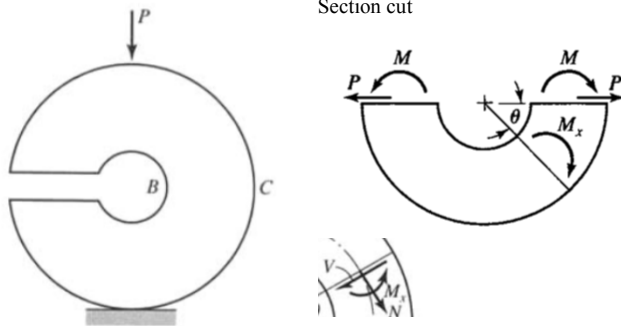


1) 9.7

**9.7.** The curved beam in Figure P9.7 has a circular cross section 50 mm in diameter. The inside diameter of the curved beam is 40 mm. Determine the stress at B for  $P = 20$  kN.



$$\sigma_{\theta, \theta} := \frac{N}{A} + \frac{M(A - r \cdot A[m])}{A \cdot r(R \cdot A[m] - A)} \quad (1)$$

$$\sigma_{\theta, \theta} := \frac{N}{A} + \frac{M(-r A_m + A)}{A r(R A_m - A)} \quad (1)$$

$$P := 20 \cdot 1000 \text{ #N}$$

$$P := 20000 \quad (2)$$

$$A[m] := \text{evalf}(2 \cdot \text{Pi} \cdot (45 - \text{sqrt}(45^2 - 25^2)))$$

$$A_m := 47.64807164 \quad (3)$$

$$A := \text{evalf}\left(\text{Pi} \left(\frac{10}{2}\right)^2\right) \text{ #mm}^2$$

$$A := 78.53981635 \quad (4)$$

$$R_m := 40 + \frac{(50 - 40)}{2}$$

$$R_m := 45 \quad (5)$$

$$r := 20$$

$$r := 20 \quad (6)$$

$$\text{\#Sum force}$$

$$F[x] := 0$$

$$F_x := 0 \quad (7)$$

$$N := -P \cdot \text{\#Sum } F$$

$$N := -20000 \quad (8)$$

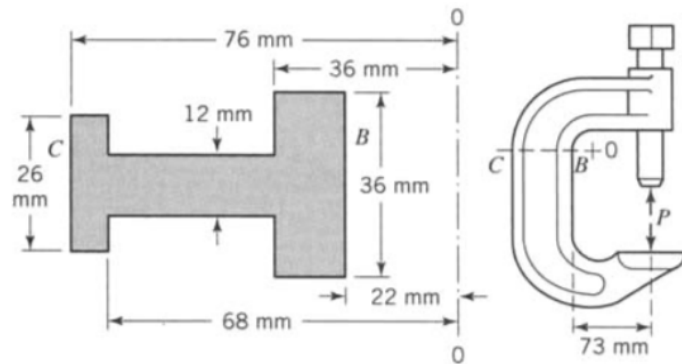
$$M := -P \cdot (24)$$

$$M := -480000 \quad (9)$$

$$\sigma_{\theta, \theta} := \frac{N}{A} + \frac{M(A - r \cdot A[m])}{A \cdot r(R \cdot A[m] - A)}$$

$$\sigma_{\theta, \theta} := -560.2253996 \quad (10)$$

2) 9.20-9.21. Ignore Bleich factors (section 9.4 is not covered in this course)  
**9.20.** A load  $P = 12.0 \text{ kN}$  is applied to the clamp shown in Figure P9.20. Determine the circumferential stresses at points  $B$  and  $C$ , assuming that the curved beam formula is valid at that section.



Assumption: that the Neutral Axis and the centroid are the same distance from the axis of curvature.



$$y_{\text{Bar}} := \frac{\sum A \cdot y}{\sum A_i}$$

$$y_{\text{Bar}} := \frac{\sum A y}{\sum A_i} \quad (1)$$

$$A[1] := (36 - 22) \cdot 36 \quad A_1 := 504 \quad (2)$$

$$A[2] := 12 \cdot (68 - 36) \quad A_2 := 384 \quad (3)$$

$$A[3] := (76 - 68) \cdot 26 \quad A_3 := 208 \quad (4)$$

$$y_{\text{Bar}} := \text{evalf}\left(\frac{A[1] \cdot (29) + A[2] \cdot (52) + A[3] \cdot (72)}{A[1] + A[2] + A[3]}\right) \quad y_{\text{Bar}} := 45.21897810 \quad (5)$$

$$R := y_{\text{Bar}} \quad R := 45.21897810 \quad (6)$$

$$r[B] := 22 \quad r_B := 22 \quad (7)$$

$$r[C] := 76 \quad r_C := 76 \quad (8)$$

$$Am[1] := \text{evalf}\left(36 \cdot \ln\left(\frac{36}{22}\right)\right) \quad Am_1 := 17.72915346 \quad (9)$$

$$Am[2] := \text{evalf}\left(12 \cdot \ln\left(\frac{68}{36}\right)\right) \quad Am_2 := 7.631865202 \quad (10)$$

$$Am[3] := \text{evalf}\left(26 \cdot \ln\left(\frac{76}{68}\right)\right) \quad Am_3 := 2.891866518 \quad (11)$$

$$Am := Am[1] + Am[2] + Am[3] \quad Am := 28.25288518 \quad (12)$$

$$A[\text{tot}] := A[1] + A[2] + A[3] \quad A_{\text{tot}} := 1096 \quad (13)$$

#Sum of Forces

$$P := 12 \cdot 1000 \quad P := 12000 \quad (14)$$

$$N := P \quad N := 12000 \quad (15)$$

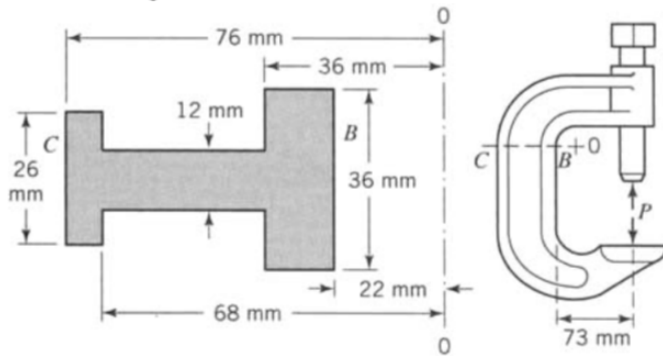
$$M := P \cdot (73 + 22) \quad M := 1140000 \quad (16)$$

$$\sigma_{\theta, \theta B} := \frac{N}{A[\text{tot}]} + \frac{M(A[\text{tot}] - r[B] \cdot Am)}{A[\text{tot}] \cdot r[B] \cdot (R \cdot Am - A[\text{tot}])} \quad \sigma_{\theta, \theta B} := 11.20930192 \quad (17)$$

$$\sigma_{\theta, \theta C} := \frac{N}{A[\text{tot}]} + \frac{M(A[\text{tot}] - r[C] \cdot Am)}{A[\text{tot}] \cdot r[C] \cdot (R \cdot Am - A[\text{tot}])} \quad \sigma_{\theta, \theta C} := 11.02428313 \quad (18)$$

3) 9.21

**9.21.** Determine the radial stress at the junction of the web and inner flange of the curved beam portion of the clamp in Problem 9.20. Neglect stress concentrations.



$$A'_m = \int_a^r \frac{dA}{r} \quad \text{and} \quad A' = \int_a^r dA \quad (9.20)$$

$$\sigma_{rr} = \frac{AA'_m - A'A_m}{t r A (R A_m - A)} M_x \quad (9.21)$$

Inner flange disst from O is 68mm

$r := 68 \text{ mm}$

$$r := 68 \quad (2)$$

$$A_{\text{prime}}[1] := 36 \cdot (14)$$

$$A_{\text{prime}_1} := 504 \quad (3)$$

$$A_{\text{prime}}[m] := \text{evalf} \left( \int_{22}^{36} \frac{361}{r} dr \right) \#(9.20)$$

$$A_{\text{prime}_m} := 177.7840113 \quad (4)$$

$$\text{Area} := 14 \cdot 36 + 12 \cdot 32 + 8 \cdot 26$$

$$\text{Area} := 1096 \quad (5)$$

$$t := 12$$

$$t := 12 \quad (6)$$

$$y_{\text{Bar}} := 45.21897810$$

$$y_{\text{Bar}} := 45.21897810 \quad (7)$$

$$R := y_{\text{Bar}}$$

$$R := 45.21897810 \quad (8)$$

$$M := 1140000 \text{ #prob2 } N \cdot m$$

$$M := 1140000 \quad (9)$$

$$A[m] := 1096 \text{ # prob2}$$

$$A_m := 1096 \quad (10)$$

$$\sigma_{rr} := \text{evalf} \left( \frac{(Area \cdot A_{\text{prime}}[m] - A_{\text{prime}}[1] \cdot A[m])}{t \cdot r \cdot Area \cdot (R \cdot A[m] - Area)} \cdot M \right)$$

$$\sigma_{rr} := -9.403741443 \quad (11)$$

3) 12.6

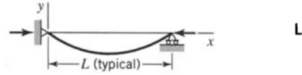
**12.6.** Three pinned-end columns each have a cross-sectional area of 2000 mm<sup>2</sup> and length of 750 mm. They are made of 7075-T5 aluminum alloy ( $E = 72.0$  GPa and  $\sigma_{PL} = 448$  MPa).

One of the columns has a solid square cross section. A second column has a solid circular cross section. The third has a hollow circular cross section with an inside diameter of 30.0 mm. Determine the critical buckling load for each of the columns.

Pinned is kind of like a ball and socket

Simple support-simple support

$$\frac{\pi^2 EI}{L^2}$$



Square Cross Section

$$I_C = \frac{bh}{12}(b^2 + h^2)$$

Circular Cross Section

$$I_C = \frac{\pi d^4}{32}$$

Hollow Circular Cross Section

$$I_C = \frac{\pi}{32}(d^4 - d_i^4)$$

$$\text{solve}(2000 = b^2, \{b\})$$

$$\{b = -20\sqrt{5}\}, \{b = 20\sqrt{5}\}$$

(1)

$$\text{solve}(2000 = \pi \cdot r^2, \{r\})$$

$$\left\{r = -\frac{20\sqrt{5}}{\sqrt{\pi}}\right\}, \left\{r = \frac{20\sqrt{5}}{\sqrt{\pi}}\right\}$$

(2)

$$r[\text{inner}] := \frac{30}{2} \# \text{mm}$$

$$r_{\text{inner}} := 15$$

(3)

$$\text{solve}(2000 = \pi \cdot r[\text{out}]^2 - \pi \cdot r[\text{inner}]^2, \{r[\text{out}]\})$$

$$\left\{r_{\text{out}} = -\frac{5\sqrt{\pi(9\pi+80)}}{\pi}\right\}, \left\{r_{\text{out}} = \frac{5\sqrt{\pi(9\pi+80)}}{\pi}\right\}$$

(4)

$$b := 20\sqrt{5}$$

$$b := 20\sqrt{5}$$

(5)

$$Iota[Sqr] := \text{evalf}\left(\frac{1}{12}b^4\right)\left(\frac{1}{1000^4}\right) \# m^4$$

$$I_{Sqr} := 3.333333333 \cdot 10^5$$

(6)

$$d := 2 \cdot \left(\frac{20\sqrt{5}}{\sqrt{\pi}}\right)$$

$$d := \frac{40\sqrt{5}}{\sqrt{\pi}}$$

(7)

$$Iota[cir] := \text{evalf}\left(\frac{\pi \cdot d^4}{32}\right)\left(\frac{1}{1000^4}\right) \# m^4$$

$$I_{cir} := 6.366197722 \cdot 10^5$$

(8)

$$d[\text{out}] := \text{evalf}\left(2 \cdot \left(\frac{5\sqrt{\pi(9\pi+80)}}{\pi}\right)\right)$$

$$d_{\text{out}} := 58.70672098$$

(9)

$$d[\text{inner}] := (r[\text{inner}] \cdot 2)$$

$$d_{\text{inner}} := 30$$

(10)

$$Iota[HollC] := \text{evalf}\left(\frac{\pi \cdot (d[\text{out}]^4 - d[\text{inner}]^4)}{32}\right)\left(\frac{1}{1000^4}\right) \# m^4$$

$$I_{HollC} := 0.000001086619772$$

(11)

$$E := 72 \cdot 10^9 \# Pa$$

$$E := 72000000000$$

(12)

$$P[\text{crit\_sq}] := \frac{\pi^2 E \cdot Iota[Sqr]}{(.75)^2} \# N$$

$$P_{\text{crit\_sq}} := 4.211031214 \cdot 10^{17}$$

(13)

$$P[\text{crit\_ci}] := \frac{\pi^2 E \cdot Iota[cir]}{(.75)^2} \# N$$

$$P_{\text{crit\_ci}} := 8.042477196 \cdot 10^{17}$$

(14)

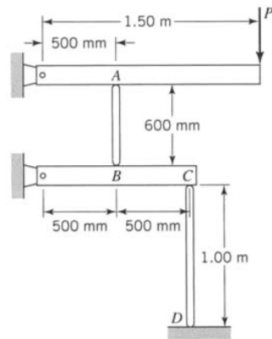
$$P[\text{crit\_Hci}] := \frac{\pi^2 E \cdot Iota[HollC]}{(.75)^2} \# N$$

$$P_{\text{crit\_Hci}} := 1.372736933 \cdot 10^6$$

(15)

4) 12.8. Assume that the horizontal members are rigid and weightless.

**12.8.** In Figure P12.8, columns  $AB$  and  $CD$  have pinned ends, are made of an aluminum alloy ( $E = 72.0$  GPa), and have equal rectangular cross sections of 20 mm by 30 mm. Determine the magnitude of  $P$  that will first cause one of the columns to buckle. Assume elastic conditions.



$$b := .03 \quad b := 0.03 \quad (1)$$

$$h := .02 \quad h := 0.02 \quad (2)$$

$$Iota[x] := \frac{1}{12} b \cdot h^3 \#m^4 \quad I_x := 2.000000000 \cdot 10^{-8} \quad (3)$$

$$b := .02 \quad b := 0.02 \quad (4)$$

$$h := .03 \quad h := 0.03 \quad (5)$$

$$Iota[y] := \frac{1}{12} b \cdot h^3 \quad I_y := 4.500000000 \cdot 10^{-8} \quad (6)$$

$$L[AB] := .6 \#m \quad L_{AB} := 0.6 \quad (7)$$

$$L[CD] := 1 \#m \quad L_{CD} := 1 \quad (8)$$

$$E := 72 \cdot 10^9 \quad E := 72000000000 \quad (9)$$

$$F[C, I_x] := \frac{\pi^2 E \cdot Iota[x]}{L[CD]^2} \quad F_{C, I_x} := 14212.23034 \quad (10)$$

$$F[C, I_y] := \frac{\pi^2 E \cdot Iota[y]}{L[CD]^2} \quad F_{C, I_y} := 31977.51827 \quad (11)$$

$$F[A, I_x] := \frac{\pi^2 E \cdot Iota[x]}{L[AB]^2} \quad F_{A, I_x} := 39478.41762 \quad (12)$$

$$F[A, I_y] := \frac{\pi^2 E \cdot Iota[y]}{L[AB]^2} \quad F_{A, I_y} := 88826.43964 \quad (13)$$

#F<sub>C, Ix</sub> is the Lowest

$$F[A_{new}] := \frac{F_{C, I_x}}{.5} \quad F_{A_{new}} := 28424.46068 \quad (14)$$

$$P := \frac{F_{A_{new}} \cdot (.5)}{1.5} \#N \quad P := 9474.820226 \quad (15)$$