MAE 3210 - Spring 2020 - Homework 5

Homework 5 is due **online** through Canvas in PDF format by 11:59PM on Wednesday April 8.

You are required to submit code for all functions and/or subroutines built to solve these problems, which is designed to be easy to read and understand, in your chosen programming language, **and which you have written yourself**. The text from your code should both be copied into a single PDF file submitted on canvas. **Your submitted PDF must also include responses to any assigned questions, which for problems requiring programming should be based on output from your code**. For example, if you are asked to find a numerical answer to a problem, the number itself should be included in your submission.

1. Develop an algorithm which, for a given function $f(x)$, interval bounds a and b with $a < b$, and a prescribed number of subintervals n, applies the multiple application trapezoidal rule to approximate the integral $\int_a^b f(x)\,dx$.

2. Develop an algorithm which, for a given function $f(x)$, interval bounds a and b with $a < b$, and a prescribed number of subintervals n, approximates the integral $\int_a^b f(x)\,dx$ according to the following procedure:

   (a) If $n = 1$, it applies the trapezoidal rule.

   (b) If n is even, it applies the multiple application Simpson's 1/3 rule.

   (c) If $n \geq 3$ and n is odd, it applies the multiple application Simpson's 1/3 rule on the first $n - 3$ subintervals, and applies the Simpson's 3/8 rule on the last three subintervals.

3. Develop an algorithm which, for a given function $f(x)$, interval bounds a and b with $a < b$, and error tolerance per subinterval tol, applies adaptive quadrature to approximate the integral $\int_a^b f(x)\,dx$ (based on the pseudocode that was presented in the recorded lectures and can be found on page 642 of the textbook).

4. Apply the algorithms you developed in questions 1-3 above to approximate

$$\int_0^1 x^{0.1}(1.2 - x)(1 - e^{20(x-1)})\,dx,$$

for varying values of n and tol. Note that this integral is not easy to evaluate analytically! Using the true value of 0.602298, plot ϵ_t as a function of n for the algorithms you developed in questions 1 and 2, and plot ϵ_t as a function of tol for the algorithm you developed for question 3. Use your best judgement to determine appropriate ranges of values for n and tol to be included in the plots.