3-18, For each of the stress states listed below, find all three principal normal and shear stresses. Draw a complete Mohr's three-circle diagram and label all points of interest.

(*a*) σ_x = -80 MPa, σ_y =-30 MPa, τ_{xy} = 20 MPa cw

sigma[x] := −80

$$\sigma_x := -80 \quad (1)$$

sigma[y] := −30

$$\sigma_y := -30 \quad (2)$$

tau[xy] := 20

$$\tau_{xy} := 20 \quad (3)$$

$center := \frac{(\text{sigma}[x] + \text{sigma}[y])}{2}$

$$center := -55 \quad (4)$$

$R := evalf\left(\text{sqrt}\left(\left(\frac{(\text{sigma}[x] - \text{sigma}[y])}{2}\right)^2 + \text{tau}[xy]^2\right)\right)$

$$R := 32.01562118 \quad (5)$$

sigma[1] := 0

$$\sigma_1 := 0 \quad (6)$$

sigma[2] := *center* + *R*

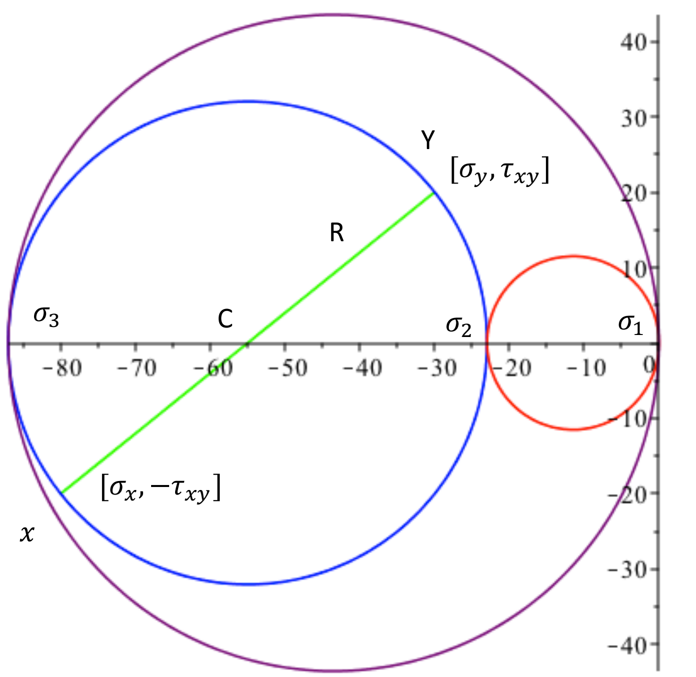
$$\sigma_2 := -22.98437882 \quad (7)$$

sigma[3] := *center* − *R*

$$\sigma_3 := -87.01562118 \quad (8)$$

with(*plottools*) :
with(*plots*) :
$c1 := circle([center, 0], R, color = blue) :$
$c2 := circle\left(\left[\frac{(center + R)}{2}, 0\right], \frac{(center + R)}{2}, color = red\right) :$
$c3 := circle\left(\left[\frac{center - R}{2}, 0\right], \frac{center - R}{2}, color = purple\right) :$

display({*c1*, *c2*, *c3*, *line*([sigma[x], tau[xy]], [sigma[y], −tau[xy]], *color* = "green")})



{NOTE: that the axis for all of these plots are will the negative τ in the **downward direction**} (*b*) σ_x = -30 MPa, σ_y =-60 MPa, τ_{xy} = 30 MPa cw

sigma[x] := -30

$$\sigma_x := -30 \quad (1)$$

sigma[y] := -60

$$\sigma_y := -60 \quad (2)$$

tau[xy] := 30

$$\tau_{xy} := 30 \quad (3)$$

$$center := \frac{(\text{sigma}[x] + \text{sigma}[y])}{2}$$

$$center := -45 \quad (4)$$

$$R := evalf\left(\text{sqrt}\left(\left(\frac{(\text{sigma}[x] - \text{sigma}[y])}{2}\right)^2 + \text{tau}[xy]^2\right)\right)$$

$$R := 33.54101966 \quad (5)$$

sigma[1] := 0

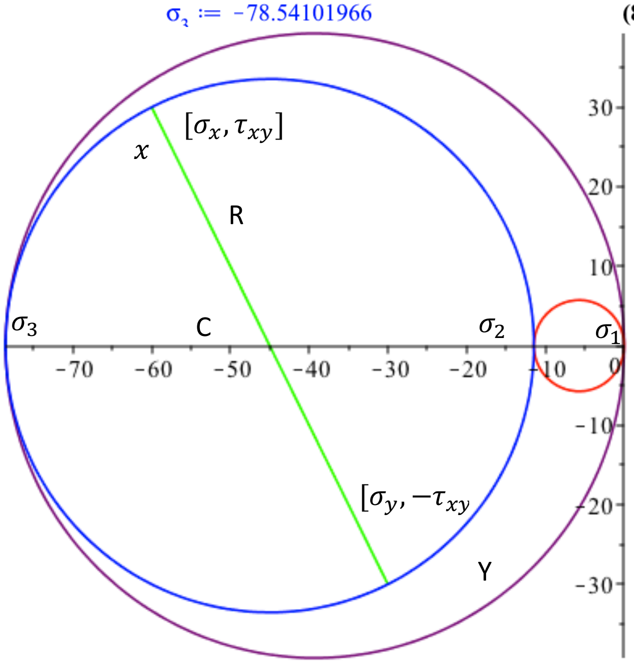
$$\sigma_1 := 0 \quad (6)$$

sigma[2] := *center* + *R*

$$\sigma_2 := -11.45898034 \quad (7)$$

sigma[3] := *center* − *R*

$$\sigma_3 := -78.54101966 \quad (8)$$



(c) $\sigma_x = 40$ MPa, $\sigma_z =$ -30 MPa, $\tau_{xy} = 20$ MPa ccw

Principal Stresses are roots of:

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma - (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) = 0 \quad (3\text{–}15)$$

$$\tau_{max} = \tau_{1/3} = \tfrac{1}{2}(\sigma_1 - \sigma_3)$$

sigma[x] := 40

$\sigma_x := 40$ **(1)**

sigma[y] := 0

$\sigma_y := 0$ **(2)**

sigma[z] := -30

$\sigma_z := -30$ **(3)**

c := 20

$\tau_{xy} := 20$ **(4)**

evalf(solve(σ^3 − (sigma[x] + sigma[y] + sigma[z])σ^2 + (sigma[x]·sigma[y] + sigma[x]·sigma[z] + sigma[z]·sigma[y] − tau[xy]^2)sigma − (−sigma[z]·tau[xy]^2) = 0, sigma))

−30., −8.28427124, 48.28427124 **(5)**

sigma[1] := 48.28427124

$\sigma_1 := 48.28427124$ **(6)**

sigma[2] := −8.28427124

$\sigma_2 := -8.28427124$ **(7)**

sigma[3] := −30

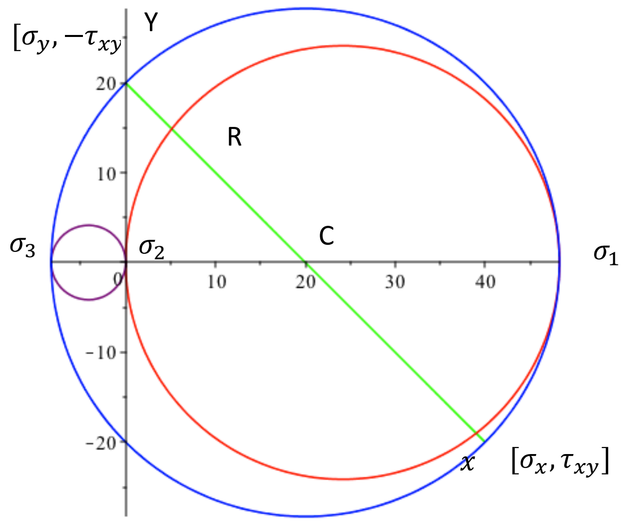
$\sigma_3 := -30$ **(8)**

center := (sigma[x] + sigma[y]) / 2

center := 20 **(9)**

R := evalf(sqrt(((sigma[x] − sigma[y]) / 2)^2 + tau[xy]^2))

R := 28.28427124 **(10)**



(*d*) $\sigma_x = 50$ MPa, $\sigma_z =$ -20 MPa, $\tau_{xy} = 30$ MPa cw

sigma[x] := 50

$$\sigma_x := 50 \tag{1}$$

sigma[y] := 0

$$\sigma_y := 0 \tag{2}$$

sigma[z] := 30

$$\sigma_z := 30 \tag{3}$$

tau[xy] := 20

$$\tau_{xy} := 20 \tag{4}$$

evalf(*solve*(σ³ − (sigma[x] + sigma[y] + sigma[z])σ² + (sigma[x]·sigma[y] + sigma[x]·sigma[z] + sigma[z]·sigma[y] − tau[xy]²)sigma − (−sigma[z]·tau[xy]²) = 0, sigma))

$$30.,\ -7.01562118,\ 57.01562118 \tag{5}$$

sigma[1] := 57.01562118

$$\sigma_1 := 57.01562118 \tag{6}$$

sigma[2] := 30.

$$\sigma_2 := 30. \tag{7}$$

sigma[3] := −7.01562118

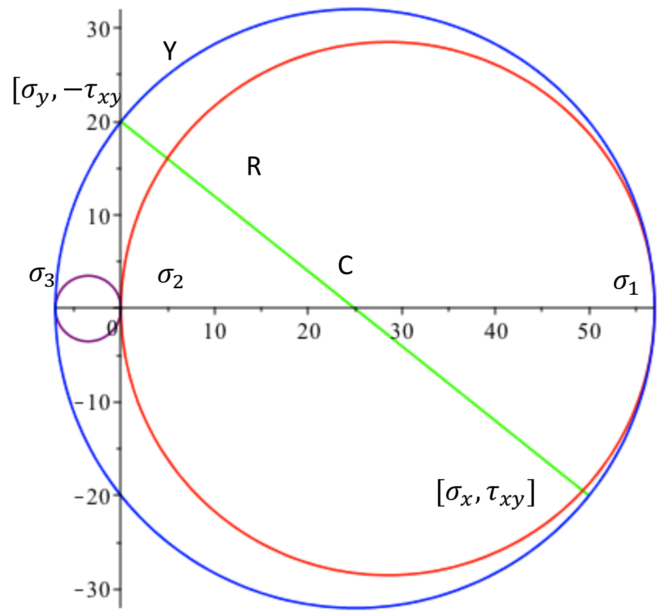
$$\sigma_3 := -7.01562118 \tag{8}$$

$$center := \frac{(\text{sigma}[x] + \text{sigma}[y])}{2}$$

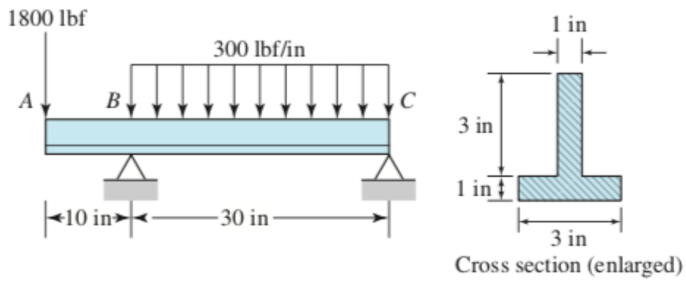
$$center := 25 \tag{9}$$

$$R := evalf\left(\text{sqrt}\left(\left(\frac{(\text{sigma}[x] - \text{sigma}[y])}{2}\right)^2 + \text{tau}[xy]^2\right)\right)$$

$$R := 32.01562118 \tag{10}$$



3-44 For the beam shown, determine



$F[C] := 3900$ #lbf

$F_C := 3900$

$F[B] := 1800 + 300\cdot 30 - F[C]$

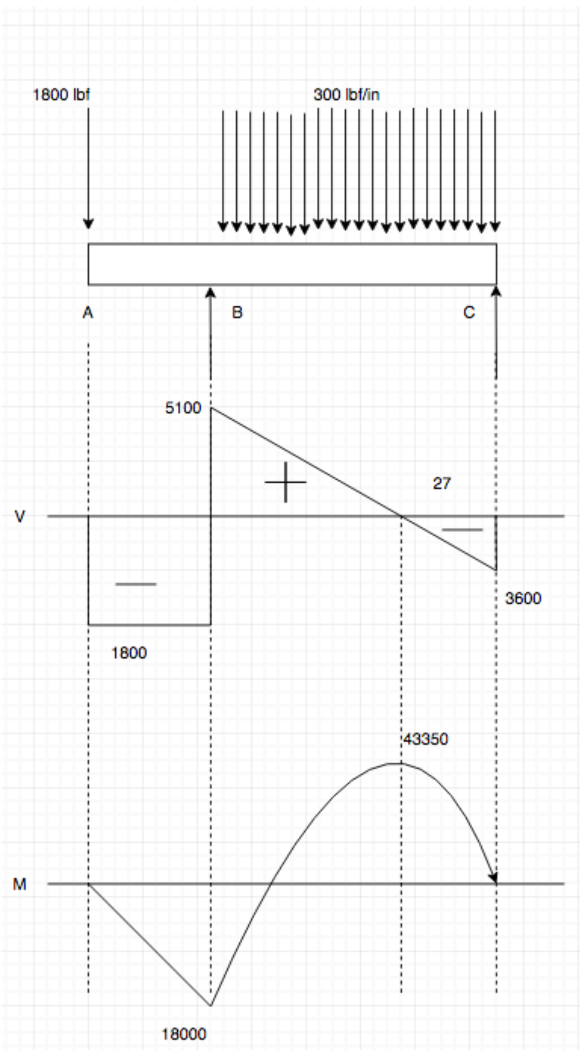
$F_B := 6900$

$evalf(solve(0 = 5100 - 300(x - 10), x))$

$27.$

$y := \int_{10}^{27} 5100 - 300(x - 10)\, dx$

$y := 43350$



find the centroid

$c2 := 1.5$

$c2 := 1.5$

$c1 := 2.5$

$c1 := 2.5$

$$\text{Iota} := \frac{1\cdot 3^3}{12} + 1\cdot 3\cdot(2.5 - 1.5)^2 + \frac{3\cdot 1^3}{12} + 3\cdot 1\cdot(1.5 - .5)^2$$

$I := 8.500000000$

$$\text{sigma} = -\frac{M\cdot y}{\text{Iota}}$$

$\sigma = -0.1176470588\, M\, y$

#@at 2.5 top

$$\text{sigma} := -\frac{-18000\cdot 2.5}{\text{Iota}}$$

$\sigma := 5294.117646$

$$\text{sigma} := -\frac{18000\cdot(-1.5)}{\text{Iota}}$$

$\sigma := 3176.470588$

$$\text{sigma} := -\frac{25350\cdot 2.5}{\text{Iota}}$$

$\sigma := -7455.882351$

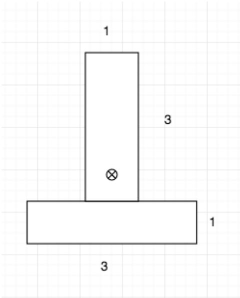
$$\text{sigma} := -\frac{25350\cdot(-1.5)}{\text{Iota}}$$

$\sigma := 4473.529411$

(a) the maximum tensile and compressive bending stresses,

MAX tensile Stress is **5294.11 PSI**

MAX compressive Stress is **-7444.88 PSI** (*b*) the maximum shear stress due to V, and



$$t[\max] := \frac{V[\max]\cdot Q}{\mathrm{Iota}\cdot b}$$

$$t_{max} := \frac{V_{max}\, Q}{\mathrm{I}\, b}$$

$$Q := y\_bar \cdot A$$

$$Q := 1.25\, A$$

$$y\_bar := 3.75 - 2.5$$

$$y\_bar := 1.25$$

$$A := 2.5\cdot 1$$

$$A := 2.5$$

$$Q := y\_bar \cdot A$$

$$Q := 3.125$$

$$t[\max] := \frac{5100\cdot Q}{\mathrm{Iota}\cdot 1}$$

$$t_{max} := 1875.000000$$

(c) the maximum shear stress in the beam.

This should happen at the neutral surface or close to it

All at the point **27 in**

$\sigma := -7455.882351$

$\sigma := -7455.882351$

$tau := \frac{sigma}{2}$

$\tau := -3727.941176$

$\sigma := 4473.529411$

$\sigma := 4473.529411$

$tau := \frac{sigma}{2}$

$\tau := 2236.764706$

$sigma := -\frac{2535.0 \cdot (-0.5)}{Iota}$

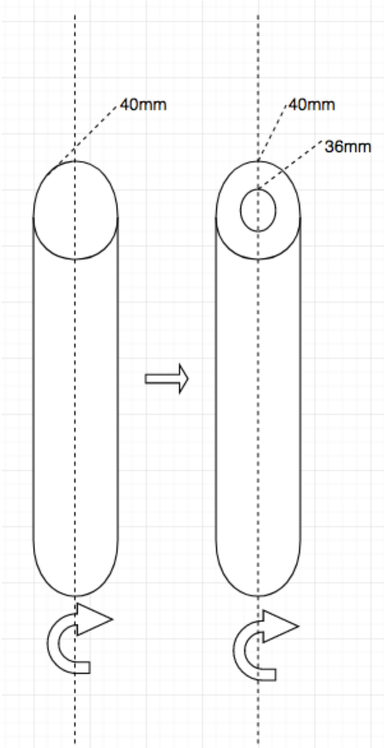
$\sigma := 149.1176470$

$tau := \frac{sigma}{2}$

$\tau := 74.55882350$

maximum shear 74.55 psi

3-62 A 40-mm-diameter solid steel shaft, used as a torque transmitter, is replaced with a hollow shaft having a 40-mm OD and a 36-mm ID. If both materials have the same strength, what is the percentage reduction in **torque** transmission? What is the **percentage reduction** in shaft weight?



 # centroid is

$C := \frac{40}{2}$ *#mm*

$$C := 20 \tag{1}$$

$\tau[\max] := \frac{T \cdot C}{J}$

$$\tau_{\max} := \frac{20\,T}{J} \tag{2}$$

$J[solid] := evalf\left(\frac{\mathrm{Pi}}{2} C^4\right)$

$$J_{solid} := 2.513274123\ 10^5 \tag{3}$$

$r[outer] := \frac{40}{2}$

$$r_{outer} := 20 \tag{4}$$

$r[inner] := \frac{36}{2}$

$$r_{inner} := 18 \tag{5}$$

$J[hollow\ ] := evalf\left(\frac{\mathrm{Pi}}{2}\left(r[outer]^4 - r[inner]^4\right)\right)$

$$J_{hollow} := 86431.49710 \tag{6}$$

#solve for T

$T := \frac{J \cdot \tau[\max]}{C}$

$$T := \frac{J\,\tau_{\max}}{C} \tag{7}$$

$T[solid] := \frac{J[solid] \cdot \tau[\max]}{C}$

$$T_{solid} := 12566.37062\ \tau_{\max} \tag{8}$$

$T[hollow] := \frac{J[hollow] \cdot \tau[\max]}{C}$

$$T_{hollow} := 4321.574855\ \tau_{\max} \tag{9}$$

$\frac{T[solid] - T[hollow]}{T[solid]}$

$$0.6561000001 \tag{10}$$

$V[solid] := (C)^2 \cdot \mathrm{Pi} \cdot L$

$$V_{solid} := 400\,\pi\,L \tag{11}$$

$V[hollow] := (r[outer])^2 \cdot \mathrm{Pi} \cdot L - (r[inner])^2 \cdot \mathrm{Pi} \cdot L$

$$V_{hollow} := 76\,\pi\,L \tag{12}$$

$evalf\left(\frac{V[solid] - V[hollow]}{V[solid]}\right)$

$$0.8100000000 \tag{13}$$

the percentage reduction in torque: 65.61%

the percentage reduction in shaft weight: 81%