model_modify = smf.ols('rent ~ bedroom+brok_amt+area+furnishing+propert
                                df).fit(cov_type = 'HC0')
sm.stats.diagnostic.het_white(results_model_modify.resid, model)
```

```
                            WLS Regression Results
==============================================================================
Dep. Variable:                   rent   R-squared:                       0.714
Model:                            WLS   Adj. R-squared:                  0.713
Method:                 Least Squares   F-statistic:                     993.3
Date:                Mon, 14 Nov 2022   Prob (F-statistic):               0.00
Time:                        21:44:08   Log-Likelihood:                -571.11
No. Observations:                2000   AIC:                             1154.
Df Residuals:                    1994   BIC:                             1188.
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
=
                 coef    std err          t      P>|t|      [0.025      0.97
5]
------------------------------------------------------------------------------
-
Intercept      8.2751      0.036    230.840      0.000       8.205       8.34
5
bedroom        0.3284      0.015     22.273      0.000       0.299       0.35
7
brok_amt       0.0073      0.001     13.058      0.000       0.006       0.00
8
area           0.0225      0.002     14.037      0.000       0.019       0.02
6
furnishing     0.3271      0.019     17.324      0.000       0.290       0.36
4
propertyage    0.0314      0.007      4.561      0.000       0.018       0.04
5
==============================================================================
Omnibus:                      305.942   Durbin-Watson:                   2.015
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1318.417
Skew:                          -0.675   Prob(JB):                    5.12e-287
Kurtosis:                       6.742   Cond. No.                         169.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.
```

Out[31]:

```
(89.06058502000613,
 1.0815402216569976e-10,
 4.611629661645695,
 5.925495423701455e-11)
```

According to WHITE TEST, we found heteroskedasticity, so we performed FGLS and HC0 to correct it. However, due to the specification of discrete sample, these two methods can not fix it.

MODEL SELECTION

In [32]:

```python
# Mallow's CP

subdat = df[['rent','bedroom','brok_amt','area','furnishing','propertyage']].co
model = smf.ols(formula='rent ~bedroom+brok_amt+area+furnishing+propertyage', d
results = model.fit()
y = df['rent']
y_pred = results.fittedvalues
```

```python
# You need to run each sub regression individually, and get the score for each
# Using subset size =1
storage_cp = pd.DataFrame(columns = ["Variables", "CP"])
k = 6 # number of parameters in orginal model (includes y-intercept)

for L in range (1, len(subdat.columns[1:])+1):
    for subset in itertools.combinations(subdat.columns[1:], L):
        formula1 = 'rent~'+'+'.join(subset)
        mr_sub = smf.ols(formula=formula1, data=df)
        mr_sub_fit = mr_sub.fit()
        y_sub=mr_sub_fit.fittedvalues
        p = len(subset)+1 # number of parameters in the subset model (includes
        cp = mallow.mallow(y, y_pred,y_sub, k, p)
        storage_cp = storage_cp.append({'Variables': subset, 'CP': cp}, ignore_
print(storage_cp.sort_values(by = "CP"))
```

```
                                             Variables           CP
30  (bedroom, brok_amt, area, furnishing, property...          6.0
25            (bedroom, brok_amt, area, furnishing)    29.099132
28         (bedroom, area, furnishing, propertyage)   114.771124
18                     (bedroom, area, furnishing)   129.402203
27     (bedroom, brok_amt, furnishing, propertyage)   131.851804
16                 (bedroom, brok_amt, furnishing)   172.746469
26           (bedroom, brok_amt, area, propertyage)   270.332618
20              (bedroom, furnishing, propertyage)   279.968494
7                            (bedroom, furnishing)   310.136475
15                       (bedroom, brok_amt, area)    327.33428
19                    (bedroom, area, propertyage)   381.645389
29        (brok_amt, area, furnishing, propertyage)   408.497444
21                    (brok_amt, area, furnishing)   410.926006
6                                  (bedroom, area)   425.205123
17                (bedroom, brok_amt, propertyage)   438.409626
5                              (bedroom, brok_amt)   531.402327
12                              (area, furnishing)   549.307724
24                 (area, furnishing, propertyage)   550.522857
8                           (bedroom, propertyage)    596.29314
0                                       (bedroom,)   673.180348
22                 (brok_amt, area, propertyage)     695.181897
9                                 (brok_amt, area)   716.136965
13                             (area, propertyage)   841.218467
2                                          (area,)   851.984847
23            (brok_amt, furnishing, propertyage)   2039.826967
10                          (brok_amt, furnishing)   2049.580598
3                                    (furnishing,)    2676.0987
14                       (furnishing, propertyage)   2677.117592
11                         (brok_amt, propertyage)   2677.902852
1                                      (brok_amt,)   2735.020114
4                                     (propertyage,)  3415.163505
```

In [33]:

```python
print(storage_cp.sort_values(by = "CP").iloc[0,0])
print(storage_cp.sort_values(by = "CP").iloc[1,0])
print(storage_cp.sort_values(by = "CP").iloc[2,0])
```

```
('bedroom', 'brok_amt', 'area', 'furnishing', 'propertyage')
('bedroom', 'brok_amt', 'area', 'furnishing')
('bedroom', 'area', 'furnishing', 'propertyage')
```

To get the best model, we calculated all possible sub-models using Mallow's CP and chose the top 3 best models to perform further tests.

In [34]:
```python
# REGRESSION COMPARISON
results_model1 = smf.ols('rent ~ bedroom+brok_amt+area+furnishing+propertyage',
results_model1.summary()
```

Out[34]:

OLS Regression Results

Dep. Variable:	rent	**R-squared:**	0.633
Model:	OLS	**Adj. R-squared:**	0.633
Method:	Least Squares	**F-statistic:**	689.2
Date:	Mon, 14 Nov 2022	**Prob (F-statistic):**	0.00
Time:	21:52:42	**Log-Likelihood:**	-526.76
No. Observations:	2000	**AIC:**	1066.
Df Residuals:	1994	**BIC:**	1099.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	8.2373	0.045	184.316	0.000	8.150	8.325
bedroom	0.3203	0.016	20.112	0.000	0.289	0.352
brok_amt	0.0068	0.001	10.525	0.000	0.006	0.008
area	0.0238	0.002	11.307	0.000	0.020	0.028
furnishing	0.3292	0.020	16.320	0.000	0.290	0.369
propertyage	0.0361	0.007	5.010	0.000	0.022	0.050

Omnibus:	334.389	**Durbin-Watson:**	2.016
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1110.104
Skew:	-0.823	**Prob(JB):**	8.79e-242
Kurtosis:	6.257	**Cond. No.**	189.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [35]:
```python
results_model2 = smf.ols('rent ~ bedroom+brok_amt+area+furnishing', df).fit()
results_model2.summary()
```

Out[35]:

OLS Regression Results

Dep. Variable:	rent	**R-squared:**	0.629
Model:	OLS	**Adj. R-squared:**	0.628
Method:	Least Squares	**F-statistic:**	845.0
Date:	Mon, 14 Nov 2022	**Prob (F-statistic):**	0.00
Time:	21:52:49	**Log-Likelihood:**	-539.27
No. Observations:	2000	**AIC:**	1089.
Df Residuals:	1995	**BIC:**	1117.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	8.3054	0.043	193.920	0.000	8.221	8.389
bedroom	0.3087	0.016	19.474	0.000	0.278	0.340
brok_amt	0.0065	0.001	10.054	0.000	0.005	0.008
area	0.0252	0.002	11.996	0.000	0.021	0.029
furnishing	0.3452	0.020	17.224	0.000	0.306	0.384

Omnibus:	333.338	**Durbin-Watson:**	2.034
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1084.164
Skew:	-0.827	**Prob(JB):**	3.77e-236
Kurtosis:	6.205	**Cond. No.**	180.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [37]:

```
results_model3 = smf.ols('rent ~ bedroom+area+furnishing+propertyage', df).fit
results_model3.summary()
```

Out [37]: OLS Regression Results

Dep. Variable:	rent	**R-squared:**	0.613
Model:	OLS	**Adj. R-squared:**	0.612
Method:	Least Squares	**F-statistic:**	790.3
Date:	Mon, 14 Nov 2022	**Prob (F-statistic):**	0.00
Time:	21:53:01	**Log-Likelihood:**	-580.83
No. Observations:	2000	**AIC:**	1172.
Df Residuals:	1995	**BIC:**	1200.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	8.0854	0.043	186.108	0.000	8.000	8.171
bedroom	0.3324	0.016	20.370	0.000	0.300	0.364
area	0.0269	0.002	12.589	0.000	0.023	0.031
furnishing	0.3308	0.021	15.964	0.000	0.290	0.371
propertyage	0.0293	0.007	3.970	0.000	0.015	0.044

Omnibus:	358.979	**Durbin-Watson:**	2.028
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1229.522
Skew:	-0.873	**Prob(JB):**	1.03e-267
Kurtosis:	6.421	**Cond. No.**	172.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

By comparing R-squared, AIC, BIC and JB-Test, we concluded that model 1 is the best model on the current stage.

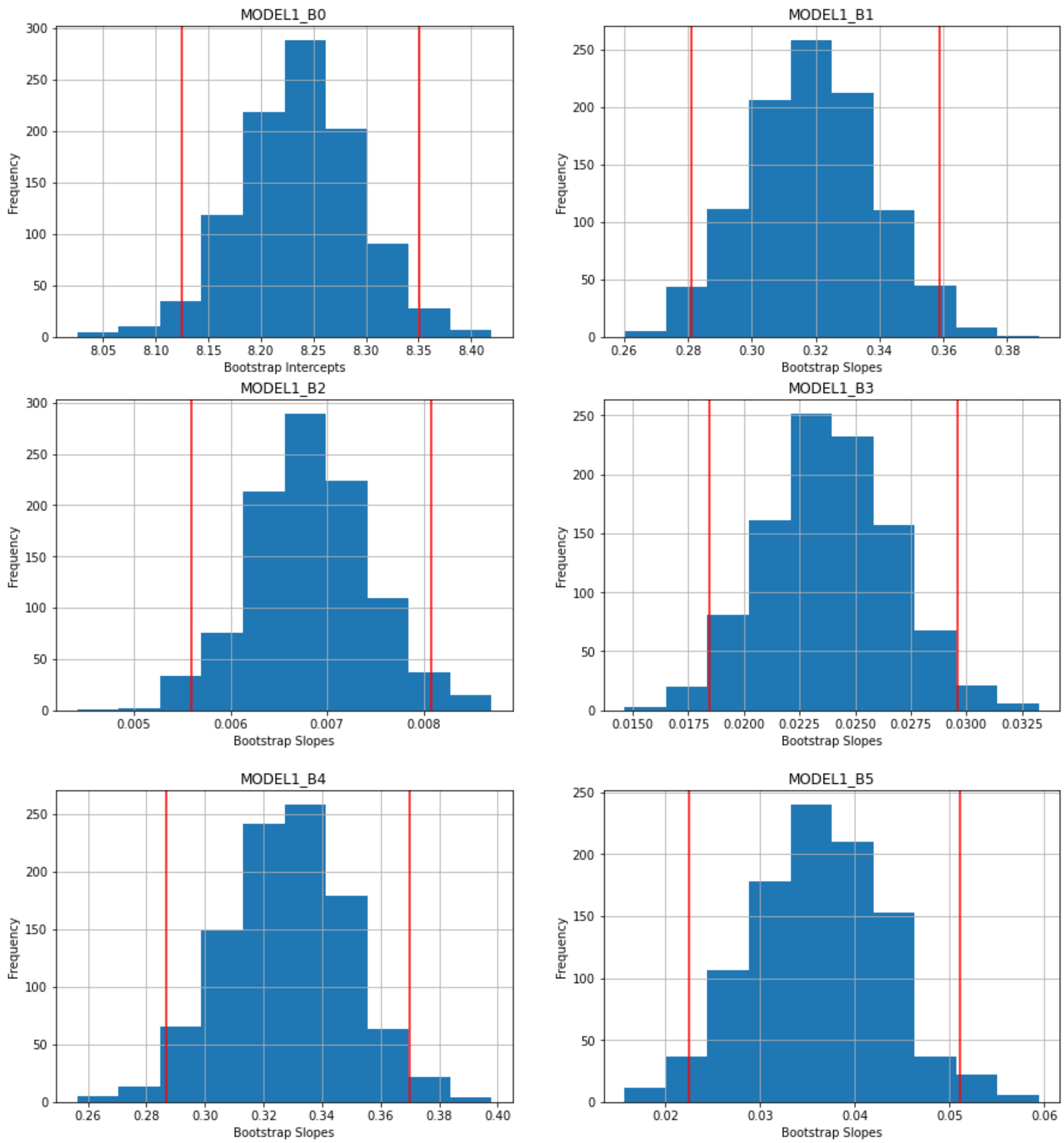
In [38]:

```
# Evaluate the robustness of coefficient estimates by bootstrapping 3 differen

# MODEL 1
mod1_coefs = pd.DataFrame(columns = ["mod1_B0","mod1_B1","mod1_B2","mod1_B3","r

for i in range(1000):

    sample = df.sample(df.shape[0] ,replace = True)

    results = smf.ols('rent ~ bedroom+brok_amt+area+furnishing+propertyage',
                      sample).fit()

    mod1_b0,mod1_b1,mod1_b2,mod1_b3,mod1_b4,mod1_b5 = results.params

    mod1_coefs = mod1_coefs.append(
```

```
        {"mod1_B0":mod1_b0, "mod1_B1":mod1_b1,"mod1_B2":mod1_b2,"mod1_B3":mod1_
         "mod1_B4":mod1_b4,"mod1_B5":mod1_b5}, ignore_index = True)

mod1_B0_u = mod1_coefs['mod1_B0'].quantile(0.975)
mod1_B1_u = mod1_coefs['mod1_B1'].quantile(0.975)
mod1_B2_u = mod1_coefs['mod1_B2'].quantile(0.975)
mod1_B3_u = mod1_coefs['mod1_B3'].quantile(0.975)
mod1_B4_u = mod1_coefs['mod1_B4'].quantile(0.975)
mod1_B5_u = mod1_coefs['mod1_B5'].quantile(0.975)

mod1_B0_l = mod1_coefs['mod1_B0'].quantile(0.025)
mod1_B1_l = mod1_coefs['mod1_B1'].quantile(0.025)
mod1_B2_l = mod1_coefs['mod1_B2'].quantile(0.025)
mod1_B3_l = mod1_coefs['mod1_B3'].quantile(0.025)
mod1_B4_l = mod1_coefs['mod1_B4'].quantile(0.025)
mod1_B5_l = mod1_coefs['mod1_B5'].quantile(0.025)

fig=plt.figure(figsize=(15,22))
ax=fig.add_subplot(4,2,1)
mod1_coefs.mod1_B0.hist()
plt.xlabel("Bootstrap Intercepts")
plt.ylabel("Frequency")
plt.title('MODEL1_B0')
plt.axvline(mod1_B0_u, color = "red")
plt.axvline(mod1_B0_l, color = "red")

ax=fig.add_subplot(4,2,2)
mod1_coefs.mod1_B1.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL1_B1')
plt.axvline(mod1_B1_u, color = "red")
plt.axvline(mod1_B1_l, color = "red")

ax=fig.add_subplot(4,2,3)
mod1_coefs.mod1_B2.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL1_B2')
plt.axvline(mod1_B2_u, color = "red")
plt.axvline(mod1_B2_l, color = "red")

ax=fig.add_subplot(4,2,4)
mod1_coefs.mod1_B3.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL1_B3')
plt.axvline(mod1_B3_u, color = "red")
plt.axvline(mod1_B3_l, color = "red")

fig1=plt.figure(figsize=(15,22))
ax=fig1.add_subplot(4,2,1)
mod1_coefs.mod1_B4.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL1_B4')
plt.axvline(mod1_B4_u, color = "red")
plt.axvline(mod1_B4_l, color = "red")

ax=fig1.add_subplot(4,2,2)
```

```
mod1_coefs.mod1_B5.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL1_B5')
plt.axvline(mod1_B5_u, color = "red")
plt.axvline(mod1_B5_l, color = "red")
```

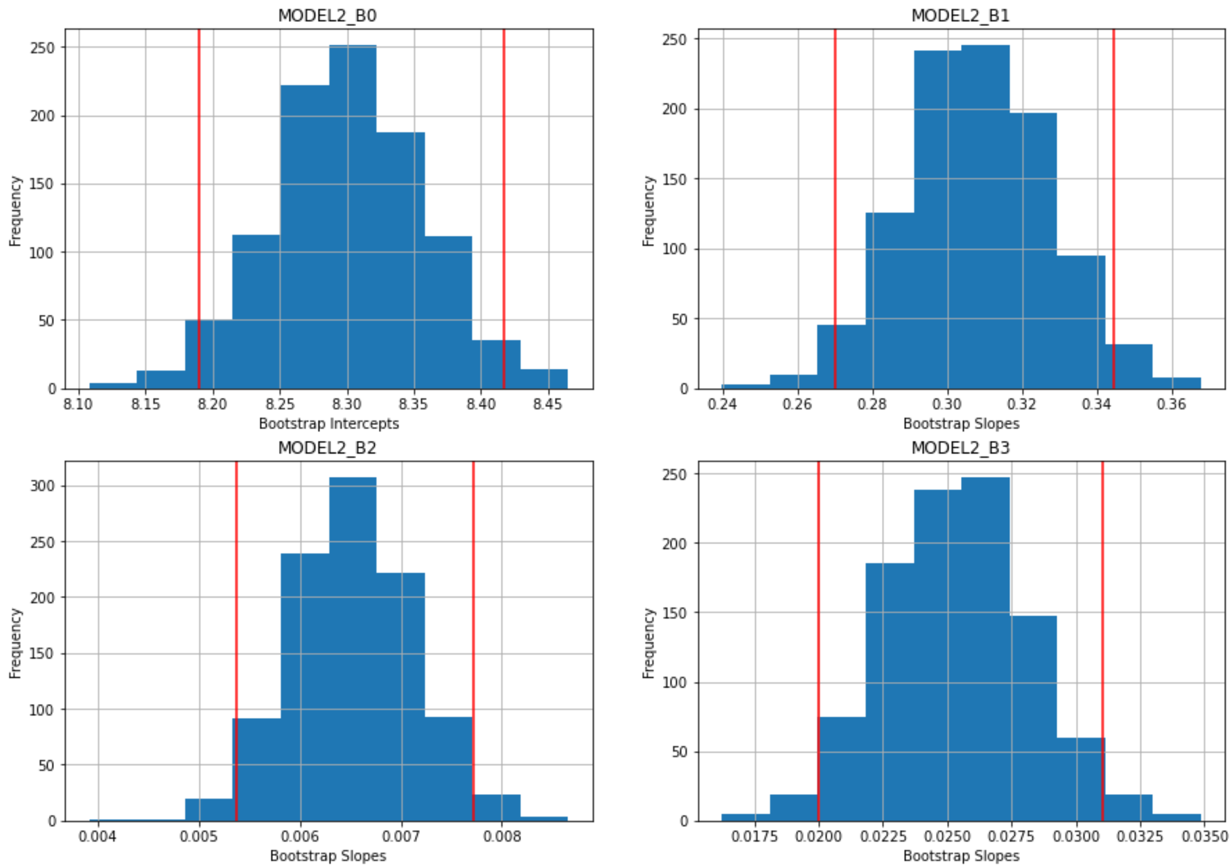
Out[38]: <matplotlib.lines.Line2D at 0x132def0a0>

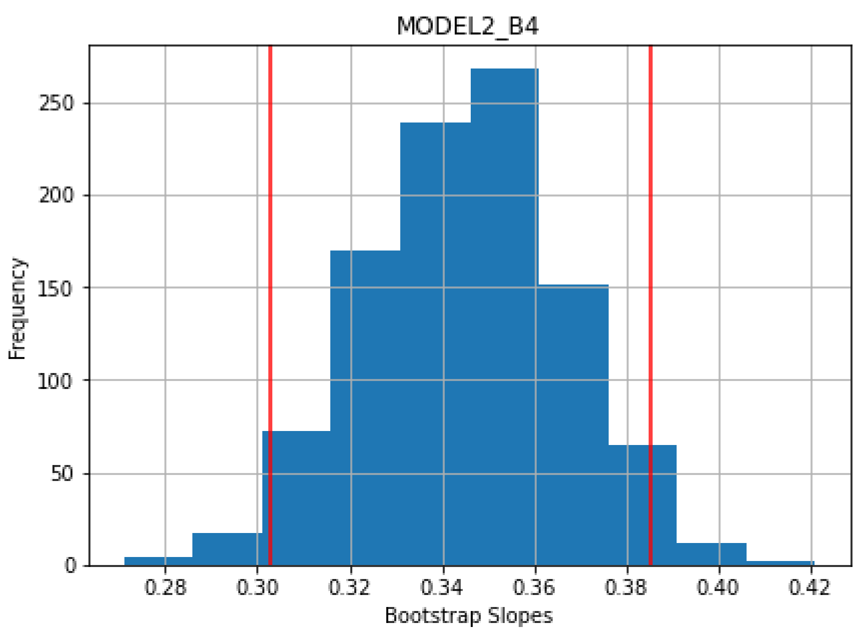


In [39]:
```
# MODEL 2
mod2_coefs = pd.DataFrame(columns = ["mod2_B0","mod2_B1","mod2_B2","mod2_B3","r

for i in range(1000):

    sample = df.sample(df.shape[0] ,replace = True)

    results = smf.ols('rent ~ bedroom+brok_amt+area+furnishing',
                      sample).fit()
```

```
    mod2_b0,mod2_b1,mod2_b2,mod2_b3,mod2_b4 = results.params

    mod2_coefs = mod2_coefs.append(
        {"mod2_B0":mod2_b0, "mod2_B1":mod2_b1,"mod2_B2":mod2_b2,"mod2_B3":mod2_
         "mod2_B4":mod2_b4}, ignore_index = True)

mod2_B0_u = mod2_coefs['mod2_B0'].quantile(0.975)
mod2_B1_u = mod2_coefs['mod2_B1'].quantile(0.975)
mod2_B2_u = mod2_coefs['mod2_B2'].quantile(0.975)
mod2_B3_u = mod2_coefs['mod2_B3'].quantile(0.975)
mod2_B4_u = mod2_coefs['mod2_B4'].quantile(0.975)

mod2_B0_l = mod2_coefs['mod2_B0'].quantile(0.025)
mod2_B1_l = mod2_coefs['mod2_B1'].quantile(0.025)
mod2_B2_l = mod2_coefs['mod2_B2'].quantile(0.025)
mod2_B3_l = mod2_coefs['mod2_B3'].quantile(0.025)
mod2_B4_l = mod2_coefs['mod2_B4'].quantile(0.025)

fig=plt.figure(figsize=(15,22))
ax=fig.add_subplot(4,2,1)
mod2_coefs.mod2_B0.hist()
plt.xlabel("Bootstrap Intercepts")
plt.ylabel("Frequency")
plt.title('MODEL2_B0')
plt.axvline(mod2_B0_u, color = "red")
plt.axvline(mod2_B0_l, color = "red")

ax=fig.add_subplot(4,2,2)
mod2_coefs.mod2_B1.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL2_B1')
plt.axvline(mod2_B1_u, color = "red")
plt.axvline(mod2_B1_l, color = "red")

ax=fig.add_subplot(4,2,3)
mod2_coefs.mod2_B2.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL2_B2')
plt.axvline(mod2_B2_u, color = "red")
plt.axvline(mod2_B2_l, color = "red")

ax=fig.add_subplot(4,2,4)
mod2_coefs.mod2_B3.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL2_B3')
plt.axvline(mod2_B3_u, color = "red")
plt.axvline(mod2_B3_l, color = "red")

fig1=plt.figure(figsize=(15,22))
ax=fig1.add_subplot(4,2,1)
mod2_coefs.mod2_B4.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL2_B4')
plt.axvline(mod2_B4_u, color = "red")
plt.axvline(mod2_B4_l, color = "red")
```

Out[39]: <matplotlib.lines.Line2D at 0x13397f1f0>



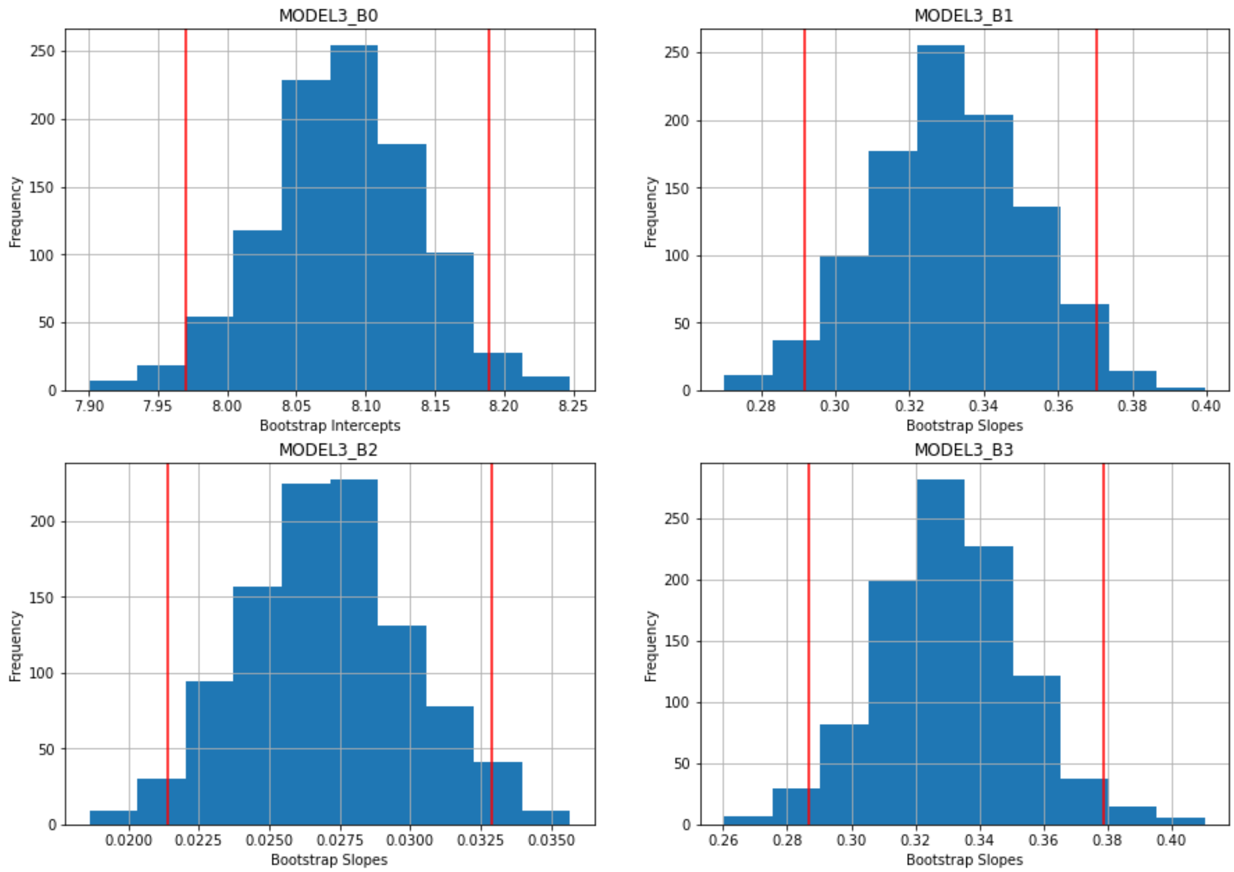


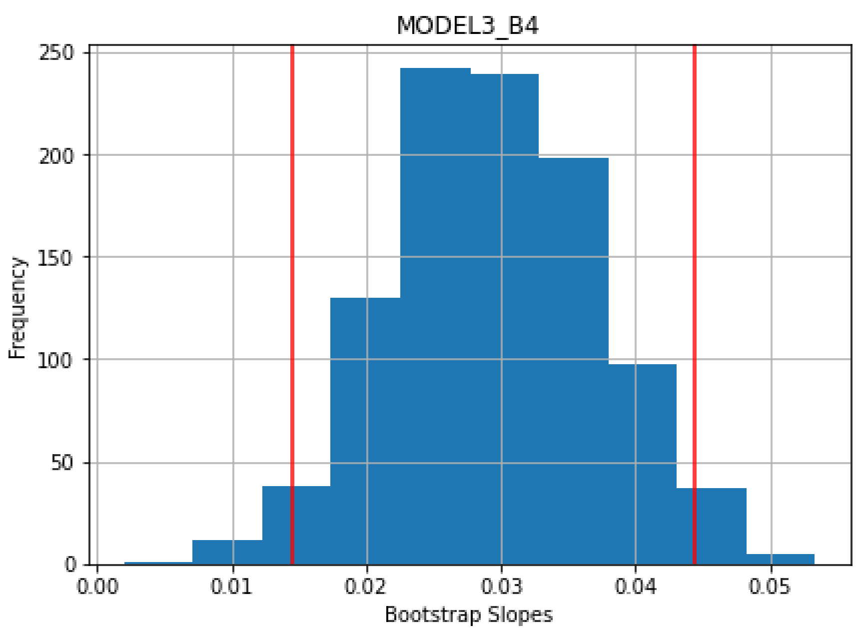
In [40]:

```
# MODEL 3
mod3_coefs = pd.DataFrame(columns = ["mod3_B0","mod3_B1","mod3_B2","mod3_B3","r

for i in range(1000):

    sample = df.sample(df.shape[0] ,replace = True)

    results = smf.ols('rent ~ bedroom+area+furnishing+propertyage',
                      sample).fit()
```

```
    mod3_b0,mod3_b1,mod3_b2,mod3_b3,mod3_b4 = results.params

    mod3_coefs = mod3_coefs.append(
        {"mod3_B0":mod3_b0, "mod3_B1":mod3_b1,"mod3_B2":mod3_b2,"mod3_B3":mod3_
         "mod3_B4":mod3_b4}, ignore_index = True)

mod3_B0_u = mod3_coefs['mod3_B0'].quantile(0.975)
mod3_B1_u = mod3_coefs['mod3_B1'].quantile(0.975)
mod3_B2_u = mod3_coefs['mod3_B2'].quantile(0.975)
mod3_B3_u = mod3_coefs['mod3_B3'].quantile(0.975)
mod3_B4_u = mod3_coefs['mod3_B4'].quantile(0.975)

mod3_B0_l = mod3_coefs['mod3_B0'].quantile(0.025)
mod3_B1_l = mod3_coefs['mod3_B1'].quantile(0.025)
mod3_B2_l = mod3_coefs['mod3_B2'].quantile(0.025)
mod3_B3_l = mod3_coefs['mod3_B3'].quantile(0.025)
mod3_B4_l = mod3_coefs['mod3_B4'].quantile(0.025)

fig=plt.figure(figsize=(15,22))
ax=fig.add_subplot(4,2,1)
mod3_coefs.mod3_B0.hist()
plt.xlabel("Bootstrap Intercepts")
plt.ylabel("Frequency")
plt.title('MODEL3_B0')
plt.axvline(mod3_B0_u, color = "red")
plt.axvline(mod3_B0_l, color = "red")

ax=fig.add_subplot(4,2,2)
mod3_coefs.mod3_B1.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL3_B1')
plt.axvline(mod3_B1_u, color = "red")
plt.axvline(mod3_B1_l, color = "red")

ax=fig.add_subplot(4,2,3)
mod3_coefs.mod3_B2.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL3_B2')
plt.axvline(mod3_B2_u, color = "red")
plt.axvline(mod3_B2_l, color = "red")

ax=fig.add_subplot(4,2,4)
mod3_coefs.mod3_B3.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL3_B3')
plt.axvline(mod3_B3_u, color = "red")
plt.axvline(mod3_B3_l, color = "red")

fig1=plt.figure(figsize=(15,22))
ax=fig1.add_subplot(4,2,1)
mod3_coefs.mod3_B4.hist()
plt.xlabel("Bootstrap Slopes")
plt.ylabel("Frequency")
plt.title('MODEL3_B4')
plt.axvline(mod3_B4_u, color = "red")
plt.axvline(mod3_B4_l, color = "red")
```

Out[40]: <matplotlib.lines.Line2D at 0x132f6fca0>





By comparing the bootstrapping result of 3 models, they are all robust.

In [41]:
```
#5-Fold Cross-validation

from sklearn.model_selection import KFold

#MODEL 1
kf = KFold(n_splits = 5)
```

```
mod1_mse = []
for train_index, test_index in kf.split(df):
    # train data over training set
    mod1_results = smf.ols('rent ~ bedroom+brok_amt+area+furnishing+propertyage
                           df.iloc[train_index]).fit()

    # test over last split
    mod1_s = (((df.iloc[test_index]["rent"]) - mod1_results.predict(df.iloc[tes

    # append test metric
    mod1_mse.append(mod1_s)

print(mod1_mse)
print('The average score of MODEL 1 is',np.mean(mod1_mse))

#MODEL 2
kf = KFold(n_splits = 5)

mod2_mse = []
for train_index, test_index in kf.split(df):
    # train data over training set
    mod2_results = smf.ols('rent ~ bedroom+brok_amt+area+furnishing',
                           df.iloc[train_index]).fit()

    # test over last split
    mod2_s = (((df.iloc[test_index]["rent"]) - mod2_results.predict(df.iloc[tes

    # append test metric
    mod2_mse.append(mod2_s)

print(mod2_mse)
print('The average score of MODEL 2 is',np.mean(mod2_mse))

#MODEL 3
kf = KFold(n_splits = 5)

mod3_mse = []
for train_index, test_index in kf.split(df):
    # train data over training set
    mod3_results = smf.ols('rent ~ bedroom+area+furnishing+propertyage',
                           df.iloc[train_index]).fit()

    # test over last split
    mod3_s = (((df.iloc[test_index]["rent"]) - mod3_results.predict(df.iloc[tes

    # append test metric
    mod3_mse.append(mod3_s)

print(mod3_mse)
print('The average score of MODEL 3 is',np.mean(mod3_mse))
```

```
[0.10560740194891065, 0.09460183821495574, 0.1131377427869036, 0.0947415512549
9978, 0.09102508350002009]
The average score of MODEL 1 is 0.09982272354115797
[0.10365670962700028, 0.0964503119120327, 0.11422844183490154, 0.0973220745661
0519, 0.0927324566853755]
The average score of MODEL 2 is 0.10087799892508305
[0.11385537670116402, 0.09928001124220427, 0.11843905548881985, 0.098896448135
32533, 0.09637891959823758]
The average score of MODEL 3 is 0.1053699622331502
```

In [43]:
```
# MODEL 1
results_mod1 = smf.ols('rent ~ bedroom+brok_amt+area+furnishing+propertyage',
                       df.iloc[:int(np.round(len(df))*0.7)]).fit()

s_mod1 = (((df.iloc[int(np.round(len(df))*0.7):]["rent"])
        - results_mod1.predict(df.iloc[int(np.round(len(df))*0.7):]))**2).mean(

print('When splitting train set and test set with 70% and 30% of model 1, the r

# MODEL 2
results_mod2 = smf.ols('rent ~ bedroom+brok_amt+area+furnishing',
                       df.iloc[:int(np.round(len(df))*0.7)]).fit()

s_mod2 = (((df.iloc[int(np.round(len(df))*0.7):]["rent"])
        - results_mod2.predict(df.iloc[int(np.round(len(df))*0.7):]))**2).mean(

print('When splitting train set and test set with 70% and 30% of model 2, the r

# MODEL 3
results_mod3 = smf.ols('rent ~ bedroom+area+furnishing+propertyage',
                       df.iloc[:int(np.round(len(df))*0.7)]).fit()

s_mod3 = (((df.iloc[int(np.round(len(df))*0.7):]["rent"])
        - results_mod3.predict(df.iloc[int(np.round(len(df))*0.7):]))**2).mean(

print('When splitting train set and test set with 70% and 30% of model 3, the r
```

```
When splitting train set and test set with 70% and 30% of model 1, the mse is
0.08824616911239616
When splitting train set and test set with 70% and 30% of model 2, the mse is
0.09032612396385653
When splitting train set and test set with 70% and 30% of model 3, the mse is
0.09209824558270897
```

By comparing the MSE-score by 5-fold Cross-validation and by splitting the data into testing and training sets, and predicting on the testing set, model 1 has the best performance in both method.

Overall, we will select model 1 (rent ~ bedroom+brok_amt+area+furnishing+propertyage) as our final model.

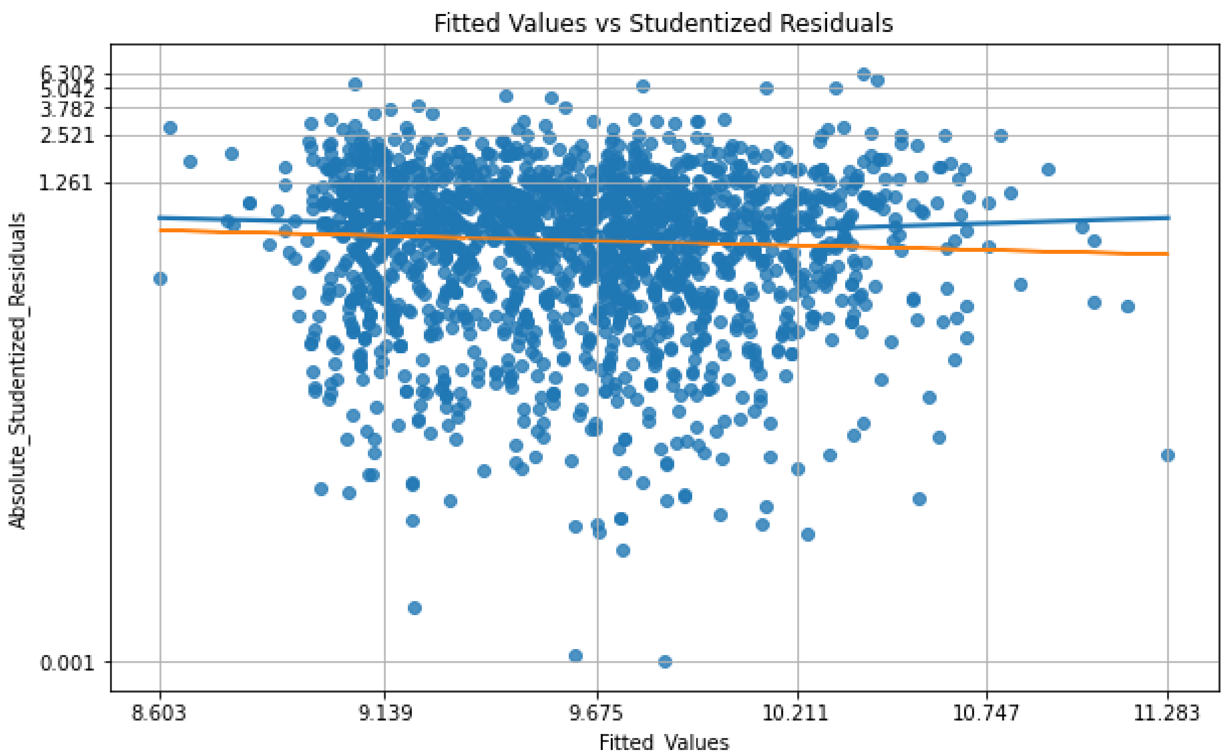
MODEL DESCRIPTION

In [44]:
```
# Fitted Values vs Studentized Residuals

def spread_level(model, data):
    df_copy = data.copy()

    # Get the studentized residuals
    df_copy["Absolute_Studentized_Residuals"] = (np.abs(model.get_influence().r
    df_copy["Fitted_Values"] = (model.fittedvalues)

    # run regression to get slope of fitted vs resid, rlm is a robust linear mo
    slreg = smf.rlm("np.log(Absolute_Studentized_Residuals) ~ np.log(Fitted_Val
    slope = slreg.params[1]

    # plot values
```

```
    fig, ax = plt.subplots(figsize = (10, 6))
    ax.set_title("Fitted Values vs Studentized Residuals")
    sns.regplot(x = "Fitted_Values", y = "Absolute_Studentized_Residuals", data
    ax.plot(df_copy.Fitted_Values.values, np.exp(slreg.fittedvalues).values)

    # Set to the logarithmic scale
    ax.set_yscale('log')
    ax.set_xscale('log')

    # convert froms scientific notation to scalar notation
    ax.yaxis.set_major_formatter(ScalarFormatter())
    ax.xaxis.set_major_formatter(ScalarFormatter())

    # Resolve overlapping label bug
    ax.minorticks_off()

    # Set tick labels automatically
    ax.set_xticks(np.linspace(df_copy["Fitted_Values"].min(),df_copy["Fitted_Va
    ax.set_yticks(np.linspace(df_copy["Absolute_Studentized_Residuals"].min(),
                              df_copy["Absolute_Studentized_Residuals"].max(),

    ax.grid()

    # return a suggested power transform of your y-variable that may correct he
    # The transform is just one minus the slope of the reegression line of you
    print("Suggested Power Transformation:", 1-slope)
```

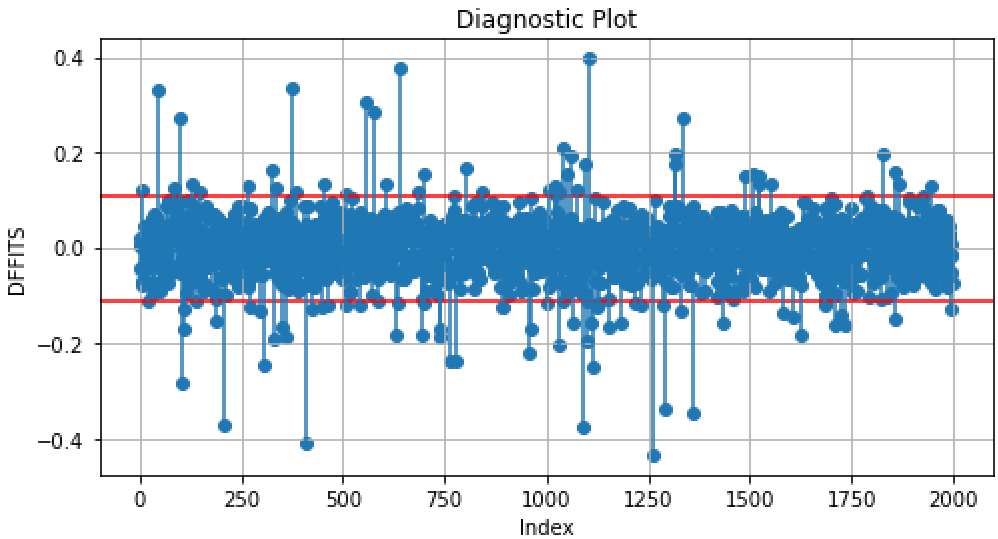
In [46]:
```
spread_level(results_model1, df)
```

```
Suggested Power Transformation: 2.3222208875550083
```

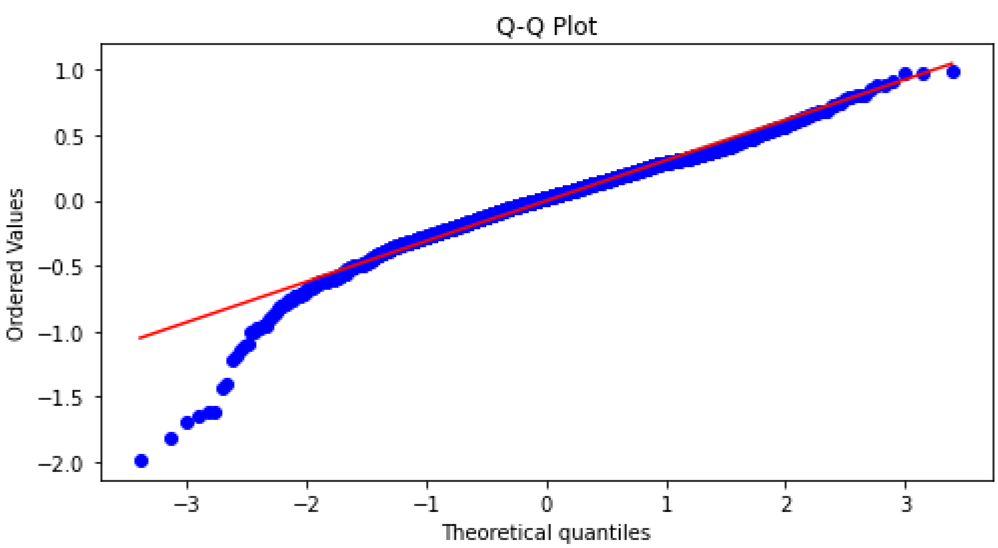


In [47]:
```
#DFFIT

dffits,threshold = results_model1.get_influence().dffits
plt.figure(figsize = (8,4))
plt.scatter(df.index, dffits)
```

```
plt.axhline(threshold,color = 'red')
plt.axhline(-threshold,color = 'red')
plt.vlines(x=df.index,ymin=0,ymax=dffits)
plt.xlabel('Index')
plt.ylabel('DFFITS')
plt.title('Diagnostic Plot')
plt.grid()
plt.show()
```

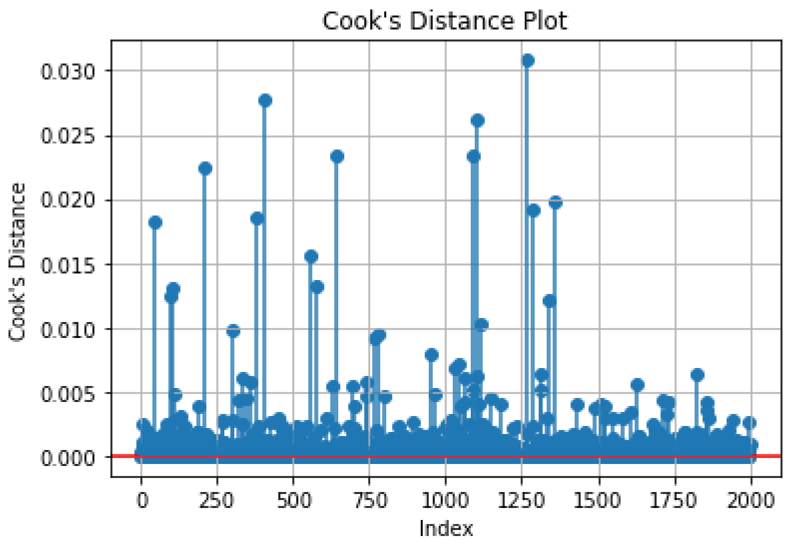


In [48]:
```
#Q-Q Plot

plt.figure(figsize = (8,4))
stats.probplot(results_model1.resid, dist = 'norm', plot = plt)
plt.title('Q-Q Plot')
plt.show()
```



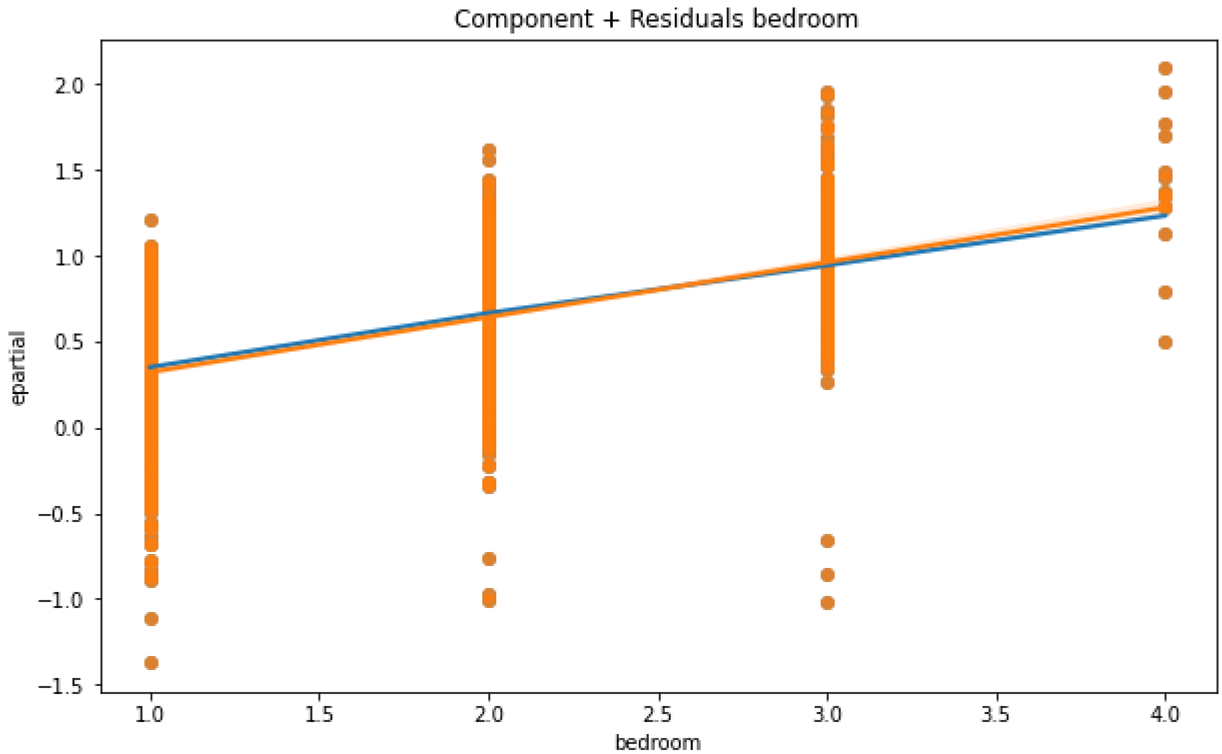
In [49]:
```
#Cook's distance Plot

distance = results_model1.get_influence().cooks_distance
plt.scatter(df.index, distance[0])
plt.axhline(0,color='red')
plt.vlines(x = df.index,ymin=0,ymax=distance[0])
plt.xlabel('Index')
plt.ylabel("Cook's Distance")
```

```
plt.title("Cook's Distance Plot")
plt.grid()
plt.show()
```

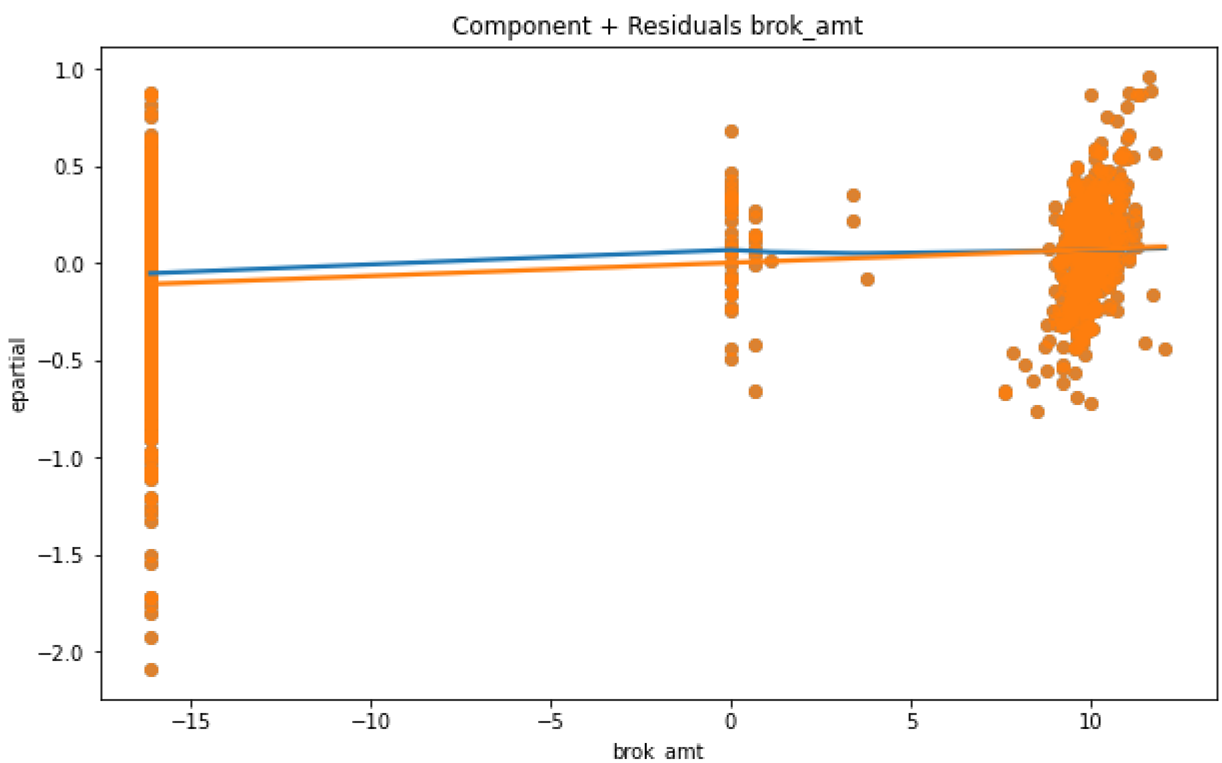


In [50]:
```
# Marginal Effects

def ccpr_plot(model, data, variable):
    df_copy = data.copy()

    df_copy["epartial"] = model.resid + model.params[variable]*data[variable]

    plt.figure(figsize = (10, 6))

    sns.regplot(x = variable, y = "epartial", data =df_copy, lowess = True)
    sns.regplot(x = variable, y = "epartial", data =df_copy)

    plt.title("Component + Residuals "+variable)
```

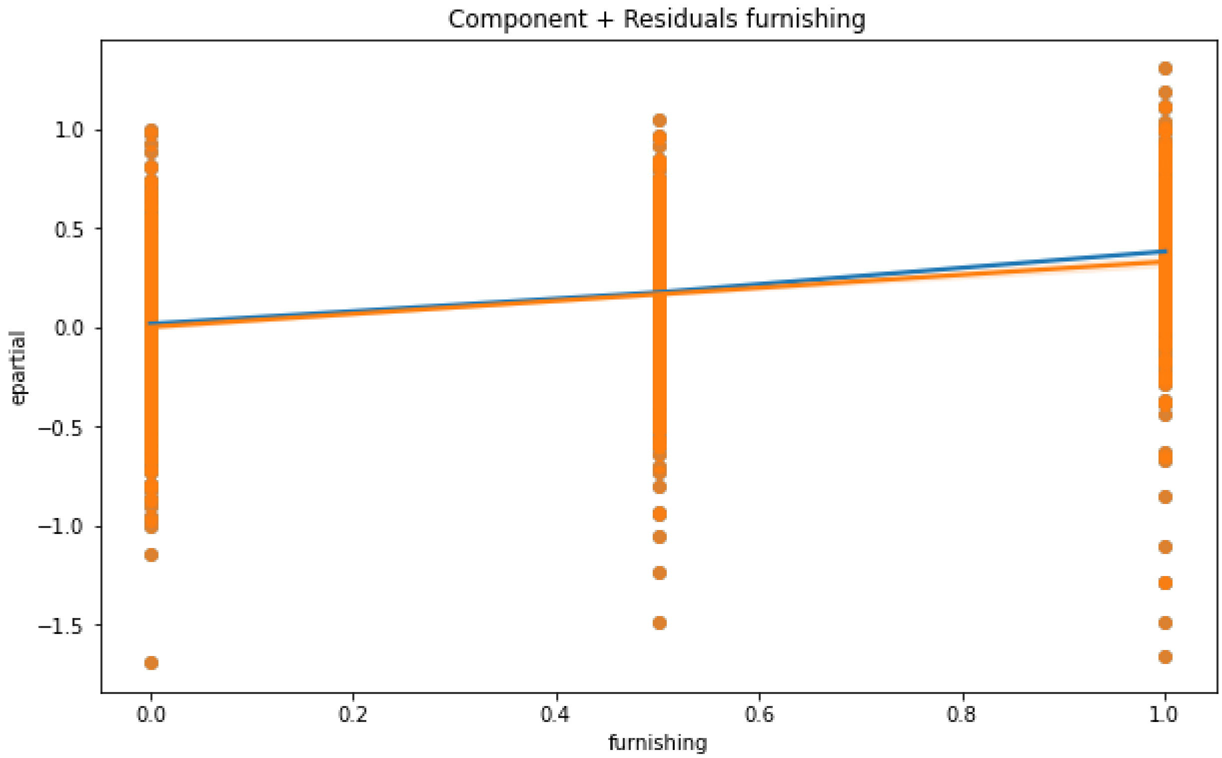
In [51]:
```
ccpr_plot(results_model1, df, "bedroom")
```



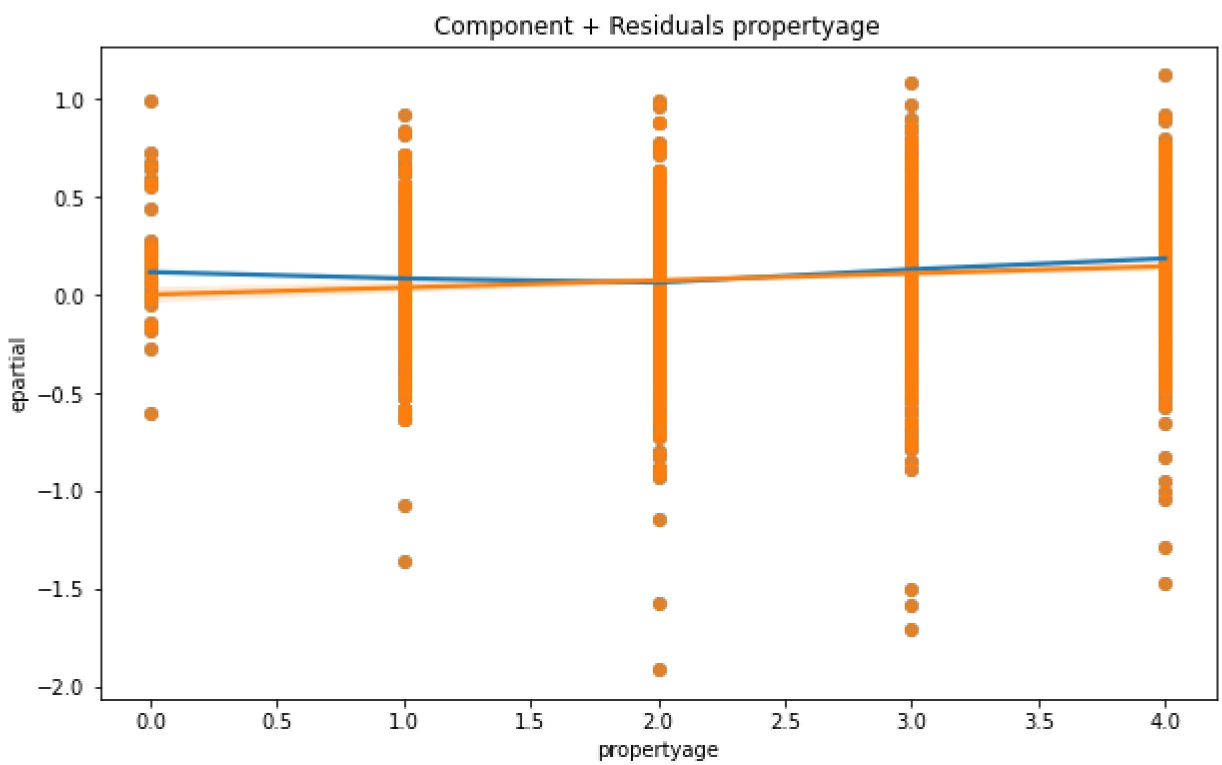
In [52]:
```
ccpr_plot(results_model1, df, "brok_amt")
```



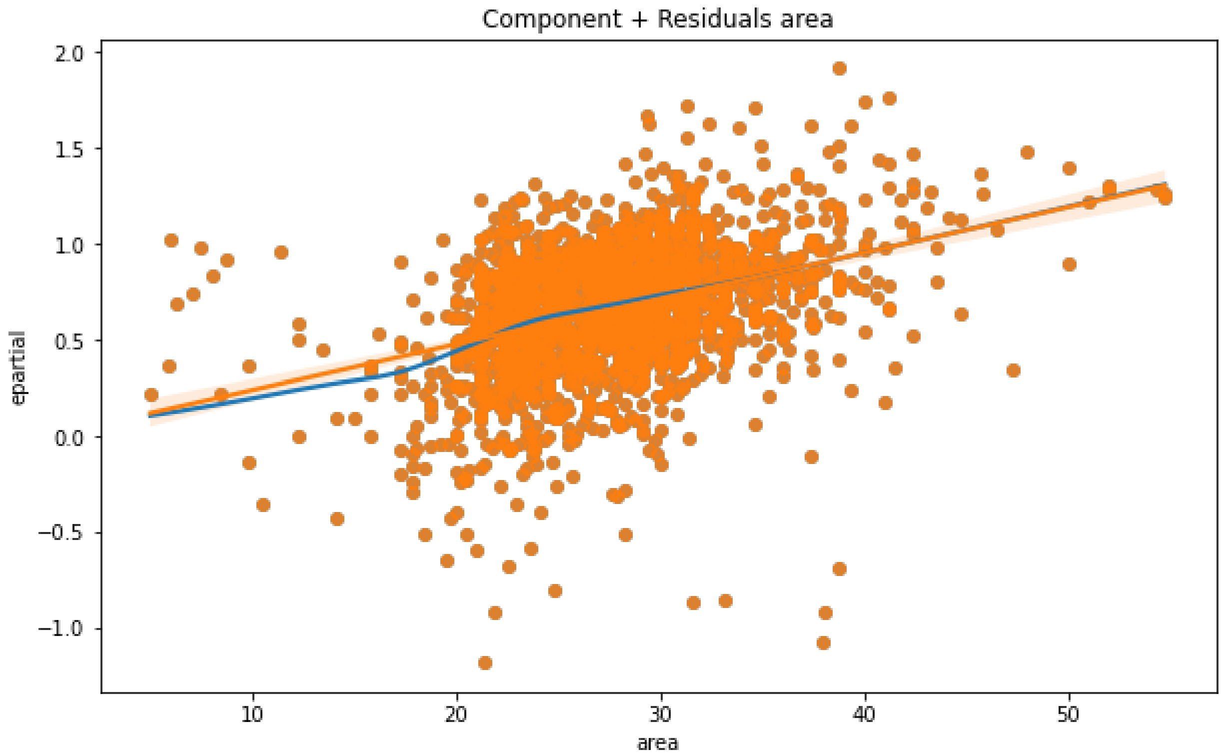
In [53]:
```
ccpr_plot(results_model1, df, "furnishing")
```



In [54]:
```
ccpr_plot(results_model1, df, "propertyage")
```



In [55]:
```
ccpr_plot(results_model1, df, "area")
```



In [56]:

```
results_model1.summary()
```

Out[56]:

OLS Regression Results

Dep. Variable:	rent	**R-squared:**	0.633
Model:	OLS	**Adj. R-squared:**	0.633
Method:	Least Squares	**F-statistic:**	689.2
Date:	Mon, 14 Nov 2022	**Prob (F-statistic):**	0.00
Time:	22:23:05	**Log-Likelihood:**	-526.76
No. Observations:	2000	**AIC:**	1066.
Df Residuals:	1994	**BIC:**	1099.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	8.2373	0.045	184.316	0.000	8.150	8.325
bedroom	0.3203	0.016	20.112	0.000	0.289	0.352
brok_amt	0.0068	0.001	10.525	0.000	0.006	0.008
area	0.0238	0.002	11.307	0.000	0.020	0.028
furnishing	0.3292	0.020	16.320	0.000	0.290	0.369
propertyage	0.0361	0.007	5.010	0.000	0.022	0.050

Omnibus:	334.389	**Durbin-Watson:**	2.016
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1110.104
Skew:	-0.823	**Prob(JB):**	8.79e-242
Kurtosis:	6.257	**Cond. No.**	189.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Our model is log(rent) = 8.2373+0.3203bedroom+0.0068log(brok_amt)+0.0238area^0.5+0.3292furnishing+0.0361prop on average, when increased bedroom number by one unit, we estimate rent increased by 32.03%; when increased brok_amt by 1%, we estimate rent increased by 0.0068%; when increased sqrt(area) by one, we estimate rent increased by 2.38%; rent of furnished houses is 32.92% higher than unfurnished ones on average; when increased propertyage by one unit, we estimate rent increased by 3.61%.

FUTURE DIRECTION

In this model, we can not fix the heteroskedasticity with feature of discrete variables and abnormal distributions. In future study, we will solve this problem by advanced technics.