The goal of this problem is to gain physical insight into transient conduction by using Energy 2Dto explore the validity of the **Lumped Capacitance Method**. To complete this problem, we will be calculating the Bi number, thermal diffusivity (α), and the Fo number for several cases and comparing temperature values using the LCM and by using Figure 5.8 for **r/ro= 0, 0.5**, and **0.75** to those obtained from Energy 2D. When using Figure 5.8 you will need to recalculate **Bi** and **Fo** with Lc=rto determine the normalized temperature. Use your best estimate for values. Assume **h= 15 W/m2-**K for the convection coefficient for calculating Bi. To complete this problem, follow the procedure given below.

(a) Download the Energy 2D file (hw06.e2d). The default settings are appropriate for Case 1.
(b) Run the simulation for 10 seconds of actual time (you can watch the timer in the top right).
(c) Open the sensor data window and record the value of each sensor at Time = 400.0 s.
(d) Change the thermal conductivity of the sensors for the next case and click Reset.
(e) Repeat steps (b)-(d) until you have completed all cases.
(f) Calculate Bi,α, Fo,TLCM,T0,T0.5, and T0.75 for each case and compare to the Energy 2D results.

Case	K	Bi	a	Fo	T_{LCM} °C	T_o	$T_{0.5}$	$T_{0.75}$	$T_{0,E2D}$	$T_{0.5,E2D}$	$T_{0.75,E2D}$
1	400	0.0125	0.012307	44.3076	57.4735	55	55	55	70.777	70.3499	71.409
2	40	0.125	0.0012307	4.43076	57.4735	65	60	55	82.609	83.140	81.211
3	8	0.625	0.0002461538	0.886153	57.4735	90	80	65	97.328	89.811	80.988

$h := 15$

$$h := 15 \tag{1}$$

$R := 1$

$$R := 1 \tag{2}$$

$L := \frac{R}{3}$

$$L := \frac{1}{3} \tag{3}$$

$k := 400$

$$k := 400 \tag{4}$$

$Bi := evalf\left(\frac{h \cdot L}{k}\right)$

$$Bi := 0.01250000000 \tag{5}$$

#.0125 < 0.1 so we can use Lumped Capacitance

rho := 25

$$\rho := 25 \tag{6}$$

$C[p] := 1300$

$$C_p := 1300 \tag{7}$$

$t := 400$

$$t := 400 \tag{8}$$

$\text{alpha} := evalf\left(\frac{k}{\text{rho} \cdot C[p]}\right)$

$$\alpha := 0.01230769231 \tag{9}$$

$F[o] := \frac{\text{alpha}}{L^2} \cdot t$

$$F_o := 44.30769232 \tag{10}$$

$T[oo] := 0$

$$T_{oo} := 0 \tag{11}$$

$T[i] := 100$

$$T_i := 100 \tag{12}$$

$A := 4 \cdot \text{Pi} \cdot R^2$

$$A := 4\pi \tag{13}$$

$V := \frac{4}{3}\text{Pi} \cdot R^3$

$$V := \frac{4}{3}\pi \tag{14}$$

$solve\left(\frac{T - T[oo]}{T[i] - T[oo]} = \exp(-(Bi \cdot F[o])), T\right)$

$$57.47350345 \tag{15}$$

$Bi\_Prime := evalf\left(\frac{h \cdot R}{k}\right)$

$$Bi\_Prime := 0.03750000000 \tag{16}$$

$F\_Prime[o] := \frac{\text{alpha}}{R^2} \cdot t$

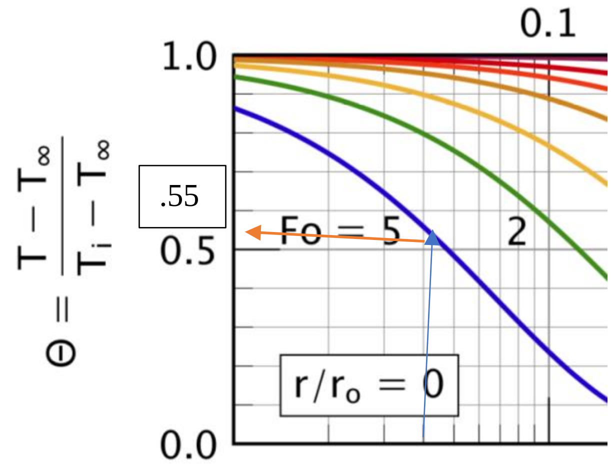
$$F\_Prime_o := 4.923076924 \tag{17}$$

$\text{theta}[FromGraph] := 0.55$

$$\theta_{FromGraph} := 0.55 \tag{18}$$

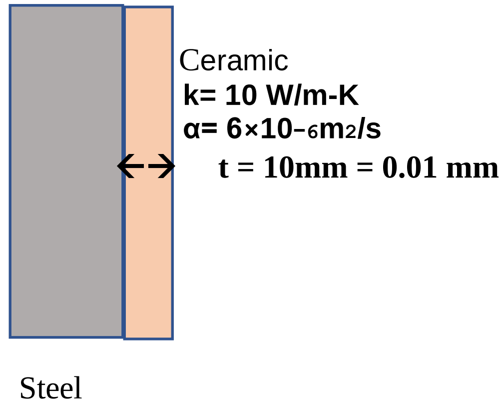
$solve\left(\text{theta}[FromGraph] = \frac{T - T[oo]}{T[i] - T[oo]}, T\right)$

$$55. \tag{19}$$



Above is an example of r/ro =0 for a sphear

2) During transient operation, the steel nozzle of a rocket engine must not exceed a maximum allowable operating temperature of **1500 K** when exposed to combustion gases characterized by a temperature of **2300 K** and a **convection coefficient of** 5000 **W/m2-K.** To extend the duration of engine operation, it is proposed that a ceramic thermal barrier coating **(k= 10 W/m-K, α= 6×10-6m2/s)** be applied to the interior surface of the nozzle. If the ceramic coating is **10 mm thick** and at an **initial temperature of 300 K**, obtain a conservative estimate of the maximum allowable duration of engine operation. Assume the nozzle radius is much larger than the combined wall and coating thickness (ie. treat it as a plane wall).



T max = 1500K
Too = 2300K
h = 5000 W/m^2*k

$h := 5000$

$h := 5000$ **(1)**

$L := .01$

$L := 0.01$ **(2)**

$k := 10\#\frac{1}{m\,K}$

$k := 10$ **(3)**

$Bi := \frac{h \cdot L}{k}$

$Bi := 5.000000000$ **(4)**

#5 !< 1 so we cant use Lumped Capacitance

$\text{alpha} := (6.0) \cdot 10^{-6}$

$\alpha := 0.000006000000000$ **(5)**

$F[o] := .7\#\frac{\text{alpha}}{L^2}$

$F_o := 0.7$ **(6)**

$T[o] := 1500$

$T_o := 1500$ **(7)**

$T[oo] := 2300$

$T_{oo} := 2300$ **(8)**

$T[i] := 300$

$T_i := 300$ **(9)**

$\text{theta} := evalf\left(\frac{T[o] - T[oo]}{T[i] - T[oo]}\right)$

$\theta := 0.4000000000$ **(10)**

$t[\max] := \frac{F[o] \cdot L^2}{\text{alpha}}$

$t_{\max} := 11.66666667$ **(11)**

11.66666667 Seconds

3) A spherical hailstone that is **5 mm in diameter** is formed in a high-altitude cloud at **-30 ◦C**. If the stone begins to fall through warmer air at **5◦C**, how long will it take before the outer surface begins to melt? What is the temperature of the stone's center at this point in time, and how much energy (J) has been transfer to the stone? A **convection coefficient of 250 W/m2-K** may be assumed, and the properties of the hailstone may be taken to be **those of ice**.

$h := 250$

$h := 250$ **(1)**

$R := \frac{.005}{2}$

$R := 0.002500000000$ **(2)**

$L := \frac{R}{3}$

$L := 0.0008333333333$ **(3)**

$k := 2.215 \# \frac{J}{m\,K}$

$k := 2.215$ **(4)**

$Bi := \frac{h \cdot L}{k}$

$Bi := 0.09405568095$ **(5)**

#5 < 0.1 so we can use Lumped Capacitance

$\text{alpha} := 1.15 \cdot 10^{-6}$

$\alpha := 0.000001150000000$ **(6)**

$F[o] := \frac{\text{alpha}}{L^2}$

$F_o := 1.656000000$ **(7)**

$T[o] := 1500$

$T_o := 1500$ **(8)**

$T[oo] := 5$

$T_{oo} := 5$ **(9)**

$T[i] := -30$

$T_i := -30$ **(10)**

$\text{rho} := 917$

$\rho := 917$ **(11)**

$A := 4 \cdot \text{Pi} \cdot R^2$

$A := 0.00007853981636$ **(12)**

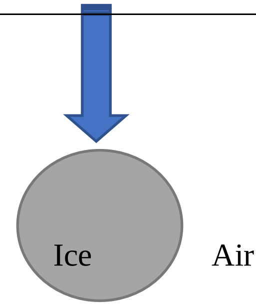
$V := \frac{4}{3} \text{Pi} \cdot R^3$

$V := 6.544984696\ 10^{-8}$ **(13)**

$C := 2100 \# \frac{j}{kg\,K}$

$C := 2100$ **(14)**

K = 2.215



Transient Conduction - Lumped Capacitance

$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right)t\right]$

$Q = (\rho V c)\theta_i\left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right]$

$\tau_t = \frac{\rho V c}{h A_s}$

$L_c = \frac{V}{A_s}$

$Bi = \frac{hL_c}{k} < 0.1$

$Fo = \frac{\alpha t}{L_c^2}$

$Q_0 = \rho c V (T_i - T_\infty)$

$solve\left(\frac{0 - T[oo]}{T[i] - T[oo]} = \exp\left(-\left(\frac{h \cdot A}{\text{rho} \cdot V \cdot C}\right)t\right), t\right)$

12.49079724 **(15)**

$t := 12.49079724$

$t := 12.49079724$ **(16)**

$\text{tau} := \frac{h \cdot A}{\text{rho} \cdot V \cdot C}$

$\tau := 0.1557875059$ **(17)**

$Q := (\text{rho} \cdot V \cdot C) \cdot (T[i] - T[oo]) \cdot \left(1 - \exp\left(-\frac{t}{\text{tau}}\right)\right)$

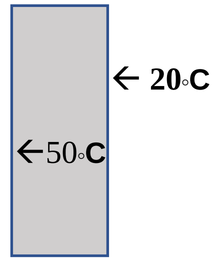
$Q := -4.411286960$ **(18)**

$Q \cdot t$

-55.10049098 **(19)**

-55.10049098 J

4) Asphalt pavement may achieve temperatures as **high as 50◦C** on a hot summer day. Assume that such a temperature exists throughout the pavement, when suddenly a rainstorm reduces the surface temperature to **20◦C**. Calculate the total amount of energy (J/m2) that will be transferred from the asphalt over a **30 min period** in which the surface is maintained at **20◦C** and the temperature deep in the pavement is **50◦C**.



$h := 15$

$$h := 15 \tag{1}$$

$$L := \frac{1}{3} \tag{2}$$

$k := 0.060$

$$k := 0.060 \tag{3}$$

$\rho := 1100$

$$\rho := 1100 \tag{4}$$

$C[p] := 920$

$$C_p := 920 \tag{5}$$

$t := \frac{30 \cdot 60}{1}$

$$t := 1800 \tag{6}$$

$\alpha := evalf\left(\frac{k}{\rho \cdot C[p]}\right)$

$$\alpha := 5.928853755\ 10^{-8} \tag{7}$$

$T[s] := 20$

$$T_s := 20 \tag{8}$$

$T[i] := 50$

$$T_i := 50 \tag{9}$$

$qPimePrime := \frac{k \cdot (T[s] - T[i])}{\mathrm{sqrt}(\mathrm{Pi} \cdot \alpha \cdot t)}$

$$qPimePrime := -98.30507689 \tag{10}$$

-98.30507689 J/m^2