Photon Mapping

Notebook:	Graphics		
Created:	13/05/2021 11:48 AM	**Updated:**	27/05/2021 12:11 PM
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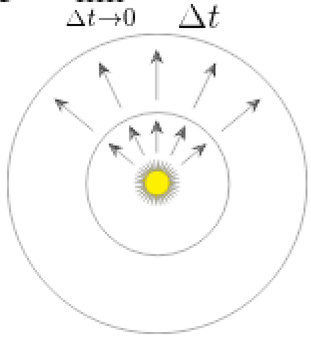
URL:	https://chat.otoyops.com/otoy/channels/nz-octane-dev-internal#		

https://graphics.cg.uni-saarland.de/courses/ris-2018/slides/17-Guiding-LightweightPM.pdf
https://cgg.mff.cuni.cz/~jaroslav/papers/2018-lwpm/2018-grittmann-lwpm-paper.pdf
https://cgg.mff.cuni.cz/~jaroslav/papers/2019-corona-caustics/2019-sik-corona-caustics-paper.pdf

- Expected Value: E(x)
    1. 离散变量x：这里：p_i为x_i的机率
       $$E(x) = \sum_{i=i}^{n} p_i x_i$$
    2. 连续变量(定义在D上面)x: 这里，p(x)为x的PDF(distribution density function)
       $$E(x) = \int_D x p(x) dx$$

- Radiant Flux/Power：Watt(Joule/sec):

$$\Phi = \lim_{\Delta t \to 0} \frac{\Delta Q(\lambda)}{\Delta t} = \frac{dQ(\lambda)}{dt}$$



例如：如图所属，大圆与小圆具有相同的Flux. 但是大圆明显单位面积上的能量较少

- Flux密度(Density)/Irradiance **E** ：单位为 瓦特/平方米：$\frac{W}{m^2}$

$$E(\vec{p}) = \lim_{\Delta A \to 0} \frac{\Delta \Phi(\vec{p})}{\Delta A} = \frac{d\Phi(\vec{p})}{dA} = \int_{\Omega} L_i(\vec{p}, \vec{\omega}) \left| cos\theta \right| d\vec{\omega}$$

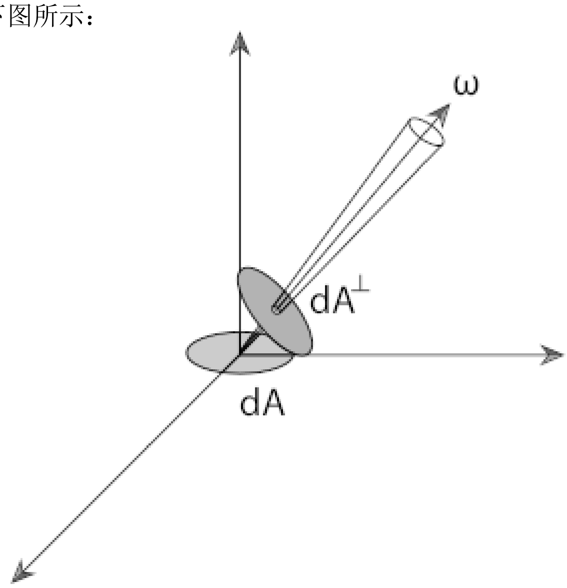
- Radiance/每天文角度的密度 **L**：单位为 $\frac{W}{sr * m^2}$

$$L_r(\vec{p},\vec{\omega}_o) = \lim_{\Delta\vec{\omega_i}\to 0} \frac{\Delta E_{\vec{\omega_i}}(\vec{p})}{\Delta\vec{\omega_i}} = \frac{dE_{\vec{\omega_i}}(\vec{p})}{d\vec{\omega_i}} = \frac{d^2\Phi(\vec{p},\vec{\omega_i})}{dA\,|\vec{n}_{\vec{p}}\cdot\vec{\omega_i}|\,d\vec{\omega_i}} = \frac{d^2\Phi(\vec{p},\vec{\omega_i})}{dA^{\perp}d\vec{\omega_i}}$$

这里：

1. $E_{\vec{\omega_i}}$ 是在 $\vec{\omega_i}$方向上的Flux密度
2. dA为平面上的投影
3. $dA^{\perp}$为 dA在 $\vec{\omega_i}$方向上的投影

见下图所示：



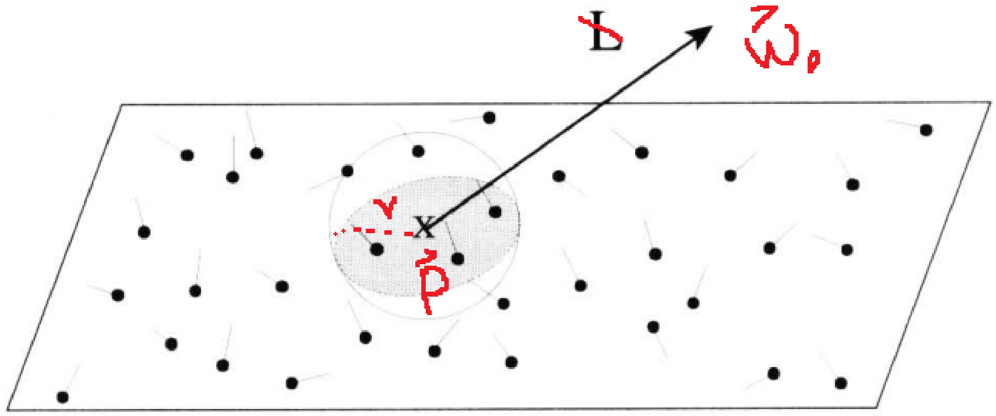
- 反射的Radiance为：

$$\begin{aligned}
L_r(\vec{p},\vec{\omega}_o) &= \int_{\Omega} f_r(\vec{p},\vec{\omega_i},\vec{\omega_o})L_i(\vec{p},\vec{\omega_i})\,|\vec{n}_{\vec{p}}\cdot\vec{\omega_i}|\,d\vec{\omega_i} \\
&= \int_{\Omega} f_r(\vec{p},\vec{\omega_i},\vec{\omega_o})\frac{d^2\Phi(\vec{p},\vec{\omega_i})}{dA\,|\vec{n}_{\vec{p}}\cdot\vec{\omega_i}|\,d\vec{\omega_i}}\,|\vec{n}_{\vec{p}}\cdot\vec{\omega_i}|\,d\vec{\omega_i} \\
&= \int_{\Omega} f_r(\vec{p},\vec{\omega_i},\vec{\omega_o})\frac{d^2\Phi(\vec{p},\vec{\omega_i})}{dA} \\
&\approx \sum_{k=1}^{N} f_r(\vec{p},\vec{\omega_k},\vec{\omega_o})\frac{\Delta\Phi_k(\vec{p},\vec{\omega_k})}{\Delta A}
\end{aligned}$$

这里：

1. 每个沿 $\vec{\omega_k}$方向的Photon的能量为 $\Delta\Phi_k(\vec{\omega_k})$
2. ΔA为收集Photon时使用的Volume在 $\vec{p}$ 的切线平面上的投影面积大小
3. 如果使用球来进行收集那么反射的Radiance为“

$$L_r(\vec{p},\vec{\omega}_o) \approx \frac{1}{\pi r^2}\sum_{k=1}^{N} f_r(\vec{p},\vec{\omega_k},\vec{\omega_o})\Delta\Phi_k(\vec{p},\vec{\omega_k})$$


L
$\vec{\omega}_0$
x
r
$\vec{p}$