1. A 40 cm diameter pipe at 75˚C is buried in a large block of Portland cement. It runs parallel with a 15˚C isothermal surface at a depth of 1 m. Plot the temperature distribution along the line normal to the 15˚C surface that passes through the center of the pipe. Compute the **heat loss** from the pipe analytically. Then obtain the solution using **either a flux plot** or Energy 2D.

k Portland cement = 0.7

$qLRatio = S \cdot k \cdot DeltaT$

$$qLRatio = S\, k\, DeltaT \quad (1)$$

$L := 1$

$$L := 1 \quad (2)$$

$$qLRatio := 115.080 \quad (3)$$

$r := .2$

$$r := 0.2 \quad (4)$$

$h := 1$

$$h := 1 \quad (5)$$

$S := \operatorname{abs}\left(\frac{2 \cdot \mathrm{Pi} \cdot L}{\cos^{-1}\left(\frac{h}{r}\right)}\right)$

$$S := 2.740838644 \quad (6)$$

$k := 0.7$

$$k := 0.7 \quad (7)$$

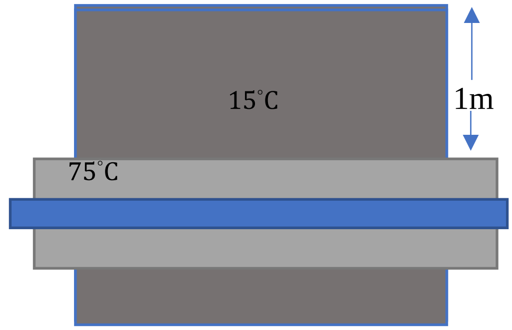
$DeltaT := 75 - 15$

$$DeltaT := 60 \quad (8)$$

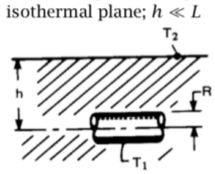
$qLRatio := S \cdot k \cdot DeltaT$

$$qLRatio := 115.1152231 \quad (9)$$

q= 115.11 W/m



5. Cylinder of radius R and length L, transferring heat to a parallel isothermal plane; $h \ll L$



$$\frac{2\pi L}{\cosh^{-1}(h/R)}$$

2. Derive the Shape factors, S, for situations 1-4 in Table 5.4.

 # Situation 1 from equ 5.69

$$s = \frac{1}{k \cdot R[t]}$$

$$s = \frac{1}{k\, R_t} \tag{1}$$

$$R[t] = \frac{L}{k \cdot A}$$

$$R_t = \frac{L}{k\, A} \tag{2}$$

$$S = \frac{A}{L}$$

$$S = \frac{A}{L} \tag{3}$$

 # Situation 2

$$R[t] := \frac{\ln\left(\frac{r2}{r1}\right)}{2 \cdot \mathrm{Pi} \cdot L \cdot k}$$

$$R_t := \frac{1}{2}\, \frac{\ln\left(\frac{r2}{r1}\right)}{\pi\, L\, k} \tag{4}$$

$$S = \frac{1}{k \cdot R[t]}$$

$$S = \frac{2\, \pi\, L}{\ln\left(\frac{r2}{r1}\right)} \tag{5}$$

 # Situation 3

$$R[t] := \frac{\left(\frac{1}{r1}\right) - \left(\frac{1}{r2}\right)}{4 \cdot \mathrm{Pi} \cdot k}$$

$$R_t := \frac{1}{4}\, \frac{\frac{1}{r1} - \frac{1}{r2}}{\pi\, k} \tag{6}$$

$$S = \frac{1}{k \cdot R[t]}$$

$$S = \frac{4\, \pi}{\frac{1}{r1} - \frac{1}{r2}} \tag{7}$$

 # Situation 4 with only 1 radius

$$S = \frac{1}{k \cdot R}$$

$$S = \frac{1}{k\, R} \tag{8}$$

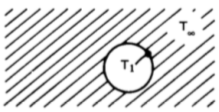
$$S = \frac{4 \cdot \mathrm{Pi} \cdot k}{k \cdot \left(\frac{1}{r}\right)}$$

$$S = 4\, \pi\, r \tag{9}$$

3. An igloo is built in the shape of a hemisphere, with an inner radius of 1.8 m and walls of compacted snow that are 0.5 m thick. On the inside of the igloo the surface heat **transfer coefficient is 3 W/m^2-K**; on the outside surface, under normal wind conditions, it is **10 W/m^2-K**. The **thermal conductivity of compacted snow is 0.15 W/m-K.** The temperature of the ice cap on which the **igloo sits is -20°C** and has the same thermal conductivity as the compacted snow. Assuming that the occupants' body heat provides a **continuous source of 320 W** within the igloo, calculate the inside air temperature when the outside air temperature is **T_∞ = −40°C.**

Hint: You will use situation 12 in Table 5.4.

4. The boundary of a spherical hole of radius R conducting into an infinite medium



$4\pi R$

$$\frac{(1/r_1)-(1/r_2)}{4\pi k}$$

R

$k[ice] := .15$

$$k_{ice} := 0.15 \quad (1)$$

$$q := \frac{DetaT}{R[inner]+R[wall\_In]+R[wall\_Out]} + \frac{DetaT}{R[inner]+R[ground]}$$

$$q := \frac{DetaT}{R_{inner}+R_{wall\_In}+R_{wall\_Out}} + \frac{DetaT}{R_{inner}+R_{ground}} \quad (2)$$

$r[inner] := 1.8$

$$r_{inner} := 1.8 \quad (3)$$

$r[out] := r[inner] + .5$

$$r_{out} := 2.3 \quad (4)$$

$T[inner] := -40$

$$T_{inner} := -40 \quad (5)$$

$T[ice] := -20$

$$T_{ice} := -20 \quad (6)$$

$$R[inner] := \frac{1}{3 \cdot 2\,\mathrm{Pi} \cdot r[inner]^2} \# \frac{1}{hA} \quad \frac{k}{W}$$

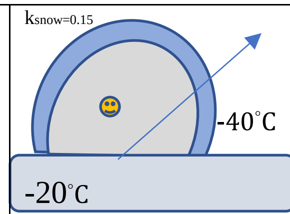
$$R_{inner} := 0.01637396533 \quad (7)$$

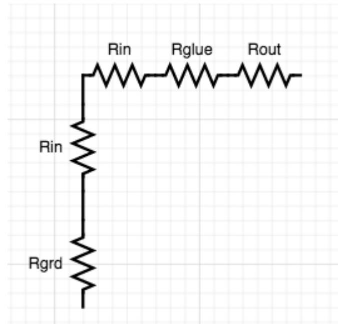
$$R[wall\_inner] := 2\left(\frac{1}{4\,\mathrm{Pi} \cdot k[ice]}\left(\frac{1}{1.8} - \frac{1}{2.3}\right)\right)$$

$$R_{wall\_inner} := 0.1281440766 \quad (8)$$

$$R[wall\_Out] := evalf\left(\frac{1}{3 \cdot \mathrm{Pi} \cdot (1.8^2)}\right)$$

$$R_{wall\_Out} := 0.03274793066 \quad (9)$$





$S := 4 \cdot \mathrm{Pi} \cdot (1.8)$

$$S := 22.61946711 \quad (10)$$

$$R[ground] := \frac{1}{k[ice] \cdot S}$$

$$R_{ground} := 0.2947313761 \quad (11)$$

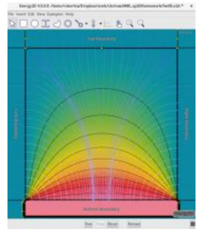
$$solve\left(320 = \frac{Ti - T[inner]}{R_{inner}+R_{wall\_inner}+R_{wall\_Out}} + \frac{Ti - T[ice]}{R_{wall\_inner}+R_{ground}}, Ti\right)$$

$$5.877480516 \quad (12)$$

Ti = 5.8774°C

4. Download the Energy 2D file (hw05.e2d). Go through the four cases by changing the properties of the 4 boundaries. For each one write what type of boundary condition this is and comment on the isotherms and heat flux lines at the boundary for each type of boundary condition. When you are finished, look at the same types of **boundary conditions and what the isotherms** and **heat flux** lines are doing at the **boundary** (**do they cross the boundary**? if they cross, are they perpendicular to the boundary?). Finally, for each type of boundary condition, describe the isotherms and heat flux lines. Note: when you right click an object, in this case these will be the blocks surrounding the center, under “Source” you have options to specify a constant temperature (when a temperature is given), “not a source” when “Not a source” is given, or a “Power source” when a volumetric heat generation is given, which will be entered in the “power density” field. An example is shown on the next page.

Case	left bound.	top bound.	right bound.	bottom bound.
1	0°C	0°C	0°C	10°C
2	0°C	0°C	0°C	50 W/m^3
3	0°C	10°C	0°C	50 W/m^3
4	Not a source	-15 W/m^3	Not a source	15 W/m^3



For example, case 1 is shown here. Each boundary is a 1st Kind or **prescribed temperature Boundary Conditio**n. The isotherms (black lines**) do not cross the boundaries** and the heat flux lines (white to blue) cross the boundaries and are perpendicular at the intersection with the boundary.

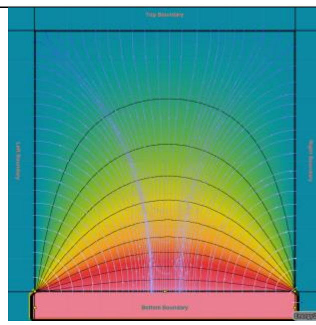
CASE 1:

boundary conditions: temperature Boundary Condition

isotherms: don’t cross Boundary

heat flux: cross boundary’s

boundary: there are 3 directions for the flux to go



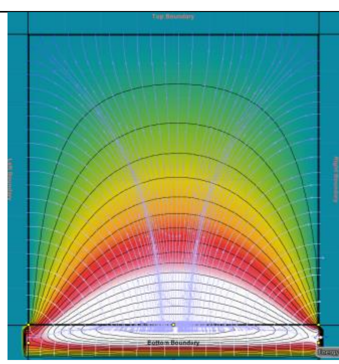
CASE 2:

boundary conditions: temperature Boundary Condition

isotherms: only cross the bottom boundary

heat flux: spread into 3 directions and are perpendicular to the boundary

boundary: bottom is a hear flux source



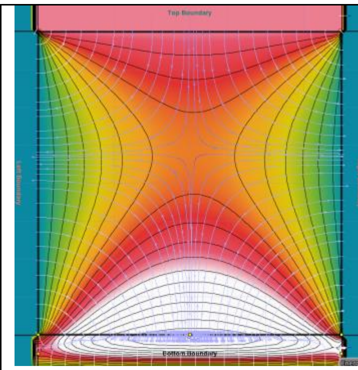
CASE 3:

boundary conditions: temperature Boundary Condition

isotherms: seem to follow the path of the awesomeness, and all bow away from the center

heat flux: The flux is always perpendicular except where the heat source is.

boundary:



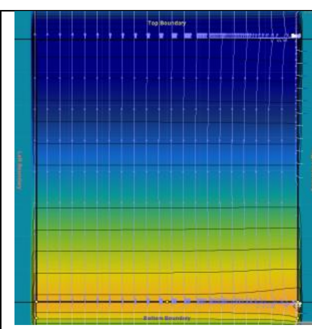
CASE 4:

boundary conditions: temperature Boundary Condition

isotherms: are perpendicular to the insulated sides

heat flux: flow from the source to the sink

boundary: flow from hot to cold 2 Law, IM not sue what to say other then one is a source and the other is a sink.



5. The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature-time history of a sphere fabricated from pure copper. The sphere, which is **12.7 mm in diameter**, is at **66°C** before it is inserted into an airstream having a temperature **of** **27°C.** A thermocouple on the outer surface of the sphere indicates **55°C** **69 s** after the sphere is inserted into the airstream. Assume, and then justify, that the sphere behaves as a space wise isothermal object (ie. the lump capacitance method is valid) and calculate the heat transfer coefficient.	$L[c] := \frac{V}{A}$ $L_c := \frac{V}{A}$ **(1)** $V := \frac{\text{Pi}\, d^3}{6}$ $V := \frac{1}{6}\pi d^3$ **(2)** $A := \text{Pi}\cdot d^2$ $A := \pi d^2$ **(3)** $d := \frac{12.7}{1000}$ $d := 0.01270000000$ **(4)** $L[c]$ 0.002116666667 **(5)** $k := 395$ $k := 395$ **(6)** $A := 0.0005067074792$ **(7)** $T[0] := 55$ $T_0 := 55$ **(8)** $T[1] := 66$ $T_1 := 66$ **(9)** $T[oo] := 27$ $T_{oo} := 27$ **(10)** $\text{rho} := 8954$ $\rho := 8954$ **(11)** $c1 := 384$ $c1 := 384$ **(12)** $evalf\left(solve\left(\frac{T[0]-T[oo]}{T[1]-T[oo]} = e^{\left(-\frac{h*69}{\text{rho}\cdot c1\cdot L[c]}\right)}, h\right)\right)$ $\frac{34.95006777}{\ln(e)}$ **(13)** $h := 34.95006777$ $h := 34.95006777$ **(14)** $B[i] := \frac{h\cdot L[c]}{k}$ $B_i := 0.0001872851733$ **(15)** **Bi = 0.000187285**