**Question 5:**

a.

To show $5n^3 + 2n^2 + 3n = \Theta(n^3)$, we need to prove there exist positive constants $c_1$, $c_2$ and a positive integer constant $n_0$ such that $c_2 n^3 \le 5n^3 + 2n^2 + 3n \le c_1 n^3$ for all $n \ge n_0$.

For upper bound $5n^3 + 2n^2 + 3n \le c_1 n^3$ :

Since $5n^3 + 2n^2 + 3n \le 5n^3 + 2n^3 + 3n^3 = 10n^3$, for all $n \ge 1$,

Then we can take $c_1 = 10$, when $n \ge 1$, $5n^3 + 2n^2 + 3n \le 10n^3$.

For lower bound $c_2 n^3 \le 5n^3 + 2n^2 + 3n$ :

Since $5n^3 \le 5n^3 + 2n^2 + 3n$, for all $n \ge 1$,

Then we can take $c_2 = 5$, when $n \ge 1$, $5n^3 \le 5n^3 + 2n^2 + 3n$.

Thus, there exist positive constants $c_1 = 10$ and $c_2 = 5$ such that:

$5n^3 \le 5n^3 + 2n^2 + 3n \le 10n^3$, for all $n \ge 1$.

Therefore, $5n^3 + 2n^2 + 3n = \Theta(n^3)$.

b.

To show $\sqrt{7n^2 + 2n - 8} = \Theta(n)$, we need to prove there exist positive constants $c_1$, $c_2$ and a positive integer constant $n_0$ such that $c_2 n \le \sqrt{7n^2 + 2n - 8} \le c_1 n$ for all $n \ge n_0$.

For upper bound $\sqrt{7n^2 + 2n - 8} \le c_1 n$ :

Square both sides: $7n^2 + 2n - 8 \le c_1^2 n^2$,

Since $7n^2 + 2n - 8 \le 7n^2 + 2n^2 = 9n^2 = 3^2 n^2$, for all $n \ge 1$,

Then we can take $c_1 = 3$, when $n \ge 1$, $\sqrt{7n^2 + 2n - 8} \le 3n$.

For lower bound $c_2 n \le \sqrt{7n^2 + 2n - 8}$ :

Square both sides: $c_2^2 n^2 \le 7n^2 + 2n - 8$

Since $2^2 n^2 = 4n^2 \leq 7n^2 + 2n - 8$, for all $n \geq 2$,

Then we can take $c_2 = 2$, when $n \geq 2$, $2n \leq \sqrt{7n^2 + 2n - 8}$.

Thus, there exist positive constants $c_1 = 3$ and $c_2 = 2$ such that:

$2n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$, for all $n \geq 2$.

Therefore, $\sqrt{7n^2 + 2n - 8} = \Theta(n)$.