Question 1:

A.

1. 10011011_2
   $= 1\times2^0 + 1\times2^1 + 0\times2^2 + 1\times2^3 + 1\times2^4 + 0\times2^5 + 0\times2^6 + 1\times2^7$
   $= 1 + 2 + 0 + 8 + 16 + 0 + 0 + 128$
   $=$ **155_{10}**

2. 456_7
   $= 6\times7^0 + 5\times7^1 + 4\times7^2$
   $= 6 + 35 + 196$
   $=$ **237_{10}**

3. $38A_{16}$
   $= 10\times16^0 + 8\times16^1 + 3\times16^2$
   $= 10 + 128 + 768$
   $=$ **906_{10}**

4. 2214_5
   $= 4 \times 5^0 + 1\times5^1 + 2\times5^2 + 2\times5^3$
   $= 4 + 5 + 50 + 250$
   $=$ **309_{10}**

B.

1. 69 ÷ 2 = 34 remainder 1
   34 ÷ 2 = 17 remainder 0
   17 ÷ 2 = 8 remainder 1
   8 ÷ 2 = 4 remainder 0
   4 ÷ 2 = 2 remainder 0
   2 ÷ 2 = 1 remainder 0
   1 ÷ 2 = 0 remainder 1
   69_{10} = **1000101_2**

2. 485 ÷ 2 = 242 remainder 1
   242 ÷ 2 = 121 remainder 0
   121 ÷ 2 = 60 remainder 1
   60 ÷ 2 = 30 remainder 0
   30 ÷ 2 = 15 remainder 0
   15 ÷ 2 = 7 remainder 1
   7 ÷ 2 = 3 remainder 1
   3 ÷ 2 = 1 remainder 1
   1 ÷ 2 = 0 remainder 1
   485_{10} = **111100101_2**

3. $6_{16} = 0110_2$
   $D_{16} = 1101_2$
   $1_{16} = 0001_2$
   $A_{16} = 1010_2$
   $6D1A_{16} =$ **0110110100011010_2**

C.

1. $1101011_2 = 01101011_2$
   $0110_2 = 6_{16}$
   $1011_2 = B_{16}$
   $1101011_2 =$ **$6B_{16}$**

2. 895 ÷ 16 = 55 remainder 15 (F)
   55 ÷ 16 = 3 remainder 7
   3 ÷ 16 = 0 remainder 3
   $895_{16} =$ **$37F_{16}$**

Question 2:

1.

```
      1 1 1
      7 5 6 6
  +   4 5 1 5
——----------------------
    1 4 3 0 3
```

$7566_8 + 4515_8 =$ **14303_8**

2.

```
        1 1 1 1 1 1
      1 0 1 1 0 0 1 1
  +           1 1 0 1
——-------------------------------
      1 1 0 0 0 0 0 0
```

$10110011_2 + 1101_2 =$ **11000000_2**

3.

```
      1 1
      7 A 6 6
  +   4 5 C 5
——----------------------
      C 0 2 B
```

$7A66_{16} + 45C5_{16} =$ **$C02B_{16}$**

4.

```
      -1 -1 -1
      3 0 2 2
  -   2 4 3 3
——----------------------
      0 0 3 4
```

$3022_5 - 2433_5 =$ **34_5**

Question 3:

A.

1. $124 \div 2 = 62$ remainder 0
   $62 \div 2 = 31$ remainder 0
   $31 \div 2 = 15$ remainder 1
   $15 \div 2 = 7$ remainder 1
   $7 \div 2 = 3$ remainder 1
   $3 \div 2 = 1$ remainder 1
   $1 \div 2 = 0$ remainder 1
   $124_{10} = 1111100_2$
   $124_{10} = \mathbf{01111100}_{\text{8 bit 2's comp}}$

2. Step 1: convert 124_{10} to binary:
   $124 \div 2 = 62$ remainder 0
   $62 \div 2 = 31$ remainder 0
   $31 \div 2 = 15$ remainder 1
   $15 \div 2 = 7$ remainder 1
   $7 \div 2 = 3$ remainder 1
   $3 \div 2 = 1$ remainder 1
   $1 \div 2 = 0$ remainder 1
   $124_{10} = 1111100_2$
   $124_{10} = 01111100_{\text{8 bit 2's comp}}$
   Step 2: flip all the bits: 10000011
   Step 3: add 1: 10000011 + 1 = 10000100
   Final answer: $-124_{10} = \mathbf{10000100}_{\text{8 bit 2's comp}}$

3. $109 \div 2 = 54$ remainder 1
   $54 \div 2 = 27$ remainder 0
   $27 \div 2 = 13$ remainder 1
   $13 \div 2 = 6$ remainder 1
   $6 \div 2 = 3$ remainder 0
   $3 \div 2 = 1$ remainder 1
   $1 \div 2 = 0$ remainder 1
   $109_{10} = 1101101_2$
   $109_{10} = \mathbf{01101101}_{\text{8 bit 2's comp}}$

4. Step 1: convert 79_{10} to binary:
   $79 \div 2 = 39$ remainder 1
   $39 \div 2 = 19$ remainder 1
   $19 \div 2 = 9$ remainder 1
   $9 \div 2 = 4$ remainder 1
   $4 \div 2 = 2$ remainder 0
   $2 \div 2 = 1$ remainder 0

$1 \div 2 = 0$ remainder 1

$79_{10} = 1001111_2$

$79_{10} = 01001111_{8\text{ bit 2's comp}}$

Step 2: flip all the bits: 10110000

Step 3: add 1: 10110000 + 1 = 10110001

Final answer: $-79_{10} =$ **$10110001_{8\text{ bit 2's comp}}$**

B.

1. $00011110_{8\text{ bit 2's comp}}$
   $= 0\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3 + 1\times2^4$
   $= 0 + 2 + 4 + 8 + 16$
   $=$ **30_{10}**

2. Step 1: flip all the bits: 00011001
   Step 1: add 1: 00011010
   Step 3: convert: $00011010_2 = 0\times2^0 + 1\times2^1 + 0\times2^2 + 1\times2^3 + 1\times2^4 = 26_{10}$
   Final answer: $11100110_{8\text{ bit 2's comp}} = -26_{10}$

   Quick way is:
   $11100110_{8\text{ bit 2's comp}}$
   $= 0\times2^0 + 1\times2^1 + 1\times2^2 + 0\times2^3 + 0\times2^4 + 1\times2^5 + 1\times2^6 + (-1)\times2^7$
   $= 0 + 2 + 4 + 0 + 0 + 32 + 64 - 128$
   $=$ **-26_{10}**

3. $00101101_{8\text{ bit 2's comp}}$
   $= 1\times2^0 + 0\times2^1 + 1\times2^2 + 1\times2^3 + 0\times2^4 + 1\times2^5$
   $= 1 + 0 + 4 + 8 + 0 + 32$
   $=$ **45_{10}**

4. Step 1: flip all the bits: 01100001
   Step 1: add 1: 01100010
   Step 3: convert: $01100010_2 = 0\times2^0 + 1\times2^1 + 0\times2^2 + 0\times2^3 + 0\times2^4 + 1\times2^5 + 1\times2^6 = 98_{10}$
   Final answer: $10011110_{8\text{ bit 2's comp}} = -98_{10}$

   Quick way is:
   $10011110_{8\text{ bit 2's comp}}$
   $= 0\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3 + 1\times2^4 + 0\times2^5 + 0\times2^6 + (-1)\times2^7$
   $= 0 + 2 + 4 + 8 + 16 + 0 + 0 - 128$
   $=$ **-98_{10}**

Question 4:

1. (b)

p	q	p ∨ q	¬(p ∨ q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(c)

p	q	r	¬q	p ∧ ¬q	r ∨ (p ∧ ¬q)
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. (b)

p	q	p → q	q → p	(p → q) → (q → p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d)

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5:

1. (b) **$(B \land D) \lor (B \land M) \lor (D \land M)$**
   (c) **$B \lor (D \land M)$**

2. (b) means if s or y, then p: **$(s \lor y) \rightarrow p$**
   (c) means y is necessary for p : **$p \rightarrow y$**
   (d) mean p if and only if s and y: **$p \leftrightarrow (s \land y)$**
   (e) means p implies either s or y: **$p \rightarrow (s \lor y)$**

3. (c) means c only if p: **$c \rightarrow p$**
   (d) means p is necessary for c: **$c \rightarrow p$**

Question 6:

1. (b) **If Joe is eligible for the honors program, then he maintains a B average.**
   (c) **If Rajiv can go on the roller coaster, then he is at least four feet tall.**
   (d) **If Rajiv is at least four feet tall, then he can go on the roller coaster.**

2. (c) $(p \lor r) \leftrightarrow (q \land r)$
   $(T \lor r) \leftrightarrow (F \land r)$
   $T \leftrightarrow F$
   F

   (d) $(p \land r) \leftrightarrow (q \land r)$
   $(T \land r) \leftrightarrow (F \land r)$
   $r \leftrightarrow F$
   Unknow

   (e) $p \rightarrow (r \lor q)$
   $T \rightarrow (r \lor F)$
   $T \rightarrow r$
   Unknow

   (f) $(p \land q) \rightarrow r$
   $(T \land F) \rightarrow r$
   $F \rightarrow r$
   T

Question 7:

Truth table:

j	l	r	¬j	¬l	¬r	l ∨ ¬r	r ∧ ¬l	¬j → (l ∨ ¬r)	(r ∧ ¬l) → j	j → ¬l	¬j → l	j → (r ∧ ¬l)
T	T	T	F	F	F	T	F	T	T	F	T	F
T	T	F	F	F	T	T	F	T	T	F	T	F
T	F	T	F	T	F	F	T	T	T	T	T	T
T	F	F	F	T	T	T	F	T	T	T	T	F
F	T	T	T	F	F	T	F	T	T	T	T	T
F	T	F	T	F	T	T	F	T	T	T	T	T
F	F	T	T	T	F	F	T	F	F	T	F	T
F	F	F	T	T	T	T	F	T	T	T	F	T

(b) **¬j → (l ∨ ¬r)**
(r ∧ ¬l) → j
Logically equivalent (Because they have the same truth values for all values.)

(c) **j → ¬l**
¬j → l
Not logically equivalent (Because when j = T, l = T, r = T, j → ¬l is F, while ¬j → l is T.)

(d) **(r ∨ ¬l) → j**
j → (r ∧ ¬l)
Not logically equivalent (Because when j = T, l = T, r = T, (r ∧ ¬l) → j is T, while j → (r ∧ ¬l) is F.)

Question 8:

1. (c) $(p \rightarrow q) \land (p \rightarrow r)$
   $\equiv (\neg p \lor q) \land (\neg p \lor r)$ [Conditional identities]
   $\equiv \neg p \lor (q \land r)$ [Distributive laws]
   $\equiv p \rightarrow (q \land r)$ [Conditional identities]

   (f) $\neg(p \lor (\neg p \land q))$
   $\equiv \neg((p \lor \neg p) \land (p \lor q))$ [Distributive identities]
   $\equiv \neg(T \land (p \lor q))$ [Complement laws]
   $\equiv \neg((p \lor q) \land T)$ [Commutative laws]
   $\equiv \neg(p \lor q)$ [Identity laws]
   $\equiv \neg p \land \neg q$ [De Morgan's laws]

   (i) $(p \land q) \rightarrow r$
   $\equiv \neg(p \land q) \lor r$ [Conditional identities]
   $\equiv \neg p \lor \neg q \lor r$ [De Morgan's laws]
   $\equiv \neg p \lor (\neg q \lor r)$ [Associative laws]
   $\equiv \neg p \lor (r \lor \neg q)$ [Commutative laws]
   $\equiv (\neg p \lor r) \lor \neg q$ [Associative laws]
   $\equiv \neg(p \text{ v } \neg r) \lor \neg q$ [De Morgan's laws]
   $\equiv (p \text{ v } \neg r) \rightarrow \neg q$ [Conditional identities]

2. (c) $\neg r \lor (\neg r \rightarrow p)$
   $\equiv \neg r \lor (\neg\neg r \lor p)$ [Conditional identities]
   $\equiv \neg r \lor (r \lor p)$ [Double negation law]
   $\equiv (\neg r \lor r) \lor p$ [Associative laws]
   $\equiv T \lor p$ [Complement laws]
   $\equiv p \lor T$ [Commutative laws]
   $\equiv T$ [Domination laws]

   (d) $\neg(p \rightarrow q) \rightarrow \neg q$
   $\equiv \neg(\neg p \lor q) \rightarrow \neg q$ [Conditional identities]
   $\equiv \neg\neg(\neg p \lor q) \lor \neg q$ [Conditional identities]
   $\equiv (\neg p \lor q) \lor \neg q$ [Double negation law]
   $\equiv \neg p \lor (q \lor \neg q)$ [Associative laws]
   $\equiv \neg p \lor T$ [Complement laws]
   $\equiv T$ [Domination laws]

Question 9:

1. (c) **$\exists x(x = x^2)$**
   (d) **$\forall x(x \leq x^2 + 1)$**

2. (b) **$\forall x(\neg S(x) \land W(x))$**
   For everyone: $\forall x$
   Was well means not sick: $\neg S(x)$
   Was well and went to work: $\neg S(x) \land W(x)$
   Everyone was well and went to work yesterday: $\forall x(\neg S(x) \land W(x))$

   (c) **$\forall x(S(x) \to \neg W(x))$**
   For everyone: $\forall x$
   Did not go to work: $\neg W(x)$
   If sick then not go to work: $S(x) \to \neg W(x)$
   Everyone who was sick yesterday did not go to work: $\forall x(S(x) \to \neg W(x))$

   (d) **$\exists x(S(x) \land W(x))$**
   Someone: $\exists x$
   Was sick and went to work: $S(x) \land W(x)$
   Yesterday someone was sick and went to work: $\exists x(S(x) \land W(x))$

Question 10:

1. (c) **True.** When x = a, b, d, e, P(x) = T, makes (x = c) → P(x) true, because F → T is T, there exists one value in the domain set. So the quantified expression $\exists x((x = c) \rightarrow P(x))$ evaluates to true.

   (d) **True.** When x = e, Q(e) = T, R(e) = T, makes Q(x) ∧ R(x) true, there exists one value in the domain. So the quantified expression $\exists x(Q(x) \land R(x))$ evaluates true.

   (e) **True.** Q(a) = T, P(d) = T, T ∧ T is T, so the quantified expression Q(a) ∧ P(d) evaluates true.

   (f) **True.** When x = b, Q(x) = F, when x = a, c, d, e, Q(x) = T. So, for all x ≠ b, Q(x) = T. $\forall x((x \neq b) \rightarrow Q(x))$ evaluates true T.

   (g) **False.** When x = c, P(x) = F, R(x) = F, makes P(x) ∨ R(x) false. So the quantified expression $\forall x(P(x) \lor R(x))$ evaluates false.

   (h) **True.** For all x in the domain set, make R(x) → P(x) true. So the quantified expression $\forall x(R(x) \rightarrow P(x))$ evaluates true. See the truth below:

	P(x)	R(x)	R(x) → P(x)
a	T	F	T
b	T	F	T
c	F	F	T
d	T	F	T
e	T	T	T

   (i) **True.** When x = a for example, makes Q(x) ∨ R(x) true, because T ∨ F is true, there exists one value in the domain. So the quantified expression $\exists x(Q(x) \lor R(x))$ evaluates true.

2. (b) **True.** When x =2, Q(2,1) = T, Q(2,2) = T, Q(2,3) =T, there exists one x for all y that works, so $\exists x \forall y Q(x,y)$ true.

   (c) **True.** When y = 1, P(1,1) = T, P(2,1) = T, P(3,1) = T, there exists one y for all x that works, so $\exists y \forall x P(x,y)$ true.

   (d) **False.** All values in S(x,y) are false, so $\exists x \exists y S(x,y)$ false.

(e) **False.** When $x=1$, $Q(1,y)$ are all false, there doesn't exist one y for all x making $Q(x,y)$ evaluate true, so $\forall x \exists y Q(x,y)$ false.

(f) **True.** When $y=1$, $P(1,1) = T$, $P(2,1) = T$, $P(3,1) = T$, there exists one y for all x that works, so $\forall x \exists y P(x,y)$ true.

(g) **False.** When $x=1$, $y=2$, $P(1,2) = F$, not for every x and every y works, so $\forall x \forall y P(x,y)$ false.

(h) **True.** When $x=2$, $y=1$, $Q(2,1) = T$, there exists at least one x one y that works, so $\exists x \exists y Q(x,y)$ true.

(i) **True.** All values in $S(x,y)$ are false, so $\neg S(x,y)$ is true, therefore $\forall x \forall y \neg S(x,y)$ true.

Question 11:

1. (c) **$\exists x \exists y(x + y = xy)$**

   There are two numbers:

   x, y

   Exist two numbers:

   $\exists x \exists y$

   Whose sum is equal to their product:

   $x + y = xy$

   So, there exist two numbers whose sum is equal to their product:

   $\exists x \exists y(x + y = xy)$

   (d) **$\forall x \forall y(((x > 0) \wedge (y > 0)) \rightarrow (\frac{x}{y} > 0))$**

   Two numbers:

   x, y

   Two positive numbers, $x > 0$ and $y > 0$:

   $(x > 0) \wedge (y > 0)$

   Every two positive numbers, means for all x,y:

   $\forall x \forall y((x > 0) \wedge (y > 0))$

   The ratio is positive:

   $\frac{x}{y} > 0$

   So, the ratio of every two positive numbers is also positive:

   $\forall x \forall y(((x > 0) \wedge (y > 0)) \rightarrow (\frac{x}{y} > 0))$

   (e) **$\forall x((0 < x < 1) \rightarrow (\frac{1}{x} > 1))$**

   Positive number less than one:

   $0 < x < 1$

   Every positive number less than one:

   $\forall x(0 < x < 1)$

   The reciprocal is greater than one:

   $\frac{1}{x} > 1$

   The reciprocal of every positive number less than one is greater than one:

   $\forall x((0 < x < 1) \rightarrow (\frac{1}{x} > 1))$

   (f) **$\forall x \exists y(y < x)$**

   There is no smallest number means: at least one number y smaller than every x.

   Two numbers:

   x, y

   y smaller than x:

   $y < x$

   At least one number y smaller than every x:

$\forall x \exists y(y < x)$

(g) **$\forall x((x \neq 0) \rightarrow \exists y(xy = 1))$**

Number other than:

$x \neq 0$

Every number other than 0:

$\forall x(x \neq 0)$

Has a multiplicative inverse, means exists one number y that their product is 1:

$\exists y(xy = 1)$

Every number other than 0 has a multiplicative inverse:

$\forall x((x \neq 0) \rightarrow \exists y(xy = 1))$

2. (c) **$\exists x(N(x) \land D(x))$**

At least one:

$\exists x$

The one is a new employee and missed the deadline:

$N(x) \land D(x)$

At least one new employee missed the deadline:

$\exists x(N(x) \land D(x))$

(d) **$\forall x(D(x) \rightarrow P(Sam,x))$**

Sam knows the phone number of everyone x:

$\forall x P(Sam,x)$

Who missed the deadline:

$\forall x(D(x) \rightarrow P(Sam,x))$

(e) **$\exists x \forall y(N(x) \land (P(x,y))$**

There is someone:

$\exists x$

Who is new employee:

$\exists x N(x)$

One knows everyone's phone number:

$\exists x \forall y(P(x,y)))$

There is a new employee who knows everyone's phone number:

$\exists x \forall y(N(x) \land P(x,y))$

(f) **$\exists x \forall y(N(x) \land D(x) \land ((x \neq y) \land (N(y) \rightarrow \neg D(y))))$**

One new employee missed the deadline:

$\exists x(N(x) \land D(x))$

Exactly one means other new employees don't miss, for all others:

$\forall y$

New employees except x don't miss the deadline:

$(x \neq y) \land (N(y) \rightarrow \neg D(y))$

Exactly one new employee missed the deadline:

$\exists x \forall y(N(x) \land D(x) \land ((x \neq y) \land (N(y) \rightarrow \neg D(y))))$

3. (c) **$\forall x \exists y((y \neq \text{Math } 101) \land T(x,y))$**

Every student:

$\forall x$

At least one class:

$\exists y$

Student x has taken class y, and y is not Math 101:

$(y \neq \text{Math } 101) \land T(x,y)$

Every student has taken at least one class other than Math 101:

$\forall x \exists y((y \neq \text{Math } 101) \land T(x,y))$

(d) **$\exists x \forall y((y \neq \text{Math } 101) \rightarrow T(x, y))$**

There is a student:

$\exists x$

Every math class:

$\forall y$

Other than Math 101:

$y \neq \text{Math } 101$

There is a student x who has taken every math class y other than Math 101:

$\exists x \forall y((y \neq \text{Math } 101) \rightarrow T(x, y))$

(e) **$\forall x \exists y \exists z((x \neq \text{Sam}) \rightarrow ((y \neq z) \land T(x,y) \land T(x,z)))$**

Everyone student:

$\forall x$

Other than Sam:

$x \neq \text{Sam}$

At least two classes y, z:

$\exists y \exists z$

Two different math classes:

$y \neq z$

Students x has taken at least two different math classes y, z:

$(y \neq z) \land T(x,y) \land T(x,z)$

Everyone other than Sam has taken at least two different math classes:

$\forall x \exists y \exists z((x \neq \text{Sam}) \rightarrow ((y \neq z) \land T(x,y) \land T(x,z)))$

(f) **$\exists x \exists y \forall z((x \neq y) \land T(\text{Sam},x) \land T(\text{Sam},y) \land (T(\text{Sam},z) \rightarrow ((z = x) \lor (z = y))))$**

Two math classes x, y:

$\exists x \exists y$

Two different math classes:

$x \neq y$

Sam has taken two math classes:

$(x \neq y) \land T(\text{Sam},x) \land T(\text{Sam},y)$

Exactly two math classes means if Sam take any class z, then z = x or z = y:

$$T(Sam,z) \rightarrow ((z = x) \lor (z = y)$$

Sam has taken exactly two math classes:

$$\exists x \exists y \forall z((x \neq y) \land T(Sam,x) \land T(Sam,y) \land (T(Sam,z) \rightarrow ((z = x) \lor (z = y))))$$

Question 12:

1. (b)
   For every patient:
   $\forall x$
   Who was given the medication or the placebo or both:
   $D(x) \vee P(x) \vee (D(x) \wedge P(x))$
   Or both meaning is include OR logic, so the logical expression:
   $\forall x(D(x) \vee P(x))$
   Negation:
   $\neg\forall x(D(x) \vee P(x))$
   Applying De Morgan's law:
   $\neg\forall x(D(x) \vee P(x)) \equiv \exists x\neg(D(x) \vee P(x)) \equiv \exists x(\neg(D(x) \wedge \neg P(x))$
   Final answer:
   - **$\forall x(D(x) \vee P(x))$**
   - **Negation: $\neg\forall x(D(x) \vee P(x))$**
   - **Applying De Morgan's law: $\exists x(\neg(D(x) \wedge \neg P(x))$**
   - **English: There exists one patient who was not given the medication and was not given the placebo.**

   (c)
   For one patient:
   $\exists x$
   Who took the medication and had migraines:
   $D(x) \wedge M(x)$
   The logical expression:
   $\exists x(D(x) \wedge M(x))$
   Negation:
   $\neg\exists x(D(x) \wedge M(x))$
   Applying De Morgan's law:
   $\neg\exists x(D(x) \wedge M(x)) \equiv \forall x\neg(D(x) \wedge M(x)) \equiv \forall x(\neg D(x) \vee \neg M(x))$
   Final answer:
   - **$\exists x(D(x) \wedge M(x))$**
   - **Negation: $\neg\exists x(D(x) \wedge M(x))$**
   - **Applying De Morgan's law: $\forall x(\neg D(x) \vee \neg M(x))$**
   - **English: Every patient either did not take the medication or did not have the migraines (or both).**

   (d)
   For every patient:
   $\forall x$
   Who took the placebo had migraines:
   $P(x) \rightarrow M(x)$
   The logical expression:

$\forall x(P(x) \rightarrow M(x))$

Apply the conditional identity:

$\forall x(P(x) \rightarrow M(x)) \equiv \forall x(\neg P(x) \lor M(x))$

Negation:

$\neg\forall x(\neg P(x) \lor M(x))$

Applying De Morgan's law:

$\neg\forall x(\neg P(x) \lor M(x))$

$\equiv \exists x\neg(\neg P(x) \lor M(x))$

$\equiv \exists x(\neg\neg P(x) \land \neg M(x))$

$\equiv \exists x(P(x) \land \neg M(x))$

Final answer:

- **$\forall x(P(x) \rightarrow M(x))$**
- **Negation: $\neg\forall x(\neg P(x) \lor M(x))$**
- **Applying De Morgan's law: $\exists x(P(x) \land \neg M(x))$**
- **English: There exists a patient who took the placebo and did not have the migraines.**

(e)

For one patient:

$\exists x$

Who had migraines and was given the placebo:

$M(x) \land P(x)$

The logical expression:

$\exists x(M(x) \land P(x))$

Negation:

$\neg\exists x(M(x) \land P(x))$

Applying De Morgan's law:

$\neg\exists x(M(x) \land P(x)) \equiv \forall x\neg(M(x) \land P(x)) \equiv \forall x(\neg M(x) \lor \neg P(x))$

Final answer:

- **$\exists x(M(x) \land P(x))$**
- **Negation: $\neg\exists x(M(x) \land P(x))$**
- **Applying De Morgan's law: $\forall x(\neg M(x) \lor \neg P(x))$**
- **English: Every patient either did not have migraines or was not given the placebo (or both).**

2. (c)

Negate the expression:

$\neg\exists x\forall y(P(x,y) \rightarrow Q(x,y))$

$\equiv \forall x\exists y\neg(P(x,y) \rightarrow Q(x,y))$ [De Morgan's law]

$\equiv \forall x\exists y\neg(\neg P(x,y) \lor Q(x,y))$ [Conditional identities]

$\equiv \forall x\exists y(\neg\neg P(x,y) \land \neg Q(x,y))$ [De Morgan's law]

$\equiv \forall x\exists y(P(x,y) \land \neg Q(x,y))$ [Double negation law]

Final answer: **$\forall x \exists y(P(x,y) \land \neg Q(x,y))$**

(d)
Negate the expression:

$\neg \exists x \forall y(P(x,y) \leftrightarrow P(y,x))$
$\equiv \forall x \exists y \neg(P(x,y) \leftrightarrow P(y,x))$ [De Morgan's law]
$\equiv \forall x \exists y \neg((P(x,y) \rightarrow P(y,x)) \land (P(y,x) \rightarrow P(x,y)))$ [Conditional identities]
$\equiv \forall x \exists y \neg((\neg P(x,y) \lor P(y,x)) \land (\neg P(y,x) \lor P(x,y)))$ [Conditional identities]
$\equiv \forall x \exists y(\neg(\neg P(x,y) \lor P(y,x)) \lor \neg(\neg P(y,x) \lor P(x,y)))$ [De Morgan's law]
$\equiv \forall x \exists y((\neg\neg P(x,y) \land \neg P(y,x)) \lor (\neg\neg P(y,x) \land \neg P(x,y)))$ [De Morgan's law]
$\equiv \forall x \exists y((P(x,y) \land \neg P(y,x)) \lor (P(y,x) \land \neg P(x,y)))$ [Double negation law]

Final answer: **$\forall x \exists y((P(x,y) \land \neg P(y,x)) \lor (P(y,x) \land \neg P(x,y)))$**

(e)
Negate the expression:

$\neg(\exists x \exists y P(x,y) \land \forall x \forall y Q(x,y))$
$\equiv \neg \exists x \exists y P(x,y) \lor \neg \forall x \forall y Q(x,y)$ [De Morgan's law]
$\equiv \forall x \forall y \neg P(x,y) \lor \exists x \exists y \neg Q(x,y)$ [De Morgan's law]

Final answer: **$\forall x \forall y \neg P(x,y) \lor \exists x \exists y \neg Q(x,y)$**