Question 7:

a) 3.1.1 a
True. The set A contains all integers that multiple of 3. Since $27 = 3 \times 9$ and 9 is an integer, then 27 is an integer multiple of 3. Thus, $27 \in A$ is true.

3.1.1 b
False. The set B contains all integers that if there is an integer y such that $x = y^2$. Since $5^2 = 25,\ 6^2 = 36$, there is no integer y such that $y^2 = 27$. Thus, $27 \notin B$, $27 \in B$ is false.

3.1.1 c
True. The set B contains all integers that if there is an integer y such that $x = y^2$. Since $10^2 = 100$ and 10 is an integer, then 100 is a perfect square. Thus, $100 \in B$ is true.

3.1.1 d
False. $E = \{3, 6, 9\}$ is not a subset of C, because $3 \in E$ and $3 \notin C$, so $E \subseteq C$ is false.
$C = \{4, 5, 9, 10\}$ is not a subset of E, because $4 \in C$ and $4 \notin E$, so $C \subseteq E$ is false.
$E \subseteq C\ or\ C \subseteq E$ indicates F or F = F. Thus, $E \subseteq C\ or\ C \subseteq E$ is false.

3.1.1 e
True. $E = \{3, 6, 9\}$, A contains all integers that multiple of 3. Since $3 \times 1 = 3$ and 1 is an integer, then 3 is an integer multiple of 3. Since $3 \times 2 = 6$ and 2 is an integer, then 6 is an integer multiple of 3. Since $3 \times 3 = 9$ and 3 is an integer, then 9 is an integer multiple of 3. Thus, $3 \in A$, $6 \in A$, $9 \in A$, every element of E is also an element of A, E is a subset of A, so $E \subseteq A$ is true.

3.1.1 f
False. A contains all integers that multiple of 3. Since $4 \times 3 = 12$ and 4 is an integer, then 12 is an integer multiple of 3, $12 \in A$. However, $12 \notin E$, there at least one element of A is not an element of E. Thus, A is not a subset of E, so $A \subseteq E$ is false.

3.1.1 g
False. The set A contains all integers that multiple of 3, the elements of A are all integers, so there is no set as an element of A. However, E is a set. Thus, E is not an element of A, $E \notin A$. So $E \in A$ is false.

b) 3.1.2 a
False. 15 is an integer, not a set. Thus, 15 is not a subset of A. $15 \subset A$ is false.

3.1.2 b

True. The set A contains all integers that multiple of 3. Since $15 = 3 \times 5$, 5 is an integer, then 15 is an integer multiple of 3, $15 \in A$. The set $\{15\}$ only has one element 15, so every element of $\{15\}$ is also an element of A, $\{15\} \subseteq A$. A has more than one elements, indicates $\{15\} \neq A$, then $\{15\}$ is a proper subset of A, $\{15\} \subset A$ is true.

3.1.2 c

True. $\varnothing$ is an empty set, empty set is a subset of every set. $C = \{4, 5, 9, 10\}$ is non-empty set, there contains elements that $\varnothing$ doesn't have, then $\varnothing$ is a proper subset of C. So $\varnothing \subset C$ is true.

3.1.2 d

True. All elements of D are in D, so every set is a subset of itself. $D \subseteq D$ is true.

3.1.2 e

False. $\varnothing$ is an empty set. B contains all integers that are perfect square, there is no set in B. Thus, $\varnothing$ is not an element of B, $\varnothing \in B$ is false.

c) 3.1.5 b

Answer is: $\{x \in \mathbb{Z}^+ : x = 3k,\ k \in \mathbb{Z}^+\}$**. The set is infinite.**

Since $3 = 1 \times 3$

$6 = 2 \times 3$

$9 = 3 \times 3$

$12 = 4 \times 3$

$\dots$

Then x is a positive integer: $x \in \mathbb{Z}^+$**,**

And x is multiple of 3: $x = 3k,\ k \in \mathbb{Z}^+$**,**

Thus, $\{3, 6, 9, 12, \ldots\}$ is $\{x \in \mathbb{Z}^+ : x = 3k,\ k \in \mathbb{Z}^+\}$ and the set is infinite.

3.1.5 d

Answer is: $\{x \in \mathbb{N}: x = 10k,\ k \in \mathbb{N}\ and\ 0 \leq k \leq 100\}$**. Its cardinality is 101.**

Since $0 = 0 \times 10$

$10 = 1 \times 10$

$20 = 2 \times 10$

$30 = 3 \times 10$

$\dots$

$1000 = 100 \times 10$

Then x is a non-negative integer: $x \in \mathbb{N}$**,**

And x is multiple of 10: $x = 10k,\ k \in \mathbb{N}$, and $0 \leq k \leq 100$,
The cardinality is $100 + 1 = 101$,
Thus, $\{0, 10, 20, 30, \ldots 1000\}$ is $\{x \in \mathbb{N}\colon x = 10k,\ k \in \mathbb{N}\ and\ 0 \leq k \leq 100\}$ and the set is finite, its cardinality is 101.

d) 3.2.1 a
True. The element 2 is in X, so $2 \in X$ is true.

3.2.1 b
True. 2 is the only element of {2}, and 2 is also an element of X, so all elements of {2} are in X, thus $\{2\} \subseteq X$ is true.

3.2.1 c
False. The elements of X are: 1, {1}, {1, 2}, 2, {3}, 4, then {2} is not an element of X, $\{2\} \notin X$. Thus $\{2\} \in X$ is false.

3.2.1 d
False. The elements of X are: 1, {1}, {1, 2}, 2, {3}, 4, then 3 is not an element of X, $3 \notin X$. Thus $3 \in X$ is false.

3.2.1 e
True. The element $\{1, 2\}$ is in X, so $\{1, 2\} \in X$ is true.

3.2.1 f
True. 1 is the element of $\{1, 2\}$, 2 is the element of $\{1, 2\}$, and 1, 2 are also elements of X, so all elements of $\{1, 2\}$ are in X, thus $\{1, 2\} \subseteq X$ is true.

3.2.1 g
True. 2 is the element of $\{2, 4\}$, 4 is the element of $\{2, 4\}$, and 2, 4 are also elements of X, so all elements of $\{2, 4\}$ are in X, thus $\{2, 4\} \subseteq X$ is true.

3.2.1 h
False. The elements of X are: 1, {1}, {1, 2}, 2, {3}, 4, then $\{2, 4\}$ is not an element of X, $\{2, 4\} \notin X$. Thus $\{2, 4\} \in X$ is false.

3.2.1 i
False. 2 is the element of $\{2, 3\}$, 3 is the element of $\{2, 3\}$. 2 is an element of X while 3 is not, so not all elements of $\{2, 3\}$ are in X, $\{2, 3\}$ is not a subset of X. Thus $\{2, 3\} \subseteq X$ is false.

3.2.1 j

False. The elements of X are: 1, {1}, {1, 2}, 2, {3}, 4, then $\{2, 3\}$ is not an element of X, $\{2, 3\} \notin X$. Thus $\{2, 3\} \in X$ is false.

3.2.1 k

False. The elements of X are: 1, {1}, {1, 2}, 2, {3}, 4, there are 6 elements, not 7.

Question 8:

3.2.4 b

Answer is: $\{\{2\}, \{1,2\}, \ \{2,3\}, \ \{1,2,3\}\}$**.**

$P(A)$ is the power set of A, $P(A) = \{\emptyset, \ \{1\}, \ \{2\}, \ \{3\}, \ \{1,2\}, \ \{1,3\}, \ \{2,3\}, \ \{1,2,3\}\}$.
$\{X \in P(A) \colon \ 2 \in X\}$ means sets X are elements of $P(A)$ where 2 is an element of X.
So sets X contains 2 are: $\{2\}, \{1,2\}, \ \{2,3\}, \ \{1,2,3\}$.
Therefore $\{X \in P(A) \colon \ 2 \in X\} = \{\{2\}, \{1,2\}, \ \{2,3\}, \ \{1,2,3\}\}$.

Question 9:

a) 3.3.1 c

Answer is: $\{-3, 1, 17\}$**.**

Since $C = \{x \in \mathbb{Z}:\ x \text{ is odd}\} = \{..., -5, -3, -1, 1, 3, 5, ...\}$
Then $A \cap C = \{-3, 0, 1, 4, 17\} \cap \{..., -5, -3, -1, 1, 3, 5, ..., 17, ...\}$
$= \{-3, 1, 17\}$

3.3.1 d

Answer is: $\{-5, -3, 0, 1, 4, 17\}$**.**

Since $C = \{x \in \mathbb{Z}:\ x \text{ is odd}\} = \{..., -5, -3, -1, 1, 3, 5, ...\}$
Then $B \cap C = \{-12, -5, 1, 4, 6\} \cap \{... -5, -3, -1, 1, 3, 5, ...\}$
$= \{-5, 1\}$
Therefore $A \cup (B \cap C) = \{-3, 0, 1, 4, 17\} \cup \{-5, 1\}$
$= \{-5, -3, 0, 1, 4, 17\}$

3.3.1 e

Answer is: $\{1\}$**.**

Since $C = \{x \in \mathbb{Z}:\ x \text{ is odd}\} = \{..., -5, -3, -1, 1, 3, 5, ...\}$
Then $A \cap B \cap C = \{-3, 0, 1, 4, 17\} \cap \{-12, -5, 1, 4, 6\} \cap \{..., -5, -3, -1, 1, 3, 5, ...\}$
$= \{1, 4\} \cap \{..., -5, -3, -1, 1, 3, 5, ...\}$
$= \{1\}$

b) 3.3.3 a

Answer is: $\{1\}$**.**

For $\bigcap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5$

Since $A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$

$A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$

$A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$

$A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$

Then $A_2 \cap A_3 \cap A_4 \cap A_5 = \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\} = \{1\}$

Therefore $\bigcap_{i=2}^{5} A_i = \{1\}$

3.3.3 b

Answer is: $\{1, 2, 3, 4, 5, 9, 16, 25\}$**.**

For $\bigcup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5$

Since $A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$

$A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$

$A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$

$A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$

Then $A_2 \cup A_3 \cup A_4 \cup A_5 = \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\}$

$= \{1, 2, 3, 4, 5, 9, 16, 25\}$

Therefore $\bigcup_{i=2}^{5} A_i = \{1, 2, 3, 4, 5, 9, 16, 25\}$

3.3.3 e

Answer is: $\{x \in \mathbb{R}: \ -\frac{1}{100} \le x \le \frac{1}{100}\}$**.**

For $\bigcap_{i=1}^{100} C_i = C_1 \cap C_2 \cap ... \cap C_{100}$

Since $C_1 = \{x \in \mathbb{R}: \ -\frac{1}{1} \le x \le \frac{1}{1}\} = \{x \in \mathbb{R}: \ -1 \le x \le 1\}$

$C_2 = \{x \in \mathbb{R}: \ -\frac{1}{2} \le x \le \frac{1}{2}\}$

$\dots$

$C_{100} = \{x \in \mathbb{R}: \ -\frac{1}{100} \le x \le \frac{1}{100}\}$

Then

$C_1 \cap C_2 \cap ... \cap C_{100} = \{x \in \mathbb{R}: (-1 \le x \le 1) \ and \ (-\frac{1}{2} \le x \le \frac{1}{2}) \ and \ ... \ and \ (-\frac{1}{100} \le x \le \frac{1}{100})\}$

$= \{x \in \mathbb{R}: \ -\frac{1}{100} \le x \le \frac{1}{100}\}$

Therefore $\bigcap_{i=1}^{100} C_i = \{x \in \mathbb{R}: \ -\frac{1}{100} \le x \le \frac{1}{100}\}$

3.3.3 f

Answer is: $\{x \in \mathbb{R}: -1 \leq x \leq 1\}$**.**

For $\bigcup_{i=1}^{100} C_i = C_1 \cup C_2 \cup ... \cup C_{100}$

Since $C_1 = \{x \in \mathbb{R}: \ -\frac{1}{1} \leq x \leq \frac{1}{1}\} = \{x \in \mathbb{R}: -1 \leq x \leq 1\}$

$C_2 = \{x \in \mathbb{R}: \ -\frac{1}{2} \leq x \leq \frac{1}{2}\}$

...

$C_{100} = \{x \in \mathbb{R}: \ -\frac{1}{100} \leq x \leq \frac{1}{100}\}$

Then

$C_1 \cup C_2 \cup ... \cup C_{100} = \{x \in \mathbb{R}: \ (-1 \leq x \leq 1) \ or \ (-\frac{1}{2} \leq x \leq \frac{1}{2}) \ or \ ... \ or \ (-\frac{1}{100} \leq x \leq \frac{1}{100})\}$

$= \{x \in \mathbb{R}: -1 \leq x \leq 1\}$

Therefore $\bigcup_{i=1}^{100} C_i = \{x \in \mathbb{R}: -1 \leq x \leq 1\}$

c) 3.3.4 b

Answer is: $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$**.**

Since $A = \{a, b\}$ and $B = \{b, c\}$,

Then $A \cup B = \{a, b\} \cup \{b, c\} = \{a, b, c\}$

$P(A \cup B)$ is the power set of $(A \cup B)$

Thus $P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

3.3.4 d

Answer is: $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$**.**

Since $A = \{a, b\}$, $P(A)$ is the power set of A, then $P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Since $B = \{b, c\}$, $P(B)$ is the power set of B, then $P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\} \cup \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

<u>Question 10:</u>

a) 3.5.1 b

Answer is (foam, tall, non-fat).

$B \times A \times C$ is the set of ordered triple,
Denoted: $\{(b, a, c) \colon\ b \in B\ and\ a \in A\ and\ c \in C\}$,
Since: foam $\in B$, tall $\in A$, non-fat $\in C$,
Thus: (foam, tall, non-fat) $\in B \times A \times C$.

3.5.1 c

Answer is {(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}.

$B \times C = \{(b, c) \colon\ b \in B\ and\ c \in C\}$,
Since: foam $\in B$, non-foam $\in B$,
non-fat $\in C$, whole $\in C$,
Then: $B \times C =$ {(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}

b) 3.5.3 b

True.

Since $\mathbb{Z}$ is the set of all integers, $\mathbb{R}$ is the set of all real numbers include all integers,
So, all elements of $\mathbb{Z}$ are also the elements of $\mathbb{R}$, $\mathbb{Z} \subseteq \mathbb{R}$.
$\mathbb{Z}^2$ is the set of ordered pairs $\{(x, y) \colon\ x, y \in \mathbb{Z}\}$,
$\mathbb{R}^2$ is the set of ordered pairs $\{(x, y) \colon\ x, y \in \mathbb{R}\}$,
Since $\mathbb{Z} \subseteq \mathbb{R}$, we know every (x, y) in $\mathbb{Z}^2$ is also an element of $\mathbb{R}^2$
Therefore $\mathbb{Z}^2 \subseteq \mathbb{R}^2$ is true.

3.5.3 c

True.

$\mathbb{Z}^2$ is the set of ordered pairs $\{(x, y) \colon\ x, y \in \mathbb{Z}\}$,
$\mathbb{Z}^3$ is the set of ordered triple $\{(x, y, z) \colon\ x, y, z \in \mathbb{Z}\}$,
Since their elements are different, indicates every element of $\mathbb{Z}^2$ is not in $\mathbb{Z}^3$,

Thus $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset$ is true.

3.5.3 e

True.

Give $A \subseteq B$, every element in A is also in B,
So for any element a: if $a \in A$, then $a \in B$.
For $A \times C$ is the set of ordered pairs: $\{(a, c): a \in A \, and \, c \in C\}$,
Since $a \in B$ and $c \in C$, then $(a, c) \in B \times C$,
Since $(a, c) \in A \times C$, $(a, c) \in B \times C$,
Therefore $A \times C \subseteq B \times C$.

c) 3.5.6 d

Answer is $\{01, 011, 001, 0011\}$**.**

Since $\{0\}^2 = \{00\}$,
$\{1\}^2 = \{11\}$,
Then $\{0\} \cup \{0\}^2 = \{0\} \cup \{00\} = \{0, 00\}$,
$\{1\} \cup \{1\}^2 = \{1\} \cup \{11\} = \{1, 11\}$,
Sinc $x \in \{0\} \cup \{0\}^2 = \{0, 00\}$
$y \in \{1\} \cup \{1\}^2 = \{1, 11\}$
Then xy: $\{01, 011, 001, 0011\}$.

3.5.6 e

Answer is $\{aaa, aaaa, aba, abaa\}$**.**

Since $\{a\}^2 = \{aa\}$,
Then $\{a\} \cup \{a\}^2 = \{a\} \cup \{aa\} = \{a, aa\}$,
Sinc $x \in \{aa, ab\}$
$y \in \{a\} \cup \{a\}^2 = \{a, aa\}$
Then xy: $\{aaa, aaaa, aba, abaa\}$.

d) 3.5.7 c

Answer is $\{aa, ab, ac, ad\}$**.**

Since $A = \{a\},\ B = \{b, c\},\ C = \{a, b, d\}$,
Then $A \times B = \{ab, ac\}$,
$A \times C = \{aa, ab, ad\}$,
Then $(A \times B) \cup (A \times C) = \{ab, ac\} \cup \{aa, ab, ad\} = \{aa, ab, ac, ad\}$.

3.5.7 f

Answer is $\{\emptyset,\ \{ab\}, \{ac\}, \{ab, ac\}\}$**.**

Since $A = \{a\},\ B = \{b, c\}$,
Then $A \times B = \{ab, ac\}$,
Since $P(A \times B)$ is the power set of $(A \times B)$, $A \times B = \{ab, ac\}$,
Then $P(A \times B) = \{\emptyset,\ \{ab\}, \{ac\}, \{ab, ac\}\}$.

3.5.7 g

Answer is:$\{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$**.**

Since $A = \{a\},\ B = \{b, c\}$,
Then $P(A)$ is the power set of A, $P(A) = \{\emptyset, \{a\}\}$,
$P(B)$ is the power set of B, $P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$,
Then:
$P(A) \times P(B) = \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$

Question 11:

a) 3.6.2 b

Proof

$(B \cup A) \cap (\overline{B} \cup A)$

$= (A \cup B) \cap (A \cup \overline{B})$ Commutative laws

$= A \cup (B \cap \overline{B})$ Distributive laws

$= A \cup \emptyset$ Complement laws

$= A$ Identity laws

2.6.2 c

Proof

$\overline{A \cap \overline{B}}$

$= \overline{A} \cup \overline{\overline{B}}$ De Morgan's laws

$= \overline{A} \cup B$ Double complement law

b) 3.6.3 b

Assume $A = \{1, 2, 3\}$ and $B = \{2\}$

Then $B \cap A = \{2\}$,

$A - (B \cap A) = \{1, 2, 3\} - \{2\} = \{1, 3\}$.

Since $A = \{1, 2, 3\}$,

$A - (B \cap A) = \{1, 3\}$,

$\{1, 2, 3\} \neq \{1, 3\}$ which means that $A - (B \cap A) \neq A$.

Therefore, $A - (B \cap A) = A$ is not an identity.

3.6.3 d

Assume $A = \{1, 2\}$ and $B = \{1, 3, 5\}$

Then $B - A = \{1, 3, 5\} - \{1, 2\} = \{3, 5\}$,

$(B - A) \cup A = \{3, 5\} \cup \{1, 2\} = \{1, 2, 3, 5\}$

SInce $(B - A) \cup A = \{1, 2, 3, 5\}$,

$A = \{1, 2\}$,

$\{1, 2, 3, 5\} \neq \{1, 2\}$ which means that $(B - A) \cup A \neq A$.

Therefore, $(B - A) \cup A = A$ is not an identity.

c) 3.6.4 b

$A \cap (B - A)$

$= A \cap (B \cap \overline{A})$ Subtraction law

$= A \cap (\overline{A} \cap B)$ Commutative laws

$= (A \cap \overline{A}) \cap B$ Associative laws

$= \emptyset \cap B$ Complement laws

$= B \cap \emptyset$ Commutative laws

$= \emptyset$ Domination laws

3.6.4 c

$A \cup (B - A)$

$= A \cup (B \cap \overline{A})$ Subtraction law

$= (A \cup B) \cap (A \cup \overline{A})$ Distributive laws

$= (A \cup B) \cap U$ Complement laws

$= A \cup B$ Identity laws