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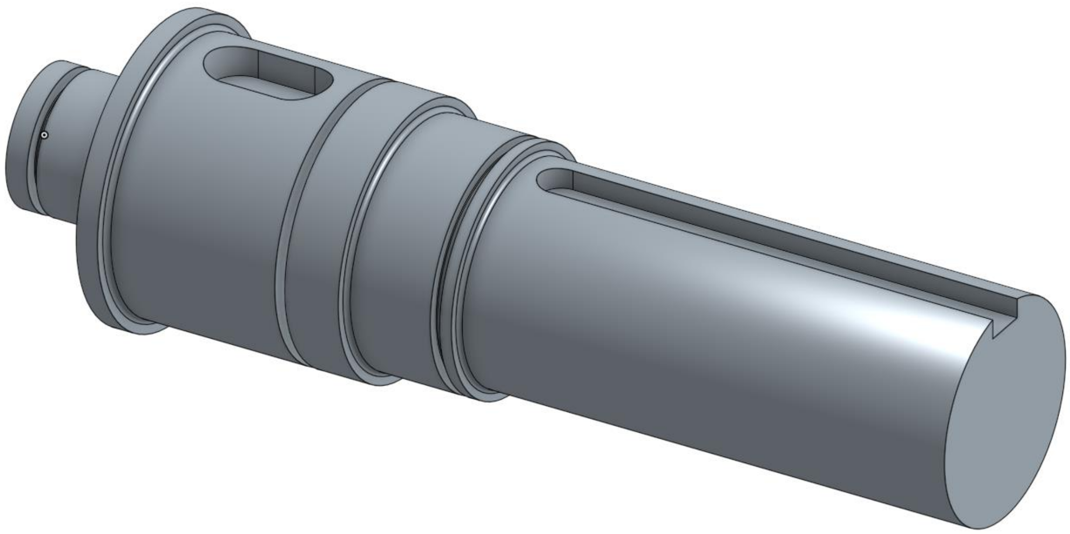
MAE 4300 Design Project

Introduction

The purpose of this design report is to analyze the output shaft of a gear box. The system was geared such that 720.29 in*lbf torque would result in 14586 in*lbf output torque. To do so, it required four gears and three shafts. The objective of this report was to analyze the last of the three shafts and determine proper tolerances such that the safety factor of each part would be exceed 1.5.

There are many obstacles to over come when developing a reliable output shaft. First the internal forces were found by determining the resulting forces acting on the shaft. Bearing must be fitted to the shaft that can withstand the Statics bearing force, from that a diameter can be determined from each point on the shaft I through R.

Output Shaft Design



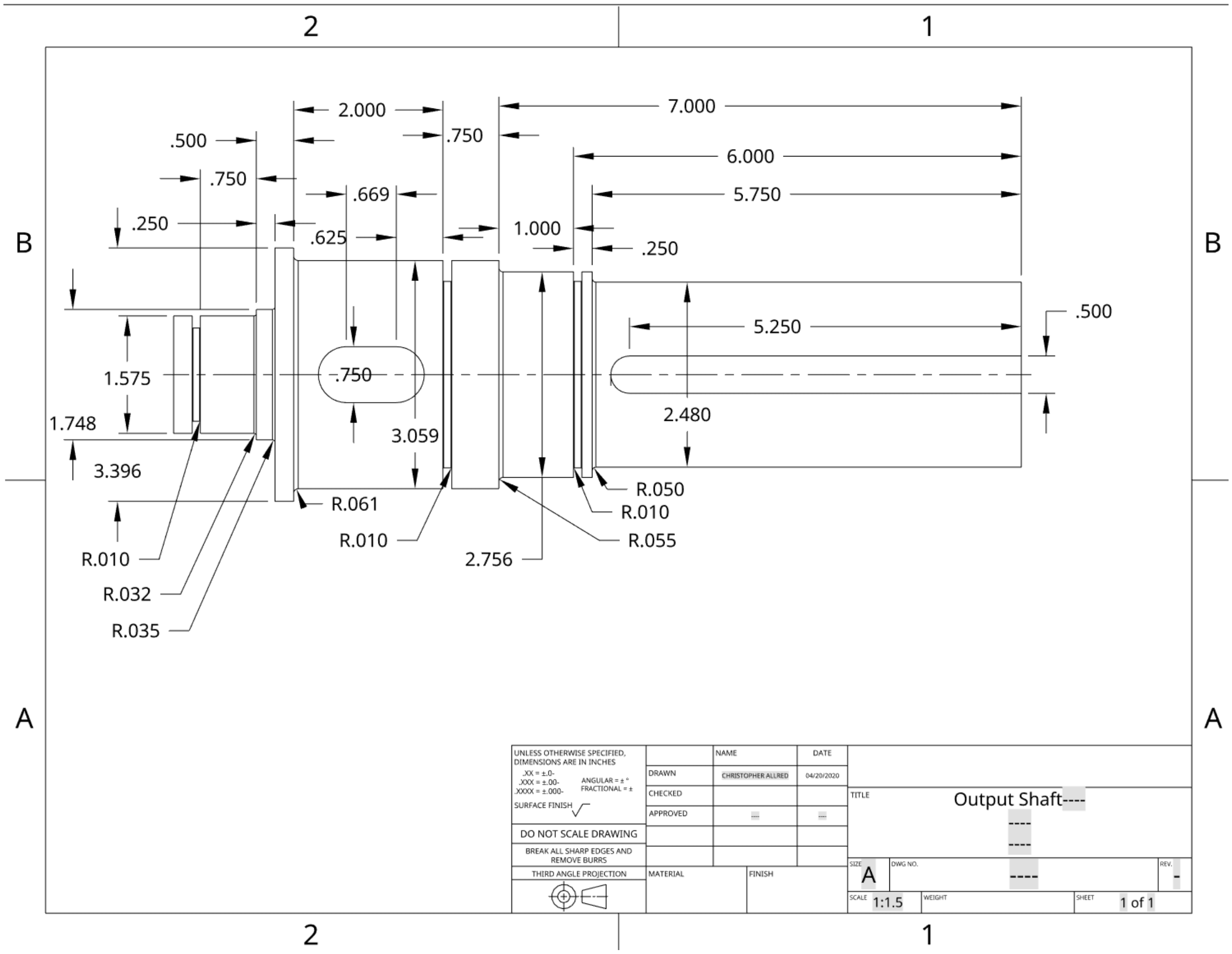


Figure 1, A sketch of the shaft labeling analysis points

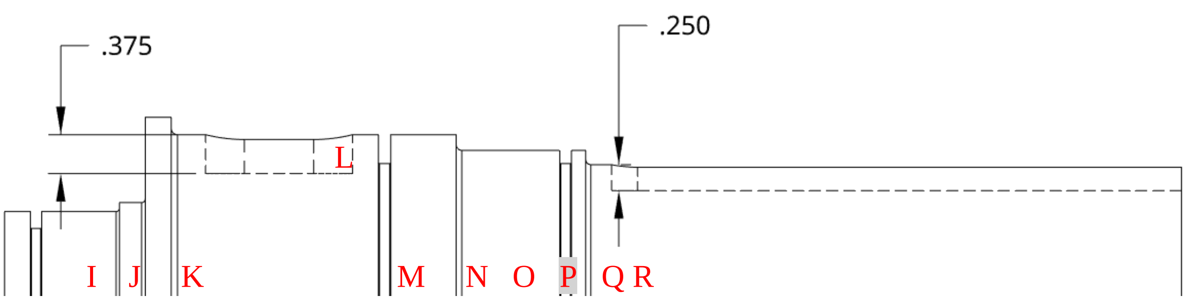


Table 1: material properties, moment, and torque at analysis points on the shaft

	Moment (in* lbf)	Torque (in*lbf)
I	539.11	0
J	898.5243	0
K	1257.9341	0
G	2695.573	14586
L	1838.001	14586
M	1553.93	14586
N	1131.8152	14586
O	721.9903	14586
P	200	14586
Q	150	14586
R	100	14586

Table 2 Stating Selected Diameters

Selected Diameters (in)						
D1	D2	D3	D4	D5	D6	D7
1.5748	1.748	3.3955	3.059	3.059	2.7559	2.48

Table 2: Listing all the safety factors at points on the shaft

Safety Factors	Against Fatigue	Against Yielding
I	7.8	11.162
J	8.32	12.04
K	28.8	45.21
L	19.71	30.942
M	2.18	3.04
N	7.03	4.519
O	4.24	2.597
P	3.24	1.866
Q	6.32	3.569
R	6.4	3.584

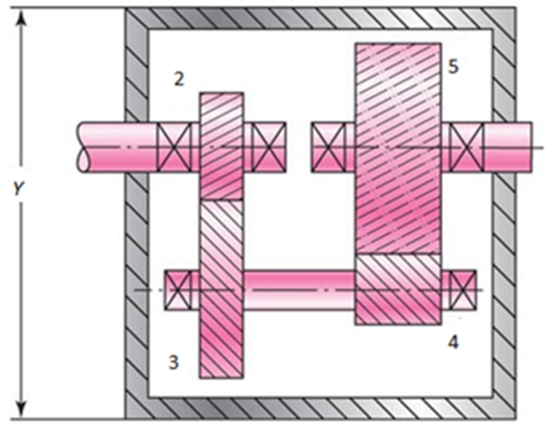
Table 3: computed bearing reaction forces, C10 loads, and bearing type

	Reaction Forces (lbf)	$C10_a$ (lbf)	Bearing Type:
A	1437.639	7822.5859	Cylindrical Roller Bearing
B	1112.5678	6053.7848	Cylindrical Roller Bearing

Table 4: Gear, Shaft key material, and dimensions

Feature	Material Name	Yield Strength (ksi)	Hight (in)	Width (in)	Length (in)	Death (in)
Key 1	1020 CD	57	¾	¾	0.6692	3/8
Key 2	1015 HR	27.5	5/8	5/8	2.0806	5/16
Gear	Not required by	contracted rubric	***	***	***	***

Calculations



Power input: $H := 20\ \text{hp}$

Input Speed: $\omega_2 := 1750\ \text{rpm}$

Reverted gear train, gear and bearing life > 12,000 hours, shaft has infinite life.

Tooth Counts: $N_2 := 16$ $N_3 := 72$ $N_4 := 16$ $N_5 := 72$

Angular Velocities: $\omega_3 := \omega_2 \cdot \dfrac{N_2}{N_3} = 388.89\ \text{rpm}$

$$\omega_5 := \omega_3 \cdot \frac{N_2}{N_3} = 86.42\ \text{rpm}$$

Rounded angular velocity: $\omega_5 := 86.42\ \text{rpm}$

Torques: $T_2 := \dfrac{H}{\omega_2} = 720.29\ \text{in lbf}$ rounded value: $T_2 := 720\ \text{in lbf}$

$T_5 := \dfrac{H}{\omega_5} = 14585.83\ \text{in lbf}$ rounded value: $T_5 := 14586\ \text{in lbf}$

Gear 5 pitch diameter: $d_{5p} := 12\ \text{in}$ Pressure angle: $\phi := 20\ \text{deg}$

Transverse Force: $W_{45t} := \dfrac{T_5}{\dfrac{d_{5p}}{2}} = 2431\ \text{lbf}$ rounded value: $W_{45t} := 2431\ \text{lbf}$

Radial Force: $W_{45r} := W_{45t} \cdot \tan(\phi) = 884.81\ \text{lbf}$ rounded value: $W_{45r} := 885\ \text{lbf}$

Distances from the left bearing center:

$L_I := 0.375\ \text{in}$

$L_J := L_I + 0.25\ \text{in} = 0.625\ \text{in}$

$L_K := L_J + 0.25\ \text{in} = 0.875\ \text{in}$

$L_M := L_K + 2\ \text{in} = 2.875\ \text{in}$

$L_L := L_M - 0.25\ \text{in} = 2.625\ \text{in}$

$L_N := L_M + 0.375\ \text{in} = 3.25\ \text{in}$

$L_O := L_M + 0.75\ \text{in} = 3.625\ \text{in}$

$L_P := L_O + 1\ \text{in} = 4.625\ \text{in}$

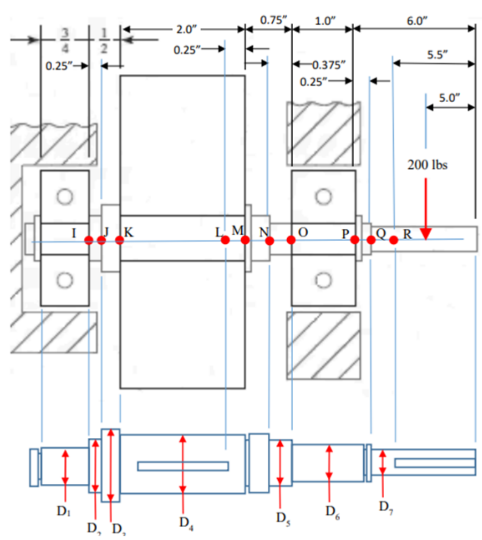
$L_Q := L_P + 0.25\ \text{in} = 4.875\ \text{in}$

$L_R := L_P + 0.5\ \text{in} = 5.125\ \text{in}$

$L_F := L_P + 1.0\ \text{in} = 5.625\ \text{in}$ Distance to load F

$L_G := L_K + 1\ \text{in} = 1.875\ \text{in}$ Distance to gear center

$L_B := L_O + 0.5\ \text{in} = 4.125\ \text{in}$ Distaance to bearing B

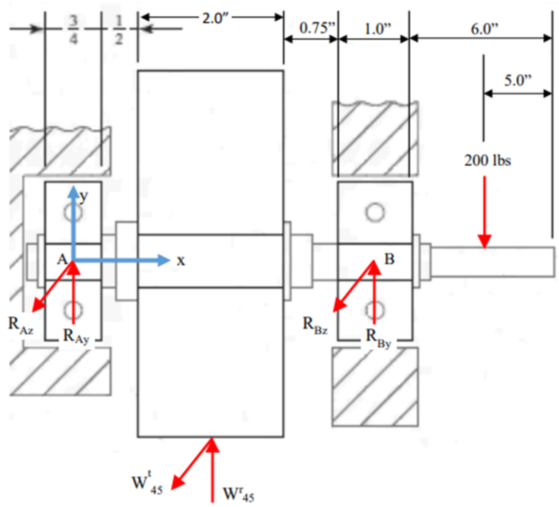


External load on output shaft: $F := 200\ \text{lbf}$

Summation of moments about A

$$R_{Bz} := -\frac{W_{45t} \cdot L_G}{L_B} = -1105\ \text{lbf}$$

$$R_{By} := \frac{F \cdot L_F - W_{45r} \cdot L_G}{L_B} = -129.55\ \text{lbf}$$



Summation of forces

$$R_{Az} := -W_{45t} - R_{Bz} = -1326\ \text{lbf}$$

$$R_{Ay} := -W_{45r} - R_{By} + F = -555.45\ \text{lbf}$$

Vector Sum forces at bearins

$$R_A := \sqrt{R_{Az}^{\ 2} + R_{Ay}^{\ 2}} = 1437.64\ \text{lbf}$$

$$R_B := \sqrt{R_{Bz}^{\ 2} + R_{By}^{\ 2}} = 1112.57\ \text{lbf}$$

Moments at analysis points:

$$M_I := \sqrt{\left(R_{Ay} \cdot L_I\right)^2 + \left(R_{Az} \cdot L_I\right)^2} = 539.11\ \text{in lbf}$$

$$M_J := \sqrt{\left(R_{Ay} \cdot L_J\right)^2 + \left(R_{Az} \cdot L_J\right)^2} = 898.5243\ \text{in lbf}$$

$$M_K := \sqrt{\left(R_{Ay} \cdot L_K\right)^2 + \left(R_{Az} \cdot L_K\right)^2} = 1257.9341\ \text{in lbf}$$

$$M_L := \left(\sqrt{\left(R_{Ay} \cdot L_L + W_{45r} \cdot \left(L_L - L_G\right)\right)^2 + \left(R_{Az} \cdot L_L + W_{45t} \cdot \left(L_L - L_G\right)\right)^2}\right) = 1838.001\ \text{in lbf}$$

$$M_M := \left(\sqrt{\left(R_{Ay} \cdot L_M + W_{45r} \cdot \left(L_M - L_G\right)\right)^2 + \left(R_{Az} \cdot L_M + W_{45t} \cdot \left(L_M - L_G\right)\right)^2}\right) = 1553.93\ \text{in lbf}$$

$$M_N := \left(\sqrt{\left(R_{Ay} \cdot L_N + W_{45r} \cdot \left(L_N - L_G\right)\right)^2 + \left(R_{Az} \cdot L_N + W_{45t} \cdot \left(L_N - L_G\right)\right)^2}\right) = 1131.8152\ \text{in lbf}$$

$$M_O := \sqrt{\left(R_{Ay} \cdot L_O + W_{45r} \cdot \left(L_O - L_G\right)\right)^2 + \left(R_{Az} \cdot L_O + W_{45t} \cdot \left(L_O - L_G\right)\right)^2} = 721.9903\ \text{in lbf}$$

$$M_P := \sqrt{\left(F \cdot \left(L_F - L_P\right)\right)^2} = 200\ \text{in lbf}$$

$$M_Q := \sqrt{\left(F \cdot \left(L_F - L_Q\right)\right)^2} = 150\ \text{in lbf}$$

$$M_R := \sqrt{\left(F \cdot \left(L_F - L_R\right)\right)^2} = 100\ \text{in lbf}$$

$$M_G := \sqrt{\left(R_{Ay} \cdot L_G\right)^2 + \left(R_{Az} \cdot L_G\right)^2} = 2695.573\ \text{in lbf}$$

Torque Analysis

$T_I := 0\ \text{in lbf} = 0$ $\quad T_J := T_I = 0$ $\quad T_K := T_J = 0$

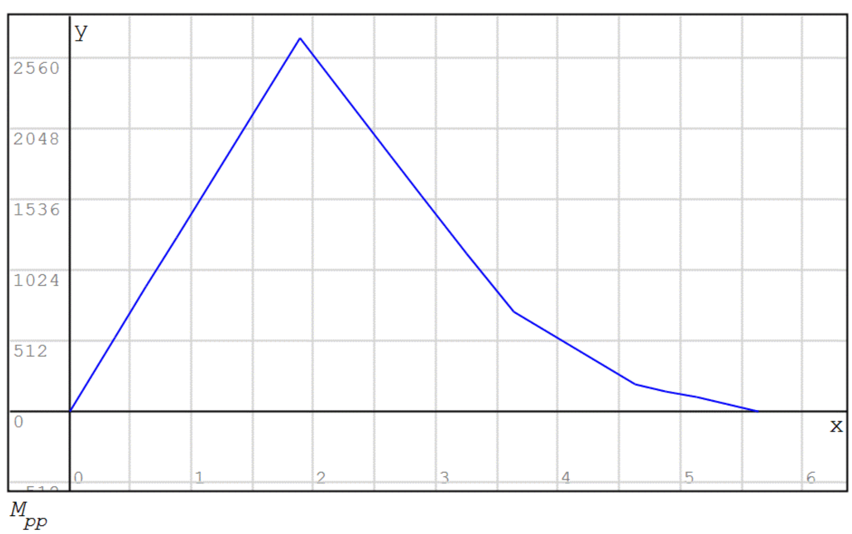
$T_G := 14586\ \text{in lbf}$ $\quad T_P := T_G = 14586\ \text{in lbf}$

$T_L := T_G = 14586\ \text{in lbf}$ $\quad T_N := T_G = 14586\ \text{in lbf}$ $\quad T_Q := T_G = 14586\ \text{in lbf}$

$T_M := T_G = 14586\ \text{in lbf}$ $\quad T_O := T_G = 14586\ \text{in lbf}$ $\quad T_R := T_G = 14586\ \text{in lbf}$

$$M_{pp} := \begin{bmatrix} 0 & 0 \\ L_I \cdot \frac{1}{\text{in}} & M_I \cdot \frac{1}{\text{in lbf}} \\ L_J \cdot \frac{1}{\text{in}} & M_J \cdot \frac{1}{\text{in lbf}} \\ L_K \cdot \frac{1}{\text{in}} & M_K \cdot \frac{1}{\text{in lbf}} \\ L_G \cdot \frac{1}{\text{in}} & M_G \cdot \frac{1}{\text{in lbf}} \\ L_L \cdot \frac{1}{\text{in}} & M_L \cdot \frac{1}{\text{in lbf}} \\ L_M \cdot \frac{1}{\text{in}} & M_M \cdot \frac{1}{\text{in lbf}} \\ L_N \cdot \frac{1}{\text{in}} & M_N \cdot \frac{1}{\text{in lbf}} \\ L_O \cdot \frac{1}{\text{in}} & M_O \cdot \frac{1}{\text{in lbf}} \\ L_P \cdot \frac{1}{\text{in}} & M_P \cdot \frac{1}{\text{in lbf}} \\ L_Q \cdot \frac{1}{\text{in}} & M_Q \cdot \frac{1}{\text{in lbf}} \\ L_R \cdot \frac{1}{\text{in}} & M_R \cdot \frac{1}{\text{in lbf}} \\ L_F \cdot \frac{1}{\text{in}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0.375 & 539.1146 \\ 0.625 & 898.5243 \\ 0.875 & 1257.9341 \\ 1.875 & 2695.573 \\ 2.625 & 1838.001 \\ 2.875 & 1553.93 \\ 3.25 & 1131.8152 \\ 3.625 & 721.9903 \\ 4.625 & 200 \\ 4.875 & 150 \\ 5.125 & 100 \\ 5.625 & 0 \end{bmatrix}$$

Plot of Peak Moments



Student ID: A02233404 Assigned Material: 1018 32 HR Steel

$S_{ut} := 58\ \text{ksi}$ $S_y := 32\ \text{ksi}$

1018 HR 58 32

Combinind Eq. 6-19 and 6-20:

$$Sef\left(S_{ut}, d\right) := 2.7 \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^{-0.265} \cdot \begin{cases} 0.91 \cdot \left(\frac{d}{\text{in}}\right)^{-0.157} & \text{if } d > 2\,\text{in} \\ 0.879 \cdot \left(\frac{d}{\text{in}}\right)^{-0.107} & \text{else} \end{cases} \cdot 0.5 \cdot S_{ut}$$

Using Eq. 6-33, 6-34, 6-35, and 6-36, we get the following functions to compute Kf and Kfs.

$$Kff\left(S_{ut}, r, K_t\right) := 1 + \frac{K_t - 1}{1 + \frac{\left(0.246 - 3.08 \cdot 10^{-3} \cdot \frac{S_{ut}}{\text{ksi}} + 1.51 \cdot 10^{-5} \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^2 - 2.67 \cdot 10^{-8} \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^3\right) \cdot \sqrt{\text{in}}}{\sqrt{r}}}$$

$$Kfs\left(S_{ut}, r, K_{ts}\right) := 1 + \frac{K_{ts} - 1}{1 + \frac{\left(0.19 - 2.51 \cdot 10^{-3} \cdot \frac{S_{ut}}{\text{ksi}} + 1.35 \cdot 10^{-5} \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^2 - 2.67 \cdot 10^{-8} \cdot \left(\frac{S_{ut}}{\text{ksi}}\right)^3\right) \cdot \sqrt{\text{in}}}{\sqrt{r}}}$$

Eq 7-5 assuming Ta = 0: $\sigma_{af}\left(K_f, M, D\right) := \frac{32 \cdot K_f \cdot M}{\pi \cdot D^3}$

Eq 7-6 assuming Mm = 0: $\sigma_{mf}\left(K_{fs}, T, D\right) := \sqrt{3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T}{\pi \cdot D^3}\right)^2}$

Eq. 6-46 for fatigue safety factor using modified Goodman $n_{ff}\left(\sigma'_a, \sigma'_m, S_e, S_{ut}\right) := \frac{1}{\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}}$

Eq. 6-49 for predicting yield (conservative approach): $n_{yf}\left(S_y, \sigma'_a, \sigma'_m\right) := \frac{S_y}{\sigma'_a + \sigma'_m}$

Eq. 11-9 for C10 bearing loads

$$ratedLoad\left(a_f, F_D, R_D, a, b, x_0, \theta, x_D\right) := a_f \cdot F_D \cdot \left(\frac{x_D}{x_0 + \left(\theta - x_0\right) \cdot \left(\ln\left(\frac{1}{R_D}\right)\right)^{\frac{1}{b}}}\right)^{\frac{1}{a}}$$

Point Calculations

The diameter for bearing 1 at D_1 is 1.5748 in

The diameter for bearing 2 at D_6 is 2.7559 in

The rest of the bearings are calculated by guess and check

From the Bearing

$D_1 := 1.5748\ in$

$D_2 := D_1 \cdot 1.11$

From the Bearing

$D_6 := 2.7559\ in$

$D_4 := \left(D_6 \cdot 1.11\right)$

$D_3 := D_4 \cdot 1.11$

$D_5 := D_4 \cdot 1$

$D_7 := 2.48\ in$

$D_1 = 1.5748\ in$

$D_2 = 1.748\ in$

$D_3 = 3.3955\ in$

$D_4 = 3.059\ in$

$D_5 = 3.059\ in$

$D_6 = 2.7559\ in$

$D_7 = 2.48\ in$

This is the Logic code to help:

```
if D_2 ≥ 1.1·D_1
  xD2D1 := "WE GOOD"
else
  xD2D1 := "NOPE"          xD2D1 = "WE GOOD"
```

```
if D_3 ≥ 1.1·D_4
  xD3D2 := "WE GOOD"
else
  xD3D2 := "NOPE"          xD3D2 = "WE GOOD"
```

```
if D_5 ≥ 1.1·D_6
  xD5D6 := "We GOOD"
else
  xD5D6 := "NOPE"          xD5D6 = "We GOOD"
```

Stress Concentration Factors are Selected
From Table 7-1

Table 7–1

First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts}.

Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

	Bending	Torsional	Axial
Shoulder fillet—sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet—well rounded (r/d = 0.1)	1.7	1.5	1.9
End-mill keyseat (r/d = 0.02)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove (use r=0.01 in)	5.0	3.0	5.0

Stress Analysis at Point I

Moment and Torque: $M := M_I = 539.1146 \text{ in lbf}$ $\quad T := 0$ $\quad K_t := 2.7$ $\quad K_{ts} := 2.2$

Selected Diameter: $d := D_1 = 1.5748 \text{ in}$

Stress Concentration Factors from Table 7-1 (Sharp Fillet): $r_d := 0.02 = 0.02$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b $\quad r_d \cdot d = 0.0315 \text{ in}$

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 2.0389$ $\quad Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 1.8125$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 2.8668 \text{ ksi}$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 0 \text{ ksi}$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 22.3532 \text{ ksi}$

Safety Factor against fatigue, Eq. 6-46: $n_{fI} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 7.8$

Safety Factor against yielding, Eq. 6-49: $n_{yI} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 11.162$

Stress Analysis at Point J

Moment and Torque: $M := M_J = 898.5243\ in\ lbf$ $T := 0$ $K_t := 1.7$ $K_{ts} := 1.5$

Selected Diameter: $d := D_2 = 1.748\ in$

Stress Concentration Factors from Table 7-1 (Rounded Fillet): $r_d := 0.02 = 0.02$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b $r_d \cdot d = 0.035\ in$

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 1.4364$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 1.3442$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 2.4613\ ksi$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 0\ ksi$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 22.105\ ksi$

Safety Factor against fatigue, Eq. 6-46: $n_{fJ} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 8.98$

Safety Factor against yielding, Eq. 6-49: $n_{yJ} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 13.001$

Stress Analysis at K

Moment and Torque: $M := M_K = 1257.9341\ in\ lbf$ $T := 0$ $K_t := 2.7$ $K_{ts} := 2.2$

Selected Diameter: $d := D_4 = 3.059\ in$

Stress Concentration Factors from Table 7-1 (Sharp Fillet): $r_d := 0.02 = 0.02$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b $r_d \cdot d = 0.0612\ in$

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 2.1671$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 1.8941$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 0.97\ ksi$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 0\ ksi$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 20.3825\ ksi$

Safety Factor against fatigue, Eq. 6-46: $n_{fK} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 21.01$

Safety Factor against yielding, Eq. 6-49: $n_{yK} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 32.99$

Stress Analysis at Point L

Moment and Torque: $M := M_L = 1838.001\ in\ lbf$ $T := 0$ $K_t := 2.14$ $K_{ts} := 3$

Selected Diameter: $d := D_4 = 3.059\ in$

Stress Concentration Factors from Table 7-1 (Key Seat): $r_d := 0.02 = 0.02$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b $r_d \cdot d = 0.0612\ in$

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 1.7826$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 2.4902$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 1.1659\ ksi$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 0\ ksi$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 20.3825\ ksi$

Safety Factor against fatigue, Eq. 6-46: $n_{fL} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 17.48$

Safety Factor against yielding, Eq. 6-49: $n_{yL} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 27.448$

Stress Analysis at Point M

Moment and Torque: $M := M_M = 1553.93\ in\ lbf$ $T := T_M$ $K_t := 5$ $K_{ts} := 3$

Selected Diameter: $d := D_4 = 3.059\ in$

Stress Concentration Factors from Table 7-1 (Grove): $r_d := 0.01 = 0.01$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b $r_d = 0.01$

$Kf := Kff\left(S_{ut}, r_d\ in, K_t\right) = 2.8784$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 2.3479$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 1.5916\ ksi$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 10.5531\ ksi$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 20.3825\ ksi$

Safety Factor against fatigue, Eq. 6-46: $n_{fM} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 3.85$

Safety Factor against yielding, Eq. 6-49: $n_{yM} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 2.635$

Stress Analysis at Point N

Moment and Torque: $M := M_N = 1131.8152\ \text{in lbf}$ $T := T_N$ $K_t := 1.7$ $K_{ts} := 1.5$

Selected Diameter: $d := D_5 = 3.059\ \text{in}$

Stress Concentration Factors from Table 7-1 (Rounded Fillet): $r_d := 0.1 = 0.1$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b $r_d \cdot d = 0.3059\ \text{in}$

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 1.5813$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 1.4336$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 0.6368\ \text{ksi}$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 6.4439\ \text{ksi}$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 20.3825\ \text{ksi}$

Safety Factor against fatigue, Eq. 6-46: $n_{fN} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 7.03$

Safety Factor against yielding, Eq. 6-49: $n_{yN} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 4.519$

Stress Analysis at Point O

Moment and Torque: $M := M_O = 721.9903\ \text{in lbf}$ $T := T_O$ $K_t := 2.7$ $K_{ts} := 2.2$

Selected Diameter: $d := D_6 = 2.7559\ \text{in}$

Stress Concentration Factors from Table 7-1 (Sharp Fillet): $r_d := 0.02 = 0.02$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 2.1478$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 1.8821$ $r_d \cdot d = 0.0551\ \text{in}$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 0.7546\ \text{ksi}$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 11.5694\ \text{ksi}$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 20.7192\ \text{ksi}$

Safety Factor against fatigue, Eq. 6-46: $n_{fO} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 4.24$

Safety Factor against yielding, Eq. 6-49: $n_{yO} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 2.597$

Stress Analysis at Point P

$T := T_P$

Moment and Torque: $M := M_P = 200\ \text{in lbf}$

$K_t := 5$ $K_{ts} := 3$

Selected Diameter: $d := D_6 = 2.7559\ \text{in}$

Stress Concentration Factors from Table 7-1 (Grove):

$r_d := 0.01 = 0.01$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b

$r_d = 0.01$

$Kf := Kff\left(S_{ut}, r_d\ \text{in}, K_t\right) = 2.8784$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 2.3247$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 0.2802\ \text{ksi}$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 14.2905\ \text{ksi}$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 20.7192\ \text{ksi}$

Safety Factor against fatigue, Eq. 6-46: $n_{fP} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 3.85$

Safety Factor against yielding, Eq. 6-49: $n_{yP} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 2.196$

Stress Analysis at Point Q

$K_t := 2.7$ $K_{ts} := 2.2$

Moment and Torque: $M := M_Q = 150\ \text{in lbf}$ $T := T_Q$

Selected Diameter: $d := D_7 = 2.48\ \text{in}$

$r_d := 0.02 = 0.02$

Stress Concentration Factors from Table 7-1 (Sharp Fillet):

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b

$r_d \cdot d = 0.0496\ \text{in}$

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 2.128$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 1.8696$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 0.2132\ \text{ksi}$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 15.7709\ \text{ksi}$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 21.0652\ \text{ksi}$

Safety Factor against fatigue, Eq. 6-46: $n_{fQ} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 3.55$

Safety Factor against yielding, Eq. 6-49: $n_{yQ} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 2.002$

Stress Analysis at Point R

Moment and Torque: $M := M_R = 100\ \text{in lbf}$ $T := T_R$ $K_t := 2.14$ $K_{ts} := 3$

Selected Diameter: $d := D_7 = 2.48\ \text{in}$

Stress Concentration Factors from Table 7-1 (Key Seat): $r_d := 0.02 = 0.02$

Corrected Fatigue Stress Concentration Factors using Eq. 6-35a and 6-35b

$Kf := Kff\left(S_{ut}, r_d \cdot d, K_t\right) = 1.7564$ $Kfs := Kfs\left(S_{ut}, r_d \cdot d, K_{ts}\right) = 2.4493$ $r_d \cdot d = 0.0496\ \text{in}$

Alternating von Mises stress from Eq. 7-5: $\sigma_{aI} := \sigma_{af}(Kf, M, d) = 0.1173\ \text{ksi}$

Midrange von Mises stress from Eq. 7-6: $\sigma_{mI} := \sigma_{mf}(Kfs, T, d) = 20.6611\ \text{ksi}$

Endurance Limit from Eq. 6-19 and 6-20: $S_e := Sef\left(S_{ut}, d\right) = 21.0652\ \text{ksi}$

Safety Factor against fatigue, Eq. 6-46: $n_{fR} := n_{ff}\left(\sigma_{aI}, \sigma_{mI}, S_e, S_{ut}\right) = 2.76$

Safety Factor against yielding, Eq. 6-49: $n_{yR} := n_{yf}\left(S_y, \sigma_{aI}, \sigma_{mI}\right) = 1.54$

$n_{fI} = 7.8$ $n_{fN} = 7.03$

$n_{yI} = 11.162$ $n_{yN} = 4.519$

$n_{fJ} = 8.98$ $n_{fO} = 4.24$

$n_{yJ} = 13.001$ $n_{yO} = 2.597$

$n_{fK} = 21.01$ $n_{fP} = 3.85$

$n_{yK} = 32.99$ $n_{yP} = 2.196$

$n_{fL} = 17.48$ $n_{fQ} = 3.55$

$n_{yL} = 27.448$ $n_{yQ} = 2.002$

$n_{yM} = 2.635$ $n_{fR} = 2.76$

$n_{fM} = 3.85$ $n_{yR} = 1.54$

```
didITWork := "NO"
 if n_fI > 1.5
  if n_yI > 1.5
   if n_fJ > 1.5
    if n_yJ > 1.5
     if n_fK > 1.5
      if n_yK > 1.5
       if n_fL > 1.5
        if n_yL > 1.5
         if n_fM > 1.5
          if n_yM > 1.5
           if n_fN > 1.5
            if n_yN > 1.5
             if n_fO > 1.5
              if n_yO > 1.5
               if n_fP > 1.5
                if n_yP > 1.5
                 if n_fQ > 1.5
                  if n_yQ > 1.5
                   if n_fR > 1.5
                    if n_yR > 1.5
                      didITWork := "YES!!!"
                    else
```

$didITWork = \text{"YES!!!"}$

Bearing Calculations

Calculating C10 bearing Loads

Eq. 11-2b:

$L_D := 12000\ \text{hr} \cdot \omega_5 = 3.9095 \cdot 10^8$

$L_{10} := 1 \cdot 10^6\ \text{rev}$

Item # NF208, Cylindrical Roller Bearing - Separable Inner Ring w/ Two Ribs, Outer Ring w/ One Rib

Caluation or Bearing A

Eq. 11-9 for C10 bearing loads Table 11-6 with a Ball Bearing

$a_{fa} := 1$ $F_{Da} := R_A = 1437.639\ \text{lbf}$ $a_a := \frac{10}{3}$ $\theta_a := 4.459$

$R_{Da} := .99$ $b_a := 1.483$ $x_{0a} := 0.02$

$x_{Da} := \frac{L_D}{L_{10}} = 62.2224$

$$C_{10a} := a_{fa} \cdot F_{Da} \cdot \left(\frac{x_{Da}}{x_{0a} + (\theta_a - x_{0a}) \cdot \left(\ln\left(\frac{1}{R_{Da}} \right) \right)^{\frac{1}{b_a}}} \right)^{\frac{1}{a_a}} = 7822.5859\ \text{lbf}$$

Caluation or Bearing B

Item # NF214, Cylindrical Roller Bearing - Separable Inner Ring w/ Two Ribs, Outer Ring w/ One Rib

$a_{fa} := 1$ $F_{Da} := R_B = 1112.5678\ \text{lbf}$ $a_a := \frac{10}{3}$ $\theta_a := 4.459$

$R_{Da} := .99$ $b_a := 1.483$ $x_{0a} := 0.02$

$x_{Da} := \frac{L_D}{L_{10}} = 62.2224$

$$C_{10a} := a_{fa} \cdot F_{Da} \cdot \left(\frac{x_{Da}}{x_{0a} + (\theta_a - x_{0a}) \cdot \left(\ln\left(\frac{1}{R_{Da}} \right) \right)^{\frac{1}{b_a}}} \right)^{\frac{1}{a_a}} = 6053.7848\ \text{lbf}$$

Key Length Calculations

Key Slot 1 Calculation

$n := 1.5$ $t := \frac{3}{4}\ \text{in}$ $D_4 = 3.059\ \text{in}$

Shaft Diameter Over	Shaft Diameter To (Incl.)	Key Size w	Key Size h	Keyway Depth
$2\frac{3}{4}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{8}$

Use a safety factor of 1.5 and assume the key material is 1006 HR steel with Sy=24 ksi.

key length for the gear exceeds 1.75-inch you can shift to a higher strength key material

Table A-20 $S_{yKey} := 57\ \text{ksi}$

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)
G10060	1006	HR	300 (43)	170 (24)
		CD	330 (48)	280 (41)
G10100	1010	HR	320 (47)	180 (26)
		CD	370 (53)	300 (44)
G10150	1015	HR	340 (50)	190 (27.5)
		CD	390 (56)	320 (47)
G10180	1018	HR	400 (58)	220 (32)
		CD	440 (64)	370 (54)
G10200	1020	HR	380 (55)	210 (30)
		CD	470 (68)	390 (57)

$S_{SY} := \frac{S_{yKey}}{\sqrt{3}} = 32.909\ \text{ksi}$ $F := \frac{T}{D_4 \cdot 0.5} = 9536.2971\ \text{lbf}$

Shear *Crushing*

$L := \frac{F \cdot n}{S_{SY} \cdot t} = 0.5796\ \text{in}$ $L_C := \frac{2 \cdot F \cdot n}{S_{yKey} \cdot t} = 0.6692\ \text{in}$

Key Slot 2 Calculation

$n := 1.5$ $t := \frac{1}{2}\ \text{in}$ $D_7 = 2.48\ \text{in}$

Shaft Diameter Over	Shaft Diameter To (Incl.)	Key Size w	Key Size h	Keyway Depth
$2\frac{1}{4}$	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{16}$

Use a safety factor of 1.5 and assume the key material is 1006 HR steel with Sy=24 ksi.

key length for the gear exceeds 1.75-inch you can shift to a higher strength key material

Table A-20 $S_{yKey} := 27.5\ \text{ksi}$

$S_{SY} := \frac{S_{yKey}}{\sqrt{3}} = 15.8771\ \text{ksi}$ $F := \frac{T}{D_4 \cdot 0.5} = 9536.2971\ \text{lbf}$

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)
G10060	1006	HR	300 (43)	170 (24)
		CD	330 (48)	280 (41)
G10100	1010	HR	320 (47)	180 (26)
		CD	370 (53)	300 (44)
G10150	1015	HR	340 (50)	190 (27.5)

Crushing

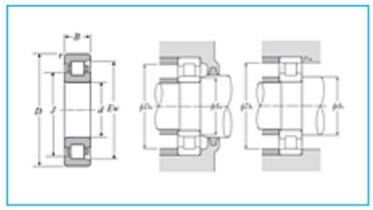
$L := \frac{F \cdot n}{S_{SY} \cdot t} = 1.8019\ \text{in}$ $L_C := \frac{2 \cdot F \cdot n}{S_{yKey} \cdot t} = 2.0806\ \text{in}$

Appendix

Selected bearings

Bearing One for D_1

Item # NF208, Cylindrical Roller Bearing - Separable Inner Ring w/ Two Ribs, Outer Ring w/ One Rib



d	1.5748 in 40.000 mm
D	3.1496 in 80.000 mm
B	0.7087 in 18.000 mm
Ew	2.7559 in 70.000 mm
J	2.1339 in 54.200 mm
r	0.0433 in 1.100 mm
da min	1.8307 in 46.500 mm
db min	1.8307 in 46.500 mm

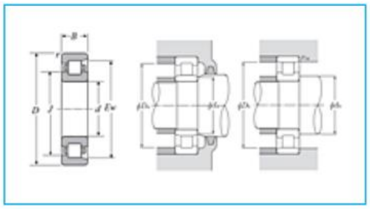
Da max	2.8937 in 73.500 mm
Db max	2.8937 in 73.500 mm
Db min	2.8346 in 72.000 mm
r1as max	0.0433 in 1.100 mm

Type	Cylindrical Roller Bearing
Bore Type	Round
Material	Hardened Alloy Steel
Cage Material	Pressed Steel
Limiting Speed - Oil	11000 RPM
Limiting Speed - Grease	9400 RPM
Configuration	One
Tolerance	ISO Class 0 (RBEC 1)

Oil Hole	w/o Oil Hole
Radial Internal Clearance	CN
Static Load Rating	9650 lbf 43000 N 43.00 kN
Dynamic Load Rating	9800 lbf 43500 N 43.50 kN
Enclosure	Open
Weight	0.847 lb 0.384 kg

Bearing Two for D_6

Item # NF214, Cylindrical Roller Bearing - Separable Inner Ring w/ Two Ribs, Outer Ring w/ One Rib



d	2.7559 in 70.000 mm
D	4.9213 in 125.000 mm
B	0.9449 in 24.000 mm
Ew	4.3504 in 110.500 mm
J	3.5276 in 89.600 mm
r	0.0591 in 1.500 mm
da min	3.0709 in 78.000 mm
db min	3.0709 in 78.000 mm
Da max	4.6063 in 117.000 mm
Db max	4.6063 in 117.000 mm
Db min	4.4882 in 114.000 mm
r1as max	0.0591 in 1.500 mm

Type	Cylindrical Roller Bearing
Bore Type	Round
Material	Hardened Alloy Steel
Cage Material	Pressed Steel
Limiting Speed - Oil	6500 RPM
Limiting Speed - Grease	5500 RPM
Configuration	One
Tolerance	ISO Class 0 (RBEC 1)
Oil Hole	w/o Oil Hole
Radial Internal Clearance	CN
Static Load Rating	21400 lbf 95000 N 95.00 kN
Dynamic Load Rating	18800 lbf 83500 N 83.50 kN
Enclosure	Open
Weight	2.571 lb 1.166 kg

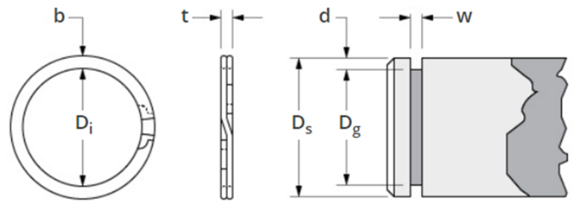
Selected Retaining Clips

$D_7 = 2.48 \text{ in}$

AT point M

Part Number	WS-318
Ring Type	External Ring
D_s: Shaft Diameter (in)	3.187
D_g: Groove Diameter (in)	3.089 +/-.006
w: Groove Width (in)	0.068 +.005/-.000
D_i: Free Inside Diameter (in)	3.061 +.000/-.030
t: Ring Thickness (in)	0.061 +/-.003
b: Ring Radial Wall (in)	0.178 +.004/-.006
# of Turns	2
Crimp	Yes
Ring Shear (lb)	25650 (safety factor of 3)
Groove Yield (lb)	11040 (groove material yield strength of 45,000 psi and safety factor of 2)

WS-318

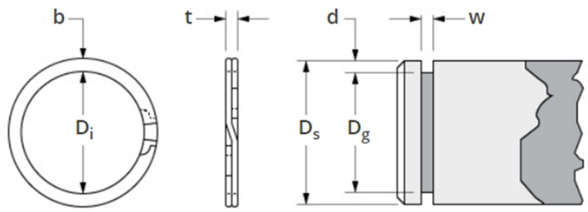


AT POINT P

$D_7 = 2.48 \text{ in}$

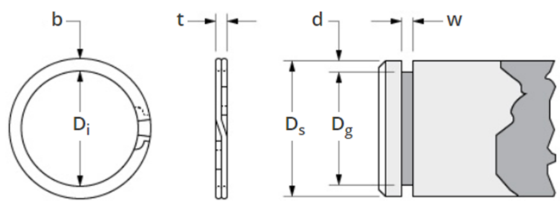
WS-256

Part Number	WS-256
Ring Type	External Ring
D_s: Shaft Diameter (in)	2.562
D_g: Groove Diameter (in)	2.476 +/-.006
w: Groove Width (in)	0.056 +.004/-.000
D_i: Free Inside Diameter (in)	2.452 +.000/-.025
t: Ring Thickness (in)	0.049 +/-.003
b: Ring Radial Wall (in)	0.148 +.003/-.005
# of Turns	2
Crimp	Yes
Ring Shear (lb)	16560 (safety factor of 3)
Groove Yield (lb)	7790 (groove material yield strength of 45,000 psi and safety factor of 2)



At D1

$D_1 = 1.5748 \text{ in}$



Part Number	WS-162
Ring Type	External Ring
D_s: Shaft Diameter (in)	1.625
D_g: Groove Diameter (in)	1.566 +/-.005
w: Groove Width (in)	0.056 +.004/-.000
D_i: Free Inside Diameter (in)	1.549 +.000/-.020
t: Ring Thickness (in)	0.049 +/-.003
b: Ring Radial Wall (in)	0.108 +.003/-.005
# of Turns	2
Crimp	Yes
Ring Shear (lb)	10510 (safety factor of 3)
Groove Yield (lb)	3450 (groove material yield strength of 45,000 psi and safety factor of 2)