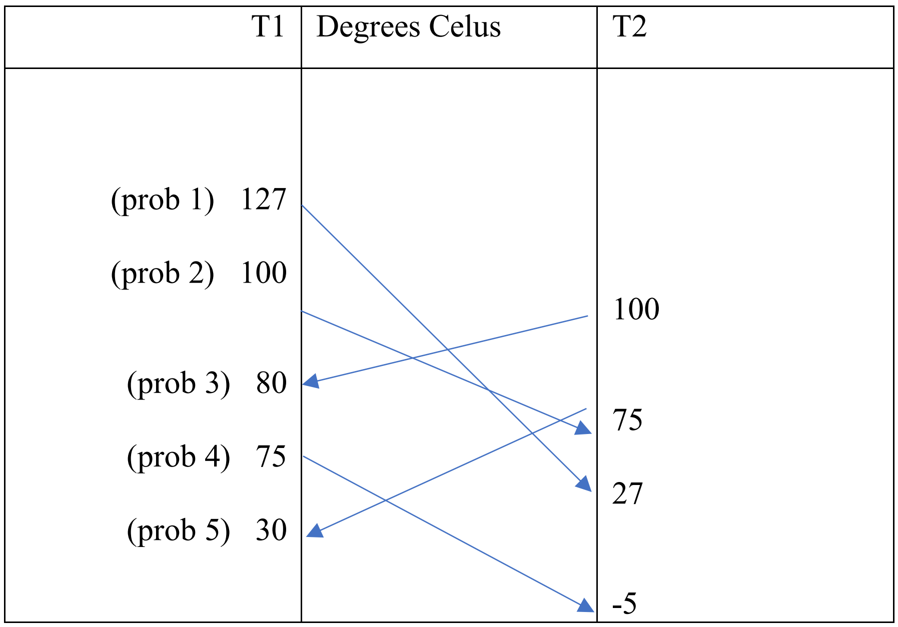
1) One-dimensional, steady-state conduction without heat generation occurs in a plane wall (T_1 on left and T_2 on right). The thermal conductivity (k) is 5 W/m^2 and the thickness (L) is 0.1 m. Determine the unknown quantities for each case in the table below and sketch the temperature distribution and indicate the direction of the heat flux for each.

case	T1	T2	dT/dx [K/m]	q'' [W/m^2]
1	**127**	27	-1000	5000
2	**100**	75	-250	1250
3	80	**100**	200	-1000
4	**75**	-5	-800	4000
5	30	**90**	600	-3000

$$q = -k * dT/dr$$



1) A solid cylindrical rod of length (L) 0.1 m and diameter (d) 25 mm is well insulated on its side, while the end faces are maintained at temperatures of 100˚C and 0˚C. What is the rate of heat transfer through the rod if it is constructed from (a) pure copper, (b) aluminum alloy 6061-T6, (c) AISI 304 stainless steel, (d) fused silica glass (SiO_2), (e) wood (oak), (f) magnesia, 85%, and (g) Silica aerogel? Use the properties tables at the end of the book	

T1			100
T2			0
D			0.025
L			0.1
		K values	q
	(a) pure copper,	398	195.367793
	(b) aluminum alloy6061-T6,	167	81.9759333
	(c) AISI 304 stainless steel,	13.8	6.77405916
	(d) fused silica glass (SiO_2),	1.41	0.69213213
	(e) wood (oak),	145	71.1767086
	(f) magnesia, 85%, and	0.067	0.03288855
	(g) Silica aerogel?	0.022	0.01079922

$$q = -k * A * dT/dr$$

3. Uniform internal heat generation at $\dot{q}$ **= 5 × 10^7 W/m^3** is occurring in a cylindrical nuclear reactor fuel rod of 50 mm diameter, and under steady-state conditions the temperature distribution is of the form **T(r) = a+br^2**, where T is in degrees Celsius and r is in meters, while **a = 800°C** and **b = −4.167 × 10^5°C/m^2**. The fuel rod properties are **k = 30 W/m-K, ρ = 1100 kg/m^3**, and **c_p = 800 J/kg-K.**

(a) What is the rate of heat transfer per unit length of the rod at **r = 0** (the centerline) and at **r = 25 mm** (the surface)?

$T\_r := a + b \cdot r^2$

$$T\_r := b\,r^2 + a \qquad (1)$$

$\frac{d}{dr} T\_r$

$$2\,b\,r \qquad (2)$$

$q[prime] := -\,k \cdot A[serf] \cdot \frac{d}{dr} T\_r$

$$q_{prime} := -2\,k\,A_{serf}\,b\,r \qquad (3)$$

$A[serf] := 2 \cdot \mathrm{Pi} \cdot r$

$$A_{serf} := 2\,\pi\,r \qquad (4)$$

$q[prime]$

$$-4\,k\,\pi\,r^2\,b \qquad (5)$$

$r := 0$

$$r := 0 \qquad (6)$$

$q[prime]$

$$0 \qquad (7)$$

$r := .025$

$$r := 0.025 \qquad (8)$$

$k := 30$

$$k := 30 \qquad (9)$$

$b := -4.167 \cdot 10^5$

$$b := -4.16700000\ 10^5 \qquad (10)$$

$q[prime]\ \#\frac{W}{M}$

$$98182.62440 \qquad (11)$$

q' = 98182.62440 W/m

(b) If the reactor power level is suddenly increased to **$\dot{q}_2$ = 10^8 W/m^3**, what is the initial time rate of temperature change at **r = 0** and **r = 25 mm**?

Find dT/dt

$T\_r := a + b \cdot r^2$

$$T\_r := b\,r^2 + a \qquad (1)$$

$\frac{d}{dr} T\_r$

$$2\,b\,r \qquad (2)$$

$b := -4.167 \cdot 10^5$

$$b := -4.16700000\ 10^5 \qquad (3)$$

$k := 30$

$$k := 30 \qquad (4)$$

$q := 10^8$

$$q := 100000000 \qquad (5)$$

$solve\left(\left(\mathrm{rho} \cdot C[p] \cdot drdt = \frac{1}{r} \cdot \frac{d}{dr}\left(k \cdot r \cdot \frac{d}{dr} T\_r\right) + q\right), drdt\right)$

$$\frac{4\,b\,k + q}{\rho\,C_p} \qquad (6)$$

$\mathrm{rho} := 1100$

$$\rho := 1100 \qquad (7)$$

$C[p] := 800$

$$C_p := 800 \qquad (8)$$

$drdt := \frac{4\,b\,k + q}{\rho\,C_p}$

$$drdt := 56.81363636 \qquad (9)$$

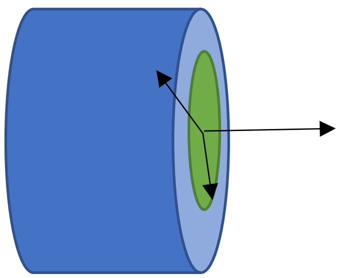
dT/dr := 56.81363636 °**C/s**

4. Passage of an electric current through a long conducting rod of **radius r_i** and **thermal conductivity k_r** results in uniform **volumetric heating at a rate of $\dot{q}$.** The conducting rod is wrapped in an electrically nonconducting cladding material of **outer radius r_o** and **thermal conductivity k_c,** and <u>convection cooling</u> is provided by an adjoining fluid (T_∞, h). For steady-state conditions, write appropriate forms of the <u>heat equations</u> for the rod and cladding. Express appropriate boundary conditions for the solution of these equations.

Heat Equation in Cylindrical

Fourier's Law: $q'' = -k\nabla T = -k\left(\hat{r}\frac{\partial T}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial T}{\partial \phi} + \hat{k}\frac{\partial T}{\partial z}\right)$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}$$



Given:

radius r_i

thermal conductivity k_r

volumetric heating at a rate of $\dot{q}$

outer radius r_o

thermal conductivity k_c,

$$\text{rho}\cdot C[p]\cdot dtdt = \frac{1}{r}\frac{\partial}{\partial r}\left(k\cdot r\frac{\partial}{\partial r}T\right) + \frac{1}{r^2}\frac{\partial}{\partial\,\text{phi}}\left(k\frac{\partial}{\partial\,\text{phi}}T\right) + \frac{\partial}{\partial z}\left(k\frac{\partial}{\partial z}T\right) + qdot$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(k\cdot r\frac{\partial}{\partial r}T\right) + qdot$$

If K is a constant

$$0 = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}T[rod]\right) + qdot$$

$$0 = \frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}T[cladding]\right) + qdot$$

Boundary Conditions:

1) At the center of the rod there is no change in Temp with respect to the radius:

$$\left.\frac{d}{dr}T\right|_{r=0} = 0$$

2) On the undersurface where the cladding meets the rod the due to conduction the temp is the same for both at steady state: T[r](r[i]) = T[c](r[i])

3) The Heat flux into the cladding is the same as the heat flux going out of the cladding

$$\left.q[cladding]''\right|_{r=ri} = \left.q[rod]''\right|_{r=ri}$$

$$\left.k\cdot r\cdot\frac{d}{dr}T\right|_{r=0} = \left.k\cdot r\cdot\frac{d}{dr}T\right|_{r=0}$$

4) Conduction will be taking place on the outer surface of the cladding.

$$h\{T[cladding](ro) - T[oo]\} = -k[cladding]\cdot\left(\left.\frac{d}{dr}T[cladding]\right|_{ro}\right)$$

5. The walls of a refrigerator are typically constructed by sandwiching a layer of insulation between sheet metal panels. Consider a wall made from fiberglass insulation of thermal conductivity **k_i = 0.045 W/m-K** and thickness **L_i = 50 mm and steel panels**, each of thermal conductivity **k_p = 60 W/m- K and thickness L_p = 3 mm**. If the wall separates refrigerated air **at $T_{\infty,i}$ = 4°C f**rom ambient air at **$T_{\infty,o}$ = 25°C**, what is the **heat gain per unit surface area?** Coefficients associated with natural convection at the inner and outer surfaces may be approximated as **$h_i = h_o$ = 5 W/m²-K.**

Air	sheet metal	insulation	sheet metal	Air
$T_{\infty,o} = 25^{\circ}$C	Lp = 3 mm kp = 60 W/m-k	L_i = 50 mm k_i = 0.045 W/m-K	Lp = 3 mm kp = 60 W/m-k	$T_{\infty,i} = 4^{\circ}$C

$$R[th] := \frac{1}{h[i]} + \frac{L[p]}{K[p]} + \frac{L[i]}{K[i]} + \frac{L[p]}{K[p]} + \frac{1}{h[i]}$$

$$R_{th} := 1.511211111 \quad (1)$$

$$L[p] := \frac{3}{1000}$$

$$L_p := \frac{3}{1000} \quad (2)$$

$$L[i] := \frac{50}{1000}$$

$$L_i := \frac{1}{20} \quad (3)$$

$$K[p] := 60$$

$$K_p := 60 \quad (4)$$

$$K[i] := .045$$

$$K_i := 0.045 \quad (5)$$

$$h[i] := 5$$

$$h_i := 5 \quad (6)$$

$$h[p] := 5$$

$$h_p := 5 \quad (7)$$

$$R[th]$$

$$1.511211111 \quad (8)$$

$$T[out] := 25$$

$$T_{out} := 25 \quad (9)$$

$$T[inner] := 4$$

$$T_{inner} := 4 \quad (10)$$

$$Q[primePrime] := \frac{(T[out] - T[inner])}{R[th]}$$

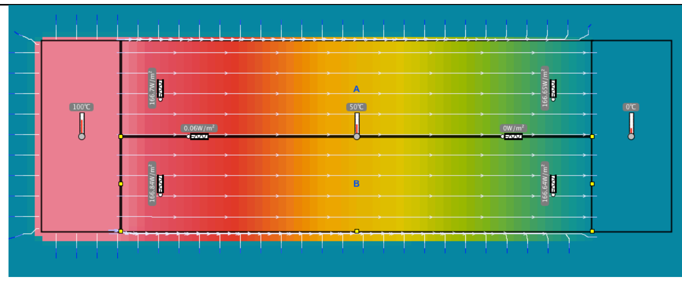
$$Q_{primePrime} := 13.89613923 \quad (11)$$

Q'' = 13.89613923 W/m^2

6. Using Energy2D and the equivalent resistance circuit method covered in class, complete the following table. The Energy2D model has been created for case 1 and is available on Canvas in the homework directory (HW02P6.e2d). For the Energy2D values, you can average the results of the heat flux sensors positioned normal to the direction of heat transfer. The temperature difference across the composite wall for each case is constant and can be obtained from the Energy2D file. Once you complete the table, do you feel the equivalent resistance method is acceptable for all cases?

$$R_{tot} = \frac{L}{K} * A$$

$$\frac{\Delta T}{\frac{L}{K}} = q''$$



case	kA (W/m-K)	kB (W/m-K)	Equiv. Res. q''	Energy2D q''	% error
1	10	10	333.333333	333.275	0.018%
2	100	1	1683.33333	1643.63	2.416%
3	1000	0.1	16668.3333	16400.25	1.635%
4	10000	0.01	166666.833	163944.1	1.661%