MAE 4300 Design Project Submitted by:

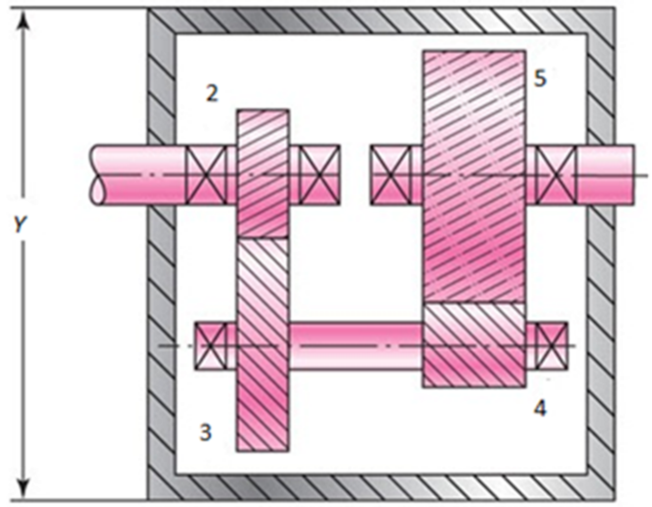
Problem 18-2
For the case study problem, design the output shaft, including complete specification of the gear, bearings, key, retaining rings, and shaft.
Source: R. G. Budynas and J. K. Keith, "Mechanical Engineering Design," 10 Ed.
All equations and procedures are from this textbook.

From the case study in Chapter 18.
Design a speed reducer. It will be a two-stage, compound reverted gear train as shown below.
Design Inputs:

Power input: $H := 20\ \text{hp}$

Input Speed: $\omega_2 := 1750\ \text{rpm}$

Reverted gear train, gear and bearing life > 12,000 hours, shaft has infinite life.



Tooth Counts: $N_2 := 16$ $N_3 := 72$ $N_4 := 16$ $N_5 := 72$

Angular Velocities: $\omega_3 := \omega_2 \cdot \frac{N_2}{N_3} = 388.89\ \text{rpm}$

$$\omega_5 := \omega_3 \cdot \frac{N_2}{N_3} = 86.42\ \text{rpm}$$

Rounded angular velocity: $\omega_5 := 86.42\ \text{rpm}$

Torques: $T_2 := \frac{H}{\omega_2} = 720.29\ \text{in lbf}$ rounded value: $T_2 := 720\ \text{in lbf}$

$T_5 := \frac{H}{\omega_5} = 14585.83\ \text{in lbf}$ rounded value: $T_5 := 14586\ \text{in lbf}$

Gear 5 pitch diameter: $d_{5p} := 12\ \text{in}$ Pressure angle: $\phi := 20\ \text{deg}$

Transverse Force: $W_{45t} := \frac{T_5}{\frac{d_{5p}}{2}} = 2431\ \text{lbf}$ rounded value: $W_{45t} := 2431\ \text{lbf}$

Radial Force: $W_{45r} := W_{45t} \cdot \tan(\phi) = 884.81\ \text{lbf}$ rounded value: $W_{45r} := 885\ \text{lbf}$

Distances from the left bearing center:

$L_I := 0.375\ \text{in}$

$L_J := L_I + 0.25\ \text{in} = 0.625\ \text{in}$

$L_K := L_J + 0.25\ \text{in} = 0.875\ \text{in}$

$L_M := L_K + 2\ \text{in} = 2.875\ \text{in}$

$L_L := L_M - 0.25\ \text{in} = 2.625\ \text{in}$

$L_N := L_M + 0.375\ \text{in} = 3.25\ \text{in}$

$L_O := L_M + 0.75\ \text{in} = 3.625\ \text{in}$

$L_P := L_O + 1\ \text{in} = 4.625\ \text{in}$

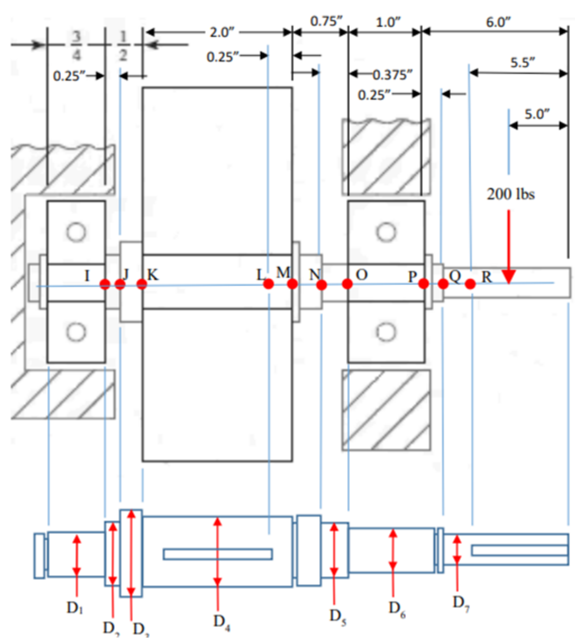
$L_Q := L_P + 0.25\ \text{in} = 4.875\ \text{in}$

$L_R := L_P + 0.5\ \text{in} = 5.125\ \text{in}$

$L_F := L_P + 3\ \text{in} = 7.625\ \text{in}$ Distance to load F

$L_G := L_K + 1\ \text{in} = 1.875\ \text{in}$ Distance to gear center

$L_B := L_O + 0.5\ \text{in} = 4.125\ \text{in}$ Distaance to bearing B

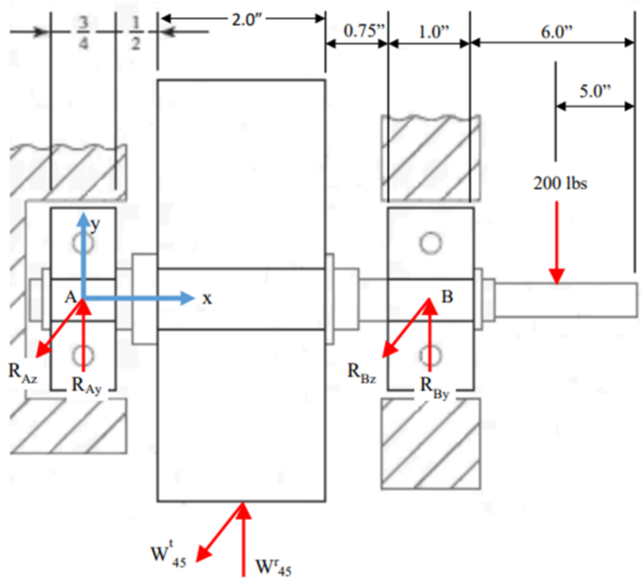


External load on output shaft: $F := 200\ \text{lbf}$

Summation of moments about A

$$R_{Bz} := -\frac{W_{45t} \cdot L_G}{L_B} = -1105\ \text{lbf}$$

$$R_{By} := \frac{F \cdot L_F - W_{45r} \cdot L_G}{L_B} = -32.58\ \text{lbf}$$



Summation of forces

$$R_{Az} := -W_{45t} - R_{Bz} = -1326\ \text{lbf}$$

$$R_{Ay} := -W_{45r} - R_{By} + F = -652.42\ \text{lbf}$$

Vector Sum forces at bearins

$$R_A := \sqrt{R_{Az}^{\ 2} + R_{Ay}^{\ 2}} = 1477.81\ \text{lbf}$$

$$R_B := \sqrt{R_{Bz}^{\ 2} + R_{By}^{\ 2}} = 1105.48\ \text{lbf}$$

Moments at analysis points:

$$M_I := \sqrt{\left(R_{Ay} \cdot L_I\right)^2 + \left(R_{Az} \cdot L_I\right)^2} = 554.18\ \text{in lbf}$$

$$M_J := \sqrt{\left(R_{Ay} \cdot L_J\right)^2 + \left(R_{Az} \cdot L_J\right)^2} = 923.6336\ \text{in lbf}$$

$$M_K := \sqrt{\left(R_{Ay} \cdot L_K\right)^2 + \left(R_{Az} \cdot L_K\right)^2} = 1293.087\ \text{in lbf}$$

$$M_L := \left(\sqrt{\left(R_{Ay} \cdot L_L + W_{45r} \cdot \left(L_L - L_G\right)\right)^2 + \left(R_{Az} \cdot L_L + W_{45t} \cdot \left(L_L - L_G\right)\right)^2}\right) = 1961.4844\ \text{in lbf}$$

$$M_M := \left(\sqrt{\left(R_{Ay} \cdot L_M + W_{45r} \cdot \left(L_M - L_G\right)\right)^2 + \left(R_{Az} \cdot L_M + W_{45t} \cdot \left(L_M - L_G\right)\right)^2}\right) = 1699.8168\ \text{in lbf}$$

$$M_N := \left(\sqrt{\left(R_{Ay} \cdot L_N + W_{45r} \cdot \left(L_N - L_G\right)\right)^2 + \left(R_{Az} \cdot L_N + W_{45t} \cdot \left(L_N - L_G\right)\right)^2}\right) = 1323.3164\ \text{in lbf}$$

$$M_O := \sqrt{\left(R_{Ay} \cdot L_O + W_{45r} \cdot \left(L_O - L_G\right)\right)^2 + \left(R_{Az} \cdot L_O + W_{45t} \cdot \left(L_O - L_G\right)\right)^2} = 985.6887\ \text{in lbf}$$

$$M_P := \sqrt{\left(F \cdot \left(L_F - L_P\right)\right)^2} = 600\ \text{in lbf}$$

$$M_Q := \sqrt{\left(F \cdot \left(L_F - L_Q\right)\right)^2} = 550\ \text{in lbf}$$

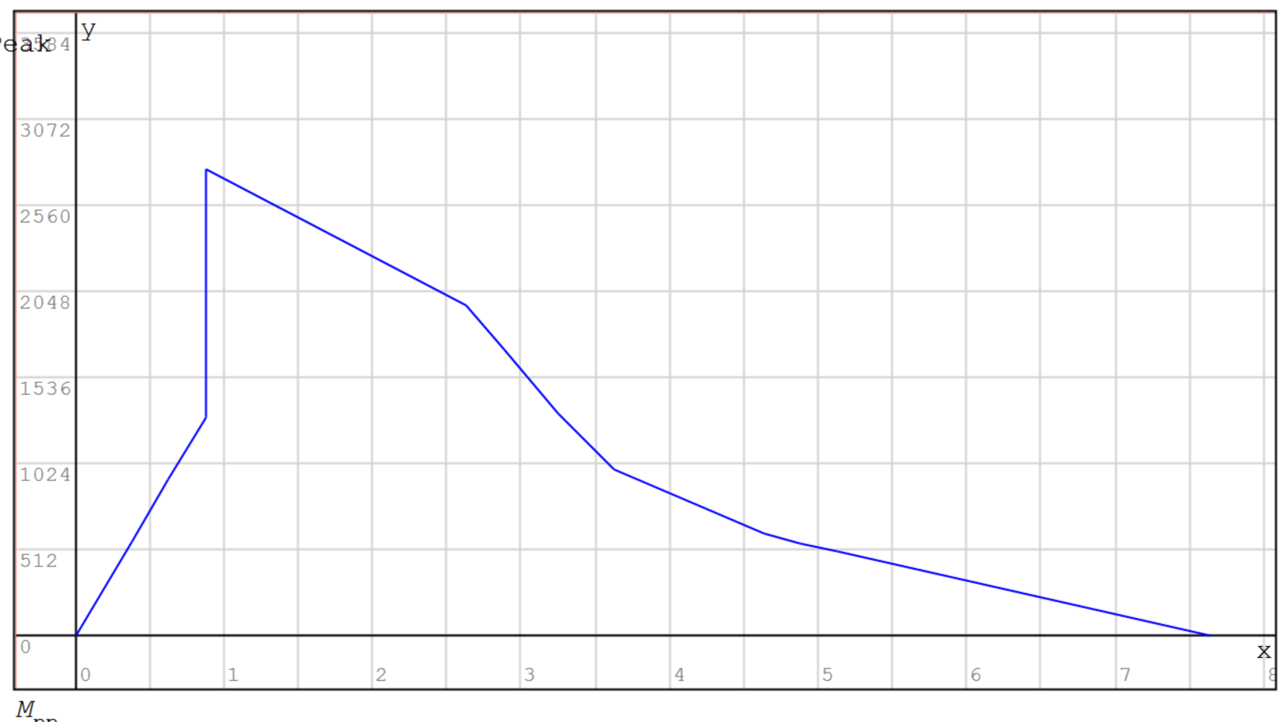
$$M_R := \sqrt{\left(F \cdot \left(L_F - L_R\right)\right)^2} = 500\ \text{in lbf}$$

$$M_G := \sqrt{\left(R_{Ay} \cdot L_G\right)^2 + \left(R_{Az} \cdot L_G\right)^2} = 2770.9007\ \text{in lbf}$$

Store lengths and Moments in Array without units

$$M_{pp} := \begin{bmatrix} 0 & 0 \\ L_I \cdot \frac{1}{\text{in}} & M_I \cdot \frac{1}{\text{in lbf}} \\ L_J \cdot \frac{1}{\text{in}} & M_J \cdot \frac{1}{\text{in lbf}} \\ L_K \cdot \frac{1}{\text{in}} & M_K \cdot \frac{1}{\text{in lbf}} \\ L_K \cdot \frac{1}{\text{in}} & M_G \cdot \frac{1}{\text{in lbf}} \\ L_L \cdot \frac{1}{\text{in}} & M_L \cdot \frac{1}{\text{in lbf}} \\ L_M \cdot \frac{1}{\text{in}} & M_M \cdot \frac{1}{\text{in lbf}} \\ L_N \cdot \frac{1}{\text{in}} & M_N \cdot \frac{1}{\text{in lbf}} \\ L_O \cdot \frac{1}{\text{in}} & M_O \cdot \frac{1}{\text{in lbf}} \\ L_P \cdot \frac{1}{\text{in}} & M_P \cdot \frac{1}{\text{in lbf}} \\ L_Q \cdot \frac{1}{\text{in}} & M_Q \cdot \frac{1}{\text{in lbf}} \\ L_R \cdot \frac{1}{\text{in}} & M_R \cdot \frac{1}{\text{in lbf}} \\ L_F \cdot \frac{1}{\text{in}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0.375 & 554.1801 \\ 0.625 & 923.6336 \\ 0.875 & 1293.087 \\ 0.875 & 2770.9007 \\ 2.625 & 1961.4844 \\ 2.875 & 1699.8168 \\ 3.25 & 1323.3164 \\ 3.625 & 985.6887 \\ 4.625 & 600 \\ 4.875 & 550 \\ 5.125 & 500 \\ 7.625 & 0 \end{bmatrix}$$

Plot of Peak Moments



Student ID: A00998548 Assigned Material: 1035 CD Steel

$S_{ut} := 80\ \text{ksi}$ $S_y := 67\ \text{ksi}$

Calculating C10 bearing Loads

Eq. 11-2b:

$$L_D := 12000\ \text{hr} \cdot \omega_5 \cdot 60 = 2.3457 \cdot 10^{10}$$

$$L_{10} := 90 \cdot 10^{6}\ \text{rev}$$

Eq. 11-9 for C10 bearing loads

$$a_{fa} := 1 \qquad F_{Da} := R_A = 1477.8137\ \text{lbf} \qquad a_a := 3 \qquad \theta_a := 4.48$$

$$R_{Da} := .99 \qquad b_a := 1.5 \qquad x_{0a} := 0$$

$$x_{Da} := \frac{L_D}{L_{10}} = 41.4816$$

$$C_{10a} := a_{fa} \cdot F_{Da} \cdot \left(\frac{x_{Da}}{x_{0a} + \left(\theta_a - x_{0a}\right) \cdot \left(\ln\left(\frac{1}{R_{Da}}\right)\right)^{\frac{1}{b_a}}} \right)^{\frac{1}{a_a}} = 8625.2573\ \text{lbf}$$

Functions used to automate the computations:

Combinind Eq. 6-19 and 6-20:

$$Sef\left(S_{ut}, d\right) := 2.7 \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^{-0.265} \cdot \left| \begin{array}{l} \text{if } d > 2\ \text{in} \\ \quad 0.91 \cdot \left(\frac{d}{\text{in}}\right)^{-0.157} \\ \text{else} \\ \quad 0.879 \cdot \left(\frac{d}{\text{in}}\right)^{-0.107} \end{array} \right. \cdot 0.5 \cdot S_{ut}$$

Using Eq. 6-33, 6-34, 6-35, and 6-36, we get the following functions to compute Kf and Kfs.

$$Kff\left(S_{ut}, r, K_t\right) := 1 + \frac{K_t - 1}{1 + \frac{\left(0.246 - 3.08 \cdot 10^{-3} \cdot \frac{S_{ut}}{\text{ksi}} + 1.51 \cdot 10^{-5} \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^{2} - 2.67 \cdot 10^{-8} \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^{3}\right) \cdot \sqrt{\text{in}}}{\sqrt{r}}}$$

$$Kfs\left(S_{ut}, r, K_{ts}\right) := 1 + \frac{K_{ts} - 1}{1 + \frac{\left(0.19 - 2.51 \cdot 10^{-3} \cdot \frac{S_{ut}}{ksi} + 1.35 \cdot 10^{-5} \cdot \left(\frac{S_{ut}}{ksi}\right)^{2} - 2.67 \cdot 10^{-8} \cdot \left(\frac{S_{ut}}{ksi}\right)^{3}\right) \cdot \sqrt{in}}{\sqrt{r}}}$$

Eq 7-5 assuming Ta = 0: $\sigma_{af}\left(K_f, M, D\right) := \dfrac{32 \cdot K_f \cdot M}{\pi \cdot D^3}$

Eq 7-6 assuming Mm = 0: $\sigma_{mf}\left(K_{fs}, T, D\right) := \sqrt{3 \cdot \left(\dfrac{16 \cdot K_{fs} \cdot T}{\pi \cdot D^3}\right)^2}$

Eq. 6-46 for fatigue safety factor using modified Goodman $n_{ff}\left(\sigma'_a, \sigma'_m, S_e, S_{ut}\right) := \dfrac{1}{\dfrac{\sigma'_a}{S_e} + \dfrac{\sigma'_m}{S_{ut}}}$

Eq. 6-49 for predicting yield (conservative approach): $n_{yf}\left(S_y, \sigma'_a, \sigma'_m\right) := \dfrac{S_y}{\sigma'_a + \sigma'_m}$

Eq. 11-9 for C10 bearing loads

$$ratedLoad\left(a_f, F_D, R_D, a, b, x_0, \theta, x_D\right) := a_f \cdot F_D \cdot \left(\frac{x_D}{x_0 + \left(\theta - x_0\right) \cdot \left(\ln\left(\frac{1}{R_D}\right)\right)^{\frac{1}{b}}}\right)^{\frac{1}{a}}$$

Shaft Diameters:

$D_1 := 0.8\ in$

$D_2 := 1.0$ in

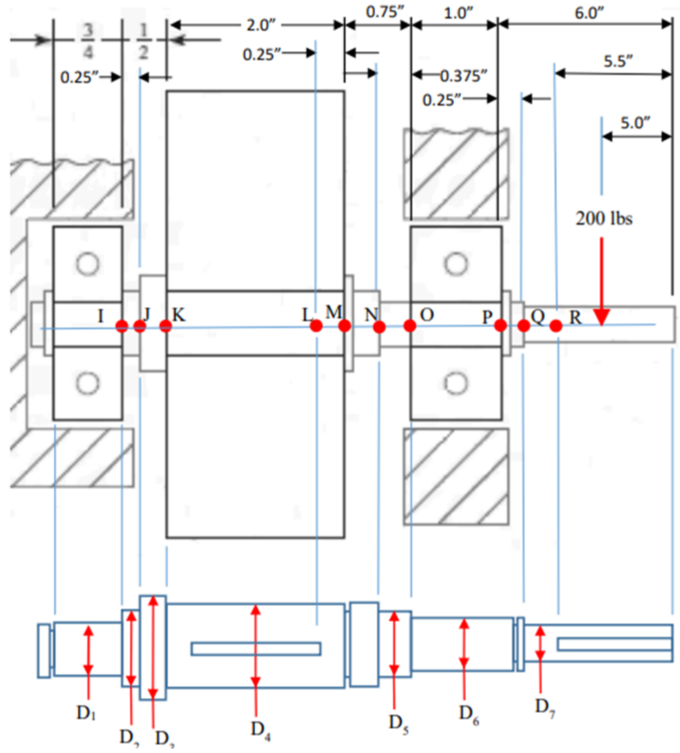
$D_3 := 3.0$ in

$D_4 := 2.5$ in

$D_5 := 2.25$ in

$D_6 := 2.0$ in

$D_7 := 1.75$ in



Stress Concentration Factors are Selected From Table 7-1

Table 7-1

First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts}.

Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

	Bending	Torsional	Axial
Shoulder fillet—sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet—well rounded (r/d = 0.1)	1.7	1.5	1.9
End-mill keyseat (r/d = 0.02)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove (use r=0.01 in)	5.0	3.0	5.0

$K_t := 2.7$ $\qquad K_{ts} := 2.2$

Stress Analysis at Point I

Moment and Torque: $M := M_I = 554.1801$ in lbf $\qquad T := 0$

Selected Diameter: $d := D_1 = 0.8$ in

Stress Concentration Factors from Table 7-1 (Sharp Fillet):

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b

$Kf := Kff\left(S_{ut}, 0.02 \cdot d, K_t\right) = 2.0286$ $\qquad Kfs := Kfs\left(S_{ut}, 0.02 \cdot d, K_{ts}\right) = 1.8056$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}\left(Kf, M, d\right) = 22.3652$ ksi

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}\left(Kfs, T, d\right) = 0$ ksi

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 30.4413\ \text{ksi}$

Safety Factor against fatigue, Eq. 6-46: $n_{fI} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 1.36$

Safety Factor against yielding, Eq. 6-49: $n_{yI} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 2.996$