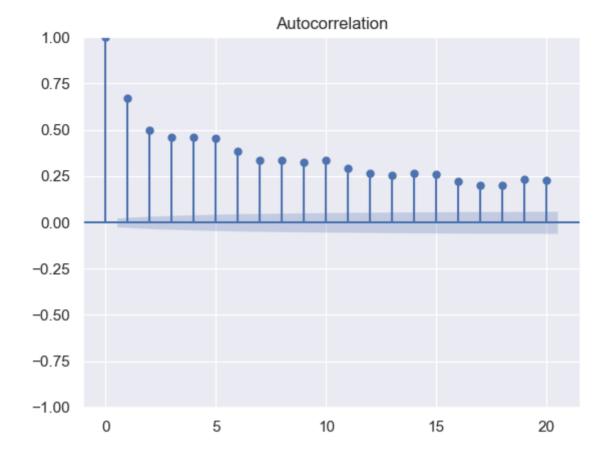
```
In [1]: # Basic libraries
        import pandas as pd
        import numpy as np
         import seaborn as sns
        import matplotlib.pyplot as plt
        # For read file from url
        import io
        import requests
        # Set font sizes in plots
        sns.set(font scale = 1.)
        # Display all columns
        pd.set_option('display.max_columns', None)
In [2]: # Read in NYSE data from url
        url = "https://raw.githubusercontent.com/ucla-econ-425t/2023winter/master/sl
        s = requests.get(url).content.decode('utf-8')
        NYSE = pd.read_csv(io.StringIO(s), index_col = 0)
        NYSE
Out[2]:
                   day_of_week DJ_return log_volume log_volatility train
               data
```

date					
1962-12-03	mon	-0.004461	0.032573	-13.127403	True
1962-12-04	tues	0.007813	0.346202	-11.749305	True
1962-12-05	wed	0.003845	0.525306	-11.665609	True
1962-12-06	thur	-0.003462	0.210182	-11.626772	True
1962-12-07	fri	0.000568	0.044187	-11.728130	True
1986-12-24	wed	0.006514	-0.236104	-9.807366	False
1986-12-26	fri	0.001825	-1.322425	-9.906025	False
1986-12-29	mon	-0.009515	-0.371237	-9.827660	False
1986-12-30	tues	-0.001837	-0.385638	-9.926091	False
1986-12-31	wed	-0.006655	-0.264986	-9.935527	False

6051 rows × 5 columns

```
In [3]: from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

plt.figure()
 plot_acf(NYSE['log_volume'], lags = 20)
 plt.show()
```



Do a similar plot for (1) the correlation between v_t and lag ℓ Dow Jones return $r_{t-\ell}$ and (2) correlation between v_t and lag ℓ Log volatility $z_{t-\ell}$.

```
In [4]: L = 5

# 一次性生成每组五个的滞后项
for s in range(1, L+1):
    NYSE[f'DJ_return_lag{s}'] = NYSE['DJ_return'].shift(s)
    NYSE[f'log_volume_lag{s}'] = NYSE['log_volume'].shift(s)
    NYSE[f'log_volatility_lag{s}'] = NYSE['log_volatility'].shift(s)

#按名称将滞后项排列在原项后
NYSE = NYSE.reindex(sorted(NYSE.columns), axis = 1)

In [5]: #将所有以原项名起头的列(抓取所有滞后)做相关系数
    corr = NYSE.filter(regex = "log_volume*|DJ_return*|log_volatility*").corr()
#画相关系数图,背景色为红蓝
    corr.style.background_gradient(cmap = "coolwarm")
```

log_volume_lag4

log_volume_lag5

-0.004202

0.002981

	DJ_return	DJ_return_lag1	DJ_return_lag2	DJ_return_lag3	DJ_return_la
DJ_return	1.000000	0.143388	-0.003597	-0.005304	-0.0055
DJ_return_lag1	0.143388	1.000000	0.143365	-0.003752	-0.0052
DJ_return_lag2	-0.003597	0.143365	1.000000	0.143336	-0.0037
DJ_return_lag3	-0.005304	-0.003752	0.143336	1.000000	0.1433
DJ_return_lag4	-0.005528	-0.005278	-0.003744	0.143389	1.0000
DJ_return_lag5	-0.014239	-0.005428	-0.005249	-0.003602	0.1433
log_volatility	0.026793	0.021996	0.017013	0.011547	0.0003
log_volatility_lag1	0.014177	0.026778	0.021991	0.016992	0.011
log_volatility_lag2	0.013557	0.014163	0.026773	0.021972	0.0169
log_volatility_lag3	0.010581	0.013561	0.014164	0.026780	0.0219
log_volatility_lag4	0.008297	0.010570	0.013557	0.014149	0.0267
log_volatility_lag5	0.009971	0.008303	0.010572	0.013568	0.0141
log_volume	0.200892	0.211669	0.108032	0.110325	0.0901
log_volume_lag1	0.047600	0.200776	0.211648	0.107846	0.1103
log_volume_lag2	0.015934	0.047396	0.200756	0.211410	0.1079
log_volume_lag3	0.008729	0.015730	0.047345	0.200523	0.2115

0.046402

0.015366

0.0464

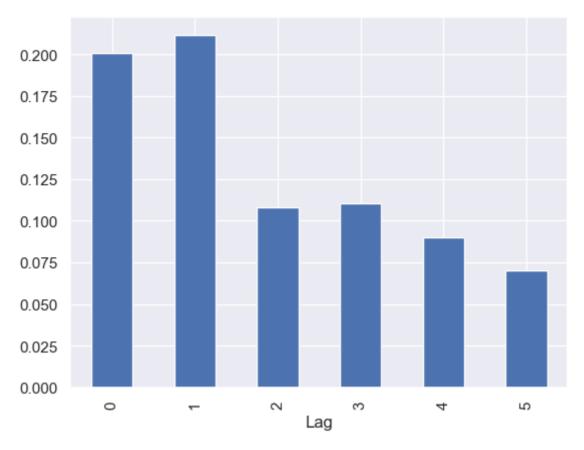
0.015549

0.007959

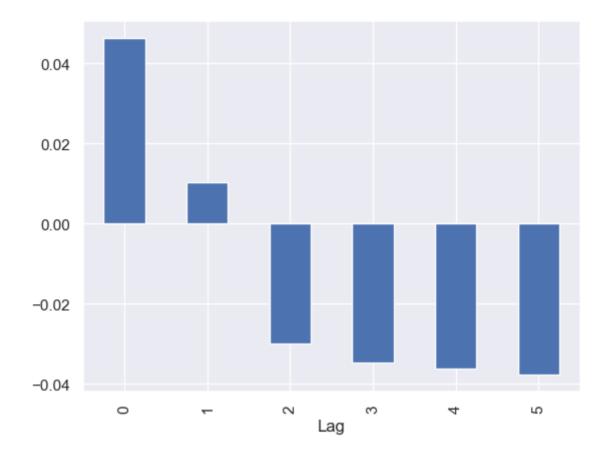
```
In [143... plt.figure()
#抓取log_volume和所有以DJ_return开头的原项和滞后项相关系数作图
corr['log_volume'].filter(regex = 'DJ_return*').plot(
    kind = 'bar',
    #将x轴坐标由原列名变成0-5
    x = range(1, L+1),
    use_index = False
    ).set_xlabel('Lag')
plt.show()
```

0.007997

-0.004333



```
In [144... plt.figure()
#抓取log_volume和所有以log_volatility开头的原项和滞后项相关系数作图
corr['log_volume'].filter(regex = 'log_volatility*').plot(
    kind = 'bar',
    #将x轴坐标由原列名变成0-5
    x = range(1, L+1),
    use_index = False
    ).set_xlabel('Lag')
plt.show()
```



Project goal: use the previous five trading days' data to forecast today's log trading volume. Use the \mathbb{R}^2 between forecast and actual values as the cross validation and test evaluation criterion.

```
In [35]: # In order to track time
         import time
         # Scikit-Learn
         from sklearn.compose import ColumnTransformer
         from sklearn.ensemble import RandomForestRegressor
         from sklearn.impute import SimpleImputer
         from sklearn.linear model import ElasticNet
         from sklearn.neural network import MLPRegressor
         from sklearn.metrics import r2 score
         from sklearn.model_selection import GridSearchCV, TimeSeriesSplit
         from sklearn.pipeline import Pipeline
         from sklearn.preprocessing import OneHotEncoder, StandardScaler
         # Tensorflow
         import tensorflow as tf
         from tensorflow import keras
         # XGBoost
         import xgboost as xgb
In [36]: NYSE_train = NYSE[NYSE['train']==True].dropna()
         print(NYSE_train.shape)
         NYSE_test = NYSE[NYSE['train']==False].dropna()
         NYSE test.shape
         (4276, 20)
```

1.baseline method: use yesterday's value of log trading volume to predict that of today

```
In [39]: r2 baseline train = r2 score(y train, X train['log volume lag1'])
         r2 baseline_train
         0.4199386914132621
Out[39]:
In [40]: r2_baseline_test = r2_score(y_test, X_test['log_volume_lag1'])
         r2 baseline test
Out[40]: 0.18026287838158628
In [41]: new row = {'Method': 'Straw Man', 'In sample R^2': r2 baseline train, 'Out of
         df = df.append(new row, ignore index = True)
         df
         /var/folders/g8/bnlxdn656x5f3c7v8y4v67lr0000gn/T/ipykernel 706/2469903245.p
         y:2: FutureWarning: The frame.append method is deprecated and will be remove
         d from pandas in a future version. Use pandas.concat instead.
           df = df.append(new_row, ignore_index = True)
              Method In sample R^2 Out of sample R^2
Out[41]:
         O Straw Man
                          0.419939
                                          0.180263
```

2. ElasticNet: Tune AR(5) with elastic net (lasso + ridge) regularization using all 3 features on the training data

```
In [11]: # 自动挑出非数值行
         cat features = X train.select dtypes(exclude = 'float64').columns
         # 自动挑出数值行
         num features = X train.select dtypes('float64').columns
         cat tf = Pipeline(steps = [
             ("encoder", OneHotEncoder(drop = 'first')),
             ("std", StandardScaler(with mean = False))
         ])
         num_tf = Pipeline(steps = [
             ("std", StandardScaler())
         1)
         # Column Transformer
         enet col tf = ColumnTransformer(transformers = [
             ('num', num_tf, num_features),
             ('cat', cat_tf, cat_features)
         ])
```

```
In [16]: enet mod = ElasticNet(
         alpha = 1.0,
         11 \text{ ratio} = 0.5,
         max_iter = 100000,
         warm_start = True,
         random state = 425,
         #selection = 'random'
In [17]: enet_pipe = Pipeline(steps = [
             ("col_tf", enet_col_tf),
             ("model", enet mod)
         ])
In [18]:
         alpha_grid = np.logspace(start = -12, stop = 2, num = 10)
         11 ratio grid = [0.0, 0.2, 0.4, 0.6, 0.8, 1.0]
         enet_tuned_parameters = {
             "model__alpha": alpha_grid,
             "model 11 ratio": 11 ratio grid
In [19]: enet_search = GridSearchCV(
           enet pipe,
           enet_tuned_parameters,
           cv = TimeSeriesSplit(5),
           scoring = 'r2',
           refit = True
In [20]: tic = time.time()
         enet_search.fit(X_train, y_train)
         toc = time.time()
         print('Execution time: ', toc-tic, "seconds")
```

/Users/apple/opt/anaconda3/lib/python3.9/site-packages/sklearn/linear_model/_coordinate_descent.py:647: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 6.779e+01, tole rance: 1.379e-02 Linear regression models with null weight for the 11 regula rization term are more efficiently fitted using one of the solvers implement ed in sklearn.linear_model.Ridge/RidgeCV instead.

model = cd fast.enet coordinate descent(

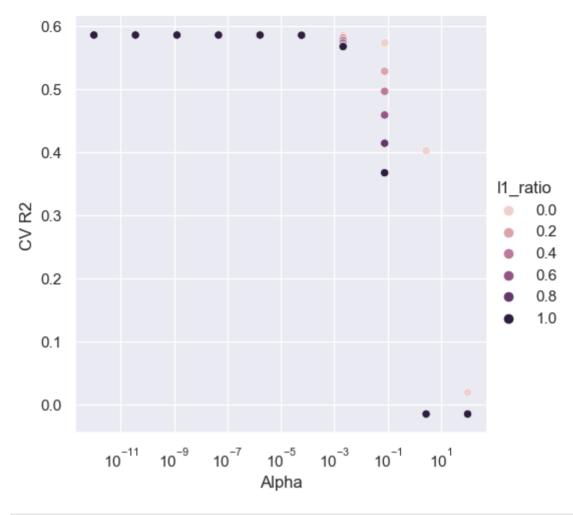
/Users/apple/opt/anaconda3/lib/python3.9/site-packages/sklearn/linear_model/_coordinate_descent.py:647: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 9.369e+01, tole rance: 1.907e-02 Linear regression models with null weight for the 11 regula rization term are more efficiently fitted using one of the solvers implement ed in sklearn.linear_model.Ridge/RidgeCV instead.

model = cd_fast.enet_coordinate_descent(
Execution time: 160.70518708229065 seconds

/Users/apple/opt/anaconda3/lib/python3.9/site-packages/sklearn/linear_model/_coordinate_descent.py:647: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 4.560e+01, tole rance: 2.278e-02 Linear regression models with null weight for the 11 regula rization term are more efficiently fitted using one of the solvers implement ed in sklearn.linear_model.Ridge/RidgeCV instead.

model = cd fast.enet coordinate descent(

```
In [21]: cv res = pd.DataFrame({
           "alpha": np.array(enet search.cv results ["param model alpha"]),
           "r2": enet_search.cv_results_["mean_test_score"],
            "l1_ratio": enet_search.cv_results_["param_model__l1_ratio"]
           })
         plt.figure()
         sns.relplot(
           # kind = "line",
           data = cv res,
           x = "alpha",
           y = "r2",
           hue = "l1 ratio"
           ) set(
             xscale = "log",
             xlabel = "Alpha",
             ylabel = "CV R2"
             )
         plt.show()
```



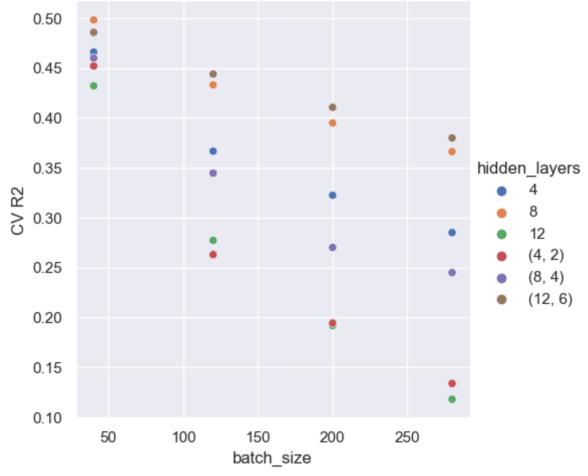
```
In [22]:
         enet_search.best_estimator_
          Pipeline(steps=[('col tf',
Out[22]:
                            ColumnTransformer(transformers=[('num',
                                                               Pipeline(steps=[('std',
                                                                                  StandardS
          caler())]),
                                                               Index(['DJ_return_lag1',
          'DJ_return_lag2', 'DJ_return_lag3', 'DJ_return_lag4',
                  'DJ_return_lag5', 'log_volatility_lag1', 'log_volatility_lag2',
                 'log_volatility_lag3', 'log_volatility_lag4', 'log_volatility_lag5',
                 'log_volume_lag1', 'log_volume_lag2', 'log_volume_lag3', 'log_volume_lag4', 'log_volume_lag5'],
                dtype='object')),
                                                              ('cat',
                                                               Pipeline(steps=[('encode
          r',
                                                                                  OneHotEnc
          oder(drop='first')),
                                                                                 ('std',
                                                                                  StandardS
          caler(with_mean=False))]),
                                                               Index(['day_of_week'], dty
          pe='object'))])),
                           ('model',
                            ElasticNet(alpha=1e-12, l1 ratio=0.0, max iter=100000,
                                        random_state=425, warm_start=True))])
In [23]:
         r2_train_enet = r2_score(
            y_train,
            enet_search.best_estimator_.predict(X_train)
          r2_train_enet
```

```
0.599718664285801
Out[23]:
In [24]: r2_test_enet = r2_score(
           y_test,
            enet search.best estimator .predict(X test)
          r2 test enet
         0.4595563133053302
Out[24]:
In [233...
         new_row = {'Method': 'ElasticNet', 'In sample R^2': r2_train_enet, 'Out of s
          df = df.append(new row, ignore index = True)
          df
          /var/folders/g8/bnlxdn656x5f3c7v8y4v67lr0000gn/T/ipykernel 1883/951802392.p
          y:2: FutureWarning: The frame.append method is deprecated and will be remove
         d from pandas in a future version. Use pandas.concat instead.
           df = df.append(new row, ignore index = True)
               Method In sample R^2 Out of sample R^2
Out [233]:
          O Straw Man
                           0.419939
                                            0.180263
                                           0.459556
           1 ElasticNet
                           0.599719
```

3. MLP: Tune Multiple layer Perceptron using all 3 features on the training data (using scikit-learn)

```
In [26]: # Column Transformer
         mlp col tf = ColumnTransformer(transformers = [
              ('num', num_tf, num_features),
              ('cat', cat tf, cat features)
         ])
In [27]: mlp mod = MLPRegressor(
           hidden layer sizes = (8, 4),
           activation = 'relu',
           solver = 'adam',
           batch size = 16,
           random state = 425
           )
In [28]: mlp_pipe = Pipeline(steps = [
              ("col_tf", mlp_col_tf),
              ("model", mlp_mod)
         ])
In [29]: hls_grid = [(4), (8), (12), (4, 2), (8, 4), (12, 6)] # hidden layer size
         bs_grid = [40, 120, 200, 280] # batch sizes
         mlp_tuned_parameters = {
           "model__hidden_layer_sizes": hls_grid,
           "model batch_size": bs_grid
           }
In [30]: | mlp_search = GridSearchCV(
           mlp_pipe,
           mlp tuned parameters,
           cv = TimeSeriesSplit(5),
           scoring = 'r2',
```

```
refit = True
In [31]: tic = time.time()
         mlp_search.fit(X_train, y_train)
         toc = time.time()
         print('Execution time: ', toc-tic, "seconds")
         Execution time: 52.096120834350586 seconds
In [32]:
         cv res = pd.DataFrame({
           "hidden_layers": np.array(mlp_search.cv_results_["param_model__hidden_laye
           "r2": mlp_search.cv_results_["mean_test_score"],
           "batch_size": mlp_search.cv_results_["param_model__batch_size"]
           })
         plt.figure()
         sns.relplot(
           # kind = "line",
           data = cv res,
           x = "batch size",
           y = "r2",
           hue = "hidden_layers"
           ).set(
             xlabel = "batch size",
             ylabel = "CV R2"
         plt.show()
         <Figure size 640x480 with 0 Axes>
```



```
Out[33]: Pipeline(steps=[('col_tf',
                           ColumnTransformer(transformers=[('num',
                                                             Pipeline(steps=[('std',
                                                                               StandardS
         caler())]),
                                                             Index(['DJ return lag1',
          'DJ_return_lag2', 'DJ_return_lag3', 'DJ_return_lag4',
                 'DJ_return_lag5', 'log_volatility_lag1', 'log_volatility_lag2',
                 'log_volatility_lag3', 'log_volatility_lag4', 'log_volatility_lag5',
                 'log_volume_lag1', 'log_volume_lag2', 'log_volume_lag3',
                 'log_volume_lag4', 'log_volume_lag5'],
                dtype='object')),
                                                            ('cat',
                                                             Pipeline(steps=[('encode
         r',
                                                                               OneHotEnc
         oder(drop='first')),
                                                                              ('std',
                                                                               StandardS
         caler(with mean=False))]),
                                                             Index(['day of week'], dty
         pe='object'))])),
                          ('model',
                           MLPRegressor(batch_size=40, hidden_layer_sizes=8,
                                        random state=425))])
In [34]: r2_train_mlp = r2_score(
           y train,
           mlp search.best estimator .predict(X train)
         r2 train mlp
         0.6039243090552505
Out[34]:
In [35]: r2_test_mlp = r2_score(
           y_test,
           mlp search.best estimator .predict(X test)
         r2 test mlp
         0.42431859461874455
Out[35]:
In [43]: new row = {'Method': 'MLP', 'In sample R^2': r2 train mlp, 'Out of sample R^
         df = df.append(new row, ignore index = True)
         df
         /var/folders/g8/bnlxdn656x5f3c7v8y4v67lr0000gn/T/ipykernel_706/1369428992.p
         y:2: FutureWarning: The frame.append method is deprecated and will be remove
         d from pandas in a future version. Use pandas.concat instead.
           df = df.append(new_row, ignore_index = True)
              Method In sample R^2 Out of sample R^2
Out [43]:
         O Straw Man
                          0.419939
                                          0.180263
          1 ElasticNet
                          0.599719
                                          0.459556
                          0.603924
          2
                 MLP
                                          0.424319
```

4. LSTM: Long Short-Term Memory networks are a special kind of RNN capable of learning long-term dependencies

```
In [72]: train val split fraction = 0.8
         train split = int(train val split fraction * int(X train.shape[0]))
         predictors = ['log_volatility', 'DJ_return', 'log_volume']
         batch size = 4
         learning rate = 0.001
         epochs = 20
         sequence length = 5
In [73]: train data = NYSE[predictors].iloc[0 : train split - L -1]
         X train2 = train data[[i for i in predictors]].values
         y train = NYSE['log volume'].iloc[L:train split]
         dataset train = keras.preprocessing.timeseries dataset from array(
            X train2,
            y_train,
            sequence_length = sequence_length,
            sampling rate = 1,
            batch size = batch size,
            shuffle = False
In [74]: # Sanity Check
         for batch in dataset train.take(1):
             inputs, targets = batch
         print("Input shape: ", inputs.numpy().shape)
         print("Target shape: ", targets.numpy().shape)
         #inputs.numpy()
         Input shape: (4, 5, 3)
         Target shape: (4,)
In [75]: val data = NYSE[predictors].iloc[(train split - L):(X train.shape[0] - 2)]
         X val = val data[[i for i in predictors]].values
         y val = NYSE['log volume'].iloc[train split:(X train.shape[0] + L -2)]
         dataset_val = keras.preprocessing.timeseries_dataset_from_array(
            X val,
            y_val,
            sequence_length = sequence_length,
            sampling rate = 1,
            batch size = batch size
In [76]:
        inputs = keras.layers.Input(
            shape = (inputs.shape[1], inputs.shape[2])
         lstm out = keras.layers.LSTM(12)(inputs)
         outputs = keras.layers.Dense(1)(lstm out)
In [77]: model = keras.Model(
            inputs = inputs,
            outputs = outputs
         model.compile(
            optimizer = keras.optimizers.Adam(learning rate = learning rate),
            loss = 'mse',
            metrics = [tf.keras.metrics.CosineSimilarity(axis = 1)]
         model.summary()
```

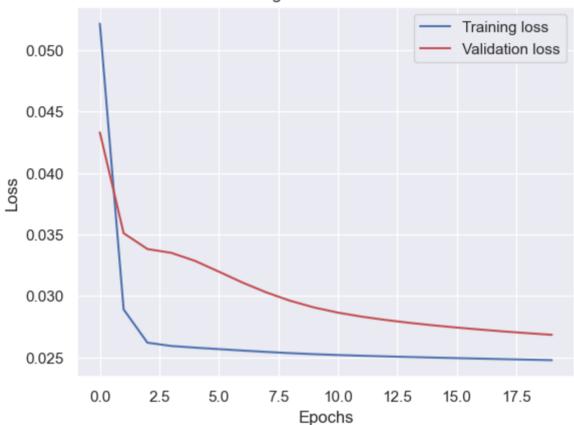
```
Layer (type)
                        Output Shape
                                           Param #
      _____
       input 3 (InputLayer)
                      [(None, 5, 3)]
                        (None, 12)
                                            768
       lstm_1 (LSTM)
                          (None, 1)
       dense_1 (Dense)
                                            13
      ______
      Total params: 781
      Trainable params: 781
      Non-trainable params: 0
In [78]: history = model.fit(
        dataset_train,
        epochs = epochs,
       validation data = dataset val,
       verbose = 2
```

```
Epoch 1/20
854/854 - 5s - loss: 0.0521 - cosine similarity: 0.1670 - val loss: 0.0433 -
val cosine similarity: 0.1612 - 5s/epoch - 6ms/step
Epoch 2/20
854/854 - 3s - loss: 0.0289 - cosine similarity: 0.5395 - val loss: 0.0351 -
val cosine similarity: 0.3879 - 3s/epoch - 3ms/step
854/854 - 2s - loss: 0.0262 - cosine similarity: 0.5694 - val loss: 0.0338 -
val cosine similarity: 0.4182 - 2s/epoch - 3ms/step
Epoch 4/20
854/854 - 2s - loss: 0.0259 - cosine similarity: 0.5735 - val loss: 0.0335 -
val cosine similarity: 0.4182 - 2s/epoch - 3ms/step
Epoch 5/20
854/854 - 2s - loss: 0.0258 - cosine similarity: 0.5776 - val loss: 0.0328 -
val cosine similarity: 0.4252 - 2s/epoch - 3ms/step
Epoch 6/20
854/854 - 2s - loss: 0.0257 - cosine similarity: 0.5811 - val loss: 0.0320 -
val_cosine_similarity: 0.4439 - 2s/epoch - 3ms/step
Epoch 7/20
854/854 - 2s - loss: 0.0255 - cosine similarity: 0.5806 - val loss: 0.0311 -
val cosine similarity: 0.4533 - 2s/epoch - 3ms/step
Epoch 8/20
854/854 - 3s - loss: 0.0254 - cosine similarity: 0.5794 - val loss: 0.0303 -
val_cosine_similarity: 0.4813 - 3s/epoch - 3ms/step
Epoch 9/20
854/854 - 2s - loss: 0.0253 - cosine similarity: 0.5753 - val loss: 0.0296 -
val cosine similarity: 0.4883 - 2s/epoch - 3ms/step
Epoch 10/20
854/854 - 2s - loss: 0.0252 - cosine similarity: 0.5747 - val loss: 0.0291 -
val cosine similarity: 0.4883 - 2s/epoch - 3ms/step
Epoch 11/20
854/854 - 2s - loss: 0.0252 - cosine similarity: 0.5747 - val loss: 0.0286 -
val_cosine_similarity: 0.5000 - 2s/epoch - 3ms/step
Epoch 12/20
854/854 - 2s - loss: 0.0251 - cosine similarity: 0.5764 - val loss: 0.0283 -
val cosine similarity: 0.4977 - 2s/epoch - 3ms/step
Epoch 13/20
854/854 - 2s - loss: 0.0251 - cosine similarity: 0.5764 - val loss: 0.0280 -
val cosine similarity: 0.5000 - 2s/epoch - 3ms/step
Epoch 14/20
854/854 - 2s - loss: 0.0250 - cosine similarity: 0.5794 - val loss: 0.0278 -
val_cosine_similarity: 0.4930 - 2s/epoch - 3ms/step
Epoch 15/20
854/854 - 2s - loss: 0.0250 - cosine similarity: 0.5794 - val loss: 0.0276 -
val cosine similarity: 0.4836 - 2s/epoch - 3ms/step
Epoch 16/20
854/854 - 3s - loss: 0.0249 - cosine_similarity: 0.5776 - val_loss: 0.0274 -
val cosine similarity: 0.4790 - 3s/epoch - 3ms/step
854/854 - 3s - loss: 0.0249 - cosine similarity: 0.5788 - val loss: 0.0272 -
val cosine similarity: 0.4883 - 3s/epoch - 3ms/step
Epoch 18/20
854/854 - 2s - loss: 0.0248 - cosine_similarity: 0.5776 - val_loss: 0.0271 -
val cosine similarity: 0.4860 - 2s/epoch - 3ms/step
Epoch 19/20
854/854 - 2s - loss: 0.0248 - cosine_similarity: 0.5788 - val_loss: 0.0270 -
val_cosine_similarity: 0.4930 - 2s/epoch - 3ms/step
Epoch 20/20
854/854 - 2s - loss: 0.0248 - cosine similarity: 0.5776 - val loss: 0.0268 -
val cosine similarity: 0.4930 - 2s/epoch - 3ms/step
```

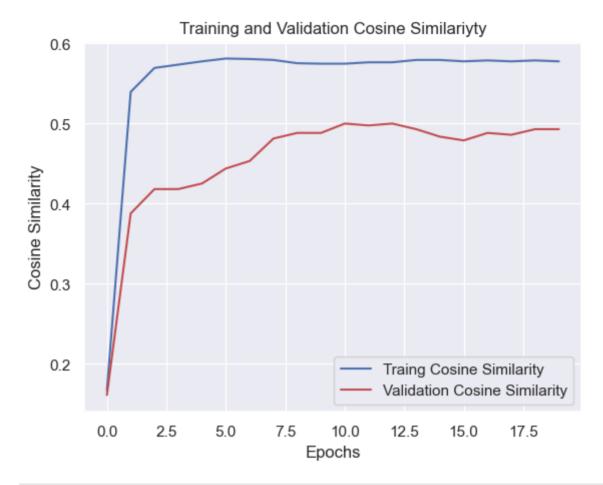
```
In [79]: def visualize_loss(history, title):
    loss = history.history['loss']
    val_loss = history.history['val_loss']
```

```
epochs = range(len(loss))
  plt.figure()
  plt.plot(epochs, loss, 'b', label = 'Training loss')
  plt.plot(epochs, val_loss, 'r', label = 'Validation loss')
  plt.title(title)
  plt.xlabel('Epochs')
  plt.ylabel('Loss')
  plt.legend()
  plt.show()
visualize_loss(history, "Training and Validation Loss")
```

Training and Validation Loss



```
In [80]:
    def visulize_cossim(history, title):
        cossim = history.history["cosine_similarity"]
        val_cossim = history.history["val_cosine_similarity"]
        epochs = range(len(cossim))
        plt.figure()
        plt.plot(epochs, cossim, 'b', label = "Traing Cosine Similarity")
        plt.plot(epochs, val_cossim, 'r', label = "Validation Cosine Similarity"
        plt.title(title)
        plt.xlabel('Epochs')
        plt.ylabel("Cosine Similarity")
        plt.legend()
        plt.show()
visulize_cossim(history, "Training and Validation Cosine Similarityty")
```



```
In [122... train_data = NYSE[predictors].iloc[0 : X_train.shape[0] + 4]
          dataset_train = keras.preprocessing.timeseries_dataset_from_array(
             train data[[i for i in predictors]].values,
             y train,
             sequence length = sequence length,
             sampling_rate = 1,
             batch_size = batch_size,
In [123...
         r2 train lstm = r2 score(
             y_train,
             np.c_[model.predict(
                dataset_train,
                batch size = batch_size,
                verbose = 2
             )].flatten()
          r2_train_lstm
          1071/1071 - 2s - 2s/epoch - 2ms/step
          0.4065061429235378
Out[123]:
In [107... | test_data = NYSE[predictors].iloc[NYSE_train.shape[0] - 5:]
          dataset_test = keras.preprocessing.timeseries_dataset_from_array(
             test_data[[i for i in predictors]].values,
             y test,
             sequence length = sequence length,
             sampling rate = 1,
             batch_size = batch_size,
```

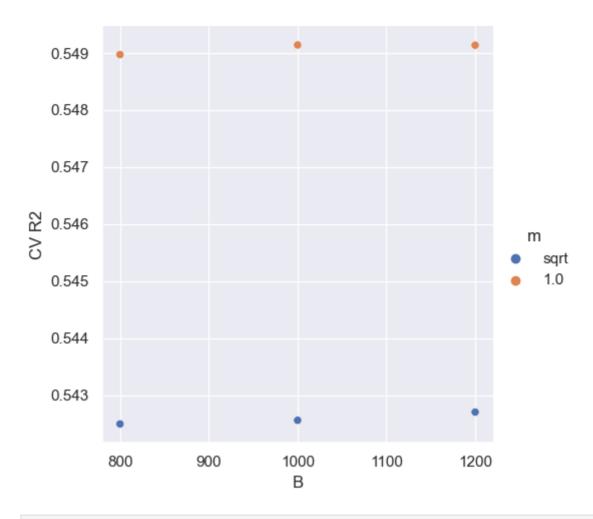
In [117... | # Sanity Check

for batch in dataset_test.take(1):

```
inputs, targets = batch
          print("Input shape: ", inputs.numpy().shape)
          print("Target shape: ", targets.numpy().shape)
          #inputs.numpy()
          #targets.numpy()
          #NYSE[predictors].iloc[NYSE train.shape[0]:(NYSE train.shape[0] + 7)]
          Input shape: (4, 5, 3)
          Target shape: (4,)
In [110...
         score, cossim = model.evaluate(
             dataset test,
             batch size = batch size,
             verbose = 2
          print('Test score: ', score)
          print('Test metric: ', cossim)
          443/443 - 1s - loss: 0.0377 - cosine similarity: 0.4531 - 974ms/epoch - 2ms/
          step
          Test score: 0.037653662264347076
          Test metric: 0.4531073570251465
In [111... | r2_test_lstm = r2_score(
             y test,
             np.c_[model.predict(
                dataset test,
                batch size = batch size,
                verbose = 2
             )].flatten()
          r2 test 1stm
          443/443 - 1s - 860ms/epoch - 2ms/step
Out[111]: 0.34550259028577346
In [235... | new row = {'Method': 'LSTM', 'In sample R^2': r2 train lstm, 'Out of sample
          df = df.append(new row, ignore index = True)
          df
          /var/folders/g8/bnlxdn656x5f3c7v8y4v67lr0000gn/T/ipykernel 1883/1975925947.p
          y:2: FutureWarning: The frame.append method is deprecated and will be remove
          d from pandas in a future version. Use pandas.concat instead.
           df = df.append(new row, ignore index = True)
Out [235]:
               Method In sample R^2 Out of sample R^2
           0 Straw Man
                           0.419939
                                            0.180263
           1 ElasticNet
                           0.599719
                                           0.459556
           2
                  MLP
                           0.603924
                                            0.424319
           3
                 LSTM
                           0.406506
                                           0.345503
```

5. Random Forest: Use the same features as in ElasticNet for the random forest

```
In [13]: rf mod = RandomForestRegressor(
           # Number of trees
           n estimators = 100,
           criterion = 'squared_error',
           # Number of features to use in each split
           max features = 'sqrt',
           oob score = True,
           random state = 425
           )
In [14]: rf pipe = Pipeline(steps = [
             ("col tf", rf col tf),
              ("model", rf mod)
         ])
In [15]: # Tune hyper-parameter(s)
         B_{grid} = [800, 1000, 1200]
         m_grid = ['sqrt', 1.0] # max_features = 1.0 uses all features
         rf tuned parameters = {
           "model n estimators": B grid,
           "model__max_features": m_grid
In [16]: rf_search = GridSearchCV(
           rf pipe,
           rf tuned parameters,
           cv = TimeSeriesSplit(5),
           scoring = 'r2',
           refit = True
In [17]: tic = time.time()
         rf search.fit(X train, y train)
         toc = time.time()
         print('Execution time: ', toc-tic, "seconds")
         Execution time: 450.868732213974 seconds
In [18]: cv_res = pd.DataFrame({
           "B": np.array(rf_search.cv_results_["param_model__n_estimators"]),
            "r2": rf search.cv results ["mean test score"],
           "m": rf_search.cv_results_["param_model__max_features"]
           })
         plt.figure()
         sns.relplot(
           # kind = "line",
           data = cv_res,
           x = "B",
           y = "r2",
           hue = "m",
           ).set(
             xlabel = "B",
             ylabel = "CV R2"
         );
         plt.show()
```

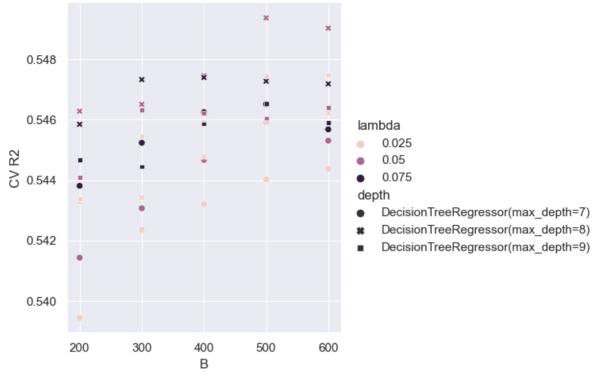


```
In [19]:
         rf_search.best_estimator_
         Pipeline(steps=[('col_tf',
Out[19]:
                            ColumnTransformer(transformers=[('num',
                                                               Pipeline(steps=[('std',
                                                                                  StandardS
          caler())]),
                                                               Index(['DJ_return_lag1',
          'DJ return lag2', 'DJ return lag3', 'DJ return lag4',
                 'DJ_return_lag5', 'log_volatility_lag1', 'log_volatility_lag2',
                  'log_volatility_lag3', 'log_volatility_lag4', 'log_volatility_lag5',
                 'log_volume_lag1', 'log_volume_lag2', 'log_volume_lag3', 'log_volume_lag4', 'log_volume_lag5'],
                dtype='object')),
                                                              ('cat',
                                                               Pipeline(steps=[('encode
          r',
                                                                                  OneHotEnc
          oder(drop='first')),
                                                                                 ('std',
                                                                                  StandardS
          caler(with_mean=False))]),
                                                               Index(['day_of_week'], dty
          pe='object'))])),
                           ('model',
                            RandomForestRegressor(max_features=1.0, n_estimators=1000,
                                                   oob score=True, random state=425))])
In [20]: r2_train_rf = r2_score(
            y_train,
            rf_search.best_estimator_.predict(X_train)
          r2 train rf
```

```
Out[20]: 0.9444562218455823
In [21]:
         r2_test_rf = r2_score(
           y test,
            rf search.best estimator .predict(X test)
          r2 test rf
          0.42534024466197673
Out[21]:
In [45]: new row = {'Method': 'Random Forest', 'In sample R^2': r2 train rf, 'Out of
          df = df.append(new row, ignore index = True)
          /var/folders/g8/bnlxdn656x5f3c7v8y4v67lr0000gn/T/ipykernel 706/1244617294.p
         y:2: FutureWarning: The frame.append method is deprecated and will be remove
         d from pandas in a future version. Use pandas.concat instead.
           df = df.append(new row, ignore index = True)
Out [45]:
                  Method In sample R^2 Out of sample R^2
          0
                Straw Man
                              0.419939
                                              0.180263
          1
                ElasticNet
                              0.599719
                                              0.459556
          2
                    MLP
                             0.603924
                                              0.424319
          3
                   LSTM
                             0.406506
                                              0.345503
          4 Random Forest
                             0.944456
                                              0.425340
         6. Boosting: Use the same features as in ElasticNet for boosting
```

```
In [23]: # Column Transformer
         bst col tf = ColumnTransformer(transformers = [
             ('num', num_tf, num_features),
             ('cat', cat_tf, cat_features)
         ])
In [24]:
         from sklearn.ensemble import AdaBoostRegressor
         from sklearn.tree import DecisionTreeRegressor
         bst_mod = AdaBoostRegressor(
           # Default base estimator is DecisionTreeRegressor with max depth = 3
           base estimator = DecisionTreeRegressor(max depth = 3),
           # Number of trees (to be tuned)
           n = 50,
           # Learning rate (to be tuned)
           learning_rate = 1.0,
           random state = 425
In [25]: bst_pipe = Pipeline(steps = [
             ("col_tf", bst_col_tf),
             ("model", bst mod)
         ])
In [26]: # Tune hyper-parameter(s)
         d_grid = [
           DecisionTreeRegressor(max_depth = 7),
           DecisionTreeRegressor(max_depth = 8),
           DecisionTreeRegressor(max depth = 9)
```

```
- 1
         B_{grid} = [200, 300, 400, 500, 600]
         lambda grid = [0.025, 0.05, 0.075]
         bst tuned parameters = {
           "model base estimator": d grid,
            "model__n_estimators": B_grid,
            "model learning rate": lambda grid
           }
In [27]: bst search = GridSearchCV(
           bst pipe,
           bst tuned parameters,
           cv = TimeSeriesSplit(5),
           scoring = 'r2',
           refit = True
In [28]: tic = time.time()
         bst_search.fit(X_train, y_train)
         toc = time.time()
         print('Execution time: ', toc-tic, "seconds")
         Execution time: 1464.862160205841 seconds
In [29]:
        cv res = pd.DataFrame({
           "B": np.array(bst search.cv results ["param model n estimators"]),
           "r2": bst_search.cv_results_["mean_test_score"],
           "lambda": bst_search.cv_results_["param_model__learning_rate"],
           "depth": bst_search.cv_results_["param_model__base_estimator"],
           })
         plt.figure()
         sns.relplot(
           # kind = "line",
           data = cv res,
           x = "B"
           y = "r2",
           hue = "lambda",
           style = "depth"
           ).set(
             xlabel = "B",
             ylabel = "CV R2"
         plt.show()
```



```
In [30]:
         bst_search.best_estimator_
         Pipeline(steps=[('col_tf',
Out[30]:
                           ColumnTransformer(transformers=[('num',
                                                             Pipeline(steps=[('std',
                                                                              StandardS
         caler())]),
                                                             Index(['DJ return lag1',
          'DJ_return_lag2', 'DJ_return_lag3', 'DJ_return_lag4',
                 'DJ_return_lag5', 'log_volatility_lag1', 'log_volatility_lag2',
                 'log_volatility_lag3', 'log_volatility_lag4', 'log_volatility_lag5',
                'log_volume_lag1', 'log_volume_lag2', 'log_volume_lag3',
                 'log_volume_lag4', 'log_volume_lag5'],
               dtype='object')),
                                                            ('cat',
                                                             Pipeline(steps=[('encode
         r',
                                                                              OneHotEnc
         oder(drop='first')),
                                                                             ('std',
                                                                              StandardS
         caler(with mean=False))]),
                                                             Index(['day_of_week'], dty
         pe='object'))])),
                          ('model',
                           AdaBoostRegressor(base_estimator=DecisionTreeRegressor(max_
         depth=8),
                                             learning_rate=0.05, n_estimators=500,
                                             random state=425))])
In [31]: r2_train_bst = r2_score(
           bst_search.best_estimator_.predict(X_train)
         r2_train_bst
         0.8220634337134233
Out[31]:
```

```
In [32]:
         r2 test bst = r2 score(
            y test,
            bst search.best estimator .predict(X test)
          r2 test bst
          0.425401004429537
Out[32]:
In [46]: new row = {'Method': 'Boosting', 'In sample R^2': r2 train bst, 'Out of samp
          df = df.append(new row, ignore index = True)
          /var/folders/g8/bnlxdn656x5f3c7v8y4v67lr0000gn/T/ipykernel 706/1878409706.p
          y:2: FutureWarning: The frame.append method is deprecated and will be remove
          d from pandas in a future version. Use pandas.concat instead.
           df = df.append(new_row, ignore_index = True)
                  Method In sample R^2 Out of sample R^2
Out[46]:
                Straw Man
                              0.419939
                                               0.180263
                ElasticNet
                              0.599719
                                               0.459556
          1
          2
                     MLP
                              0.603924
                                               0.424319
          3
                    LSTM
                              0.406506
                                               0.345503
            Random Forest
                                               0.425340
          4
                              0.944456
                 Boosting
                              0.822063
                                               0.425401
```

Summary

```
In [47]:
           df
Out [47]:
                    Method In sample R^2 Out of sample R^2
           0
                  Straw Man
                                                     0.180263
                                  0.419939
           1
                  ElasticNet
                                  0.599719
                                                     0.459556
           2
                       MLP
                                  0.603924
                                                     0.424319
           3
                      LSTM
                                  0.406506
                                                     0.345503
              Random Forest
                                  0.944456
                                                     0.425340
                    Boosting
                                  0.822063
                                                     0.425401
```

From the result we can see, In sense of in-sample \mathbb{R}^2 , Random Forest has the highest 0.94, however it's too high and clearly overfitting. Also, LSTM is weaker than the baseline method, this might be not well-trained. In sense of out-of-sample \mathbb{R}^2 , Boosting performed slightly better than Random Forest, while elasticnet perform the best, this again tell us simple method can do a great job. besides, LSTM perform weakly but still better than baseline method, this might also be not well-trained.