Homework 3: DH Parameters and Wheeled Vehicle Kinematics

ECE/MAE 5930 Mobile Robotics

1 Fundamental Relationship Between Quaternions and Rotation Matrices (15 pnts)

Every three-dimensional rotation can be represented using a single axis with a rotation about that axis. We discussed in class how this axis and rotation can be used to fundamentally define a quaternion (see LaValle Section 4.2.2 - Quaternions). In fact, LaValle equation (4.21) relates the quaternion parameters to that axis and rotation.
The relationship between a single axis and rotation can also be used to define rotation matrices. This actually forms a fundamental part of what is known as screw theory – any transformation can be represented as a translation and rotation about a single axis similar to a screw providing rotation motion and translation due to the pitch of the screw threads. An infinitely small screw motion is called a twist. For pure rotation, this twist can be written using purely the axis of rotation and the rotation amount. Given that the axis is written as $v = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$, the twist is the skew-symmetric matrix defined as

$$\hat{v} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}. \tag{1}$$

The rotation matrix can be found using the matrix exponential (*expm(...)* in Matlab), rotation θ, and rotation axis as

$$R = exp\{\hat{v}\theta\}. \tag{2}$$

For this reason, $\hat{v}\theta$ is known as the exponential coordinates of a rotation. Multiplying it out there are three exponential coordinates $\zeta_1 = v_1\theta$, $\zeta_2 = v_2\theta$, and $\zeta_3 = v_3\theta$. Note that, similar to representing the rotation with angles, mapping from exponential coordinates to rotation matrices is a many-to-one mapping.

1.1 Relation Through Example (10 pnts)

Show through example the relation between these different representations. Create a rotation matrix from LaValle equation (3.42) using $\frac{\pi}{4}$ for the value of each angle. Calculate the quaternion. From the quaternion, calculate the axis of rotation and angle of rotation. Form the exponential matrix $\hat{v}\theta$. Take the matrix exponential and show that the resulting rotation matrix is equivalent to before.
Give the results of each step:

- Initial rotation matrix
- Calculated quaternion

- Axis of rotation
- Exponential Coordinates
- Resulting rotation matrix

1.2 Many-to-one Mapping (5 pnts)

Find an alternative value for ζ which results in the same rotation matrix.

2 Kinematic Models

2.1 Implement simulations (10 pnts)

Implement Matlab simulations for which implement the kinematics for the following vehicle models. The simulation should allow for the defined input variables to be changed. The models are:

1. Simple Car / Bicycle: Equation (13.15) in LaValle
2. Differential Drive: Equation (13.16) in LaValle
3. Simple Unicycle: Equation (13.18) in LaValle
4. Better Unicycle: Equation (13.46) in LaValle

A framework already exists which you can continue to develop. If you use that framework, the "Simple Car" and "Simple Unicycle" have already been implemented for you!
Create a function that simulates each model and clearly identify which function simulates which vehicle.

2.2 Constant Rotation – Constant Inputs (12 pnts)

For each of the models in Problem 2.1 choose a value for $x(0)$ and **constant inputs** to have the vehicles move in a circle of radius 10.
For each of the models, turn in a plot showing each of the following plotted vs time:

- Translational velocity
- Rotational velocity
- Inputs

Also turn in a plot showing the path of the robot over the simulation time.

2.3 Constant Rotation – Varying Inputs (6 pnts)

Repeat problem 2.2 for the "Better Unicycle" model with the following variations:

- Have the initial state equal to zero (this includes velocities).
- Create a time varying profile for the inputs.

Note: Don't spend too much time on this. Just loosely attempt to create inputs that will make the vehicle converge to the radius of 10. This will likely include segments where you speed up and then maintain. In the next section of the course we will develop methods for designing the control inputs. If you could not get a radius of 10 then simply explain what you tried and you will get credit.

2.4 Extra credit

You can receive extra credit for the following

- Repeat problems 2.1 through 2.3 with one of the following models (15 points each)
  - Continuous-steering car: Equation (13.48) modified to have non-unit velocity (i.e. reintroduce the translational velocity term)
  - Smooth differential drive: Equation (13.49)
- Updated model (15 points): Create a plot more accurately depicting the vehicle for one of the models.
- Updated integration (15 points): Euler integration is not an excellent solution. Implement a new solution which can directly replace the "integratorEuler(...)" function call in "testVehicle.m." Show plots depicting that the new solution has less error than the "integratorEuler(...)" when compared to the "integrateODE(...)" solution.
- Advanced model (25 points): Implement a dynamic model such as the car with tire skidding in equation (13.111) of LaValle. Explain the model and the chosen parameters. Create a function call as done in problem 2.1 to show it working.

3 Kinematic Constraints (9 pnts)

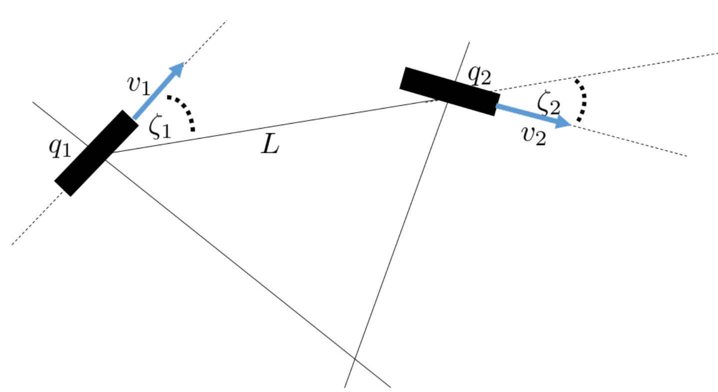

Figure 1: A diagram for a bicycle in which both wheels can be steered.

Figure 1 depicts an advanced bicycle model in which both the front and rear wheels can be steered. The following parameters describe the elements in the figure:

- q_i is the center of wheel i

- ζ_i is the orientation of wheel i from the line extending between q_1 and q_2. Note that positive angles are with respect to a z axis coming out of the page (i.e. ζ_2 in Figure 1 would have a negative value and ζ_1 would have a positive value).
- v_i is the 2D velocity at point q_i
- $L = 1$ is the length between points q_1 and q_2

Assume that the rear wheel (q_1) is traveling forward at a velocity of 2 m/s, $\zeta_1 = 0.1$ rad and $\zeta_2 = -.3$ rad. Answer the following

1. What is the instantaneous radius of motion for each wheel (Note: use the sine rule)?
2. What is the rotational velocity at q_1?
3. What is the velocity of wheel 2 (i.e. v_2)?