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**MAE 3440: HW #2**

Due January 16, 2020

1. One-dimensional, steady-state conduction without heat generation occurs in a plane wall ( $T_1$  on left and  $T_2$  on right). The thermal conductivity ( $k$ ) is  $5 \text{ W/m}^2$  and the thickness ( $L$ ) is  $0.1 \text{ m}$ . Determine the unknown quantities for each case in the table below and *sketch* the temperature distribution and indicate the direction of the heat flux for each.

case	$T_1$	$T_2$	$dT/dx \text{ [K/m]}$	$q''_x \text{ [W/m}^2\text{]}$
1	400 K	300 K		
2	100°C		-250	
3	80°C		+200	
4		-5°C		4000
5	30°C			-3000

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2. A solid cylindrical rod of length ( $L$ ) 0.1 m and diameter ( $d$ ) 25 mm is well insulated on its side, while the end faces are maintained at temperatures of 100°C and 0°C. What is the rate of heat transfer through the rod if it is constructed from (a) pure copper, (b) aluminum alloy 6061-T6, (c) AISI 304 stainless steel, (d) fused silica glass ( $\text{SiO}_2$ ), (e) wood (oak), (f) magnesia, 85%, and (g) Silica aerogel? Use the properties tables at the end of the book.

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3. Uniform internal heat generation at  $\dot{q} = 5 \times 10^7 \text{ W/m}^3$  is occurring in a cylindrical nuclear reactor fuel rod of 50 mm diameter, and under steady-state conditions the temperature distribution is of the form  $T(r) = a + br^2$ , where  $T$  is in degrees Celsius and  $r$  is in meters, while  $a = 800^\circ\text{C}$  and  $b = -4.167 \times 10^5^\circ\text{C/m}^2$ . The fuel rod properties are  $k = 30 \text{ W/m-K}$ ,  $\rho = 1100 \text{ kg/m}^3$ , and  $c_p = 800 \text{ J/kg-K}$ .

(a) What is the rate of heat transfer per unit length of the rod at  $r = 0$  (the centerline) and at  $r = 25 \text{ mm}$  (the surface)?

(b) If the reactor power level is suddenly increased to  $\dot{q}_2 = 10^8 \text{ W/m}^3$ , what is the initial time rate of temperature change at  $r = 0$  and  $r = 25 \text{ mm}$ ?

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4. Passage of an electric current through a long conducting rod of radius  $r_i$  and thermal conductivity  $k_r$  results in uniform volumetric heating at a rate of  $\dot{q}$ . The conducting rod is wrapped in an electrically nonconducting cladding material of outer radius  $r_o$  and thermal conductivity  $k_c$ , and convection cooling is provided by an adjoining fluid ( $T_\infty$ ,  $h$ ). For steady-state conditions, write appropriate forms of the heat equations for the rod and cladding. Express appropriate boundary conditions for the solution of these equations.

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5. The walls of a refrigerator are typically constructed by sandwiching a layer of insulation between sheet metal panels. Consider a wall made from fiberglass insulation of thermal conductivity  $k_i = 0.045$  W/m-K and thickness  $L_i = 50$  mm and steel panels, each of thermal conductivity  $k_p = 60$  W/m-K and thickness  $L_p = 3$  mm. If the wall separates refrigerated air at  $T_{\infty,i} = 4^\circ\text{C}$  from ambient air at  $T_{\infty,o} = 25^\circ\text{C}$ , what is the heat gain per unit surface area? Coefficients associated with natural convection at the inner and outer surfaces may be approximated as  $h_i = h_o = 5$  W/m<sup>2</sup>-K.

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6. Using Energy2D and the equivalent resistance circuit method covered in class, complete the following table. The Energy2D model has been created for case 1 and is available on Canvas in the homework directory (HW02P6.e2d). For the Energy2D values, you can average the results of the heat flux sensors positioned normal to the direction of heat transfer. The temperature difference across the composite wall for each case is constant and can be obtained from the Energy2D file. Once you complete the table, do you feel the equivalent resistance method is acceptable for all cases?

Table 1: Property sets for comparison

case	$k_A$ (W/m-K)	$k_B$ (W/m-K)	Equiv. Res. $q''$	Energy2D $q''$	% error
1	10	10			
2	100	1			
3	1000	0.1			
4	10000	0.01			